ANALYSIS OF EDDY CURRENT RESPONSE DUE TO FLAWS IN IMPERFECTLY CONDUCTING MATERIALS

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INTRODUCTION

The fundamental theory for the prediction of eddy current response due to the presence of conducting samples of rather simple geometries has been well established. However, for any practical case the mathematics of the problem often preclude an exact analytical solution from being attained. Techniques based solely on numerical methods [1,2] have proven to be quite valuable, but have the disadvantage of concealing the dependence of the basically simple eddy current response on the physical parameters of the test-coil and material configuration. Several approximations can be made to simplify the mathematics of the problem, and thus allow solutions to be found that illustrate the fundamental functional relationships of the physical problem. A loss of generality is an inherent shortcoming of this approximate technique, but utilization of practical values of conductivities, permeabilities, dimensions, and frequencies of operation allow the results to remain meaningful.

THEORY

In previous work an approximate analytical expression has been derived for the change in impedance both for a single turn coil near an imperfectly conducting half space (as shown in Fig. 1) [3] and for a similar coil surrounding a conducting cylindrical sample (as shown in Fig. 2) [4]. The results of these investigations can then be used to calculate the change in impedance due to a small flaw in the conducting material. A first order approximation for this change in complex impedance can be expressed in terms of the electric field at the position of the flaw [5]. For flaws small enough that the fields do not vary greatly over their volume, the expression may be further approximated by using just the value of the field at the position of the centroid of the flaw (r_c, z_c) [5].

$$\Delta Z \approx \frac{\sigma V_F}{i^2} \vec{E}_0(r_c, z_c) \cdot \vec{E}_0(r_c, z_c) \tag{1}$$

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where σ is the conductivity of the material, V_F is the volume of the flaw, and i is the current at the terminal of the coil. For the planar geometry shown in Fig. 1 with a small flaw located at $r=r_c$ and $z=z_c$ the electric field can be approximated by [5]:

$$\vec{E}_0 = \hat{\phi} \frac{\mu i r_0}{2} \omega e^{-(1+j)z_c/\delta}[1+j]\delta I(r_c) \quad (2)$$

where μ is the permeability of the material, r_0 is the radius of the coil, ω is the angular frequency, δ is the skin depth $(=\sqrt{2/\omega\mu\sigma})$. I(r) contains the dependence on the radial position of the flaw and is given by:

$$I(r) = \int_0^\infty J_1(\alpha r)J_1(\alpha r_0)e^{-\alpha \ell}\alpha d\alpha$$

$$= -\frac{1}{\pi(rr_0)^{3/2}(x^2-1)} \times \left[\frac{x}{2}Q_{1/2}(x) - \frac{1}{2}Q_{1/2}(x)\right] \quad (3)$$

