THE EXISTENCE OF LOW LOSS LAMB MODES IN HIGHLY ATTENUATIVE MEDIA

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INTRODUCTION

A great deal of work on Lamb waves has been devoted to their application to materials which are assumed to be elastic and hence have negligible material attenuation (see e.g. [1-2]). However for materials such as polyethylene, the assumption of elasticity may be invalid as the amplitude of the Lamb waves will decrease with propagation distance, a phenomenon not accounted for in the elastic treatment. From a dispersion curve point of view, the interest lies in determining whether the inclusion of attenuation will greatly change their characteristics. In most, if not all, circumstances the bulk shear wave attenuation for any given material is considerably larger than that for the bulk longitudinal wave. Given that Lamb waves can be modelled by the superposition of such bulk waves, it is believed that the shear wave attenuation will be the dominant factor on the character of the dispersion curves.

In this paper we will present theoretical results showing the effect of introducing attenuation on dispersion curve predictions. The attenuation will be introduced gradually, from zero, up to a value which corresponds to that of a material known as High Performance Polyethylene (HPPE) which is currently being used in a project at Imperial College [3], and then to a value considerably greater than this. We will then go on to discuss the practical implications of the dispersion curves, such as mode selection. Subsequently, results from experiments carried out to verify the existence of one of the low attenuation modes identified as being of potential NDE use are presented.

LAMB WAVE MODEL

The Lamb wave model used for the dispersion curve predictions is based on the model used by Lowe[1]. The model assumes that the acoustic field within a plate can be described by the superposition of four bulk plane waves (2 longitudinal and 2 shear), these waves being called partial waves. The waves are summed in such a way that the boundary conditions of zero traction at the plate surfaces are satisfied. In order to model the attenuation of the material, each partial plane wave is assumed to have a complex wavenumber

$$k = k_r + ik_i \qquad (1)$$

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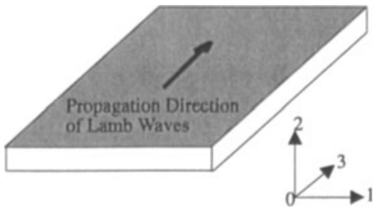


DIR^N	LONG. BULK vel. m/s	LONG. BULK att. Np/wl	SHEAR BULK vel. m/s	SHEAR BULK att. Np/wl
0-1	2344	0.055	953	0.286
0-2	2480	0.058	1024	0.215
0-3	2344	0.055	953	0.286

Figure 1. Bulk wave properties for high performance polyethylene

where k_r is the real part of the wavenumber representing the propagation of the wave and k_i is the imaginary part representing the attenuation of the wave. A further assumption is that the attenuation k_i is assumed to be frequency dependent, the dependence being linear. This assumption was made on the basis that it seemed to represent fairly well the attenuation behaviour of real materials used in the laboratory at Imperial College. The method used for solving the resulting characteristic frequency equation is that formulated by Lowe[1] and is based on a global matrix technique.

PROPERTIES OF HIGH PERFORMANCE POLYETHYLENE (HPPE)

The material used for the dispersion curve predictions is High Performance Polyethylene (HPPE) which is a form of high density polyethylene specifically produced for use in water and gas pipes. Given that the dispersion curve predictions require both bulk longitudinal and bulk shear wave attenuation values to be inputted to the model, these were measured, along with the corresponding bulk velocities in three orthogonal directions of the plate. The thickness of the plate was 12.7mm. The results are shown in Fig.1, the attenuation being expressed in Nepers per wavelength. The attenuation has been normalised with respect to wavelength because the attenuation for each wave type is constant per wavelength. It can be seen that the attenuation of bulk shear waves is considerably larger than that of bulk longitudinal waves and that the material is slightly anisotropic. However, the anisotropy is not large and therefore it was assumed that the material was isotropic. The material properties in the 0-3 direction were used in the predictions since this was the direction of wave propagation in the experiments.

DISPERSION CURVE PREDICTIONS AT DIFFERENT ATTENUATIONS

In all the following dispersion curve predictions, the same bulk longitudinal and bulk shear wave velocities are used throughout. The dispersion curves were calculated for a 12.7mm thick plate as used in the experiments and therefore have been plotted using a frequency axis rather than the conventional frequency-thickness axis.

