IMPROVED PROBE-FLAW INTERACTION MODELING, INVERSION
PROCESSING, AND SURFACE ROUGHNESS CLUTTER

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INTRODUCTION

In Reference 1 a first comparison was made of measured eddy current signals with calculations based on nonuniform probe-field interaction theory. These calculations followed the basic analysis developed in Reference 2. They used interrogating field distributions calculated by Dodd and Deeds theory for the air core coils of Reference 3. (Note that Fig. 6 in Reference 1 and Fig. 7 in Reference 3 should be interchanged). In Reference 1 theoretical and experimental plots of the flaw profile curve (a plot of $\Delta Z$ versus distance along the mouth of a surface breaking flaw) were found to be in good agreement, with regard to shape, for several selected EDM notch samples in aluminum. An iterative procedure was also developed for systematically varying the length, depth, and opening width to obtain a best fit to the experimental data. In the present paper a full inversion procedure is developed and illustrated for approximately rectangular-shaped EDM notches. The mathematical structure of the inversion problem is first examined and a solution is proposed. Physical reasoning, based on the form of the flaw profile curves, is then used to simplify the approach and to provide guidance in selection of the most suitable probe geometry. Other topics briefly addressed include, possible improvements in the theory for the region with $a/\delta$ close to unity and for more realistic flaw shapes (i.e., semi-elliptical, rather than rectangular), inaccuracies due to errors in the probe scan path, and background clutter due to surface roughness, machining marks, and microstructure.

THE FLAW PROFILE CONCEPT

Figure 1 shows some examples of raster scan images of flaw response signals of different types. These were constructed by plotting amplitude contours for the component of the flaw $\Delta Z$ that is normal to liftoff. The gray scale is coded, with darker regions corresponding to larger amplitudes. These images provide a clear visual distinction between the various types of flaws. There is, however, no accurate dimensional information directly available from these plots, although they do give a rough idea of the surface extent of the flaw.

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Fig. 1. Eddy Current Images for Various Defects (after Copley\textsuperscript{5}).
The present paper is concerned exclusively with surface-breaking flaws (case (a) in the figure) and presents a quantitative procedure for inverting the measured data. This involves use of the flaw profile (i.e., a plot of the amplitude and phase of $\Delta Z$ along the vertical line in Fig. 1(a)). Identification of this line on the image is easily made, because it passes through the two peaks of the image pattern. Excellent perspective images of this double-peaked response for flat surface-breaking flaws have been obtained by Rummel. To summarize, the flaw profile is obtained by first making a raster scan of the amplitude and phase of $\Delta Z$, identifying the line of the flaw profile as described above, and then plotting the amplitude and phase of $\Delta Z$ along this line. (For inversion from $\Delta Z$ theory, it is necessary to have experimental data on both the amplitude and the absolute phase of $\Delta Z$). In this procedure, errors in locating the line of the flaw profile have minimal effect on inversion accuracy, because this line corresponds to the ridge passing through the two peaks of the image plot.

In the following section a formal algebraic procedure is described for inverting the data available in the flaw profile. It is shown that an essential initial step in this process is an accurate determination of the surface length of the surface-breaking flaw. To perform this part of the inversion algebraically it is first necessary to estimate the surface length and then refine by iteration. Information concerning the first estimate is readily available from the flaw amplitude profile (Figs. 2 and 3). These figures show the surface opening, or mouth, of surface-breaking flaws of arbitrary sub-surface shape. Superposed on each figure is an eddy current pattern due to the probe. To picture the $\Delta Z$ amplitude response as this eddy current pattern is scanned along the mouth of the flaw, it should be recalled that there is always a "dead spot" at the center of an eddy current pattern, just as at the center of a hurricane. As the probe moves in from the left in Fig. 2 the $\Delta Z$ response increases when the flaw begins to intercept the current loops. This continues until the dead spot reaches the end of the flaw. Near this point in the scan the probe can no longer sense the end of the flaw, and a flat spot occurs in the flaw profile. The $\Delta Z$ response increases again as the probe moves further onto the flaw, and flattens off when the eddy current pattern moves entirely onto the central part of the flaw. In this region the probe does not sense the ends of the flaw, and the response is independent of position. The same sequence repeats in reverse order at the other end of the flaw. For a flaw with surface length $2c$ much greater than the
probe diameter the flaw amplitude profile has the form shown in Fig. 2, with inflection points corresponding to the ends of the flaw. When $2c$ is much shorter than the probe diameter (Fig. 3) the response is quite different. In this case the flaw does not extend over more than a small part of the eddy current pattern, and the inflection points do not appear. Instead, the entire flaw lies in the dead spot when the probe is centered over the flaw. The amplitude response becomes small at this point, as shown in the figure. Clearly, this is not a good choice of probe geometry for determining $2c$. The detailed shapes of the flaw amplitude profile will, of course, depend on the shape of the interrogating field generated by the probe (Fig. 4), since this determines the cross-section profile of the eddy current pattern in Figs. 2 and 3. However, the general features remain the same, and the surface length of a flaw can be measured experimentally by taking flaw profiles with probes of smaller and smaller diameters until a profile similar to that of Fig. 2 is obtained.

