

2012

Essays on product return management and closed loop-supply chain network design

Nan Gao

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Essays on product return management and closed loop-supply chain network design

by

Nan Gao

A dissertation submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Major: Industrial Engineering

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2012

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ACKNOWLEDGEMENTS

Some people say the most read part of a Ph.D thesis is the acknowledgements section. With that being said, I am glad that I have the opportunity to express my gratitude to the people who have supported me in many ways during this adventure.

My first debt of gratitude must go to my advisor, Dr. Sarah M. Ryan. She patiently provided the vision, encouragement and invaluable suggestions necessary for me to proceed through the doctoral program and complete my dissertation. She has been a supportive adviser to me throughout my Ph.D study, but she has always given me great freedom to pursue independent work. I could not have imagined having a better advisor and mentor for my Ph.D study.

I also want to thank my committee, Dr. Frank L. Montabon, Dr. Jo Min, Dr. Lizhi Wang, and Dr. Guiping Hu for their support, guidance and insightful comments. Their guidance has served me well and I owe them my heartfelt appreciation. It has been a great privilege to spend three years in the Department of Industrial and Manufacturing Systems Engineering at Iowa State University, and its members will always remain dear to me.

The good and bad moments of my doctoral journey have been shared with many friends in Ames. Special thanks go to Yongfeng Ai, Zhaoyang Duan, Xiaopeng Ning, Chunyan Dai, Shan Jin, Xiaoli Yang, Walter Bennette, Peihan Zhong, Wenbo Shi, Yanyi He, Pan Xu, Yonghan Feng and Yuanfeng Cai. They are always the source of laughter, joy, and support.

Finally, I would like to dedicate this dissertation to my parents and my sister. Their love provided my inspiration and was my driving force. I owe them everything and wish I could show them just how much I love and appreciate them.

ABSTRACT

This dissertation focuses on managerial and operational challenges associated with product return management and CLSC network design. The possibility of product return plays an important role in consumer's purchase decisions. It also motivates firms to extend their forward-only supply chain network structures to a Closed-Loop Supply Chain (CLSC) network and handle both forward and reverse flows of products. While the configuration of the CLSC network is a complex problem comprised of the determination of the optimal locations and capacities of factories, warehouses and collection centers, this problem becomes even more complex under the potential regulations on carbon emissions.

This dissertation follows a three-paper format. With a focus on product return management, the first paper studies the roles that pricing and return policy play in the product exchange process for refurbished products. We first apply netnography to study consumer attitudes, general opinions and experiences concerning refurbished electronics purchases, and then propose an analytical model that considers customers' purchasing and return behavior as a result of the firm's decisions regarding the pricing and return policy for refurbished products. The numerical results suggest that sellers should deliberately consider the market segmentation conditions, consumer valuation, and cost factors when choosing the appropriate price and return policy for refurbished products.

The second and third paper focus on different aspects of CLSC network design. The second paper investigates a problem to design facility configurations that are robust to variations in possible carbon regulations and their cost and constraint implications. We establish a two-stage, multi-period stochastic programming model to include uncertain demand and return quantities and then extended it to incorporate the uncertainties in carbon regulation policy by the robust optimization method. We propose a hybrid model to account for either carbon tax or cap-and-trade regulatory policies and derive tractable robust counterparts under box and ellipsoidal uncertainty sets. Implications for network configuration, product allocation and transportation configuration are derived. We also present computational results that illustrate how the problem formulation under an ellipsoidal uncertainty set allows the decision

maker to balance the trade-off between robustness and performance.

The third paper formulates and solves an integrated model for product return management and CLSC network design considering uncertain carbon cost. We build a robust optimization model to address the carbon cost uncertainty, and develop a piecewise linear approximation for the nonlinear profit as a function of the refund. The results of the robust model are compared with those of deterministic models where no or only nominal carbon cost is considered. Extensive parametric analyses illustrate the impact of the cost, revenue and consumer profile parameters on the optimal refund, profit and network topology.

CHAPTER 1. GENERAL INTRODUCTION

1.1 Motivation

Product returns are an essential part of the exchange process between firms and consumers. Large retailers can have return rates that exceed 10 % of their sales and the annual costs of commercial returns exceed \$100 billion (Atasu et al., 2008). For this reason, many firms tend to view product returns as necessary evils, preventing them from recognizing potential value of those activities. A critical challenge for managers is to understand how a return policy, that allows refunds for any reason, affects the consumer purchase decision as well as firm's overall profit. For particular type of product (e.g., a refurbished product), such understanding might be critical to enhance the value added by recovery activities.

Meanwhile, product returns have motivated firms to plan their supply chain structures to handle both the forward flows of products to consumers and the reverse flows from consumers back to the firm for reprocessing. This leads to the idea of Closed-Loop Supply Chain (CLSC) management. According to Guide and Van Wassenhove (2009), CLSC management focuses on “the design, control, and operation of a system to maximize value creation over the entire life cycle of a product with dynamic recovery of value from different types and volumes of returns over time”(p.10). As the network design is one of the most important strategic decisions in firms' planning and operation, a firm that wishes to proactively design a CLSC needs to consider all the possible factors that will affect the outcomes of its decisions.

In recent decades, concerns over global climate change are increasingly focusing attention on both the fuel costs and the carbon emissions that result from transporting goods. Although subject to political vagaries, regulation of carbon emissions is becoming inevitable. For example, in 2005 the European Union instituted a carbon emission trading scheme (EU ETS) for the energy-intensive industries with the aim of reducing greenhouse gas emissions by at least 20% below 1990 levels (Bohringer et al., 2009).

Also, the New Zealand Emissions Trading Scheme (NZ ETS) was introduced in 2009 to reduce the carbon emissions in that country (Jiang et al., 2009). Australia's government also announced a carbon tax plan, aimed at lowering carbon by discouraging the use of fossil fuels and increasing investment in renewable energy (Siriwardana et al., 2011). Those emission regulations aim to eventually reduce substantial emission in all economic sectors, of which transportation is a major source of emissions. According to U.S. Energy Information Administration (EIA), 33.2% of carbon emission are from the transportation sector (EIA., 2009). Therefore, it is not surprising that several countries including the United States, Japan and Canada are discussing the implementation of cap-and-trade systems that would include the transportation sector. The EU ETS does not include road transport but does cover the aviation sector in 2012 (Flachsland et al., 2011).

Thus, a firm that needs to design a CLSC network that involves heavy logistic activities should anticipate possible carbon regulations. Moreover, the firm should understand that the recovery activities are also triggered by the pricing and return policies set by the firm. Depending on the firm's role in a CLSC, it might be interested in different aspects of its management:

1. **Product return management:** for sellers (either retailers or manufacturers) who sell directly to the consumer, the decision maker might be interested in understanding how the purchasing and return behavior would be affected by the return policy as well as how to optimize the return related process.
2. **CLSC network design:** for the manufacturers who are responsible for collecting the returns from existing retailer, the decision maker might be interested in how to design the CLSC network when carbon emission regulations are taken into consideration. Also, how to design a network that is responsive to retailers' demands and returns?
3. **Integration of product return management and CLSC network design:** for the manufacturers who own the retail store or otherwise have the control over the return policy, the firm might be interested in how to combine the product return policy with CLSC network design decisions to improve profitability.

The goal of this dissertation is to provide insights regarding managerial and operational challenges associated with product return management and CLSC network design. New insights are derived by

considering both issues together. The main contributions of this dissertation are as follows: For the product return management problem, we combine qualitative and quantitative research methods to verify some existing assumptions concerning refurbished products, and we also bring a marketing perspective to the problem of pricing refurbished products through a focus on factors related to the demand faced by the seller. For the CLSC network design problem, we explicitly address the effects of uncertain carbon emission regulations on the CLSC network configuration by incorporating two such policies into a hybrid model. Also, we model the CLSC network design problem with the combination of both robust optimization and stochastic programming methodologies. The carbon regulation parameters characterized by prices or tax rates are modeled with uncertainty sets while the demands and returns are represented by discrete probabilistic scenarios. Finally, we contribute to the literature by formulating an integrated model for product return management and CLSC network design subject to uncertain carbon cost. The extensive numerical results improve our understanding of the effect of various parameters on both problem aspects. In addition, it contributes to a solution method designed for models with quadratic terms as well as ellipsoidal uncertainty.

1.2 Thesis Structure and Overview

This thesis is structured as follows: it consists of three main chapters, preceded by this general introduction and followed by a general conclusion. Each of those main chapters is a journal article, with the first two under review and the third in preparation for submission.

The second chapter, titled “Optimal Pricing and Return Policies for Refurbished Products Considering Tactical Consumers”, focuses on understanding the roles that pricing and return policy play in the product exchange process for the refurbished products (Gao and Ryan, 2012). This chapter first uses netnography to study consumers’ attitudes, general opinions and experiences concerning refurbished electronics purchases, and then develops an analytical model that considers the customer’s purchasing and return behavior as a result of the firm’s decisions regarding the pricing and return policy for refurbished products. The results suggest that sellers should deliberately consider the market segmentation conditions, consumer valuation, and cost factors when choosing the appropriate price and return policy for refurbished products. The combinations of either low price with relatively strict return policy or

high price with lenient return policy appear to be preferable under a wide variety of conditions. Even though this study looks at a special product type (refurbished electronics), it sheds some light on the optimal design of the return policy for more general product types.

The third chapter, titled “Robust Design of a Closed-loop Supply Chain Network for Uncertain Carbon Regulations and Random Product Flows”, addresses a multi-period capacitated closed-loop supply chain (CLSC) network design problem subject to uncertainties in the demands and returns as well as the potential carbon emission regulations (Gao and Ryan, 2011b). This chapter extends the deterministic setting in Gao and Ryan (2011a) and two promising regulatory policy settings are considered; namely, (a) a carbon cap and trade system, or (b) a tax on the amount of carbon emissions. A traditional CLSC network design model using stochastic programming is extended to integrate robust optimization to account for regulations of the carbon emissions caused by transportation. We propose a hybrid model to account for both regulatory policies and derive tractable robust counterparts under box and ellipsoidal uncertainty sets. Implications for network configuration, product allocation and transportation configuration are obtained via a detailed case study. We find that the optimal network configuration balances the trade-offs between investment, transportation and carbon emission costs if the carbon regulation is incorporated. More facilities will be opened and the total expected cost will increase as the uncertainty level increases. Moreover, the share of transportation by the low-emitting modes will increase as the regulation policy uncertainty level increases. We also present computational results that illustrate how the problem formulation under an ellipsoidal uncertainty set allows the decision maker to balance the trade-off between robustness and performance. The proposed method yields solutions that provide protection against the worst case scenario without being too conservative.

