

## USE OF THE ANALYTIC SIGNAL IN ULTRASONIC IMAGING

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### INTRODUCTION

Ultrasonic inspection of materials is an important technique in the nondestructive testing program at the Oak Ridge Y-12 Plant. One of the aims of the program is to develop procedures whereby an estimate of the three-dimensional structure of a subsurface artifact may be determined. Toward this goal, traditional C-scans and B-scans have been employed to obtain two-dimensional projection and depth information of a scanned surface. The latest development at the Y-12 Plant has been to simultaneously combine the C-scan and B-scan information and improve the B-scan resolution by using the analytic signal.

Use of the analytic signal to improve the quality of B-scans was proposed by Gammell.<sup>1,2</sup> It was shown by Heyser<sup>3-5</sup> that the magnitude squared of the analytic signal is proportional to the total energy of a bounded oscillating system. It was argued that since the analytic signal is a measure of the total energy, it is a better indicator of the position of sources than either the kinetic energy or potential energy because maxima and minima in the latter correspond to a temporary partition of the total energy while maxima in the total energy unambiguously correspond to sources.

In the following section, a software implementation and a hardware implementation of the analytic signal are described. Following this are found discussions of the reconstruction algorithm and the accuracy of the reconstruction. The final section describes known and suspected problems in the procedure and future directions of the program.

### GENERATING THE ANALYTIC SIGNAL

It is well known<sup>6</sup> that the imaginary part of a complex function analytic over a domain can be recovered from a knowledge of the real part

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of the function over the domain. Similarly, the real part can be deduced given the imaginary part. It can be shown<sup>3,6,7</sup> that the real and imaginary parts of the function are related by the Poisson integral formula, i.e., they are Hilbert transforms of each other. This fact is exploited to generate the analytic signal.

The total energy of a bounded oscillating system is made up of the sum of kinetic energy term (proportional to the square of a generalized velocity) and a potential energy term (proportional to the square of a generalized coordinate). The square root of the total energy can be written as a complex function with the square roots of the kinetic energy and potential energy as the real and imaginary parts. The magnitude squared of this function is the total energy.

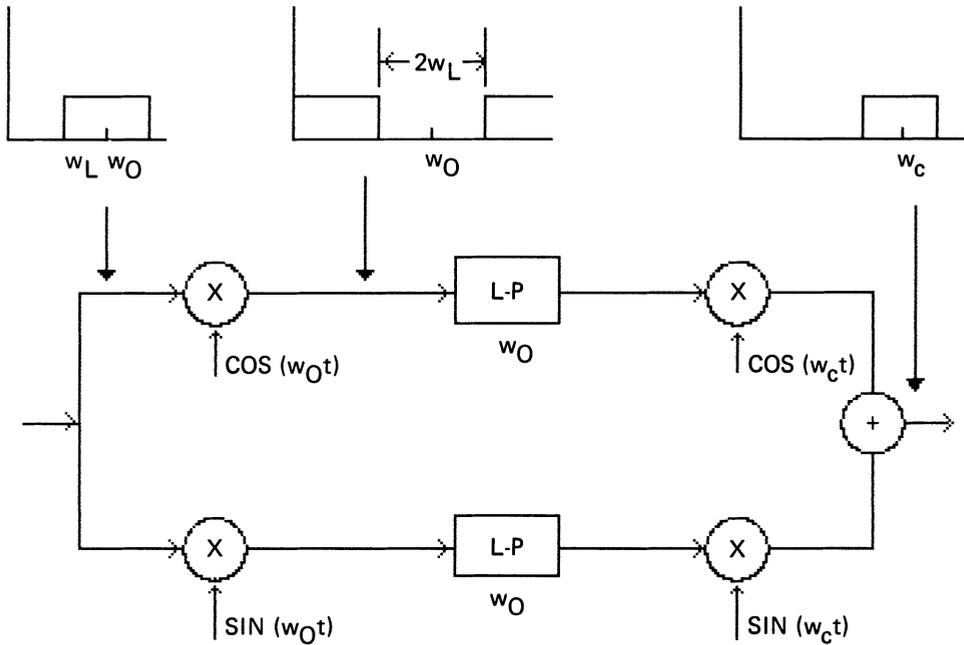
The Fourier transform of the real or imaginary part is related to the Fourier transform of the analytic signal in that they are proportional for positive frequencies and the transform of the analytic signal is identically zero for negative frequencies. The analytic signal is thus seen to be a single side band signal. These properties suggest two methods to generate it from the output of an ultrasonic transducer.

An ultrasonic transducer is sensitive to the local pressure field, and therefore, generates a signal proportional to the local displacement, i.e., square root of potential energy. Using the relations between the transforms, the Fourier transform of the analytic signal can be obtained by computation of the Fourier transform of the transducer output. The complex analytic signal is then obtained by inverting the computed transform. The desired energy information is then found by computing the magnitude squared of the complex signal. All of the preceding is, of course, implemented with Fast Fourier Transform algorithms. Computation time on a PDP 11/23 is approximately three seconds for a 256 point waveform. Using an array processor, it is estimated that the time could be reduced to 0.3 seconds.

