A NEURAL NETWORK APPROACH FOR SOLVING INVERSE PROBLEMS IN NDE

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INTRODUCTION

Solution to inverse problems is of interest in many fields of science and engineering. In nondestructive evaluation [1], for example, inverse techniques are used to obtain quantitative estimates of the size, shape and nature of defects in materials. Inverse scattering problems in electromagnetics deal with estimation of scatterer information from knowledge of incident and scattered fields. Inverse problems are frequently described by Fredholm integral equations in the form

$$\int_a^b k(x,y)z(y)dy = u(x) \qquad (c \leq x \leq d) \tag{1}$$

where u(x) represents the measured data, z(y) represents the source function or the system states or parameters, and k(x,y) represents the kernel of the transformation. The objective of inverse problem is then to solve for the source or state function from known measurements. This problem is sensitive to the system parameters z, to the shape of the kernel k, and to the accuracy of the measurements u.

The solutions to these equations are generally ill-posed and various methods of solution have been proposed. The paper presents a novel strategy for solving Fredholm integral equations using Hopfield type neural networks. The major advantage of this method is the stability of the solution that comes from the high degree of parallelism and interconnectivity encountered in the neural networks.

The algorithm consists formulating the problem in the form of an error minimization equation. By comparing the error function to the energy function of the Hopfield neural network, the circuit parameters of the network are estimated. The operation of the network is simulated for obtaining the solution to the integral equation. Results of simulation for an inverse scattering problem in electromagnetics are presented.

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PROBLEM FORMULATION

Representing the system parameters function z in Eq. (1) as

$$z(x) = \sum_{i=1}^{N} v_i R_i(x) \qquad (2)$$

where R_i is a basis function, chosen to be sine or cosine function, Eq. (1) can be written in matrix form as

$$F\underline{V} = \underline{g} + \underline{n} \qquad (3)$$

where $F_{ij} = \int_a^b k(x, y_j) R_i(x) dx$ and $g_i = u(x_i)$.

The solution $\underline{V}$ is obtained by minimizing the cost function E where

$$E = \frac{1}{2}(F\underline{V} - \underline{g})^T (F\underline{V} - \underline{g}) + \lambda \underline{V}^T D \underline{V} + \lambda_1 (\Psi \underline{V} - z_p)^T (\Psi \underline{V} - z_p) \qquad (4)$$

The first term in the cost function is the model error, the second term is a smoothness constraint included to minimize the ill-posedness of the problem [2], and the third term represents the boundary conditions. Ψ is a matrix whose elements are basis functions R_i. z_p is the system state at a given point p, λ and λ_1 are Lagrange multipliers. In electromagnetic scattering problems, z_p represents the complex permittivity and can be expressed as

$$z_p = [\epsilon_p \ \sigma_p/\omega\epsilon_0]^T$$

The value of λ is chosen to compensate for the measurements noise in the cost function. Iterative technique for arriving at the optimal value of λ is given in [3]. λ_1 is chosen high enough to ensure the boundary conditions.

The cost function in Eq. (4) can be written in matrix form as

$$E = -\frac{1}{2}\underline{V}^T T^{\dagger} \underline{V} - \underline{V}^T \underline{I}^{\dagger} \qquad (5)$$

where:

$$\begin{aligned} T^{\dagger} &= -F^T F - \lambda D - \lambda_1 \Psi^T \Psi \\ \underline{I}^{\dagger} &= F^T \underline{g} + 2\lambda_1 \Psi^T \underline{\psi}_p \end{aligned}$$

NEURAL NETWORK IMPLEMENTATION

One method of performing this minimization is to use a deterministic Hopfield network [5]. A schematic of the network is presented in Fig. 1. Each neuron in the network is represented by an operational amplifier and the relation between the output v_i and input u_i of the i'th amplifier is given by a transfer function s with $v_i = s(u_i)$. The input of each amplifier is connected to ground through a resistor ρ_i in parallel with a capacitor C_i to simulate the delay of the response of a biological neuron. The output of neuron i is

