Photoneutron resonances in Oxygen 16

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PHOTOBUTRON RESONANCES IN OXYGEN 16

by

James Edward Griffin

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
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Approved:

Signature was redacted for privacy.

In Charge of Major Work

Signature was redacted for privacy.

Head of Major Department

Signature was redacted for privacy.

Dean of Graduate College

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Ames, Iowa

1963
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I. INTRODUCTION

The interaction of electromagnetic radiation with matter has for many years been used as an aid in the investigation of the nature of matter. The early quantum theory began with attempts to explain the electromagnetic radiation spectrum found to be in equilibrium with a black body. Observation and explanation of the photoelectric effect soon followed and the quantization of atomic mechanical systems was inferred by observation of the absorption and emission of electromagnetic radiation from such systems. Energy levels within the systems were found to be related to the frequencies of emission or absorption and the Rydberg-Ritz principle was developed. It was found reasonable to quantize the electromagnetic radiation field and to represent radiation from atomic systems by discrete bundles of energy called photons.

The nucleus of the atom was soon discovered to be capable of the emission and absorption of photons, often of unprecedented energy, and it appeared reasonable that such electromagnetic interaction could lead to knowledge of the structure of the nucleus. The energy of photons emitted from the nucleus was assumed to represent the separation of energy levels within the nucleus (apart from a slight degradation of photon energy resulting from the requirement that momentum be conserved in the process) and it was only
necessary to develop quantum mechanical theories which would predict the existence of observed levels within the nucleus. Such theories are, at the present time, incomplete and somewhat inadequate and it is reasonable to continue the experimental study of the nuclear interaction with electromagnetic radiation.

One method of investigation is the study of the nuclear photoeffect. If photons of energy higher than the binding energy of nuclear components (nucleons) are directed at nuclei, a process analogous to the photoelectric effect can take place, the photon being absorbed by the nucleus with the subsequent emission of a nucleon or aggregation of nucleons. The resultant nucleus is sometimes radioactive, in which cases the extent to which photons have been absorbed in the nuclear photoeffect can be measured by counting the number of radioactive nuclei which result from bombardment of a large population of parent nuclei with photons of known energy and intensity. On other occasions a similar bombardment of photons is performed and the products of the photodisintegration are detected directly.

Early photodisintegration experiments (1) were performed by allowing gamma rays from naturally radioactive nuclei to impinge upon light nuclei with low binding energies and observing directly the emission of nucleons. While such experiments are still of interest, their scope is limited due to the relatively few photon energies
available. The invention and development of high energy electron accelerators has made possible investigation of the nuclear photoeffect over essentially the entire energy range of interest on a wide variety of nuclei. In most instances the photon source for such experiments has been the bremsstrahlung spectrum of photons resulting when the high energy electrons are allowed to strike a target within the accelerator.

Difficulties have persisted however since, unlike the early experiments where the photon source was essentially monochromatic, the photons from accelerators appear with an energy range extending from zero to the kinetic energy of the electrons in the accelerator (2, p. 242ff). This has in the past necessitated a rather unsatisfactory mathematical treatment of the data and the absorption spectra derived from them have had large uncertainties, especially at high energies. This difficulty has led to the recent development of sources of fairly monochromatic high energy photons (3, 4). Experiments with such systems are at present limited in energy range and technically difficult.

An alternative solution to the problem, an improved mathematical treatment of the data, has been developed recently in this laboratory by Dr. B. C. Cook (5). The development of the data from the experiment described in this paper will constitute the first application of this
mathematical method to photonuclear data and in some respects will constitute a test of the validity of the mathematical method. A discussion of the mathematical techniques involved will be given in section III of this paper.

In section II a brief outline of the theory of photonuclear reactions culminating in a discussion of recent calculations relating specifically to Oxygen 16 (hereafter referred to as $^{16}\text{O}$) will be given.

Previous accelerator photonuclear experiments have on occasion suffered in accuracy due to instability of the electron energy control systems. Recent technical developments in the energy control system of the Iowa State University synchrotron have made possible extended photon bombardments with exceedingly stable energy control. The energy control system will be discussed in section IV.

As a result of the three recent developments mentioned above, mathematical data treatment, theoretical calculations, and accelerator energy control, it was decided to undertake a new measurement of the photon absorption spectrum of the nuclear reaction in which the stable nucleus $^{16}\text{O}$ absorbs electromagnetic radiation and emits a single neutron, the residual nucleus being radioactive Oxygen 15. The reaction is written schematically, $^{16}\text{O}(\gamma,n)^{15}\text{O}$. 
II. OUTLINE OF PHOTONUCLEAR THEORY

A. Absorption Cross Section and Sum Rules

A number of review articles on the theory of photonuclear reactions have appeared within the last few years (6, 7). Probably the most useful of these are by D. H. Wilkinson (8), J. S. Levinger (9), and the book Nuclear Photo-disintegration (10) by J. S. Levinger. In this section a general outline of the current status of the theory will be presented.

The extent to which a quantum mechanical system interacts with a radiation flux of a particular energy can be defined by the ratio of transitions per second resulting from photon absorption to the photon flux at that energy. This ratio has dimensions of length squared and has come to be known as the cross section \( \sigma \) for the process in question.

\[
\sigma = \frac{\text{transitions per second}}{\text{photon flux}} \left( \frac{\text{cm}^2}{\text{nucleus}} \right). \tag{1}
\]

The vector potential of the plane wave describing the incident photons can be written (10, p. 14)

\[
\vec{A}(\vec{r}) = A_0 \vec{u} \exp i(\omega t - \vec{k} \cdot \vec{r}) \tag{2}
\]

where \( \vec{u} \) is the unit polarization vector

\( \vec{k} = \text{photon propagation vector}. \)
For incident photons with energies below the $\pi$ meson creation energy (approx. 140 MeV) the radiation wavelength can be considered to be large with respect to the linear extent of nuclear wave functions representing initial and final states of the nucleus. Under these conditions expression 2 can be expanded in powers of $\hat{k} \cdot \hat{r}$ and the perturbation energy resulting from interaction of the leading term with a bound particle of charge $e$ is (11, p. 233)

$$H(r,t) \propto e(\hat{A}(\vec{r}) \cdot \hat{p}) = eA_0(t)\hat{u} \cdot \hat{p}$$  (3)

where $\hat{p}$ is the momentum operator of the particle and the exponential time dependence has been represented by $A_0(t)$. Since $\hat{p} = M\hat{r}$, this term in the expansion is proportional to the time derivative of $D = e(\vec{u} \cdot \vec{r})$, the $\vec{u}$ component of the electric dipole moment of the system, and the interaction is the electric dipole, or $E1$ interaction. Successive terms in the expansion yield higher order electric and magnetic multipole interactions. If the incident photon polarization is assumed to be in the $z$ direction then equation 3 reduces to

$$E(z,t) \propto A_0(t)\dot{D} = A_0(t)e\dot{z}.$$  (4)

From time dependent perturbation theory (12, p. 172), the transition probability is known to be proportional to the square of the interaction matrix element $\langle i | \hat{H} | f \rangle$ between initial and final nuclear states. Using the Heisenberg
relationship (12, p. 113) and the Bohr frequency condition
\[ \hbar \omega = (E_f - E_i) \]

\[ \langle i | \hat{D} | f \rangle = \frac{(E_f - E_i)}{\hbar} \langle i | D | f \rangle = i \omega \langle i | D | f \rangle = i \omega (D_{if}). \] (5)

The integral of the E1 absorption cross section, equation 1, over the dipole absorption frequencies (or energies) can be written in terms of the dipole matrix element \( D_{if} = e z_{if} \) 
(10, p. 7)

\[ \int \sigma_{E1} \, d\omega = \text{const.} \, k(z_{if})^2 \]

where \( k \) is the incident photon energy. One can now define an oscillator strength

\[ f_{if} = \frac{2 \hbar \omega}{\hbar^2} (z_{if})^2 \]

and it has been shown that the sum of \( f_{if} \) over all final states is equal to unity.

\[ \sum_f f_{if} = 1. \] (8)

This is the Thomas-Reiche-Kuhn (referred to as TRK) dipole sum rule and can be derived from the closure properties of quantum mechanics (13, p. 46).

The extension of the single particle TRK sum rule to a nucleus consisting of \( Z \) protons and \( N \) neutrons has been treated by J. S. Levinger and H. A. Bethe (14) and others
The results are, in general, that the sum of the oscillator strengths, equation 8, for nuclei will be $NZ/A$, ($A = N+Z$) plus another somewhat smaller term due to two-body exchange forces within the nucleus. The integrated cross section can be written

$$\int_0^{150 \text{ Mev}} \sigma d\omega = 0.060 \frac{NZ}{A} (1 + 0.8x) \text{ Mev-barns}$$

(9)

$x = \text{fraction of attractive exchange forces.}$ (1 barn = $10^{-24} \text{ cm}^2$.)

A more general calculation, based on dispersion theory, has been presented by Gell-Mann, Goldberger, and Thirring (referred to as GGT) (18). This expression applies to all multipole absorption and has been derived under the assumption that for very high photon energies the $N$ neutrons and $Z$ protons in a bound nucleus behave as free nucleons. The additive term has been evaluated by experiment and is thought to be valid only to about 30 percent. The GGT expression is

$$\int_0^{150 \text{ Mev}} \sigma_{\text{tot.}} d\omega = 0.060 \frac{NZ}{A} \left[1 + 0.1 \frac{A^2}{NZ}\right] \text{ Mev-barns.}$$

(10)

For $^{16}O$, $60 \frac{NZ}{A}$ is equal to 240 and, using either expression (with $x = 0.5$ in equation 9) one has the very approximate result

$$\int_0^{150 \text{ Mev}} \sigma d\omega = 336 \text{ Mev-millibarns.}$$

(11)
B. The Collective Model

Early accelerator measurements of the nuclear photoeffect (19) indicated the presence of a large resonance in the (γ, n) cross section in the region 10-20 MeV which could not be ascribed to the onset of multiple reactions (i.e. (γ, 2n) or (γ, pn)). This was found to be true for several nuclei, notably carbon, copper, and tantalum. The cross section at resonance was found to be much larger than that which could result from interaction of a photon with a single particle bound within the nucleus. The implication was that the absorption was the result of a collective motion of all of the bound nucleons.

In 1944 A. Migdal (20) made calculations of the extent to which a nucleus could be polarized by a spatially constant electric field. In 1948 M. Goldhaber and E. Teller (21) presented three models of collective nuclear vibrations resulting from electric dipole excitation. In the second of the three models, the proton and neutron fluids oscillate against each other as separately compressible fluids within a rigid boundary at the nuclear surface. This model gives a resonance energy proportional to $A^{-1/3}$ in reasonable agreement with experiment. The model was extended by Steinwedel, Jensen and Jensen (22) by the inclusion of a symmetry energy density in the nucleus. It is significant that each of these approaches stressed the electric dipole
character of the interaction. The resultant energy for
the giant resonance is

\[ E_m = \left[ 34.6 \left( \frac{NZ}{A^2} \right) k \left( \frac{R^2}{R^2} \right) \right]^{\frac{1}{2}} \text{ Mev.} \]  \hspace{1cm} (12)

In this expression \( k \) is the coefficient of the asymmetry
energy density term in Weizsacker's semi-empirical mass
formula (approx. 20 Mev), \( R \) is the nuclear radius and \( M \) is
the nucleon mass. K. Danos (23, 24, 25) further extended
the calculations by taking into consideration coulomb inter-
actions within the proton fluid and examining higher energy
resonances resulting from this slightly perturbed hydro-
dynamical model. He finds that 85 percent of the classical
(\( TRK \)) summed oscillator strength NZ/A should appear in the
fundamental resonance described by equation 12, the remaining
oscillator strength appearing in \( E1 \) overtones starting at
2.86 \( E_m \), \( E2 \) transitions starting at 1.6 \( E_m \) and higher
multipoles. It was suggested that these high energy res-
onances may not be observed due to high damping occurring
at these high energies.

