ANALYSIS OF FLAT COILS' SYSTEM WITH DISPLACED SENSORS FOR EDDY CURRENT NDE OF FERROMAGNETIC METALS

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INTRODUCTION

The existing theoretical models for the eddy current method [1-3] frequently cannot accommodate new developments, especially in terms of new coil designs. Among them is a use of flat coils with displaced exciting and sensor coils which already have found practical applications in some new eddy current instruments (4). It can also be useful for the eddy current testing of pipes in oil and gas wells in order to localize the area of inspection (Fig. 1).

However, there has not been any theoretical investigation for this, so called 'nonaxial' design (Fig. 2). So, we will try here to build an analytical model which will allow us to analyze relationships between eddy current field and various parameters of coils and metal and eventually optimize parameters of inspection.

In the future analysis we are going to make some standard assumptions which practically will not effect the accuracy of results but will considerably facilitate the process of investigation.

First, we will substitute real dimensions of coils with their equivalent values according to (3). This approximation will allow us to assume a current source located above conductive ferromagnetic half-space as a delta function and apply the well known Fourier-Bessel method to solve an appropriate Helmoltz equation.

Second, due to the usually small size of coils and small currents in exciting coils the magnitude of the magnetic field on the surface of metal is assumed to be less than .1Oe. This means that magnetic permeability is roughly constant, and the model itself remains linear [5].

Third, coils located inside of pipe should be small enough to localize the inspection, and therefore the radius of pipe is usually much bigger than radiuses of the coils, which makes surface of pipe flat for those coils. This means we can use the result of analysis derived for half-space.

