SIMULATION OF LAMB WAVE PROPAGATION USING PURE MODE EXCITATION

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INTRODUCTION

A combination of studies of dispersion behavior (wave mode properties) with time domain Finite Element modelling constitutes a very useful approach to developing guided wave NDE techniques. The dispersion curves and the mode shapes reveal a lot of information about how a guided wave will behave in a plate and can be used to judge modes and frequencies that are likely to be sensitive to defects. However, because of their modal nature, the dispersion curves cannot predict how a guided wave will react with a local geometric change in a plate such as a defect, which is of great importance to Lamb wave testing. Finite element modelling can fill in this information.

Since linear behavior is assumed, each mode can be studied separately and the results combined using the superposition principle to understand the entire system. This information is used both as a design and a calibration tool. Knowing how the various modes behave allows the optimum mode and frequency to be chosen for the testing requirements. In addition, the sensitivity of the experimental setup can be predicted quickly and inexpensively. These predictions can also provide a guideline to confirm when all of the experimental equipment is functioning correctly, since the modelling serves as a controlled virtual experiment. The number of Finite Element calculations needed to thoroughly analyze the system can be reduced by generating an input signal that has a wide frequency bandwidth. Once the response of the system over a wide frequency band is known, the response of the system to an input signal that contains a subset of that frequency band can be calculated immediately without re-running a Finite Element calculation.

In order to best understand the interaction of a specific mode with a simulated defect, it is advantageous if all of the energy in the simulated incident wave is confined to a single selected mode. If the incident signal consists of several modes, separating the effects of the different modes is difficult. This paper explores two
methods to excite a ‘pure mode’ by applying a time-varying displacement profile of the desired mode at the end of the plate. Since the modes of a plate are orthogonal at any frequency, if the applied displacement profile properly matches the displacement profile of the mode, only the desired mode will be generated [1]. The simpler of the two methods to produce a pure mode will be referred to as ‘center mode shapes.’ This technique, which assumes that the mode shape is constant over the frequency bandwidth of the input signal, works well to preferentially excite fundamental modes at low frequency-thicknesses. However, there are other locations on the dispersion curves where a more sophisticated technique, such as the one referred to as ‘exact mode shapes’ in this paper, is required. The exact mode shape technique accounts for the change in the displacement profile of the mode as the frequency changes and therefore concentrates the energy in the chosen mode throughout the excitation frequency range.

This paper discusses the procedures that are used to generate pure modes and compares the results from the two types of generation. It presents a summary and example application of techniques which have been developed in reference [2]. Three different input signals generated at separate locations on the dispersion curves will be used to evaluate the two techniques. These locations are shown in Figure 1, which shows the group velocity dispersion curves for an aluminum plate. The first two points, (a) and (b), are on the A0 mode at 1.0 and 5.0 MHz-mm respectively. The third location, marked (c), is on the S2 mode at its maximum group velocity, which occurs near 6.8 MHz-mm.

TWO DIMENSIONAL FOURIER TRANSFORM ANALYSIS

In order to analyze the purity of the mode that is excited, a two dimensional transform (2-D FFT) technique will be used. This technique has already been documented in the literature[2,3], however the concepts are summarized here since they are very relevant to the analysis of the work presented.

Lamb waves are sinusoidal in both time and space, the wave fields being governed by the complex exponential, $e^{i(kx - \omega t)}$, where $k$ is the wavenumber, $x$ is the distance in the direction of propagation, $\omega$ is the circular frequency, and $t$ is the time. Therefore, a two dimensional Fourier transform, which involves both time and space,
Figure 2. A schematic diagram showing how the two-dimensional transform technique[3] determines the amplitude of propagating Lamb wave modes by first transforming time into the frequency domain and then space into the wavenumber domain.

may be used to resolve the amplitudes of the propagating Lamb wave modes, even though several modes can exist at any frequency.

