# EXTENDED MAGNETIC POTENTIAL METHOD FOR QUASI-STATIC ELECTROMAGNETISM AND EDDY CURRENT PHENOMENA

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## INTRODUCTION

### Background

In-service eddy current (EC) inspection of aircraft engine components is one of the most demanding NDE applications. Any defect in such high-performance components will have serious consequences, and hence reliable periodic inspections are required. Yet, it is known that significant improvements to the current implementation are desirable in various aspects. One of our recent modeling activities is directed toward developing a software simulator that is able to predict the performance of inspections and, hence, to examine any improvement ideas such as new probe designs on computers.

The major challenge for such modeling tasks is to deal with the geometrical complication of parts and probes. Consider a conceptual EC inspection system as illustrated in Figure 1. The system may consist of a part and a probe. The part may be of any shape, and contain geometrical singularities such as edges and cracks. The probe may be also of complex constructions, and its constituent objects, *i.e.*, cores and coils, may have arbitrary shapes. It should be noted that this probe-part combination must be treated as one system as a whole because of the small probe lift off that is characteristic to EC inspections.

To solve electromagnetic equations for this type of systems, we have chosen the boundary element method (BEM) as the most suitable numerical technique. The BEM is as flexible as the conventional 3D finite element method (FEM) in dealing with arbitrary component geometry, and yet, since it uses Green's functions, it does not require excessive amounts of computational resources unlike the FEM. The volume integral method, often used in half-space and cylindrical geometries, is less flexible in handling arbitrary geometry because it requires analytically soluble, reference problems [1].

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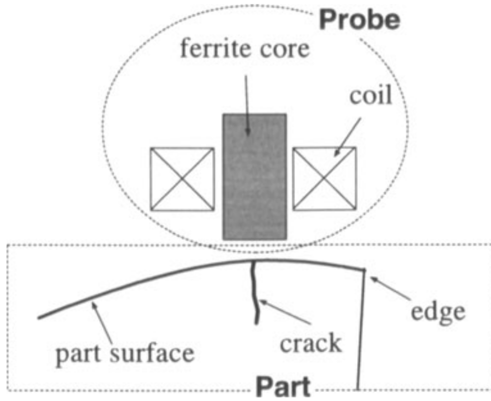


Figure 1. Illustration for a schematic eddy current inspection system. The proposed modeling algorithm allows the introduction of ferrite cores with minimal expense. It makes no assumptions on object geometry.

Purpose of the paper

There are as many BEM approaches to electromagnetism as there are a variety of underlying boundary integral equation (BIE) formulations. The most popular among those is the Stratton-Chu (SC) approach [2]. In this paper, however, we present an alternative BIE formulation that uses an extended use of the scalar magnetic potential method. It is our contention that the so-called extended magnetic potential (EMP) method is superior to the SC approach for our eddy current problems.

The focus of this paper is to present the extended scalar magnetic potential formulation for quasi-static electromagnetism. (See also Reference [1].) We will derive the governing BIEs for each of the air, core, and part regions of the system described in Figure 1. We will mention the benefits of our method over the SC formulation, but defer the details to our companion papers [3,4] that will describe the software implementation of the EMP formulation, its numerical tests, and its application to computer-aided EC probe designs. Throughout the paper, we assume that the quasi-static condition holds, known to be valid in most in-service EC inspections.

The following section contains the main contribution of this paper, where we present the extended magnetic scalar potential formulation. First, we introduce magnetic scalar potentials $\psi$ in air and core regions in such a way that their multi-valuedness can be avoided. Second, we extend the potential $\psi$ into the metal region via a field equation and a continuity condition. Third, we list the complete set of our BIEs which will be solved subsequently via the BEM. The subsequent section is devoted to discussing the merits of the extended potential method, and the status of its applications. The last section is for conclusions.

## EXTENDED MAGNETIC SCALAR POTENTIAL METHOD

### Single-Valued Scalar Potentials in Air and Core Regions

Let us first consider coils in air. Let $\vec{j}$ denote the current density of the coils, which we consider fixed by assuming constant drive current. In air, they generate magnetic field $\vec{H}^0$ according to

$$\vec{\nabla} \times \vec{H}^0 = \vec{j}\,. \tag{1}$$

When ferrite cores are present, the magnetic field $\vec{H}$ is modified from $\vec{H}^0$ but still satisfies the same equation

$$\vec{\nabla} \times \vec{H} = \vec{j}\,. \tag{2}$$

In Equations (1) and (2), the displacement current is dropped under the quasi-static condition. Taking the difference between (1) and (2), one finds that a scalar function $\psi$ can be introduced such that