In addition to the resonance energy \( E_m \) and the total
area under the cross section curve it is of interest to
examine the sharpness of the resonance curve. It is common
practice to describe sharpness by \( \Gamma \), the full width at
half maximum of the resonance curve.

The collective model has not been well suited to
calculations of \( \Gamma \). The hydrodynamical dipole oscillations
must be coupled through a sort of nuclear viscosity to other nuclear motions and to the emission of nucleons. Some estimates of nuclear damping have been made (26), and Danos (25) and K. Okamoto (27) have used the collective model to explain observed broadening, and even splitting, of the giant resonance for deformed nuclei, or nuclei with large electric quadrupole moments. B. M. Spicer (28) has pointed out that the splitting of the giant resonance should be more readily observable in nuclei with Z between 9 and 30 and he has presented five examples (F$^{19}$, Mg$^{25}$, Si$^{29,30}$, V$^{51}$, Cu$^{63}$) of experimental verification of the prediction.

With regard to O$^{16}$ the collective model seems to have little to offer. Expression 12 for the giant resonance energy is more applicable to heavier nuclei (the asymmetry of O$^{16}$ is, of course, zero). It predicts an energy of 28 Mev for the O$^{16}$ giant resonance and this is roughly 20 percent higher than experimental values. It does predict, however, that the giant resonance of O$^{16}$ will not be broadened or split due to deformation since the electric quadrupole moment of O$^{16}$ is zero.

C. The Independent Particle Model

The independent particle (IPM), or nuclear shell model, has resulted in correct predictions of many of the ground state and low energy level properties of nuclei (29). It seems reasonable to investigate the applicability of the
shell model to the high energy nuclear photoeffect. Nuclear shell theory treats each nucleon as an independent particle experiencing a short range central attractive force resulting from the average effect of all of the remaining nucleons. In this central potential each nucleon is assumed to be capable of describing an orbit of well defined energy and angular momentum. The nucleons fill shells or levels in the potential in accordance with the Pauli principle.

Calculations of the nuclear photoeffect for a harmonic oscillator potential, an infinite square well, and a finite square well respectively have been done by S. S. Wu (30), A. Z. Reifman (31), and J. L. Burkhardt (32). D. H. Wilkinson has extended the finite square well calculations (33) and has demonstrated that essentially all of the El oscillator strength is exhausted in transitions from the last filled shell levels to virtual shell levels in the continuum. Transitions from unfilled shells do not contribute appreciably to the oscillator strength. The well known angular momentum, parity, and isotopic spin selection rules for El transitions are assumed to hold (10, p. 67), i.e. for El absorption the total angular momentum must change by one or zero units with zero to zero transitions forbidden, there must be a change in parity, and for self conjugate (T3=0) systems the isotopic spin must change by one unit. Wilkinson showed that El transitions from the last filled shell which are allowed by the selection rules go to levels
sufficiently close together as to blend into an apparent giant resonance. Since all shells below the uppermost filled shell are also filled, transitions between these shells are forbidden by the Pauli principle and excitations between these shells can contribute no oscillator strength to the sum, equation 8, but in the same way downward transitions, which contribute to the sum negatively, are also forbidden. In this way oscillator strength is "transferred" upward through the shells and the oscillator strength from the uppermost filled shell results from a collective absorption by all the nucleons in the shell with the excitation of one nucleon to the next allowable unfilled level.

While Wilkinson's IPM calculations were able to account for the existence of a giant resonance, roughly correct oscillator strength, and reasonable widths \( f \) for the giant resonance, the giant resonance energy resulting from these calculations was far too low. Wilkinson used a well depth, or harmonic oscillator level spacing consistent with the nucleon binding energies of the nuclei treated.

When a nucleon is elevated by \( E_1 \) photon absorption from the last filled shell to a virtual shell level a hole is left in the filled shell and such holes can be treated in many ways like single particles in the shell. In 1957 J. P. Elliott and B. H. Flowers (34) presented calculations of the effect of particle-hole interactions on the odd
parity states of $^1\!^6_\text{O}$ with direct application to the nuclear photoeffect. Similar calculations have been described by G. E. Brown and M. Bolsterli (35) and applied to $^1\!^6_\text{O}$ by G. E. Brown, L. Castillejo and J. A. Evans (referred to as BCE)(36).

The procedure, in general, has been to use experimental results for the energies of shell levels of oxygen 15 and oxygen 17 as the unperturbed hole and particle energy levels respectively. Particle-hole interactions cause configuration mixing and the resulting $E_1$ energy levels are linear combinations of the single particle-single hole $E_1$ levels. The calculations take into consideration spin-orbit coupling. Use of the experimental unperturbed particle and hole energies results in an initial increase in the excitation energies and the calculations indicate that excitations to $T = 1$ states are further increased in energy by the particle-hole interaction.

Figure la shows the simple harmonic oscillator shell levels for $^1\!^6_\text{O}$ with $\hbar \omega = 17$ Mev, plus the next two unpopulated levels. Figure lb shows the same levels with degeneracy removed by spin-orbit coupling. Transitions from the lp shell to the unfilled levels of the 2s1d shell are shown by vertical lines and the approximate unperturbed (by particle-hole interaction) transition energies are shown. These transitions correspond to transitions predicted by Wilkinson although the energies are higher because of the
Figure 1. Oxygen 16 harmonic oscillator shell levels
larger simple harmonic oscillator energy used. The $^3_1/2^d_5/2$ transition is not shown because it requires a change of two units of angular momentum. The Elliott-Flowers and BCE predictions resulting from linear combinations of these transitions are tabulated in Table 1 along with predicted oscillator strengths and level widths where available.

The BCE transitions are listed for calculations resulting from the use of an ordinary force and for an exchange mixture postulated by J. M. Soper, $V = (0.3 + 0.43P_M$

$+ 0.27P_B)V(\bar{F}_1-\bar{F}_2)$, where $P_M$ and $P_B$ are the Majorana and Bartlett exchange operators. The Soper mixture results are expected to be in better agreement with experiment but the general trend of the levels is somewhat insensitive to the exchange mixture chosen. These calculations indicate that the entire E1 oscillator strength will be exhausted at energies below 30 Mev. The level at highest energy is the most symmetric combination of single particle states, i.e. the state most similar to the collective giant resonance in which all the protons move together. The low intensity level in the vicinity of 13.5 Mev cannot, of course, be observed in $^{16}(\gamma,n)^{15}$ experiments because it lies below the mass threshold (approx. 16 Mev) for the reaction.

The particle-hole interaction coupled with empirical unperturbed shell levels has been employed recently in a thesis by V. Gillet and the results published by V. Gillet and N. Vinh Tau (37, 38). (In reference 37, the published version of reference 38, the section on electric quadrupole
Table 1. Calculated energies, dipole strengths, and level widths for E1 T=1 transitions in $^{0}_{16}$

<table>
<thead>
<tr>
<th>Elliott and Flowers</th>
<th>Brown, Castillejo, Evans</th>
<th>Gillet and Vinh-Mau</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force mixture</td>
<td>Ordinary Soper Approximation</td>
<td>Approximation I</td>
</tr>
<tr>
<td>25.2</td>
<td>32</td>
<td>5.8</td>
</tr>
<tr>
<td>22.6</td>
<td>67</td>
<td>12.0</td>
</tr>
<tr>
<td>20.4</td>
<td>0</td>
<td>0.02</td>
</tr>
<tr>
<td>17.3</td>
<td>1</td>
<td>0.14</td>
</tr>
<tr>
<td>13.1</td>
<td>0</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Table 2. Calculated energies and relative strengths of E2 T=0 and T=1 transitions in $^{0}_{16}$ (Energies of $1f_{7/2}$ level are referred to ground level for $^{0}_{16}$)

<table>
<thead>
<tr>
<th>$1f_{7/2}$ = -0.3 Mev</th>
<th>$1f_{7/2}$ = 6.7 Mev</th>
<th>$1f_{7/2}$ = 13.7 Mev</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy Mev</td>
<td>% Relative strength</td>
<td>Energy Mev</td>
</tr>
<tr>
<td>2$^+$ T=0</td>
<td>12.6</td>
<td>89</td>
</tr>
<tr>
<td>2$^+$ T=1</td>
<td>53.2</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>48.2</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>23.7</td>
<td>40</td>
</tr>
</tbody>
</table>
transitions, of interest here, has been deleted.)

Five nuclear force parameters, the range, well depth, and the Feisenberg, Majorana, Wigner, and Bartlett exchange operators (subject to the condition that $W + X + H + B = 1$) were varied simultaneously in such a way as to achieve a minimum error in the calculation of known levels in $^{16}_0$. The resulting parameters were employed to calculate new levels. Two separate calculations were made. Approximation I corresponds to diagonalization of one particle-one hole interaction configurations in a limited subspace as per Elliott and Flowers. Approximation II is equivalent to linearization of the equations of motion in a Hartree-Fock time dependent theory. The results of these calculations for El $T=1$ levels are included in Table 1. Figure 1c shows three E2 transitions considered to have significant oscillator strengths if the first ($7/2^-$) state listed by F. Ajzenberg-Selove (39) is assumed to be $1f_{7/2}^-$. In Figure 1d the same transitions are shown with the $1f_{7/2}^-$ level seven Mev higher (cf. 38, p. 7) which is more consistent with nuclear shell theory. A third and extreme situation with the $1f_{7/2}^-$ level raised 14 Mev, Figure 1e, is calculated by the authors. In Table 2 the results of these calculations are presented for the three assumed energies of the $1f_{7/2}^-$ level. Both $T=0$ and $T=1$ levels are presented as no isotopic spin selection rule applies for E2 transitions. The two highest states are almost pure $1s^-1ld_{5/2}$ and
ls⁻¹ld₃/₂ transitions and their energy difference should reflect the ld₃/₂⁻ld₅/₂ spin-orbit splitting. The lowest T = 1 and T = 0 levels tabulated depend primarily on the transition lp⁻¹lf₇/₂. It is evident that the T = 1 levels are elevated while the T = 0 levels are depressed by the particle-hole interaction. In the high energy calculations, notably the extreme case mentioned above, two particle-two hole interactions should become important, and they have not been taken into consideration.

Very recently J. H. Carver, D. C. Peaslee, and R. E. Taylor (40) have examined the problem of nuclear El overtones in the light of the upward shift of the giant resonance energy due to particle-hole interactions. They suggest that the oscillator strength for three-quantum absorption into the first El overtone may be as high as 25 percent of that for the giant resonance. The El overtone energy is thought to be near three times the unperturbed harmonic oscillator spacing plus one-half or one-third of the upward shift of the fundamental. For O¹⁶ (ℏω = 17 Mev) this places the El overtone near 45 Mev. The suggestion is made that, since the classical hydrodynamical E2 resonance falls between the El fundamental and the first overtone, the two may have been confused in the interpretation of experimental results. A calculation of the El integrated cross section to 30 Mev, taking into consideration the first El overtone, is presented and plotted as a function of the nucleon
number A. The result for $\sigma^1_{16}$ (if an ordinate label is changed from millibarns to barns) appears to be about 590±60 Mev millibarns.

