DECONVOLUTION BY DESIGN - AN APPROACH TO THE INVERSE PROBLEM OF ULTRASONIC TESTING

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INTRODUCTION

In this paper we present some preliminary results on a new approach to the problem of characterizing flaws using ultrasonics. The approach takes advantage of the fact that we have control over the time waveform of the probing pulse in an ultrasonic test. It also takes advantage of some special properties of the inverse Gaussian function and an effective, stable, continuous deconvolution procedure which is based on this special function. The procedure also has the special feature that the error in the resultant of the deconvolution, which contains all available information about the flaw-scatterer, can be estimated in a powerful way.

First we present the problem formulation and the analytical reasoning. We then discuss the inverse Gaussian function, the deconvolution procedure based on this probe function, and point out some of the special features of the probe function and the procedure. We also present some numerical tests and results using this procedure, demonstrate that the tools necessary to implement the procedure are within grasp, and present some preliminary experimental results.

PROBLEM FORMULATION

As shown in Figure 1, the input to the probing transducer is $V_o(t)$; the generated ultrasonic wave interacts with the test object and flaw contained therein and the modified ultrasonic field is detected by a transducer resulting in a voltage waveform V(t). This output waveform V(t) may be considered to be the result of a convolution of $V_o(t)$ with the transducer's generating and receiving functions T and T' and the dynamic Green's function, G, of the structure which completely characterizes the test object and flaw. Denoting the convolution operation by *, then

$$V = V_o*T*G*T'. \qquad (1)$$

It should be pointed out that, in general, the Green's function is a second order tensor function of vector fields (source and detector

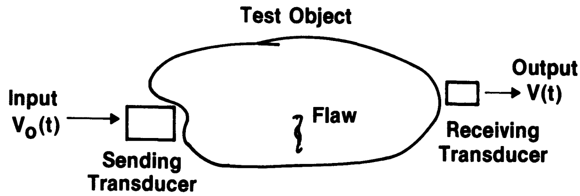

Figure 1. General schematic of an ultrasonic test.

locations) and time. However, we assume the transducers convert one particular component of the Green's function such as vertical force or tangential displacement, to voltage and vice versa. In addition, we assume a point source and point detector test configuration, which can be either a pulse-echo or a pitch-catch arrangement. The point source/point detector assumption is valid as long as the transducer size is small compared with the dimensions of the test object and small compared to the wavelengths of interest. These assumptions reduce equation (1) to a scalar equation for a given location of source and detecter.

In ultrasonic testing, there are several different inverse problems of interest. The explicit determination of the transducer transfer functions T and T' is one such inverse problem. For example, using a known source and a structure with a known Green's function the transform of the transduction could be deduced. This is essentially the method used at the National Bureau of Standards (NBS) to calibrate transducers as receivers [1]. Another example of an inverse problem in ultrasonics is the use of the Born approximation to attempt to size defects; see for example [2,3]. In this example, essentially an approximate Green's function is assumed, it being the Green's function for a weak spherical scatterer. Then a characterization of the flaw is attempted by using the measured voltages from the scattered field and the assumed Green's function. This procedure leads to mixed results [2].

The particular inverse problem in ultrasonics which we are interested in here is the determination of the actual Green's function (or impulse response) of the flaw and medium from the known waveforms V_o and V. This Green's function contains all the information about the scatterer which is available to us from an ultrasonic test. Also, flaw characterization will be more efficient using an actual Green's function than using a generic approximation such as the Born approximation. The determination of G is an important problem to address. In our formulation we define a probing pulse, L, by

$$L = V_o * T * T' \quad (2)$$

Then eq (1) becomes

$$L * G = V. \quad (3)$$

The measured waveform, V, is the convolution of the Green's function, G, and the probing pulse, L. L incorporates those features of the test over which we have some reasonable control so that the form of L is subject to design. Generally, L and V are smooth functions but G has very sharp features as is shown in [4]. One of the problems in solving this inverse problem is to develop a procedure robust enough to give the sharp features of G from the relatively smooth waveforms L and V.

A key question is: What form for L would make finding G easiest? Common wisdom frequently suggests that the ideal probing pulse would be a Dirac-delta function. It is, of course, not possible to generate a true delta function and for an arbitrary approximation there is no mechanism for doing an error analysis to find out how measurement errors will propagate.

