RESPONSE OF AN ELASTIC PLATE TO SURFACE LOADS AND BURIED DISLOCATION SOURCES

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INTRODUCTION

The response of an elastic plate to sources of acoustic emission is a problem of current interest in the development of nondestructive inspection methods. So far the two main approaches that have been used to calculate the response are the ray theory and the normal mode technique. In this paper we present another method based on a classical integral transform technique. Working in the frequency domain, the spectral response is computed by summing the residues at the roots of the Rayleigh-Lamb spectrum. An FFT inversion gives the transient response of the plate.

PROBLEM DESCRIPTION

A dislocation source is characterized by its depth, direction of slip and the inclination of the slip plane. We locate a coordinate system as shown in Fig. 1, and introduce parameters λ, δ, r, θ, where,

δ = Dip angle, angle between slip plane and x_1x_2 plane.
λ = Rake angle, angle between slip direction and x_2 axis.
r, θ = Cylindrical coordinates of the observation point.

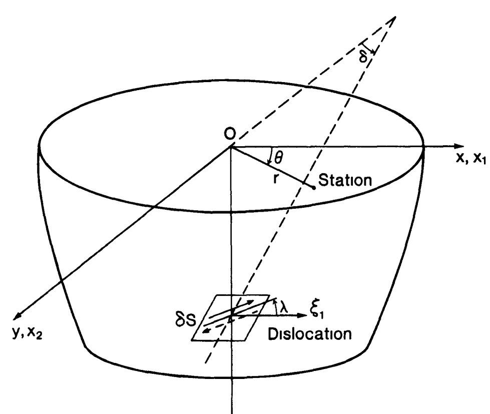

Fig. 1. Dislocation Geometry

The elastodynamic field generated by the dislocation is described by means of a representation theorem (Mal, 1972). This theorem relates the displacement vector at any point in the surrounding medium to the discontinuous motion that is taking place across the dislocating zone in the form,

$$U_k(\underset{\sim}{x},\omega) = \iint_{S(\underset{\sim}{\xi})} \Big[U_i(\underset{\sim}{\xi},\ \omega)\Big]_-^+ \quad T_{ij}^{(k)} n_j \ dS(\underset{\sim}{\xi}) \tag{1}$$

where,

$U_k(\underset{\sim}{x},\omega)$ = displacement in k-direction at position $\underset{\sim}{x}$.

$\Big[U_i(\underset{\sim}{\xi},\ \omega)\Big]_-^+ = D_i\ F(\omega\ )$ = displacement jump across dS.

$F(\omega)$ = Fourier time transform of the time dependence of the dislocation given by $f(t)$.

D_i = x_i component of the final static dislocation.

$T_{ij}^{(k)}(\underset{\sim}{x},\ \underset{\sim}{\xi})$ = ij-component of stress at $\underset{\sim}{\xi}$ due to a unit force applied in k-direction at $\underset{\sim}{x}$.

n_j = x_j-component of unit normal vector to $dS(\underset{\sim}{\xi})$

$dS(\underset{\sim}{\xi})$ = elemental dislocation area.

All quantities above are Fourier time transformed in the following manner. We first introduce a complex frequency $\omega - i\varepsilon$ by adding to the real frequency ω a small imaginary constant, $-i\varepsilon$, where ε is real and positive. For any general field variable $P(\underset{\sim}{x},t)$ and its transform $Q(\underset{\sim}{x},\omega - i\varepsilon)$ we define a Fourier transform pair by,

$$Q(\underset{\sim}{x},\ \omega - i\varepsilon) = \int_0^\infty P(\underset{\sim}{x},t)\ \exp(-i(\omega - i\varepsilon\ )t)\ dt$$

$$P(\underset{\sim}{x},t) = (1/2\pi) \int_{-\infty}^{\infty} Q(\underset{\sim}{x},\ \omega - i\varepsilon)\ \exp(\varepsilon t + i\ \omega t)\ d\omega \tag{2}$$

Every infinitesimal area δS contributes a displacement given by,

$$\delta U_k(\underset{\sim}{x},\ \omega) = D_i F(\omega)\ T_{ij}^{(k)} n_j \quad \delta S\ . \tag{3}$$

