WAVELET EXPANSIONS IN VOLUME INTEGRAL METHOD OF EDDY-CURRENT MODELING

Bing Wang, John P. Basart and John C. Moulder
Center for NDE and
Department of Electrical Engineering and
Computer Engineering
Iowa State University
Ames, Iowa 50011

INTRODUCTION

Eddy current nondestructive evaluation is a widely used in-plant NDE technique in which the flaw information is extracted from the impedance change of a coil placed above the metal testpiece. To obtain quantitative information about flaw size and shape, we would like to have a theoretical model which is able to predict the impedance change of a coil for different flaws in the test geometry. Because of its importance, this eddy current forward problem has been studied extensively for many years. For simple flaw shapes and geometry, it is possible to obtain analytical solutions. However, for flaws with irregular shapes and complex geometry, an analytical solution usually is not available so we must find a numerical solution. There have been several numerical models in the literature, e.g., the finite element method[1], the boundary element method[2], the volume integral method[3-5] and methods based on variational formulas[6].

Among these models, the volume integral method has shown good potential due to its capability of modeling a three dimensional flaw with arbitrary shape. Its principal advantage lies in its efficiency: only the volume of the flaw has to be discretized in this method. It is well known that the eddy current modeling problem is a solution of the underlying diffusion equation with given boundary conditions. The volume integral method transforms this differential equation with complex boundary conditions to an integral equation which is more suitable for numerical solution. The are two key components in the volume integral model. First, for the given geometry we need to calculate the Green's function by solving a partial differential equation. Second, the solution of the integral equation can be changed to the solution of a linear system after a discretization process. Although it is relatively straightforward to use the volume integral method, there sometimes is a problem of high computational resource requirement when it is used to model a large 3D flaw. Because the element size must be much smaller than the skin depth for an accurate solution, a large number of elements is required to model a flaw with dimensions much larger than the skin

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depth. This usually results in the inability of the volume integral method to model large flaws with conventional computing resources, such as PCs and workstations.

To solve this problem, we apply the wavelet transform to reduce the dimensions of the system matrix that we must invert. The physical basis for the application of the wavelet expansion comes from the fact that the unknown total field tends to have an abrupt change near the boundary of the flaw, and to have a slow change near the center of the flaw. This suggests that if we express the unknown total field with a wavelet expansion, then many of the high resolution coefficients will be close to zero, which enables us to compress the system matrix.

The organization of the paper is as follows: first we briefly review the volume integral model and wavelet theory, then we explain how to apply the wavelet transform to the solution of the integral equation and discuss some related problems. At the end of the paper we give some numerical examples and compare the theoretical result from the wavelets method with experimental data obtained from eddy current measurements on a thin aluminum plate with a flat bottom hole on the side opposite the coil.

THE VOLUME INTEGRAL METHOD

The volume integral method was originally used in geophysical prospecting studies by Raiche and Weidelt and was introduced to the field of eddy current NDE by Dunbar[4,5]. McKirdy[6] improved the accuracy of the volume integral method by analytically integrating some ill-behaved terms of the primary Green's function in the singular element. The volume integral method replaces the governing diffusion equation for the eddy current problem by the following integral equation[4]

$$\mathbf{E}^0(\mathbf{r}) = \mathbf{E}(\mathbf{r}) - \int_{V'} \overline{\mathbf{G}}(\mathbf{r},\mathbf{r}') \cdot \delta\sigma(\mathbf{r}')\mathbf{E}(\mathbf{r}')dv', \tag{1}$$

where $\mathbf{E}^0(\mathbf{r})$ is the incident field which, in our sense, is the electric field induced in the unflawed testpiece by a probe coil. $\mathbf{E}(\mathbf{r})$ is the unknown total field. $\delta\sigma(\mathbf{r}')$ is the conductivity change in the flawed testpiece, which is nonzero in the flaw volume, and zero outsize the flaw volume. $\overline{\mathbf{G}}(\mathbf{r},\mathbf{r}')$ is the Green's function for the test geometry, which is the solution of the following partial differential equation

$$[\nabla^2 + k^2]\overline{\mathbf{G}}(\mathbf{r},\mathbf{r}') = \delta(\mathbf{r}'), \tag{2}$$

where k is a complex number and is related to the skin depth δ

$$k = \frac{1+i}{\delta}. \tag{3}$$

Given the incident field, the conductivity changes, and the Green's function, we can solve (1) to obtain the total field. Once the total field is calculated, we can compute the impedance change in the coil by using the following reciprocity principle formula[4]

$$\Delta Z = \frac{-1}{I^2}\int_{V'} \delta\sigma(\mathbf{r})\mathbf{E}^0(\mathbf{r}) \cdot \mathbf{E}(\mathbf{r})dv'. \tag{4}$$

In the numerical solution of the volume integral model, the equation is discretized by dividing the flaw volume into many rectangular elements with identical sizes. We can then assume both the incident and the total field are constant within each element. The integration of the Green's function in each element can be performed either numerically or analytically. Depending upon the complexity of the geometry, analytical integration of the Green's function may or may not be possible. But for any singular elements, it is necessary to integrate the primary Green's function analytically to ensure the stability of the result.