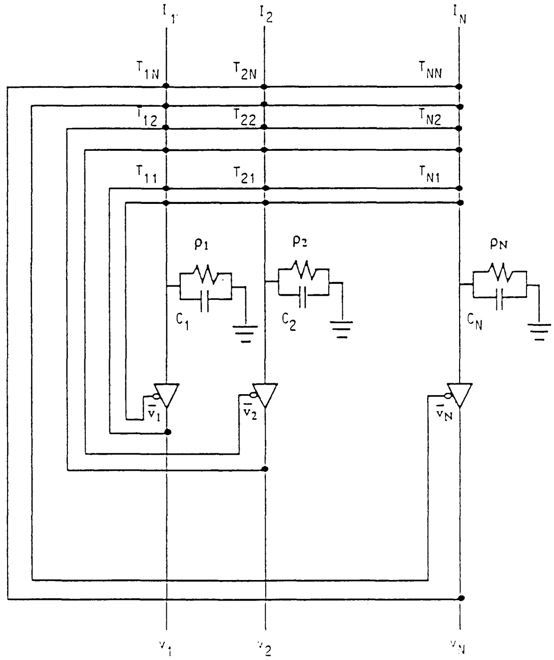

Figure 1. Hopfield neural network

connected to the input of neuron j by a synapse represented by a conductance $T_{ij} = 1/R_{ij}$. If the synapse is inhibitory, the conductor is connected to an inverted version of the output of the amplifier.

The differential equation describing the dynamics of amplifier i is given by

$$C_i \frac{du_i}{dt} = \sum_j \frac{v_j - u_i}{R_{ij}} - \frac{u_i}{\rho_i} + I_i \tag{6}$$

The performance of the network is governed by the global function which is the energy function E given by [5]

$$E = -\frac{1}{2} \sum_i \sum_j T_{ij} v_i v_j + \sum_i \frac{1}{R_i} \int_0^{v_i} s_i^{-1}(\xi) d\xi - \sum_i I_i v_i \tag{7}$$

The time derivative of the energy function is always less than or equal to 0 if the transfer function of the amplifiers is monotonically increasing. A simple transfer function is to choose a linear transfer function such that

$$s_i = \begin{cases} \alpha u_i & | u_i | \leq \gamma/\alpha \\ \gamma & u_i > \gamma/\alpha \\ -\gamma & u_i < -\gamma/\alpha \end{cases} \tag{8}$$

where α is the amplifier gain and γ is the saturation level.

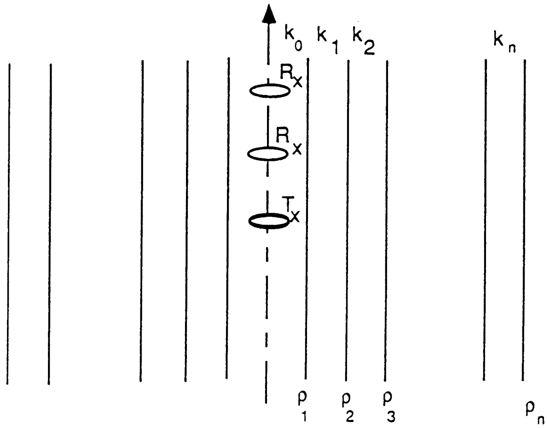

Figure 2. Coaxial cylinders model

For this amplifier transfer function, the neural network energy function can be written as

$$E = -\frac{1}{2}\underline{V}^T T \underline{V} + \frac{1}{2}\underline{V}^T T_r \underline{V} - \underline{V}^T \underline{I} \tag{9}$$

where

$$\begin{aligned} T_r &:= \text{diag}\ [1/(\alpha_1 R_1)\ 1/(\alpha_2 R_2) \ldots 1/(\alpha_n R_n)] \\ \underline{V}^T &:= [v_1\ v_2 \ldots v_N] \\ \underline{I}^T &:= [I_1\ I_2 \ldots I_N] \end{aligned}$$

