THEORY OF EDDY CURRENTS IN METAL MATRIX COMPOSITES

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INTRODUCTION

Eddy current nondestructive characterization of composite materials is complicated by the anisotropic nature of their electrical conductivities. Earlier calculations for anisotropic materials show that eddy current distributions can be quite different from the rather simple patterns produced in isotropic media, and this can have a profound effect on probe response [1-3]. It follows that proper interpretation of eddy current data in terms of microstructural material properties requires understanding of how microstructure affects electrical anisotropy and how this anisotropy is reflected in the response of an eddy current probe.

In this paper we present two models pertaining to eddy current characterization of continuous filament metal matrix composites. The first model deals with the effects of the spatial distribution of fibers on the macroscopic conductivity tensor, and the second is an extension of earlier theories for uniaxially anisotropic materials to a more general biaxial case. Applications are illustrated by a study of volume fraction measurement for a titanium alloy reinforced with silicon carbide fibers.

THEORY OF THE CONDUCTIVITY TENSOR

The problem addressed here is that of calculating the effective conductivity tensor for an array of parallel fibers in a metal matrix. In the special case where the fibers form a regular array, like a square or hexagonal lattice, the problem reduces to that solved by Rayleigh nearly 100 years ago [4]. In the practical case, however, irregular fiber distributions are of concern and a more general calculational approach is needed.

Although the model we will describe is quite general, our initial application focuses on a particular type of composite with particular structural abnormalities. The material is fabricated by laying down a sheet of the metal, then a layer of fibers, another sheet of metal, etc. The metal/fiber sandwich is then hot pressed so that metal flows around the fibers and recrystallizes to form a continuous matrix. The problem that can occur is that fibers tend to move within rows leading to regions where the fibers are closely spaced within a row and other regions where the spacing is large, as illustrated in Fig. 1. To determine the effects...
of such abnormal microstructure on eddy current probe response, we need a model that can handle irregular distributions like those shown in the figure.

In general, the effective DC conductivity tensor for regions like those shown in Fig. 1 can be calculated in terms of the electrostatic potential $\Phi$ on the surfaces of the fibers. It has been shown that the $i$'th component of the current density, averaged over a cell containing $N$ fibers, is given by [5]

$$\langle j_i \rangle = \sigma_M E_i - \frac{\sigma_F - \sigma_M}{V} \sum_{j=1}^{N} \int_{S} \phi_{n_i} dS_j,$$  \hspace{1cm} (1)

where $E_i$ is the $i$'th component of the applied field, $\sigma_M$ is the conductivity of the metal, $\sigma_F$ is that conductivity of the fiber (which we take to be negligible), $V$ is the volume of the cell, $\Phi$ is the electrostatic potential on the surface of the j' th fiber, $n_i$ is the $i$'th component of the surface normal, and the integral is over the surface of the j' th fiber. If the applied field is given and we can calculate the potential $\Phi$, then we can calculate the average current density, substitute in the anisotropic form of Ohm's law, which is

$$\langle j_i \rangle = \sum_k \sigma_{ik} E_k,$$  \hspace{1cm} (2)

and solve for the components of the conductivity tensor. The theory of the local conductivity tensor for a cell therefore reduces to solving the Laplace problem for $\Phi$ for an arbitrary distribution of $N$ fibers in the cell.

To solve for the potential we first assume that the fibers form a square array outside the cell, so that Rayleigh's solution applies in that region. We then use the integral form of Laplace's equation and substitute Rayleigh's solution in the integrals over fibers outside the cell, resulting in the following equation for the potential on the surface of the i' th fiber:

$$\phi_i = -E\phi_i + \sum_{j=1}^{N} \int_{S} \left[ \phi_j \frac{dG}{dn} - G \frac{d\phi_j}{dn} \right] dS_j + \sum_{j=1}^{N} \int_{S} \left[ \phi_j \frac{dG}{dn} - G \frac{d\phi_j^R}{dn} \right] dS_j,$$  \hspace{1cm} (3)

where $G = 1/|x_j - \bar{x}_j|$ and $\phi_j^R$ is Rayleigh's potential for a square array of fibers. Next we expand $\phi_i$ in a Fourier series in $\phi_i$, the azimuthal angle about the center of the i' th fiber, and carry out the indicated integrations to obtain a matrix equation for the Fourier coefficients.