Equation (3) forms the basis of our analysis of a point source of Acoustic Emission (AE). Clearly, the determination of the stress components $T_{ij}^{(k)}$ is the major task here. To this end it is convenient to introduce the scalar potentials $\phi^{(k)}$, $\psi^{(k)}$ and $\chi^{(k)}$, which are related to the displacement vector $u^{(k)}$ through,

$$\underset{\sim}{U}^{(k)} = \underset{\sim}{\nabla}\phi^{(k)} + \underset{\sim}{\nabla} \times \hat{e}_3 \chi^{(k)} + \underset{\sim}{\nabla} \times \underset{\sim}{\nabla} \times \hat{e}_3 \psi^{(k)}\ . \tag{4}$$

The potentials themselves satisfy Helmholtz equations.

The approach followed in calculating the stresses is similar to the Haskell-Thompson (1953) matrix formulation. This method simultaneously yields the surface displacement due to the dislocation as well as to an applied surface point force on the plate. The details of the procedure can be found in Mal, Kundu and Xu (1984) and in Vasudevan and Mal (1984). Since we are working in the frequency domain it is possible to split the force and dislocation spectra into their symmetric and antisymmetric components. This allows us to compute the individual modal response in the time domain. The expressions for the stresses together with (3) then give the final dislocation response. Their general form is given by,

$$I = \int_0^\infty (F(k)/D(k))\, kJ_n(kr)\, dk\,, \tag{5}$$

where the integer n takes on the values 0, 1 or 2. We can write (5) as integrals over $-\infty\,,\infty$ where J_n is replaced by $H_n^{(2)}(kr)$. The path of integration is then closed by a large semicircle in the lower half complex k-plane, so that I can be replaced by the sum of residues since no branch cuts are present. For each frequency this procedure gives the spectral response. Having reached a sufficiently large frequency where the spectra have decreased considerably we invert it into the time domain through an FFT procedure.

In evaluating the spectral response special consideration must be given to singularities present in the integrals at certain discrete values of frequency. These are the cutoff frequencies for each real mode and also includes the minima of the second symmetric (S), the third antisymmetric(AS) and the sixth AS modes. The mathematical difficulties caused by these singularities are resolved through the use of the complex Fourier transform discussed above, which effectively introduces a dissipative mechanism and serves to avoid the condition of resonance. The effect of the complex perturbation in the frequency is removed after inversion into the time domain as in (2). This procedure also automatically excludes roots belonging to the negative group velocity regions of the spectrum. Owing to the rapid decay of the residue expressions the contribution of roots lying beyond a certain value of imaginary wavenumber can be safely ignored for numerical calculations.

NUMERICAL RESULTS

Here we outline the numerical procedures for the case of a glass plate with properties, H = .96cm and 1.0cm, α = 5.76 km/sec., β = 3.49 km/sec., ρ = 2.3 gm/cc, where H is the plate thickness, α the P-wave speed, β the S-wave speed and ρ the density. A thickness of 0.96cm was used for the force response problem and that of 1.0cm for the dislocation problem. These values indicate a Poisson's ratio of 0.21. A Taylor series expansion of the wavenumber about real frequency was used to compute its new value for a small imaginary increment in the frequency. The Taylor series theoretically breaks down in the vicinity of all the singular points discussed earlier. Use of approximate expressions for the dispersion relations near these points shows that the series works well within one frequency sampling division of the singular point.