As shown schematically in Figure 2, the two dimensional transform requires time histories from multiple (typically 64) equally spaced points in space. The desired portion of each signal is gated in time and transformed into the frequency domain. For each frequency, the responses at all of the monitoring points are collected and a second transform is performed to convert the spatial information into the wavenumber domain. In the frequency-wavenumber domain, each of the modes that is present appears as a peak in amplitude. The height of the peaks can be used to quantify the amount of energy present in each mode as seen in the monitoring region. The width of the peaks in the frequency domain is controlled by the frequency bandwidth of the input signal and the width of the peaks in the wavenumber domain (as seen in 2-D FFT plot) is controlled by the length of the monitoring region. Since a short monitoring region is unable to differentiate between similar wavenumbers, the peaks will spread over a wide wavenumber range. As the length of the monitoring region increases, the width of the peaks (in the wavenumber domain) will decrease until the monitoring region is infinitely long and the peaks contain only one wavenumber for each frequency component.

CENTER MODE SHAPES

The ‘center mode shape’ technique represents a first attempt to generate a pure mode by prescribing a displacement profile at the boundary of a finite element mesh[2]. For the technique to work, the displacement profile of the desired mode must be known. The mode shape, which specifies the stress and displacement profiles through the thickness of the plate, is a part of the modal information for a plate that can be extracted from the dispersion curves. The general purpose dispersion curve tracing program Disperse [4], which was developed by the authors, was used to generate the mode shapes presented in this paper.

The displacement profiles in the two directions in the plane of the model are calculated at the center frequency of the chosen input signal. Then the input time signal at each node at the end of the Finite Element mesh is simply scaled according to
the amplitude of the displacement at that location in the mode shape profile. The process is applied in both displacement directions, and a phase difference is introduced between the two components as appropriate. The scaling operates in the time domain and produces a series of inputs to be used by the FE model. This process is shown schematically in Figure 3 for the A0 mode at low frequency. Since the Lamb wave modes are orthogonal to each other, if the displacement profile is correct for a single mode, only that mode is excited [1].

This technique works well to generate some modes. For example, Figure 4 shows the 2-D FFT results and a sample time history when the A0 mode is generated at 1.0 MHz-mm (point (a) in Figure 1), using a 12 cycle Gaussian windowed tone-burst. The dispersion curves for the system, which were originally shown in the frequency-group velocity domain in Figure 1, have been converted into the frequency-wavenumber domain and overlaid on the contour plot below the two dimensional transform to help identify the modes that are generated. The jagged edges on the contours are an artifact of the interpolation of the surface plot; in reality, the contours are smooth. Only one peak appears in the 2-D FFT plot revealing the presence of only one mode. Consequently, the sample time trace from a monitoring point on the surface of the plate is clean and easy to interpret.

However, when the frequency of the input signal is increased or the number of cycles in the input signal is reduced, this basic strategy is problematic. Figure 5 displays the results when the A0 mode is generated at 5.0 MHz-mm (point (b) in Figure 1), using a 2 cycle Gaussian windowed tone-burst. In addition to the desired A0 mode, the A1, A2, and A3 modes are also generated, as evidenced by the humps in
Figure 5. The (a) 2-D FFT and (b) sample time history for the generation of the A0 mode at 5.0 MHz-mm using center mode shapes and a 2 cycle Gaussian windowed tone-burst.

Figure 6. The (a) 2-D FFT and (b) sample time history for the generation of the S2 mode at its maximum group velocity using center mode shapes and a 10 cycle Gaussian windowed tone-burst.

The generation of additional modes in these two cases can be attributed to the fact that the input displacement profiles do not match the mode at all of the frequency
Figure 7. A contour view and profile of the displacements in the direction of propagation of the S2 mode as a function of frequency-thickness and position through the thickness of the plate using (a) exact mode shapes and (b) center mode shapes. The amplitudes shown in the contour view account for generation by a 10 cycle Gaussian windowed tone-burst at 6.8 MHz-mm.

components that are present in the excitation signal. Implicit in the direct scaling of a single time history is the assumption that the mode shapes are constant for all frequencies. However, in reality, the mode shapes change with frequency. Therefore, the center mode shapes technique will impose an incorrect mode shape at all frequencies where the mode shapes are different from that at the center frequency of the input signal. The residual components at these frequencies generate additional undesired modes.