A function that has special properties in regard to the choice of form for L is the inverse Gaussian function [5,6],

$$K(\sigma,t) = \frac{\sigma}{2\sqrt{\pi t^3}} e^{-\sigma^2/4t} \qquad (4)$$

In considering this function for a probing pulse, t is the time variable and σ is a shape parameter for the waveform; see figure 2. The function can be considered as a particular C^∞ approximation to a delta function.

DECONVOLUTION WITH AN INVERSE GAUSSIAN KERNEL

Many of the special properties of the inverse Gaussian function as a kernel in deconvolution problems were studied in the context of a one dimensional heat conduction problem [7]. Consider a semi-infinite rod with diffusivity a^2. The distance along the rod is given by x and the boundary is at x=0. The temperature field of the 1-D rod as a function of position and time is w(x,t). The temperature history at $x=\ell$, $w(\ell,t)$, is given by the convolution of the inverse Gaussian function, $K(\sigma,t-\tau)$,

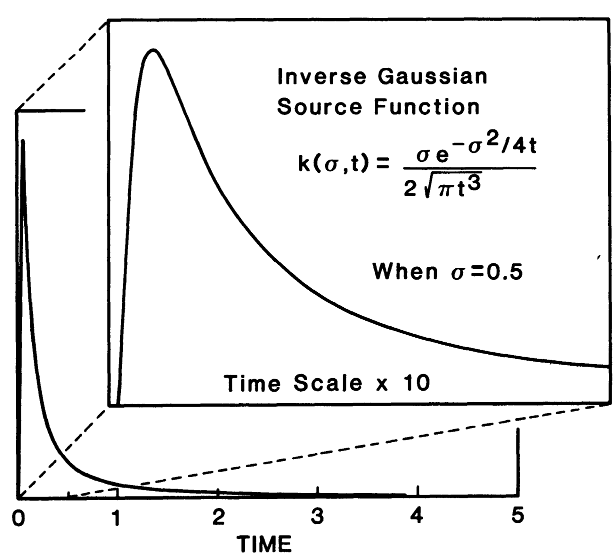

Figure 2. Inverse Gaussian function.

where $\sigma=\ell/a$, with the temperature history applied to the boundary at x=0, i.e.; b(t)=w(0,t), presuming that the rod is initially at a uniform temperature, say zero. That is,

$$w(\ell,t) = \int_0^t K(\sigma,t-\tau)b(\tau)d\tau = K*b \qquad (5)$$

with boundary conditions

$$w(x,0) = 0 \text{ at } t = 0$$
$$w(0,t) = b(t) \text{ for } t>0.$$

The temperature history at the end, b(t), can be discontinuous.

An inverse problem of significant interest is to find, b(t), the boundary temperature history, from the resulting temperature history at $x=\ell$, $w(\ell,t)$. An optimum method for solving this problem has been rigorously given recently [7]. While the problem of determining b(t) is ill-posed, b(t) can be approximated by $b_e(t)$ where

$$b_e(t) = w(e,t). \qquad (6)$$

That is, the temperature history at points as close as we wish to x=0 can be determined in a systematic, stable way using the procedure in [7] and the procedure can be carried out as a continuous deconvolution. This concept of continuous deconvolution involves determining $b_e(t)$ by using smaller and smaller values of e in a systematic, continuous way. It is particularily useful when the measured $w(\ell,t)$ is noisy because the progress of the deconvolution can be monitored and artifacts in b(t) can be avoided, for example, by a-priori information. This procedure [7] is based on special properties of K and is not generally possible with other functions. Also based on special properties of K, it is possible to determine strong bounds on errors in $b_e(t)$ due to errors in $w(\ell,t)$. These bounds are in terms of the L^∞-norm.

By assuming a probing pulse L to be an inverse Gaussian function, K, the entire framework of the thermal problem can be carried over to the problem of determining the mechanical Green's function of a test object. By using a probing pulse as defined by eq. (2) which is an inverse Gaussian function for our ultrasonic test, we have transformed the ultrasonic inversion problem into a deconvolution with an inverse Gaussian kernel.

