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Inventory management in a manufacturing/remanufacturing hybrid system with condition monitoring

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1. INTRODUCTION

1.1 Motivation

Traditional supply chains consist of manufacturers, who process, assemble and sell products to customers. Once the product has been sold, the ownership of the product is transferred on to the customer. Typically after a possible warranty period, the repair, maintenance and eventual disposal of the product is then the responsibility of the customer. The reverse processing activities of inspection, parts remanufacturing, and materials recycling can substantially reduce the material and energy consumed by producing goods. Although these activities have a beneficial environmental impact, customers fail to participate in the remanufacturing efforts by producers or third parties because they often lack incentives.

Remanufacturing has received tremendous attention from companies over the last few decades. Although one side of the coin is to extend the life of used products and achieve a sustainable environment, there is an economic aspect to it that is attractive. A lot of companies seem to be making huge profits in the remanufacturing business today. But, one thing that drew so much attention to remanufacturing in the past few decades is the quality of the final product. It can be said that a remanufactured product is ready for a second life, performing as new [16].

To encourage remanufacturing, several environmental and economic thinkers have proposed a concept called "servicizing" [28]. In this paradigm, producers become service providers who provide the use and maintenance of products while retaining ownership; customers become clients who pay fees to receive the benefits the products provide. Instead of extensive buying and disposing of products, servicizing includes the obligation to dispose of used products responsibly, while reusing them and their constituent parts and materials as much as possible. However, because the provider retains responsibility for the product while it is in use by different client firms, the service paradigm also creates the need for better information and communication technology to increase the provider's knowledge of the product condition. Monitoring the condition of the equipment enhances the ability of the service provider to make better replacement decisions (when to replace the product in the fleet to avoid failures) and better inventory management decisions (how much remanufactured stock to maintain so the customer is ensured a working product at all times). This thesis aims at optimizing the replacement and inventory decisions of the service provider in order to minimize the long-run overall cost per unit time.

1.2 Background

Most companies that adopt remanufacturing rely on return of used products from the customers to process them to 'as good as new' condition. These companies could be original equipment manufacturers (OEMs) which adopt various collection techniques to acquire these used products or service providers which retain the ownership of the product throughout its lifecycle and thereby take possession of the product towards the end of its cycle. Providing product-based services, termed as servicizing, is a strategy in which the producers provide the use and maintenance of products while retaining ownership and the prospective customers, or clients, pay the fees to receive the services of products. This strategy minimizes repeatedly buying and disposing of the products. Providing product-based services requires the producer to extend its responsibility for the product both during and after the use phase. For example, heavy equipment manufacturers offer "power by the hour" contracts to major customers and the service contracts frequently include replacement of the initial machine with newer or better ones, and the machines coming off the fleet due to end of lease are remanufactured extensively [1]. Service providers must choose when to take old products out of service, and decide whether to remanufacture them or replace them with newly manufactured products. Decisions regarding maintenance, repair, replacement, and remanufacturing all are complex due to uncertainty in the product's condition. This uncertainty is especially significant to a servicizer, who must ensure that its equipment remains in proper working order to provide continuous service to geographically dispersed client firms.

Servicizing motivates the use of condition monitoring. Condition monitoring is the process of monitoring a parameter of condition in machinery, such that a significant change is indicative of a developing failure. Companies use sensors, information and communication technology to increase visibility of the product's condition and environment while in use. For example, the large earth-moving equipment and mining equipment produced by Caterpillar, Inc., frequently is equipped with remote monitoring devices along with communication equipment that transmits the data to a server. Under a service agreement, software algorithms determine when to perform service on a particular machine based on remotely-monitored fuel burn and load cycles. Condition monitoring helps make better decisions on the maintenance of the product and also helps determine the 'remanufacturability' of the return. The remanufacturing leadtimes and costs of the products depend strongly on their condition. Condition monitoring helps reduce the number of failures and replace the product while it is still in a 'remanufacturable' state.

Remanufacturing facilities operate together with a manufacturing plant in satisfying the demand. These types of systems are known as *hybrid manufacturing and remanufacturing systems*. Remanufacturing involves a reverse flow of products which makes the inventory management in hybrid systems quite complex. In most cases, remanufacturing is less costly than manufacturing a new product. However, the entire market demand cannot be satisfied by remanufacturing since the return rate is lower than the demand rate due to possible failures.

Therefore, it is crucial to have a good coordination between the manufacturing and remanufacturing processes. It is highly challenging due to the uncertainty in the quantity, quality and also the timing of the returns. Many techniques have been proposed to influence the quantity and timing of the returns but the variability in the returned product quality remains an issue. This is because the remanufacturing effort required to make the product 'as good as new' depends on the condition (quality) of the product. It is highly unlikely that all returned products are in the same working condition and require the same remanufacturing effort to bring them back to 'as good as new' condition. Thus it is very important that we include in our model, the differences in the cost and leadtime of the remanufacturing process based on the condition of the return.

1.3 Objective

In this thesis, we consider a fleet of products that is condition monitored at frequent intervals and the replacement decisions (whether to replace the product or wait) are made by the service-provider based on the condition information. These decisions are made at instants called decision epochs. For each replacement, there is a demand for a new product to fill in the gap in the fleet. This demand can be fulfilled by remanufacturing a returned product or manufacturing a new one. An ideal model would contain a policy that can both make optimal replacement decisions and maintain optimal inventory level for the system. This thesis considers these policies and decisions separately. The replacement decisions are made based on a replacement policy that has been proven optimal in [5]. We show the procedure to calculate the replacement and failure rates of the system with condition monitoring and illustrate with a numerical example. The focus of the study is then directed to the management of inventory. When a product in the fleet is replaced with a new one, it is sent back into the loop to be remanufactured. The product returns are of varying quality (condition). And, when a customer returns a product, it is required to be replaced with an 'as good as new' one immediately. Thus we try to analyze the inventory level of the remanufactured products in order to ensure that the customer has a working product available at all times. We follow a continuous-review base stock policy to maintain an optimal stock level for the serviceable inventory [14]. The primary aim is to minimize the long-run average cost per unit time. We illustrate with an example to study the effect of the serviceable inventory level and the remanufacturable disposal levels on the cost function.

1.4 Thesis Organization

The rest of the thesis is organized as follows. Chapter 2 presents an overview of the most relevant academic literature. Chapter 3 presents our model with details of all the notations. Chapter 4 describes in detail the procedure adopted to calculate the replacement rates for each state. And Chapter 5 introduces the inventory management policy and describes the procedure adopted to obtain the optimal base stock level of the serviceable inventory along with a numerical example. This chapter also discusses the results obtained under different assumptions about inventory holding costs, in detail. This is followed by Chapter 6 which concludes this study with a few remarks and scope for further research.

2. LITERATURE REVIEW

The importance of remanufacturing and its environmental relevance has been stressed greatly in recent literature. The increasing environmental concern has motivated many firms to emphasize waste reduction and make remanufacturing an integral part of their marketing strategy. Rather than selling the products and allowing them to be discarded, many producers provide the use and maintenance of products while retaining ownership. Providing these product-based services is called *servicizing*. This strategy has gained attention for its expected economic and environmental benefits and also encouraged research in many areas. However, because the provider retains responsibility for the product while it is in use by different client firms, the service paradigm also creates the need for better information and communication technology (ICT) to increase the provider's knowledge of the product condition. This encourages the use of condition monitoring equipment to monitor the product at frequent intervals. By doing this, the producer is able to make better decisions on when to replace the machine in the fleet with a new one and thereby have product returns with better 'remanufacturability'. Most remanufacturing processes work together with a manufacturing unit to satisfy customer demand. Some literature about such manufacturing/remanufacturing hybrid systems is also discussed in this section.

Recent literature has examined servicizing with remanufacturing from both the environmental and the economic points of view. Sundin et al. [7] focused on selling services or functions instead of physical products. They asserted that this practice, combined with remanufacturing, could be a way of closing material flows in present society. Their analysis showed that it is preferable that products aimed for service-selling be designed to be remanufactured. Cooper [8] also focused on the sustainability aspect of product life. He developed a theoretical model to demonstrate how, by contributing to efficiency and sufficiency, longer product life spans may secure progress toward sustainable consumption. Sundin and Bras [9] also discussed the economic and environmental benefits of functional sales, their term for servicizing, used in connection with product remanufacturing. Their research elucidated these benefits and argued for why products to be used for functional sales should be remanufactured.

Most literature indicates that greater knowledge about the outcome of the remanufacturing process facilitates the integration of inventory from production and replenishment. Ferrer [2] discussed the decision making process based on yield information. He compared the scenarios where the decision maker chooses from making an early inspection (to avoid dependence on condition monitoring and be able to make decisions earlier by not having to look for a responsive supplier), responsive supplier (in case of last minute orders due to insufficient inventory in the remanufacturing unit) and condition monitoring (which would give precise information of the outcome of remanufacturing). His results suggested that condition monitoring would be useful in the case of high yield variance and high repair and inventory costs. Ferrer and Ketzenberg [3] also examined procurement decisions subject to yield information and supplier lead time. They developed four decision models to evaluate the impact of these factors and indicated that for products with few parts, better yield information is quite valuable whereas increasing supplier responsiveness provides trivial returns. Ryan, Padakala & Wu [4] have assessed the value of condition monitoring in replacement under the proportional hazards model for product life. They showed how frequent monitoring of the product decreases the cost of the optimal replacement policy, and this cost decrease justifies the investment in information and communication technology required for frequent monitoring.

Condition information allows better decisions concerning the future of the product – whether to replace it immediately or continue to use it. There has been plentiful research on optimal repair and replacement policies for deteriorating systems including many studies that stress condition monitoring. Earlier research revolved around optimal replacement and inspection policies to maintain a system subject to deterioration. Ohnishi et al. [17] and Luss [18] studied a Markovian deteriorating system and derived an optimal policy for the system to minimize the total long-run average cost. The condition information of the system was assumed known only through inspections. The optimal policy included scheduling of future inspections and making replacement decisions based on the condition information that minimizes the total cost. These are situations where the inspection process yields the precise condition information. Research has been done on systems in which the condition information available is only probabilistically related to the actual internal state of the system. Smallwood and Sondik [19], Satia and Lave [20] and Ellis et al. [21] studied the optimal maintenance policies for such systems. The optimal policy describes when to inspect and when to repair so as to minimize the long-run cost. The system to be controlled is characterized as a Markov decision process. There are numerous research papers on optimal condition-based maintenance policies with partial or imperfect condition information [24-27]. Research has further been extended to systems with fixed inspection intervals to monitor the condition. Barbera et al. [22] discuss a condition based maintenance model with exponential failures, and fixed inspection intervals. The condition of the equipment is monitored at equidistant time intervals and if the variable indicating the condition is above a threshold an instantaneous maintenance action is performed. Chiang and Yuan [23] propose a state-dependent maintenance policy for a multi-state continuous-time Markovian deteriorating system. Under the maintenance policy, the system is inspected at each period to identify the current state and then the action is chosen: do-nothing, repair and replacement. In this study, we adopt the replacement policy derived by Makis and Jardine [5]. They examine a replacement problem for a system subject to stochastic deterioration. They describe a case where upon failure of the system, it is replaced by a new one and a failure cost is incurred. And if the system is replaced before failure a smaller cost is incurred. They use Cox's proportional hazards model to describe the failure rate of the system and specify an optimal replacement policy which minimizes the long-run expected average cost per unit time. This replacement policy, proven optimal by Makis and Jardine, is applied to our model to maintain the fleet of servicized products. Following the same replacement policy, Wu and

Ryan [6] have further extended the computations for the optimal replacement rates to an arbitrary number of states.

As much as companies are trying to extensively perform remanufacturing, it cannot satisfy the entire demand. Hence, remanufacturing mostly operates together with the manufacturing process in order to fill in the gaps. Some recent literature focuses on such manufacturing/remanufacturing hybrid systems and highlights the complexities involved in satisfying the demand, primarily focusing on the inventory management. Van der Laan et al. [10] focus on the production planning and inventory control in hybrid systems where manufacturing and remanufacturing operations occur simultaneously. They compare the traditional systems without remanufacturing to PUSH and to PULL controlled systems with remanufacturing, and derive managerial insights into the inventory related effects of remanufacturing. Ying and Zu-Jun [11] propose two models to minimize the total cost per time unit for ordering remanufacturing and manufacturing lots and holding returned and new/remanufactured items in stock. They determine the optimal lot sizes for the manufacturing of new items and the remanufacturing of returned items in a hybrid manufacturing/remanufacturing system. Kiesmuller [12] derives a new approach for controlling a hybrid stochastic manufacturing/remanufacturing system with inventories and different leadtimes.

The study of hybrid systems has drawn attention due to the complexities involved in the coordination between the remanufacturing and manufacturing process. More importantly, the management of inventory and the coordination of production decisions have been heavily studied and discussed over the past few years. Liu et al. [13] deal with the inventory control problem for the hybrid production system. In their model, the global serviceable inventory is managed by the (s, S) continuous review replenishment policy. The changes of inventory state under the stochastic demand and product returns are illustrated with the Markov quasi-birth-death (QBD) processes and the hybrid inventory system is formulated as a Markov decision model. Aras et al. [14] also follow a continuous review base stock policy in managing the serviceable inventory in their model. They focus their study on the stochastic nature of product returns and in particular, the variability in the condition of the returns. They present an approach for assessing the impact of quality-based categorization of returned products and the incorporation of returned product quality in the remanufacturing and disposal decisions. They also show that prioritizing higher quality returns in remanufacturing is, in general, a better strategy and we follow this strategy in our model. However, since the fleet in our system is condition monitored, the inspection stage is eliminated.

To summarize, this thesis studies the inventory management in a manufacturing/remanufacturing hybrid system. There is a fleet of servicized products maintained by the replacement policy proved optimal by Makis and Jardine (1992). And, the inventory is managed by a continuous review base stock policy as described in Aras et al. (2004). A detailed description of the model and the assumptions are given in the next chapter.