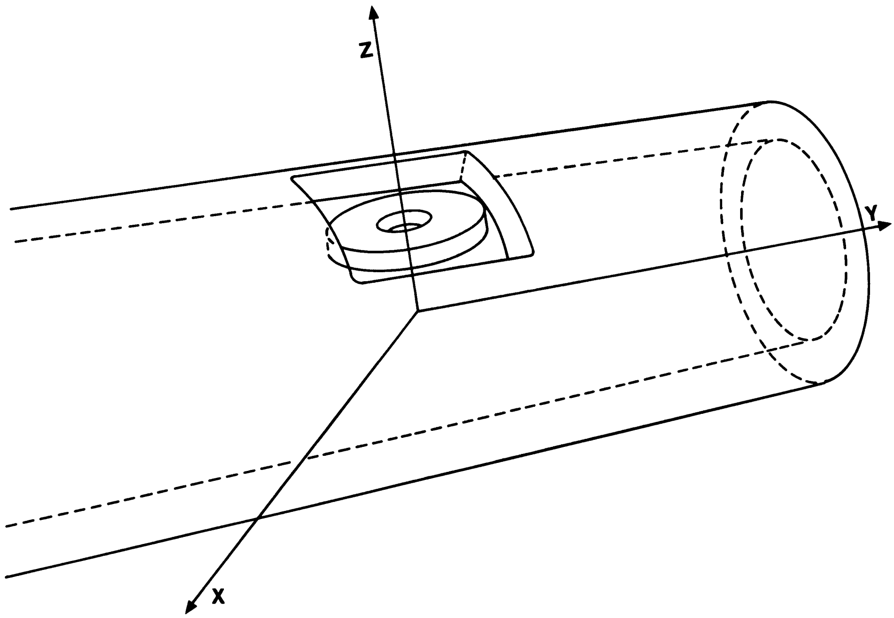

FIG. 1

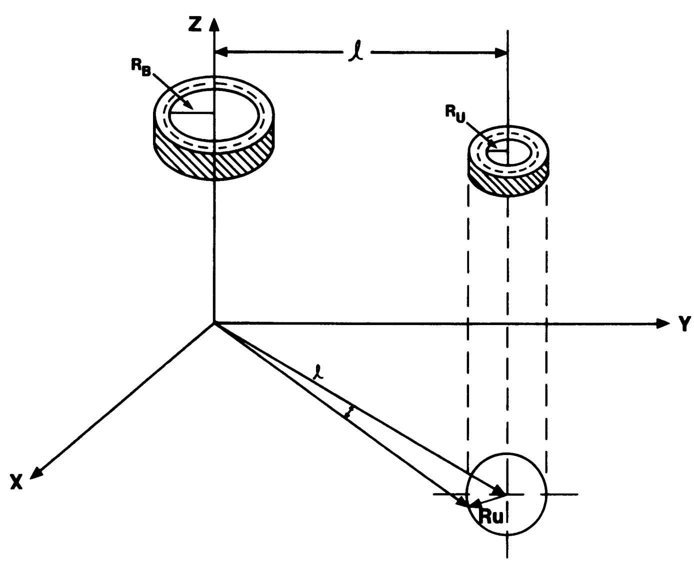

FIG. 2

We begin with the Helmholtz equation for vector-potential:

where:

$$\nabla^2 A + k^2 A = - \mu\mu_o J_{cT} \qquad (1)$$

A is a vector-potential
B = rot A

$$k^2 = - j\omega\mu\mu_o\sigma; \; J_{cT} = i\delta(r - R_B)\delta(z)$$

and where

ω is a frequency of excitation
μ_o is a permeability of metal
μ is a permeability of the free space
σ is a conductivity
J_{cT} is a density of the exciting current
i is a magnitude of the exciting current
δ is a delta function
r, z are variables in cylindrical coordinates
R_B is a equivalent radius of the exciting coil determined according to [2]

In cylindrical coordinates for an axi-symmetry problem (Fig. 2), the equation (1) becomes two dimentional:

$$\frac{1}{R}\frac{\partial}{\partial r}\left(r\frac{\partial A}{\partial r}\right) + \left(k^2 - \frac{1}{r^2}\right) A + \frac{\partial A}{\partial z^2} = - \mu\mu_o J_{cT}; \qquad (2)$$

$$A\,(r,z) = A_o + A_{BH} \qquad (3)$$

where:

A_o is vector-potential in air

A_{BH} is vector-potential due to presence of metal

Equation (2) can be solved using Fourier-Bessel method [2] for both components of the equation (3):

$$A_o(r,z) = \frac{\mu_o i R_B}{2} \int_o^\infty J_1(\lambda R_B)\, J_1\,(\lambda r) L^{-\lambda|z - h|} d\lambda \qquad (4)$$

$$A_{BH}(r,z) = \frac{\mu_o i R_B}{2} \int_o^\infty F_1 J_1(\lambda R_B)\, J_1(\lambda r) L^{-\lambda(z+h)} d\lambda \qquad (5)$$

where:

$$F_1 = \frac{(\mu^2\lambda^2 - q^2)(1 - e^{-2qT})}{(\mu\lambda + q)^2 + (\mu\lambda - q)^2\, e^{-2qT}};$$

and

$q^2 = \lambda^2 - k^2$; h is a distance between exciting coil and metal

T is wall thickness of metal

$k^2 = -j\omega\mu\mu_o\sigma$; $j = \sqrt{-1}$;

In practical terms it is more convenient to operate with induced voltage rather than with vector-potential. Using a standard transition from one parameter to another [2], the equations (4) and (5) respectively become:

$$U_o = \frac{-j\omega\mu_o \; i \; R_B}{2} \int_o^\infty J_1(\lambda R_B) \; G(\lambda, R_u, L) \; L^{-\lambda|z - h|} \; d\lambda \quad (6)$$

$$U_{BH} = \frac{-j\omega\mu_o \; i \; R_B}{2} \int_o^\infty F_1 \; J_1(\lambda R_B) \; G(\lambda, R_u, L)^{-\lambda(z + h)} \; d\lambda \quad (7)$$

where:

R_B is radius of source coil

R_u is radius of sensor coil

G is a function related to position of sensor in space

L is a distance between the sensor and the source

As we can see the equations (6) and (7) contain function "G" related to a position of the sensor coil. In the traditional case, where both coils have the same axis, "G" is a Bessel function where the radius of sensor coil is part of argument [1-3]. In this more general case, function "G" depends upon both radius of sensor and upon distance between the coils.