**GENERAL STRUCTURE OF $\Delta Z$ THEORY**

The inversion procedure treated here is based on nonuniform field probe-flaw interaction theory for rectangular surface-breaking cracks.\(^1\)\(^2\) This approach is justified by the good agreement of theory with experiment for approximately rectangular shaped EDM notches in aluminum.\(^7\) Figure 5 shows a comparison of theoretical flaw amplitude profiles with experimental results obtained by J.C. Moulder at the National Bureau of Standards,\(^7\) and with a measurement made at Stanford by F. Muennemann, using the method described in Reference 4. As plotted, the theory is fitted to the National Bureau of Standards data at the center of the flaw. The amplitude of the curves is the only adjustable parameter. If taken directly as calculated (i.e., with no adjusted parameters) the theoretical curves would be scaled by a factor of approximately 0.75.

This theoretical model is based on the assumption that the interrogating field at the surface of an unflawed work piece appears essentially
unchanged in the mouth of the flaw. Also, the magnetic field inside the flaw is approximated as if the walls of the flaw were perfectly conducting.\textsuperscript{1,2} (For a "closed" flaw the side walls are assumed to be separated by a distance that is very small compared to the other flaw dimensions). The $x$-component of the magnetic field in the mouth of the flaw (Fig. 2) is expanded in a Fourier sine series ($X$ is the coil position and $x$ is the field measurement point)

$$H_x(X,x) = \sum_n B_n \sin \frac{n\pi(x + c)}{c}$$  \hspace{1cm} (1)

$$B_n(X,c) = \frac{1}{c} \int_{-c}^{c} H_x(X,x) \sin \frac{n\pi(x + c)}{c} \, dx$$  \hspace{1cm} (2)

and the quasistatic magnetic field inside the flaw is obtained by solving the Laplace problem for the interior region, with perfectly conducting walls. This step is easily treated analytically only for a flaw of rectangular shape, but can be carried out numerically for other flaw geometries. Once the interior magnetic field has been obtained, all of the field quantities appearing in Eq. (2) of Reference 2 can be found and $\Delta Z$ can be calculated, as described in that paper. In the general case this leads to a $\Delta Z$ of the form

$$\Delta Z(X) = \sum_{m,n} B_m(X,c) B_n(X,c) F_{mn}(c,a,\Delta u)$$  \hspace{1cm} (3)

where $X$ is the position of the probe in the flaw mouth (Fig. 2), the $B_n$'s are the expansion coefficients in Eq. (2), $a$ is the flaw depth, $\Delta u$ is the flaw opening width, and the $F_{mn}$'s are functions characteristic of the flaw itself. The expansion coefficients depend on the position $X$ of the probe because the field on the left-hand side of Eq. (1) varies with probe position. In writing Eq. (3) it has been assumed that the conductivity $\sigma$ of the work piece and the probe frequency are specified.

By collapsing the pairs of subscripts in Eq. (3) according to the following scheme

$$11 = 1$$

$$12 = 21 = 2$$

$$22 = 3, \text{ etc.}$$

the equation can be rewritten as

$$\Delta Z(X) = \sum_I \beta_I(X,c) F_I(c,a,\Delta u)$$  \hspace{1cm} (4)

$$\beta_1 = B_1^2$$

$$\beta_2 = B_1 B_2, \text{ etc.}$$

This constitutes a set of linear equations for the characteristic functions of the flaw, with known (from measured data) inhomogeneous terms on the left-hand side. The coefficients $\beta_I$ of this set of equations can be
calculated from the interrogating field of the probe (Eq. (1)), but only after the surface length $2c$ of the flaw has been determined. Once the $\beta_i$'s are found, Eq. (4) can be solved for the characteristic functions of the flaw. The depth $a$ and opening width $\Delta u$ of the flaw can then be obtained from the form of the characteristic functions. Since there is an infinite number of characteristic functions in Eq. (4), the set of equations must be truncated in order to solve. The number of equations included can then be increased until the solutions stabilize. Each of the characteristic functions may be used independently to determine $a$ and $\Delta u$, thereby giving a redundancy check on the accuracy of the assumed flaw shape against the measured data. To illustrate, the characteristic functions for a closed rectangular flaw with very large $a/\delta$ are