The fourth chapter, titled “Robust Design of a Closed-loop Supply Chain Network for Uncertain Carbon Regulations and Random Product Flows”, focuses on an integrated profit maximization modeling framework for CLSC network design problem and product return management. This integration can benefit a manufacturer who has control over the return process; for example, by owning the retail stores. In addition, our model includes the uncertain cost of carbon emissions, which may result from potential environmental regulations. We build a robust optimization model to address the carbon cost uncertainty, and develop a piecewise linear approximation for the nonlinear profit as a function of the refund. The results of the robust model are compared with those of deterministic models where no

carbon cost is considered or only nominal carbon cost is considered. Extensive parametric analyses illustrate the impact of the cost, revenue and consumer profile parameters on the optimal refund, profit and network topology. We find that different parameters vary in their impact on the refund provided, the profit and the network topology. For policy implications, we found that uncertainty in carbon regulation has the effect of reducing the optimal refund and thus, results in less returns. This might not benefit the environment as the products might go to landfill instead of being recycled or reused.

Finally, in Chapter 5 we conclude the thesis with a summary of the results and suggest some future research directions.

CHAPTER 2. OPTIMAL PRICING AND RETURN POLICIES FOR REFURBISHED PRODUCTS CONSIDERING TACTICAL CONSUMERS

2.1 Introduction

Refurbished cell phones, laptops and other categories of refurbished electronics are increasingly common. Such products are offered for sale at a reduced price and explicitly labeled as reprocessed (e.g., refurbished, returned, remanufactured, or reconditioned). The refurbished products originate from various sources such as consumer returns, damage during manufacture/shipping, trade-ins or end of life take-backs (Guide et al., 2006). After they are shipped to the manufacturer, they are inspected and/or refurbished or remanufactured, then repackaged for sale at a discounted price.

Products labeled as refurbished may be completely new items that were returned to the seller simply because the original purchaser changed his/her mind. Alternatively, they may have had technical problems that were repaired by the manufacturer, followed by full testing. The consumer is unlikely to know the source of refurbished products. A recent study by Ovchinnikov (2011) found that not knowing the product history caused great uncertainty in consumer attitudes towards purchasing refurbished products. One important mechanism to mitigate such uncertainty is the return policy. Marketing and economics research have confirmed that a return policy reduces consumer risk and provides a better match between the consumer's needs and product quality (Mann and Wissink, 1988, 1990; Heiman et al., 2001). Generally speaking, consumers place a higher value on their ability to return the items when they face uncertainty (Anderson et al., 2009). From the consumer's point of view, the question is whether the initial cost advantage from purchasing a refurbished product outweighs the perceived increased chance of unsatisfactory performance. Even when they can rely on the return policy to mitigate such unpredictability, they still have to balance the trade-offs between the benefits from the refurbished product and the loss from the potential return.

In practice, manufacturers offer various pricing and return policies for refurbished products. For example, if consumers are unsatisfied for any reason with a refurbished product purchased from HP, they may return it with no questions asked for a restocking fee of 25% of the purchase price paid¹. Dell, Best Buy and Sony also have similar return policies for the refurbished products with restocking fees of 10% - 25% of the purchase price. Their refurbished products are usually priced at 75%-85% of the comparable new product price. However, manufacturers like Epson and Fujitsu price the refurbished product at 60% to 70% of the comparable new product price but do not allow any returns of refurbished products². These observations raise the question of what factors drive the pricing and return policy decisions concerning the refurbished products. These decisions will affect profits gained from both refurbished and new products. A low price and/or overly generous return policy could attract more purchases of refurbished products at the expense of new product sales. On the other hand, a combination of high price and strict return policy could result in lost revenue from budget-minded and risk-averse customers.

The goal of this paper is to provide the seller with insights into how the price and return policies for refurbished products should be designed to maximize profit. To do so, we must understand what attitudes, general opinions and experiences consumers may have concerning purchasing refurbished products; and what important factors influence their decisions whether or not to purchase them.

To understand consumer's attitudes, we conducted an empirical qualitative study using the netnography method introduced by Kozinets (2002). Netnography uses the online community to study consumers' attitudes, perceptions and feelings based on the observation of textual discourse. It is able to offer "thick descriptions" of real life consumers, and has been adapted to study consumers' experiences in the cosmetic (Langer and Beckman, 2005), music (Giesler and Pohlmann, 2003) and tourism (Hsu et al., 2009) industries. Our empirical study confirms the observation of Ovchinnikov (2011) and finds that the majority of consumers feel it is somehow "risky" to purchase refurbished products. We also find heterogeneity in consumer attitudes towards refurbished products. There exist consumers who will never consider purchasing refurbished products and consumers who will only consider the refurbished versions. They constantly compare the savings from buying refurbished rather than the comparable

¹http://www.hp.com/sbso/buspurchase_refurbished_faq.html#return

²<http://www.shopfujitsu.com/www/content/products/notebooks/ordering.php>

new products. But they also seek some mitigation of the potential uncertainty, such as a warranty/return policy or positive reputation of the purchasing channel and seller. For example, consumers feel more comfortable knowing that they can return or exchange the refurbished product and perceive less risk from purchasing a refurbished product that has been certified by the manufacturer.

Based on the empirical findings, a single period analytical framework is built up to model three types of consumers: those who would purchase only new products, those who would purchase only refurbished products and those (labeled tactical) who consider both types of products. The tactical consumer is subject to valuation uncertainty, characterized by a discrete random variable, regarding the refurbished product. We characterize the consumer demand for the refurbished products as a function of price and return policy. We present three return policy design strategies for the seller: strict, lenient, and intermediate. We then develop a detailed profit maximization model for each to explore how the market segmentation conditions, consumer valuation, and attributes of both new and refurbished products affect the seller's optimal pricing and return policy.

Our analytical and numerical studies reveal that different parameters vary in their impacts on the seller's optimal price for refurbished products and return policy decisions. Production or refurbishing cost-related factors and consumer expected valuation have more impact on the pricing decision, while allowing or not allowing return depends more on the salvage value of returned products and the market segmentation conditions. Furthermore, the combinations of either low price with relatively strict return policy or high price with lenient return policy appear to be preferable under a wide variety of conditions.

The contributions of our paper are as follows. We combine qualitative and quantitative research methods to verify some existing assumptions concerning refurbished products, and we also bring a marketing perspective to the problem of pricing refurbished products through a focus on factors related to the demand faced by the seller. We combine three aspects of refurbished product marketing that are typically examined separately: product return, consumer purchase/return behavior, and market segmentation. To the best of our knowledge, this is the first paper that simultaneously addresses all three aspects in the refurbished product context. The remainder of the paper is organized as follows. The related research is discussed in Section 2. Section 3 provides the empirical study and Section 4 introduces the basic model notation and assumptions. In Section 5 we analyze the model and present three different strategies, including results on the optimal price and restocking fee. Numerical analysis and

the main results are shown in Section 6. We conclude with a discussion of our results and managerial insights in Section 7.

2.2 Literature Review

Our work is related to a stream of papers that address consumers' valuation uncertainty by instituting return policies. Davis et al. (1995) find that the profitability of a money back guarantee is influenced by the salvage value of the item. Davis et al. (1998) show that the seller should implement a low-hassle return policy when the product cannot be consumed in a short period of time, the seller can cross-sell the product, or the returned product has a high salvage value. Wood (2001) finds that a lenient return policy will increase purchase rates and product return rates for customers in remote purchase environments. Anderson et al. (2009) illustrate how varying the cost of returning an item affects a firm's profits, so that an optimal return policy requires the company to balance both demand and cost involved in the return policy. Petersen and Kumar (2009) offer a summary of prior research focused on customer product return behavior from different perspectives. They show that a customer's product return behavior positively affects his or her future buying behavior; thus, designing the return policy should be an important aspect of product management. Within the operations management field, Liu and Xiao (2008) compare the use of a return policy with inventory rationing in the context of revenue management and derive conditions on how the firm should choose different policies. Shulman et al. (2009) study how to use price, restocking fees and information optimally to improve the operational efficiency. Ketzenberg and Zuidwijk (2009) study the optimal price, return policy and quantity decisions for new products. In their model, they allow returned products to be recovered and to be sold as good-as-new ones and they find that an intermediate return policy is preferred under various conditions. Akçay and Boyacı (2011) investigate the price, ordering quantity and refund decisions for the new product. They show that selling with "money-back-guaranteed" increases retail sales and profit. Our work differs from theirs by considering the seller's pricing and restocking fee decisions in the refurbished product context.

This paper is also related to the variety decisions (including quantity and pricing) of serving consumers with various quality levels of products. Such issues are addressed by classical literature such

as Mussa and Rosen (1978) and Moorthy and Png (1992). They study the optimal pricing of products that are differentiated by quality when consumer valuation of quality is heterogeneous. In a refurbishing context with no return policy involved, Debo et al. (2005) study the joint pricing and production technology selection problem and derive the manufacturer's optimal remanufacturing decisions as well as conditions on the viability of remanufacturing. Vorasayan and Ryan (2006) study the pricing and quantity issues involved in selling new and refurbished products. They characterize different situations to refurbish none, some, or all of the returned products by using a queueing network. Jin et al. (2007) investigate the profitability of selling remanufactured products by using a nonlinear utility function and find that customer segmentation drives the remanufacturing decision. Atasu et al. (2008) show that the profitability of a remanufacturing system strongly depends on factors including the remanufacturing cost savings, the green segment size, market growth rate, and consumer valuations. Ghosh et al. (2010) optimize the production rate for new products and the price for refurbished products in queueing network model under heavy traffic. Most of this work assumes that consumers have a deterministic discounted valuation for refurbished products. Our work complements this line of research by incorporating valuation uncertainty towards refurbished products and the use of a return policy to mitigate it. We also consider the three different market segments faced by the seller.

The combination of qualitative and quantitative research methods adds to the emerging area of research on consumer behavior in the context of remanufacturing. This stream of study focuses on how the consumer behavior affects the pricing and remanufacturing strategy. Agrawal et al. (2009) investigate the effect of remanufactured products on the perceived value of new products. They find that the perceived value of new products will decrease if the OEM sells refurbished products and increase if a third-party remanufacturer sells them. Guide Jr and Li (2010) use an auction to study consumer's willingness-to-pay for both new and remanufactured products. Ovchinnikov (2011) use a survey to study the switching behavior from new to remanufactured products and finds that the fraction of customers who switch has an inverted U-shape with respect to the decrease in the price of remanufactured product.