Now we will try to find the explicit expression for "G". For that purpose we will take projections of both exciting and sensor coils on "X-Y" space and present "X" and "Y" in parametric form as a function of one parameter "t" (Fig. 3):

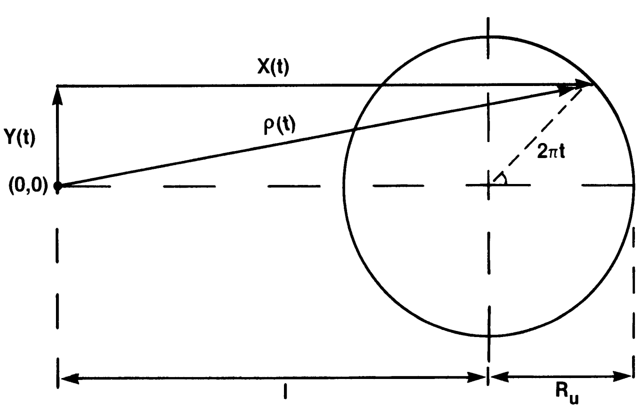

FIG. 3

$$x(t),\ y(t) \quad 0 \leq t \leq 1$$

$$x(t) = L + R_u \cos 2\pi t \tag{8}$$

$$y(t) = R_u \sin 2\pi t \tag{9}$$

$$\rho(t) = \sqrt{x^2(t) + y^2(t)}$$

$$= \sqrt{(L + R_u \cos 2\pi t)^2 + (R_u \sin 2\pi t)^2} \tag{10}$$

$$\frac{dx}{dt} = - 2\pi R_u \sin 2\pi t \tag{11}$$

$$\frac{dt}{dt} = 2\pi R_u \cos 2\pi t$$

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} =$$

$$= 2\pi R_u dt \tag{12}$$

and

$$G(\lambda, R_u, L) = \int_o^\infty J_1[\lambda\rho(t)]ds$$

$$= 2\pi R_2 \int_o^1 J_1[\lambda\sqrt{L^2 + 2LR_u \cos 2\pi t + R_u^2}]dt \tag{13}$$

Now we have derived a general expression for the function "G". As we can see the traditional case is accommodated by a more general expression (13). It can easily be verified by taking a distance between coils "L" equal zero:

$$G(\lambda, R_u, L) = 2\pi R_u J_1 (\lambda R_u) \tag{14}$$

In order to eliminate the unnecessary coefficients, induced voltage is usually normalized to voltage in air. In the general case, for the flat coils when "L" is arbitrary, the expression for normalized induced voltage is:

$$U^*_{BH} = \frac{\int_o^\infty F_1 J_1 (\lambda R_B)[\int_o^\infty J_1(\lambda\sqrt{L^2+2L R_u \cos 2\pi t+R_u^2})dt]e^{-\lambda(z+h)}d\lambda}{\int_o^\infty J_1 (\lambda R_B) [\int_o^\infty J_1 (\lambda\sqrt{L^2+2LR_u \cos 2\pi t+R_u^2})dt]e^{-\lambda|z-h|}d\lambda} \tag{15}$$

and for normalized magnitude:

$$|U^*_{BH}| = \sqrt{(J_m U^*_{BH})^2 + (R_e U^*_{BH})^2} \qquad (16)$$

and phase:

$$\phi = TAN^{-1}\left(\frac{J_m U^*_{BH}}{R_e U^*_{BH}}\right) \qquad (17)$$

It will now be interesting to see how distance between exiting and sensor coils affects parameters of induced voltage for a case of eddy current measurements of wall thickness of pipes. Because nominal wall thickness of pipes used in the oil and gas industry is in a range of .25 - .55 inches, and initial magnetic permeabilities are between 30 and 110 [4], operational frequencies should remain low (somewhere around 10 - 50 Hz), to provide a sufficient depth of penetration. The typical amplitude and phase characteristics of the normalized induced voltage as a function of wall thickness for the different distances between coils, calculated by formulas (16) and (17), are shown in Fig. 4 and Fig. 5 respectively.