Because the fiber arrangement is arbitrary, the coefficients are different for each fiber in the cell, and the order of the matrix is $N$, the number of fibers, times the number of terms in the Fourier sum. The calculation is completed by numerically solving the matrix equation.

We applied this method to a large number of randomly generated fiber arrays with volume fractions ranging from about 0.2 to about 0.5, which is the range of densities illustrated in Fig. 1. To generate these distributions we first placed the fibers in a regular array with the proper spacing between rows and with the spacing within a row determined by the volume fraction. The fibers were then randomly displaced with the constraint that the displacement in the vertical direction be a small fraction of the inter-row spacing. For high volume fractions, this gives us distributions in which the fibers tend to bunch together within rows, as in Fig. 1.
Fig. 1. Computer simulations of fiber distributions: (a) high volume fraction, (b) low volume fraction
Fig. 2 is a plot of the diagonal components of the conductivity tensor, normalized to the conductivity of the matrix, as a function of the volume fraction of fibers in the cell. The coordinate system is such that the x direction is along the fibers, y is across the fibers in a row, and z is across the fibers perpendicular to the rows. Off-diagonal components were also calculated but are not shown because they are smaller by two or three orders of magnitude.

The spread in the data for the transverse components (yy and zz) is caused by different random distributions at a given volume fraction. There is no scatter in the data for the component along the fibers because that component is a function of volume fraction only; it is otherwise independent of fiber arrangement. The straight lines are least squares fits to the data.

One point to notice is that the z component, which is across the rows of fibers, varies more rapidly with volume fraction than the other components. This is a direct result of our constraining fiber motion to be largely within rows, leading to nearly close packed rows at high volume fraction. It is also evident that the transverse conductivities are not equal, that their ratio varies with volume fraction, and that we need an eddy current model for biaxial anisotropy to properly describe the fields in these materials. The development of a biaxial model is described in the next section.

PROBE RESPONSE MODEL

Our model of eddy currents in a biaxial anisotropic medium is based on the recent work of Wait [6]. If we consider the half space \( z < 0 \) with a diagonal conductivity tensor, Wait's analysis shows that the two-dimensional Fourier transform of the electric field has components of the form

![Fig. 2. Calculated conductivity tensor normalized to the matrix conductivity. Points with the same symbol correspond to different fiber arrangements. The straight lines are least squares fits.](image)
\[ e_x(k,z) = ae^{\lambda z} + be^{\chi z}, \]  

where \( a \) and \( b \) are constants to be determined by the boundary conditions at \( z=0 \); similar expressions exist for the components of the transform of \( \vec{H} \). The propagation parameters \( \lambda \) and \( \chi \) are complicated functions of the components of the conductivity tensor and the wave vector \( \vec{k} \) [6]. If two of the conductivity components are equal, the material is uniaxially anisotropic, and Wait shows that the analysis simplifies to a form equivalent to the TE/TM modal theory of Bowler, et al. [2].

We have extended Wait's theory to the solution of the boundary value problem for a half space with an arbitrary current source in the space \( z > 0 \). Substitution in the reciprocity expression [7] for the probe impedance leads to the form

\[
\Delta Z = \frac{2i\omega}{\mu_0 I^2} \int \Gamma(\vec{k}) \vec{z} \cdot \vec{H}_s(\vec{k}) \vec{z} \cdot \vec{H}_s(-\vec{k}) \frac{d^2k}{k},
\]

where \( \vec{H}_s(\vec{k}) \) is the transform of the vector potential associated with the current source in free space, and the reflection function \( \Gamma \) depends on the conductivity tensor, but is independent of \( \vec{H}_s(\vec{k}) \).

MEASUREMENT OF VOLUME FRACTION

The application addressed in this section is the measurement of fiber volume fraction. Given that the arrangement of fibers is unknown, and that the conductivity of the matrix may vary due, for example, to porosity, the question we asked was the following: what are the effects of fiber arrangement and matrix conductivity on an eddy current measurement of volume fraction?

To answer this question we used the two models described above to calculate the probe response as a function of volume fraction for a number of different fiber arrangements and matrix conductivities. All calculations were done for a titanium alloy matrix with nominal conductivity of 1% IACS, for fibers with 150 micron diameter, a nominal volume fraction of 0.34, and a fiber conductivity which is negligible compared to the conductivity of the matrix.