D. Rapprochement

In 1957 D. W. Brink (41) showed that the shell model wave function of the state reached by the dipole nuclear photoeffect contains the collective vibration described by Goldhaber and Teller. More recently V. V. Balashov (42) has pointed out that the particle-hole, or nucleon-nucleon correlations necessary to correct the independent particle model giant resonance predictions play an important collectivizing role in the theory. The dipole (and higher multipole) resonances are characterized by strong mixing of a large number of single particle states. The author attempts to base a collective dipole excitation on a shell model taking into account residual nucleon-nucleon interactions rather than on a hydrodynamical model. The main conclusions of the model are identical with the results of the procedure presented by Brown and Bolsterli (35), and it can perhaps be generalized more readily.

It appears that introduction of nucleon-nucleon correlations into either model is necessary if the model is to describe correctly anything more than the most gross features of the photonucleon cross section. A decision regarding the validity of one model or another is
unimportant. One uses the model which yields the most accurate predictions. At the present time the hydrodynamical collective model probably does not include sufficient refinements to enable detailed calculations of cross section structure for light nuclei.
III. ANALYSIS OF PHOTONUCLEAR YIELD DATA

A. Development of the Problem

In Figure 2 a typical photonuclear experimental set-up is depicted. Electrons of kinetic energy $T_0$ are allowed to strike a target within the accelerator vacuum chamber. The resulting bremsstrahlung radiation is directed through a sample containing a very large number of nuclei of the type under investigation and then into a radiation monitoring device or dosemeter. The radioactivity developed in the sample by photonuclear transmutation or the number of nucleons emitted from interacting nuclei is measured quantitatively. The peak energy of the photon spectrum is equal to the kinetic energy $T_0$ of the electrons. The yield $A(T_0)$ of radioactivity is measured for many equally spaced increments of $T_0$ throughout the range of interest. The yield, $A(T_0)$, is a function of the shape and peak energy of the bremsstrahlung spectra and of the nuclear cross section for photon absorption. The yield $A(T_0)$ can be expressed

$$A(T_0) = \frac{\text{Number of detector counts at } T_0}{\text{Monitor response at } T_0} = \int_0^{T_0} n(T_0, k) \sigma(k) dk$$

where $n(T_0, k)$ is the number of photons of energy $k$ in range $k$ to $k+dk$ entering the sample per unit monitor response, $\sigma(k)$ is the nuclear cross section per nucleus.
Figure 2. Experimental set-up
for the reaction under investigation as a function of the photon energy \( k \), \( n \) is the number of nuclei per square centimeter exposed to the radiation.

The function \( N(T_0, k) \) can be written

\[
N(T_0, k) = \frac{N(T_0, k) f_s(k)}{F(T_0)}
\]

where \( N(T_0, k) \) is a function proportional to the bremsstrahlung spectrum at energy \( T_0 \),

\( f_s(k) \) is a function which corrects the spectrum seen by any part of the sample for photon absorption ahead of that part of the sample, and

\( F(T_0) \) is a monitor response function which normalizes the monitor response to one unit out for one unit of radiative energy into the sample.

Equation 13 can now be rewritten

\[
Y(T_0) = \int_{E_t}^{T_0} X(T_0, k) s(k) dk
\]

where \( Y(T_0) = A(T_0) F(T_0) \) is called the reduced yield,

\( s(k) = n f_s(k) \sigma(k) \) is called the reduced cross section.

The lower limit of integration has been changed from 0 to \( E_t \), the threshold energy for the nuclear reaction or the lowest energy at which \( \sigma(k) \) becomes non-zero.

Equation 15 is a Volterra equation of the first kind and the solution sought is, of course, \( s(k) \). If expression 15 is thought of as an integral operator operating on \( s(k) \) to yield \( Y(T_0) \) then it can be shown (45) that this
operator has no bounded inverse. Suppose \( s(k) \) is a solution to equation 15 and add to it the function \( f_m = \sin mk \). For any integrable kernel (excluding \( N(T_0, k) = \delta(T_0 - k) \))

\[
\lim_{m \to \infty} \left[ \int_a^b N(T_0, k) \sin(mk) \, dk \right] = 0 \tag{16}
\]

hence an infinitesimal change in \( g_m \) can cause a finite change in \( f_m \) and the system is unstable. This demonstrates, in principle, the difficulty involved in use of the rather imprecisely measured \( Y(T_0) \) for an accurate determination of \( s(k) \).

Since \( Y(T_0) \) consists of a sequence of data points rather than a known function it is reasonable to replace the integral by a sum of terms averaged over the data intervals.

\[
Y_i = \sum_{j=1}^{m} N_{ij} s_j \tag{17}
\]

where \( s_j \) is the average value of \( s(k) \) over the \( j \)th interval

\[
s_j = \frac{1}{\Delta k} \int_{k_j - \Delta k}^{k_j} s(k) \, dk, \tag{18}
\]

and \( N_{ij} \) is the number of photons in the \( j \)th interval resulting from the \( i \)th spectrum, (or from electrons with kinetic energy \( T_i \)),

\[
N_{ij} = \int_{k_j - \Delta k}^{k_j} N(T_i, k) \, dk. \tag{19}
\]
Functional approximations to the bremsstrahlung spectra $N(T_1,k)$ are available (2, 44, 45) and the $N_{ij}$ can be calculated. Since there are no photons of energies higher than the incident electron kinetic energy

$$N_{ij} = 0 \quad j > 1 \quad (20)$$

and $N_{ij}$ can be represented by a triangular matrix. $Y_1$ and $s_j$ can be represented by column vectors and equation 17 can be rewritten

$$\begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \\ \vdots \\ Y_m \end{pmatrix} = \begin{pmatrix} N_{11} & 0 & \cdots & 0 \\ N_{21} & N_{22} & 0 \\ \vdots & \vdots & \ddots & \vdots \\ N_{m1} & \cdots & \cdots & N_{mm} \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \\ \vdots \\ s_m \end{pmatrix} \quad (21)$$

The system of equations represented by 21 can be written

$$Y_1 = N_{11} s_1$$
$$Y_2 = N_{21} s_1 + N_{22} s_2$$
$$Y_3 = N_{31} s_1 + N_{32} s_2 + N_{33} s_3$$
$$\text{etc.}$$

The first equation can be solved for $s_1$, which, substituted in the second gives $s_2$ and so on. This is the "photon difference" method presented by L. Katz and A. G. W. Cameron (46). Determination of successive $s_1$ from $Y_1$ with individual errors results in rapid build-up of cumulative errors for larger energies.
The Penfold, Leiss method (47) involves an inversion of the $N$ matrix to obtain an inverse operator $N^{-1}$, which, when applied to equation 17 yields the $s_j$. The difficulty here is just related to the problem of obtaining a bounded inverse operator for the integral expression, small deviations in the $Y_i$ cause the resultant $s_j$ to oscillate wildly at higher energies, notably beyond the giant resonance.

Attempts have been made to smooth the data points $Y_i$ and understandably smoother cross-sections have resulted, but such smoothing can certainly result in removal of physical information from the problem.

Recently a solution to the problem has been presented by R. Malvano, A. Molinari and V. Omni (48) in which an artificial kernel function composed of straight lines is used. The method is being examined in this laboratory.

E. Least Structure Analysis of Data

The solution developed recently by E. C. Cook (5, 49) can be termed the "least structure" solution of the photoneutron yield problem. For background material on this procedure the reader is also referred to reference (43).

Each data point $Y_i$ is taken a sufficient number of times so that a valid standard deviation can be determined for it. Actually, percent standard deviations are calculated. The percent standard deviations for all data points are added and divided by the number of data intervals to
yield a measure of the overall precision of the experiment. The errors on a given $Y_i$ result from two factors, counting statistical errors $e_i$, and systematic errors $\epsilon_i$. The standard deviation of a radioactivity count is the square root of the total number of counts. Since the total number of counts increases with energy in a photonuclear experiment the percent counting error $e_i$ decreases with increasing energy. In an experiment with reasonable counting rates, $e_i$ becomes negligible with respect to $\epsilon_i$ after the first five or ten data points above threshold energy. The errors can be summed over the $m$ yield points in the following way.

$$\sum_{i=1}^{m}(e_i^2 + e_i^2) = m \sum_{i=1}^{m} \frac{\epsilon_i^2 (e_i^2 + e_i^2)}{\epsilon_i^2}$$

$$= m \epsilon_{\text{ave}}^2 \sum_{i=1}^{m} \frac{(\epsilon_{\text{ave}}^2 + e_i^2)}{\epsilon_{\text{ave}}^2} = \chi^2 \sum_{i=1}^{m} \frac{1}{w_i}.$$

Since the systematic errors are not energy dependent, $\epsilon_{\text{ave}}$ is evaluated by averaging the standard deviations of all yield points where $e_i$ is negligible. The parameter $\chi^2$ is then a measure of the overall systematic error of the experiment. The energy dependent factors $w_i$ are calculated using the $\epsilon_{\text{ave}}$ and the counting statistics. The $w_i$ are 1 for all points where $e_i$ are negligible. An expectation value $\Delta \chi^2$ of the probable error of $\chi^2$ is also established (rather arbitrarily) as an aid in computation. $\Delta \chi^2$ might be
chosen to be about one percent of $\chi^2$.

Consider now a sequence $s'_j$ such that

$$Y_i = E_{ij}s'_j.$$  \hspace{1cm} (24)

(The summation sign has been dropped, sums are taken on repeated indices.)

That sequence $s'_j$ is considered to be a solution which is the smoothest sequence consistent with the requirement that the sequence $Y_i$ (equation 24) match the measured $Y_i$ to within $\Delta \chi^2$ of the summed experimental errors, i.e.

$$\sum_{i=1}^{m} \left( \frac{Y_i - Y_i^2}{\chi^2} - \chi^2 \right) \leq \Delta \chi^2. \hspace{1cm} (25)$$

The sequence $s'_j$ can be smoothed by minimizing the sum of the squares of its second differences. Define a "structure" function

$$\mathcal{S} = \sum_{i=2}^{m-1} \rho_k \delta_{ki} (s'_{i+1} - 2s'_i + s'_{i-1})^2. \hspace{1cm} (26)$$

The function $\mathcal{S}$ is now minimized (actually the problem is just formulated as an extremum problem) by taking the variation with respect to $s'_i$ subject to the constraints imposed by equation 25. The $\rho_k$ are weighting factors to be discussed later. For the purpose of the variation $\Delta \chi^2$ is considered to be zero and the constraint is imposed by the Lagrangian multiplier method (50, p. 43).
\[
\delta \left[ \sum_{i=2}^{m-1} P_k \delta_{k1} (s_{i+1}^i - 2s_i^i + s_{i-1}^i)^2 + \lambda^{-1} \left( \sum_{i=1}^{m} \left( \frac{N_i s_i^i - Y_i^i}{\overline{Y_i^2}} \right)^2 - \chi^2 \right) \right] = 0
\] (27)

where \(\lambda^{-1}\) is a Lagrangian multiplier and \(Y_i^i\) has been replaced by its value from equation 24. The solution to the system of algebraic equations resulting from the variation is

\[
Y_i^i = \begin{bmatrix} \frac{N_{ij}}{\lambda} \left( \frac{W_{ij}}{\lambda} \right) \delta_{ik} P_k \delta_{kj} N^{-1}_{qp} S_{pj} \end{bmatrix} s_j^i = M_{ij} s_j^i
\] (28)

where \(N^{-1}_{qp}\) is the inverse of the transpose of \(N_{ij}\) and \(S_{pj}\) is a smoothing matrix which contains a maximum of five non-zero terms in each row located symmetrically about the diagonal. The parameter \(\lambda\) now becomes a measure of the smoothing applied to the \(s_j^i\) and \(\lambda = 0\) just gives the original unsmoothed expression.