NUMERICAL TESTS OF THE PROCEDURE

We numerically simulated an experimental procedure of applying a point force probe pulse with inverse Gaussian time dependence, and used the resulting output waveform and the continuous deconvolution of [7] to calculate the Green's function. To do this numerical experiment it is necessary to have access to an exact Green's function for some structure. In [8,9] ray theory methods were used to formulate the Green's tensor for an infinite elastic plate. Computational software for calculating the exact Green's function for a plate, given the source and observation locations, was also developed. We have made use of the software resulting from [9] to calculate specific Green's functions and then convolved these with the inverse Gaussian probe pulse to obtain a simulation of ideally measured resultant time displacement waveforms for various test configurations. The result of the convolution of the probe pulse with the exact Green's function is treated as an output, V, from which we seek to determine the approximate Green's function by our continuous deconvolution operation on V. By comparing the deconvolution results with the known exact Green's function in each case we can assess the performance of the procedure. All of the numerical experiments were done with the G_{33} component of the Green's tensor which describes the normal displacement due to a normal applied force. The Green's functions presented here are those corresponding to a Heaviside function input to the test configurations with the observation point at epicenter and two plate-thicknesses away and on the same and opposite side of the plate as the input probe pulse. The specific K used had σ=0.5. In figures 3-5, waveforms simulating the measured displacement at the observation point, the deconvolved estimate of the Green's function and the exact Green's function are presented in normalized units.

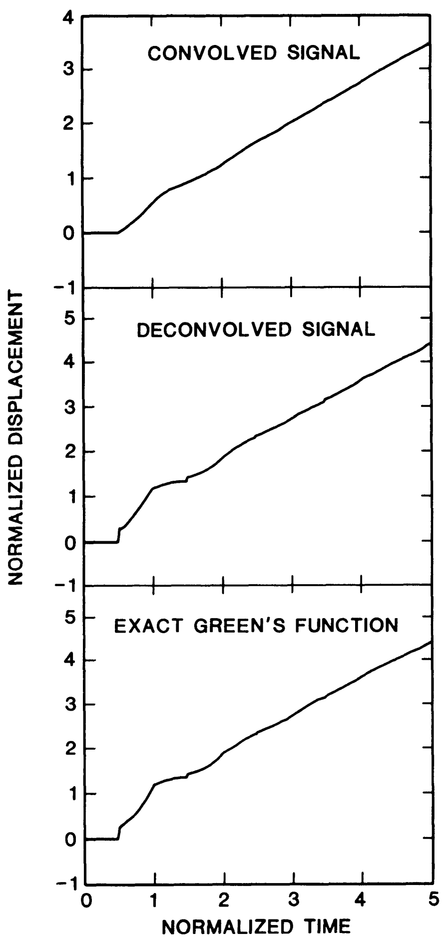

Figure 3. Deconvolution of epicentral response.

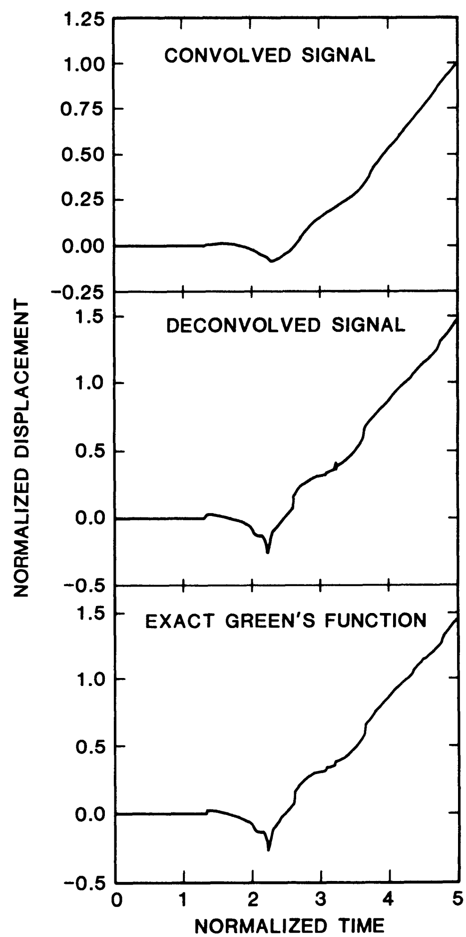

Figure 4. Deconvolution of response at two thicknesses away from epicenter with source and receiver on opposite sides.

