MODE THEORY OF THE NONSPECULAR REFLECTION PHENOMENA OF A GAUSSIAN ULTRASONIC BEAM INCIDENT ONTO AN ELASTIC PLATE

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INTRODUCTION

Nonspecular reflection effects of a bounded ultrasonic beam incident from a liquid onto an elastic structure have been the subject of a great deal of interest during last decades for material characterization [1-10]. It refers to phenomena where the reflected beam has an intensity profile different from that of the incident beam, including a lateral beam displacement, one or several minimum intensity area and a trailing field (Fig. 1). This phenomena occurs when the incident beam is phase-matched to one of the leaky waves supported by the structure. Numerous theoretical studies, based on the calculation of the reflection coefficient, have successfully explained the nonspecular reflection profile of a bounded beam incident at a critical angle [1, 3, 6-10]. In this paper, we present a mode theory for analyzing these nonspecular reflection effects. This approach, which gives a good physical insight, has been recently used to study the excitation of Lamb waves by the a bounded beam [11].

MODE THEORY AND PERTURBATION METHOD

To study the nonspecular effects, we consider a Gaussian beam reflected from a Liquid/Solid/Vacuum (L/S/V) structure as shown in Fig. 1. The beam is incident at a critical angle $\theta_i = \theta_n = \arcsin (c_0 / v_n)$ where c_0 is sound velocity in liquid and v_n is that of a given mode supported by the elastic waveguide. For the sake of simplicity, the beam is assumed to propagate in the yz plane along the z direction and to be uniform along the y direction, i.e., $\partial/\partial x = 0$ (2-dimensional model). The profile of the incident beam is characterized by an effective width 2w. With the assumption of $2w \gg \lambda$ (the wavelength in the liquid), the incident beam is considered parallel and its particle displacement is written as,

$$u_i (0, z) = U_{i0} \exp[-(z/w_0)^2 - ik_n z] \qquad (1)$$

where $w_0 = w \sec (\theta_i)$, U_{i0} is the amplitude and k_n is the wavenumber of the *n*th guided mode along the z direction. Assume u_r the particle displacement associated with the reflected wave, the boundary conditions to be satisfied at the liquid/solid interface (y = b/2) for the normal components of the displacement and the stress are, respectively,

$$(u_i + u_r) \cos\theta_n = a_n u_{ny} (b/2) \qquad (2a)$$

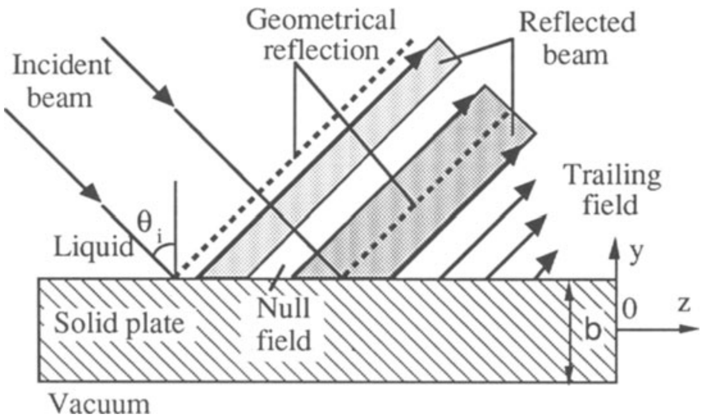


Fig. 1 Schematic diagram of the nonspecular reflection phenomena of a bounded beam rincident onto a liquid/solid/vacuum (L/S/V) structure.

$$T_{yy}\,(b/2,\, z) = -\, i\omega\, Z_L\, (-u_i + u_r) \tag{2b}$$

Here Z_L is the acoustic impedance of the liquid and $a_n\, u_{ny}$ is the displacement associated with the *n*th eigenmode. Writing Eq. (1a) implies that only the *n*th guided mode is excited by the incident wave. It can be readily realized by choosing the incident angle according to Snell's law $\theta_i = \theta_n = \arcsin\,(c_0 / v_n)$, especially when the width of the incident beam is large compared to the wavelength. Also, it is assumed in Eq. (1a) that the excited displacement of the leaky guided mode, when the layer immersed, is the same as that in the stress-free condition. This approximation based perturbation principle is acceptable when the liquid density is much smaller than that of the elastic waveguide. From Eq. (1), the reflected field at the liquid/solid interface can be expressed as,

$$u_r\,(o,\, z) = u_i\,(o,\, z) + a_n\, u_{ny}\,(b/2) / \cos\theta_n \tag{3}$$

which shows clearly that the reflected field is the superposition of a specular field $u_i\,(o, z)$ due to the incident wave and a leaky field caused by the guided mode. The interference of the two fields generally gives rise to a nonspecular field. So, the physical origin of the nonspecular reflection phenomena is readily illustrated, using the mode theory and a simple perturbation method. To determining $u_r\,(o, z)$, we shall find out the excited field of the guided mode.