Shown in fig. 2 is the total displacement response due to the first two symmetric and the first two antisymmetric modes at a radial distance of twenty plate thicknesses (r=20H) for a vertical surface load. The time dependence of the force is a full period sine pulse of duration one microsecond. The abscissa is a non dimensional time t^* and the ordinate is a nondimensional displacement U^* where,

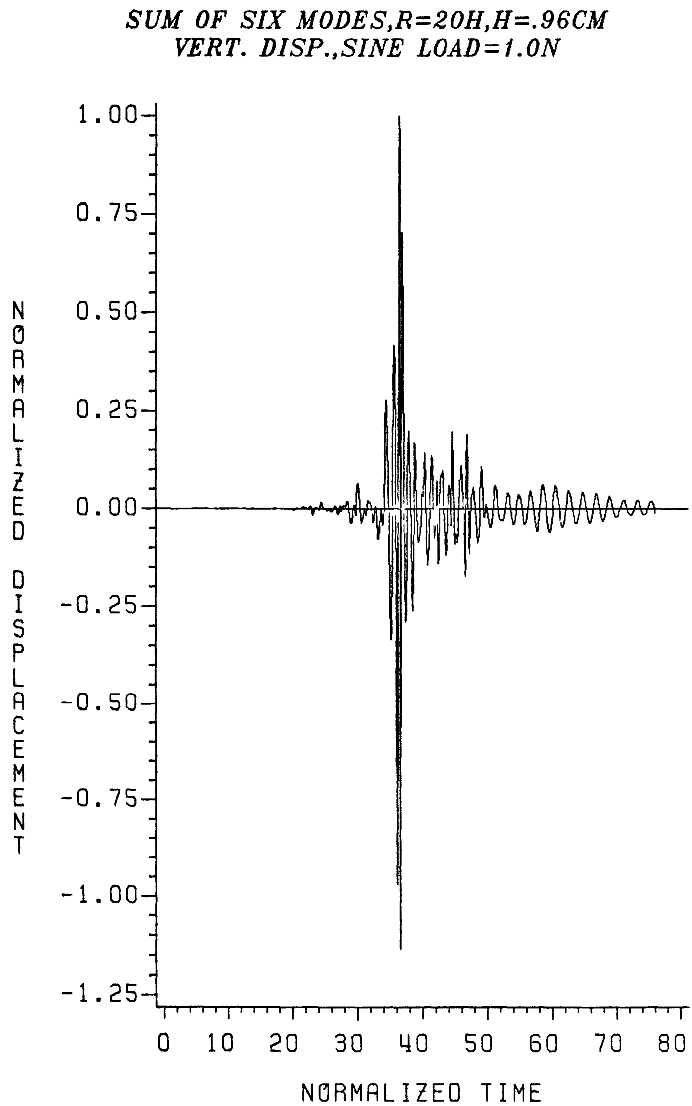

Figure 2: Total vertical displacement - r=20H
6 modes; sine load ; C = .1370d+12

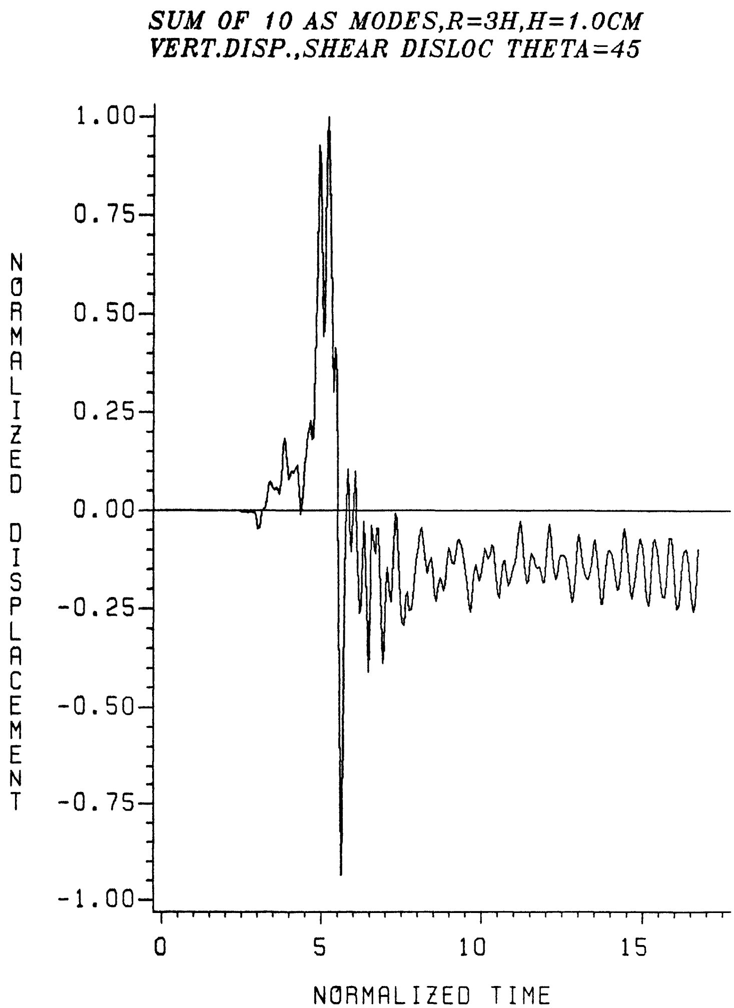

Figure 3: vertical displacement r=3H
Shear dislocation; C =.7770d+08

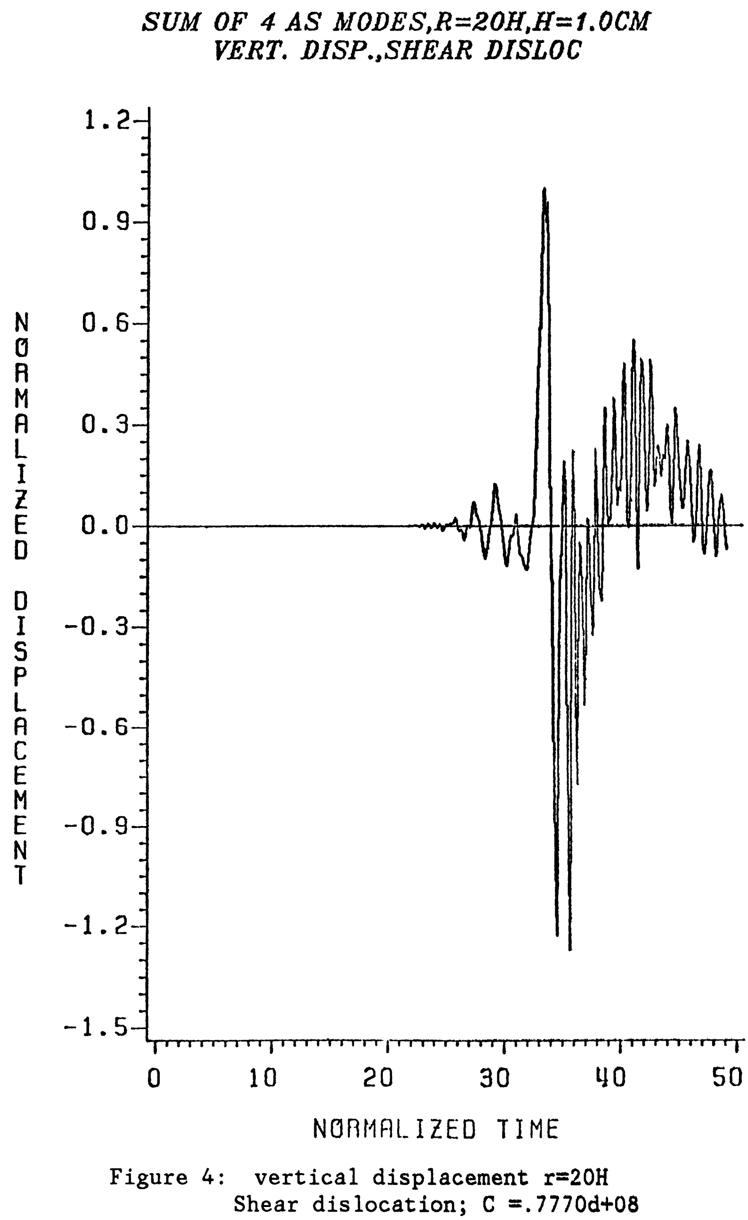

Figure 4: vertical displacement r=20H
Shear dislocation; C =.7770d+08

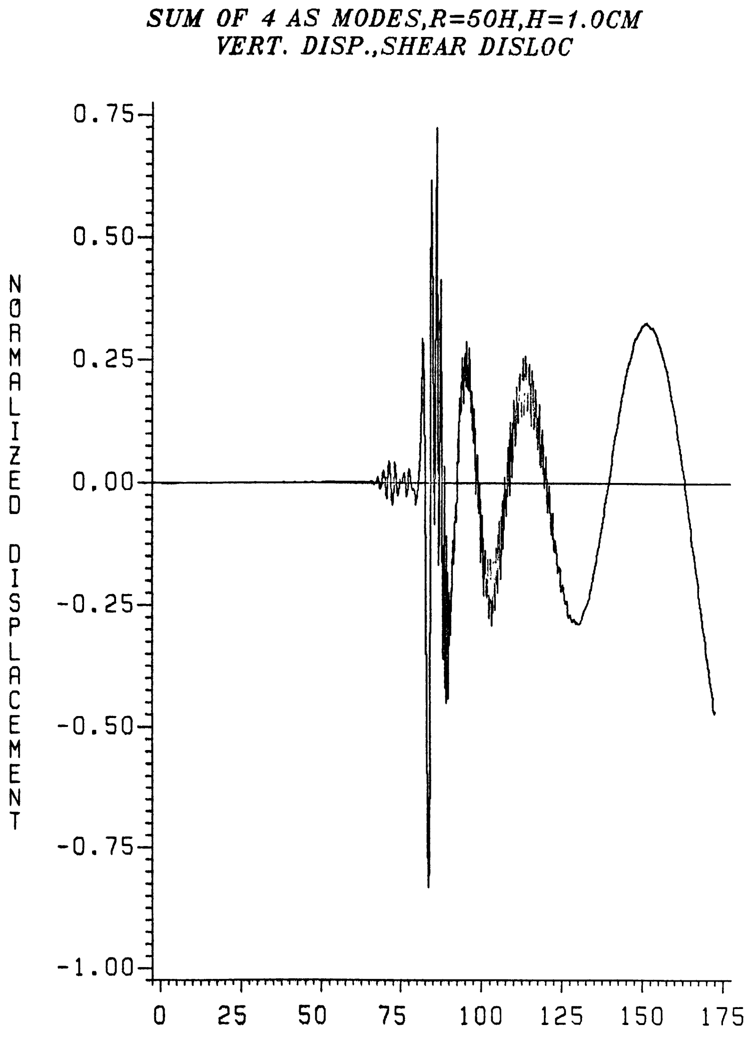

Figure 5: Vertical displacement r=50H
Shear dislocation; C = .3027d+08