After discretization, the integral equation becomes a matrix equation

$$(\mathbf{I} - \mathbf{C})\mathbf{E} = \mathbf{E}^0, \tag{5}$$

where $\mathbf{I}$ is the identity matrix, $\mathbf{E}^0$ and $\mathbf{E}$ are the vectors containing the incident and total fields and each element of $\mathbf{C}$ is itself a matrix which is derived from the integration of the Green's function in each volume element.

WAVELETS AND MULTIRESOLUTION DECOMPOSITION

Wavelets are new families of basis functions which are derived from the scaling and translation of a same function - the mother wavelet. Due to their unique time-frequency localization property, wavelets have been applied to various areas such as nonstationary signal analysis and image compression. Some background material on wavelets can be found in Ref. [7-11]. The wavelet basis function can be expressed as

$$\psi^{a,b}(x) = |a|^{-1/2}\psi(\frac{x-b}{a}), \tag{6}$$

where $\psi(x)$ is the mother wavelet, and a, b are the scaling and translation indices, respectively.

The continuous wavelet transform maps a function in a one-dimensional spatial domain to a two-dimensional spatial-frequency domain, which we call the wavelet domain. This continuous transform is redundant and thus is not appropriate for the purpose of basis expansion. To construct a group of orthogonal wavelet base, we can discretize the wavelet transform by sampling on a grid in the wavelet domain. Usually we use scales of powers of 2 and integer translations. Also, the shape of the mother wavelet must satisfy some conditions for the construction of an orthogonal wavelet basis. Simply put, the mother wavelet for an orthogonal wavelet basis is derived from the scaling function, which is the solution of the dilation equation, and two conditions, the approximation condition and the orthogonality condition.

A group of orthonormal wavelet base constitutes a multiresolution analysis in the square integrable function space. A set of subspaces $\{V_j\}_{j\in Z}$ is said to be a multiresolution approximation of $L^2(R)$ if it possesses the following properties

$$V_j \subset V_{j+1} \quad \forall j \in Z,$$
$$\bigcup_{j\in Z} V_j \text{ is dense in } L^2(R),$$

$$\bigcap_{j\in Z} V_j = \varnothing,$$

$$f(x) \in V_j \Leftrightarrow f(2x) \in V_{j+1} \quad \forall j \in Z,$$

$$f(x) \in V_j \Leftrightarrow f(x - 2^{-j}n) \in V_j \quad \forall j, n \in Z,$$

where Z denotes the set of integers. Notice that the basis functions for subspace V_j are scaled and translated versions of the scaling function, not the mother wavelet. Any function in $L^2(R)$ can be projected into the subspace V_j by taking the inner-product of the signal and the basis functions for subspace V_j. Thus we can obtain a multiresolution expansion of the original signal by using orthonormal wavelet basis functions.

The significance of having a multiresolution expansion of the signal is that we can separate slow-changing components and fast-changing components of the signal even when they are overlapped either in the frequency or in the spatial domain. The spatial-frequency localization property of the wavelet transform has been widely used in data compression, noise reduction and pattern recognition. Recently, it has been applied in the method of moments and in the time-harmonic electromagnetic scattering problem. In this paper, we apply the wavelet basis to the solution of the volume integral equation mentioned above, in which the role of wavelets is as the basis function which has the capability to reduce the data dimension.

WAVELET EXPANSIONS IN VOLUME INTEGRAL METHOD

The application of the wavelet basis is based on the following two observations on the physical nature of the eddy current problem: First, the incident electric field distribution is relatively smooth due to the diffusive nature of eddy currents; second, the total electric field distribution tends to have fast changes near the flaw boundary and slow changes near the center of the flaw. Now, if we express the incident field and the total field by their wavelet expansions, then many high resolution expansion coefficients should be close to zero, which provides us the opportunity to compress the linear system we must solve.

