

SURFACE WAVE MODES ON SPHERICAL CAVITIES

EXCITED BY INCIDENT ULTRASOUND

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INTRODUCTION

It has been shown both experimentally and theoretically¹ that ultrasonic waves propagate circumferentially around the surface of cavities in an elastic medium, besides being reflected from its "flash points". Surface wave returns were seen to decisively influence the time structure of the echo return from incident ultrasonic pulses. Nagase² has solved a characteristic equation applicable to the spherical cavity problem, from which it could be shown³ that the surface of a spherical cavity supports a Rayleigh-type and two (P and S) Franz-type surface waves, of known speeds and dispersions. On the other hand, the complex eigenfrequencies of cavities were recently obtained numerically⁴. We have used these numerical results in order to satisfy Nagase's solutions, presented in the form of propagation constants of the surface waves as series of fractional powers of the frequency, and have obtained in this way a mode number assignment for all the complex eigenfrequencies. Using this, we calculate dispersion curves for the Rayleigh, P and S- type surface wave phase velocities; their knowledge will permit an accurate interpretation of ultrasonic scattering experiments¹, which previously could be analyzed in a qualitative way only.

CAVITY EIGENFREQUENCIES AND SURFACE WAVES

Surface waves generated on target objects during the scattering of an incident wave, with an amplitude of the form (for the case of a spherical target)

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$$A_{sc} = A_0 e^{i(\nu + \frac{1}{2})\theta - i\omega t}, \quad (1)$$

where θ is the polar angle, ω the circular frequency, and $\nu + \frac{1}{2}$ the frequency-dependent complex angular propagation constant, were first extensively discussed by Franz⁵. It was subsequently⁶ shown that equating ν to a real integer mode number n (physically corresponding to a phase matching of the surface wave after each encirclement of the sphere), ν being a known analytic function of frequency, leads to solutions which are the complex eigenfrequencies of the spherical object.

The wave amplitude scattered from a spherical cavity is generally given by the mode series³

$$A_{sc} = \sum_n (B_n/D_n) h_n^{(1)}(kr) P_n(\cos\theta). \quad (2)$$

The Watson transformation⁵ converts this series into a contour integral in the ν -plane, to be evaluated at the poles of the integrand. The latter are found as the zeros of the denominator, i.e., as the roots of the equation $D_\nu = 0$. This is the characteristic equation of the problem, which for real $\nu=n$ gives the complex eigenfrequencies, but which for a fixed real wave number k gives the complex surface wave propagation constant ν as a function of frequency. The characteristic equation was solved in this latter sense by Nagase² for an evacuated spherical cavity of radius a in a solid medium. He obtained as the solutions the propagation constants in the form of asymptotic series of fractional powers of the frequency, for three types of surface waves:

1) Dilatational waves

$$\nu = x + \alpha \frac{r}{2} x^{1/3} - iu + \beta \frac{r^2}{120} x^{-1/3} + \left(\frac{r^3}{40} - 1\right) \frac{x^{-1}}{70} + \dots, \quad (3)$$

$$\begin{aligned} x &= my = k_P a, & m &= c_S/c_P, \\ y &= k_S a, & k_P &= \omega/c_P, & k_S &= \omega/c_S, \\ \alpha &= e^{i\pi/3}, & \beta &= e^{-i\pi/3}, & r &= (3\rho_1)^{2/3} \end{aligned} \quad (4)$$

where $\rho_1 = 2.383\dots$, $\rho_2 = 5.510\dots$, $\rho_3 = 8.647\dots$ are the zeros of the Airy function, giving rise to a multiplicity of surface waves; finally,

$$u = \frac{4m^3 (1-m^2)^{1/2}}{(2m^2-1)^2}. \quad (5)$$

In the above, c_P and c_S are the dilatational (P) and shear (S) elastic wave speeds, respectively. Note that Nagase's phase factors α, β appear to be incorrect; they have been corrected here in order to agree with those of Franz and Galle⁷ who obtained corresponding results for the acoustic limit, $m = 0$.

2) Shear waves

$$v = y + \alpha r y^{1/3} + v + \beta \frac{r^2}{120} y^{-1/3} + \left(17 - \frac{3}{M} + 32M\right) \frac{vr}{6} y^{-2/3} + \left(\frac{r^3}{2800} + 128M^2 - 80M - \frac{1539}{70} - \frac{2}{M}\right) y^{-1} + \dots, \tag{6}$$

$$v = 4(1-m^2)^{1/2}, \quad M = 1 - m^2. \tag{7}$$

3) Rayleigh waves

$$\text{Re}v = k_R a + O(k_R^0), \quad k_R = \omega/c_R, \tag{8}$$

$$\lim_{k \rightarrow \infty} \text{Im}v = -Ae^{-Bk_R}, \quad A, B > 0 \tag{9}$$

where c_R is the Rayleigh-wave speed on a flat surface of the solid medium.