$$\vec{H} = \vec{H}^0 + \vec{\nabla}\psi\,, \tag{3}$$

and if $\vec{j}$ is divergence-free,

$$\nabla^2\psi = 0\,. \tag{4}$$

Equation (4) can be integrated to yield the standard Green's formula

$$\psi = \int_S dS'\left[\left(-\nabla'_n G_0\right)\psi + G_0 \nabla'_n \psi\right], \tag{5}$$

where $\vec{n}$ is the outward-directed normal and where $G_0 = 1/4\pi r$. Clearly, Equations (3)-(5) hold in both air and core regions.

<u>Extension of the Magnetic Scalar Potential into Conducting Parts</u>

Next, we will add conducting objects into our consideration. Since the analog of Equation (2) is

$$\vec{\nabla} \times \vec{H} = \sigma\vec{E} \tag{6}$$

in this region, scalar potentials are not usually introduced. Notice, however, that it is possible to extend the potential $\psi$ from the air side into metals via continuation. Let us introduce a scalar $\Psi$ in conductors and specify it by the differential equation

$$\left[\nabla^2 + k^2\right]\Psi = 0, \tag{7}$$

and by the continuity condition

$$\Psi = \psi\,, \tag{8}$$

imposed on the interface between the air and the metal. In Equation (7), $k = (1+i)/\delta$ where $\delta$ is the skin depth, as usual. Equations (7) and (8) determines $\Psi$ uniquely, when $\psi$ is given outside. Equation (7) implies the Green's formula

$$\Psi = \int_S dS'\left[\left(-\nabla'_n G\right)\psi + G \nabla'_n \Psi\right] \tag{9}$$

similar to (5), except that $G = e^{ikr}/4\pi r$.

Given $\Psi$ , an analog of Equation (3) can be written in the metal,

$$\vec{H} = \vec{h} + \vec{\nabla}\Psi \ , \tag{10}$$

with a new vector field $\vec{h}$ . Since $\vec{h} \neq \vec{H}^0$ in general, we must derive a governing equation for $\vec{h}$ . We can do so by subtracting the Maue formula [5] for $\vec{\nabla}\Psi$ ,

$$\vec{\nabla}\Psi = \int_S dS'\left[\left(\vec{\nabla}'G\right)\times\left(\vec{n}'\times\vec{\nabla}\Psi\right)-\left(\vec{\nabla}'G\right)\nabla_n\Psi - k^2 G\vec{n}'\Psi\right], \tag{11}$$

from the Stratton-Chu formula [2] for $\vec{H}$,

$$\vec{H} = \int_S dS'\left[\left(\vec{\nabla}'G\right)\times\left(\vec{n}'\times\vec{H}\right)-\left(\vec{\nabla}'G\right)H_n - \sigma\, G\,\vec{n}'\times\vec{E}\right]. \tag{12}$$

The Governing Boundary Integral Equations

We finally couple the integral representations with interface conditions, and derive the entire set of the BIEs for our extended scalar magnetic potential formulation. The interface conditions across the boundaries are

$$B_n, \vec{H}_t = \text{ continuous}, \tag{13}$$

where $B_n$ denotes the normal component of the magnetic flux density $\vec{B}$, and where $\vec{H}_t$ is the tangential components of $\vec{H}$. Since $\vec{B} = \mu\vec{H}$ where $\mu$ is the permeability, the normal derivatives can be expressed as

$$\nabla_n\psi = B_n/\mu - H_n^0, \tag{14}$$

in the air and core side, and

$$\nabla_n\Psi = B_n/\mu - h_n, \tag{15}$$

in the conductor side. The tangential continuity of (13) implies that $\psi$ is continuous across the core-air interfaces, and that

$$\vec{h}_t = \vec{H}_t^0 \tag{16}$$

across the metal-air interface because of (8).