2.3 Empirical Study

According to Kozinets (2002), netnography is “a new qualitative research methodology that adapts ethnographic research techniques to study cultures and communities that are emerging through computer-mediated communications.” We conducted a netnographic study to understand consumer’s attitudes, opinions and experiences regarding purchasing refurbished electronics. The specific research questions were: What attitudes, experiences, and feelings do consumers express concerning refurbished electronics? What are the important factors that drive a consumer’s decision to purchase refurbished electronics or not? We chose the forum of CNET.com, the leading website for consumer technology products, to collect posts related to refurbished electronics in general, as well as MacRumors.com, a leading consumer forum on Apple products, to study purchasing attitudes within a specific brand. We obtained a total of 140 posts from both forums by limiting a search of the years 2005 to 2011 (first quarter) to posts that contained “refurbished” in their titles. The complete threads were directly copied, then coded and categorized based on consumer attitude and purchase experience before further analysis and interpretation (by the author). The body of each post was analyzed multiple times to identify consumer attitudes towards purchasing refurbished products. Below we summarize findings and provide sample quotes to support them, and provide links to the quotes in Appendix A1.

Most consumers perceive uncertainty about purchasing refurbished products. Most entries began with a question like “Is it safe to buy a refurbished product?” or “Should I buy refurbished?” The respondents posted their opinions and experiences. One typical response is: *“It is a 50/50, toss of the coin issue when it comes to buying refurbished anything. Sometimes you make out OK, sometimes you don’t.”*

One reason for such uncertainty is that refurbished products come from various sources. One contributor mentioned: *“‘Refurbished’ is a crap shoot. Refurbished products come from a variety of sources. There may have been nothing at all wrong with it. It may have an intermittent problem that does not immediately make its presence known. It may have an overheating problem and work fine for a few minutes, then completely quit working and is returned for that reason... After saying all this, in my experience, the odds are with you in the long run. Not every device that you buy ‘refurbished’ will be a defective device. You’re going to get several good ones for every bad one you get, but expect a bad one*

every now and then.”

Some consumers value a refurbished product very closely to the new counterpart. One typical response is: *“Apple brings the products back to as-new specifications. They even replace the outer cases on products like the iPods, so you don’t even have to worry about getting one with cosmetic damage.”*

Overall, we identified mixed attitudes towards purchasing refurbished electronics. Certain consumers will never again purchase them because of a previous negative experience. For example, *“Refurbished computers – I wouldn’t do it again.”* Others will never even consider purchasing a refurbished product. *“I have always stayed away from refurbished products they weren’t as good.”* But there are also consumers who, out of satisfaction with previous purchases, have decided to only purchase refurbished products. For example, one contributor wrote, *“I almost always buy refurbished rather than brand new.”* One reason is that they believe the refurbished product has been thoroughly tested whereas only a small sample of the new products have been tested. *“High-volume production lines typically omit 100% testing of all production items, opting instead to bear the cost of a small percentage being returned to make them serviceable. Thus, such ‘refurbished’ items are more thoroughly tested than a new off-the-shelf unit at a dealer; boding well for buying such a unit... I’m a proponent of refurbished electronics and have great success with them while saving money!”*

Noticeably, consumers tend to compare a new product with a refurbished one within a specific model. A common question posed is “should I buy a new or refurbished one?” For example, a common post is like: *“The camcorder (Canon Vixia HF200) is \$749 new but the refurbished one is \$399! Is the price worth the camcorder being refurbished?”* Several factors, of which cost saving is the most common, influence the consumer’s purchasing decision. For example, one contributor suggests that *“You should be able to save 30% - 50% on a refurbished electronics purchase.”* Moreover, the refurbished product should be covered a warranty or return policy. One typical response is: *“The key is making sure you’re comfortable with where you’re getting the item from and making sure they have some kind of warranty (whether it’s exchange or return).”* Furthermore, the refurbished product should be certified and produced by the manufacturer. Typical words are: *“I would not hesitate to buy a factory refurbished product, especially if I’m buying it from the manufacturer.”*

Entries from MacRumors.com also show that uncertainty-mitigation mechanisms are very impor-

tant. For example, one contributor asked: *“I think I’m going to buy a refurbished Macbook (Aluminium or Pro), but what is the return policy? If I get it, and I’m not happy can I return it? What if it has a scratch or dead pixels? Can I return it for a replacement?”* As the MacRumors contributors usually purchase the refurbished products directly from Apple’s online store, their concerns are more specific about the hassle of a possible return or exchange. For example, if they purchase the refurbished products online, can they return it to the local store? Finally, return of a refurbished product might not be due to functional defect, but because they feel the quality is not worth the price. One typical response is: *“I recently bought a refurbished 24” ACD and it had a big piece of dust between the glass and the screen. It was very noticeable to me, so I returned it immediately for a full refund. I tried for a second refurbished 24” ACD and it had a smaller piece of dust, but it also had a dead pixel (stuck on red), so I returned it as well. I just couldn’t justify paying \$599 for a display that wasn’t perfect.”*

To summarize these qualitative findings, it is clear that consumers have heterogeneous attitudes towards refurbished electronics and many feel it is more risky to purchase refurbished than new. Their *ex post* valuation could turn out to be “as good as new” or “someone else’s headache”. This is consistent with the observation of Ovchinnikov (2011) that such uncertainty stems from not knowing the history of refurbished products. When they decide to purchase the refurbished product, they want enough cost savings to compensate for such uncertainty. At the same time, they seek risk-mitigation mechanisms. They rely on the purchasing channel, the seller’s reputation and the warranty/return policy to counterbalance the unpredictability of refurbished product quality.

2.4 Model Assumptions and Notation

The following assumptions are made throughout the paper.

Assumption 1. *Consumers are interested in a specific brand and model of product.*

As indicated by the empirical study, we focus on the situation where the consumers have a specific brand and model in mind but have not decided whether to buy a new or refurbished one. The total number of consumers in the market is normalized to 1. Each consumer is interested in purchasing at most one unit of the product.

Assumption 2. *There are three segments of potential customers: (1) New product consumers (denoted as N-consumers) who will never consider purchasing a refurbished product; (2) Refurbished product consumers (denoted as R-consumers) who would only purchase a refurbished product; and (3) “Tactical” consumers (denoted as T-consumers) who will consider purchasing either a new or a refurbished product. The proportions of consumers who fall into these categories are β^N , β^R and β^T , respectively ($\beta^N + \beta^R + \beta^T = 1$). Within each market segment, consumer valuations of the new product are uniformly distributed on $[0, 1]$.*

Our empirical findings from the netnographic study indicate the existence of these market segments, assuming the stated preferences of consumers match their real market behaviors. The existence of N-consumers is also mentioned in Agrawal et al. (2008) while R-consumers are mentioned in Atasu et al. (2008) where they were labeled “green consumers.” Note that we can model any degree of customer segmentation by this setting. If $\beta^T = 1$, all consumers in the market consider purchasing either a new or a refurbished product. On the other hand, if $\beta^T = 0$, the market is perfectly segmented into N-consumers and R-consumers. Thus, the proportion of consumers who are tactical reflects the potential for cannibalization of demand between the new and the refurbished product. The firm could identify the proportion of consumers of each type by marketing research.

Assumption 3. *The decision variables are the price, p_r , of the refurbished products and the restocking fee, denoted by f , where $0 \leq f \leq p_r$. The hassle cost for the consumers to return the refurbished product is normalized to zero. The return policy is characterized by f .*

The returns of refurbished products are due to heterogeneous consumer expectations before and after the purchase. We assume the R-consumers have more knowledge about the functionality of the refurbished products, and do not experience any “gaps” between their expectations and the actual valuation of the refurbished products. Thus, all the R-consumers will keep the refurbished products they purchased. A T-consumer will perceive lower than expected value from the refurbished product (called a “lemon”) with probability δ and will receive “completely as new” (designated as a “peach” product) with probability $1 - \delta$.

Assumption 4. *The price of a new product p_n is exogenously given, and $p_n \geq \delta\phi$, where ϕ represents the lemon’s degradation in value ($0 < \phi < 1$) by T-consumers. We do not consider returns of the new*

product and assume the monopolist seller has an ample supply of the refurbished products.

The exogenous price assumption is based on the observation that refurbished products appear later than the corresponding new products so that their pricing can be delayed. Also, there is a noticeable time lag between when product returns increase to stable volumes and when refurbished products become available (see Figure 2 in Guide et al. (2006)). The inequality in Assumption 4 states that the expected degradation in valuation ($\delta\phi$) is less than the new product price. This is reasonable given that the values of δ and ϕ are expected to be small and p_n should be high.

In practice, the seller needs to build an inventory of refurbished products from the return of new products. This, however, is beyond the scope of our single period model. We assume that the supply of the refurbished product is not an issue for the seller, either due to a high volume of returned products or by labeling a certain amount of new products as refurbished when facing a limited supply of refurbished products (Jin et al., 2007).

Assumption 5. *When a refurbished product is returned, the seller obtains a net salvage value $v \leq c_r < c_n$, where c_r (resp. c_n) is the marginal cost of producing a refurbished (resp. new) product.*

Assumption 5 is consistent with previous research; for example, Ketzenberg and Zuidwijk (2009). Note that we consider only the marginal production costs for new product and refurbished product, which is consistent with the closed-loop supply chain literature (Savaskan et al., 2004; Atasu et al., 2008). The notation is summarized in Table 2.1.

2.5 Analysis

The seller decides on the price of the refurbished product and the associated return policy. Then, each consumer observes the seller's decision and decides which product to purchase based on maximizing his/her own surplus. Therefore, our model is a Stackelberg game in which the seller is the leader and the consumers are the followers. The sequence of events is (see Figure 2.1):

- (i). The seller sets the price and restocking fee for the refurbished product.
- (ii). Based on their utility functions, the N-consumers decide whether to buy a new product or not to buy, the R-consumers decide to buy a refurbished product or not to buy, and the T-consumers decide whether to buy a new or a refurbished product or neither.

Table 2.1 Model notation

Symbol	Definition
<i>Decision Variables</i>	
p_r	Sale price for the refurbished product
f	Restocking fee for the returned refurbished product
<i>Parameters</i>	
β^N	Proportion of N-consumers in the market
β^R	Proportion of R-consumers in the market
β^T	Proportion of T-consumers in the market
δ	Proportion of “lemon” refurbished products
ϕ	Degradation in valuation for the “lemon” refurbished product
p_n	Sale price for the new product
c_n	Marginal cost of producing the new product
c_r	Marginal cost of processing the refurbished product
v	Net salvage value of a returned refurbished product

(iii). A T-consumer who purchased the refurbished product decides whether to keep the product or return it for a refund of p_r less the fee f .