The coefficient $a_n(z)$ of the excited mode is governed by a mode amplitude equation, obtained from the reciprocity relation and the orthoganality condition of eigenmodes [12, 13],

$$4\, P_n \left(\frac{d}{dz} + ik_n\right) a_n\,(z) = f_n\,(z) \tag{4}$$

where P_n is the average power flow associated with the *n*th guided mode and f_n is the exciting force applied on the liquid/solid interface ($y = b/2$) from both the incident and reflected waves,

$$f_n\,(z) = i\omega \left[u^*_{ny}(y)\;\; T_{yy}\,(y,\, z)\right]_{y\,=\,b/2} \tag{5}$$

In the case where the liquid is nonviscous, only normal stress T_{yy} is transferred onto the surface of the guide. Using the boundary conditions (2) and substituting Eq. (5) into Eq. (4) leads to the mode amplitude equation [13],

$$[\frac{d}{dz} + ik_n + \alpha_n]\, a_n(z) = -\frac{\omega^2 Z_0 w_0 u^*_{ny} u_i}{2P_n} \tag{6}$$

where α_n is the attenuation per unit length, stemming from the energy leakage of the given mode into the surrounding liquid during its propagation along the interface liquid/solid [11, 13]. It is proportional to the normal components at the interface y = b/2. According to Eq. (6), once the input force term due to the incident wave is given, the mode amplitude can be found by resolving the differential equation. For the incidence of a Gaussian beam given by Eq. (1), the amplitude of the excited mode can be determined by using the integral technique, as

$$a_n(z) = -u_i(0, z)\, (\sqrt{\pi}/4 P_n)\, \omega^2 w_0 Z_0 u^*_{ny} \exp(\gamma_n^2)\, \mathrm{erfc}(\gamma_n) \tag{7}$$

where $\gamma_n = -z/w_0 + \alpha_n w_0$.

REFLECTED FIELDS OF A GAUSSIAN BEAM ONTO L/S/V STRUCTURES

Substituting α_n of Eq. (9) into Eq. (11) yields the acoustic field of a Gaussian beam reflected from a L/S/V structure ,

$$u_r(0, z) = u_i(0, z)\, [1 - \sqrt{\pi}\, (\alpha_n w_0) \exp(\gamma_n^2)\, \mathrm{erfc}(\gamma_n)] \exp(-ik_n z) \tag{8}$$

Eq. (8) shows the reflected acoustic field is a function of only two parameters: the scale factor - z/w_0 and $h_n = \alpha_n w_0$, related to the wave leakage into the liquid. The identical expression was previously given by Bertoni and Tamir [1] in a simple case of L/S (liquid/solid) structure where the solid is a semi-infinite substrate supporting only one leaky surface wave, i.e. Rayleigh wave. Notice that the analytical solution of Eq. (8) is now generalized from a L/S structure to L/S/V structure. The elastic waveguide under investigation is no longer limited to a substrate and can be a single layer or multiple layers.

Fig. 2 presents the reflected profiles of a 12 mm-wide Gaussian beam incident from water onto an aluminium substrate (v_l = 6290 m/s, v_s = 3280 m/s) at the Rayleigh angle $\theta_R = 29°$. When the value of $h_R = \alpha_R w_0$ increases with the frequency [12], the reflected profiles become nonspecular and undergo a lateral displacement. Similar phenomena are observed in the beams reflected from elastic plates. Figs. 3 and 4 illustrate the reflected profiles associated with the fundamental Lamb modes S_0 and A_0. More complicated than the incidence at the Rayleigh angle, the Lamb angle in L/S/V structure is a function of the guided modes and fb (frequency x plate thickness). However, as can be expected, the reflected profiles associated with S_0 and A_0 modes (Figs. 3c and 4c) approach to that of the Rayleigh wave (Fig. 2c) when fb becomes great.