3. MODEL INTRODUCTION AND NOTATION

3.1 Model description

In this thesis, we consider a fleet of products in service. The objective is to ensure that each client has a working product available at all times. The products are monitored fully at discrete intervals and the condition information is used in making replacement decisions. It is assumed that the life of the product follows the Proportional Hazards Model [15].

A proportional hazards model consists of two parts: the underlying hazard function, describing how hazard changes over time and the effect parameters, describing how hazard relates to other factors. In the proportional hazards model, it is assumed that effect parameters multiply hazard and it is possible to estimate the effect parameters without any consideration of the hazard function. We use the proportional hazards model to describe the failure rate of the system.

In our model, the failure rate of the system depends both on the age of the system and on the values of a diagnostic stochastic process Z. It is assumed that the system deteriorates continuously over time and there is a positive probability of failure at every instant. The transition times and thus the replacement decisions are dependent on the present state and action taken. The state of the system is defined by the value of Z; we confine our attention to a two-state case where Z can assume the values $\{0, 1\}$. We assume that the values of the process Z are available only at discrete time points called decision epochs and all replacement decisions are made only at these points. The product can be preventively replaced only at these decision instants. However, in case of failure, the replacement is made immediately. All failed products are discarded. When a product is taken out of service due to replacement or failure, it is replaced immediately with a new or remanufactured product. The condition of the product is monitored and the condition monitoring equipment gives us the value of Z at discrete intervals. All replacement decisions are made based on this condition information at each decision epoch.

We study a way to efficiently maintain the stock of 'new' products in order to ensure the client has a working product available at all times. The stock of serviceable products is managed by a simple continuous review base stock policy. This policy aims at keeping the inventory position at the base stock level x. This inventory position would include the serviceable inventory of remanufactured products, the Work-in-progress inventory (products being remanufactured) and all the outstanding manufacturing orders. And each time a demand is served from the serviceable inventory, a returned product is pulled into the remanufacturing process. The serviceable inventory is replenished by either remanufacturing the replaced product or manufacturing a new one.

The system considered is a joint manufacturing and remanufacturing system with three inventories. The remanufactured products are stored in the serviceable inventory, and the preventively replaced products are stored in one of the two remanufacturable inventories based on their state. The preventively replaced products are categorized into two groups based on the remanufacturing effort needed to bring them back to 'as good as new' condition. The products requiring less remanufacturing effort are grouped as Type 0 products and the remaining lower quality products are categorized as Type 1. The flow of products through the system is shown in Figure 3.1. Since the fleet of products is monitored from time to time, the condition of the product is known at the time of replacement. Therefore the product is readily categorized without the need of any further inspection and sent to either the Type 0 inventory or the Type 1 inventory based on its quality (or condition).

The priority is to satisfy the customer demand. This is done from the stock of remanufactured products first and manufacturing is considered viable only when the serviceable inventory is unable to satisfy the demand. The serviceable inventory is managed by a simple continuous review base stock policy which aims at maintaining the inventory position at least at a base stock level x at all times. Whenever the inventory position drops below x, i.e., when a demand is served from the serviceable inventory, preventively replaced products are pulled into the remanufacturing process for remanufacturing.

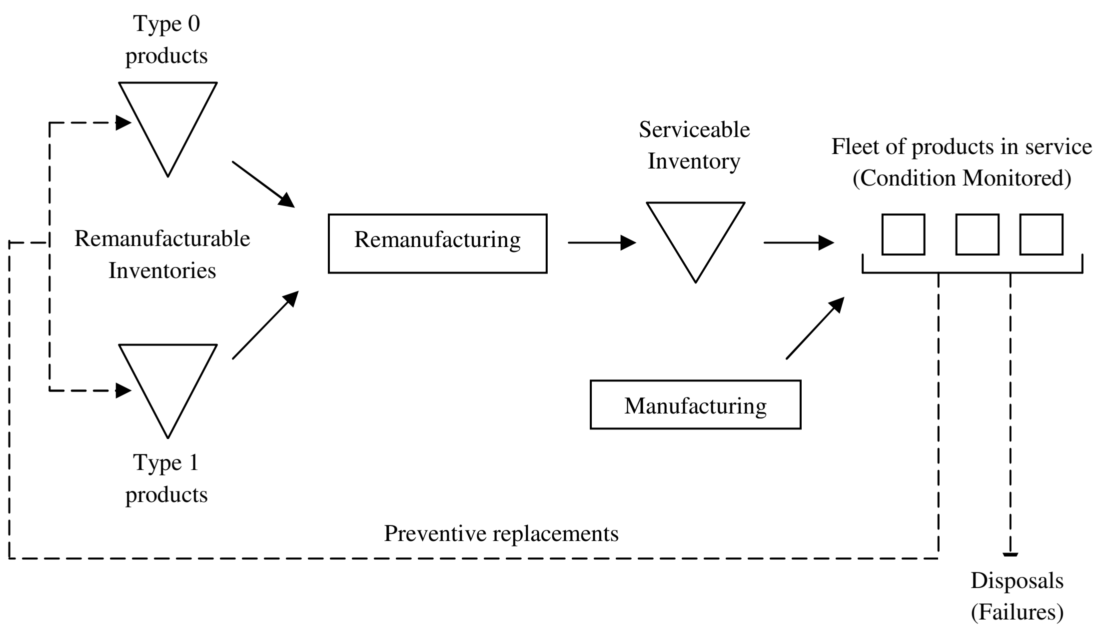

Figure 3.1: The manufacturing/remanufacturing hybrid system with categorized inventories

The serviceable inventory position is always maintained at a base stock level x and the disposal decisions of Type 0 and Type 1 returns are controlled by disposal levels Q_0 and Q_1, respectively. The disposal levels determine the maximum number of items that can be held at the storage facilities prior to remanufacturing. Any product returned after the associated remanufacturable inventory reaches its disposal level is discarded. To focus on optimizing x, we try different ways to reduce the effect of these disposal levels in our model (Chapter 5).

Since there are two remanufacturable inventories, when there is a need for a product to be pulled into the remanufacturing process, there is some flexibility. That is, the priority can be given to either Type 0 products or Type 1 products to be remanufactured. Aras et al. describe the Type 0-first strategy as pulling the Type 0 products first whenever a demand occurs. Only when the Type 0 inventory is zero, we begin to pull Type 1 products into the remanufacturing process. The Type 1-first strategy is a mirror image of the Type 0-first in terms of its priorities. Aras et al. [14] proved that the Type 0-first strategy is the better of the two strategies. Under Type 0-first strategy, more of high quality returns are remanufactured, which results in significant reductions in remanufacturing cost as well as WIP and serviceable inventory holding cost and further more the shorter leadtime enables faster satisfaction of customer demand and eventual reduction in the manufacturing costs. Therefore we adopt this strategy in this study. It is also possible that both Type 0 and Type 1 inventories are empty which means that from that point on, every demand would result in an *outstanding remanufacturing order.* All future replacements are then processed immediately as they arrive, irrespective of their type.

Also, the remanufacturing leadtimes for Type 0 and Type 1 products differ because the remanufacturing effort required for the two product types varies. Type 0 products require less remanufacturing effort to bring them to the "as good as new" condition. This means, (i) Type 0 average remanufacturing leadtime is shorter; (ii) Type 0 unit remanufacturing cost is lower; and (iii) Type 0 unit disposal cost is equal to or lower than that of Type 1. The customer demand that cannot be satisfied immediately from serviceable inventory is satisfied from the manufacturing plant. Also, the inventories have holding costs based on their value, i.e., the holding cost for the products that are yet to be remanufactured is less than the holding costs of the remanufactured products.

To summarize, the model in this thesis uses two policies – replacement policy and inventory policy. The optimal replacement policy minimizes the overall cost per unit time and formulation by Makis and Jardine enables us to determine the overall replacement rate and failure rate and also the preventive replacement rates for each state of the system. The inventory model uses the replacement and failure information to optimize the base stock level of the serviceable inventory in order to minimize the long-run average cost per unit time.

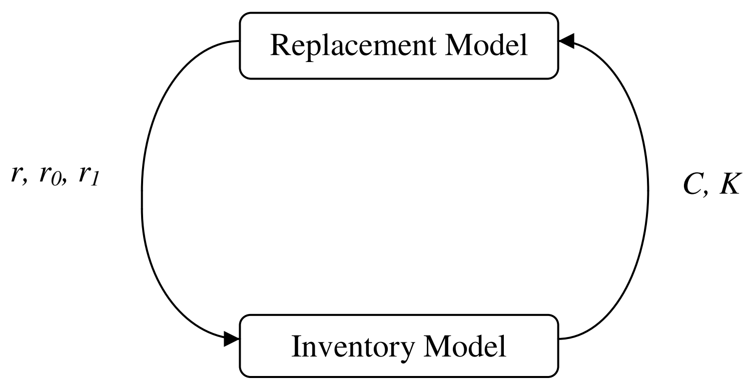

Figure 3.2: Relationship between the replacement model and the inventory model

The Figure 3.2 shows the decisions from each model used in the other. The overall replacement rate (r) and the preventive replacement rates (r_0, r_1) are calculated from the formulation used in the replacement model. And, these rates enable the inventory decisions. In turn, the remanufacturing (replacement) cost (C) and the additional cost to manufacture a new product (K) are used from the inventory model to make replacement decisions. There is a mutual coordination between the two models which helps make optimal decisions for the system.

3.2 Assumptions

We assume the service provider has a fleet of products currently in service by clients and an inventory of serviceable products to be managed in order to satisfy the demand. In addition, we assume:

(i) The service provider is responsible for ensuring that the client always has a product available in working order.

(ii) The time to failure of the product follows the proportional hazards model; the product can be preventively replaced only at an observation epoch but must be replaced immediately if it fails between observation epochs. Replacement is instantaneous.

(iii) The continuous time Markov chain *Z,* which describes a product's condition, is a pure birth process, i.e., whenever a transition occurs, the state of the system always increases by one. If the system has n states, the state $n\text{-}1$ is absorbing. This thesis considers a two-state system.

(iv) When a product is replaced, priority is given to satisfying the client's demand with remanufacturing; manufacturing would be viable only when remanufacturing is not possible. This is based on the conventional wisdom that remanufacturing is cheaper than manufacturing.

(v) Manufacturing is instantaneous. This thesis does not consider the leadtime associated with the manufacturing process. Instead, an aggregate manufacturing cost m is assumed to be incurred that includes all the material, production, inventory and possible back-ordering (or lost sales) costs.

(vi) All preventive replacements are categorized and stored in one of the two remanufacturable inventories based on their state (quality); Type 0 represents higher quality products (or products that require less remanufacturing effort in terms of leadtime and cost) and Type 1 represents the remaining lower quality products. We give priority to Type 0 products first whenever there is a demand.

3.3 Notation

We assume the following notation:

Input parameters

$h_0(\cdot)$	: Baseline failure rate
$\psi(\cdot)$	: A positive function dependent only on the state of the system
Δ	: Monitoring interval
N	: Number of products in the fleet
C	: Average cost of remanufacturing a preventively replaced product
K	: Additional cost to manufacture a new product
μ_0, μ_1	: Remanufacturing processing rates
δ_0, δ_1	: Unit disposal costs
h	: Remanufacturable inventory holding cost for Type 0 and Type 1 products
h_0^s, h_1^s	: Unit serviceable inventory holding costs
h_0^w, h_1^w	: Unit WIP holding costs
m	: Average manufacturing cost

α : Opportunity cost of capital

Intermediate parameters

The following parameters play an intermediate role in obtaining the output.

z_k : Condition of the system at time point $k\Delta$ after the last replacement

T : Time to failure of the product

$\bar{R}(k, Z_k, t)$: Survivor function given the age $k\Delta$ and condition Z_k

g : Optimal expected average cost of the replacement policy per unit time

T_d : Replacement time associated with expected average cost d

$k_i\Delta$: Replacement time associated with specific condition i

$W(j,i)$: Expected residual time to replacement given the age $j\Delta$ and $Z_j = i$

$Q(j,i)$: Expected residual time to failure given the age $j\Delta$ and $Z_j = i$

r : Overall replacement rate

r_f : Overall failure rate

r_p : Overall rate of preventive replacements

r_0, r_1 : Replacement rates for products in state 0 and state 1 respectively

$M(j,0)$: Probability of preventive replacement of the system in state 0 and age $j\Delta$

$N(j,1)$: Probability of preventive replacement of the system in state 1 and age $j\Delta$

$\bar{I}_0, \bar{I}_1$: Average remanufacturable inventory on-hand per unit time

$\bar{I}_s$: Average serviceable inventory on-hand per unit time

$\bar{W}_0, \bar{W}_1$: Average WIP inventory per unit time for Type 0 and Type 1 products

$\bar{R}_0, \bar{R}_1$: Average number of returned products remanufactured per unit time

$\bar{D}_0, \bar{D}_1$: Average number of returned products disposed per unit time

$\bar{M}$: Average number of products manufactured per unit time

Output Parameters

x : Base stock level of serviceable inventory

Q_0, Q_1 : Disposal levels of remanufacturable inventories

$\bar{C}(x, Q_0, Q_1)$: Long-run average cost per unit time for the inventory policy

The next chapter explains in detail the proportional hazards model and the procedure adopted to solve for the replacement rates and failure rates of the system. Chapter 5 explains in detail the inventory model and how we derive at the optimal base stock level for the serviceable inventory.

4. OPTIMAL REPLACEMENT POLICY

4.1 Summary

This chapter describes the proportional hazards model and the procedure adopted to calculate the rates of replacement and failure. We consider a system which deteriorates continuously over time and is subject to failure at any instant. A fleet of products is considered to be in service by a service-provider and the product lifetime is assumed to follow the proportional hazards model. The failure rate depends on the age of the system and also on the values of concomitant variables describing the effect of the environment in which it operates. The condition of the product is observed fully and thus the values of the concomitant variables are known at discrete time points where the decisions are made. Based on the condition information, the product can be preventively replaced only at these decision instants. However, in case of failure, the replacement is made immediately. We assume the service provider follows a replacement policy that has been proven optimal for the proportional hazards model. We describe the method to calculate the overall replacement and failure rates as well as the preventive replacement rates for each state.