The first case we will look at is the case where shear waves cannot exist. Therefore, only longitudinal waves can propagate. This case is discussed because of its implications later on in the paper. It is further assumed that no attenuation is present in the material. The phase velocity dispersion curves for this particular set-up are shown in Fig.2a. At cut off, all modes are through-thickness longitudinal bulk waves and as the frequency increases all modes tend towards the longitudinal bulk velocity. We shall refer to this as the fluid-plate case as these dispersion curves can be found in a plate consisting of a fluid (i.e. a medium which is unable to support shear stresses) (see e.g. [4]).

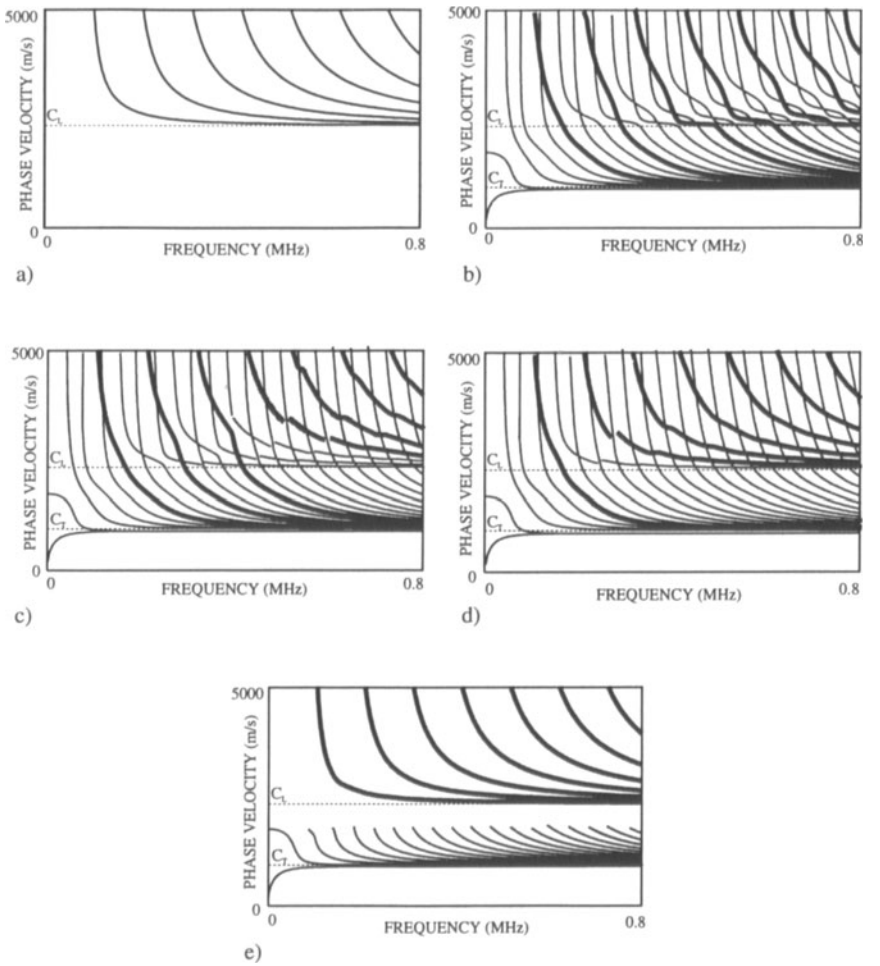


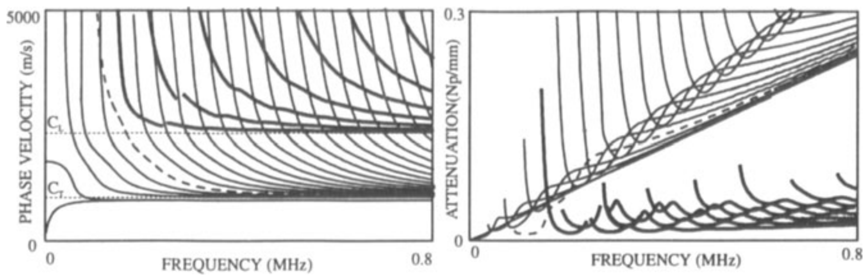
Figure 2. Dispersion curves for a) fluid-plate; b) zero attenuation; c) half attenuation (α_L = 0.0275Np/wavelength, α_S=0.143Np/wavelength); d) full attenuation (α_L = 0.055Np/wavelength, α_S = 0.286Np/wavelength); e) four times attenuation (α_L = 0.220Np/wavelength, α_S = 1.144Np/wavelength). Longitudinal asymptotic modes shown in bold.