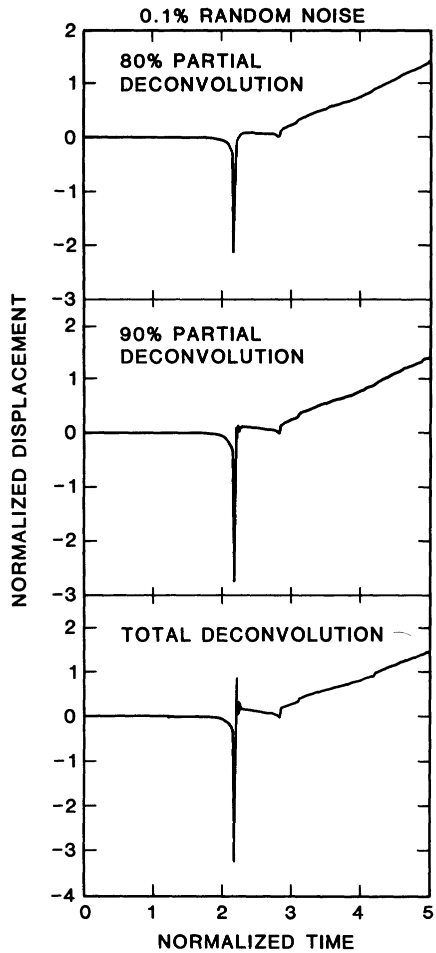

Figure 5. Deconvolution of response at two thicknesses away from epicenter with source and receiver on same side.

Additional examples including the case of a delta Green's function and partial deconvolutions in the presence of noise were presented in [10]. It is clear that the procedure is capable of recovering the sharp features of G even from rather smooth outputs.

DESIGN FOR PHYSICAL IMPLEMENTATION

Based on the encouraging results from the numerical experiments, we have begun work on implementation of the procedure for ultrasonic testing. We wish to design a system such that the probe pulse

$$L = V_o * T * T' = K(\sigma, t). \qquad (7)$$

Two approaches for this design have been identified. One approach is to separately design a voltage pulser and transducers; the other is a semi-inverse technique.

In the first approach, one can design a voltage pulser such that the voltage input to the generating transducer is inverse Gaussian and the generating and receiving transducer functions are high-fidelity. That is, if

$$V_o \approx K(\sigma,t)$$
$$T \approx 1 \text{ and} \qquad (8)$$
$$T' \approx 1$$

then

$$L \approx V_o \approx K(\sigma,t).$$

It is expected that the pulser design will not be too difficult. For example, figure 2 of [11] shows a pulse form which qualitatively looks much like K even though this was not a design criteria for the circuit which produced the pulse.

Considerable progress has also been made on very high-fidelity receiving transducers. Figure 6 shows a comparison of theoretical and measured waveforms of the displacement of a plate due to a point-force step-function input. For the experimental results, a breaking-glass capillary source was used at various plate thicknesses away from an NBS conical transducer [12] and on the same and opposite side of a large glass plate as the transducer [4]. The similarity of the waveforms demonstrates that $T' \approx 1$ for this transducer. Reciprocity arguments might also suggest that the transducer would be a high-fidelity driver as well.

A second approach to the design of the probing pulse is to use a semi-inverse technique. An initial function could be selected and computed, say $K(\sigma,t)$. Currently available digital to analog devices are capable of producing a suitable V_o although a broad band power amplifier would be necessary to sufficiently drive a transducer. The driving and receiving transducers would be placed on a structure with a known Green's function and inverse Green's function, G^{-1}. A plate with transducers at epicenter would be a suitable choice since G and G^{-1} are known for this case [13]. The resulting output signal could be captured by an analog to digital device and transferred to a computer where this waveform could be convolved with G^{-1} to obtain L, the waveform of the probing pulse. This L could be compared to $K(\sigma,t)$. Using this result and a priori information such as independently determined T and T' for the transducers, a better estimate for V_o could be obtained and tried. It seems likely that a combination of these two approaches will produce an optimum probing pulse.