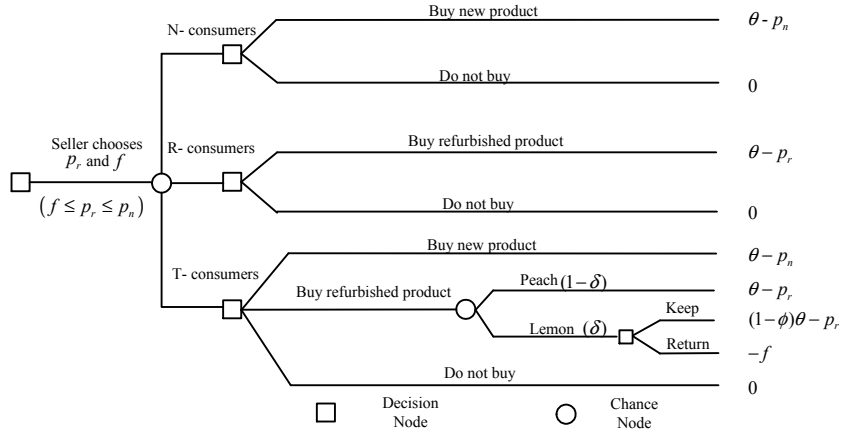


Figure 2.1 Sequence of events

An N-consumer may either demand superior performance of the product or have had a bad experience with a refurbished product. He or she obtains utility $\theta - p_n$ from the new product and 0 utility from the refurbished product and, thus, will purchase the new product only when his/her utility exceeds its price; that is, $\theta > p_n$. The proportion of the N-consumers who will purchase the new product is $1 - p_n$. Because p_n is given exogenously, for a given β^N , the seller gains a net revenue of $\beta^N(1 - p_n)(p_n - c_n)$ from the sales to N-consumers. For simplicity, we omit this portion of profit from the seller’s objective

function, as it is not influenced by the decision variables.

The R-consumer may care more about the functionality of the product or be more environmentally conscious and, therefore, value the refurbished product at least as highly as the new product. Hence, an R-consumer receives utility $\theta - p_n$ from a new product and at least $\theta - p_r$ from the refurbished one. Assuming $p_r < p_n$, the R-consumer will always prefer to buy a refurbished product and will purchase one if his/her utility exceeds that price; that is, $\theta > p_r$. The proportion of the R-consumers who will purchase the refurbished product is $1 - p_r$. Because the purchase behavior of the N- and R-consumers will not be affected by the seller's return policy, we focus our analysis on the behavior of the T-consumers.

The T-consumer makes two sequential decisions: (1) whether to buy either the new product or the refurbished product, and (2) conditioned on buying the refurbished product, whether to return it or keep it. The T-consumer will keep the peach products and consider returning the lemon products. They will return the lemon if $(1 - \phi)\theta - p_r \leq -f$, and keep it otherwise. Therefore, the expected utility before purchasing the refurbished product is given by $U_r^T \equiv (1 - \delta)(\theta - p_r) + \delta \max[(1 - \phi)\theta - p_r, -f]$.

Backward induction reveals that they prefer the refurbished product if their *ex ante* expected utility is greater than the utility they derive from purchasing a new product; i.e., $U_r^T \geq \theta - p_n$. Define θ_h to be the valuation level for which the T-consumer is indifferent between purchasing the new and refurbished products, and θ_l as the level for which the consumer is indifferent between purchasing the refurbished product and not purchasing at all. We assume that these values are endogenously determined by the seller's price and return policy; thus, θ_h and θ_l are functions of p_r and f . The proportions of the T-consumers who purchase the new and refurbished products, respectively, are given by $1 - \theta_h$ and $\theta_h - \theta_l$.

To characterize the effect of restocking fees, we first define the consumer with valuation equal to $\bar{\theta}$ as indifferent between keeping the lemon refurbished product and returning it; that is, $\bar{\theta} \equiv \frac{p_r - f}{1 - \phi}$. Thus, consumers with valuation less than $\bar{\theta}$ will return the lemon refurbished products, which consumers with valuation greater than $\bar{\theta}$ will keep them.

We can then calculate the seller's profit as a function of p_r and f . Since a consumer's valuation is a private information, the seller must set the price and restocking fee for the refurbished product and then allow each consumer to choose the product he/she favors by comparing the surplus gained from each. Based on the comparison of $\bar{\theta}$, θ_h and θ_l , we can describe the seller's approach as offering either a strict,

a lenient, or an intermediate return policy.

2.5.1 Strict Return Policy

Under this strategy, the seller discourages the T-consumer from returning any lemon refurbished products. For a T-consumer, the expected surplus from purchasing the refurbished product is $\hat{U}_r^T(\theta) \equiv (1-\delta)(\theta-p_r) + \delta((1-\phi)\theta-p_r)$. According to the definitions of θ_h and θ_l , their values under this strategy must satisfy $\hat{\theta}_h - p_n = \hat{U}_r^T(\hat{\theta}_h)$ and $\hat{U}_r^T(\hat{\theta}_l) = 0$. Thus,

$$\hat{\theta}_h = \frac{p_n - p_r}{\delta\phi}, \quad (2.1)$$

$$\hat{\theta}_l = \frac{p_r}{1 - \delta\phi}. \quad (2.2)$$

Therefore, if the seller sets the price and restocking fees so that $\bar{\theta} \leq \hat{\theta}_l \leq \hat{\theta}_h$, no returns of refurbished products will occur because all of the consumers will prefer to keep the lemon product. The comparison of $\bar{\theta}$, $\hat{\theta}_h$ and $\hat{\theta}_l$ is shown in Figure 2.2.

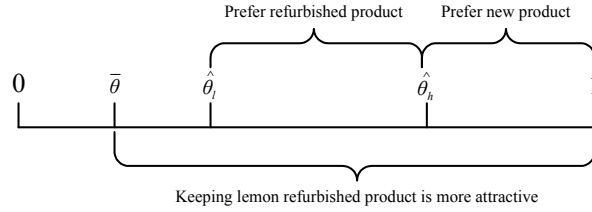


Figure 2.2 Comparison between $\bar{\theta}$, $\hat{\theta}_h$ and $\hat{\theta}_l$ under the strict return policy

Under the strict return policy, the seller solves the following problem:

$$\max_{p_r} \hat{\Pi} = \left(\beta^R(1-p_r) + \beta^T \left(\frac{p_n - p_r}{\delta\phi} - \frac{p_r}{1 - \delta\phi} \right) \right) (p_r - c_r) + \beta^T \left(1 - \frac{p_n - p_r}{\delta\phi} \right) (p_n - c_n) \quad (2.3)$$

$$s.t. \quad p_r \leq (1 - \delta\phi)p_n \quad (2.4)$$

$$(1 - \delta\phi)f - (1 - \delta\phi)p_r \geq 0 \quad (2.5)$$

$$p_r \geq p_n - \delta\phi \quad (2.6)$$

The objective is to maximize the profit from selling both products. The first term in the objective function is the profit from selling refurbished products to the R-consumers and the T-consumers. The

second term is the profit from selling new products to the T-consumers. Constraint (2.4) ensures $\hat{\theta}_l \leq \hat{\theta}_h$ and constraint (2.5) ensures $\bar{\theta} \leq \hat{\theta}_l$. Constraint (2.6) guarantees that the T-consumers have nonnegative demand for new products; that is, $\hat{\theta}_h \leq 1$. Assumption 4 and constraint (2.6) imply that $p_r \geq 0$, while $f \geq 0$ is implied by constraint (2.5). Moreover, $p_r \leq p_n$ is implied by constraint (2.4).

It is straightforward to verify concavity of $\hat{\Pi}(p_r)$ by its second derivative, and we then form the Lagrangian using multipliers $\hat{\lambda}_1$, $\hat{\lambda}_2$, $\hat{\lambda}_3$ and $\hat{\lambda}_4$:

$$\begin{aligned} \hat{L}(p_r, f) = & \hat{\Pi} + \hat{\lambda}_1((1 - \delta\phi)p_n - p_r) + \hat{\lambda}_2((1 - \delta\phi)f - (1 - \delta)\phi p_r) \\ & + \hat{\lambda}_3(p_r - p_n + \delta\phi) + \hat{\lambda}_4(p_r - f) \end{aligned}$$

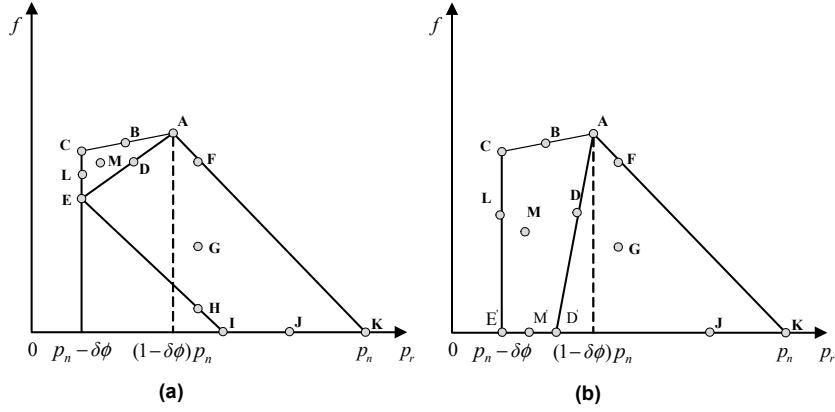


Figure 2.3 (a) Possible options for the seller when $p_n \geq 1 - (1 - \delta)\phi$; (b) Possible options for the seller when $p_n \leq 1 - (1 - \delta)\phi$;

By setting different combinations of $\hat{\lambda}_1$ and $\hat{\lambda}_3$ equal to 0, the KKT conditions generate systems of equations that can be solved to identify options A, B and C for the seller under this strategy (see Figure 2.3). On the line containing segment AC, $\hat{\theta}_l = \bar{\theta}$, so no one will return the lemon refurbished products. We ignore the region above this line because there is no need to set the restocking fee any higher. At point C, $\hat{\theta}_h = 1$, so T-consumers buy only refurbished products. At point A, $\hat{\theta}_h = \hat{\theta}_l = \bar{\theta}$, so T-consumers buy only new products. The line containing segment AC has slope $0 < \frac{(1 - \delta)\phi}{1 - \delta\phi} < 1$. The values of the decision variables at points A, B and C along with conditions under which each is optimal under the strict return policy are given in Appendix A2.

Theorem 1. *Under the strict return policy, (i) If $c_r < \min[2(1 - \delta\phi)p_n - 1, (1 - \delta\phi)c_n]$, the point A cannot be optimal.*

(ii) If $c_r > \max [(1 - \delta\phi)c_n - 2\delta\phi(1 - p_n), 2(p_n - \delta\phi) - 1]$, the point C cannot be optimal.