If we now introduce the ability for shear waves to propagate (while still maintaining zero attenuation), then the phase velocity dispersion curves are modified and are shown in Fig.2b. The fact that shear waves can now exist means that coupling between longitudinal and shear waves can occur [4-5] and the result of the new coupling conditions is that we have many more modes than before within the same frequency range and all modes with the exception of the fundamental modes tend towards the bulk shear wave velocity. It should also be noted that as the frequency increases, the modes seem to plateau around the bulk longitudinal wave velocity creating a terrace effect. This plateauing behaviour is temporary as the modes then continue down towards the bulk shear wave velocity.

By introducing half the amount of attenuation measured from the HPPE plate (i.e. longitudinal attenuation α_L= 0.0275 Np/wavelength and shear attenuation α_S = 0.143 Np/wavelength), we obtain the phase velocity dispersion curves shown in Fig.2c. The curves can be divided into two interesting regions, one at the low frequency end (below about 0.3MHz) and the other above this frequency. At the low frequency end the dispersion curves are very similar to those for the zero attenuation case. However, at the high frequency end, significant changes have started to take place. All the longitudinal cut-off modes are now asymptotic to the bulk longitudinal wave velocity. One possible explanation for this phenomenon is that because the shear attenuation is so much higher than the longitudinal attenuation, it is the shear waves which will be most affected by the inclusion of attenuation. The shear waves will tend to be "killed off" by the large attenuation and we are essentially left with longitudinal waves, which as we know will tend to produce fluid-plate dispersion curves (i.e. curves asymptotic to the bulk longitudinal wave velocity). The reason why this phenomenon is occurring at the higher frequencies is that the shear attenuation increases with frequency so that the modes at the higher frequencies will suffer larger attenuations than those at the lower frequencies. These large attenuations are then sufficient to make the mode heavily dominated by the longitudinal wave. At the lower frequencies, this is not the case. If we were to increase the amount of attenuation input into the model, we would expect from the previous argument that the onset of the longitudinal asymptotic mode behaviour would occur at a lower and lower frequency. As a sidenote, it can be seen that various discontinuities exist on the dispersion curves. These were areas in which the characteristic equation could not be solved and there are several possible explanations for this. One explanation is that the rate of change of attenuation was so large in these areas that convergence to a solution was extremely difficult. Another explanation is that in these discontinuous regions, the wavenumber becomes purely imaginary. These solutions cannot be found using the existing software since it was developed purely for finding propagating modes.

If the attenuation is now increased to the full amount for HPPE (i.e. longitudinal attenuation α_L= 0.055 Np/wavelength and shear attenuation α_S = 0.286Np/wavelength) then the phase velocity dispersion curves of Fig.2d are produced. As predicted, the onset of the longitudinal asymptotic modes now occurs at a lower frequency than before.

By increasing the attenuation to four times the full amount (i.e. longitudinal attenuation α_L = 0.220Np/wavelength, shear attenuation α_S = 1.144Np/wavelength) the phase velocity dispersion curves of Fig.2e are obtained. We can see that by this stage all longitudinal cut-off modes have become longitudinal asymptotic. Note that the shear asymptotic modes have not been plotted to their cut-off frequencies. This was because the attenuation for these modes were so high that solutions could not be found. Therefore, if the attenuation is sufficiently high, it is possible to have two distinct groups of Lamb modes; one which is longitudinal at cut-off and longitudinal asymptotic at high frequency; the other which is shear at cut off and shear asymptotic at high frequency. If we now compare the longitudinal asymptotic modes with those



a) b)

Figure 3. a) Phase velocity and, b) attenuation dispersion curves for the case of full attenuation (α_L = 0.055Np/wavelength, α_S = 0.286Np/wavelength). Longitudinal asymptotic modes shown in bold; longitudinal cut-off, shear asymptotic modes shown dotted.

of the fluid-plate in Fig.2a, we indeed see that the curves are practically identical, hence reinforcing the idea of the shear waves being "killed off" by high attenuation in these longitudinal cut-off modes.