PRELIMINARY RESULTS

As pointed out above, one possible implementation of a specially designed probing waveform is to use two high-fidelity transducers, one as a sender and one as a receiver, and analog circuitry to generate a voltage pulse in the form of an inverse Gaussian. We have set up a test configuration using two NBS conical transducers on opposite surfaces of a 2.52 cm thick aluminum plate. A simple capacitive discharge pulse circuit was used to generate the probing ultrasonic wave. Shown in figure 7 are the input voltage pulse V_o (dashed line) and the received pulse V (solid line). We consider this preliminary result very promising when the result is compared with the expected waveforms.

CONCLUSIONS

In this paper, we have outlined a new approach to inverse problems which inevitably arise in attempts to quantitatively evaluate flaws by

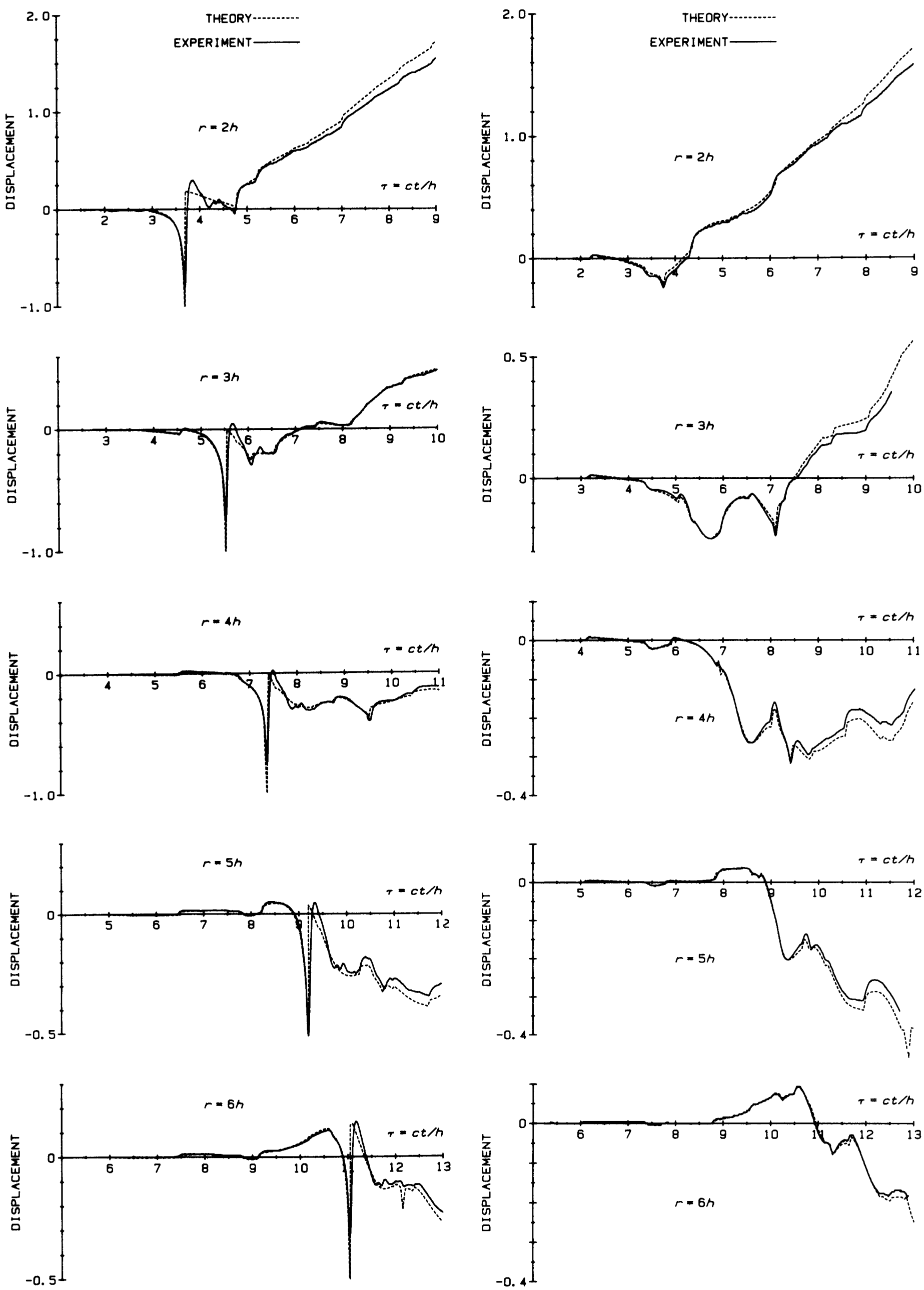

Figure 6. Comparison between waveforms theoretically predicted and those measured with an NBS conical transducer. On the left, the receiver and step-force input are on the same side; on the right, the opposite side at various distances r separating the source and receiver measured in plate thicknesses h.

ultrasonic testing. The focus has been on determining the impulse response function of the flaw and host since this function contains all the information about the flaw available to us from an ultrasonic test.