All proofs are provided in Appendix A5. Theorem 1(i) indicates that when the cost of refurbishing is low relative to either the price or the cost of new products, the seller should price the refurbished products low enough to attract some demand from the T-consumers. Theorem 1(ii) states that when the refurbishing cost is high, the seller should price the refurbished products high enough that some T-consumers prefer the new products.

2.5.2 Lenient Return Policy

Under this strategy, the seller sets the price and restocking fees such that every T-consumer who receives a lemon product will return it. The expected surplus from buying and returning the refurbished product is $\check{U}_r^T(\theta) \equiv (1 - \delta)(\theta - p_r) + \delta(-f)$. According to the definitions of θ_h and θ_l , under this strategy their values are:

$$\check{\theta}_h = \left(\frac{p_n - (1 - \delta)p_r}{\delta} - f \right), \quad (2.7)$$

$$\check{\theta}_l = \left(\frac{\delta f}{1 - \delta} + p_r \right). \quad (2.8)$$

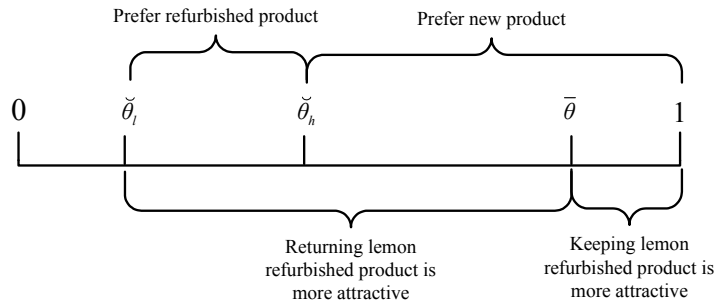


Figure 2.4 Comparison between $\bar{\theta}$, $\check{\theta}_h$ and $\check{\theta}_l$ under the lenient return policy

Thus, if the seller sets the price and restocking fees so that $\check{\theta}_l \leq \check{\theta}_h \leq \bar{\theta}$, all of the lemon refurbished products will be returned (see Figure 2.4).

Under this strategy, the seller solves the following problem:

$$\begin{aligned} \max_{p_r, f} \check{\Pi} = & \delta\beta^T \left(\frac{p_n - (1-\delta)p_r}{\delta} - f - \frac{\delta f}{(1-\delta)} - p_r \right) (f + v - c_r) \\ & + \beta^T \left(1 - \frac{p_n - (1-\delta)p_r}{\delta} + f \right) (p_n - c_n) \\ & + \left((1-\delta)\beta^T \left(\frac{p_n - (1-\delta)p_r}{\delta} - f - \frac{\delta f}{(1-\delta)} - p_r \right) + \beta^R (1 - p_r) \right) (p_r - c_r) \end{aligned} \quad (2.9)$$

$$s.t. \quad (1 - (1-\delta)\phi)p_r - \delta\phi f \geq (1-\phi)p_n \quad (2.10)$$

$$\delta f + (1-\delta)p_r \leq (1-\delta)p_n \quad (2.11)$$

$$\delta f + (1-\delta)p_r \geq p_n - \delta \quad (2.12)$$

$$f \geq 0 \quad (2.13)$$

The first term of the objective function is the profit or loss due to consumer returns, while the second term is the profit from selling the new products to the T-consumers, and the last term is the profit from selling refurbished products to the T-consumers and R-consumers. Constraint (2.10) ensures $\bar{\theta} \geq \check{\theta}_h$. Constraint (2.11) guarantees that $\check{\theta}_h \geq \check{\theta}_l$. Constraint (2.12) provides for a nonnegative demand for the new products from the T-consumers; i.e., $\check{\theta}_h \leq 1$. We do not need to include constraint $f \leq p_r$ because this condition is always implied by constraint (2.10) and $p_r \leq p_n$. We also can omit constraint $p_r \leq p_n$ because this condition is implied by constraints (2.11) and (2.13). Moreover, $p_r \geq 0$ is implied by constraints (2.10) and (2.13).

We then form the Lagrangian with multipliers $\check{\lambda}_1, \check{\lambda}_2, \check{\lambda}_3$ and $\check{\lambda}_4$.

$$\begin{aligned} \check{L}(p_r, f) = & \hat{\Pi} + \check{\lambda}_1(1 - (1-\delta)\phi p_r - \delta\phi f - (1-\phi)p_n) + \check{\lambda}_2((1-\delta)p_n - \delta f - (1-\delta)p_r) \\ & + \check{\lambda}_3(\delta f + (1-\delta)p_r - p_n + \delta) + \check{\lambda}_4 f \end{aligned}$$

By setting different combinations of $\check{\lambda}_1, \check{\lambda}_2, \check{\lambda}_3$ and $\check{\lambda}_4$ to zero, these conditions lead to all possible options A and D through K for the seller under the lenient return policy when $p_n \geq 1 - (1-\delta)\phi$ (see Figure 2.3(a)). When $p_n \leq 1 - (1-\delta)\phi$, the possible options for the seller are A, D, D', F, G, J, K (see Figure 2.3(b)). On the line containing segment AE, $\check{\theta}_h = \bar{\theta}$. $\check{\theta}_h = \check{\theta}_l$ on the line containing segment AK, and there is no demand for refurbished products. The line containing EI in Figure 2.3(a) is where $\check{\theta}_h = 1$ and there is no demand for new products. We should also note that at point A, $\check{\theta}_h = \check{\theta}_l = \bar{\theta}$. The lines containing segments AK and EI have the same slope equal to $-\frac{(1-\delta)}{\delta}$. The line containing segment

AE has a positive slope of $\frac{1 - (1 - \delta)\phi}{\delta\phi}$. The value of p_r at point E, which is $p_n - \delta\phi$, is a lower bound for p_r . Each option's value of the decision variables and conditions under which points A and D (D') through K are optimal under the lenient return policy are given in Appendix A3.

Theorem 2. *Under the lenient return policy,*

- (i). *If $c_r < \min [2(1 - \delta)p_n - 1, \delta v + (1 - \delta)c_n]$, then points A and G are not optimal.*
- (ii). *If $c_r > \max \left[v\delta + (1 - \delta)c_n, 1 - \frac{2(1 - p_n)}{1 - \delta} \right]$, then points H and I are not optimal.*
- (iii). *If $c_r > 2p_n - 1$, then points E and F are not optimal. If $p_n < 1 - (1 - \delta)\phi$, then point E cannot be optimal.*
- (iv). *If $c_n < 2p_n - 1$, then points J and K are not optimal.*

Theorem 2(i) shows that when the refurbishing cost is low, serving the T-consumers with only the new product is not optimal for the seller under strategy 2. Moreover, Theorem 2(ii) and (iii) show that when the refurbishing cost is high, serving the T-consumers with only the refurbished products is not favorable. Theorem 2(iii) also indicates that when the price of the new product is low, serving the T-consumers with only refurbished products is not optimal. Theorem 2(iv) also indicates that when the cost of the new product is low, encouraging returns by charging the consumer no restocking fees for the refurbished product is not optimal. Note that point A is common to both the strict and the lenient return policies, as it results in selling only new products to T-consumers.

2.5.3 Intermediate Return Policy

Under the third strategy, the seller sets the price and restocking fees so that $\check{\theta}_l \leq \bar{\theta} \leq \hat{\theta}_h$ (see Figure 2.5). Therefore, some T-consumers who purchase the refurbished products and receive “lemons” will return them while others will not.

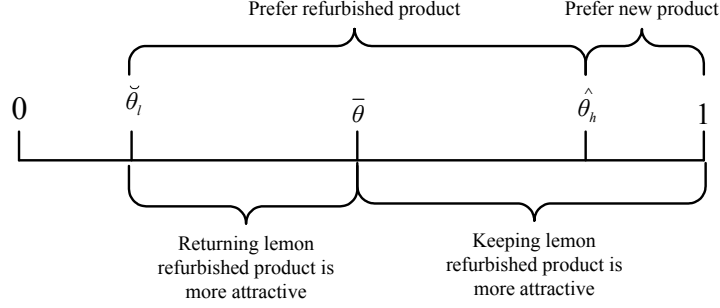


Figure 2.5 Comparison between $\bar{\theta}$, $\hat{\theta}_h$ and $\check{\theta}_l$ under the intermediate return policy

Under this intermediate strategy, the seller solves the following problem:

$$\begin{aligned} \max_{p_r, f} \tilde{\Pi} = & \beta^T \left(1 - \frac{p_n - p_r}{\delta\phi} \right) (p_n - c_n) + \delta\beta^T \left(\frac{p_r - f}{1 - \phi} - \frac{\delta f}{(1 - \delta)} - p_r \right) (f + v - c_r) \\ & + \left(\beta^T \left(\frac{p_n - p_r}{\delta\phi} - \delta \left(\frac{p_r - f}{1 - \phi} \right) - (1 - \delta) \left(\frac{\delta f}{(1 - \delta)} + p_r \right) \right) + \beta^R (1 - p_r) \right) (p_r - c_r) \end{aligned} \quad (2.14)$$

$$s.t. \quad (1 - \delta)\phi p_r - (1 - \delta\phi)f \geq 0 \quad (2.15)$$

$$(1 - (1 - \delta)\phi)p_r - \delta\phi f \leq (1 - \phi)p_n \quad (2.16)$$

$$p_r \geq p_n - \delta\phi \quad (2.17)$$

$$f \geq 0 \quad (2.18)$$

The objective function terms represent the same profit components as in the previous models. Constraint (2.15) ensures that $\bar{\theta} \geq \check{\theta}_l$, and constraint (2.16) enforces that $\hat{\theta}_h \geq \bar{\theta}$. Constraint (2.17) guarantees that there are nonnegative demands for the new products from the T-consumers. We can omit the constraint $f \leq p_r$ because it is implied by constraint (2.15). We need not include constraint $p_r \leq p_n$ as this condition is implied by constraint (2.16).