The complex eigenfrequencies of a spherical cavity in steel, calculated⁴ by solving the characteristic equation, are presented in Fig. 1, plotted in the complex x -plane. We have shown previously⁸ that

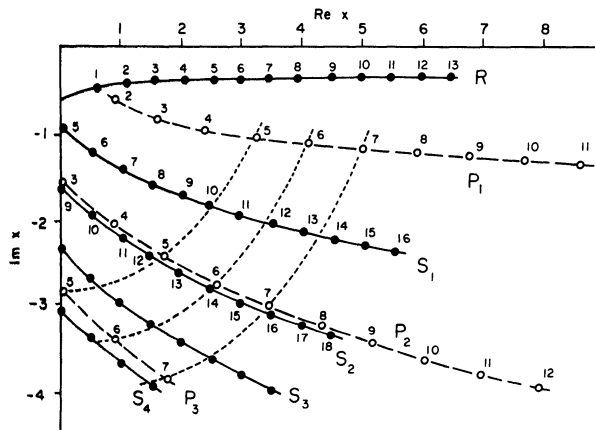


Fig. 1. Complex eigenfrequency pattern of a spherical cavity in steel. Solid curves indicate eigenfrequency layers belonging to Rayleigh (R) and shear-type (S) surface waves, dashed curves those of dilatational (P) type. Dotted curves connect points of given mode number n .

the layers in which these eigenfrequencies occur, correspond to successive resonances (caused by phase matching) of one given surface wave each; we thus labeled the layers by $R, P_1, P_2, \dots, S_1, S_2, \dots$ according to the type of surface wave they belong to.

Inserting these numerical eigenfrequencies in Eqs. (3), (6) and (8) indeed leads to integer solutions $v=n$, and the eigenfrequencies in Fig. 1 are labeled by these n -values. We also entered some dotted lines in the figure to indicate curves of given n . Note that the condition

for obtaining $\nu=n$ in a consistent way led us to the values of the ratio $m \equiv c_S/c_P = 0.48$ and $c_R/c_D = 0.46$, showing how a careful analysis of the eigenfrequency pattern can determine the a-priori unknown ratios of the wave speeds of the material.

DISPERSION CURVES

Identifying $\text{Re}(\nu + \frac{1}{2}) = k_1^{(s)}$ with the dimensionless propagation constant of the surface wave of type $i = P, S$, or R , we find the surface wave phase and group velocities as

$$c^{(s)}/c_P = x/\text{Re}(\nu + \frac{1}{2}), \tag{10}$$

$$c^{(s)g}/c_P = 1/(d \text{Re}\nu/dx), \tag{11}$$

where we used c_P for normalization purposes. If the amplitude is represented as $\exp(-\theta/\theta^{(s)})$, one finds the phase and group decay angles as

$$\theta^{(s)} = 1/\text{Im}\nu, \tag{12}$$

$$\theta^{(s)g} = x/(d \text{Im}\nu/dx), \tag{13}$$

respectively. The corresponding dispersion curves may be obtained using Eqs. (3), (6) and (8). A simplifying approximation consists in taking, e.g., in Eq. (10), the real part of the eigenfrequency for x and the corresponding n -value for ν , which we used for obtaining the dispersion curves shown in Fig. 2.

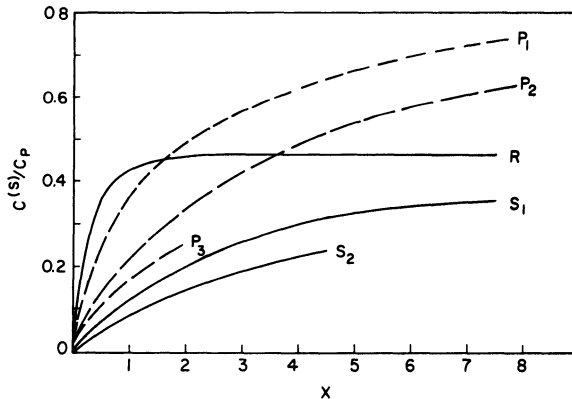


Fig. 2. Phase velocity dispersion curves of the R, P and S type surface waves on a spherical cavity in steel.

CONCLUSIONS

We have shown here how a numerical calculation of the complex eigenfrequencies of a cavity in a solid medium, and an analytic calculation of the complex wave numbers of surface waves propagating around the cavity, can be tied together by the fact that the eigenfrequencies

correspond to cavity resonances caused by the phase matching of the surface waves. The ensuing dispersion curves of the surface waves must be utilized in order to accurately interpret experiments in which e.g. the arrival times of short surface-wave pulses¹ are measured for the purpose of determining the size a of the cavity.

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