The single side band nature of the analytic signal leads to a hardware implementation. Gammell<sup>2</sup> indicates an apparatus that uses a single balanced mixer to modulate the raw waveform away from baseband and a sharp bandpass filter to remove the lower side band. The envelope of the resulting single side band modulated signal is the magnitude of the analytic signal.

The disadvantage of Gammell's circuit is the sharp cutoff filter required to remove the lower side band. A circuit due to Weaver,<sup>8</sup> shown in Figure 1, offers a way around this at the expense of more components.

The transducer signal is input at the left and is assumed to be bandlimited and bounded away from DC. The input signal is split and the resulting two waveforms are mixed with sine and cosine waves at the center frequency of the original band. Mixing with the center frequency, causes one of the side bands to be centered around DC and the other to be at twice the center frequency. Since the input signal is bounded away from DC, the two bands are separated by twice the lowest frequency present in the original spectrum. This is shown in the middle spectrum in Figure 1. If the input spectrum contains low frequency components, then the space between the bands fills in.



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Fig. 1. Single side band modulator circuit.

To obtain the single side band signal, the mixed signals are low pass filtered to remove the upper side band. The advantage of this circuit over Gammell's is that since the spectrum of most commercial transducers are naturally bandlimited and bounded away from DC, the low pass filter can take advantage of the dead zone between the bands and does not need as sharp a cutoff. This translates into lower Q poles and simpler construction. A circuit is being implemented at Y-12 that uses 6-pole elliptic filters to achieve unwanted side band suppression of 50 dB.

A second mixing with a carrier whose frequency is large compared to the center frequency of the transducer followed by the summing junction results in the desired single side band signal. The magnitude of analytic signal can be recovered by full wave rectification followed by low pass filtering. Care must be exercised in selecting the cutoff frequency of this last filter since too high a cutoff will not provide sufficient filtering while too low a cutoff will cause a loss of time resolution.

## RECONSTRUCTION

Once the analytic signal has been generated, it is a simple matter to locate the position of a scatterer. The waveform reflected from an isolated artifact and its analytic signal are shown in Figure 2. It is

seen that the analytic signal is unipolar and has a single peak. The time of arrival of this peak is proportional to the distance from the transducer to the artifact. By knowing the coordinates (relative to some laboratory system) of the transducer and the direction of its sound field, the position of the reflector can be deduced. By repeating this procedure at many points around the artifact, and by using transducers at several different orientations, an image of the surface of the artifact can be built up.

The major advantage in using the analytic signal to determine transducer to reflector distance lies in the ability to separate closely spaced flaws. Figure 3 shows the waveform reflected from two artifacts separated by about 0.063 mm (2.5 mils) in water and the corresponding analytic signal. It is clear that a determination of the separation of the two reflectors is impossible from the transducer signal while the separation of the two returns of energy in the analytic signal is obvious. It should be mentioned that the transducer used in Figures 2 and 3 had a center frequency nominally of 30 MHz.

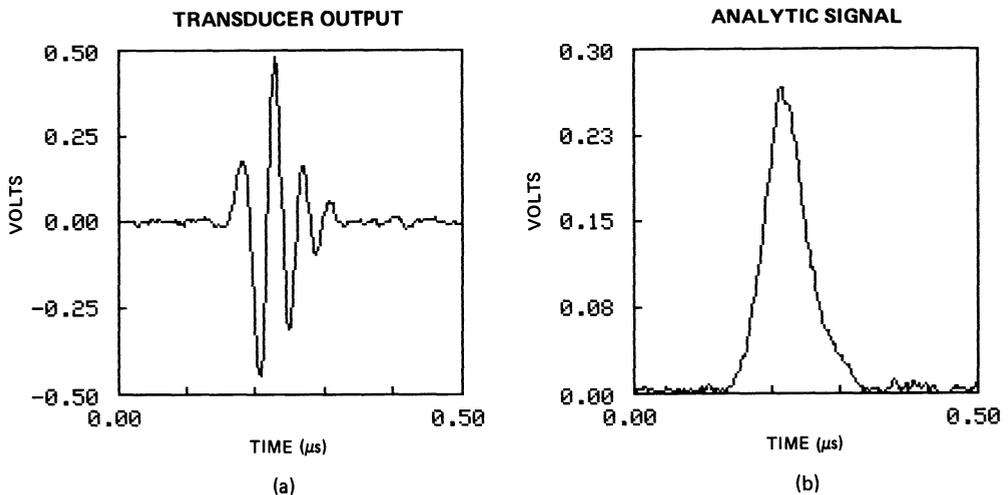


Fig. 2. Waveform and analytic signal from isolated reflector.

