ELASTIC WAVE SCATTERING FROM AN INTERFACE CRACK IN A LAYERED HALF SPACE UNDER WATER

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INTRODUCTION

Wave propagation in layered elastic media has been the focus of much work both in seismology and for nondestructive evaluation (NDE) applications. In much of this work, the layers are assumed to be perfectly bonded to each other. Structures are not always ideal however, and can contain flaws such as interface cracks or debonding between layers. One concern in NDE is the location and characterization of these regions of debonding. With this purpose in mind, in this paper we investigate the elastic wave scattering by a crack at the interface between a layer and a half space.

In Achenbach et al. [1] and Keer et al. [2], the plane strain problem of scattering by a crack parallel to the free surface of a half space was considered. The structure considered here reduces to this case in the appropriate limit. Comparison in this limiting case with results in [2] has been used as one check on the results presented here. The corresponding antiplane problem was considered in Ryan and Mall [3].

The antiplane and plane strain problems of an interface crack between a layer and half space were investigated by Neerhoff [4] and Yang and Bogy [5], respectively. Reference [5] was also used as a check on the calculations of the present work.

In the present paper, we present a short summary of the analytic solution of Gracewski and Bogy [6]. In this reference, results are presented for three incident loadings: 1) uniform normal and shear loading applied to the upper solid surface, 2) plane waves incident from the liquid with angle $\theta_L$ measured from the vertical, and 3) an incident beam with Gaussian profile. Results were given for a structure which has the same material parameters for the layer and the half space. Here we give results for a Gaussian beam incident on a layered half space consisting of two different materials.
The two-dimensional plane strain structure to be considered consists of an elastic layer of thickness \( H \) bonded below to an elastic half space (substrate) and in contact above with a liquid half space (see Fig. 1). The layer is perfectly bonded to the solid half space except along the region \(|x|<a\), where a stress-free crack of length \( 2a \) is located. We assume the crack faces remain traction free over the entire load cycle. In order to achieve this condition in reality the crack would have to be initially opened by a static preload or some other means, and our dynamical solution would be superposed on that initial state. Both solids are assumed to be isotropic, homogeneous, and linearly elastic and the liquid is compressible and inviscid.

Quantities referring to the substrate and liquid are distinguished from those on the layer by the addition of a prime or a subscript (or superscript) \( L \) respectively.

![Liquid - solid layer - solid substrate structure with an interface crack of length 2a.](image)

Because of linearity, the principle of superposition can be used to reformulate the problem in a more convenient form. The total solution will be expressed as the sum of its incident and scattered parts, i.e.,

\[
\tau = \tau^I + \tau^S, \quad u = u^I + u^S.
\]
The total field (no superscript) is defined as the solution of the problem formulated above subject to a given disturbance. The incident field (superscript I) is the solution corresponding to the given disturbance on a similar structure, but without a crack. And the scattered field solution (superscript S) is the difference between the total and incident solutions. The interface conditions for the incident field are given by equations (1) with \( a = 0 \). The interface and boundary conditions for the scattered field are identical to those in equations (1) except along the crack where

\[
\tau^{S}_{xz} = \tau^{I}_{xz} - \tau^{S}_{zz} = \tau^{I}_{zz} \text{ at } z = H, \quad |x| < a.
\]

To obtain integral equations for the scattered field solution, a dynamical generalization of Betti's reciprocal theorem will be used. This theorem for plane strain and steady harmonic motion can be stated as

\[
\iint_{\partial D} \left( f^{A} \cdot \mathbf{u}^{B} - f^{B} \cdot \mathbf{u}^{A} \right) dA = \frac{1}{\mu} \int_{\partial D} \left( \tau^{B} \cdot \mathbf{u}^{A} - \tau^{A} \cdot \mathbf{u}^{B} \right) \cdot \mathbf{n} \, ds \quad (4)
\]

where the superscripts A and B distinguish between fields corresponding to two different sets of surface tractions and body forces (acting at the same frequency) and \( \partial D \) is a region within the domain of the elastic body with boundary \( \partial D \) and outward unit normal \( \mathbf{n} \). The quantity \( \mathbf{u} \) represents a vector with components \( \mathbf{u}_{i} \).