For simplicity, we start with the following one dimensional integral problem

$$E^0(x) = E(x) - \int_\Omega G(x,x')\delta\sigma(x')E(x')dx', \tag{7}$$

where Ω is the flaw region. Now expanding the unknown total field in the solution region with an orthonormal wavelet basis $\psi_i(x)$ gives

$$E(x) = \sum_{i=0}^{N-1} a_i \psi_i(x), \tag{8}$$

where we have combined the scaling index and translation index into one index and we let the first basis function be a constant term. Substituting (8) into (7), it follows that

$$E^0(x) = \sum_{i=0}^{N-1} a_i \psi_i(x) + \int_\Omega G(x,x')\delta\sigma(x') \sum_{i=0}^{N-1} a_i \psi_i(x')dx'. \tag{9}$$

After rewriting the right hand side, we have

$$E^0(x) = \sum_{i=0}^{N-1} a_i \psi_i(x) + \sum_{i=0}^{N-1} a_i \langle G(x,x')\delta\sigma(x'), \psi_i(x') \rangle, \tag{10}$$

where $< >$ denotes the inner-product operation on the solution domain. Now, taking the inner-product of the above equation with the basis functions $\{\psi_j(x)\}$, we have the following N equations

$$\langle E^0(x), \psi_j(x) \rangle = \sum_{i=0}^{N-1} a_i \delta_{ij} + \sum_{i=0}^{N-1} a_i \langle \langle G(x,x')\delta\sigma(x'), \psi_i(x') \rangle, \psi_j(x) \rangle \quad j = 0, \ldots, N-1. \tag{11}$$

This linear system can be rewritten as

$$\sum_{i=0}^{N-1} a_i C_{ij} = b_j \quad j = 0, \ldots, N-1, \tag{12}$$

where $b_j = \langle E^0(x), \psi_j(x) \rangle$ is the wavelet expansion coefficient for the incident field, and $C_{ij} = \delta_{ij} + \langle \langle G(x,x')\delta\sigma(x'), \psi_i(x') \rangle, \psi_j(x) \rangle$, where the second term is the two-dimensional wavelet expansion coefficient for the product of the Green's function and the conductivity change.

Extension of the above approach to the three dimensional case is straightforward. Here we just indicate that the final matrix equation can be written as

$$\sum_{i=0}^{N_x-1} \sum_{j=0}^{N_y-1} \sum_{k=0}^{N_z-1} a_{ijk} \overline{C}_{ijklmn} = b_{lmn} \quad i = 0, \ldots, N_x - 1, \; j = 0, \ldots, N_y - 1, \; k = 0, \ldots, N_z - 1, \tag{13}$$

which is analogous to (12). Notice that here each element of $\overline{\mathbf{C}}$ is itself a 3 by 3 matrix which is derived from the wavelet expansion coefficients of the nine elements of the dyadic Green's function. Based on the physical nature of this problem, this transformed matrix can be rendered sparse by thresholding. The inverse of this sparse matrix requires much less memory and computation.

To apply this method, there are still some additional problems needing further consideration. The first problem is how to compute the wavelet transform of the system matrix without fully storing it in the computer memory, since this matrix could be very large for a large flaw. To solve this problem, we utilized Mallat's pyramid algorithm[10] for the fast wavelet transform. Notice that in the pyramid algorithm, the computation at any resolution level is solely based on the result of the immediately higher resolution level. Thus, we are able to compute a wavelet transform of size N from the wavelet transforms with a smaller size M if both N and M are powers of 2. Based on this relationship, we can compute the wavelet transform of the whole matrix in two steps. In the first step, we divide the system matrix into many submatrices with identical sizes and compute the wavelet transform for each of them. The result is stored in a sparse matrix data structure. In the second step, the wavelet transform of the whole matrix is obtained by using the result of the first step.

The second problem concerns the selection of the mother wavelet. There have been a number of orthonormal wavelet bases developed by various researchers. We need to choose a wavelet basis which can give us the greatest data reduction. It is important to indicate that all the orthonormal wavelet bases, except the Haar basis, have a support larger than the grid separation, which causes a boundary problem since we need to know some values outside the solution region in order to compute some of the expansion coefficients. This boundary problem can be avoided by using a periodic extension. But this again may cause an edge effect. More discussion on these problems can be found in [12]. In this paper, we use the Haar basis in all wavelet transforms.

NUMERICAL EXAMPLE AND COMPARISON WITH EXPERIMENTAL DATA

To illustrate the result of the wavelet approach, we present a numerical example and compare the theoretical result with the experiment. The problem here is to detect the pitting corrosion on the bottom side of a 1-mm thick 2024 aluminum plate. The flaw has a cylindrical shape with a height of 0.275 mm and a radius of 3.15 mm. The probe is a 504-turn air-core coil with an inner radius of 3.8 mm and an outer radius of 5.635 mm. The height of the coil is 2.4 mm and the liftoff is 0.229 mm. A Hewlett-Packard 4194A impedance analyzer was used to measure the impedance change. The test frequency ranges from 2.5 kHz to 50 kHz, with a step size of 2.5 kHz.