PRACTICAL IMPLICATIONS

From a practical point of view, if a mode is to be used for inspection purposes, then one of the most important properties is the attenuation. The dispersion curves for attenuation are shown in Fig.3 together with the phase velocity dispersion curves for the case of full attenuation.

Turning our attenuation to the attenuation dispersion curves, we can see that there are two groups of modes with low attenuation, those which are longitudinal asymptotic (bold) and those which are longitudinal at cut-off and shear asymptotic (striped). In this particular case only one longitudinal cut-off, shear asymptotic mode exists. This mode has a localised low attenuation region near its cut-off position. As the frequency increases, the attenuation increases rapidly to join up with the high attenuation shear cut-off, shear asymptotic modes. Note that if the bulk shear attenuation is high enough, this mode will join the longitudinal asymptotic group of modes and hence have considerably lower attenuation at higher frequencies.

Given that there are a considerable number of low attenuation modes, the question of which mode or modes to use for NDE must be addressed. In many circumstances, exciting multiple modes may not be a problem if the modes have sufficiently different velocities and attenuations as this would tend to give modes which are well separated in the time domain, hence allowing individual modes to be identified. However, in this particular case, the longitudinal asymptotic modes tend to be grouped closely together in attenuation, phase velocity and group velocity space, and as a result individual signals may be difficult to resolve. If we turn to the longitudinal cut-off, shear asymptotic mode which is the second order symmetric mode (s_2 using the standard convention for Lamb waves in elastic media) then the situation becomes somewhat more favorable. Near its cut-off frequency, there is a region of low attenuation which is fairly well isolated from neighbouring modes. Hence if the mode is excited within this frequency region, a single mode will propagate and in fact it will propagate with the lowest attenuation of all the modes (except the fundamental modes which have lower attenuation albeit at impractical (very low) frequencies).

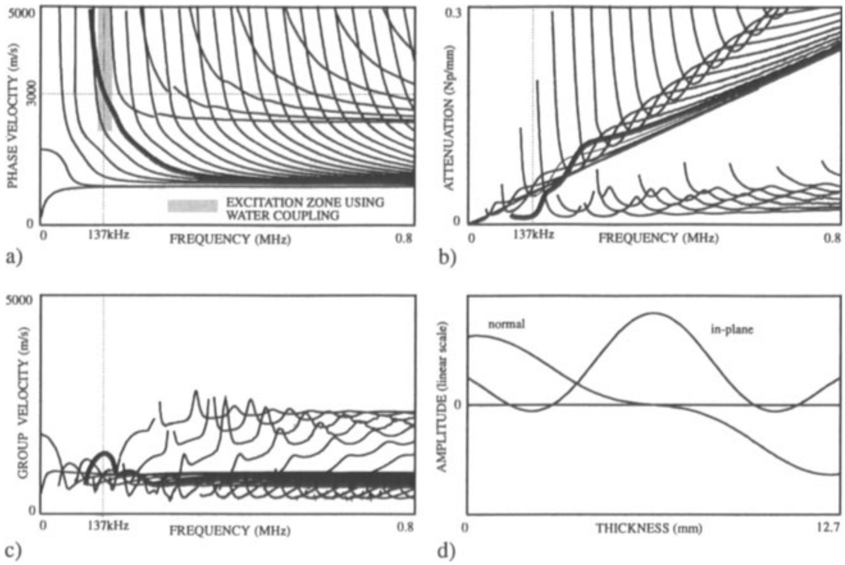


Figure 4. Dispersion curves for 12.7mm thick HPPE plate highlighting s_2 mode.
a) phase velocity; b) attenuation; c) group velocity; d) displacement mode shapes of s_2 mode at 137kHz.