To obtain the desired integral equations let fields A and B correspond to the scattered field solution and to the Green's function solution on the uncracked structure, respectively. (The derivation of this Green's function solution is given in Appendix B of Gracewski [7].) Evaluating equation (4) for the domain of the structure in Figure 1, we obtain a set of singular integral equations for the scattered field. After some rearrangement, these integral equations can be written for \( |x_p| < a \) as

\[
\tau^{I}_{zz}(x_p, H) = -\frac{1}{\pi} \int_{-a}^{a} \frac{du^{S}_{1}(x)}{dx} \left[ \int_{0}^{\infty} \frac{K_{11}(k)}{k} \cos k(x-x_p) \, dk \right] \, dx + \frac{1}{\pi} \int_{-a}^{a} \frac{du^{S}_{1}(x)}{dx} \left[ \int_{0}^{\infty} \frac{K_{12}(k)}{k} \sin k(x-x_p) \, dk \right] \, dx
\]

\[
\tau^{I}_{xz}(x_p, H) = \frac{1}{\pi} \int_{-a}^{a} \frac{du^{S}_{1}(x)}{dx} \left[ \int_{0}^{\infty} \frac{K_{21}(k)}{k} \sin k(x-x_p) \, dk \right] \, dx + \frac{1}{\pi} \int_{-a}^{a} \frac{du^{S}_{1}(x)}{dx} \left[ \int_{0}^{\infty} \frac{K_{22}(k)}{k} \cos k(x-x_p) \, dk \right] \, dx
\]

\[
\tau^{S}_{zz} = \tau^{I}_{zz} - \tau^{S}_{zz} = \tau^{I}_{zz} \text{ at } z = H, \quad |x| < a.
\]

and the \( K_{ij}(k) \) (which are explicitly given in reference [7]) are functions of the Green's function solution. The contour of integration along the real axis of the complex \( k \) plane must be chosen appropriately to avoid poles and branch cuts of the \( K_{ij}(k) \).

The additional constraint that arises from the condition that there can be no jump in displacement at the crack tip is

\[
\int_{-a}^{a} \frac{du^{S}_{1}(x)}{dx} \, dx = 0
\]
Before solving the singular integral equations (5), we must first determine the order of the singularity of the unknown function \( d/dx \begin{bmatrix} u^S \end{bmatrix} \) at the crack tips. By following Muskhelishvili [8], the singularities of the unknowns can be determined by examining the dominant part of equations (5).

For the present case, the unknown functions have integrable singularities at the end points. Both unknown functions must have the same order singularity, and therefore they can be written as

\[
\frac{d}{dx} \begin{bmatrix} u_i(x) \\ v_i(x) \end{bmatrix} = \frac{U_i(x)}{(a-x)^\gamma}, \quad \gamma = \gamma_R + i\gamma_I
\]

where \( 0 < \gamma_R < 1 \)

\[\text{where} \quad u_i(x) \text{ is a bounded, continuous function that can always be represented as an infinite series.}\]

Solving the dominant part of equation (5) for \( y \), we obtain

\[
y = \frac{1}{2} + \frac{i}{2\pi} \ln(1+\beta)
\]

where

\[
\beta = \frac{\mu(1-2\nu') - \mu'(1-2\nu)}{\mu(2-2\nu') + \mu'(2-2\nu)}
\]

which agrees with the interface crack tip singularity obtained by several authors for related static problems.

The real part of \( \gamma \) defines a square-root singularity, while the imaginary part results in an oscillating singularity. For the rest of this paper the material combinations will be chosen so that \( \beta \) is zero and only the square-root singularity will be considered. This puts the following restriction on the material parameters:

\[
\frac{\mu'}{\mu} = \frac{1-2\nu'}{1-2\nu}
\]

Since many material combinations come close to satisfying this restriction, approximate results for many real materials can be obtained.

The singular integral equations (5) along with (6) must next be discretized to obtain a set of algebraic equations. These algebraic equations were obtained by applying the Gauss-Chebyshev quadrature formula developed by Erdogan et al. [9]. The final set of equations can then be solved numerically for any given incident disturbance once the incident field stresses along the crack faces are known.

RESULTS AND CONCLUSIONS

Numerical results will be presented for three material combinations: 1) water-iron layer-nickel substrate, 2) water-nickel layer-iron substrate, and 3) water-iron layer-iron substrate. Material parameters which satisfy the constraint \( \beta = 0 \) are given in Table 1.