As Fig. 4 indicates, an amplitude of normalized voltage is considerably higher when normalized distance between coils "L/RB" is increased from zero to "L/RB"=5 ("L/RB"=0 corresponds to the traditional case with both coils having the same axis; normalization is made to radius of exciting coil "RB" as it is commonly accepted in the NDE literature). A similar situation exists with phase characteristics. As we can see in Fig. 5 sensitivity of phase to the

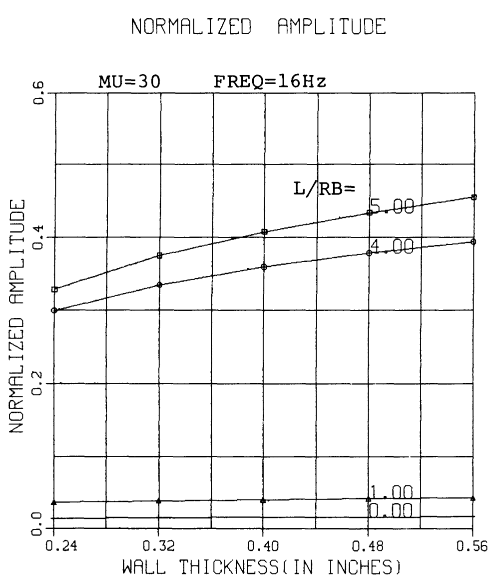

FIG. 4

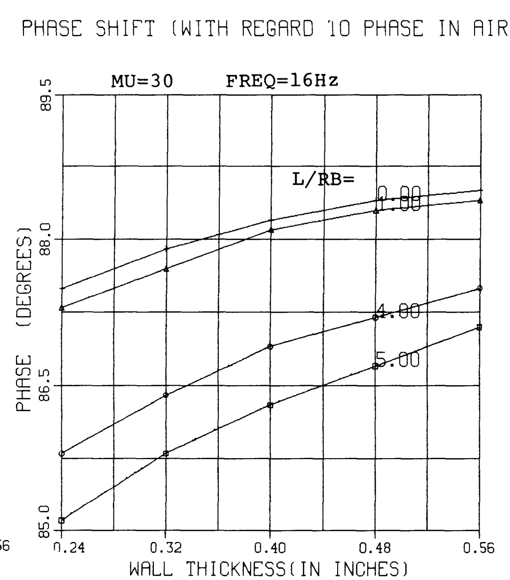

FIG. 5

change of wall thickness is also much higher with displacement of the coils. Similar amplitude and phase characteristics are observed on other frequencies (Fig. 6 and Fig. 7) and other magnetic permeabilities (Fig. 8 and Fig. 9).