Now for some value of \(\lambda\) expression 28 can be operated upon by the inverse of \(Y\) yielding a sequence \(s_j^i\),

\[
k^{-1}_{ki} Y_{ki}^i = \delta_{ki} s_j^i = \delta_{ki} s_j^i = s_j^i
\] (29)

Then, using equation 24 a set of \(Y_{ki}^i\) is calculated and a \(\chi^2\) (the first term of equation 25) is calculated and compared to \(\chi^2\). If the condition of equation 25 is not satisfied the calculations are redone for a new value of \(\lambda\). The sequence is repeated until an acceptable value of \(\chi^2\) results. The iteration is, of course, not based upon random selections.
of $\lambda$, but a plot of $\lambda$ versus $X^2$ provides a basis for a rational iteration procedure.

The question of the validity of the results of the procedure might be examined by postulating a "true" set of $s_j$ and determining the extent to which a set of calculated $s'_g$ can vary from the $s_j$.

$$Y_1 = N_{ij} s_j = M_{ik} s'_k.$$  \hspace{1cm} (30)

$$M^{-1} N_{ij} s_j = M^{-1} M'_{ik} s'_k = \delta_{gk} s'_k = s'_g.$$ \hspace{1cm} (31)

If the product $M^{-1} N$ were to represent a unit matrix or delta function then the $s'_g$ and $s_j$ would be identical. In fact, $M^{-1} N$ is a very narrow function of energy at low energies and becomes broader, while maintaining constant area, at higher energies. $M^{-1} M$ has been called the "resolution function" because its width indicates the extent to which a narrow resonance in the true cross section could be expected to be resolved by this procedure. Programs such as this have, on occasion, been suspected of anomalous behavior such as the creation of resonances or peaks where, in fact, there were none. Examination of the resolution function should reveal any such tendencies. If large oscillations appeared at the sides of the main curve anomalous cross section resonances might appear. In section V some resolution functions resulting from analysis
of the data will be shown. They have not shown tendencies to introduce unwarranted variations in the results.

A weighting factor $P_k$ has been included in equation 26. Early calculations indicated that the individual terms of equation 25 were not contributing uniformly to $\chi^2$ but that errors tended to be larger in one region than another. In effect, more smoothing than necessary was being applied in some regions and less than necessary in others. In an attempt to force the program to smooth uniformly, so that the resulting approximation to the cross section will be equally good, say, in the giant resonance region, as it is far above the giant resonance, several empirical functions $P_k$ were tried. Finally the $P_k$ were based on the expression

$$P_k = \arctan \frac{\Delta s_k \text{ unsmoothed}}{s_k \text{ smoothed}}.$$  \hspace{1cm} (32)

The nonuniformity was not large and the results with no $P_i$ would probably have been acceptable.

The $\Delta s_k$ mentioned in equation 32 is the error calculated for $s_k$. These errors are calculated in the computer program and are defined by

$$\Delta s_j = \left[ \sum_{k} \left( \frac{1}{M_{jk}} \frac{1}{M_{jk} - 1} (\Delta Y_k)^2 \right)^{\frac{1}{2}} \right]^{\frac{1}{2}}$$ \hspace{1cm} (33)

where the $\Delta Y_k$ are the estimated standard deviations in the yields. Implicit in this result is the assumption that all sets of data with the same $\chi^2$ will result in the same $\lambda$
and $M$ matrix. This is not strictly true. For a given set of data with given statistical errors, slightly different distribution of errors results in a statistical distribution of $\lambda$'s and $\Omega$'s. Consequently a term like $((\Delta M_{j k}^{-1})^2 Y_k^2)^{1/2}$ should be added to the error in equation 33. At present there is no convenient method of calculating this term.
IV. EXPERIMENTAL TECHNIQUE

A. Synchrotron Energy Control

Precision in measurement of yield points $Y_1$ is dependent to a large extent upon the stability and reproducibility of the electron kinetic energy $T_0$ during the sequence of measurements. The kinetic energy of a relativistic electron is related to its momentum $p$ by the expression

$$(pc)^2 = T(T + 2m_0c^2)$$

(34)

where $c =$ velocity of light and $m_0c^2 =$ rest energy of the electron. If the electron is in a magnetic field its momentum can be expressed as

$$p = (2.998 \times 10^{-4})B \gamma \text{ MeV/c}$$

(35)

where $\gamma$ is the instantaneous radius of curvature of the electron path in cm and $B$ is the magnetic induction (or flux density) at the electron and normal to the plane of its path in gauss. So determination of the electron kinetic energy within the synchrotron requires a determination of the orbit radius and the z-component of magnetic induction at the orbit.

Electrons become highly relativistic when they are accelerated to energies above a few mev and their velocities very closely approximate the velocity of light. Electrons in the Iowa State University synchrotron are locked in
phase with a very-high-frequency rf energy source during the acceleration period and they make one orbit revolution for each rf cycle. Since they are traveling with velocity \( c \), the orbit circumference must be (on the average) just the wavelength of the rf energy source. The frequency of the rf source can be measured and maintained to high precision, hence the orbit radius can be established to the same precision. During this experiment the rf frequency was 171.4 mc per sec and the corresponding orbit radius was 27.84 cm.

It remains necessary to make an accurate determination of \( B(t) \) during the acceleration cycle of the synchrotron. In Figure 2, superimposed on the drawing of the electron orbit, is a dashed annular line labeled "energy control pick-up coil". This represents a loop of copper of the configuration shown, very accurately scribed on a piece of printed circuit board and bonded to the synchrotron pole face outside of the vacuum chamber and directly below the electron orbit. This loop samples the magnetic induction in the region of the electron orbit at all azimuthal angles. The voltage induced in this loop can be expressed

\[
V(t) = AB(t) \text{ volts} \tag{36}
\]

where \( A \) is the area of the annular coil. The determination of \( B(t) \) depends upon an accurate integration of this voltage wave with respect to time.
In many synchrotron and betatron laboratories this integration is performed with Miller integration circuits (51, 52, 53) using high quality chopper stabilized operational amplifiers. Figure 3a shows the electronic circuit normally used in such systems. The output of the integration circuit is a voltage function of time which is the analog of the electron momentum. This voltage is fed to a circuit in which it is compared with a fixed dc voltage which represents the analog of the desired electron energy. When the voltages are equal the comparator delivers an output pulse which triggers a fast electronic circuit which destroys the orbit stability conditions within the synchrotron and electrons with the desired energy strike the target causing bremsstrahlung radiation (54, 55).

Such circuits have been beset with difficulties. Even though the amplifiers are very stable, the dc level of the output wave is subject to slow drift, \( V_d(t) \), and this drift voltage, when added to the signal voltage, causes comparison at the wrong level and energy drift. This situation can be improved by the addition of a resistor \( R_f \) in parallel with the integrating capacitor but such a resistor deteriorates the quality of integration. Even with such resistors these circuits are subject to low frequency noise, \( V_n(t) \), on the output resulting from transient disturbances of the circuit. Such transients cause a smearing of the energy resolution of the system.
Figure 3. Energy control circuits
Circular electron accelerators (betatrons, synchrotrons) accelerate particles only on the positive slope portion of the $B(t)$ wave after $B(t)$ has passed through zero. In order to achieve higher energies the magnets of such machines are sometimes biased with a dc field so that $B(t)$ passes through zero somewhere near the negative peak of the time varying wave. Such a bias level is shown on the output wave form of Figure 1a. The feedback resistor $R_f$ tends to hold the average dc level of the integrator output wave at zero, and for a biased machine this is, of course, no longer a valid analog of the electron momentum.

The drift, noise, and bias problems could all be resolved if the output level of the integration circuit could be clamped at zero volts until the accelerator field passes through zero and integration started at that instant. Such a system has been reported (56) but the integration process was allowed to proceed as in Figure 3a and gating was performed on the output. This placed an unnatural constraint upon the system voltage levels and when the gate was opened a sharp step voltage was observed followed by the normal integration process.

In Figure 3b the gated integrator circuit presently used in this laboratory is depicted. The integrating resistance is split into two equal components and a fast high quality silicon switching transistor is connected from the junction to ground. During the period of time
immediately preceding passage of $B(t)$ through zero ("zero time" in accelerator parlance) current is delivered to the base of the transistor and the negative signal from the pick-up coil is clamped a few millivolts from ground potential. As the field passes through zero a pulse is generated in a tiny coil of wire wrapped around a filament of ferrite in the field (57). This "zero time" pulse triggers a fast rectangular gate generator the output of which is coupled to the base of the clamping transistor. The transistor clamping action is terminated, the voltage rises to its correct value, and integration proceeds. The integration gate is sufficiently long to allow integration until the field has reached its maximum value at which time it terminates and the clamp transistor resumes its clamping role. The trailing edge of the integration gate triggers a reset gate. This gate is fed to a high transconductance triode, the plate of which is connected to the output as shown. Application of this gate causes the discharge of the integrating capacitor through the silicon diode $d_1$ at the amplifier input. This diode from the amplifier input to ground does not affect the integration performance for two reasons, it is a silicon diode and as such requires about 0.5 volts forward voltage before it will conduct and the input of this operational amplifier is a virtual ground point and as such is not affected by anything but a very low impedance to ground. The silicon diode $d_2$
holds the output level just below ground potential during this discharge period so that when the reset gate is removed the voltage across the integrating capacitor is very nearly zero. Between the end of the reset gate and the beginning of the next integration the system is idling with no constraints placed upon the levels so that when the clamp is removed, integration of the applied signal can proceed with no disturbance other than the normal transient response of the amplifier.

It is obvious from the foregoing description that the demands made of the clamping transistor are rather rigorous. During clamping the transistor must conduct quite heavily in order to hold the voltage within a few millivolts of zero, otherwise the integrator will integrate this voltage producing an output other than zero at zero time. (This in fact happens, the transistor is not perfect. The output level at the end of reset is adjustable to a few millivolts negative so that it just reaches zero at zero time.) At zero time the transistor must terminate its clamping action very quickly. If the stored charge in the transition region takes even a few microseconds to be swept out, the resulting turn-off delay will introduce an initial momentum error. After the transistor is cut off, any leakage current introduced by it into the integration network would cause errors of integration. In effect, the transistor
must represent an almost perfect switch. There are available a very few PNP silicon planar epitaxial high speed switching transistors which meet the requirements. One such transistor is the Fairchild 2N995. The measured cut-off leakage current is less than $5 \times 10^{-9}$ amperes, the turn-off time from heavy conduction is about $10^{-7}$ seconds and the transistor will clamp the input signal to about 20 millivolts. Examination of Figure 3b will reveal that the transistor appears to be drawn in the circuit backwards. This is so, the transistor performs this particular function better with the roles of collector and emitter reversed. The gain is reduced by this procedure, but the gain-bandwidth product remains essentially constant so that quicker turn-off is achieved at the expense of more excitation current during clamping.