The steps of the wavelet method are as follows:

1. Compute the incident field distribution by using Dodd and Deed's result[13].
2. Compute the integral of the Green's function in each element. The Green's function for thin plate geometry has been derived in [14]. For the singular elements, both analytical and numerical integrations are used to stabilize the result. For the nonsingular elements, only numerical integration is used.
3. Compute the wavelet transform of the system matrix by using the two-step algorithm. Store the thresholded result in a sparse matrix data structure.
4. Invert the transformed system matrix by using a sparse matrix LU decomposition routine.
5. Compute the wavelet transform of the incident field.
6. Compute the wavelet coefficients of the total field by backsubstitution.
7. Take the inverse wavelet transform to obtain the total field.
8. Calculate the impedance change by using the reciprocity principle formula.

Fig. 1 is a map of the transformed system matrix after applying a threshold of 0.05. A black spot denotes a nonzero element and white space denotes zero elements. After thresholding, only 0.73% of all the matrix elements are nonzero. The thresholded matrix also shows some interesting patterns. First, many nonzero elements are near the matrix diagonal, which indicates a strong correlation between basis functions with overlapped supports. Second, there are more nonzero elements in the upper-left corner where low resolution basis functions are located and fewer nonzero elements in the bottom-right corner where high resolution functions are located.

Fig. 2 shows a comparison of the theoretical and the experimental results for the real and imaginary parts of the impedance change at zero offset(Coil centered over the flat bottom hole). The theoretical result is obtained by using a threshold of 0.05. The general correspondence between the theoretical and the experimental results is very good with a difference of less than 10%.

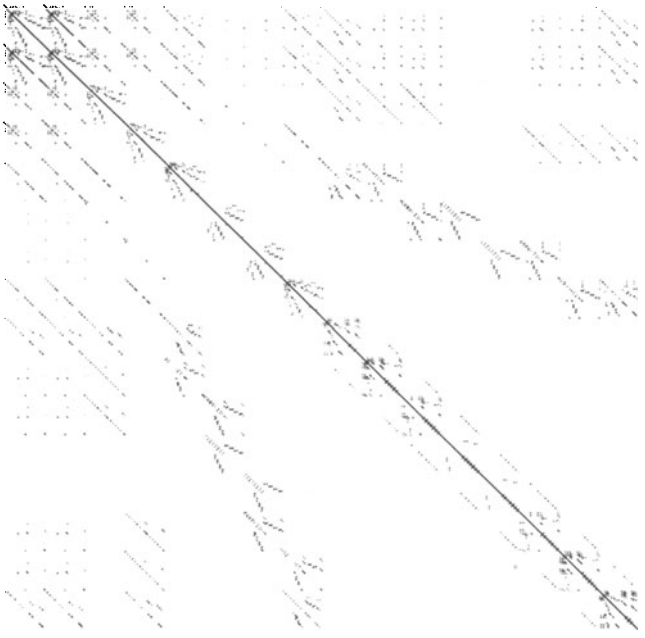

Fig. 1. The transformed system matrix map after using a threshold of 0.05. A black spot denotes a nonzero element and a white space denotes a zero element. Only 0.73% of all the matrix elements are nonzero after thresholding.

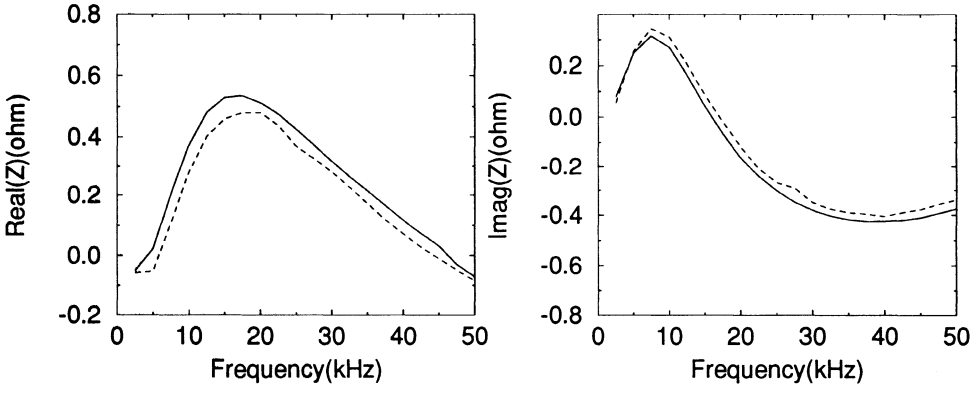

Fig. 2. Comparison of theoretical and experiment results. The left and the right pictures show the real and imaginary parts of the coil impedance change over a 1-mm 2024 aluminum plate with a flat hole on the bottom side. The experimental result is plotted with a dashed line and the theoretical result is plotted with a solid line. The threshold used is 0.05.

CONCLUSION

In this paper we applied the wavelet expansion to the solution of the volume integral equation for the eddy current modeling problem. It is shown that the wavelet method has good potential in reducing the memory space and computation load requirements. As indicated by the numerical result, it is possible to achieve a large reduction in computational resources required with a negligible loss in accuracy by using the wavelet method. Some further work is still desirable to fully utilize the benefits of this method.

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