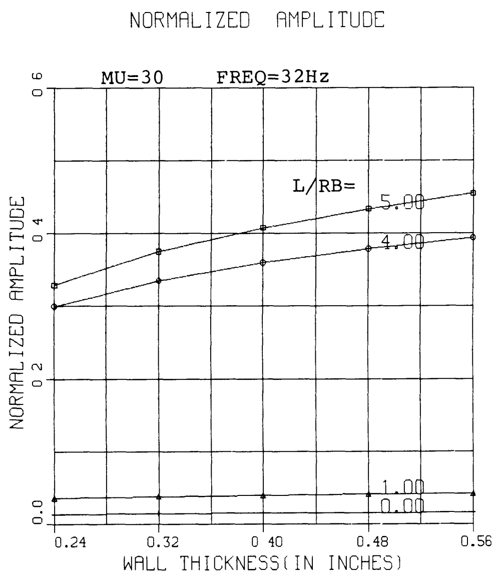

FIG. 6

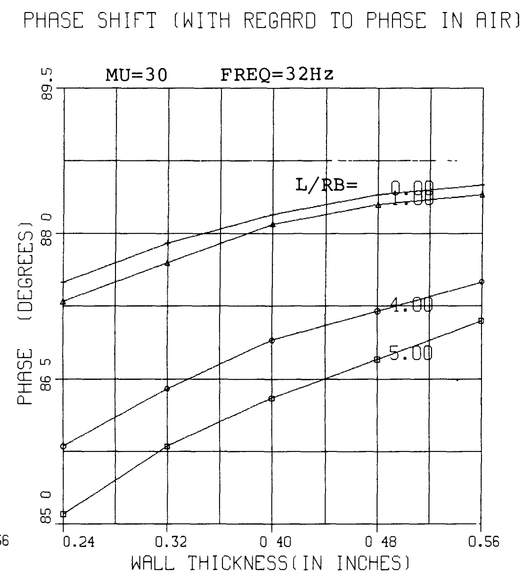

FIG. 7

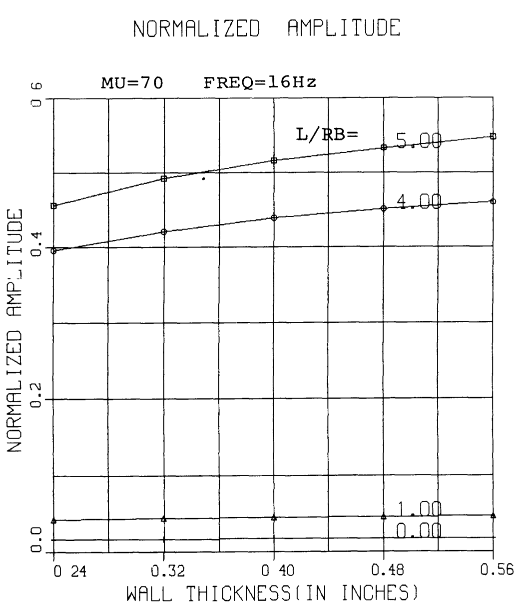

FIG. 8

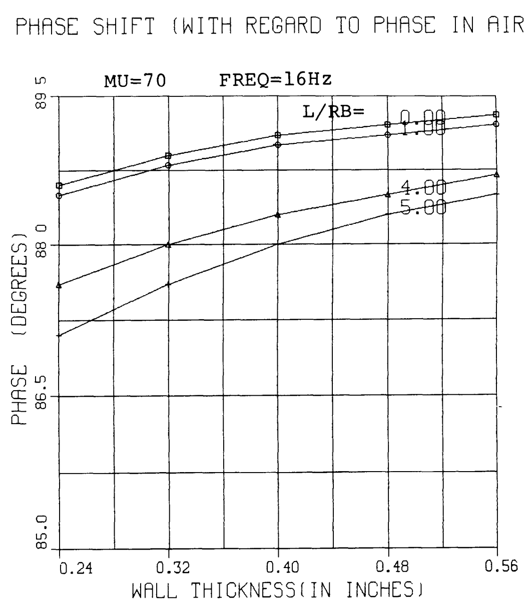

FIG. 9

In general terms all these results indicate that displacement of exciting and sensor coils can be very beneficial, at least for the thickness measurements, because both amplitude and phase changes are much higher than for the traditional case although absolute value of the response signal gets weaker with increased distance between coils. Therefore, if that factor is not very critical, displacement of coils can be recommended as a good way to improve the performance of eddy current instruments.

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DISCUSSION

J.L. Fisher (Southwest Research Institute): First of all, a comment. In our work we see empirically the kind of thing that you are talking about. We also have the probes spaced apart. So I am curious if you have any physical explanation for why the sensitivity changes using your sensitivity parameter.

S. Marinov: It is very hard to explain those results physically because we used a quite artificial parameter to define quality of the system. It is not simply the certain distance between two coils where we have maximum phase shift, but it is a distance where the phase due to the changes of the wall thickness is big, and the phase shift due to the permeability changes is small. Yes, you are right, it is quite reminiscent of the remote eddy current technique in case of the circumferential coils which you are going to talk about, I believe.

It is essentially the same phenomenon except coils are turned on 90-degrees here compared with the circumferential coils. The only physical explanation I can see is that the phase shift changes linearly, and we can vary the angle of the slope by varying the distance between coils. This is similar to the circumferential coils where, also, by varying distance between coils, we can vary the stiffness of the slope.

J.L. Fisher: It is less linear.

S. Marinov: Yes, but it is still linear. I mean the stiffness of the slope could be changed by varying different parameters, not only spacing between the coils. But again, it is an arbitrary parameter we used just in order to define the quality of the system.