$$F_{mn} = \frac{4c^2}{\pi} \frac{(1+i)}{\sigma\delta} \frac{\tanh(n\pi/2)a/c}{n} \tag{5}$$

$$F_{mn} = 0$$

If $c$ has already been found, these functions determine $a$ directly. For a semi-elliptical shape, the characteristic functions are best found by numerical methods. This requires solving the interior Laplace problem for applied fields in the mouth of the flaw corresponding to the individual terms in Eq. (1). Since the characteristic functions are then unavailable in closed algebraic form, graphs (or look up tables) giving them as functions of $a$ and $\Delta u$ must be constructed. Only a few of these graphs will be needed to perform inversion and to check the accuracy of the assumed flaw shape.

Performance of the inversion procedure described above requires an initial determination of the surface length $2c$. A condition for the correct value of $c$ is obtained by noting that the set of linear equations (Eq. 4) must be a self-consistent set, regardless of the depth and opening width of the flaw. The standard test for consistency is stated in terms of the $\beta_i$'s and the $\Delta Z$'s. This gives a set of conditions on $c$, because the $\beta_i$'s are functions of $c$ only. The procedure may be applied to a truncated version of Eq. (4) and tested with estimates of $c$, starting from the visual determination of Fig. 2.

INVERSION

The inversion protocol outlined above requires a substantial effort in numerically evaluating fairly large determinants. (The theoretical curves shown in Fig. 5 required keeping 64 terms for a precision of a few percent, although the curves were still recognizable with only 8 terms). Furthermore, care must be taken in choosing the values of $X$ used for the $\Delta Z$ terms on the left-hand side of the truncated version of Eq. (4). An optimum choice is one giving the greatest sensitivity to the surface length $2c$ in the self-consistency test. Here, more physical approaches to the problem, similar to the visual method for obtaining the flaw length from the inflection points of the flaw amplitude curve of Fig. 2, will be presented.

Length Inversion

In implementing the inflection point inversion procedure of Fig. 2 it is important to test the sensitivity of the method to changes in the
flaw depth $a$ and width $\Delta u$. Flaw amplitude profiles were calculated for a variety of flaw parameters, and it was found that the inflection point positions did not change noticeably with $a$ and $\Delta u$. It was also observed that extension of the outer slopes of Fig. 2 to the horizontal axis gave intercept points accurately spaced by the flaw length $2c$. Both the inflection point and the slope intercept methods for length inversion require a proper choice of the probe radius. Figure 6 illustrates this point by showing a continuous transition from the single-peaked response of Fig. 2 to the double-peaked response of Fig. 3. For $L$ greater than $2r_{AVE}$ the peaks of the double-peaked response still give a fair indication of the ends of the flaw, but for smaller $L$'s the flaw profile curve shape is governed almost entirely by the shape of the coil field. It can also be seen from Fig. 6 that the inflection point never occurs exactly at the end of the flaw. To allow for this correction, a theoretical inversion curve has been constructed in Fig. 7, using the distance between either the inflection or the outer maximum points as the abscissa. A second curve was constructed using the distance between the two slope intercept points. Plots of the NBS experimental points on these curves show a greater precision for the slope intercept method. A comparison of actual and predicted lengths is given in Fig. 8, and the same information is tabulated in Fig. 9.

**Depth and Width Inversion**

A search was made for similar visual features of the amplitude and phase profile curves that might be used to separately identify $a$ and $\Delta u$, but none were found that could be clearly associated with either $a$ or $\Delta u$. Figure 10 is a theoretical inversion chart that was found to allow simultaneous inversion of the data to obtain these flaw parameters. The figure is a plot of contours of constant $a$ and $\Delta u$ against the amplitude and phase of $\Delta Z$ when the coil is centered over the flaw -- that is, at the center point of the flaw profile. Figures 11 and 12 are plots of actual and predicted flaw dimensions obtained by applying this procedure to the NBS data. The same information is also tabulated in Fig. 9.
FLAW INVERSION DATA

(a) LENGTH

<table>
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<th>FLAW</th>
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<td></td>
<td>SLOPE</td>
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<td>3.51r_ave</td>
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<tr>
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(b) DEPTH

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(c) WIDTH

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<tr>
<td>5</td>
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Fig. 8. Length Inversion Data.

