RADIOGRAPHIC IMAGE ENHANCEMENT BY WIENER DECORRELATION

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INTRODUCTION

The primary focus of the application of image processing to radiography is the problem of segmentation. The general segmentation problem has been attacked on a broad front [1, 2], and thresholding, in particular, is a popular method [1, 3-6]. Unfortunately, geometric unsharpness destroys the crisp edges needed for unambiguous decisions, and this difficulty can be considered a problem in filtering in which the object is to devise a high-pass (sharpening) filter. This approach has been studied for more than 20 years [7-13].

The inverse filter can be devised either in the spatial domain or the frequency domain. Both cases require either a model of the degradation process (geometric unsharpness), or a set of "ideal" image/ degraded image pairs from which an inverse filter can be calculated by some estimation method (such as least squares). In the case of radiography with an incoherent source, a simple model of the imaging process leading to a frequency domain solution can be developed.

The remainder of this paper describes a model of the radiographic process and the consequences of certain conditions and simplifying assumptions. A particular method of inverse filtering (Wiener filtering) is applied to some digitized radiographs and the results are discussed.

IMAGING PROCESS

The formation of radiographic images on a plane located at $z = Z$ can be shown [14] to be a superposition integral of the form

$$i(x,y,Z) = \int \rho(x',y') \exp\left(-\int_0^{Z/w} \mu(x'+ut,y'+vt,wt) dt\right) \frac{Z}{2\pi w^3} dx'dy'$$

(1)

where $u$, $v$, and $w$ are the direction cosines with respect to the $x$-, $y$-, and $z$- axes, $\rho(x',y')$ is the intensity of an isotropic source at $z = 0$, $\mu(x'', y'', z'')$ is the attenuation coefficient along the ray from
(x', y', 0) to (x, y, Z), and R is the distance from (x, y, Z) to 
(x', y', 0).

Consider the case of a single, thin attenuator located at z = z'', as 
shown in Figure 1. Equation (1) reduces to

\[ i(x,y,Z) = \int \rho(x',y') \ T(x'+\alpha x', y'+\beta y') \ \frac{Z}{2\pi R^2} \ dx'dy' \]

where \( t = z''/\omega, \) a constant. If R is not a strong function of (x', y'),
then the solid angle term (the third multiplicand in the integral) can be
removed from the integral. Now, the direction cosines are given by
\( u = (x-x')/R, \ v = (y-y')/R, \) and \( w = Z/R. \) Making these substitutions in
equation (2) yields

\[ i(x,y,Z) = \Omega \int \rho(x',y') \ T(x'+\frac{z''}{Z}(x-x'), y'+\frac{z''}{Z}(y-y')) \ dx'dy' \]

where \( \Omega \) is the solid angle term.

Write \( k = z''/Z, \) \( p = ((1-k)/k)x', \) \( q = ((1-k)/k)y'. \) Equation 3 now
becomes

\[ i(x,y,Z) = \Omega \left( \frac{k}{1-k} \right)^2 \int \rho(\frac{k}{1-k} p, \frac{k}{1-k} q) \ T(kx+p, ky+q) \ dpdq \]

This is the correlation integral of \( \rho(kx/(1-k), ky/(1-k)) \) and \( T(kx, ky), \)
and penumbra is seen to be generated by the action of correlation. The
first term is the inverted projection of the source onto the detection
plane through a pinhole located in the object plane. The second term is
the transmission function projected onto the detection plane by a point
source at the origin. Thus, the prescription for recovering the
transmission function is to obtain a pinhole image of the X-ray source, a
radiograph of the desired object, and then perform a decorrelation.

![Fig. 1. Imaging geometry.](image-url)
It might be noted here that correlation and convolution are not identical operations. Convolution differs from correlation in that one of the operands is inverted with respect to the origin. Consequently, convolution is commutative while correlation is not. In addition, convolution and correlation can be equivalent only if at least one of the operands is symmetric.

DECORRELATION

The Fourier transform of equation 4 is easily found to be

\[
I(r, s) = 2\pi \sum_{k} P(k, k) \left( \frac{1-k}{k} r, \frac{1-k}{k} s \right) T(k, k)
\]

where \( r, \text{ and } s \) are the transform variables, \( P(k, k) \) is the transform of \( \rho(k, k) \), \( T(k, k) \) is the transform of the transmission function, \( I(k, k) \) is the transform of the image function, and \( \dagger \) indicates complex conjugation. Clearly, if the transforms of the measured image and the pinhole image are generated, in the absence of noise and/or measurement error, \( T(k, k) \) can be recovered by division.

Unfortunately, ideal conditions are unlikely, so zeros in \( T(k, k) \) do not usually cancel zeros in \( P(k, k) \). This leads to unrealistic oscillations in the decorrelated image. Many methods to mitigate this result exist and one simple technique is Wiener filtering [15, 16]. This method has been used by several authors [7-12] with holographic systems to perform deconvolution. The filter is implemented in this paper with a digital computer and some interesting properties are revealed.