Again, the Hessian matrix for the profit function is positive definite. Therefore, the function $\tilde{\Pi}(p_r, f)$ is concave. We then form the Lagrangian using multipliers $\tilde{\lambda}_1$, $\tilde{\lambda}_2$, $\tilde{\lambda}_3$ and $\tilde{\lambda}_4$.

$$\begin{aligned} \tilde{L}(p_r, f) = & \tilde{\Pi} + \tilde{\lambda}_1((1 - \delta)\phi p_r - (1 - \delta\phi)f) + \tilde{\lambda}_2((1 - \phi)p_n - (1 - (1 - \delta)\phi)p_r + \delta\phi f) \\ & + \tilde{\lambda}_3(p_r - p_n + \delta\phi) \end{aligned}$$

By setting different combinations of $\tilde{\lambda}_1$, $\tilde{\lambda}_2$, $\tilde{\lambda}_3$ and $\tilde{\lambda}_4$ to zero, these conditions lead to all possible options L, M and A through E for the seller under the intermediate return policy when $p_n + (1 - \delta)\phi - 1 \geq 0$ (see Figure 2.3(a)). When $p_n + (1 - \delta)\phi - 1 \leq 0$, the possible options for the seller are A, B, C, D, D', E', M'

(see Figure 2.3(b)). Note the points in common with the other two policies. The intermediate policy reduces to the strict policy on line containing segment AC. Similarly, it shares a boundary with the lenient policy along AE (AD'). The line containing segment CL represents $\hat{\theta}_h = 1$, so T-consumers buy only refurbished products. Each option's value of the decision variables and conditions under which option L, M, A through E (D', E', M' as well) are optimal under the intermediate policy are given in Appendix A4.

Theorem 3. *Under the intermediate return policy,*

- (i). *If $c_r < \min [2(1 - \delta\phi)p_n - 1, \delta v + (1 - \delta)c_n, (1 - \delta\phi)c_n]$, then point A is not optimal.*
- (ii). *If $c_r > \max [(1 - \delta\phi)c_n - 2\delta\phi, 2(p_n - \delta\phi) - 1]$, then points C and L are not optimal.*
- (iii). *If $\frac{v}{c_r} > \frac{1 - \phi}{1 - \delta\phi}$, then points B and C are not optimal.*
- (iv). *If $(1 - \phi)c_r - (1 - \delta\phi)v > 2(1 - \phi)(1 - p_n)$, then point E is not optimal.*
- (v). *If $v\delta\phi + (1 - \phi)c_r < (1 - \phi)c_n$, then point D is not optimal.*

Theorem 3(i) shows that serving the T-consumers with only new products will not be favorable when the cost of refurbishing is low. Theorem 3(iii) also indicates that if the ratio of salvage value to the cost of refurbishing exceeds a threshold value, discouraging the return of refurbished product will not be optimal. Theorem 3(ii) and (iv) show that if the refurbishing cost is high or the salvage value is too low, serving the T-consumers with only refurbished products while allowing return is not optimal. Theorem 3(v) illustrates that when both the cost of refurbishing the product and the salvage value are low, serving the T-consumers with both new and refurbished products and encouraging returns will not be an optimal choice for the seller.

To summarize, the seller has several options for setting the price and restocking fees of refurbished products, shown in Figure 2.3. Corollary 1 summarizes the conditions under which the pricing extremes A, C, L and E cannot be optimal for the seller under any return policy.

Corollary 1. (i). *If $c_r < \min [2(1 - \delta\phi)p_n - 1, \delta v + (1 - \delta)c_n, (1 - \delta\phi)c_n]$, then point A is not optimal.*

(ii) *If $c_r > \max \left[(1 - \delta\phi)c_n - 2\delta\phi(1 - p_n), 2p_n - 1, \frac{1 - \delta\phi}{1 - \phi} + 2(1 - p_n) \right]$, then points C, L and E are not optimal.*

Corollary 1 indicates that the refurbishing cost plays an important role in deciding with which products to serve the T-consumers. When the refurbishing cost is low relative to the price of the new

product, the salvage value and the cost of producing a new product, serving the T-consumers with only the new product by charging a high p_r will not be optimal. On the other hand, when the refurbishing cost is high, serving the T-consumers with only the refurbished product by pricing them cheaply will not be favorable.

2.6 Numerical Study

The analytical results provide some guidance concerning conditions under which the different strategies and their options may be optimal, but leave many questions unanswered. In particular, we wish to provide insight into the types of situations in which the different options are optimal overall. We characterize the seller's pricing decision in terms of the product variety choice provided to the T-consumers. The illustrations of all the options are provided in Figure 2.6. Because the expressions for the decision variables and profit under the different options are too complex to decipher analytically, we performed an extensive numerical experiment to validate our analytical findings and to better understand the optimal price, return policy and profit.

Product Variety Return Policy	Serve the T- consumer with only new products	Serve the T- consumer with both new and refurbished products	Serve the T-consumer with only refurbished products
Strict	A/F/K	B	C
Intermediate		M/M'	L/E'
Lenient		D/G/J/D'	E/H/I

Figure 2.6 Characteristics of different options for the seller

The numerical study consists of 32,000 total instances resulting from a full factorial combination of the values listed in Table 2.2. The value of β^N is fixed at 0.1, and $\beta^R = 0.9 - \beta^T$. These parameters cover a wide range of plausible values and could represent all the possible variations in market segment conditions (in terms of β^T and β^R), consumer valuation degradation (in terms of ϕ and δ), product segment (in terms of p_n) and cost factors (in terms of c_n , c_r and v). We set ϕ and δ to at most 0.5 to satisfy Assumption 4. For each parameter combination, we optimized $\hat{\Pi}$, $\check{\Pi}$ and $\tilde{\Pi}$, chose the largest

as the optimal profit Π^* , and recorded the corresponding optimal refurbished product price p_r^* and restocking fee f^* . With the exception of 4 particular instances where there was no feasible solution under the strict return policy, we obtained an optimal solution under each policy for all instances.

Table 2.2 Parameter value settings in numerical experiments

Parameter	Values
β^T	{0.1, 0.35, 0.6, 0.85}
δ	{0.1, 0.2, 0.3, 0.4, 0.5}
ϕ	{0.1, 0.2, 0.3, 0.4, 0.5}
p_n	{0.5, 0.6, 0.7, 0.8, 0.9}
c_n	{ $0.1p_n$, $0.3p_n$, $0.5p_n$, $0.7p_n$ }
c_r	{ $0.1c_n$, $0.3c_n$, $0.5c_n$, $0.7c_n$ }
v	{ $0.1c_r$, $0.35c_n$, $0.6c_r$, $0.85c_r$ }

Table 2.3 shows the average values of β^R , β^T , p_n , ϕ , δ , EV, c_n/p_n , c_r/c_n and v/c_r for which each of options B, C, G, J, K, L and M were optimal (the remaining options were not optimal for any combination of parameters tested). The average values of Π^* , p_r^* , f^* , p_r^*/p_n , and f^*/p_r are shown in Table 2.4 as well as the average profit from selling new products to T-consumers (denoted as Π_{NT}), profit from selling refurbished products to the T-consumers (Π_{RT}), profit/loss due to the consumer returns (Π_{Return}) and profit from sale of the refurbished products to the R-consumers (Π_{RR}). Note that the values of f under options B, C, and K are not really meaningful because no refurbished products are returned in those cases (either because none are sold to T-consumers, or because of the strict return policy). We also calculated the frequency with which each option was optimal. In the following sections we first examine the seller's product variety decision and return policy decisions separately, and then observe their interactions.

2.6.1 Product Variety Decision for the T-consumers

Serving T-consumers with only new products (Option K) is optimal when the population of R-consumers is large, refurbishing cost is high, and the salvage value of returned product is low. From the seller's point of view, extreme high refurbishing cost and low salvage value make it less profitable to serve the T-consumer with refurbished products. Charging a very high price for the refurbished product results in the T-consumers purchasing only the new product. As the market size of R-consumers is very

Table 2.3 Average parameter values for which options are optimal

Option	β^R	β^T	p_n	ϕ	δ	EV	c_n/p_n	c_r/c_n	v/c_r
B	0.308	0.592	0.625	0.349	0.377	0.870	0.259	0.476	0.365
C	0.420	0.480	0.735	0.250	0.283	0.931	0.429	0.370	0.399
G	0.616	0.284	0.644	0.452	0.438	0.803	0.325	0.528	0.561
J	0.560	0.340	0.533	0.418	0.352	0.855	0.364	0.380	0.532
K	0.654	0.246	0.512	0.341	0.380	0.871	0.408	0.597	0.449
L	0.419	0.481	0.748	0.381	0.234	0.910	0.436	0.360	0.811
M	0.298	0.602	0.633	0.417	0.338	0.860	0.258	0.457	0.814

Table 2.4 Average profit, price and restocking fees

Option	Frequency at Optimality	Π^*	p_r^*	f^*	p_r^*/p_n	f^*/p_r	Π_{NT}	Π_{RT}	Π_{Return}	Π_{RR}
B	0.111	0.166	0.516	0.131	0.828	0.255	0.046	0.056	0.0000	0.063
C	0.610	0.139	0.666	0.130	0.906	0.197	0.000	0.068	0.0000	0.071
G	0.017	0.165	0.564	0.068	0.877	0.122	0.042	0.003	0.0001	0.119
J	0.039	0.176	0.516	0.000	0.970	0.000	0.055	0.004	-0.0002	0.117
K	0.052	0.160	0.512	0.000	1.000	0.000	0.037	0.000	0.0000	0.123
L	0.137	0.137	0.658	0.203	0.880	0.309	0.000	0.065	0.0003	0.072
M	0.137	0.166	0.515	0.161	0.816	0.312	0.047	0.056	0.0004	0.063

large, the seller can extract considerable profit from those consumers to compensate for the absence of refurbished product sales to the T-consumers. As shown in Table 2.4, R-consumers are the major source of profit under option K; thus, a large population of R-consumers is an important factor in identifying this option as optimal.

Under options C and L, the seller will serve the T-consumers with only refurbished products. *Serving T-consumers with only refurbished products is optimal when the refurbishing cost is low, new product production cost and price are high and the T-consumers' expected value degradation of refurbished products is low.* Because the T-consumers value the refurbished product highly, they can be charged a higher price (see Table 2.4). With this high price, the R-consumers also pay more for the refurbished product. Also, the high cost of the new products erodes the profitability of selling them. Table 2.4 shows that the profits from T-consumers and R-consumers are nearly the same under options C and L. But the overall profit is relatively low. Thus, even though the seller charges a high price to both segments, the lack of new product sales is felt.

Under options B, G, J and M, the seller will serve the T-consumers with both new and refurbished products. *Serving the T-consumers with both new and refurbished products is valuable under low production cost for new products, low to medium refurbishing cost and medium to high value degradation of the refurbished products.* As long as neither the cost of new products or the refurbishing cost is high, those options can be applied. Low production cost for the new product will ensure the profitability from selling it, while the price for refurbished products varies among options B, G, J and M, which are differentiated by return policy as explained below. As the seller is able to differentiate the market to the maximum degree, options B, G, J and M generate high profits.