4.2 Proportional Hazards Model

Let $Z = \{Z_t, t \geq 0\}$ be a stochastic (diagnostic) process that reflects the effect of the operating environment on the system and thus influences the time to failure of the product. In the proportional hazards model, it is assumed that the failure rate of a system is the product of a baseline failure rate $h_0(\cdot)$ dependent on the age of the system and a positive function $\psi(\cdot)$ dependent only on the value of the concomitant variables (i.e., the product condition).

Thus the hazard rate at time t can be expressed as

$$h(t, Z_t) = h_0(t)\psi(Z_t) \text{ for } t \geq 0$$

and the survivor function is given by

$$P(T > t \mid Z_s, 0 \leq s \leq t) = \exp\left(-\int_0^t h_0(s)\psi(Z_s)ds\right), t \geq 0.$$

As in [5], it is assumed that condition information is available only at time points $0, \Delta, 2\Delta, \ldots$ in a given replacement cycle, and we let Z_j be the condition at time point $j\Delta$ after the last replacement. Although condition information is available only at integer multiples of Δ, Z_t may shift among its discrete values at any time. For simplicity, it is assumed that Z_t is a two-state continuous time Markov chain that starts after each replacement in state 0 and moves to absorbing state 1 in amount of time that is exponentially distributed with rate q. Wu and Ryan [6] show how to extend these computations to an arbitrary number of states.

Then, for $t \in [0, \Delta]$, we have the survivor function

$$P(T > j\Delta + t \mid T > j\Delta, Z_1, \ldots, Z_j) = \exp\left(-\int_{j\Delta}^{j\Delta+t} h_0(s)\psi(Z_s)ds\right) \equiv \bar{R}(j, Z_j, t).$$

For each value of Z, we can specify the survivor function as

$$\begin{aligned}
\bar{R}(j,0,t) &= \exp\left(-\int_{j\Delta}^{j\Delta+t} \psi(Z_s)h_0(s)ds\right) \\
&= \int_0^t \exp\left(-\int_{j\Delta}^{j\Delta+s} \psi(0)h_0(u)du - \int_{j\Delta+s}^{j\Delta+t} \psi(1)h_0(u)du - v_0 s\right) v_0 ds \\
&\quad + e^{-v_0 t} \exp\left(-\int_{j\Delta}^{j\Delta+t} \psi(0)h_0(u)du\right)
\end{aligned}$$

$$\bar{R}(j,1,t) = \exp\left(-\psi(1)\int_{j\Delta}^{j\Delta+t} h_0(s)ds\right). \qquad \ldots(4.1)$$

4.3 Optimal Replacement Policy

When a product is taken out of service, it is replaced immediately from the stock. The stock is replenished by either remanufacturing the replaced product or producing a new one. Thus, we consider C to include the cost of remanufacturing a product that has been replaced before failure and K to include the additional cost to manufacture a new one. The purpose of monitoring is to reduce the failure rate and in turn reduce the average cost per unit time. The expected rate of preventive replacements in a system with condition monitoring would be higher than in the system with age-based replacements. Also, the failure rate is comparatively lower in a system with condition monitoring.

The objective is to calculate the replacement rates for each value of Z using the replacement policy by Makis and Jardine [5]. The value of Z is available only at observation epochs and at each of these decision instants, there are two possible actions – replacement or non-replacement. A state is defined as (j, z), where j is the number of monitoring intervals since the last replacement and $z = Z_j$ is the condition of the product of age $j\Delta$. Decision 0 denotes immediate replacement, and decision $+\infty$ corresponds to non-replacement (i.e., wait and see). Makis and Jardine (1992) provide the details of computing the optimal policy for this system. They show that the optimal replacement policy δ is a non-increasing function of state and is given by

$$\delta(j,z) = \begin{cases} +\infty \text{ if } K\left[1-\overline{R}(j,z,\Delta)\right] < g\int_0^\Delta \overline{R}(j,z,t)dt \\ 0 \text{ otherwise.} \end{cases} \quad \ldots(4.2)$$

Here, the computation of the optimal policy parameters for this system is shown. If the value of g were known and no failure would occur, then the optimal replacement time for a specific condition z would be $j\Delta$, where j is the minimum integer that satisfies the inequality:

$$K\left[1-\bar{R}(j,z,\Delta)\right] \geq g\int_0^{\Delta} \bar{R}\left(j,z,t\right) dt \qquad \ldots(4.3)$$

According to Makis and Jardine (1992), g can be found as a fixed point.

We can define

$$\phi(d) = \left[C + KP\left(T_d \geq T\right)\right] \big/ E\left[\min\left\{T, T_d\right\}\right] \qquad \ldots(4.4)$$

For any $x_0 \geq 0$, let $x_n = \phi\left(x_{n-1}\right)$, for n = 1, 2 … . Then, $\lim_{n\to+\infty} x_n = g$.

4.3.1 Fixed Point Algorithm

From [4], the fixed-point algorithm is described as below

Step 1: Initialize $n = 0$ and $g = g_0$ with an arbitrary positive value.

Step 2: For $d = g_n$, use (4.3) to find the replacement time $j_i\Delta$ associated with the specific condition i , i.e.,

$$j_i = \min\left\{ j \geq 0 : K\left[1-\bar{R}(j,i,\Delta)\right] \geq d\int_0^{\Delta} \bar{R}(j,i,t)dt \right\}, i = 0,1$$

Step 3: Use the replacement policy obtained in Step 2 and Equation (4.3) to calculate $g_{n+1} = \phi(g_n)$.

Step 4: If $g_{n+1} = g_n$, stop with $g^* = g_n$; otherwise, set $n \leftarrow n+1$ and go to Step 2.

In order to apply the algorithm, it is necessary to compute $P\left(T_d \geq T\right)$ and $E\left[\min\left\{T, T_d\right\}\right]$.

For $j \geq 0$ and $i \in S$,

$$W(j,i) = E\left[\min\{T, T_d\} - j\Delta \mid (j,i)\right]$$

which is the expected residual time to replacement given that the age of the system is $j\Delta$ and $Z_j = i$. Then,

$$W(0,0) = E\left[\min\{T, T_d\}\right] = 1/\lambda_r'(\Delta) \qquad \ldots(4.5)$$

where $\lambda_r'(\Delta)$ is the preventive replacement rate based on the monitoring interval Δ.

Similarly,

$$Q(j,i) = P\left(T_d \geq T \mid (j,i)\right)$$

given that the age of the system is $j\Delta$ and $Z_j = i$.

Then,

$$Q(0,0) = P\left(T_d \geq T\right). \qquad \ldots(4.6)$$

For cases where Z does not change state after some time *v*, the density of the time of failure, *T*, is defined as

$$f_i(v,t) = \frac{d}{dt}\left[-\exp\left(-\psi(i)\int_v^t h_0(u)\,du\right)\right], \; i = 0,1$$

given $Z_t = i$ for all $t \geq v$.

The procedure to obtain $W(0,0)$ and $Q(0,0)$ recursively [4] is given in Section 4.3.2.

4.3.2 Procedure to compute the expected time to replacement

Let X_j be the time spent by the process in state *j* before transiting into state $j+1$. Then, X_j is exponentially distributed with rate, say, v_j.

The residual time to replacement given the current state of the system as zero,

$$W\left(j,0\right)=E\left[\min\left\{T,T_d\right\}-j\Delta\middle|\left(j,0\right)\right].$$

The survivor function of T conditioned on $j\Delta$ and Z_k is

$$P(T>j\Delta+t\mid T>j\Delta,Z_1,\ldots,Z_j)=\exp\left(-\int_{j\Delta}^{j\Delta+t}h_0(s)\psi(Z_s)ds\right).$$

Let $T_R=T-j\Delta$. Then for a given *t*, the survival probability is

$$P(T_R>t\mid j\Delta,Z_j)=P(T>t+j\Delta\mid j\Delta,Z_j)=\exp\left(-\int_{j\Delta}^{j\Delta+t}h_0(s)\psi(Z_s)ds\right).$$

The residual time to replacement varies depending on the current state of the system and also on the amount of time the system spends in a particular state. Let the current state of the system be zero. Then, X_o is the time spent in state zero before transition to state 1.

A. If $X_0=s<\Delta$, that is if the system changes its state before the next decision epoch,

$$W\left(j,0\right)=\begin{cases}T_R \text{ if } T_R\leq\Delta\\ \Delta+W\left(j+1,1\right) \text{ if } T_R>\Delta\end{cases}$$

(i) $0<t\leq s$

$$F_0(j,t)=P(T_R\leq t)=1-\exp\left(-\psi\left(0\right)\int_{j\Delta}^{j\Delta+t}h_0\left(u\right)du\right)$$

(ii) $s < t \le \Delta$

$$F_1(j,t) = P(T_R \le t) = 1 - \exp\left(-\psi(0)\int_{j\Delta}^{j\Delta+s} h_0(u)\,du - \psi(1)\int_{j\Delta+s}^{j\Delta+t} h_0(u)\,du\right)$$

(iii) $t > \Delta$

$$F_2(j,t) = P(T_R \le t) = F_1(j,t)$$

Putting the above equations together,

$$\left(W(j,0) \mid X_0 = s < \Delta\right) = \int_0^s t d\left(F_0(j,t)\right) + \int_s^\Delta t d\left(F_1(j,t)\right) + (\Delta + W(j+1,1))(1 - F_1(j,\Delta))$$

B. If $X_0 = s \ge \Delta$, that is, if the system does not change its state for the entire monitoring interval

$$W(j,0) = \begin{cases} T_R \text{ if } T_R \le \Delta \\ \Delta + W(j+1,0) \text{ if } T_R > \Delta \end{cases}$$

Therefore,

$$\left(W(j,0) \mid X_0 = s \ge \Delta\right) = \int_0^\Delta t d\left(F_0(j,t)\right) + (\Delta + W(j+1,0))(1 - F_0(j,\Delta))$$

Combining A and B, we have

$$\begin{aligned} W(j,0) &= \int_0^\Delta v_0 e^{-v_0 s}\left(\int_0^s t d\left(F_0(j,t)\right) + \int_s^\Delta t d\left(F_1(j,t)\right) + (\Delta + W(j+1,1))(1 - F_1(j,\Delta))\right) ds \\ &\quad + \int_\Delta^\infty v_0 e^{-v_0 s}\left(\int_0^\Delta t d\left(F_0(j,t)\right) + (\Delta + W(j+1,0))(1 - F_0(j,\Delta))\right) ds \\ &= \int_0^\Delta v_0 e^{-v_0 s}\left(\int_0^s t d\left(F_0(j,t)\right) + \int_s^\Delta t d\left(F_1(j,t)\right) + (\Delta + W(j+1,1))(1 - F_1(j,\Delta))\right) ds \\ &\quad + e^{-v_0 \Delta}\left(\int_0^\Delta t d\left(F_0(j,t)\right) + (\Delta + W(j+1,0))(1 - F_0(j,\Delta))\right) \end{aligned} \quad \ldots(4.7)$$

For example, let us assume $h_0(u) = 2u$, $\psi(z) = \exp(z)$, $\Delta = 1$.

Then it follows

$$F_0(0,t) = 1 - \exp\left(-\exp(0)\int_{j\Delta}^{j\Delta+t}(2u)\,du\right) = 1 - \exp(-t^2 - 2tj\Delta)$$

$$\frac{\partial}{\partial t}\left(F_0(j,t)\right) = f_0(j,t) = 2(t + j\Delta)\exp(-t^2 - 2tj\Delta)$$

$$\begin{aligned} F_1(j,t) &= 1 - \exp\left(-\psi(0)\int_{j\Delta}^{j\Delta+s} h_0(u)\,du - \psi(1)\int_{j\Delta+s}^{j\Delta+t} h_0(u)\,du\right) \\ &= 1 - \exp\left(-e^1 t^2 - 2e^1 tj\Delta + (e^1 - 1)(s^2 + 2sj\Delta)\right) \end{aligned}$$

$$\frac{\partial}{\partial t}\left(F_1(j,t)\right) = f_1(j,t) = 2e^1(t + j\Delta)\exp\left(-e^1 t^2 - 2e^1 tj\Delta + (e^1 - 1)(s^2 + 2sj\Delta)\right)$$

…(4.8)

From (4.7),

$$\begin{aligned} W(j,0) = \int_0^{\Delta} v_0 e^{-v_0 s}\left(\int_0^s tf_0\,dt + \int_s^{\Delta} tf_1\,dt + (\Delta + W(j+1,1))(1 - F_1(j,\Delta))\right)ds \\ + e^{-v_0\Delta}\left(\int_0^{\Delta} t f_0\,dt + (\Delta + W(j+1,0)) * (1 - F_0(j,\Delta))\right) \end{aligned}$$

…(4.9)

where

$$1 - F_0(j,\Delta) = \int_{\Delta}^{\infty} f_0(j,t)\,dt\ ,\ 1 - F_1(j,\Delta) = \int_{\Delta}^{\infty} f_1(j,t)\,dt$$

When the system is in state 1,

$$W(j,1) = \begin{cases} T_R & \text{if } T_R \le \Delta \\ \Delta + W(j+1,1) & \text{if } T_R > \Delta \end{cases}$$

$$F_1^1(j,t) = P(T_R \le t) = 1 - \exp\left(-\psi(1)\int_{j\Delta}^{j\Delta+t} h_0(u)\,du\right)$$

$$\frac{\partial}{\partial t}\left(F_1^1(j,t)\right)=f_1^1(j,t)=2a(t+j\Delta)\exp(-a(t^2+2tj\Delta))$$

$$W(j,1)=\int_0^\Delta tf_1^1(j,t)dt+(\Delta+W(j+1,1))(1-F_1^1(j,\Delta) \qquad \ldots(4.10)$$

4.3.3 Procedure to compute the probability that replacement is due to failure

Let the current state of the system be zero. Then the residual time to failure, given the age $j\Delta$ and state zero can be defined as

$$Q(j,0)=P\left(T\le T_d \middle| (j,0)\right)$$

Let $T_R = T - j\Delta$.