where $Q_{1/2}$ is the Legendre function of half order and

$$x = \frac{\ell^2+r^2+r_0^2}{2rr_0}$$

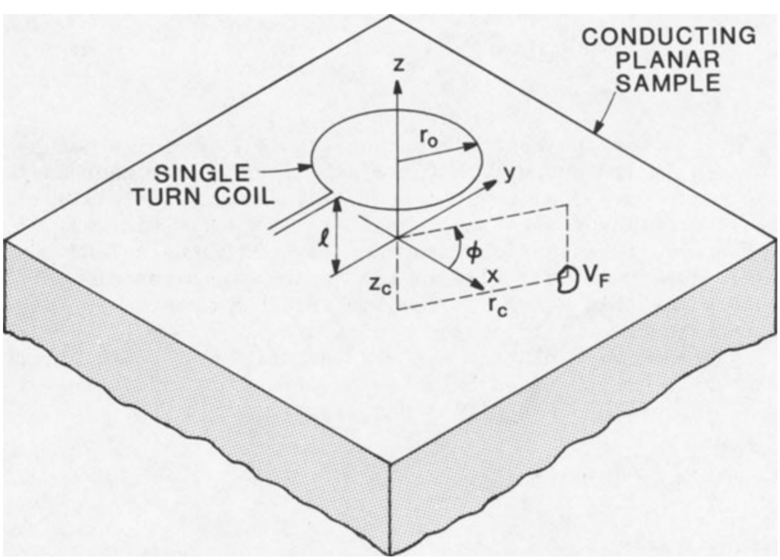

Fig. 1 Planar sample geometry

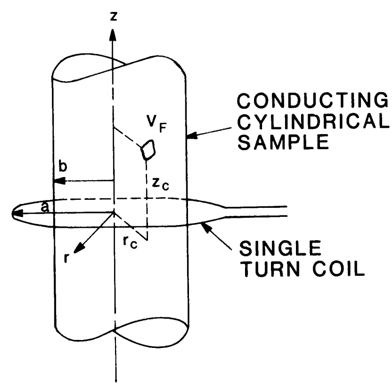

Fig. 2 Cylindrical sample geometry

The total change in complex impedance can thus be expressed as:

$$\Delta Z = V_F \omega \mu r_0^2 [I(r_c)]^2 e^{-2z_c/\delta} e^{-j2[z_c/\delta)-(\pi/4)]} \qquad (4)$$

In similar fashion the fields in a conducting cylindrical sample with a coaxial coil (as shown in Fig. 2) can also be approximated [6] as

$$\vec{E}_0 \approx \frac{j\omega\mu_0 ia}{\pi} \frac{I_1(\sqrt{j}\ \kappa r)}{\sqrt{j}\ \kappa b I_0(\sqrt{j}\ \kappa b)} \int_0^\infty \frac{K_1(\lambda a)}{K_1(\lambda b)} \cos\lambda z \, d\lambda \ \hat{\phi}, \qquad (5)$$

where I_1 and K_1 are modified Bessel functions of order one, $\kappa=\sqrt{\omega\mu\sigma}$, a is the radius of the coil, b is the radius of the sample, and λ is the integration variable.

Using the previous expression for the change in impedance and this expression for the electric field:

$$\Delta Z \approx \frac{\sigma V_F \omega^2 \mu_0^2}{\pi^2} \left[\frac{\sqrt{j}\ I_1(\sqrt{j}\ \kappa r)}{\kappa b I_0(\sqrt{j}\ \kappa b)}\right]^2 \cdot \left[a \int_0^\infty \frac{K_1(\lambda a)}{K_1(\lambda b)} \cos\lambda z \, d\lambda\right]^2 \Bigg|_{r_c, z_c} \qquad (6)$$

The right hand side of this equation can be separated into two parts, one that is just dependent on the depth of the flaw (F_d), and a second that is only a function on its axial position (F_a).

$$\Delta Z \approx \frac{\sigma V_F \omega^2 \mu_0^2}{2\pi^2} \left[\left(\frac{\delta}{b}\right)^2 \frac{b}{r_c} e^{-2r_d/\delta} e^{-j2(r_d/\delta - \pi/4)} \right] \cdot \left[\frac{B}{B^2 + \left(\frac{\beta z_c}{a}\right)^2} \right]^2 , \quad (7)$$

$$\Delta Z \approx \frac{\sigma V_F \omega^2 \mu_0^2}{\pi^2} F_d F_a$$

where $B(\beta) \doteq 0.815\beta - 0.794$, $r_c - b = -r_d = -$ the flaw depth from the surface, and $\beta = a/b$.

RESULTS

The expressions for the total change in complex impedance for each geometry can be divided into changes in real and imaginary parts and examined as a function of the depth of the flaw and either its radial or axial position. The dependence of both the real and imaginary parts on the depth of the flaw is shown in Fig. 3 for both the planar and cylindrical cases. Fig. 4 illustrates the effect of the radial position of the flaw on the magnitude of the eddy current response for the planar geometry. Similarly, the variation of the magnitude of the response as a function of axial position for the cylindrical case is shown in Fig. 5. In each of these two cases it should be noted that the effect is one on the overall magnitude, thus affecting both the real and imaginary parts.