$$t^* = t\alpha/H \tag{6}$$

$$U^* = U/\left[(4\pi^3\mu\beta^2C)/H^2\right]. \tag{7}$$

Here C is the maximum positive value for that plot (shown in the figure) and μ is the shear modulus. From (7) and knowing U^*, U can be calculated in suitable units. In fig. 2, the sharp peak is the Rayleigh wave arrival coming mainly from the first antisymmetric mode. Higher modes account for the persistant oscillations seen after the peak value. The next three figures (3-5) show the vertical displacement at three stations at distances r=3H, 20H and 50H due to an embedded shear dislocation of depth z=H/2. The rake angle and dip angle are both zero indicating that the dislocation is parallel to the plate surface. At the smallest distance the static displacement is reached quickly whereas at the largest distance the static value cannot be seen within the time window of the plot. This is mainly due to the dispersion effects of the plate.

The advantages of the technique lies in its efficiency and versatility. Once the roots of the plate dispersion equation have been obtained and stored for a given Poisson's ratio, solutions at different station distances and for different source types can be obtained inexpensively. The inversion of the spectra into time domain through the use of FFT is almost trivial to perform. Thus, this method altogether avoids numerical integration, a time consuming process that is used in some existing schemes (e.g., Weaver and Pao, 1983). The method works over an arbitrarily large range of distances from epicenter. Physically acceptable dissipative mechanisms can be easily incorporated in the model since we employ a frequency-domain approach.

ACKNOWLEDGEMENT

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