1. If $T_0 = s < \Delta$, then

$$Q(j,0)=\begin{cases}1 & if\ T_R\le\Delta\\ Q(j+1,1) & if\ T_R>\Delta\end{cases}$$

2. If $T_0 = s > \Delta$, then

$$Q(j,0)=\begin{cases}1 & if\ T_R\le\Delta\\ Q(j+1,0) & if\ T_R>\Delta\end{cases}$$

Therefore,

$$Q(j,0)=\int_0^\Delta v_0e^{-v_0s}\left(\int_0^s f_0(j,t)dt+\int_s^\Delta f_1(j,t)dt+Q(j+1,1)(1-F_1(j,\Delta))\right)ds$$
$$+e^{-v_0\Delta}\left(\int_0^\Delta f_0(j,t)dt+Q(j+1,0)(1-F_0(j,\Delta))\right). \qquad \ldots(4.11)$$

When the current state of the system is 1,

$$Q(j,1)=\begin{cases}1 \text{ if } T_R \le \Delta \\ Q(j+1,1) \text{ if } T_R > \Delta\end{cases}$$

Therefore,

$$Q(j,1)=\int_0^{k_1\Delta - j\Delta} f_1^1(j,t)dt \qquad \ldots(4.12)$$

For each value of Z, we can also derive the survivor function equation as below

$$\bar{R}(j,0,t)=\int_0^t\left[1-F_1(j,t)\right]v_0e^{-v_0s}ds+e^{-v_0t}\left[1-F_0(j,t)\right]$$

$$\bar{R}(j,1,t)=1-F_1^1(j,t)$$

4.3.4 Procedure to calculate the replacement and failure rates

The point of interest is to calculate the failure rate, the overall replacement rate and also the rates of preventive replacement for each value of *Z*. From the recursive process described above, we have the values of $W(0,0)$ and $Q(0,0)$ which are the times to preventive replacement and failures, respectively, for a newly replaced system.

The overall replacement rate can be defined as the reciprocal of the replacement time

$$r=\frac{1}{W(0,0)}$$

The failure rate can be defined as the ratio of failure time to the replacement time

$$r_f=\frac{Q(0,0)}{W(0,0)}$$

And the rate of preventive replacements $r_p = r - r_f = \dfrac{1-Q(0,0)}{W(0,0)}$

For state i, let the replacement rate be r_i, then $\sum_i r_i = r_p$

Let $\quad M(j,0) = P(\textit{Preventively replace when Z is in state } 0 \mid j)$

$N(j,1) = P(\textit{Preventively replace when Z is in state } 1 \mid j)$

where the age of the system is $j\Delta$.

Then, the replacement rate for $Z = 0$,

$$r_0 = M(0,0)/W(0,0)$$

and the replacement rate for $Z = 1$,

$$r_1 = N(0,0)/W(0,0)$$

The recursive procedure which could be applied to obtain $M(0,0)$, $N(0,0)$ is described below.

For a given Δ, we calculate the preventive replacement times j_0 and j_1 from the algorithm in Section 4.3.1.

<u>Computation of the replacement rate for state 0:</u>

For state zero, we can define

$$M(j,0) = 1 \; for \; j \geq j_0$$

$$M(j,1) = M(j,2) = 0, \forall j$$

For $j < j_{o,}$ the calculation of $M(j,0)$ is described below.

1. If $S_0 = s_0 \ge \Delta$

$$M\left(j,0\right) = \begin{cases} 0 \text{ if } T_R \le \Delta \\ M\left(j+1,0\right) \text{ if } T_R > \Delta \end{cases}$$

$$M_0^0 = \left(M(j,0) \mid S_0 \ge \Delta\right) = M(j+1,0)(1-F_0^0(j,\Delta))$$

2. If $S_0 = s_0 < \Delta$

$$M(j,0) = 0$$

Hence,

$$M(j,0) = \int_{\Delta}^{\infty} v_0 e^{-v_0 s} M_0^0 \, ds_0 \qquad \text{…(4.13)}$$

<u>Computation of the replacement rate for state 1</u>

For state 1, we can describe the computation of $N(0,0)$ as below

$$N(j,0) = 0 \;\; for \;\; j \ge j_0$$

$$N(j,1) = 1 \;\; for \;\; j \ge j_1$$

For $j<j_o$, the calculation of $N(j,0)$ is described below.

1. If $S_0 = s_0 \ge \Delta$

$$N\left(j,0\right) = \begin{cases} 0 \text{ if } T_R \le \Delta \\ N\left(j+1,0\right) \text{ if } T_R > \Delta \end{cases}$$

$$N_0^0 = \left(N(j,0) \mid S_0 \ge \Delta\right) = N(j+1,0)(1-F_0^0(j,\Delta))$$

2. If $S_0 = s_0 < \Delta$, $S_1 = X_0 + X_1 > \Delta$

$$N(j,0) = \begin{cases} 0 \text{ if } T_R \le \Delta \\ N(j+1,1) \text{ if } T_R > \Delta \end{cases}$$

$$N_1^0 = \left(N(j,0) \mid s_0, s_0 < \Delta < S_1\right) = N(j+1,1)(1 - F_1^0(j,\Delta))$$

3. If $S_1 = s_1 \le \Delta$

$$N(j,0) = 0$$

Therefore,

$$N(j,0) = \int_\Delta^\infty v_0 e^{-v_0 s} N_0^0 ds_0 + \int_0^\Delta v_0 e^{-v_0 s_0} e^{-v_1(\Delta - s_0)} N_1^0 ds_0 \qquad \text{…(4.14)}$$

For $j < j_1$, the calculation of $N(j,1)$ is described below.

1. If $X_1 = r \ge \Delta$

$$N(j,1) = \begin{cases} 0 \text{ if } T_R \le \Delta \\ N(j+1,1) \text{ if } T_R > \Delta \end{cases}$$

$$N_0^1 = \left(N(j,1) \mid X_1 \ge \Delta\right) = N(j+1,1)(1 - F_0^1(j,\Delta))$$

2. If $X_1 = r < \Delta$,

$$N(j,1) = 0$$

Therefore,

$$N(j,1) = e^{-v_1 \Delta} N_0^1 \qquad \text{…(4.15)}$$

The above recursive procedure, contributed by Xiang Wu (personal communication, July 22, 2008), is used to obtain the values of the replacement rates necessary for the management of inventory in our system.

4.4 Numerical Example

Let us consider a numerical example to calculate the replacement rates for each value of Z. In the replacement model, K should represent the difference in the manufacturing and remanufacturing costs of a product. In our model, we have two types of products with remanufacturing costs $c_0 = 3$ and $c_1 = 4.5$ and manufacturing cost $m = 15$. The incremental failure costs are $K_0 = m - c_0$ and $K_1 = m - c_1$ respectively. The formulation to calculate the replacement rates is for a single product type and hence we consider K to be the average of K_0 and K_1 weighted by the probabilities of Type 0 product returns and Type 1 product returns respectively. These probabilities are obtained by solving for the preventive replacement rates for each state. We have the preventive replacement rate for Type 0 as r_o and for Type 1 as r_1. And the overall preventive replacement rate is given by $r_p=r_0+r_1$. Now, the probability of obtaining a Type 0 return would be $p_0=r_0/r_p$ and the probability of obtaining return would be $p_1=r_1/r_p$.

But, in order to calculate the replacement rates, we need the value of K. We start off with an approximate and assumed value and move back and forth, following recursive iterations to obtain at the accurate weighted average of K_0 and K_1 as shown in Table 4.0. Similarly, C is the weighted average of the replacement costs c_0 and c_1. We first assume the probabilities p_0'=0.3 and p_1'=0.7. After iteration 1, we obtain the actual probability values for those values of C and K. By using those values, we calculate C and K again and use them in the second iteration. We continue the recursive iterations till the assumed values and the actual values are the same.

Table 4.0: Recursive iterations to obtain *C* and *K*

p_0'	p_0'	C	K	r_0	r_1	p_0	p_1
0.3	0.7	4.2000	10.8000	0.2989	0.6198	0.3253	0.6747
0.3253	0.6747	4.0120	10.9880	0.3747	0.6645	0.3606	0.6394
0.3606	0.6394	3.9592	11.0408	0.3747	0.6645	0.3606	0.6394
0.3606	0.6394	4.0000	11.0000	0.3747	0.6645	0.3606	0.6394

According to the procedure, we should stop at values C=3.9591 and K=11.0408. But, we tested for round of values C=4 and K=11 and obtained the same probabilities. Hence, for this example, we have $K=11$ and $C=4$. Also, let

$$h_0(u)=0.7u\,;\ \Delta=0.1\,;\ \psi(z)=e^z$$

Let us assume the product's condition follows a Markov chain with two states $\{0,1\}$ and the transition probability matrix $P=\begin{pmatrix}0.45 & 0.55\\ 0 & 1\end{pmatrix}$ from which it follows that $v_0=-\ln(0.45)$.

Let us initialize $g_o=11$ (arbitrary) and illustrate the first iteration for finding the g.

1. Initialize $n=0$ and $g_o=11$
2. For $d=g_0$, and $i=0,1$

$$j_0=\min\left\{j\geq 0: K\left[1-\overline{R}(j,0,\Delta)\right]\geq d\int_0^{\Delta}\overline{R}(j,0,t)dt\right\}=10$$
$$j_1=\min\left\{j\geq 0: K\left[1-\overline{R}(j,1,\Delta)\right]\geq d\int_0^{\Delta}\overline{R}(j,1,t)dt\right\}=4$$

3. Using the replacement policy obtained above, we then calculate $g_1=\phi(g_0)$

$$\phi(g_0)=\left[C+KP\left(T_{g_0}\geq T\right)\right]\Big/E\left[\min\left\{T,T_{g_o}\right\}\right]$$

In order to calculate $\phi(g_0)$, we need the values of $W(0,0)$ and $Q(0,0)$

From equations (4.9) and (4.11), we have

$$W(0,0) = 0.7260$$

$$Q(0,0) = 0.2455$$

Therefore,

$$g_1 = \phi(g_0) = 9.2571$$

The complete results are shown in Table 1. The fixed point procedure converges after two iterations at $g = 9.2295$.

The overall replacement rate is $$r = \frac{1}{W(0,0)} = 1.3773 \qquad \ldots(4.16)$$

And the failure rate is $$r_f = \frac{Q(0,0)}{W(0,0)} = 0.3382 \qquad \ldots(4.17)$$

Table 4.1: Computation of g using the fixed point algorithm

g_n	j_0	j_1	$W(0,0)$	$Q(0,0)$	$\phi(g_n)$
11	10	4	0.6955	0.2216	9.2571
9.2571	11	4	0.7260	0.2455	9.2295
9.2295	11	4	0.7260	0.2455	9.2295

Therefore, the overall preventive replacement rate is $r_p = r - r_f = 1.0392$

Our point of interest is to calculate the preventive replacement rates for state 0 and state 1, which are given by

Replacement rate for $Z = 0$: $\quad r_0 = M(0,0)/W(0,0)$

Replacement rate for $Z = 1$: $\quad r_1 = N(0,0)/W(0,0)$

From the equations (4.13) and (4.14), we have

$$M(0,0) = 0.2720$$

$$N(0,0) = 0.4824$$

Therefore,

$$r_0 = 0.3747$$

$$r_1 = 0.6645 \qquad \ldots(4.18)$$

The next chapter shows how the above replacement rates are used in calculating the base stock level of the serviceable inventory that minimizes the average cost per unit time.

5. INVENTORY MANAGEMENT IN HYBRID SYSTEM

5.1 Summary

This chapter describes the management of the serviceable inventory in the hybrid system. Whenever a replacement occurs, the gap in the fleet needs to be filled with a new product from the serviceable inventory. This is done primarily by remanufacturing under the assumption that remanufacturing is much cheaper than manufacturing a new product. The manufacturing option is considered only when the remanufacturing process is unable to satisfy the demand. The product returns are of two types as explained in Chapter 3. All the preventive replacements in state 0 are said to be of Type 0 (higher quality) and all the preventive replacements that occur in state 1 are said to be of Type 1 (lower quality). And each product type has a different inventory to sit in, before being remanufactured. As soon as a product is taken out of the fleet, it is sent to its corresponding inventory unless it is failed. All failed products are discarded immediately. A numerical example is illustrated to show the procedure of determining the optimal base stock level of the serviceable inventory that minimizes the long-run average cost per unit time. The decision parameters are the remanufacturable disposal levels for Q_0 and Q_1 and the serviceable inventory base stock level x. These are the decision parameters for the base stock model.

5.2 Base stock policy

In our system, the demand and returns are modeled as independent Poisson processes with rates Nr and Nr_p respectively. This is an approximation because "demand" is created by failures and returns are preventively replaced products and these processes result from the policy in Chapter 4. And the demand rate is assumed to be greater than the return rate because failed products are discarded. Also, the processing rates required for remanufacturing a higher quality product (μ_0) and a lower quality product (μ_1) are assumed to have different means because the remanufacturing effort required to bring them back to "as good as new" condition varies with their quality. This means that the average remanufacturing lead time is shorter for higher quality products compared to the lower quality ones $(1/\mu_0 < 1/\mu_1)$. Also, the unit remanufacturing cost for Type 0 products is less as compared to the Type 1 products $(c_0 < c_1)$.

The two remanufacturable inventories have holding costs (h) associated with them, which do not vary with the type of the product they are holding. However, the unit holding costs for the remanufacturable inventories and the serviceable inventory are different because of the difference in the value of the products they store. WIP inventory is considered to have approximately 50% value added and the serviceable inventory obviously has all the value added. The unit serviceable inventory holding costs for Type 0 and Type 1 products are given by

$$h_0^s = h + \alpha c_0$$

$$h_1^s = h + \alpha c_1 \qquad \ldots(5.1)$$

where α is the opportunity cost of capital. We also have unit-holding costs for Work-In-Process (WIP) inventory. At each stage, there are products being processed in the remanufacturing unit which haven't acquired their complete 'value' yet and therefore we assume that an average product has half of the value-added.