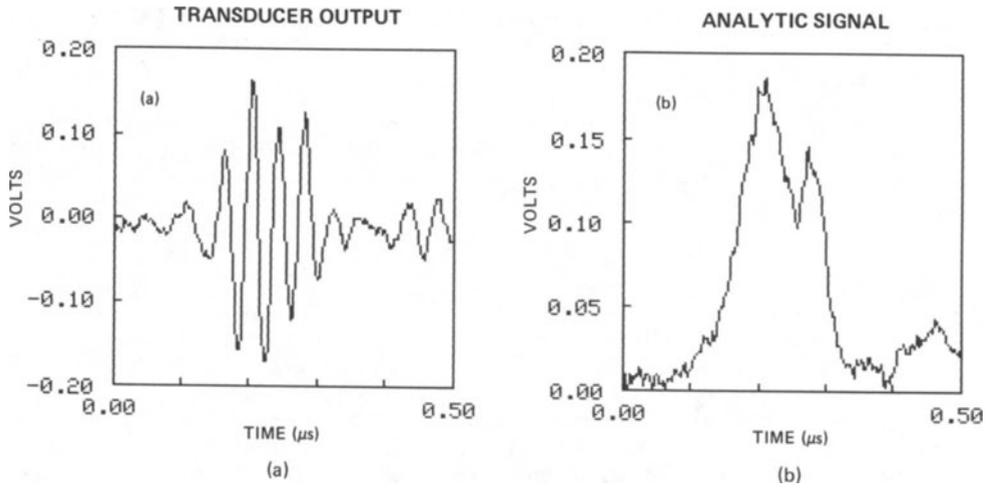


Fig. 3. (a) Waveform and (b) analytic signal from isolated reflector.

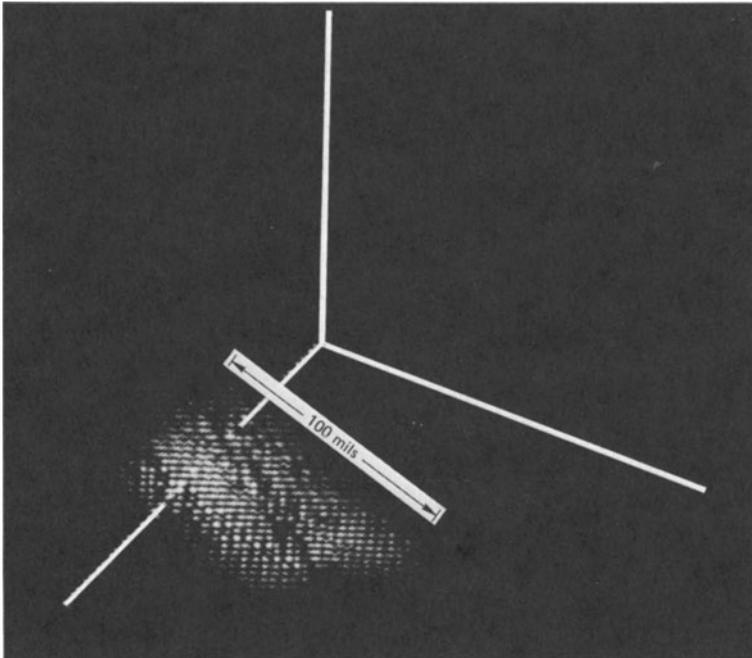


Fig. 4. Reconstruction of flat copper foil.

The ability of the analytic signal to resolve two reflectors strongly depends on the frequency and bandwidth of the transducer, since the width of the energy pulse is determined by these quantities. The peaks in Figure 3b are separated by approximately 70 ns. This separation would be lost with a much lower frequency transducer. A rough, but reasonable, estimate of the resolution of this technique is the wavelength of the sound.

Figure 4 shows a reconstruction of a flat trapezoidal copper foil immersed in a water bath. The longest side is approximately 2.5 mm (100 mils) in length. The reconstruction is composed of 1024 B-scans arranged in a 32 by 32 array. The spacing between scans is 0.15 mm (6 mils).

#### FUTURE DIRECTIONS

Two problem areas in this reconstruction technique have been identified: Depth resolution and the merging of images from several transducers. The apparent thickness of the foil in Figure 4 is due to the finite rise and fall times of the transducer. With the 30 MHz transducer used to produce the figure, flat planar interfaces have an apparent thickness of about 0.05 mm (2 mils). In an effort to reduce this effect, which is more pronounced with lower frequency transducers, an attempt is being made to model the reconstruction as a superposition of reference images and to compute, in effect, an impulse response.

The problem of merging images obtained from different transducers is basically a problem of determining the orientations of the transducers and their positions. This problem is complicated by the fact that the sound field is not necessarily coaxial with the transducer body. Pulse-echo measurements of the scattering from spherical reflectors as a function of transducer position ought to shed light on this problem.

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