During photonuclear experiments the base line level of the output signal is monitored and has been observed to remain constant to within five millivolts for hours and days. The amplitude of the output wave is over 100 volts for full energy so that the output wave level stability is accurate to well better than one-tenth of one percent.

The performance of the level comparator is monitored in much the same manner and has been seen to have comparable stability. The voltage reference supply during the experiment was a KayLab Absolute DC Power Supply model 30 C 15 which uses a standard cell as its internal reference
transient dies out at about 10 millivolts and the method chosen of adjusting the transient oscillation on the baseline results in the linear extrapolation of the response intersecting the baseline about 0.3 microseconds before zero time. More accurate linearity checks are under consideration.

Because of the character of the hysteresis loop of the ferrite in the zero time peaker strip the output pulse is delayed slightly from true magnetic field zero. This delay is a function of $B(t)$ at zero field and consequently it is a function of the synchrotron bias level. The delay of zero time was measured for 50 amperes bias by measuring the momentum and delay time of electron acceptance for a number of different injection voltages ranging from 10 kilovolts to about 80 kilovolts. The straight line through the momentum points was extrapolated to zero and indicated a delay of zero time of 50 microseconds. The slope of this line was found to be 10.065 kev/c per microsecond.

With this information regarding the linearity and the actual zero time, the momentum, or energy calibration of the system can be accomplished in two ways. The system can be used as an absolute energy measurement device. The area of the pickup coil has been measured accurately and the voltage induced in it can be related to the magnetic induction at the electron orbit by an expansion of the
field in cylindrical coordinates about the median plane of the orbit. The radial gradient of the z-component of the field is applied to the calculation as a boundary condition and this has been measured by several techniques in this laboratory. The time constant of the integration stage can in turn be measured quite accurately by measurement of the slope of the linear response to a dc input stimulus. Such measurements have been made and preliminary results show encouraging agreement with some known nuclear mass thresholds.

For the purposes of this experiment the energy calibration of the synchrotron was accomplished by using the known zero intercept and the sharp break in the activation curve of $^{0}_{16}$ known to exist at 17.27 kev (59). The reference voltage at which this break occurs was measured and found to be 29.854±0.015 volts on the output wave.

The integration wave is now known to have a zero intercept of -2.7 microseconds, or 29 kev/c. The electron momentum can be expressed as

$$ p = CV + 0.029 \text{ kev/c} \quad (37) $$

where $V$ is the output voltage and $C$ is a constant to be calculated. 17.270 kev corresponds to a momentum of 17.781 kev/c. Insertion of this momentum into expression 37 together with the corresponding voltage results in a value of $C = 0.5946$. With these constants reference voltages
for all required electron energies were calculated.

It must be revealed at this point that a slight error was made in the original calculation and energy increments which were thought to be 0.500 Mev during the experiment were later found to be 0.4853 Mev. All energy scales presented in this paper have been plotted according to the corrected energy scale although some computer data associated with the experiment is incorrectly labeled.

The performance of the energy control system was tested by measurement of the threshold of the reaction $^{63}\text{Cu}(\gamma,n)^{62}\text{Cu}$ both with and without bias. The threshold energy is known to be 10.833±0.017 Mev (59) from mass data. The measured threshold energies found under the two conditions of measurement differed slightly and the differences were found to be consistent with the direction of zero intercept shift but slightly larger (approx. 50 kev) than that which could be accounted for by peaker strip zero shift. By measurement of the shift of the 17.27 Mev $^{16}\text{O}(\gamma,n)^{15}\text{O}$ break with and without bias and comparison with the copper shift it was established that the shift was not in integration, but rather an unaccountable shift in zero intercept time. Since the voltage wave shape from the pick-up coil is different with and without bias this test was considered to be a good test of integration performance. During the four month period over which these tests were being performed
the gated integrator circuit was undergoing modification and improvement and it now appears that the bias-no bias shift resulted from differing amounts of stored charge in the collector junction of the clamping transistor resulting in different turn-off delays. Before the oxygen experiment the clamping transistor was replaced by the very much improved transistor mentioned earlier and measurements of the 17.27 kev break in the $^{16}O(\gamma, n) ^{15}O$ yield curve periodically over a period of four months preceding the oxygen experiment indicate self consistency and agreement with measurements of the $^{65}Cu(\gamma, n) ^{62}Cu$ threshold of about 10 kev. Since the relative energy calibration of the synchrotron appears to be affected by zero time shifts resulting from changes in bias level all calibration points and data were taken with 50 amperes dc bias current in the synchrotron magnet. Before the experimental data points were taken the synchrotron zero time peaker strip was changed from permalloy to the ferrite strip described earlier (57) resulting in a marked improvement of stability of zero time with changes in bias. The ferrite has a smaller hysteresic loop so the zero pulse is always closer to true zero regardless of the B(t) wave shape.

B. Measurement of Radiation Dosage

The kernel of the integral equation 13 is expressed in equation 14 as the radiation spectrum seen by the sample
divided by a function \( P(T_0) \), the monitor response function. Since \( P(T_0) \) is not a function of \( k \) it can be removed from the integral and the equation rewritten as in equation 15 with the reduced yield defined as the product of measured counts per indicated dose times the dosemeter response function.

As indicated in Figure 2 the radiation dosage is measured by directing photons through the sample, then into the dosemeter. The dosemeter used in this experiment is a modification of a standard ionization chamber developed at the National Bureau of Standards and described in a monograph by J. S. Pruitt and S. R. Domen (60). The output of the chamber is charge resulting from radiation ionization of the gas within the chamber. This charge can be collected on a polystyrene capacitor connected in integrating feedback configuration across a Cary model 31 vibrating reed electrometer circuit. The output voltage of the electrometer circuit is the collected charge divided by the feedback capacitance.

The standard ionization chamber designed by NES has been calibrated to an absolute accuracy of about three percent by comparison with a calorimeter and other absolute radiation measurement devices. The absolute calibration curve is given in the monograph (60) and from it one can get factors relating the output charge in coulombs to the
total incident energy in Mev for each experimental energy. These factors will be labeled \( A_i \) coulombs per Mev so that the electrometer output voltage at a particular energy divided by \( A_i \) will be proportional to the total energy incident upon the chamber at that energy.

The HBS calibration was done using bremsstrahlung radiation filtered only by the vacuum chamber wall while the radiation spectrum reaching the monitor in this experiment has been filtered by the presence of the sample also. Consequently measurements have been made in this laboratory of the monitor output with the sample in place and with the sample removed. The radiation intensities were normalized in each case by an off-beam intensity monitor shielded from scattered radiation from the sample. In this way a second series of factors \( B_i \) were developed which correct the monitor response for the presence of the sample. (Note here that the sample absorption is not simply the nuclear absorption of the photoeffect, but includes all other absorption mechanisms down to very low energies, such as Compton scattering, pair production, etc.) The measured sample absorption factors varied smoothly from 0.832 at 15 Mev to 0.898 at 65 Mev.

Since the chamber used in the experiment was not identical to the HBS chamber it was necessary to measure yet another correction factor \( C_i \), the ratio of the response
of the chamber used to that of an actual replica of the NBS chamber. This was done in a manner similar to the previous measurement except that instead of an off-beam monitor, a thin walled transmission ionization chamber was placed in the beam and used for normalization. The larger chambers were shielded from scattered radiation from the transmission chamber and only allowed to receive the direct beam. The ratio of the experimental chamber response to the NBS chamber response was unity up to about 15 Mev, then fell off smoothly to 0.980 at 64.5 Mev.

The local chamber was substituted for the NBS chamber because the NBS chamber is not hermetically sealed and variations in atmospheric pressure and humidity cause variations in response. The local chamber is sealed under positive pressure of dry nitrogen. Since the response is a function of the pressure a meter is affixed to the chamber and the pressure noted periodically. Some difficulties have arisen recently with this chamber comparison method. The transmission chamber response is not linear with beam intensity for reasons which are not yet clear and care must be taken to normalize the chamber responses at the same intensities. To avoid all such problems the NBS chamber is being modified so that it can be sealed and operated under positive pressure.

The ratio of activity developed in samples to dosemeter
response at fixed energy was measured at several different intensities and found to be constant indicating that the non-linearity is in the transmission chamber and not in either of the dosemeter ionization chambers. This can be seen by examination of Figure 4 in which counts per dose has been plotted as a function of time for several different bombardment intensities at 35.75 Mev. The plot was made primarily to demonstrate counting losses resulting from high counting rates but one can see that the counts per dose calculations at the fourth minute, where counting losses are negligible, have a standard deviation of 0.23 percent for a dosemeter, or beam intensity, ratio of about 5 to 1. The standard deviations at later times get larger again because of increasing counting statistical errors as the activity dies out.

The output voltage of the electrometer, divided by the three correction factors \( A_1, B_1, \) and \( C_1, \) now represents the total energy in the bremsstrahlung spectrum incident upon the sample for electron energy \( T_1. \) The monitor response can now be normalized to unity by dividing by this total energy which is just

\[
\int_{0}^{T_1} \frac{\Phi(T_1, k) dk}{k} = \int_{0}^{T_1} \Phi(T_1, k) dk = \Phi_1
\]  

(38)
Figure 4. Counts per dose as a function of time for several bombardment intensities
where \( \sigma(T^,k)/k \) is the bremsstrahlung cross-section with the inverse energy dependence made explicit. The integrals of the intensity spectra \( g_i \) have been tabulated by computer for the Schiff integrated over angle expression (44).

The product of the constants \( A_1, B_1, C_1, g_1 \) is now proportional to the monitor response function \( F(T_0) \), equation 14.

The ionization chamber has provision for insertion of a standard ionizing source of strontium 90 as a check on the entire dose monitor. Standard source readings were taken at frequent intervals during the experiment and reproducibility appeared to be about 1 percent. These readings were taken, however, by reading the millivolt meter on the front of the Cary electrometer, a rather inaccurate way of recording data. Further comments on dosimeter reproducibility appear in the following section.

C. Activity Computer

The actual measurement of the nuclear photoeffect in this experiment is performed indirectly by measurement of the decay of the radioactive product \( \text{O}^{15} \). The voltage output of the dosimeter represents the integral of the total energy incident upon the sample but since some of the \( \text{O}^{15} \) atoms created early in the bombardment will have decayed by the end of the bombardment this dosimeter
reading will not be proportional to the resulting radioactivity unless the radiation dosage remains constant during bombardment. Proportionality could be established for all conditions by placing a resistance across the electrometer feedback capacitor such that the RC time constant is the same as the $\beta^{15}$ mean life. In this way the dosimeter output will build up and decay in just the same way as the resultant activity.