By comparing Eqs. (5) and (9), we can derive the design values for the weights and external input currents of the neural network as

$$\begin{aligned} T &= T^\dagger + T_r \\ \underline{I} &= \underline{I}^\dagger \end{aligned} \tag{10}$$

APPLICATION TO ELECTROMAGNETIC SCATTERING

The Hopfield network parameters were derived for the inverse problem in electromagnetic scattering described in [6]. In this problem, it is required to reconstruct the permittivity and conductivity of finite number of subannuli of different wave numbers as shown in the axisymmetric geometry in Fig. 2. The excitation source is an incident wave launched from a circular loop located on the axis of symmetry and the measurements are taken of the scattered electric field using circular loops at different locations along the axis. Formulation of the forward problem for the axially symmetric cylinders is given by Chew [7].

The problem has to be solved iteratively, and the governing equation is

$$E_\phi(\rho, z) - E_\phi^0(\rho, z)$$

$$= \int_{Z'=-\infty}^{\infty} \int_{\rho=R_I}^{R_0} G^0\left[(\rho-\rho'),(z-z')\right]\ k_0^2 \delta\epsilon_c\ E_\phi(\rho',z')\ d\rho' dz' \tag{11}$$

$$= \int_{Z'=-\infty}^{\infty} \int_{\rho=R_I}^{R_0} G^0\left[(\rho-\rho'),(z-z')\right]\ k_0^2 \delta\epsilon_c\ E_\phi^0(\rho',z')\ d\rho' dz' \tag{12}$$

where Eq. (12) is obtained using Born approximation. E^0 is the electric field corresponding to initial known complex permittivity distribution ϵ_c^0, and E is the electric field corresponding to a distribution $\epsilon_c^0 + \delta\epsilon_c$, and G is the Green's function. For axially symmetric problem with a circular loop of radius a as the source, the relation between electric field and Green's function is given by

$$E_\phi^0 = j\omega\mu I\ a\ G^0 \tag{13}$$

Taking Fourier transforms, Eq. (12) can be written as

$$g(R,a,k_z) - g^0(R,a,k_z) = 2\pi k_0^2 \int_{R_I}^{R_o} d\rho'\ \delta\epsilon_c(\rho')\ g^0(\rho',a,k_z) g^0(R,a,k_z) \tag{14}$$

which can be put in the form

$$h_c(k_z) = \int_{R_I}^{R_o} d\rho' Q_c(\rho') K_c(\rho,k_z) \tag{15}$$

Eq. (15) is a Fredholm integral equation and can be solved using the neural network implementation described above. The discrete-form matrix equation will be in terms of complex variables. The equation can be transformed into an equation in real variables by considering the real and imaginary parts separately.

Another method for converting Eq. (15) into a real valued equation consists of using only amplitude measurements of the scattered electric field [8]. This results in the equation

$$\int_{R_I}^{R_o} d\rho' Q_c(\rho') K_c(\rho',k_z) = \delta g_c^0(k_z) \tag{16}$$

with $\Re\{\frac{\delta g^0}{g^0}\} = \delta \log |g^0|$ Therefore,

$$\int_{R_I}^{R_o} d\rho' \left\{ \Re\left\{\frac{K_c}{g^0}\right\} \Re\left\{Q_c(\rho')\right\} - \Im\left\{\frac{K_c}{g^0}\right\} \Im\left\{Q_c(\rho')\right\} \right\} = \delta \log |g^0| \tag{17}$$

which is in the form of a Fredholm equation.

The step by step algorithm for performing the inversion is given below.