2.6.2 Return Policy Decision

Return of the lemon refurbished product occurs under options G, J, L, and M. Those options are characterized by relatively high salvage value for returned product. Average conditions for options M and L are very similar to options B and C (under which no returns occur), except the salvage value under options L and M are higher. Therefore, *allowing or not allowing return depends on the salvage value of returned products.* Allowing return will be favored when the salvage value of returned products is not too low but the strict return policy should be employed when the salvage value of returned product is very low. This conclusion is consistent with the results from Davis et al. (1998) and Shulman et al. (2009), where they find the salvage value plays an important role in determining the leniency of the return policy.

Among the options where T-consumers purchase both products, options G and J represent the most generous return policy while option M represents the intermediate return policy where only some of the T-consumers will return the lemon products. Conditions where options G and J are optimal are characterized by a relatively small proportion of T-consumers and relatively low expected valuation for refurbished products. Under option G, allowing the T-consumers to return all of the lemon products will protect them from the low valuation. With 12.2% restocking fees on average, the seller incurs a slight profit from the return. Under option J, the T-consumer can return the refurbished product for free at the expense of paying a very high price. Because of the very low refurbishing cost, the seller actually incurs a slight loss from the returns. Thus, *a generous return policy will be favored when the proportion of T-consumers is small and their expected value degradation of refurbished products is relatively high.*

A small number of T-consumers ensures that the return quantities are not too high while minimal unit loss ($f + v - c_r$) or even a slight unit profit results from the returns.

Under options L and M, the seller will charge a restocking fee of nearly 30% on average. This is very close to the real life observation where some sellers (e.g., HP) set the restocking fees around 25%. Lowering the price of the refurbished product and increasing the restocking fee (compared to option B or C) induces some but not all T-consumers to return the lemon refurbished product. Options L and M are optimal where there is a large segment of T-consumers. Because a large population of T-consumers makes it unaffordable for the seller to allow all of them to return the lemon products, the intermediate return policy is appropriate. Thus, *an intermediate return policy is preferred when T-consumers form a large proportion of the market and their value degradation of refurbished products is low to medium*. Table 2.4 shows that the seller actually gains a tiny profit under the intermediate return policy due to the small amount of returns, the restocking fee charged and high salvage value.

Among these combinations of parameter values, option C was optimal in 61% of the cases, followed in frequency by options B, L and M, and lastly by options G, J and K. This is because in the numerical experiment, the size of R-consumer and T-consumer segments both varied from 0.1 to 0.9. In reality, the proportion of R-consumers is expected to be small. If the values of β^T are at least 0.5 or 0.6, we expect the optimality frequency of options B and M to increase relative to the other options.

2.6.3 Combined Decisions

An examination of the seller's product variety and return policy decisions together indicates that serving the T-consumers with only refurbished products and offering a lenient return policy (options E, H or I) is not a good combination, as large amounts of returns would erode the seller's profit. The intermediate return policy (including its strict and lenient boundary cases) is appropriate if the seller decides to serve the T-consumers with both new and refurbished products. Moreover, either the combination of low price with relatively strict return policy (options B/M) or high price with lenient return policy (options G/J) appear to be favorable under a wide variety of conditions. A combination of high price and strict return policy (option K) might be preferred under very special market conditions (e.g., when the proportion of R-consumers is very large). The combination of low price and strict return policy reflects the practice of Epson and Fujitsu while a high price with strict return policy falls in line with

other manufacturers (e.g., HP and Apple).

Observing the two decisions together reveals additional insights. First, the R-consumers are “angels” for the seller while the T-consumers are not. The profit is usually higher when the proportion of R-consumers in the market is large (e.g., options G, J and K). Because the R-consumers value the refurbished product at least as highly as the new product and will always keep it, they will be a stable source of profit. The T-consumers, in contrast, not only have a discounted valuation of the refurbished product, but they will also return it in some cases. Second, allowing returns may expand the market. Atasu et al. (2008) find that the seller can charge a high price for the remanufactured product when the consumer’s valuation is high. But under option G, the seller is able to charge a relatively high price even when the consumer’s expected valuation is low. Third, losses resulting from allowing the consumers to return the refurbished products may be offset by charging a high price, as in option J.

2.7 Discussion

When a seller offers both new and refurbished products, he or she should be aware of the heterogeneous attitudes the consumer may have towards refurbished products. Our empirical study indicates that there exist consumers who will never consider purchasing refurbished products and consumers who will only consider purchasing the refurbished version of a given model. But the majority of consumer will consider purchasing both products and have uncertainty concerning their valuation of refurbished electronics. Those tactical consumers want enough cost savings to compensate for the valuation uncertainty and seek various mechanisms to mitigate such unpredictability.

Motivated by the empirical study, we focused on two strategic decisions of the seller; namely, the selling price and restocking fee for the refurbished products. We analyzed three different strategies for the seller and explored the optimal price and restocking fees in a numerical study. Optimality conditions and numerical analysis show that different combinations of price and restocking fees are favored under different conditions. The combinations of either low price and strict return policy or high price and lenient return policy are prevalent. Such results are consistent with observations from real life³. We also show that the combination of low price and strict return policy might be favored under very

³http://www.hp.com/sbso/buspurchase_refurbished_faq.html#return <http://www.shopfujitsu.com/www/content/products/notebooks/ordering.php>

special conditions. Moreover, parameters vary in their impacts on the variety decision and restocking fee decision. The price of new products, cost savings from refurbishing and the tactical consumers' expected valuations of refurbished products have more impact on the product variety decision. The salvage value of returned products and the prevalence of the tactical consumers have more impact on encouraging return or not. By protecting the consumer against a lemon product, allowing returns enables the seller to serve the tactical consumers with refurbished products and charge a relatively high price even when their expected valuation is low. In this way, both the seller and the consumer benefit from allowing returns. The results also suggest that sellers should deliberately consider the market segment conditions, consumer valuations, and cost factors to choose the appropriate price and restocking fees for refurbished products.

There are several ways to extend the current research. The first is to take a dynamic perspective. This paper considers only a single time period assuming there are already enough refurbished products available to serve different market segments. An interesting extension would be to consider a multi-period model in which the refurbished products are explicitly linked to consumer returns of new products in a previous period. Another extension is to incorporate the limited inventory of refurbished products. It would be worthwhile to investigate how the limited inventory would change the consumer behavior and the seller's decisions. Although the results here are obtained under a linear utility function, the insights regarding the pricing and restocking fee decision might be generalized beyond this special case (see Jin et al. (2007) for the case of nonlinear utility function). Moreover, manufacturers such as Dell use different items to represent different quality conditions of refurbished products. For example, a "reconditioned" product may refer to an effectively new product, while a "remanufactured" product may have been returned to the OEM and subjected to an extensive remanufacturing process. Whether the manufacturer should provide this information is worth investigating.

CHAPTER 3. ROBUST DESIGN OF A CLOSED-LOOP SUPPLY CHAIN NETWORK FOR UNCERTAIN CARBON REGULATIONS AND RANDOM PRODUCT FLOWS

3.1 Introduction

Environmental and economic factors have motivated firms to plan their supply chain structures to handle both forward and reverse flows of products. Activities in the reverse supply chain occur due to commercial and consumer returns, or to capture the potential profits derived from remanufacturing and resale. For example, the annual costs of commercial returns in the US exceed \$100 billion (Atasu et al., 2008). Usually, these items are shipped back to the manufacturer from the retailer. The reverse flows are also compelled by various regulations (Atasu et al., 2009). Many state-operated programs in the US require the manufacturer to collect and recycle electronic waste (e-waste) (Gui et al., 2010). This leads to the idea of closed-loop supply chain (CLSC) management. According to Guide and Van Wassenhove (2009), CLSC management focuses on “the design, control, and operation of a system to maximize value creation over the entire life cycle of a product with dynamic recovery of value from different types and volumes of returns over time” (p.10). One of the most important strategic decisions in a firm’s CLSC management is its network design. As the CLSC network is expected to be in use for a considerable amount of time, the firm should consider all the possible factors that will affect the design decisions.

One important factor is the potential environmental regulation. In recent decades, concerns over global climate change are increasingly focusing attention on both the fuel costs and the carbon emissions that result from transporting goods. Although subject to political vagaries, regulation of carbon emissions is becoming inevitable. Compared to a more rigid command-and-control policy, market-based environmental mechanisms that put a price on greenhouse gas emissions are usually favored

because they provide incentives for emission reduction. The market-based approach has been proven effective in controlling sulphur dioxide in the US and has been instituted elsewhere, to reduce the carbon emissions. For example, in 2005 the European Union instituted a carbon emission trading scheme (EU ETS) for the energy-intensive industries with the aim of reducing greenhouse gas emissions by at least 20% below 1990 levels (Bohringer et al., 2009). Also, the New Zealand Emissions Trading Scheme (NZ ETS) was introduced in 2009 (Jiang et al., 2009). In 2011, Australia's government announced the details of a carbon tax plan (Siriwardana et al., 2011). Such emission regulations aim to eventually emission reductions in all economic sectors, among which transportation is a main source of emissions. According to the U.S. Energy Information Administration (EIA), 33.2% of carbon emission are from the transportation sector (EIA., 2009). Therefore, it is not surprising that several world regions including California and Canada are discussing cap-and-trade systems that would include the transportation sector (Flachsland et al., 2011).

A firm that wishes to proactively design a CLSC in anticipation of market mechanisms to control carbon emissions faces multiple forms of uncertainty. The first question is the type of policy (carbon tax or cap-and-trade system) that may be administered. In major carbon emitting nations such as the United States, China and Japan, there are extensive debates over which regulatory policy will be favored. Even if the firm could know which policy will be applied, it still faces considerable uncertainty about the magnitudes of incentives or penalties and the stringency of constraints. Carbon emission permit prices elsewhere have exhibited considerable volatility. In the EU ETS, the permit price increased from around 7 euros in January, 2005 to above 30 euros in April, 2006, before crashing to below 10 euros within 3 days. It then rose again and stabilized above 15 euros for about 4 months before decreasing to nearly zero by mid-2007 (Benz and Trück, 2009). Such behavior implies that estimation of credible probability distributions for carbon prices based on historical data might be very difficult. Second, forecasting consumer demand is a perennial challenge even with the aid of historical or market research information to inform the construction of a probability distribution, and forecasting return flows is even harder.

Designing a CLSC network involves long-term decisions to invest in fixed facilities such as manufacturing or remanufacturing plants, warehouses, and collection facilities. It also involves decisions concerning the transportation modes between different facilities. The goal of this paper is to formulate

a tractable CLSC network design problem, and solve it to obtain a facility configuration that is robust to variations in possible carbon emission regulations while enabling responsiveness to the random variations in retailer demands and returns.