The problem was actually solved in a slightly different manner. The feedback capacitor of the electrometer was reduced to a very small value and shunted by a resistor so that voltage out of the electrometer circuit is proportional to the instantaneous value of the incident energy. (The small capacitor integrates over synchrotron burst periods but the time constant is very small with respect to the total bombardment period or the activity half-life.) This output is fed to the input of the electronic circuit shown in Figure 5. The operational amplifier is a chopper stabilized Philbrick model USA-3 and $B_1$ and $B_2$ are adjustable feedback ratios. If the gain of the amplifier is assumed to be infinite and it is assumed that no current flows into the amplifier input, the differential equation for the circuit is

$$\left(\frac{d}{dt} + \frac{B_2}{B_1 R_1 C_f}\right)v_o(t) = -\frac{1}{B_1 R_1 C_f} v_1(t).$$

(39)
OPERATIONAL AMPLIFIER

\[ A \to \infty \]

\[ \text{Figure 5. Circuit diagram of activity computer} \]
At a particular photon energy the probability of creation of a radioactive atom by nuclear photoabsorption is proportional to the product of the reaction cross section \( \sigma \), the photon intensity \( I(t) \), and the number of nuclei per unit area \( N_0 \). (A negligible number of atoms are transformed so \( N_0 \) is essentially constant.) If the number of radioactive atoms is \( N(t) \) and the radioactive decay constant is \( \lambda \) then the differential equation for \( N(t) \) is

\[
\frac{dN}{dt} + \lambda N(t) = \sigma N_0 I(t).
\] (40)

Equations 37 and 38 are isomorphic so if \( V_i(t) \) is proportional to \( I(t) \) then \( V_o(t) \) can be made proportional to \( N(t) \), the number of radioactive atoms, by adjustment of the feedback ratios \( B_1 \) and \( B_2 \). In the circuit used, \( B_1 \) is adjustable with a precision potentiometer and \( B_2 \) is unity. The ratio \( R_f/R_1 \) was 10 in the circuit used giving an overall voltage "gain" of 10 if the concept of gain in such a circuit is meaningful. Finally, the number of radioactive atoms is proportional to the activity through the decay constant \( \lambda \).

The output of the activity computer is delivered to an expanded scale Brown recorder where a record is made of each bombardment. Because of the expanded scale the data can be read to about 1 part in 5000.

With the strontium 90 standard source in place a
series of 3 minute dose accumulations were performed with the activity computer decay time set to match the 123 second half-life of $^{16}O$. The result of about thirty such runs performed over a period of about four hours indicated a standard deviation in Brown recorder reading of 0.2 percent.

D. Samples and Photon Beam Geometry

Boric acid, $H_3BO_3$, was chosen for the samples because it is readily available in chemically pure form, it is easy to handle, and it contains a large percentage of oxygen. The nuclear photoeffect on Boron creates only activities with very short or very long half-lives so negligible interference results from its presence. The hydrogen cannot, of course, contribute any activity. Samples were prepared by compressing 35 grams of boric acid in a 1 1/8 inch diameter form using a 20 ton hydraulic press. Upon removal from the form the samples increased slightly in radius to a very uniform 2.870 cm. The average length of the samples was 3.703 cm and the density was found to be 1.464±0.001 grams per cubic centimeter. While the sample densities were exceedingly uniform, slight inequities in initial weighing resulted in a slight variation in length. This was taken care of by irradiating each sample a minimum of three times at 35.75 Mev and counting the activities developed. An average response for each sample was calculated and compared to the average response for all of the
samples. In this way a sample normalization factor was determined for each sample. These correction factors were typically one-half percent with the largest around two percent. The experiment was started with 49 acceptable samples and several were rejected during the experiment after they had been dropped and chipped.

The samples were sufficiently hard so that they could be handled and inserted into sample holders without damage. Slight shaving at the edges during the experiment was not considered important because the photon beam passed through the samples axially being entirely eclipsed by the samples. The beam diameter emerging from the far end of a sample was measured by x-ray photograph and found to be 1 inch and quite precisely centered.

The photon beam was collimated ahead of the sample as shown in Figure 2. The hole in the collimator is conical of half angle 0.021 radians with the apex at the synchrotron target. This was done to minimize scattering from the collimator walls which has been suspected of affecting dosimetry. The collimator is 4 inches outside diameter and it is placed as close as possible to the synchrotron vacuum chamber. Since the collimator extends well into the synchrotron magnetic field it was constructed of lead shot bonded together by epoxy resin so that eddy current heating would be minimized. The length of the collimator was
calculated to give a photon intensity attenuation factor of $10^4$ using the lowest mass absorption factor for lead, 0.045, and taking into account the reduction in density resulting from the use of lead shot. The collimator is 25 cm long, as indicated in Figure 2.

The collimator was centered on the photon beam using the copper cross and photographic methods. Careful checks were made for beam position or direction shifts at the upper end of the energy scale. The beam position was found to remain fixed to well within a millimeter at the leading edge of the collimator under various conditions of intensity reduction throughout the energy range. It appears very unlikely that beam movement can be responsible for any observed structure in the observed photoneutron cross section.

Beyond the sample the radiation monitor chamber is shielded from scattered radiation by 4 inches of lead. The direct beam passes into the lead house through a circular hole which is 1/2 inch larger in diameter than the beam. The effectiveness of shielding the dosemeter from scattered radiation was measured by comparing the dosemeter reading with the collimators open to the reading with both collimators blocked. The ratio was found to be 3000 to 1 and the shielding was considered to be adequate.
E. Radioactivity Counting System

The $^{0}\text{15}$ nucleus decays to stable $^{N}\text{15}$ by emission of a positron which very quickly annihilates with the emission of two 0.51 Mev photons. The annihilation photons were detected by placing the bombarded samples in an accurately reproducible position in contact with the front face of a 3 by 3 inch thallium activated sodium iodide scintillation crystal coupled to an EMI 9531B multiplier phototube. Output pulses from the phototube were delivered through a standard White cathode follower circuit to a Hamner model 1303 non-overloading amplifier. Pulses of constant height and width from the amplifier integral discriminator output were delivered to two Eldorado model SC-750 decimal scalers. The counts indicated by the two scalers were recorded and compared with each other as a precaution against scaler malfunction. The multiplier phototube was operated with 1000 volts supplied by a Keithlay model 241 chopper stabilized high voltage supply with a specified stability of 0.005 percent per hour. A 50,000 ohm thermistor was placed in series with the phototube dynode resistance divider string and adjusted to minimize gain shifts of the phototube-crystal combination resulting from temperature changes. The scintillation counter system was shielded against background radiation by four inches of lead and the background counting rate during the experiment was typically 1800
counts for three minutes. At frequent intervals during the experiment the entire counting system was checked by counting radiation from a standard Na\textsuperscript{22} source. If necessary, the amplifier discriminator level was adjusted as a result of a standard count so that a counting stability better than ±0.3 percent was maintained throughout the experiment. The decrease in the expected standard count resulting from the 2.58 year half-life of Na\textsuperscript{22} was taken into account in these adjustments. In order to obtain the best possible counting statistics for data points just above threshold the amplifier gain was set as high as was possible without introducing large background counts due to phototube noise. Counting rates reached 100,000 counts for the three minute counting period at about 20 Mev and the maximum rate above 30 Mev was typically 1,300,000 counts. At energies above about 28 Mev the synchrotron injection timing was detuned slightly in order to hold bombardment intensity roughly constant.

In order to examine the counting system for counting losses at high counting rates five bombardments were made at 35.75 Mev with intensities from well below the level used in the experiment to the maximum obtainable beam intensity. The arbitrary reading of the activity computer for these runs varied from 1.540 to 8.175 while the readings during the experiment were typically 3.6 above 30 Mev. The samples were counted for 50 second periods with 10 second intervals for 7 minutes and counts per dose for these
periods were calculated. A plot of the results has been presented in Figure 4 and it is quite clear that counting losses occur at high counting rates for the high intensity runs. As was pointed out earlier, the convergence of all of the curves to a very small spread of values after a few minutes indicates that the initial variation in counts per dose values results primarily from counting rate errors and not from non-linearity of the radiation monitoring equipment. The data plotted in Figure 5 were used to make a counting rate correction factor curve and these correction factors were applied to all of the experimental data requiring them. The correction factor was unity up to 500,000 counts/3 min. and varied smoothly up to a one percent correction to 1,450,000 counts/3 min. The experimental counting correction factor curve was found not to agree with a theoretical correction factor curve based on a fixed dead time analysis of counting. The implication was that as the counting rate increased the dead time also increased making an experimental evaluation of the correction factors necessary.

P. Measurement of Yield Curve

The yield curve for the reaction $^{16}\text{O}(\gamma,n)^{15}\text{O}$ was measured by bombarding the boric acid samples with photons from the synchrotron and measuring the yield of radioactive $^{15}\text{O}$ at each energy. The total number of counts for three minutes divided by the output reading of the activity
The computer at the end of bombardment at a particular energy is the yield \( A(T_0) \) defined in equation 13. Data were taken from 15 Mev, below the reaction threshold, to 62 Mev in increments of 0.4833 Mev. The yield at each energy point was measured a minimum of three times and each of the samples was bombarded a minimum of three times at 35.75 Mev for sample normalization purposes.

Since radioactivity created in the sample is decaying during the bombardment little is gained by bombardments longer than one mean lifetime of the radioactive nucleus created. The mean lifetime of \( ^{15}O \) is very close to 177 seconds so the samples were bombarded for three minutes, one minute was allowed to elapse during which the sample was removed from the synchrotron and placed in the radioactivity detector, and the radioactive decays of \( ^{15}O \) were counted for three minutes.

It is possible that activities with longer mean lives than \( ^{15}O \) can be produced weakly in the samples by higher order reactions (i.e. \( ^{16}O(\gamma, p2n)^{13}N \), \( ^{16}O(\gamma, \alpha n)^{11}C \), etc.) at high energies. To prevent such activities from contributing spurious counts to the low energy yield points, where counting rates are very low, ten samples were set aside and used only at energies below 19 Mev. The measured background with one of these samples, not recently bombarded, in the counter was not sensibly different from the no-sample background.
Accumulation of the data, exclusive of preparatory and auxiliary measurements, was accomplished in about 200 consecutive hours of synchrotron operation.

After multiplying each yield number by its sample normalization factor and counting rate correction factor a percent standard deviation was calculated from the three data at each energy. For all energies above about 20 Mev the counting statistical errors were negligible with respect to the system errors and the system errors did not appear to be correlated with energy. An average percent standard deviation was calculated for all energy points above 20 Mev and this average error was considered to be an indication of the overall precision of the experiment. The percent standard deviation for the entire experiment was found to be 0.91 percent. On the basis of this error the data were examined for individual points falling more than three standard deviations from the mean. Three such data points were found out of approximately three hundred points and they were discarded and replaced by subsequent remeasurement of the points. The individual data runs were labeled A, B, and C, and an average yield was calculated for each point. The percent standard deviation for the average yield function was considered to be $0.91/\sqrt{3}$ or 0.53 percent. Figure 6 is a plot of the average yield data, error flags representing percent standard deviation would be about the size of the dots on the figure.
Figure 6. $^{16}(\gamma,n)^{15}$ radioactive yield data
As mentioned previously, higher order activities can contribute weakly to counting rates at higher energies as can the 71 second activity resulting from the reaction \(^{16}\text{O} (\gamma, 2n)^{14}\text{N}\). The cross section for the \((\gamma, 2n)\) reaction, integrated to 32.5 Mev, has been reported to be less than 0.2 percent of the \((\gamma, n)\) reaction cross section integrated to the same limit (61). In order to correct for these extraneous activities bombardments were made at 30.5, 40.5, 50.5, and 60.5 Mev, the resultant activity was measured until the counting rate was comparable to background and the results plotted on semilog paper. No evidence was seen of the 71 second activity. After the 123 second \(^{15}\text{O}\) activity had decayed to a negligible level a longer lived tail appeared with an apparent half-life of 15 minutes. This apparent activity is almost certainly a poorly resolved combination of 10 minute \(^{13}\text{N}\) and 20.5 minute \(^{11}\text{C}\). A straight line through the longer lived activity points was extended back to the beginning of the count period and the initial activity rate was compared to that of \(^{15}\text{O}\). In this way a correction factor for the three minute \(^{15}\text{O}\) count was developed for each of the above energies and a smooth curve was drawn through the points. Extraneous activity correction factors were then taken from the smooth curve at each bombardment energy and applied to the data. These correction factors varied smoothly from unity below 25 Mev
to about 0.986 at 60 Mev and for the purpose of data treatment they are labeled $D_1$.