Therefore, the WIP holding costs are represented by

$$h_0^w = h + \alpha c_0 / 2$$

$$h_1^w = h + \alpha c_1 / 2 \qquad \text{…(5.2)}$$

The average serviceable holding cost is determined by the composition of the serviceable inventory. Thus,

$$h_s = h_0^s \left(\frac{\overline{R}_0}{\overline{R}_0 + \overline{R}_1} \right) + h_1^s \left(\frac{\overline{R}_1}{\overline{R}_0 + \overline{R}_1} \right) \qquad \text{…(5.3)}$$

where $\overline{R}_0$ and $\overline{R}_1$ are the number of returned products remanufactured of Type 0 and Type 1 respectively. Here $\overline{R}_0$ and $\overline{R}_1$ and consequently h_s depend on the choice of the decision variables (x, Q_0, Q_1). The remanufacturing process does not have a limitation on the capacity, i.e., there is no waiting time for processing. Any demand that is not satisfied by the remanufacturing process is met by resorting to manufacturing with an average manufacturing cost of *m* per unit.

In our model, we consider the policy variables as the serviceable base stock level *x* and the remanufacturing disposal levels Q_0 and Q_1. The objective is to optimize these inventory levels (decision variables) so as to minimize the long-run operating cost per unit time of the system, $\overline{C}(x, Q_0, Q_1)$, which is represented as

$$\overline{C}(x, Q_0, Q_1) = h\overline{I}_0 + h\overline{I}_1 + h_s\overline{I}_s + h_0^w\overline{W}_0 + h_1^w\overline{W}_1 + c_0\overline{R}_0 + c_1\overline{R}_1 + \delta_0\overline{D}_0 + \delta_1\overline{D}_1 + m\overline{M} \qquad \text{…(5.4)}$$

5.3 Continuous-time Markov chain representation

As in [14], we use a continuous-time Markov chain representation for the hybrid system in calculating the average cost and determining the optimal policy parameters. The Markov chain is then considered to have a five-dimensional state variable denoted as

$$X(t) = (I_0(t), I_1(t), W_0(t), W_1(t), B(t) : t \geq 0)$$

where:

$I_0(t)$: Type 0 remanufacturable inventory at time *t*;

$I_1(t)$: Type 1 remanufacturable inventory at time *t*;

$W_0(t)$: Type 0 WIP inventory at time *t*;

$W_1(t)$: Type 1 WIP inventory at time *t*;

$B(t)$: Number of outstanding remanufacturing orders at time *t*;

And a finite state space

$$S = \{(i_0, i_1, w_0, w_1, b) : i_0 = 0,...,Q_0, i_1 = 0,...,Q_1, w_0 = 0,...,x, w_1 = 0,...,x, b = 0,...,x\}$$

The above state space gives us a potentially large number of states but it is important to note that not all states are feasible. The conditions below need to be satisfied for a state to exist.

i. An outstanding order exists only when both the remanufacturable inventories are zero. That is, $B(t) > 0$ only if $I_0(t) = I_0(t) = 0$.

ii. A continuous review base stock policy is used to manage the serviceable inventory which implies that at any instant, the sum of serviceable on-hand inventory, WIP inventory and the outstanding remanufacturing orders should equal the base stock level s. That is, $x = I_s(t) + W_0(t) + W_1(t) + B(t)$.

Considering the current state of the system to be (i_0, i_1, w_0, w_1, b), we study the different states the system could go to, in the next time interval and their corresponding transition rates (listed in Table 5.1).

According to Aras et al., the Markov chain is irreducible and ergodic and, therefore, has a limiting distribution. In [14], the computation procedure for the limiting distribution, the marginal distribution of each state and, therefore, the long-run average system cost is described. And, by considering the parameter space large enough to contain the optimal solution, we resort to an enumerative search procedure to find the optimal policy (x, Q_0, Q_1) and the associated optimal cost $\overline{C}^*$.

The limiting probability is equal to the long-run fraction of time the process is likely to be in a particular state. Let $\pi(j), j \in S$ be the limiting probability of our continuous-time Markov chain where $j = (i_0, i_1, w_0, w_1, b)$. The following system of linear equations can be solved to obtain these steady-state probabilities.

$$\pi(j)\sum_{k \neq j} q_{jk} = \sum_{k \neq j} q_{kj}\pi(k) \qquad j \in S$$

$$\sum_{j \in S} \pi(j) = 1 \qquad \ldots(5.5)$$

where q_{jk} represents the infinitesimal transition rate from state *j* to state *k*.

Table 5.1: Transition rates from state (i_0, i_1, w_0, w_1, b)

To state	Condition	Rate	Event
$(i_0 - 1, i_1, w_0 + 1, w_1, b)$	$i_0 > 0$	r	Demand
$(i_0, i_1 - 1, w_0, w_1 + 1, b)$	$i_0 = 0, i_1 > 0$	r	Demand
$(i_0, i_1, w_0, w_1, b + 1)$	$i_0 = 0, i_1 = 0$	r	Demand
$(i_0 + 1, i_1, w_0, w_1, b)$	$i_0 < Q_0$	r_0	Type 0 returns
$(i_0, i_1 + 1, w_0, w_1, b)$	$i_1 < Q_1$	r_1	Type 1 returns
$(i_0, i_1, w_0 + 1, w_1, b - 1)$	$b > 0$	r_0	Type 0 returns
$(i_0, i_1, w_0, w_1 + 1, b - 1)$	$b > 0$	r_1	Type 1 returns
$(i_0, i_1, w_0 - 1, w_1, b)$	$w_0 > 0$	$w_0 \mu_0$	Type 0 remanufactured
$(i_0, i_1, w_0, w_1 - 1, b)$	$w_1 > 0$	$w_1 \mu_1$	Type 1 remanufactured

It would be informative to understand the probability distribution of each of the state variables ignoring the information of the others. Therefore the marginal probabilities are calculated as below, and thereby used in estimating the long-run average system cost per unit time.

$$P\{I_0 = n\} = \sum_{(i_1, w_0, w_1, b)} \pi(n, i_1, w_0, w_1, b)$$

$$P\{I_1 = n\} = \sum_{(i_0, w_0, w_1, b)} \pi(i_0, n, w_0, w_1, b)$$

$$P\{I_s = n\} = \sum_{\{(w_0, w_1, b): w_0 + w_1 + b = x - n\}} \pi(i_0, i_1, w_0, w_1, b)$$

$$P\{W_0 = n\} = \sum_{(i_0,i_1,w_1,b)} \pi(i_0, i_1, n, w_1, b)$$

$$P\{W_1 = n\} = \sum_{(i_0,i_1,w_0,b)} \pi(i_0, i_1, w_0, n, b)$$

$$P\{B = n\} = \sum_{(i_0,i_1,w_0,w_1)} \pi(i_0, i_1, w_0, w_1, n)$$

…(5.6)

And from the above marginal probability distributions, we can calculate the following.

The remanufacturing inventory for Type 0 products: $\bar{I}_0 = E[I_0] = \sum_{i_0=0}^{Q_0} i_0 P\{I_0 = i_0\}$

The remanufacturing inventory for Type 1 products: $\bar{I}_1 = E[I_1] = \sum_{i_1=0}^{Q_1} i_1 P\{I_1 = i_1\}$

WIP inventory for Type 0 products: $\bar{W}_0 = E[W_0] = \sum_{w_0=0}^{x} w_0 P\{W_0 = w_0\}$

WIP inventory for Type 1 products: $\bar{W}_1 = E[W_1] = \sum_{w_1=0}^{x} w_1 P\{W_1 = w_1\}$

Number of outstanding orders: $\bar{B} = E[B] = \sum_{b=0}^{x} b P\{B = b\}$

Number of disposed Type 0 products: $\bar{D}_0 = \gamma_0 P\{I_0 = Q_0\}$

Number of disposed Type 1 products: $\bar{D}_1 = \gamma_1 P\{I_1 = Q_1\}$

Number of remanufactured Type 0 products: $\bar{R}_0 = \gamma_0 - \bar{D}_0$

Number of remanufactured Type 1 products: $\bar{R}_1 = \gamma_1 - \bar{D}_1$

On-hand serviceable inventory: $\bar{I}_s = x - \bar{W}_0 - \bar{W}_1 - \bar{B}$

Demand not satisfied by remanufacturing: $\bar{M} = \lambda P\{I_s = 0\}$

…(5.7)

By calculating all the above terms, we can obtain the cost function as in (5.4). We calculate the cost function for different states and thereby obtain the optimal base stock level of the serviceable inventory which minimizes the long-run average cost. We consider the parameter space large enough to contain the optimal solution and thereby resort to an enumerative search procedure.

5.4 Numerical Example

The example below illustrates all of the above calculations.

Let us consider 2 products in the fleet. Therefore, *N*=2.

Remanufacturing processing rates: $\mu_0 = 5$, $\mu_1 = 2.5$

Holding cost for the remanufacturable inventory: $h = 0.5$

Opportunity cost of capital: $\alpha = 0.1$

Disposal costs: $\delta_0 = \delta_1 = 0$

The demand rate of the system would be the overall rate at which the products are being replaced. From the calculations in Chapter 4, we have the replacement rate for a single product is $r = 1.3773$ (Eqn. 4.16). For the fleet of 2 products, we have the overall replacement rate as Nr = 2.7546. The overall return is the rate at which the products are preventively replaced, i.e., r_p. In our system, since the products are condition monitored at frequent intervals, we already have the condition information of the product during replacement. The return rate for Type 0 and Type 1 products are the preventive replacement rates we obtained for state 0 and state 1 in Chapter 4. Therefore, we have the replacement rates for a single product as $r_0 = 0.3747$ and $r_1 = 0.6645$ and the overall replacement (return) rates are $Nr_0 = 0.7494$ and $Nr_1 = 1.3290$.

The first step here would be to solve (5.5) and obtain the steady state probabilities. We combine both the equations together as $[\pi][Q] = 0$ where $[Q]$ is the transition rate matrix and $[\pi]$ is the steady state probability matrix. In order to obtain the probabilities, we need $[Q]$ and this is obtained in the following fashion. Each element of the Q matrix is the transition rate of the system from state j to state k.

Let us first assume the simplest case where $x = 1, Q_0 = 1, Q_1 = 1$. Now, the state space would be $S = \{(i_0, i_1, w_0, w_1, b) : i_0 = 0,1, i_1 = 0,1, w_0 = 0,1, w_1 = 0,1, b = 0,1\}$. As mentioned earlier, there are a few conditions that need to be satisfied for a state to exist. For example, consider the state $j = (i_0, i_1, w_0, w_1, b) = (0,1,0,0,1)$. This state cannot exist since it does not satisfy the condition which says: $b > 0$ if and only if $i_0 = i_1 = 0$. Let us consider another state $j = (i_0, i_1, w_0, w_1, b) = (0,0,1,0,1)$. This state also cannot exist since it fails to satisfy the other condition which says $w_0 + w_1 + b \le x$. Thus the state space has thirteen states and the possible transitions of the system from its current state are shown in Figure 5.0.

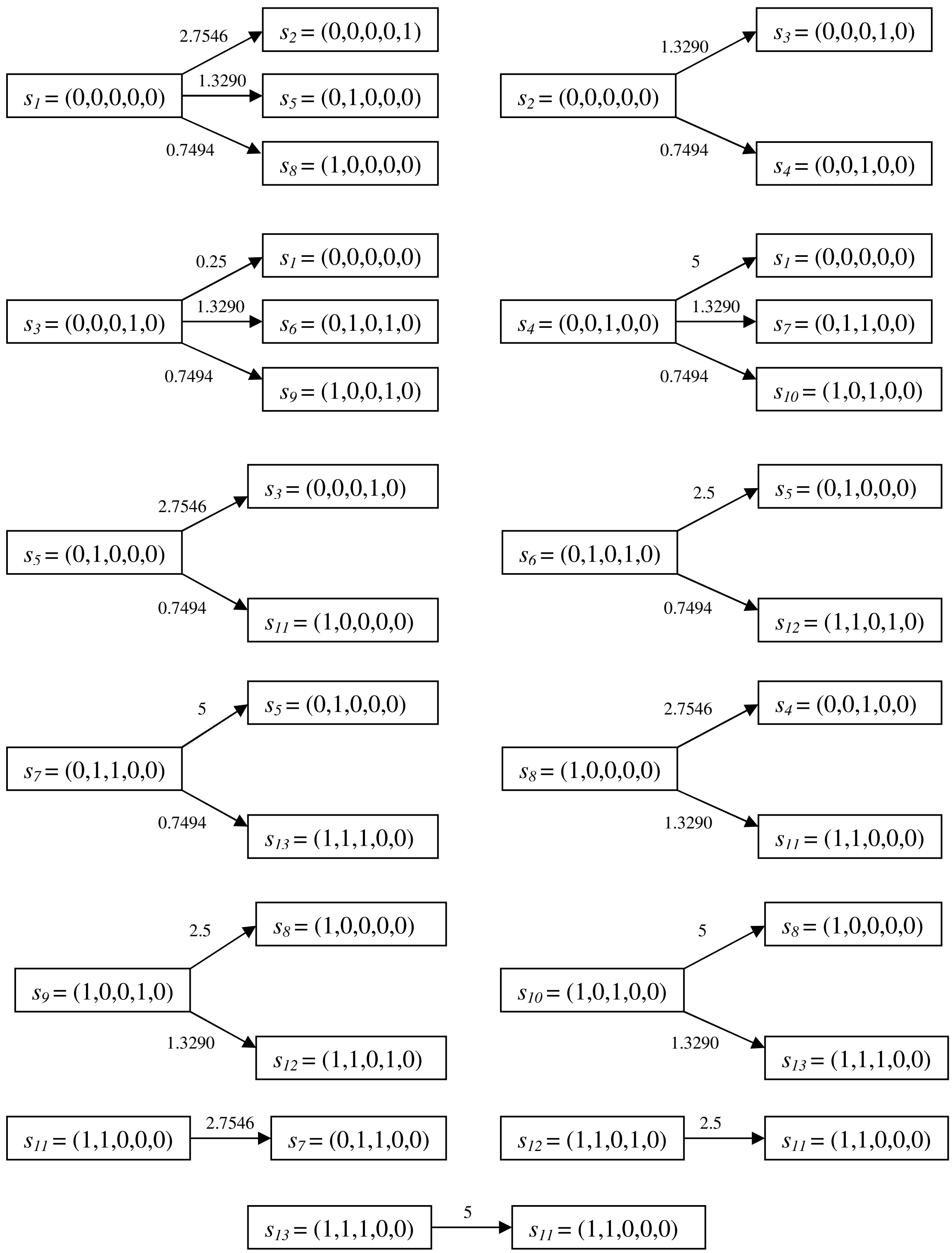

Figure 5.0: Transition rates of each state for x=1, Q_0=1, Q_1=1

The transition matrix is obtained from the transition rates given in Table 5.1. The diagonal elements of the matrix are the negative of the sum of all the elements of that row. Since we know the values of $r = 2.7546$, $r_0 = 0.7494$, $r_1 = 1.3290$, $\mu_0 = 5$, $\mu_1 = 2.5$ the transition matrix is given as below.