EXACT MODE SHAPES

The generation of additional modes can be eliminated if the input signal accounts for the frequency dependent changes in mode shapes. We achieve such an input by operating in the frequency domain, using a technique we call exact mode shapes. This technique operates by combining the frequency dependent displacement profiles of a mode with the frequency spectrum of the desired input signal [2].

The first step of the process involves choosing the desired waveform, for example a 10 cycle Gaussian windowed tone burst centered at a particular frequency, and calculating its frequency spectrum using a Fourier transform. The displacement profiles of the desired mode are then calculated for every frequency component at which there is significant amplitude (> 0.1 percent of the maximum amplitude of the frequency response). The amplitudes of the displacements are scaled so that the total strain energy held in the mode is the same for every frequency in the bandwidth of the excitation signal.

The second step multiplies the frequency spectrum of the input signal and the frequency dependent displacement profiles. For each node through the thickness of the finite element mesh, the multiplication process steps through all of the significant frequency components in the input signal. The value of each frequency component in the signal is multiplied by the value of the mode shape displacement at that location in the thickness of the plate and at that frequency. The result can be considered to be a matrix whose rows contain the significant frequency components, whose columns
contain the nodes through the thickness of the plate, and whose values are the displacement. A contour view of the amplitude of this matrix can be seen in Figure 7a, which plots the magnitude of the displacement in the direction of propagation of the S2 mode (i.e. the in-plane displacement) at its maximum group velocity in an aluminum plate (point (c) in Figure 1) as a function of frequency-thickness at 30 positions through the plate. For comparison, Figure 7b shows the same matrix as it would look when the center mode shapes technique is used. Below each of the contour plots, and sharing the frequency-thickness axis, is a strip showing the displacements in the direction of propagation of the S2 mode that were used for these calculations. There is a significant change in the displacement of the S2 mode around its maximum group velocity, which will cause the frequency spectra applied to the various nodes to vary also.

The final step that is used for the exact mode shapes technique converts the scaled frequency spectra into time histories using inverse Fourier transforms. The resulting time histories are then applied to the end nodes of a Finite Element mesh. As the Finite Element model marches forward in time, a pure mode is generated.

The benefit of using exact mode shapes can be seen in Figure 8 and Figure 9, which show the results of applying exact mode shapes to the two cases with which the center mode shapes technique had difficulties (see Figure 5 and Figure 6). For both of the cases, A0 generated over a wide frequency bandwidth and S2 generated at its maximum group velocity, the 2-D FFT results indicate the presence of only one generated mode. The time traces are also much simpler to interpret than they were for the center mode shapes cases. The effect of dispersion can be seen for both cases; the low frequency components of the A0 mode to arrive first and the tone burst envelope of the S2 mode has increased in duration, decreased in amplitude, and changed shape as various frequency components interfere. However, the influence of secondary modes is not evident as it was for the other generation method.
CONCLUSIONS

Attempting to generate a mode without considering its changes in mode shape can lead to the generation of undesired modes. The presence of additional modes complicates the analysis of guided wave behavior. However, a technique presented in this paper, exact mode shapes, can create a pure mode excitation for Finite Element models by accounting for changes in the mode shape that occur with frequency.

Accounting for the frequency dependent changes in mode shapes has an especially important effect on two types of cases: when wide frequency bandwidths (i.e. a few cycles) are used and when higher order modes are being modelled. The wide frequency bandwidth case benefits because the displacement profiles can change significantly over the frequency range and other modes can be generated when there are large discrepancies between the actual and imposed mode shapes. The high order mode case benefits because there are usually many other possible modes in the frequency range being studied, so any small errors in the profiles will cause complicated multi-mode signals to be generated.

Although in experiments, an exact displacement profile can not be specified through the thickness of plate in order to generate a pure mode, various other techniques, such as incident angle or interdigital transducers, can be combined with time gating to ensure that a single mode is received. The results of Finite Element models that use pure mode excitation predict how the modes that are generated will react so that an optimum test procedure can be developed quickly and inexpensively.

REFERENCES