In Section III, page 24, the relationship which exists between the yield function and the reduced yield, equation 15, was described. The reduced yield for the $i^{th}$ energy is obtained by multiplying the yield by the monitor response function. In Section IV B, page 49, the monitor response function was shown to be proportional to the product of the energy dependent factors $A_1$, $B_1$, $C_1$, and $\bar{\Phi}_1$. Since the yield data has in it already the counts for the $i^{th}$ energy divided by the photon dose it is only necessary to multiply by the product $A_1B_1C_1\bar{\Phi}_1$ and the extraneous activity correction factor $D_1$ to get the reduced yield function. The meaning of these factors, in review, are

- $A_1$ - Ionization chamber charge per incident energy
- $B_1$ - Sample absorption factor
- $C_1$ - Ratio of NBS chamber response to response of chamber used
- $\bar{\Phi}_1$ - Total energy under bremsstrahlung curve for $i^{th}$ energy
- $D_1$ - Extraneous activity correction for $i^{th}$ energy.

The primary objective of this experiment is to examine the $^{16}$O photoneutron cross section for structure at energies above the giant resonance using a mathematical technique lacking firmly established precedent. It is known that the resultant cross section is particularly sensitive to small
variations in the reduced yield \( Y_1 \). Since all of the above factors are energy dependent care must be taken not to introduce structure into the data due to imprecision in the correction factors. To preclude this possibility the above factors were all multiplied together to yield one factor for each energy. For lack of a better name these factors as a function of energy will be called the reducing function. First differences of the reducing function were plotted as a function of energy and a smooth curve was fitted to the points. A new reducing function was created using as first differences points taken from the smooth curve. Application of the smoothed reducing function to the yield data could not introduce any significant point to point variation in the reduced yield which was not already in the data.

Figure 7 shows the reduced yield resulting from application of the smoothed reducing function to the average yield data of Figure 6. The smoothed reducing function factors were applied to each of the individual data A, B, and C, so that four complete reduced yield functions were available for least structure analysis.

In the NBS report by Pruitt and Domen (60) some extreme limits on the dosemeter response factors \( A_i \) were given which differed slightly in energy dependence from the average values used above. In order to examine the effect of such changes on the derived cross section two
Figure 7. $^{16}\text{(y,n)}^{15}$ reduced yield data

$^{16}(\gamma,n)^{15}$ REDUCED YIELD
AVERAGE OF THREE RUNS
$\sigma = 0.5\% \times 2 = 2.69 \times 10^{-3}$
MAY 13, 1962
additional smoothed reduction functions were calculated using the extreme values and the average data was reduced using these functions. The results of the least structure process on all of the reduced yields will be presented in the next section.
V. EXPERIMENTAL RESULTS

A. Least Structures Results

The experimental reduced yields with their associated errors were analyzed by the least structure method described in Section III. The calculations were done on the MURA IBM 704 computer at Madison, Wisconsin through the AEC Ames Laboratory data link. Upwards of fifty computer runs were made on different combinations of the data and the results consistently showed the same general structure. The effects of the least structure smoothing can be seen by examination of Figure 8. Figure 8a shows the result of operating on the reduced yield with the inverse $H$ matrix with no smoothing ($\lambda=0$), Figure 8b is the same operation but with some slight smoothing and Figure 8c is a converged solution by the $\chi^2$ test. These figures are photographs of cross sections plotted by the computer and the ordinates are not all the same since the computer selects plotting units to best fit the range of data to be presented. Nevertheless it is quite clear that the technique has produced a smooth and reasonable cross section curve from the data while the matrix inversion method alone produced a highly oscillatory and unsatisfactory solution.

In Figure 9a, b, and c are shown converged solutions for the individual sets of reduced yield data, runs A, B, and C. The degree of similarity in these curves is
Figure 8a. Cross section calculation from averaged yield data using no smoothing, $\lambda = 0$

Figure 8b. Cross section calculation from averaged yield data, slightly smoothed but not converged solution.

Figure 8c. Cross section calculation from averaged yield data, almost converged solution.
Figure 9. $^{16}(\gamma,n)^{15}$ converged least structure cross sections for runs A, B, and C.
indicative of the similarity of results in a large majority of computer runs made on the data. The error flags shown are the errors calculated by the computer program according to equation 35. The values of the ordinates of the graphs are not indicative of the absolute partial cross section for the reaction mainly because of arbitrariness in the proportionality between counts recorded and actual radioactivity in the sample.

Figure 10 shows a converged least structures solution for the averaged reduced yield points. The upper horizontal error flags over the peaks of Figure 10 represent standard deviations in the energies of the tops of the peaks resulting from comparison of the three curves A, B, C. The lower horizontal error flags are the full widths at half maximum of the resolution functions, equation 31, for each of the peaks.

In Figure 11 the resolution functions corresponding to the cross section resonances of Figure 10 are shown. The shape varies smoothly as a function of energy and the resolution function corresponding to the resonance at 45 Kev is deleted so that, in addition to the general trend, the degree of overshoot can be seen more clearly. While there exists a slight overshoot at higher energies, it does not appear possible that this overshoot could result in reflection of the giant resonances as small resonances at higher energy.
Figure 10. $^{16}(\gamma,n)^{15}$ converged least structure solution
Figure 11. Resolution functions
It is not clear at the present time that optimum information is obtained from converged (i.e. $\bar{\chi}^2 = \chi^2$) cross section calculations. Slight undersmoothing shows structure which appears in some cases to be valid but which is lost in smoothing to a converged solution. The three individual yield curves have been analyzed using the resolution function required for a converged solution of the averaged data ($\lambda = 0.37 \times 10^{-17}$).

Since the errors on the individual curves are higher than that of the average, use of this resolution function results in undersmoothed results. The results are presented in Figure 12a, b, and c. In all of these curves slight structure can be seen just above the 25 Mev resonance. The positions of the five uppermost resonances on these curves have been averaged to give the mean values and errors on the high energy resonances.

The actual experimental bremsstrahlung spectrum is only approximated by the Schiff spectrum used in the analysis. The possibility exists that use of an incorrect spectrum in analysis may introduce or affect the shape or position of high energy resonances in the cross section. In order to investigate the effect of spectrum shape on cross section the averaged data were analyzed using two modified spectra. In one case the Schiff expression was used but the spectra were calculated for energies 11 Mev
Figure 12. Least structure cross sections from individual runs with $\lambda = 0.37 \times 10^{-17}$
higher than the experimental energies. The resulting relative cross section, shown in Figure 13a, is larger than normal due to the decrease in the number of photons at each energy but the cross section shape and resonance position remains essentially unchanged. In the second spectrum modification the Schiff spectrum was modified to have an increased number of photons at the tip in agreement with recent calculations by R. T. Deck, C. J. Mullin and C. L. Hammer (62). This modification, Figure 13b, resulted in a slight lowering of the energy of the sharp resonances below the giant resonance but essentially no change in the appearance of the high energy resonances. Figure 13c is just the average result of Figure 10 presented for comparison.

In Figure 14a and b the cross sections calculated using the extreme limits of the dosemeter response function are shown. Again Figure 14c is a reproduction of Figure 10 for comparison. In Figure 14a the cross section in the giant resonance region is essentially unchanged while the minimums between the high energy resonances dip to excessively negative values. This evidently results from a slight depression of the yield curve at high energy using these limiting values of the dosemeter response function. In Figure 14b the opposite extreme response function was used and the high energy cross section is raised slightly above the average value. In any event the shape and
Figure 13. Least structure cross sections from averaged data using extreme dosemeter response functions
Figure 14a. $^{16}(\gamma,n)^{15}$ cross section by least structure calculation using wrong starting energy, 27 Mev, for calculation of bremsstrahlung spectra

Figure 14b. $^{16}(\gamma,n)^{15}$ cross section by least structure calculation using increased tip value on bremsstrahlung spectra

Figure 14c. $^{16}(\gamma,n)^{15}$ cross section, converged least structure solution of averaged data
Using modified Schiff spectrum

Converged solution averaged data
position of the high energy resonances appear to be essentially independent of these rather broad changes in dosimeter response function.

The existence of structure in the cross section above the giant resonance seems now to be verified by its consistent appearance in all calculated cross sections and by its apparent insensitivity to many experimental effects which might be suspected of causing the appearance of apparent structure.

B. Comparison with Other Data

Several measurements of the $^{16}(\gamma, n)^{15}$ cross section below 30 Mev have been made (63, 64) and an excellent bibliography has been compiled by E. Toms (65). The cross section derived from the present work will be compared to several of the most recent results achieved by methods other than the measurement of radioactivity.

The energy spectrum of neutrons from the reaction $^{16}(\gamma, n)^{15}$ has been measured in a neutron time of flight experiment by F. W. K. Firk and K. F. Lokan using an electron linear accelerator at Harwell, England (66). In Figure 15a the ($\gamma, n$) cross section derived by Firk and Lokan is presented. Below 20 Mev the cross section is complicated slightly by excited state transitions and some other phenomenon associated with the water target used, according to the authors.
Figure 15a. $^1_{^{16}}(\gamma,n)^{15}$ cross section from neutron time of flight experiment of Firk and Lokan

Figure 15b. $^{16}_{^{16}}(\gamma,n)^{15}$ cross section from direct neutron detection experiment of Bolen and Whitehead

Figure 15c. Undersmoothed result of this experiment
$^{16}(\gamma, n)^{15}$
FIRK AND LOKAN
NEUTRON TIME OF FLIGHT

$^{16}(\gamma, n)^{15}$
BOLEN AND WHITEHEAD
DIRECT NEUTRON DETECTION

MOI6S
UNSMOOTHED RESULT
OF THIS EXPERIMENT

ENERGY MEV
CROSS SECTION - ARBITRARY UNITS
The $^{16}\text{(}\gamma,n\text{)}$ cross section below 30 Mev has been measured recently by L. N. Bolen and W. D. Whitehead (67) using an electron synchrotron at the University of Virginia. In this experiment the ejected neutrons were detected directly and the cross section obtained from the yield curve by inverting the $N$ matrix (Penfold-Leiss method). Some smoothing of the cross section was obtained by analyzing the data in interlacing 1 Mev bins and more by drawing smooth gaussian curves through the resulting cross section points. The cross section obtained in this experiment is presented in Figure 15b and the curve shown is a smooth curve drawn through the data points presented rather than the gaussian curves presented by the authors.