$$Q = \begin{bmatrix}
-4.8330 & 2.7546 & 0 & 0 & 1.3290 & 0 & 0 & 0.7494 & 0 & 0 & 0 & 0 & 0 \\
0 & -2.0784 & 1.3290 & 0.7494 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2.5 & 0 & -4.5784 & 0 & 0 & 1.3290 & 0 & 0 & 0.7494 & 0 & 0 & 0 & 0 \\
5 & 0 & 0 & -7.0784 & 0 & 0 & 1.3290 & 0 & 0 & 0.7494 & 0 & 0 & 0 \\
0 & 0 & 2.7546 & 0 & -3.5040 & 0 & 0 & 0 & 0 & 0 & 0.7494 & 0 & 0 \\
0 & 0 & 0 & 0 & 2.5 & -3.2494 & 0 & 0 & 0 & 0 & 0 & 0.7494 & 0 \\
0 & 0 & 0 & 0 & 5 & 0 & -5.7494 & 0 & 0 & 0 & 0 & 0 & 0.7494 \\
0 & 0 & 0 & 2.7546 & 0 & 0 & 0 & -4.0836 & 0 & 0 & 1.3290 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.5 & -3.8290 & 0 & 0 & 1.3290 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & -6.3290 & 0 & 0 & 1.3290 \\
0 & 0 & 0 & 0 & 0 & 0 & 2.7546 & 0 & 0 & 0 & -2.7546 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.5 & -2.5 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & -5
\end{bmatrix}$$

Solving for the steady-state probabilities,

$$\pi_1 = 0.1127; \quad \pi_2 = 0.1493; \quad \pi_3 = 0.1522; \quad \pi_4 = 0.0328;$$

$$\pi_5 = 0.1809; \quad \pi_6 = 0.0623; \quad \pi_7 = 0.0657; \quad \pi_8 = 0.0437;$$

$$\pi_9 = 0.0298; \quad \pi_{10} = 0.0039; \quad \pi_{11} = 0.1214; \quad \pi_{12} = 0.0345; \quad \pi_{13} = 0.0109$$

From the steady-state probabilities, we obtain the marginal probabilities as in (5.6) and the cost terms from the equations in (5.7). Hence we have the long-run average cost per unit time $\overline{C}(1,1,1) = 28.2446.$

This is just a sample calculation for $x = 1, Q_0 = 1, Q_1 = 1$. We similarly calculate the cost function for the entire parameter space until we find the optimal solution. Since we do not have an explicit function that could give us the optimal solution, we adopt a search procedure. We consider the parameter space large enough to contain the optimal solution and thereby obtain our optimal base stock and disposal levels that minimizes the long-run average cost per unit time. Table 5.2 shows the costs for different values of x, Q_0 and Q_1.

Table 5.2 Effect of *x* on cost for $Q_0 = Q_1$ (all holding costs included; optimal values in bold)

x	$Q_0=Q_1=1$	$Q_0=Q_1=2$	$Q_0=Q_1=3$	$Q_0=Q_1=4$	$Q_0=Q_1=5$
1	28.2446	27.1758	27.0367	27.2262	27.5571
2	23.3965	22.2917	21.7971	21.545	21.407
3	22.1275	21.4842	21.177	21.0119	20.9163
4	21.5984	21.2296	21.0497	**20.9529**	**20.8974**
5	21.3272	21.1137	21.0106	20.9568	20.9272
6	21.181	21.0591	**21.0026**	20.9751	20.9616
7	21.1037	21.0371	21.0091	20.9976	20.9937
8	21.066	**21.0331**	21.0223	21.0203	21.022
9	21.0512	21.0387	21.038	21.0415	21.022
10	**21.0496**	21.0491	21.054	21.0603	21.0665
11	21.0551	21.0612	21.0691	21.0766	21.0832
12	21.0641	21.0736	21.0825	21.0903	21.0967
13	21.0744	21.0852	21.0942	21.1016	21.1135

We assess the cost impact of the disposal levels of remanufacturable inventories by drawing a comparison of the optimal costs for each value of Q_0 and Q_1. Figure 5.1 shows how the disposal levels affect the cost function.

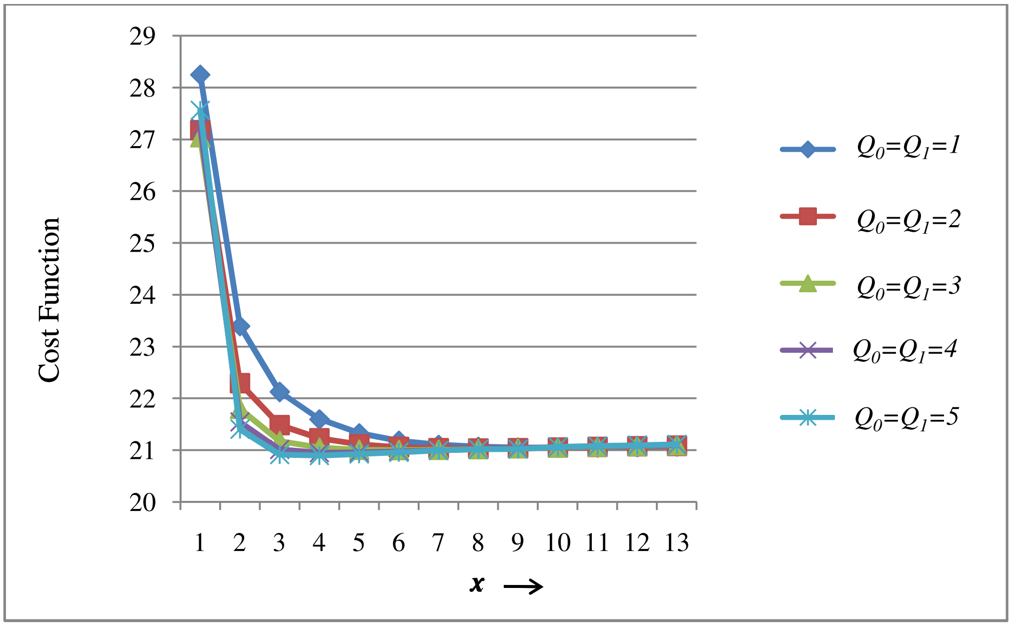

Figure 5.1: Effect of base stock inventory level *x* on the cost function (all holding costs included)

The above plot shows the pattern in which the cost function decreases up to a certain value of '*x*' and begins to increase again. For a given disposal level of the remanufacturable inventories, that *x* is said to be optimal. To get a closer picture, the smaller values of *x* are ignored and another plot is drawn for $Q_0 = Q_1 = 1$ (Figure 5.2).

From Table 5.2, it can be observed that the optimal base stock inventory position that minimizes the long-run average cost per unit time is x=10 for $Q_0 = Q_1 = 1$ and decreases to x=4 at $Q_0 = Q_1 = 5$. We can also observe that as Q_0 and Q_1 increase, the cost decreases. When the remanufacturable inventory levels are low, it seems to be optimal to maintain a high serviceable inventory level. This is to fill the gap and satisfy the demand. On the other hand, if we have higher values of Q_0 and Q_1, it reduces the need for a high base stock level in the serviceable inventory because there will be enough number of products to be remanufactured in order to satisfy the demand.

From Table 5.2, it can also be noted that as the values of Q_0 and Q_1 increase, the optimal cost decreases. This is because higher Q_0 and Q_1 means that we always have a product ready to be remanufactured whenever demand arises. To be able to satisfy the demand with a remanufactured product would minimize the need for manufacturing a new product, and therefore minimize the overall cost. Also, according to Type 0-first strategy, higher inventory of Type 0 remanufacturable products will decrease the need to remanufacture a Type 1 product or have an outstanding order and thereby decrease the overall cost.

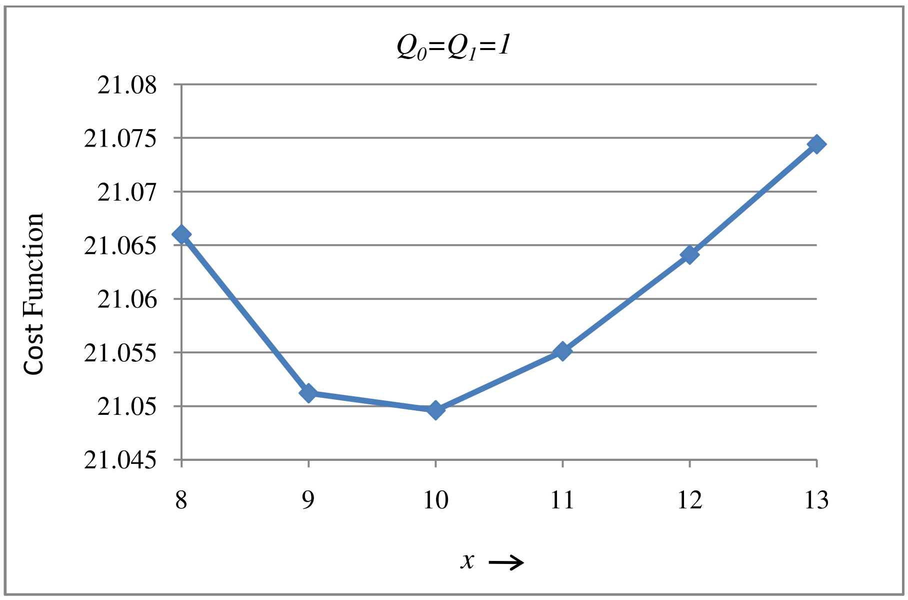

Figure 5.2: Effect of base stock inventory level x on the cost function for Q_0=Q_1=1

Our point of interest is the optimal management of the serviceable inventory to minimize the cost. Hence, we try to minimize the effect of the remanufacturable inventory disposal levels on the base stock level. We attempt to do this in different ways. First, we assume the holding cost for the remanufacturable inventories (h) to be zero. By doing this, we shift the focus from Q_0 and Q_1 to the base stock level x. We are trying to determine the optimal base stock level when there is no constraint on the remanufacturable inventory levels. Table 5.3 shows the costs for the parameter space. We can observe that the optimal base stock level drifts as we increase the disposal levels from $Q_0=Q_1=1$ to $Q_0=Q_1=24$. The cost at the optimal x also decreases monotonically as we increase the disposal levels Q_0 and Q_1 with $Q_0=Q_1$.

Table 5.3: Effect of x on cost when the holding cost $h=0$ (Optimal values in bold)

x	$Q_0=Q_1=1$	$Q_0=Q_1=2$	$Q_0=Q_1=3$	$Q_0=Q_1=4$
1	27.4594	25.8858	25.2626	24.9792
2	22.6180	21.2702	20.5800	20.1642
3	21.2563	20.4386	20.0011	19.7349
4	20.6242	20.1116	19.8261	19.6488
5	20.2585	19.9241	19.7329	19.6123
6	20.0297	19.8061	19.6762	19.5933
7	19.8814	19.7298	19.6407	19.5835
8	19.7836	19.6800	19.6187	19.5792
9	19.7184	19.6474	19.6051	**19.5779**
10	19.6749	19.6261	19.5971	19.5784
11	19.6458	19.6124	19.5926	19.5798
12	19.6265	19.6037	19.5902	19.5816
13	19.6137	19.5983	19.5892	19.5834
14	19.6054	19.5951	**19.5891**	19.5852
15	19.6001	19.5933	19.5893	19.5868
16	19.5968	19.5923	19.5898	19.5882
17	19.5947	19.5919	19.5904	19.5894
18	19.5936	**19.5918**	19.5909	19.5784
19	19.5929	19.5919	19.5914	19.5798
20	19.5926	19.5921	19.5919	19.5816
21	**19.5925**	19.5923	19.5893	19.5834
22	19.5926	19.5923	19.5898	19.5852
23	19.5927	19.5924	19.5904	19.5868
24	19.5928	19.5926	19.5909	19.5882

The values in Table 5.3 show that higher the inventory, the lower is the cost and lower is the base stock level. This trend is similar to the case where the holding cost h=0.5. But, however, it can be observed that the cost values are lower as compared to the first case. This is because of the obvious impact of making the holding cost zero. Also, being functions of *h,* both the WIP holding cost and the serviceable inventory holding cost decrease and thereby decrease the overall cost function . The plot in Figures 5.3a, 5.3b is drawn to study the behavior of the cost function with respect to *x*.