CONCLUSION

A modal analysis has been presented to study the nonspecular effects of a Gaussian beam reflected from liquid/solid/vacuum (L/S/V) structure. Owing to a perturbation method, the analytical expression of the reflected field given by Bertoni and Tamir for liquid/solid (L/S) structure was generalized to L/S/V one. In comparison with previous theories, the modal analysis gives a good physical feel for the affecting factors. Besides, numerical results have been demonstrated for the beam incidence at the Rayleigh angle and Lamb angles associated with the fundamental modes S_0 and A_0.

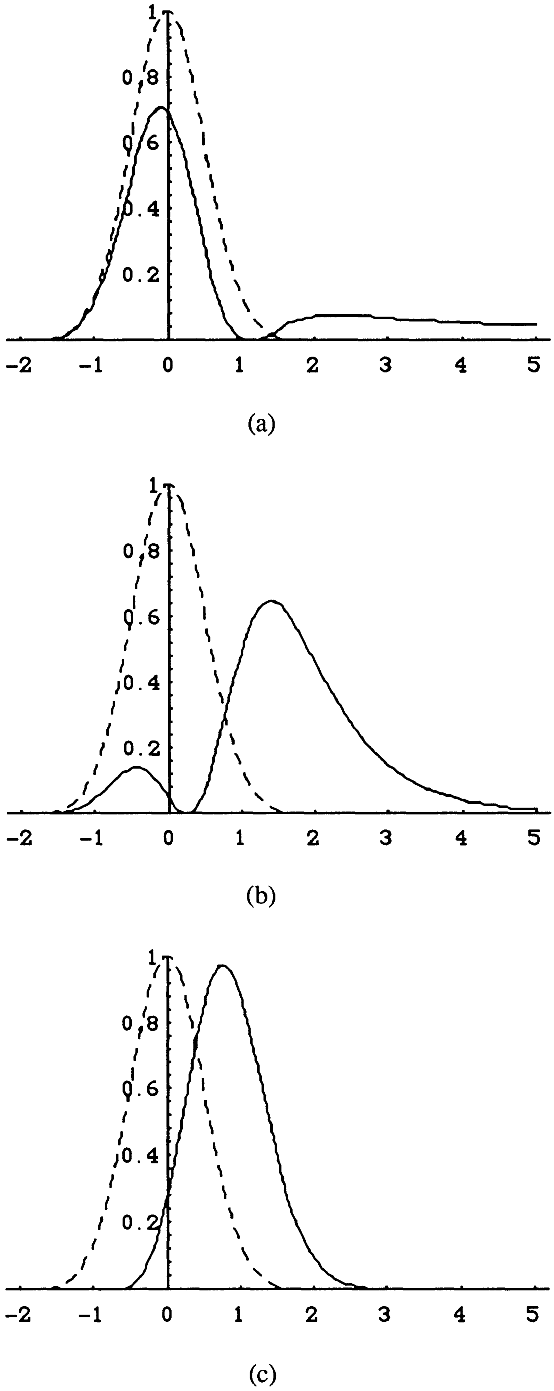


Fig. 2 Incident (dashed curves) and reflected (solid curves) profiles at Rayleigh angle θ_R = 29° for (a) $\alpha_R w_0 = 0.1$, (b) $\alpha_R w_0 = 0.6$ and (c) $\alpha_R w_0 = 2$.