Two of the most important points implied by this approach are:

1) to reexamine many of the plausible assumptions made in attempts to solve the inverse problem in ultrasonics which turn out to be suspect under scrutiny;

2) to take advantage of those aspects of the test procedure which we have control over and to design those aspects, including the pulse waveform and transducers, to optimize the results of the necessary inversion procedure.

The use of the inverse Gaussian function as a probing pulse is an exciting prospect. The inversion procedure for this pulse waveform has at least two powerful features. The procedure permits continuous deconvolution which is a systematic numerical procedure that supports resolution and minimization of artifacts in the results. The procedure also provides a powerful bound on errors that result from deconvolution in the presence of noise.

Results of numerical experiments have also been presented which demonstrate that the procedure gives very nice estimates of the Green's function. The results of physical experiments indicate that the approach is also a practical one and that the tools necessary for implementation are within reach.

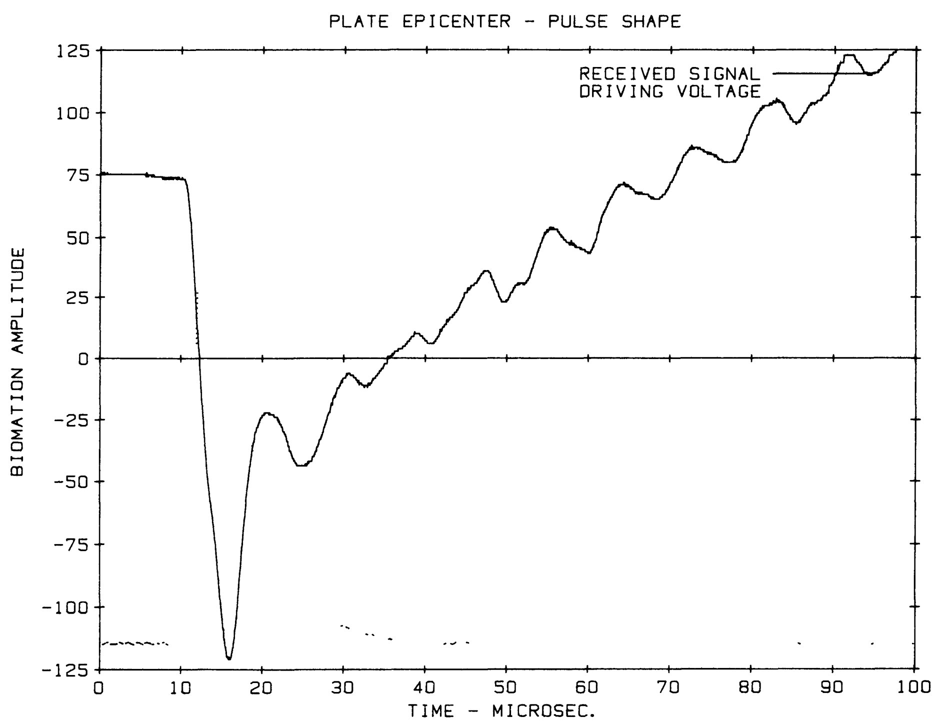

Figure 7. Driving voltage waveform and output waveform from an NBS conical transducer at epicenter on an aluminum plate.

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