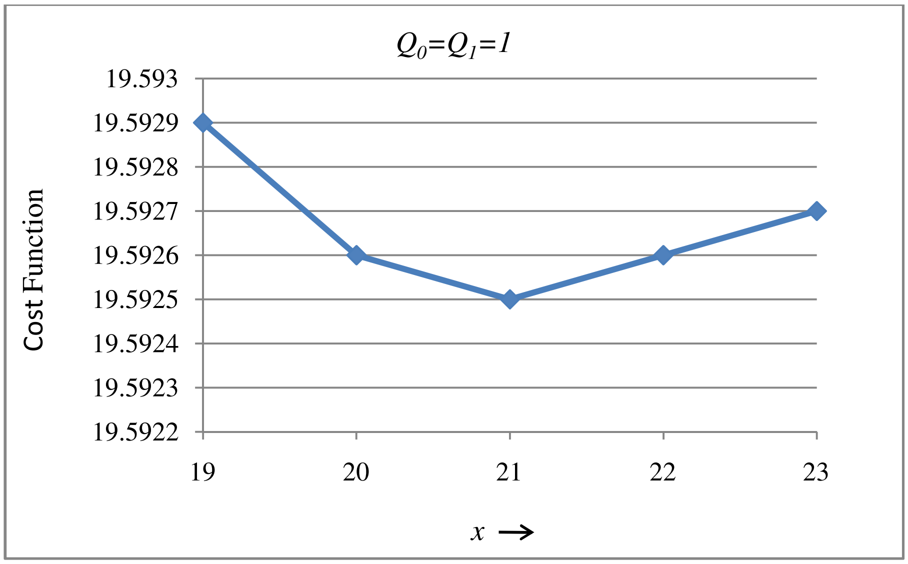

Figure 5.3a: Effect of base stock inventory level x on the cost function (h=0) for Q_0=Q_1=*1*

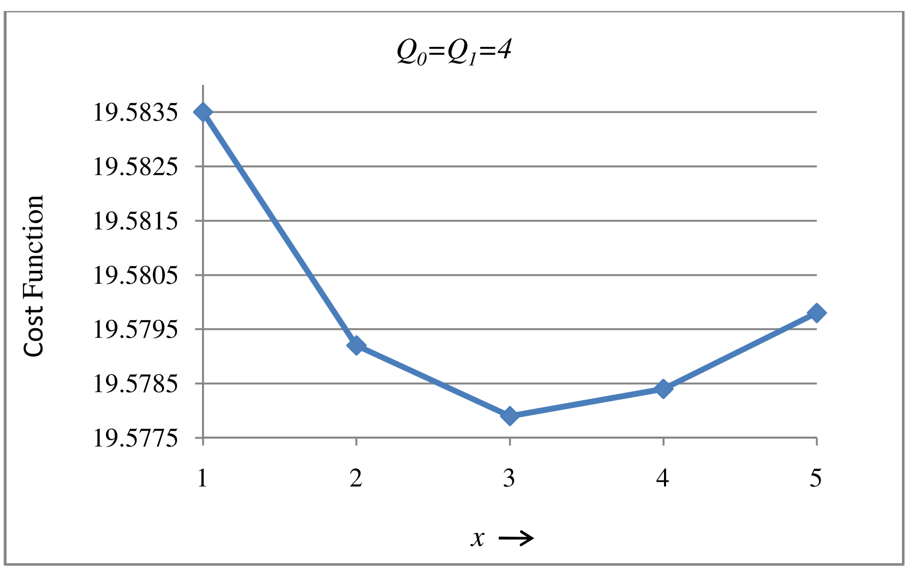

Figure 5.3b: Effect of base stock inventory level x on the cost function (h=0) for Q_0=Q_1=*4*

From the plot, we can observe that for Q_0=Q_1=1, the serviceable inventory is optimal at x=21 whereas for Q_0=Q_1=4, it decreases to x=9. The costs in Table 5.3 show that it is more profitable to have higher remanufacturable inventories. Also, it can be observed that the cost at optimal x decreases as the values of Q_0 and Q_1 increase. However, in realistic situations, it is not possible for the remanufacturer to hold a very high number of products without any holding costs. Hence there might have to be a compromise depending on the space limitations.

As we know, the Work-in-progress inventory holding cost and the serviceable inventory holding cost are both functions of h. By making the remanufacturable inventory holding cost, h=0, we also decrease the values of the WIP holding cost and the serviceable inventory holding cost. To avoid that, we consider another case where we simply eliminate the holding cost terms from the cost function. We originally have the cost function equation as

$$\overline{C}(x,Q_0,Q_1) = h\overline{I}_0 + h\overline{I}_1 + h_s\overline{I}_s + h_0^w\overline{W}_0 + h_1^w\overline{W}_1 + c_0\overline{R}_0 + c_1\overline{R}_1 + \delta_0\overline{D}_0 + \delta_1\overline{D}_1 + m\overline{M}.$$

By eliminating the holding cost from the cost function we have,

$$\overline{C}(x,Q_0,Q_1) = h_s\overline{I}_s + h_0^w\overline{W}_0 + h_1^w\overline{W}_1 + c_0\overline{R}_0 + c_1\overline{R}_1 + \delta_0\overline{D}_0 + \delta_1\overline{D}_1 + m\overline{M}$$

By doing this, we do not alter the other holding costs in any manner. Table 5.4 lists the cost values for the parameter space considered. Figures 5.4a and 5.4b show the effect of x on the cost function for different disposal levels when the holding cost is eliminated from the cost function.

Table 5.4: Effect of x, Q_0 and Q_1 on the cost when h is eliminated from the cost function

(Optimal values in bold)

x	$Q_0=Q_1=1$	$Q_0=Q_1=2$	$Q_0=Q_1=3$	$Q_0=Q_1=4$	$Q_0=Q_1=5$
1	27.8847	26.3514	25.7445	25.4691	25.3324
2	23.2407	21.9584	21.3034	20.9096	20.6443
3	22.033	21.2817	20.8812	**20.6383**	**20.4769**
4	21.5345	21.0893	**20.8426**	20.6900	20.5875
5	21.2821	21.013	20.8605	20.7649	20.7001
6	21.1485	20.9857	20.8923	20.8334	20.7934
7	21.0800	**20.9831**	20.9274	20.8923	20.8684
8	21.0485	20.993	20.9615	20.9418	20.9284
9	**21.0383**	21.0089	20.9927	20.9828	20.9761
10	21.0399	21.0268	21.0201	21.0163	21.0139
11	21.0479	21.0445	21.0436	21.0435	21.1191
12	21.0587	21.061	21.0634	21.0654	21.1894
13	21.0703	21.0758	21.0799	21.0829	21.2596

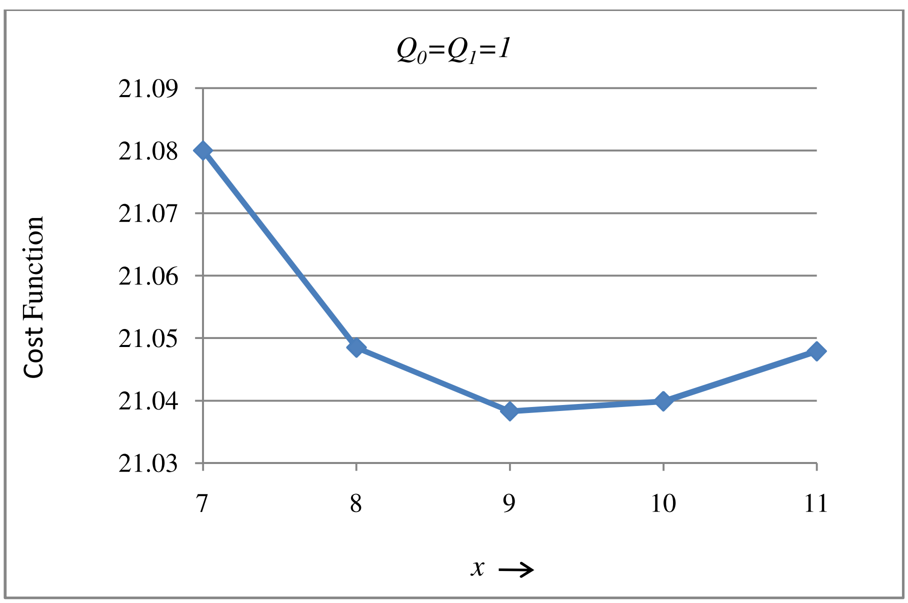

Figure 5.4a: Effect of base stock inventory level x on the cost for $Q_0=Q_1=1$

(h eliminated from cost function)

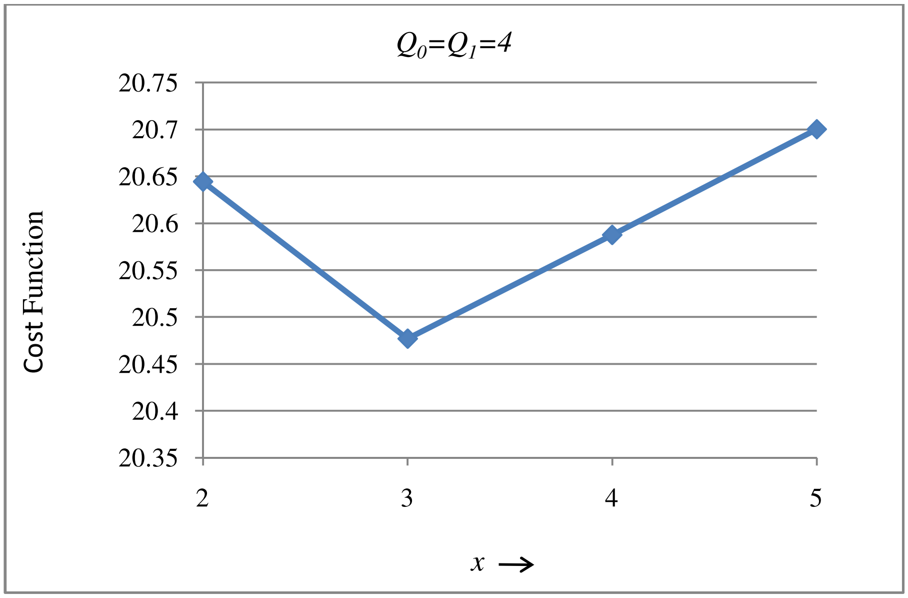

Figure 5.4b: Effect of base stock inventory level x on the cost function for Q_0=Q_1=4

(h eliminated from cost function)

For each disposal level, the pattern of the above plots (Figure 5.4a and 5.4b) seems to be similar to that of previous cases but the optimal base stock levels are lower. Also, as the disposal levels increase, the over-all cost seems to be decreasing. Unlike the previous case where h=0, this case does not change the WIP and serviceable inventory holding costs. Since the serviceable inventory cost would remain the same, the optimal value of x is lower compared to the previous case because the cost would be minimal at lower inventories. Hence, the remanufacturer will be better off holding less serviceable inventory and more remanufacturable inventory in this case. The remanufacturer can hold as many remanufacturable products as possible since the cost decreased with increasing values of Q_0 and Q_1. As we see from Table 5.4, the value of x decreases as we increase the disposal levels. In realistic situations, the remanufacturer decides the disposal level based on the space limitations since the cost continuously decreases for higher values of Q_0 and Q_1.

We try another case to completely shift the focus onto the serviceable inventory. In this case, we totally eliminate all other costs from the cost function except the serviceable inventory holding cost. Now, the cost function will be

$\bar{C}(x, Q_0, Q_1) = h_s \bar{I}_s + c_0 \bar{R}_0 + c_1 \bar{R}_1 + m\bar{M}$ (without the WIP holding costs and the disposal costs)

Table 5.5 lists the cost values for this case. We then plot the effect of *x* on the cost function and determine the optimal value.

Table 5.5: Effect of *x, Q_0 and Q_1* on cost when all holding costs are eliminated

x	$Q_0=Q_1=1$	$Q_0=Q_1=2$	$Q_0=Q_1=3$	$Q_0=Q_1=4$
1	27.6090	26.0547	25.4300	25.1589
2	23.0222	21.7175	21.0493	20.6467
3	21.874	21.1136	**20.7069**	**20.4597**
4	21.4121	20.9629	20.7133	20.5585
5	21.1830	20.9119	20.7579	20.6612
6	21.0655	**20.9015**	20.8073	20.7478
7	21.0087	20.9111	20.8549	20.8195
8	20.9861	20.9302	20.8983	20.8784
9	**20.9828**	20.9531	20.9367	20.9266
10	20.9900	20.9767	20.9698	20.966
11	21.0025	20.9991	20.9981	20.9979
12	21.0171	21.0194	21.0218	21.0237
13	21.032	21.0374	21.0810	21.0445

Here again, the cost decreases as the disposal levels increase. This is due to the elimination of holding costs from the cost function. Also, there could be as many products as possible in the remanufacturable inventory and also in the remanufacturing process (WIP inventory) since that does not affect the cost function in any manner. It only decreases the overall cost because higher levels of remanufacturable inventories minimize the need for manufacturing a new product. Now, all the remanufacturer is paying for is the serviceable inventory. Hence it is justified for the values of *x* to decrease as we increase the disposal levels.