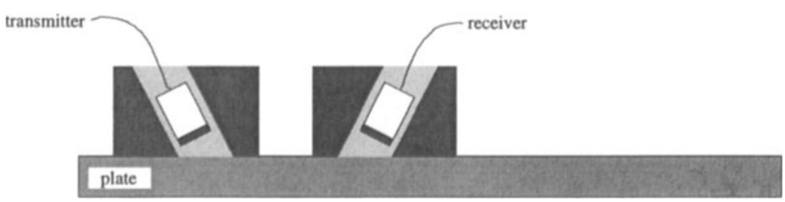


Figure 5. Schematic of set-up used for experimental verification of s_2 mode

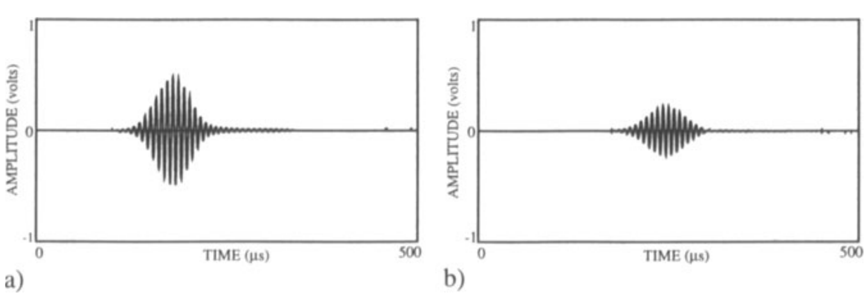


Figure 6. Time domain response of plate a) at first receiver position; b) at second receiver position.

Table 1. Comparison of predicted and measured group velocities and attenuations for s_2 mode.

	PREDICTED	MEASURED	% DIFF.
GROUP VEL.(m/s)	1370	1364	0.4
ATT.(Np/mm)	0.0105	0.0090	17

EXPERIMENTAL VERIFICATION OF A LOW ATTENUATION MODE

From the previous discussion, we have seen that the longitudinal cut-off, shear asymptotic mode s_2 was very favorable for practical NDE use. Hence this mode was chosen for experimental verification. The phase, group and attenuation dispersion curves are replotted in Figs.4a-c, with the required mode shown in bold. Given that the minimum in attenuation corresponds to a maximum in group velocity (and hence a point of minimum dispersion), this point was selected for excitation. The frequency was 137kHz and the phase velocity 3000m/s. At this frequency, the attenuation is 10.5Np/m. If we compare this value to the bulk longitudinal attenuation (3.2Np/m) and bulk shear attenuation (41.1Np/m), we can see that the Lamb wave value lies between the two bulk values as expected and is closer to the bulk longitudinal attenuation. Fig.4d shows the displacement mode shapes for the excitation point. We can see that there is a large normal component of displacement at the plate surface which would favor an excitation set-up consisting of normal perturbations to the surface of the plate. The mode can therefore be excited using fluid coupling and water was consequently chosen as the coupling medium. The transducers used were 1 inch, 0.5MHz centre frequency transducers which were excited using a 137kHz, 20 cycle tone burst from an arbitrary function generator via a power amplifier and were sufficiently broadband to get a good output at 137kHz. Using the coincidence principle, the angle of the transducers in water was set at 30°. The excitation zone produced by the angular bandwidth of the finite sized transducers and the frequency bandwidth of the excitation signal was calculated roughly and is marked on the dispersion curves. A simple pitch-catch arrangement was used and is shown in Fig.5.

The measurement procedure consisted of capturing the received signal in one position and then moving the receiver to another position and once again capturing the response. The separation distance between the two receiver positions was noted down and the time of flight measured. The captured time domain responses are shown in Figs.6a and 6b. It can be seen that the dispersion is very low so the group velocity could be measured simply from the time of flight. The attenuation α was obtained by measuring the amplitudes of the two received signals. The results were compared to those predicted from the dispersion curves and are shown in Table 1. We can see that the correlation in group velocity is very good whereas the agreement in attenuation is less good. However, given that attenuation measurements are inherently difficult to make as they depend on amplitude measurements which in turn are very sensitive to the inclination angle of both transmitter and receiver (see e.g. [6]), the error of 17% is considered to be sufficient for verification purposes. The predicted attenuation was found to be larger than that measured. One possible reason for this is that the bulk shear attenuation that was used in the predictions may have been too high as it was taken from the 0-3 direction rather than say the 0-2 direction which had an attenuation 25% smaller.

CONCLUSION

In this paper we have looked at the effect of introducing material attenuation into dispersion curve predictions. We have identified that the bulk shear wave attenuation is the dominant factor in changing the nature of dispersion curves of polyethylene. The main effect of the attenuation is to alter the coupling conditions between longitudinal and shear waves such that fluid-plate type modes start to appear. We have also seen that there are two classes of mode with relatively low attenuation. For practical inspection, the mode corresponding to the s_2 mode in an elastic plate which is longitudinal through-thickness at cut-off and is asymptotic to the bulk shear velocity at high frequencies is attractive and the properties of this mode have been verified experimentally.

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