The theory used in the inversion procedures described above is for the large $a/\delta$ regime.\(^9\) To perform this inversion satisfactorily in a factory environment it will be necessary to have an adequate signal-to-noise ratio for the experimental data. It is therefore important to consider the sources of noise and clutter in the large $a/\delta$ regime. A general treatment of these questions has been given by Bahr and Cooley,\(^10\) where it is shown that the principal limitation comes from liftoff and tilt contributions to signal clutter. Liftoff clutter in the quadrature channel of a phase discrimination detection circuit has been shown to decrease slowly with respect to the flaw signal as the frequency is
increased in the large $a/\delta$ regime. However, it is often stated that the ultimate limiting factor in this regime is clutter due to surface roughness, machining marks, and conductivity variations due to microstructure. Although there exists no quantitative experimental data regarding this point, it is of such great practical importance that a beginning needs to be made on examining the basic theory.

One approach to liftoff analysis is to formulate the problem in terms of the surface impedance of the work piece. A second method, more easily applied to the roughness problem, relies on the volume integral formulation of the $\Delta Z$ formula for flaws in nonmagnetic metals,

$$\Delta Z = \frac{1}{I^2} \int_{V_F} \sigma \cdot E^* \, dV$$

This formula is applied to liftoff by considering the "flaw" to be a thin slice of thickness $h$ removed from the surface of the work piece, and taking the perturbed (primed) field to be the same as the unperturbed field. The same approach can be used for surface roughness by allowing $h$ to vary with position on the surface according to the roughness function. This procedure shows that roughness clutter and liftoff clutter have the same phase and frequency dependence. Similarly, conductivity variations can be treated by allowing the conductivity $\sigma$ to vary with position in the work piece.

The method given in the previous paragraph is applicable only in situations where the height of the surface roughness is small compared with the skin depth of the work piece. When the surface roughness is large compared with skin depth the best method is to consider, for example, a surface scratch as a large $a/\delta$ surface flaw with an opening $\Delta u$ larger than its depth $a$. This case is considered in the general treatment of two-dimensional flaws (Ref. 12). In Fig. 2 of that reference the flaw opening contribution is the third term in the $\Delta Z$ formula cited. At high frequencies this term dominates the others for wide open flaws, such as scratches. It has the same phase angle and frequency dependence as liftoff.
Fig. 11. Depth Inversion Data.

Fig. 12. Width Inversion Data.
CONCLUSION

Agreement has been found between theoretical predictions and experimental measurements of $\Delta Z$ flaw profiles for EDM simulations of rectangular-shaped surface-breaking cracks interrogated with a nonuniform air-core probe field. Differences between theory and experiment are approximately 20 to 30 percent in amplitude. This can be attributed to: (1) approximations in the theory, (2) imperfect rectangularity of the flaws, (3) errors in measurement of $\Delta Z$. Discrepancies of approximately $10^\circ$ in the phase of $\Delta Z$ were also observed. Based on this theory, an inversion procedure was proposed and applied to the experimental data presented in Reference 7, with accuracy of approximately 5 percent for $c$, 15 to 20 percent for $a$ and 20 to 30 percent for $\Delta u$. An important consideration in performing this type of inversion is testing the flaw with a series of probes of different radius, to find the optimum probe for determination of the flaw length.

Improvements needed to permit application of this procedure to realistic flaws in the field include extension of the theory to semi-elliptical flaws and to the analysis of flaw signal response in the region of $a/\delta$ close to one. Variational methods appear to be an attractive option for the second problem. Unpublished work in 1981 describes how this method can be applied to flaws in perfectly conducting work pieces, but subsequent efforts to extend this to finite conductivity have so far been unsuccessful. Another direction for future work is application of this inversion procedure to two-port, or reflection, probes. For such probes, the $\Delta Z$ formula is modified by taking the primed fields to be those of the (small) detection coil and the unprimed fields (for the unflawed work piece) to be those of the (large) excitation coil. If the field of the excitation coil is uniform over the mouth of the flaw, it can be removed from the integral and inversion can be performed essentially as described above. Otherwise the excitation field provides another weighting function, whose effect must be allowed for in the inversion.

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REFERENCES


6. R. D. Rummel, Assessment of the effects of scanning variations and eddy current probe type on crack detection, this volume.