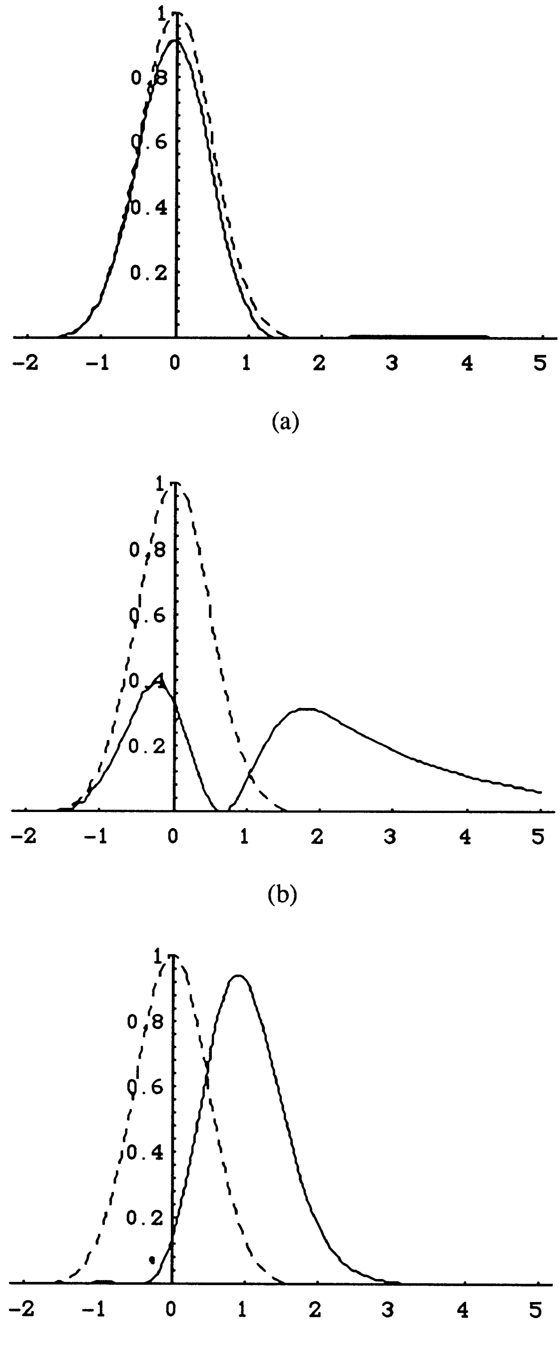
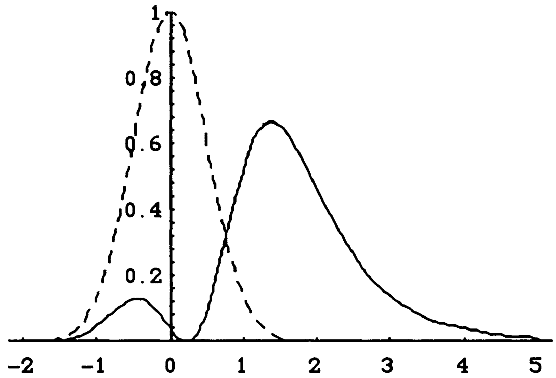
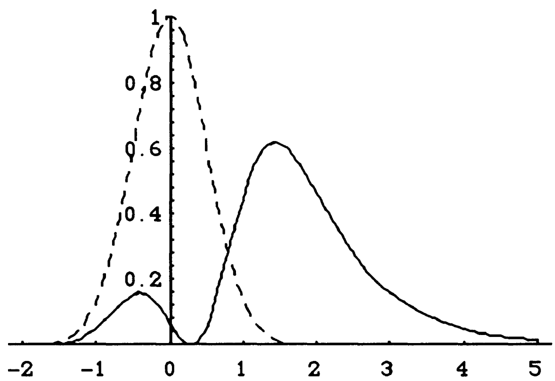


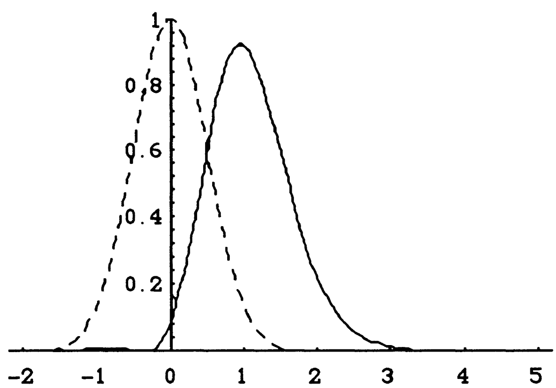
Fig. 3 Incident (dashed curves) and reflected (solid curves) profiles at Lamb angles of the mode S_0 in a 1 mm-thick plate: (a) $\theta_i = 15.5°$ (fb = 1.0 MHz x mm), (b) $\theta_i = 16.9°$ (fb = 2.0 MHz x mm) and (c) $\theta_i = 18.9°$ (fb = 7.0 MHz x mm).



(a)



(b)



(c)

Fig. 4 Incident (dashed curves) and reflected (solid curves) profiles at Lamb angles of the mode A_0 in a 1 mm-thick plate: (a) $\theta_i = 38°$ (fb = 1.0 MHz x mm), (b) $\theta_i = 32.2°$ (fb = 2.0 MHz x mm) and (c) $\theta_i = 29.1°$ (fb = 7.0 MHz x mm).

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