Figures 2 a) and b) show the mode I and mode II stress intensity factors respectively as a function of the nondimensional frequency \( \tilde{\omega} = \omega H/C_R \) for an incident normal plane wave. For these plots we have taken \( C_R \) to be the Rayleigh wave speed of an iron half space and \( H/a = 2.0 \). The shape of
Table I. Values Used for Material Parameters

<table>
<thead>
<tr>
<th>Material</th>
<th>$\nu$</th>
<th>$\rho$ (g/cm$^3$)</th>
<th>$c_d$ (km/s)</th>
<th>$c_s$ (km/s)</th>
<th>$c_R$ (km/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iron</td>
<td>0.28</td>
<td>7.7</td>
<td>5.72</td>
<td>3.16</td>
<td>2.92</td>
</tr>
<tr>
<td>Nickel</td>
<td>0.31</td>
<td>8.8</td>
<td>5.24</td>
<td>2.75</td>
<td>2.55</td>
</tr>
<tr>
<td>Water</td>
<td>1.0</td>
<td>1.48</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

these curves for the three material combinations are similar, but the location and the magnitude of the resonant peaks vary. A comparison of graphs of this kind for the range of crack sizes of interest could be used to predict the range of frequencies that will give the maximum response in an NDE test.

Figure 2 a) Mode I and b) Mode II stress intensity factors versus normalized frequency for a plane wave incident from the water onto a layered structure for $\theta_L = 0.0$ degrees. Responses for three material combinations are presented.

Figure 3 shows the real and imaginary parts of the scattered field normal displacement along the liquid-solid interface for a Gaussian beam incident at an angle $\theta_L = 30.1$ degrees from the liquid onto a nickel layer-iron substrate structure. The beam with normalized half beam width $\hat{\theta}_0 = W_0/a = 20.0$ is centered above the crack. For these plots $\hat{H} = H/a = 2.0$ and $\hat{\omega} = 1.25$. This value of $\hat{\omega}$ was chosen since it is near the resonant peak for this structure.
The largest peaks in these plots occur close to the crack center. The oscillations to the right and left of these peaks have a wavelength equal to the Rayleigh wavelength which can be expressed in non-dimensional form as

$$\hat{\lambda}_R = \frac{2\pi\hat{H}/\hat{\omega}}{.}$$

The magnitudes of the peaks of these oscillations slowly decay with distance from the crack. Therefore, we conclude that the oscillations are the result of forward and back-scattered leaky Rayleigh waves.

In Figure 3, results are given for the water-iron layer-iron substrate structure for the same parameters as in Figure 3 except here for $\theta_L = 30.4$ degrees. As in Figure 3, there is a sharp peak above the crack at $x = 0.0$ and the forward and back-scattered waves are evident. The magnitude of the forward and backward scattered waves for this case are lower than those in Figure 3.

In order to get the total response, the scattered field must be added to the incident field response. Total field responses for the water-iron structure can be found in reference [6]. For maximum response, the
wavelength must be on the order of the layer thickness and crack length. Since the beamwidth must be much larger than a wavelength, the incident beam will be much larger than the crack length. To avoid this constraint, a focussed incident beam can be used.

REFERENCES


DISCUSSION

Chairman Bahr: Any questions, comments?

Mr. Jim Rose (Ames Laboratory): Yes. I was very interested in what this oscillating singularity was. Does it have an analog in the crack in free space or the crack in the half space? What is it?

Ms. Gracewski: For a crack in a free space, there would not be an oscillating singularity. There has been discussion about whether the oscillating singularity is realistic, because within a very small distance from the crack tip, there is overlap between the crack surfaces. That's one of the reasons why I avoided dealing with the oscillating singularity here since I don't know if the results would be meaningful. A different model for the crack which takes contact into account may be needed.

Mr. Bahr: Yes, sir.

Mr. Anil Govada (ALCOA): Did you look at different crack tip radii?

Ms. Gracewski: This is just an infinitesimal crack. It takes no space.

Mr. Govada: The crack tip is just a point then.

Ms. Gracewski: Right.

Mr. Govada: Thanks.
Mr. Thomas Derkacs (TRW): Your conclusion seems to be that you can detect the presence of a disbond by running a surface wave along the surface of the part.

Ms. Gracewski: That's the goal, yes. Like I showed in the last two slides, though, the effect of the crack on the total field response may be difficult to distinguish experimentally for a Gaussian incident beam. This is because the beam width must be much larger than a wavelength while the crack depth is on the order of a wavelength. Therefore the beamwidth is much larger than the crack. To avoid this restriction, a focussed beam can be used.

If, in an experiment, the scattered field (i.e. the change in the total field due to the crack) is measured, the crack can more easily be located and characterized.

Mr. William M. Visscher (Los Alamos): You said that in order to get rid of the oscillating singularity, you had to put some constraints on the material parameters. Were those realistic constraints for the system you were considering?

Ms. Gracewski: I think that for many material combinations, it's close to being realistic. Not all material combinations will satisfy that constraint, but I chose material combinations for which Beta was approximately zero. For the numerical results, I was using nickel and steel and modified the constants a few percent to get the Beta equal to zero.