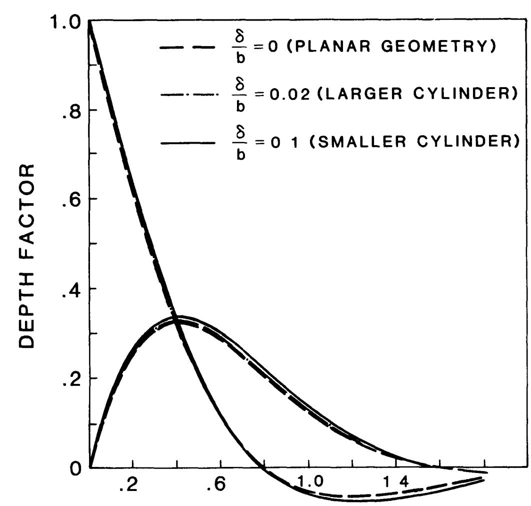

Fig. 3 Real and imaginary parts of the change in impedance as a function of flaw depth.

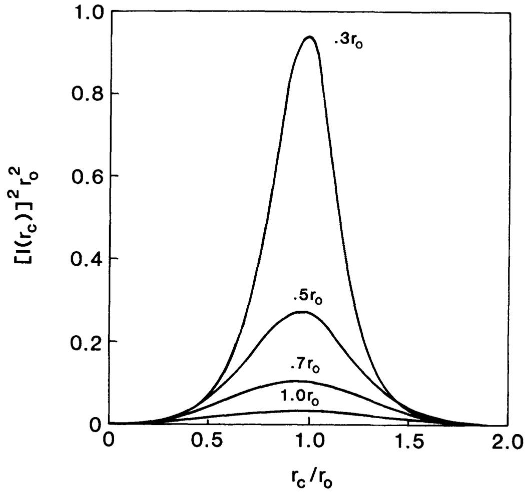

Fig. 4 Magnitude of eddy current response as a function of radial position of the flaw - planar case.

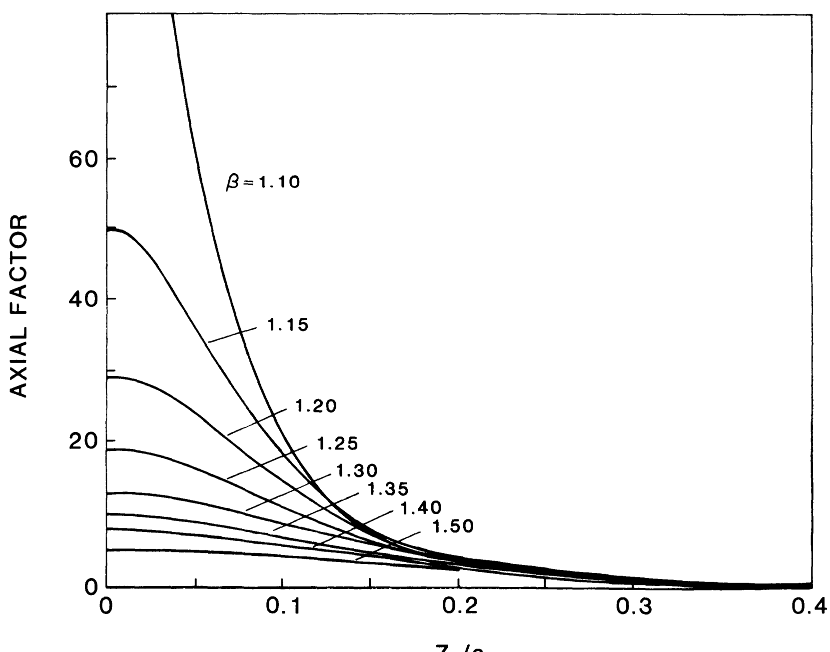

Fig. 5 Magnitude of eddy current response as a function of axial position of the flaw - cylindrical case.

CONCLUSIONS

A first order approximation has been derived for the change in complex impedance for a single turn coil both near an imperfectly conducting half space, and coaxial with a cylindrical conducting sample due to a small flaw in the conducting material. The dependence of this change in impedance on the position of the flaw as well as the geometrical and physical parameters of the coil and sample has been investigated and representative results have been given.

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