Fig. 3 summarizes the first set of calculations. For this application we used the linear fits to the conductivity shown in Fig. 2 and computed the normalized impedance (the impedance divided by the coil reactance in free space) as a function of volume fraction and frequency for a cylindrical coil with axis normal to the surface of the material. Calculations were also performed for tangent coils but will not be discussed here because the results are qualitatively similar.

As is evident in Fig. 3, the impedance loci for different frequencies as a function of volume fraction all seem to lie on the same curve, which looks very much like the impedance curve for an isotropic conductor. In fact, if we compare the curve in Fig. 3 with calculations for an isotropic conductor as a function of conductivity, the two curves are almost identical. What this tells us is that, in spite of its anisotropy, probe response for the composite is almost exactly the same as that for a conductor with some effective isotropic conductivity. This means that if we simply ignore the fact that we are working with an anisotropic material and measure its effective conductivity in the usual way, the result can be directly related to volume fraction.
Fig. 3. Normalized impedance loci as a function of volume fraction at several frequencies; calculations are based on average conductivities from Fig. 2.

But recall that the calculations shown in Fig. 3 were done using the straight line fits from Fig. 2, and therefore do not account for the fact that different fiber distributions give different conductivities at the same volume fraction. To see what effect this might have on the measurement of volume fraction, we did additional calculations using linear fits through the minimum and maximum conductivities at each volume fraction.

The result is shown in Fig. 4, where we plot volume fraction as a function of the effective isotropic conductivity that we would measure using an eddy current conductivity meter. Variations in conductivity caused by variations in the arrangement of fibers produce some uncertainty in the measurement of volume fraction. These data indicate an uncertainty of about ±15%.

As mentioned earlier, these calculations were done for a circular coil with axis normal to the surface. We can show mathematically that if we were to use a probe with a high degree of anisotropy such that the induced current is strongly unidirectional, and if we orient this probe so that current flows along the fibers, then variations in fiber arrangement will cause no problems. The reason for this is that the conductivity in the fiber direction depends only on the volume fraction and not on the geometrical arrangement of fibers. The message is that with proper attention to probe design, we can safely ignore the effects of fiber arrangement in the measurement of volume fraction.

There is, however, another problem. Suppose the conductivity of the matrix varies due, for example, to variations in porosity. Can we distinguish such effects from variations in volume fraction?
Fig. 4. Volume fraction vs. effective isotropic conductivity normalized to the matrix conductivity. The effective isotropic conductivity is that indicated by an eddy current conductivity meter. Data sets correspond to linear fits through the minimum, average and maximum points of the data in Fig. 2.

The answer is sometimes. In Fig. 5, we show the impedance locus for the composite as a function of volume fraction for the nominal matrix conductivity of 1% IACS. The second curve is the locus as a function of matrix conductivity from 0.5% IACS to 1% at the nominal volume fraction of 0.34. The parts of the curves corresponding to high volume fraction and high matrix conductivity overlap, so if our measurement falls in this region we do not know whether we are seeing a volume fraction or a matrix conductivity effect or some combination of the two. We did similar calculations at other frequencies, for different coil sizes, for tangent probes and for separate exciter/receiver coils to try to find some technique for separating the two effects. The result, however, was always the same - there is always a region of ambiguity where we cannot distinguish volume fraction and matrix conductivity effects.

Although we have not exhausted the possibilities, it seems that these results are telling us that we cannot rely on a single eddy current measurement for an unambiguous determination of volume fraction or matrix conductivity. A single eddy current scan can tell us when something is wrong, but it may not tell us exactly what is wrong. What we need, then, is some type of supplementary test to help determine which effect we are seeing. This is something we plan to look for in the future.

SUMMARY

In summary, then, we have developed two models pertaining to eddy current inspection of metal matrix composites. One model gives us the conductivity tensor for an arbitrary fiber arrangement, and the other uses the conductivity data in the calculation of probe response. The results of numerical studies show that, with proper attention to the design of a probe, we can minimize the effect of fiber arrangement on
a volume fraction measurement. But the calculations also show that we cannot distinguish volume fraction and matrix conductivity effects, at least not with a single measurement. Our principal conclusion is that we need to look for supplementary measurements to help separate the two effects.

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