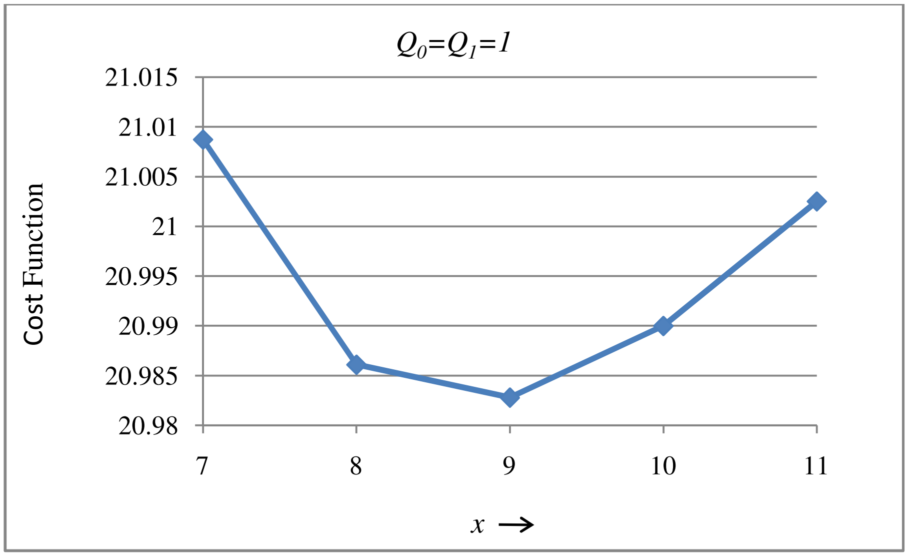

Figure 5.5a: Effect of base stock inventory level *x* on the cost function for $Q_0=Q_1=1$ (All holding costs eliminated)

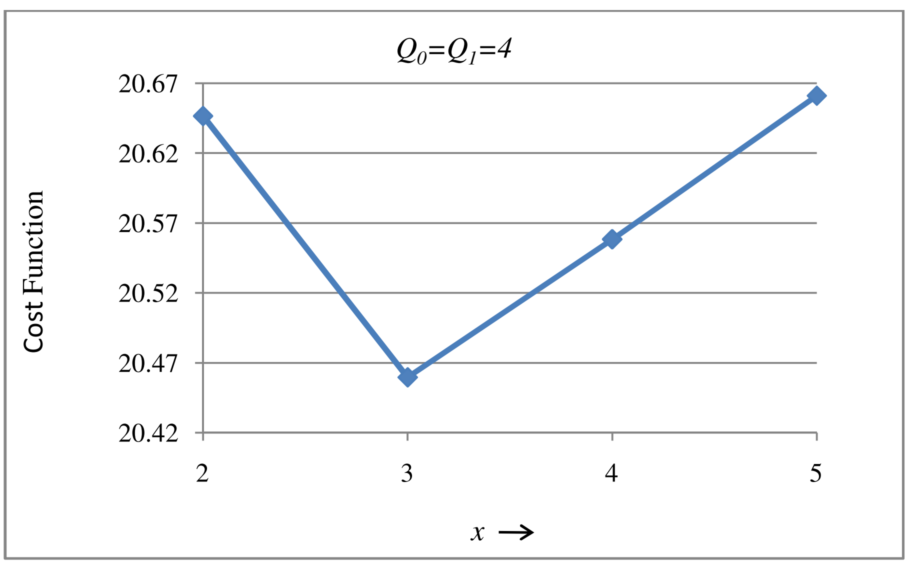

Figure 5.5b: Effect of base stock inventory level *x* on the cost function for $Q_0=Q_1=4$ (All holding costs eliminated)

The cost function, however, follows the same trend as in other cases. The cost is least at a particular value of x which we consider to be optimal for that disposal level. The entire focus is on the serviceable inventory and all the above cases are able to give us the optimal base stock level for a certain disposal level that minimizes the long-run cost per unit time.

Here, we draw a comparison of the cost values at the optimal x for all the four cases. These values are optimal only for $Q_0=Q_1=1$. This is just to give a better comparison of each situation.

For $Q_0=Q_1=1$,

Table 5.6: Comparison of the optimal base stock level and cost for all the cases

	x^*	$\overline{C}^*(x, Q_0, Q_1)$
Case 1: All holding costs included	10	28.2446
Case 2: Remanufacturable holding cost h=0	21	**27.4594**
Case 3: Remanufacturing holding cost h=0.5 but eliminated from the cost function	9	27.8847
Case 4: All holding costs eliminated from the cost function	9	27.6090

From Table 5.6, we can observe that when the remanufacturable inventory holding cost, h is zero, the cost is the least. This is obvious since the WIP and serviceable inventory holding costs are functions of h and making h=0 decreases their value and in turn decreases the cost function.

So far, in our comparisons, we considered both the disposal levels to be equal. It would be interesting to know how each of the disposal levels individually affects the cost function.

Below is the table which shows the effect of Q_0 on the cost for the case where all the holding costs are included in the cost function.

Table 5.7: Effect of Q_0 on the cost (holding cost eliminated from the cost function)

s	$Q_0=Q_1=1$	$Q_0=2; Q_1=1$	$Q_0=3; Q_1=1$	$Q_0=5; Q_1=1$	$Q_0=10; Q_1=1$
1	28.2446	27.6077	27.4187	27.3412	27.3333
2	23.3965	23.0525	22.9697	22.9435	22.9417
3	22.1275	21.9394	21.8943	21.8802	21.8792
4	21.5984	21.4882	21.4617	21.4534	21.4529
5	21.3272	21.2599	21.2437	21.2387	21.2384
6	21.181	21.1393	21.1294	21.1264	21.1262
7	21.1037	21.078	21.0719	21.0701	21.07
8	21.066	21.0504	21.0467	21.0457	21.0456
9	21.0512	**21.0421**	**21.04**	**21.0394**	**21.0394**
10	**21.0496**	21.0446	21.0435	21.0432	21.0432
11	21.0551	21.0527	21.0522	21.0521	21.0521
12	21.0641	21.0632	21.0631	21.0632	21.0632
13	21.0744	21.0745	21.07253	21.0743	21.0743

The important thing to note here is Q_0 does not seem to have any significant effect on the cost function beyond $Q_0=Q_1=5$. If we observe the cases when $Q_0=1$ and $Q_0=5$, we notice that increasing Q_0 decreases the cost and also the optimal base stock level. But beyond $Q_0=5$, for all higher values of Q_0 and Q_1, the cost does not differ. This could be due to the Type 0-first strategy we follow in this thesis. Whenever there is a demand, the Type 0 products are pulled first and therefore they are not 'held' in the inventory for a long time to effect the cost function. This could be better understood by observing the effect of Q_1 on the cost (Table 5.8).

From the Table 5.8, we can observe that increasing the disposal level for Type 1 remanufacturable inventory increases the cost. This is again due to the priority strategy used in the inventory policy. Type 1 products are pulled into the remanufacturing process only when the Type 0 inventory is zero and there is a probability for the Type 1 products to sit in the inventory for a longer time. Also, as the Q_1 increases, the value of the optimal base stock level decreases.

Table 5.8: Effect of Q_1 on the cost function

s	$Q_0=Q_1=1$	$Q_0=1; Q_1=2$	$Q_0=1; Q_1=3$	$Q_0=1; Q_1=4$	$Q_0=1; Q_1=10$
1	28.2446	27.7674	27.7371	27.9138	30.3305
2	23.3965	22.6837	22.3215	22.126	21.9139
3	22.1275	21.7074	21.486	21.3639	21.2164
4	21.5984	21.3636	21.2375	21.1681	21.0885
5	21.3272	21.1964	21.1268	21.0894	21.0525
6	21.181	21.1103	21.0739	21.0556	**21.0437**
7	21.1037	21.0684	21.0518	**21.0447**	21.047
8	21.066	21.0516	**21.0467**	21.0461	21.0561
9	21.0512	**21.0491**	21.0508	21.0539	21.0674
10	**21.0496**	21.0543	21.0595	21.0644	21.079
11	21.0551	21.0632	21.0702	21.0758	21.0901
12	21.0641	21.0737	21.0811	21.0867	21.1
13	21.0744	21.0842	21.0914	21.0967	21.1087

All the above cases and comparisons give a good understanding of the effect of each parameter on the cost function. This thesis is concluded with some remarks in the next chapter.

6. CONCLUSION

The major challenge of servicizing is the decision making regarding both product replacements and inventory management. Since the customer is ensured a working product at all times, these decisions become very important in a servicizing scenario and thus the research in these areas. Also, the stochastic nature of the product quality becomes an issue in the remanufacturing scenario. Condition monitoring plays a major role in helping reduce the number of failed products and obtaining better quality remanufacturable returns.

6.1 Thesis Contribution

This thesis solves the problem of managing the inventory in a manufacturing/ remanufacturing hybrid system managing a fleet of products in service that are condition monitored at discrete intervals. The system is defined as a product-based service scenario where the products in service are monitored for their working condition and the decision to replace it or not is made based on the condition information available at discrete intervals. These replacement decisions are made by following a replacement policy defined by Makis and Jardine (1992). Now, the problem left unresolved is the management of inventory in the hybrid system with manufacturing and remanufacturing processes working together to satisfy the customer demand. Review of literature on the various inventory policies revealed that a good fit for this scenario would be the continuous-review base stock policy because of the following characteristics of the system in this thesis.

(a) Demand occurs one at a time.

(b) Remanufacturing leadtimes are known.

(c) Remanufacturable products are pulled 'one at a time' into the remanufacturing process.

(d) Demand can be approximated with a continuous distribution (Poisson's distribution in our case)

According to this policy, whenever a product is taken out of service, one 'as good as new' product is given back to the customer from the serviceable inventory as a replacement. And whenever a replacement is made, a product from the remanufacturable inventory is pulled into the remanufacturing process to replenish the serviceable inventory. It is like a 'purchase order' (remanufacturing) being placed when a 'sale' (replacement) is made. But, since the remanufacturing process had a leadtime (which is known), we require to maintain an inventory of products for ready replacements. Also, the demand rate is higher than the return rate. Hence, we also need to integrate a manufacturing process to fill in the gap when the remanufacturing process alone is unable to satisfy the demand.

Literature shows that this policy has been applied to highlight the importance of categorizing products before remanufacturing. The stochastic nature of the leadtimes of the remanufacturing process was focused on. However, this thesis applies this model to a service paradigm with condition monitoring which stands apart in the following aspects. Condition monitoring eliminates the effort of inspecting the returns and categorizing them. Also, availability of the condition information at frequent intervals has control over the return rates of products in each state. That is, the service provider is able to make better decisions concerning the replacement of the product with condition information as opposed to waiting for failure. This also minimizes the over-all cost since we would have products returning in two different states. For these products, neither are the holding costs same nor the remanufacturing leadtimes. The remanufacturing effort required to process a low quality product is higher than that required for a high quality one. This means that the higher quality products have lower remanufacturing leadtime, lower remanufacturing cost, and lower WIP holding cost (since the time spent in the manufacturing process is lower). Hence it wouldn't be realistic to assume that all the costs for returns of two different states are the same. This thesis considers the idea by Aras et al. (2004) where the products are categorized based on their quality (condition). The continuous-review base stock policy has been chosen to manage the inventory in our system and the formulation for a two-state system to calculate the cost has been adopted from [14].

Using the base stock policy, we were able to make inventory decisions for the servicizing scenario and find the optimal base stock level for the serviceable inventory position. We also varied the assumptions for the holding costs to study the change in the behavior of cost function with the decision parameters. All the holding costs were eliminated one at a time and also all together, in order to study the serviceable inventory in particular and determine the optimal base stock level ignoring the effect of all other parameters. It has been demonstrated with a numerical example that in the case where the remanufacturable inventory has a zero holding cost, the overall cost is minimum compared to other cases where the holding costs were ignored from the cost function. The study of the behavior of cost function with different assumptions concerning the holding costs gave us a detailed understanding of nature of different scenarios.

6.2 List of Assumptions

All the assumptions made in this thesis in both the replacement model and the inventory model are listed below.

- The customer has a working product available at all times. No demand is lost.
- Life of the product follows the Proportional Hazards Model.

- Condition of the product is observed at finite intervals and condition information is available only at these instants.
- All replacement decisions are made based on the age and condition of the system and only at decision instants.
- System deteriorates continuously over time and failure can occur at any instant. All failures are discarded and replaced immediately.
- The system can exist in two states only, defined by the value of Z.
- Customer demand and product returns are modeled as independent Poisson processes
- Remanufacturing returns an 'as good as new' product
- All the demand is satisfied by the remanufacturing process before resorting to manufacturing. Remanufacturing is assumed to be cheaper than manufacturing.
- Remanufacturing costs and leadtimes are assumed to vary with the condition of the product. Higher the quality of the return, lesser the remanufacturing effort required for processing it to 'as good as new' condition and lesser the cost and leadtime associated with it.
- All returns have their respective inventories depending on their condition.
- Type 0-first pull strategy is followed in remanufacturing the product returns. Whenever a demand occurs, Type 0 product is pulled into the remanufacturing process. In the case where Type 0 inventory is zero, the Type 1 products are pulled. If both the inventories are zero, the next available inventory is processed regardless of its type.
- The holding costs for the remanufacturable inventories of Type 0 and Type 1 products are the same.
- Manufacturing is instantaneous.

6.3 Limitations and Future work

This thesis enhances the current literature on replacement policy and inventory management, but further research is warranted.

This thesis assumes the demand and return rates to be processes from the replacement policy. This is an approximation. More accurate information on the distribution of demand and return rates will improve the current model and help us achieve more accurate results. Also, it enables us to integrate the replacement policy and the inventory policy into one whole which can address these two problems.

The inventory model in the thesis assumes all the returns to fall into only two categories. This suits our model since the replacement model is assumed to have two states only. However, Wu and Ryan (2008) have extended the same formulation for arbitrary number of states. It would be intriguing to integrate these states as categories in the inventory model and have '*n*' number of categories, one for each state or a range of states. This would expand the usage of this model.

This thesis assumes the manufacturing process is instantaneous. Though this assumption makes the model simpler, it is, however, not realistic. For better applications, the material flow and stock policies associated with manufacturing could be made more explicit. This would however add complexity to the computations.

Remanufacturing is assumed to be cheaper than manufacturing in this thesis. This needn't be true in all cases. There exist products where manufacturing a new product would cost much less than remanufacturing a used product due to issues related to parts availability, parts compatibility etc.

Another aspect in the inventory model that could be further researched is the holding costs of the remanufacturable inventories. This thesis considers the holding cost of the product returns prior to remanufacturing are all the same. This could be looked at, in one way, as all the products being of no use before being processed and in the manufacturer's point of view, mean the same. But however, the higher quality products consist of more value than the lower quality ones and the holding costs might differ in a certain cases.

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