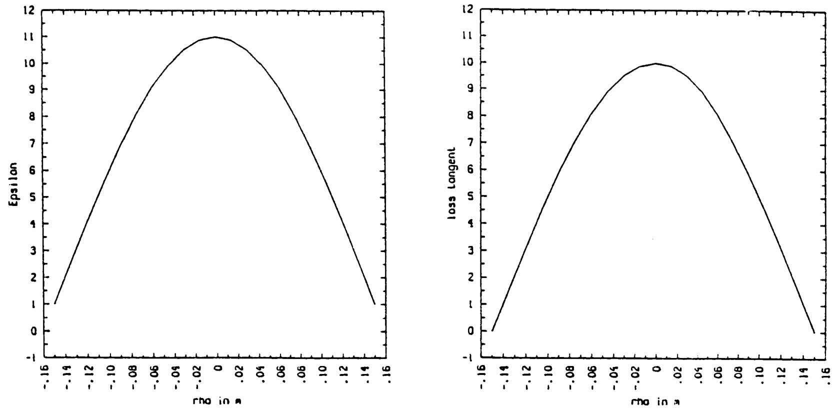

Figure 3. True distributions.

1. Obtain the measurements of electric field at different location and normalize them to get G(R,z) and compute Fourier transform to get $g(R, k_z)$.
2. Assume initial guess for ϵ_c.
3. Solve for g^0 from the forward model.
4. Substitute g^0 in either equation (15) or (17).
5. Cast the integral equation in a discretized form and use a neural network implementation to obtain solution for ϵ_c.
6. Go to 3.

RESULTS

The algorithm was implemented by simulating the Hopfield network and using a modified gradient descent algorithm [9]. Figure 3 shows the true permittivity ϵ, and loss tangent $\frac{\sigma}{\omega \epsilon_0}$. Figures 4, and 5 show the initial values, and reconstructed values after first, second and third iterations. Figures 6, and 7 show the reconstructed values using the amplitude of the scattered wave.

CONCLUSIONS

Preliminary results indicate that the proposed method has considerable promise. The major advantage of this approach is that it is guaranteed to be stable and is potentially capable of deriving the globally minimum error solution. Moreover, the method is eminently suited for optical or VLSI implementation, thereby offering solutions in real time.

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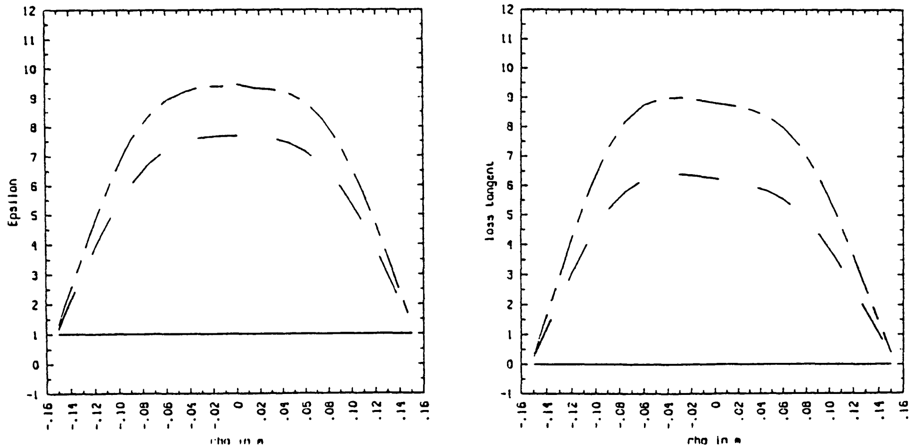

Figure 4. Initial value, 1st, and 2nd iterations.