It is now straightforward to derive the governing BIEs by collocating the following set of the integral representations: in the core,

$$\psi = \int_{S_c} dS'\left[\left(-\nabla_n' G_0\right)\psi + G_0\left(B_n/\mu_c - H_n^0\right)\right], \tag{17}$$

in the air

$$\psi = -\int_{S_c} dS'\left[\left(-\nabla_n' G_0\right)\psi + G_0\left(B_n/\mu_0 - H_n^0\right)\right] - \int_S dS'\left[\left(-\nabla_n' G_0\right)\psi + G_0\left(B_n/\mu_0 - H_n^0\right)\right], \tag{18}$$

and in the conducting media,

$$\Psi = \int_S dS' \left[ \left( - \nabla'_n G \right) \psi + G \left( B_n / \mu - h_n \right) \right], \tag{19}$$

$$\vec{h} = \int_S dS' \left[ \left( \vec{\nabla}' G \right) \times \left( \vec{n}' \times \vec{H}^0 \right) - \left( \vec{\nabla}' G \right) h_n - k^2 G \left\{ \frac{\vec{n}' \times \vec{E}}{i\omega\mu} - \vec{n}' \psi \right\} \right], \tag{20}$$

where $\mu_0, \mu_c$, and $\mu$ are the permeabilities of air, core, and metal, respectively, and where $S_c$ [$S$] is the core [part] surface. The unknowns on the core-air interfaces are $\{\psi, B_n\}$ which one can determine by collocating (17) and (18). On the air-metal interfaces, there are five surface unknowns

$$\left\{ \psi, B_n, h_n, \vec{n} \times \vec{E} \right\}, \tag{21}$$

that can be determined by the five BIEs from (18), (19), and (20). It is straightforward to generalize Equations (17)-(20) for multiple object cases.

DISCUSSIONS

Features of the Proposed Algorithm

Our scalar potential formulation described in the previous section has several intended and unintended features that distinguish it from the SC approach.

Originally, the scalar potentials were introduced in order to reduce excessive computational tasks when ferrite cores are introduced. This objective was clearly met by the simplicity of Equations (17) and (18). Moreover, this is achieved without the multi-valuedness problem since we use Equation (3).

In our method, the majority of the governing equations is scalar-like, as seen in Equations (17)-(19). Notice, also, that all the governing BIEs, including those from (20), have at most strongly singular kernels, to which a well-established numerical technique can be applied. Consequently, our BIEs are amenable to the conventional BEM techniques.

The continuation of the potential $\psi$ into metals via Equations (7) and (8) makes the interface conditions uniform between the air/core and air/metal interfaces. Particularly, this removes the explicit use of the more conventional interface conditions

$$\vec{E}_t, \vec{H}_t = \text{continuous} \tag{22}$$

from the formulation. It turned out, somewhat unexpectedly, that the extended scalar magnetic potential method distinguishes itself from the SC formulation in edge problems. As described in our companion paper [3], we studied edge phenomena by the two methods, one with the proposed approach, and the other with the Sratton-Chu formulation. Our method yielded intuitively correct eddy current flows near the edge, and moreover predicted correct edge impedance signals that agreed with experimental data. The SC method, however, failed to give correct results for the edge problem, when edge nodes are treated equally with other nodes. It appears that it is necessary to impose special boundary conditions on edge nodes to stabilize the SC-based computation. Our EMP formulation yields correct results with no special treatment of edge nodes.

Status of Software Implementations

The present work is primarily motivated by the task for developing a computer-aided EC probe design procedure. This is, therefore, the first and most natural application of the present formulation. The scalar potential has made it particularly easy to introduce ferrite-cores into probe models. See Reference [4] for details.

## CONCLUSIONS

This paper describes, at the basic level, an EC modeling formulation based on an extended use of the magnetic scalar potential. In this approach, we assume that the quasi-static condition holds, and introduce scalar functions $\psi$ and $\Psi$ in air/cores and in conductors, respectively, to express a part of the magnetic field $\vec{H}$ as their gradients. They are related to $\vec{H}$ via Equations (3) and (10). The complete governing equations are given in Equations (17)-(20), which can be cast into BIEs and solved by the standard BEM techniques.

As mentioned briefly in the previous section, computer codes have been written to obtain numerical results based on the proposed scheme. Our companion papers [3,4] describe the details on numerical implementations and applications. Several distinguishing features of the extended scalar magnetic potential method are found: (a) Thanks to the scalar potential description, it is not exceedingly costly, unlike the Stratton-Chu approach, to include any number of ferrite cores in the computation. (b) The governing BIEs are amenable to the conventional BEM thanks to their at most strongly singular kernels. (c) Most importantly, the present formulation distinguishes itself from the Stratton-Chu approach in its ability to yield correct results around part edges without requiring any special treatment of edge nodes.

In a broader perspective, our BEM approach is less computationally intensive than the general 3D finite element method (FEM). Our method is also more efficient than the volume integral method (VIM) for arbitrary constituent geometry. See the discussions in Reference [1].

Based on these observations, we conclude that the present formulation is proper and the most effective for the model studies of general geometry eddy current inspections as depicted in Figure 1.

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