The Wiener filter calculates an estimate of the true image given the measured image and a model of the degradation process. The filter equation for decorrelation is given by

\[
T'(r, s) = C \frac{P(r, s)}{|P(r, s)|^2 + D_n(r, s)/D_T(r, s)} I(r, s)
\]

where \( C \) is the collection of all constant terms in equation 4, \( T' \) is the estimate of \( T(k, k) \), and \( D_n \) and \( D_T \) are the power spectra of the noise process and the true transmission function, respectively. This filter is optimal in the sense that it minimizes the mean square error between \( T(x, y) \) and \( T'(x, y) \).

Equation 6 differs from the usual Wiener filter equation by having the numerator \( P \) rather than \( P' \). However, a property of the Fourier transform is that the conjugate of the transform equals the transform of the function inverted through the origin. Thus correlation is equivalent to convolution with the inverted image. Consequently, the pinhole image of the X-ray source can be used directly, since it is inverted through the pinhole.

Equation 6 includes the power spectra of the noise and the true image, neither of which are known. We have followed the suggestion of Gonzalez and Wintz [1] and replaced that term with an adjustable constant in calculating the decorrelations below. The equation used is

\[
T'(r, s) = C \frac{P(r, s)}{|P(r, s)|^2 + K^2} I(r, s)
\]

APPLICATION TO RADIOGRAPHS

Figures 2 and 3 show a pinhole image of an X-ray source and the Fourier transform of the image. It is seen that Figure 2 is not the image of a symmetric function. The image has been scaled logarithmically
Fig. 2. Pinhole image of X-ray source.

Fig. 3. Fourier transform of Figure 2.

in Figure 3 so that the peak component is bright white. As expected, the pinhole image behaves as a low pass filter, however significant high-frequency components are present. Furthermore, numerous regions of small amplitudes are also evident, that, if not cancelled by zeros in the transform of the transmission function, will lead to spatial oscillations in the decorrelated result.

Figures 4-6 each show four images. The image in the upper left is the original radiograph of some lead numerals and is identical in each in order to serve as a reference. Figure 4b shows the effects of small amplitudes in the transform of the pinhole image; spatial oscillations are seen around the 60. The constant used was approximately 4% of the peak amplitude of the pinhole transform. A constant of 16% and 32% of the peak amplitude was used in Figures 4c and 4d, respectively. Note that the letters have sharply defined borders and their gray level is uniform. The separation between the 6 and the 0 is maintained, but the distinction between the 3 and 5 is lost. The latter, however, is obviously faint to begin with.

Figure 5 shows the progression of K from 40% to 60% of peak amplitude. The sharp border remains, but the overall image quality appears to be degrading with increasing K. In Figure 6, K increases from 80% to 140% of peak amplitude, and the images show increasing blurring.

The backgrounds in Figures 4-6 are wire meshes of 60/inch and 35/inch (hence the numerals). The triangular mesh above and to the left of the 60 is 24/inch, and above and to the left of the 35 is 20/inch. It can be seen that the intersections of the triangular patches are degraded by the decorrelation operation in Figure 4b, darkened in Figure 4c, and blurred beyond distinction in Figures 4d, 5, and 6. The 60/inch mesh is enhanced in Figure 4a, and lost thereafter. The 35/inch mesh appears to be best enhanced in Figure 4c.

These effects are understandable in terms of equation (7). When K is small, the equation implements direct decorrelation with the observed spurious oscillations occurring because of incompletely cancelled poles in the transfer function. As K increases, the large-amplitude near-origin region of \( \tilde{P}(r, s) \) remains dominant while the poles become masked.
Fig. 4. Clockwise from upper left: a) Original image, b) $K = 0.04$, c) $K = 0.16$, d) $K = 0.32$.

Fig. 5. Clockwise from upper left: a) Original image, b) $K = 0.40$, c) $K = 0.48$, d) $K = 0.60$. 
As $K$ grows even larger, it begins to dominate $p'(r,s)$ and equation (7) approaches

$$T'(r, s) = C P(r,s) I(r,s)$$

where $C$ now includes the factor of $K^2$ from the denominator. Equation 8 is a simple convolution and, in fact, is almost exactly the opposite of the desired operation.

From the above, it is seen that the choice of $K$ in equation 7 changes the action of the filter from pure decorrelation (which in this model is, in effect, a high-pass operation) to pure convolution (which is a low pass model). Judicious choice of $K$ enhances specific spatial frequency components of the image, allowing the same filter to bring out different features. Thus, the filter can also exhibit bandpass characteristics.

SUMMARY

It has been shown how a simple model of the radiographic imaging process has led to a straightforward method of enhancement. Wiener filtering was employed as a method to mitigate the effects of grain noise in the film and digitization error, and it was found that the same filter form could be used as a low pass, bandpass, and high pass filter by the adjustment of a single parameter. Correct choice of the parameter allowed edge definition combined with field flattening, smoothing, and selective feature enhancement.
ACKNOWLEDGEMENT


REFERENCES

15. Rafael C. Gonzalez and Paul Wintz, op. cit., Chapter 5.