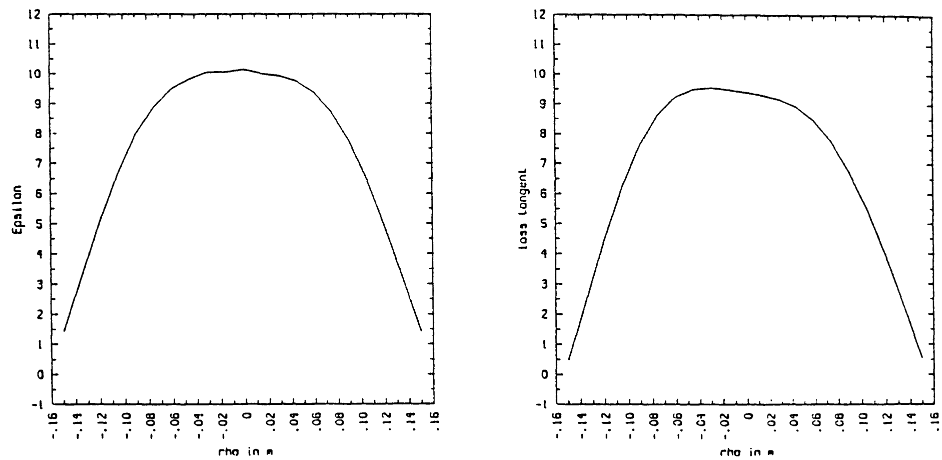

Figure 5. 3rd iteration.

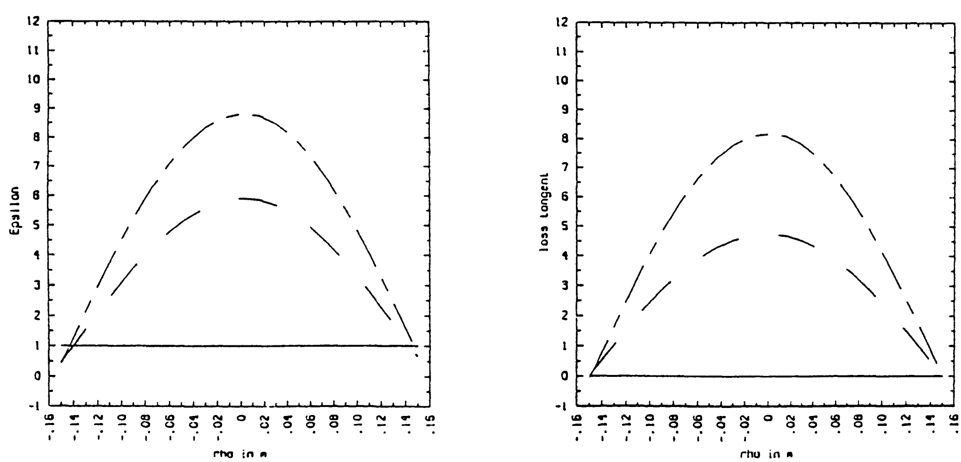

Figure 6. Initial value, 1st, and 2nd iterations using amplitude of measurements.

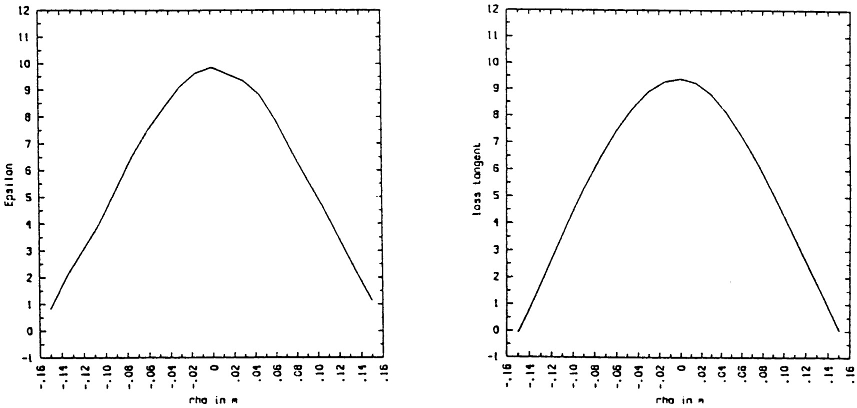

Figure 7. 3rd iteration using amplitude of measurements.

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