In Figure 15c a cross section derived from the present data is presented. It is essentially the same as the cross section of Figure 10 except that it is undersmoothed somewhat by our present $\chi^2$ standards. This undersmoothed curve has been shown to illustrate some of the problems involved in deriving an acceptable cross section as well as for comparison with the other work. In all of our converged solutions there appears an unresolved bulge on the low energy side of the first giant resonance. In undersmoothed data this bulge is resolved into a definite resonance at 21 Mev and this is to be compared with the resonance obtained by Firk and Lokan at exactly the same energy.
Our giant resonance peaks do not agree exactly with those of Firk and Lokan probably because of their much finer energy resolution (or bin width) in this region. Above the 24.5 Mev giant resonance peak the converged solution of Figure 10 diminishes smoothly through some unresolved structure towards an apparently large resonance near 33 Mev. Our undersmoothed cross section however shows two definite resonances in this region in almost exact agreement with the cross section data presented by Bolen and Whitehead. There seems to be reason to believe that some of the structure observed in undersmoothed results is physically significant. It should be noted that the data points presented by Bolen and Whitehead correspond in some respects to our method of analysis with the Lagrangian multiplier \( \lambda \) set equal to zero, i.e. no smoothing. Actually their cross section was obtained by analyzing the data in interlacing 1 Mev bins using no smoothing.

The total \( \gamma \) absorption cross section for \( O^{16} \) has been measured in Russia by L. E. Lazereva and group (68). They have reported four resonance peaks at 22.3, 23.05, 24.3 and 25.15. The measurement was made between 18.9 and 26.6 Mev. The three uppermost peaks reported are probably contained but not resolved in our rather broad resonance at 24.5 Mev.

Very few cross section experiments have been performed on oxygen above the giant resonance. A cloud chamber
experiment has been performed by A. N. Gorbunov and V. A. Osipova (69) and the (γ,n) cross section is presented to 75 Mev. These results are shown in Figure 16a. The two main resonances at 22.4 and 24.5 Mev are shown and a relatively large tail extends from the 25 Mev resonance to 75 Mev but no structure is resolved above the giant resonance. The authors point out that the peak of the angular distribution of particles begins to shift toward 70° at energies above 24 Mev and above 25 Mev the peak is definitely established near 70 degrees. This seems to indicate an onset of E2 transitions near 24 Mev.

The results of an inelastic electron scattering experiment at Orsay, France, have been reported by D. B. Isabelle and G. R. Bishop (70). In Figure 16b one result of this experiment is presented. The curve presented is the charge scattered at an angle of 70 degrees with incident electron energy of 150 Mev. The radiation tail calculated by the authors has been subtracted from the curve. The intensity distribution at 70 degrees favors demonstration of E2 transitions while E1 transitions are suppressed. The salient features of the curve are the peak at 19.4 Mev and the two peaks at 45.5 and 49 Mev. In a more recent publication the authors submit that the resonance observed at 19.4 Mev results from an E2 transition (71). Figure 16c is again a reproduction of Figure 10 for comparison. Bishop and Isabelle also report a definite resonance,
Figure 16a. $^{16}(\gamma,n)^{15}$ cross section from cloud chamber experiment of Gorbunov and Osipova

Figure 16b. $^{16}$ levels from electron scattering experiment of Isabelle and Bishop

Figure 16c. $^{16}(\gamma,n)^{15}$ cross section converged result of this experiment
assumed to be El, at 21.0 Mev in agreement with levels shown in Figure 15.

These higher energy results agree in general with our finding that sizeable cross section exists above the giant resonance and that some structure exists in this region.

C. Summary of Results

Since absolute cross sections were not measured in the experiment the results can perhaps be expressed most concisely by tabulating the energy of each of the resonances observed and stating, for those resonances above 30 Mev, the area under the resonance as a percentage of the cross section integrated to 30 Mev. These data are tabulated in Table 5 along with levels reported by Bishop and Isabelle, Firk and Lokan, and by Bolen and Whitehead. Areas are derived from the averaged yield cross section data and energies are the average energies of the three individual cross sections calculated with $\lambda = 0.37 \times 10^{-17}$. Errors on the energies above 25 Mev are the standard deviations of the three results used and systematic energy errors have not been taken into account. Below 25 Mev energy resolution is of course limited by bin width.
Table 3. Observed resonance energies

<table>
<thead>
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<th>Energy Mev (this exp.)</th>
<th>Percent of cross section integrated to 30 Mev</th>
<th>Pirk and Lokan</th>
<th>Bishop and Isabelle</th>
<th>Bolen and Whitehead</th>
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<td>17.4±0.25</td>
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<td>17.3</td>
<td>17.5</td>
<td>17.3</td>
</tr>
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<td>-</td>
<td>19.1</td>
<td>19.5</td>
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<td>-</td>
<td>21.7</td>
<td>22.6</td>
<td>22.3</td>
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<tr>
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</tr>
<tr>
<td>38.9±0.5</td>
<td>7.9</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>45.1±1.1</td>
<td>4.9</td>
<td>-</td>
<td>44.8</td>
<td>-</td>
</tr>
<tr>
<td>50.4±1.0</td>
<td>2.4</td>
<td>-</td>
<td>49.3</td>
<td>-</td>
</tr>
<tr>
<td>57.9±1.3</td>
<td>4.7</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

<sup>a</sup>Positions of apparent levels which are not resolved in converged solutions.
VI. CONCLUSIONS

The results of the present experiment indicate that the \( (\gamma,n) \) cross section integrated from 30 to 60 Mev, i.e. well above the giant resonance region, is approximately 40 percent of the cross section integrated from threshold energy to 30 Mev. This is somewhat higher than the 15 percent allocated to the region above the giant resonance by M. Danos, page 10. According to the Wilkinson shell theory of the giant resonance, page 13, the entire El oscillator strength is exhausted by 30 Mev. Any portion of the observed high energy cross section resulting from El transitions is in conflict with this shell model prediction.

The \( ^{0}\text{H}_{16}(\gamma,n)^{0}\text{H}_{15} \) cross section integrated to 32.5 Mev has been reported to be 53.3\( \pm \)4.7 Mev mb. by E. Breuer and W. Pohlit (61). This is in essential agreement with other reports of the integrated cross section to approximately the same limits (67, 72, 73). If the integrated cross section to 30 Mev is assumed to be 50 Mev mb. then the integral to 60 Mev will be about 70 Mev mb. The \( (\gamma,p) \) cross section integrated to 32 Mev has been reported by W. R. Dodge and W. C. Barber (74) to be about 10 percent larger than reported \( (\gamma,n) \) cross sections integrated to the same limit. If it is assumed that this ratio of \( (\gamma,p) \) to \( (\gamma,n) \) cross section holds to 60 Mev, the cross section for \( (\gamma,p) \) plus \( (\gamma,n) \) reactions integrated to 60 Mev will be about
150 Mev mb. This integrated cross section falls far short of the sum-rule figure of 336 Mev mb., equation 11, page 8. There must then be appreciable cross section resulting from absorption processes between 60 and 150 Mev. The quasi-deuteron interactions and more complicated nucleon cluster interactions are known to contribute cross section in this region.

An attempted evaluation of the significance of the relative profusion of levels observed can only be conjecture until such time as definite multipolarities can be established. From a shell model point of view it seems reasonable to start by associating the 45.1 and the 50.4 Mev levels with E2 transitions from the $1s_{1/2}$ state to the $1d_{3/2}$ and $1d_{5/2}$ states as shown in Figure 1. This assumption is in reasonable agreement with the calculations of V. Gillet presented in Table 2. Since Bishop and Isabelle have reported the 19.3 Mev level to be a $T=0$ E2 transition (71) some credence might be given to the middle column of Table 2 and our relatively strong resonance at 32.7 Mev might be associated with the predicted 30.7 Mev $1d_{5/2}^{-1}1f_{7/2}$ transition. In this scheme the resonance at 21.0 Mev may be substituted for the 19.3 Mev resonance in the family of five E1 transitions predicted in the giant resonance region. This plus the remaining lower energy resonances observed at 17.4, 22.4, and 24.5 Mev are in reasonable agreement with the
particle-hole $E1$ levels predicted by Elliott and Flowers, Table 1, column 1. This leaves the very prominent resonance at 38.9 Mev and the uppermost resonance at 57.9 unaccounted for.

While the expressions derived from collective models have not been successful in predicting structure in the region of the giant resonance for oxygen, collective predictions regarding higher order oscillations based on the observed giant resonance structure may be of interest. By employing the formalism described by Carver, Peaslee, and Taylor, referred to as CPT, (40) for calculation of nuclear $E1$ overtones, Bishop and Isabelle have recently suggested that their levels at 44.8 and 49.3 may be $E1$ overtones (75). V. Danos (24, 25) has suggested that $E1$ overtones should start at $2.86E_m$ ($E_m$ is energy of giant resonance) based on hydrodynamical considerations. This factor can be incorporated into the CPT formalism by modifying their expression

$$E_m = 40A^{1/3} + 7.5 \text{ Mev} \quad (41)$$

for the giant resonance, to read

$$E_{E1} = (2.86)(40)A^{1/3} + 3 \text{ Mev.} \quad (42)$$

The added constants are related to the energy elevations resulting from the two nucleon exchange forces and the selection of 3 Mev for the second constant is rather
arbitrary. If equation 41 is solved for $40A^{-1/3}$ and the result is inserted into equation 42, an expression for $E_{3m}$ in terms of $E_m$ results. If the energies of the two main peaks of the giant resonance are inserted into this expression, the result is 45.7 and 51.7 Mev for nuclear E1 overtones.

Danos has also stated that nuclear collective E2 transitions should begin at $1.6 E_m$. Equation 41 can again be modified using this factor and using some intermediate energy, say 5 Mev, as additive constant.

$$E_{2m} = (1.6)(E_m-7.5) + 5 \text{ kev}.$$ (43)

Then E2 collective oscillation energies can be calculated based on the observed giant resonance energies and they occur at 28.2 and 32.2 Mev, very near resonances observed in the experiment.

The energies calculated by Gillet for E2 transitions from the $1s_{1/2}$ state to the $1d_{3/2}$ and $1d_{5/2}$ states are based on a rather imprecise experimental evaluation of the energy of the $1s$ level in oxygen (76). One might assume that this evaluation is far too low and that the $1s$ level is near an energy consistent with equally spaced harmonic oscillator levels as shown in Figure 1. The two E2 transitions in question are then brought to energies near 30 and 35 Mev. In Figure 1e the $1f_{7/2}$ level is shown elevated to an energy near its corresponding harmonic oscillator energy and the
transition energy is about 58 Mev. This simple nuclear shell picture provides three E2 transition energies between 30 and 40 Mev in reasonable agreement with the predictions based on collective motion discussed above and with resonances observed in the experiment. The resonances at 45 and 50 Mev are then left free to fill the suggested role of E1 overtones. In this very simple way nearly all of the significant cross section structure observed in the experiment can be accounted for. The very broad resonance near 57 Mev is not accounted for although its existence seems to be well established.

Very little more can be said about the high energy resonances until measurements are refined and multipoolarities are established experimentally.

The utility of the least structure method of analysis appears to be firmly established. The problem of the extent to which useful information is lost in converged solutions will have to be investigated intensively in the near future.

The number of evidently valid new levels in the oxygen nucleus and the suggestion that the present techniques can be extended to reveal even more structure should serve to stimulate further experimental and theoretical investigation of oxygen and other low Z nuclei in the near future.
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