We propose a two-stage, multi-period stochastic programming model in which the demands for new products and returns of those products are discrete random variables. Then we extend this formulation to incorporate two carbon regulation policies: tax or cap and trade. By analyzing the similarity in the effects of the two regulation policies, we propose a hybrid model that could account for them both. The carbon prices or tax rates are characterized as uncertain parameters within certain sets, and a robust optimization method based on Ben-Tal and Nemirovski (1999, 2000) is adopted to handle such uncertainty. Based on the possible primary scenarios the decision maker has, tractable forms of a robust counterpart under box and ellipsoidal uncertainty sets are developed. A case study shows how the optimal network configuration balances the trade-offs among investment costs, transportation costs and carbon emission costs. The network configurations obtained under the “carbon-incorporated” model are different from those obtained under a “carbon-free” model. More facilities will be opened to reduce the distance traveled, and transportation modes with lower carbon emission rates will be favored as the uncertainty in either carbon emission regulation policy increases (in terms of carbon permit price or carbon tax). The total expected carbon emissions and total cost will also increase as the product flow variability increases. Simpler formulations under deterministic demands and returns as well as nominal carbon prices or tax rates are also derived. Numerical experiments show how, if the ellipsoidal uncertainty set is adopted, the decision maker can balance the trade-off between robustness and cost by changing the size of the ellipsoidal set. Also, compared to the nominal carbon prices or tax rates model, the robust model yields solutions that provide protection under the worst-case scenario without being overly conservative. This paper contributes to the literature by formulating the network design problem with multiple types of uncertainty. To the best of our knowledge, this is the first paper that solves the CLSC network design problem with the combination of robust optimization and stochastic programming to address the effects of uncertain environmental regulations.

The rest of the paper is organized as follows. In the next section, we review the literature related to our work. In Section 3, we provide the two-stage, multi-period stochastic programming model without consideration of carbon emissions, then extend it to include the possible carbon emission regulations.

Hybrid model of both possible regulation policies is provided in Section 4, where we also proposed the tractable robust counterparts under box and ellipsoidal uncertainty sets. We present case studies and computational results in Section 5 and finish the paper with concluding remarks in Section 6.

3.2 Literature Review

Supply chain network design problems have been relatively well-studied both for forward-only supply chain and for CLSCs (see Melo et al. (2009) and Akçalı et al. (2009) for reviews). Traditionally, mixed-integer programming (MIP) models are commonly used. These models range from simple uncapacitated facility location models to complex capacitated multi-stage or multi-commodity models. Their common objective is to determine the least cost system design, which usually involves making tradeoffs among fixed opening costs of facilities and variable transportation costs. Various solution methods have been developed to solve the network design problem but only a few studies have considered the uncertain nature of various input parameters in a strategic planning horizon through scenario-based stochastic programming (Santoso et al., 2005; Listes, 2007). Those papers use probabilistic optimization methods which take advantage of known or estimated probability distributions for the data. But these scenario-based optimization methods encounter difficulty if a discrete probability distribution of the uncertain parameters is largely unknown (Bertsimas et al., 2004).

To overcome this shortcoming, a robust optimization methodology was first developed by Soyster (1973) and then further developed by Mulvey et al. (1995), El Ghaoui and Lebret (1997), and Ben-Tal and Nemirovski (Ben-Tal and Nemirovski, 1999, 2000). This approach has also been applied to network design problems. For the robust network flow problem, Mudchanatongsuk et al. (2007) developed a method to solve a network flow problem under transportation cost and demand uncertainty. They defined an affine function for the arc flows in terms of the uncertain demand and then transformed the model into a MIP problem. Atamturk and Zhang (2007) described a two-stage robust optimization approach for solving network flow and design problems with uncertain demand, including both capacity allocation and routing decisions. That work focuses on the network flow problems and the selection of locations for the facilities are not involved. Pishvaei et al. (2011) proposed a robust optimization model for handling the inherent uncertainty of customer demands and transportation costs in a CLSC net-

work design problem. Their model is a single stage robust optimization problem with box uncertainty, which can be converted to an equivalent mixed-integer linear program. Baron et al. (2011) apply robust optimization to the problem of locating facilities in a network facing uncertain demand over multiple periods. They use the box and ellipsoidal uncertainty sets to characterize the demand uncertainty. The latter two papers consider only uncertainties in the demand and/or cost data, and the effects of carbon emission regulations are not considered.

This paper is also related to operational and strategic impacts of supply chain decisions on carbon emissions. Benjaafar et al. (2009) presented an extension of the lot sizing model that accounts for carbon emissions under various regulatory policies. Also, with the increase of environmental consciousness, those environmental parameters have also been taken into account when designing the supply chain network (Chaabane et al., 2008; Diabat and Simchi-Levi, 2009; Ramudhin et al., 2008). These authors have used deterministic models to study the network design problem when different regulations are taken into account. But because they focus on the impact of subcontracting and production activities with a predetermined supply chain network, the effect of carbon regulations on the network configuration is not addressed. Hoen et al. (2010) examined the effect of two regulation mechanisms on the transport mode selection decision when a single mode must be selected for all transport of a single item. In their simplified setting, they found that introducing an emission cost for freight transport via either a direct emission tax or a market mechanism such as cap and trade is not likely to result in significant changes in transport modes and hence will not reduce emissions much.

This paper differs from previous research in several ways. First, we explicitly address the effects of uncertain carbon emission regulations on the CLSC network configuration by incorporating two such policies into a hybrid model. Second, this paper models the CLSC network design problem with the combination of both robust optimization and stochastic programming methodologies. The carbon regulation parameters characterized by prices or tax rates are modeled with uncertainty sets while the demands and returns are represented by discrete probabilistic scenarios.

3.3 A Two-stage Multi-period Stochastic Programming Model for CLSC Network Design

In this paper, we consider a firm that has to design a CLSC network for a single product. The primary decisions regard the investment in fixed facilities in anticipation of forward and reverse flows between facilities over multiple periods. The firm must decide the locations of factories for manufacturing new and recovering returned products. It will open separate warehouse and collection facilities for distributing new products and collecting returned products, respectively. Modification of the model for combined facilities to handle forward and reverse flows is straightforward. In each period, the warehouses will satisfy the retailer demands, and returns will occur due to buyer remorse, product malfunction and other reasons. The returned products are first shipped to the collection center, and then transported to the factories for inspection and recovery. Several transportation modes allow the firm to accommodate the flows between facilities. Each mode has different cost and emission implications. The network topology is illustrated in Figure 3.1.

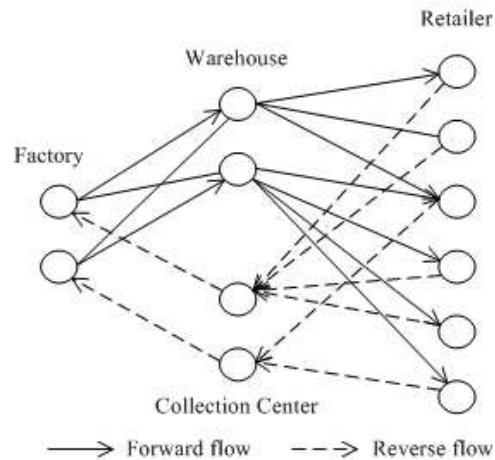


Figure 3.1 Closed-loop supply chain network structure

This problem has a two-stage, multi-period structure. It has a two-stage structure because the first-stage facility investment decisions must be made before the realization of demand and return scenarios. It is multi-period because transportation flows can vary in response to changing demand and return quantities (and in the robust extension, to changing carbon regulation parameters). After this basic model is set up, we will extend it to incorporate the uncertainty from carbon emission regulations. In

the extended model, investment decisions also must be made within the first-stage prior knowing which type of regulation will be used. The product flow decisions for each subsequent period constitute the second stage, after all uncertainties are realized. The following notation will be used throughout this paper.

Sets and Indices

\mathcal{P} set of potential factories for manufacturing new and recovering returned products, $p \in \mathcal{P}$

\mathcal{W} set of potential warehouses for distributing new products, $w \in \mathcal{W}$

\mathcal{L} set of potential collection centers for returned products, $l \in \mathcal{L}$

\mathcal{K} set of retailer locations, $k \in \mathcal{K}$

\mathcal{M} set of transportation modes, $m \in \mathcal{M}$

\mathcal{T} set of time periods, $t \in \mathcal{T}$

\mathcal{S} set of alternative scenarios of retailer demands and returns, $s \in \mathcal{S}$

\mathcal{A} set of all the arcs in the network $\mathcal{A} \equiv \{ij : (i \in \mathcal{P}, j \in \mathcal{W}) \text{ or } (i \in \mathcal{W}, j \in \mathcal{K}) \text{ or } (i \in \mathcal{K}, j \in \mathcal{L}) \text{ or } (i \in \mathcal{L}, j \in \mathcal{P})\}$

\mathcal{F} set of potential facilities, $\mathcal{F} = \mathcal{P} \cup \mathcal{W} \cup \mathcal{L}$

\mathcal{N} set of all the nodes in the network, $\mathcal{N} = \mathcal{F} \cup \mathcal{K}$

Parameters

ω_t^s probability of scenario s in period t , $s \in \mathcal{S}, t \in \mathcal{T}$

d_{kt}^s new product demand of retailer k under scenario s in period t , $k \in \mathcal{K}, s \in \mathcal{S}, t \in \mathcal{T}$

μ_t^s return rate in period t under scenario s , $s \in \mathcal{S}, t \in \mathcal{T}$

r_{kt}^s returns of retailer k under scenario s in period t , $r_{kt}^s = \mu_t^s d_{kt}^s$, $k \in \mathcal{K}, s \in \mathcal{S}, t \in \mathcal{T}$.

c_{ijm} unit transportation cost from node i to node j using transportation mode m , $ij \in \mathcal{A}, m \in \mathcal{M}$

f_i the investment cost for building facility, $i \in \mathcal{F}$

Λ_{it} maximum capacity of facility i in period t , $i \in \mathcal{F}, t \in \mathcal{T}$

β_{ij} distance (km) from node i to node j , $ij \in \mathcal{A}$

τ_m carbon emission factor (g/ton-km) for transportation mode m , $m \in \mathcal{M}$

w unit weight of product (ton)

α_t carbon tax rate in period t (dollar per ton), $t \in \mathcal{T}$

ϕ_t average spot price of emission allowance in period t (dollar), $t \in \mathcal{T}$

κ_t number of carbon permits firm received from allocation in period t , $t \in \mathcal{T}$

Decision Variables

x_{ijm}^s the amount of product transported from node i to node j using transportation mode m under scenario s in period t , $ij \in \mathcal{A}$, $m \in \mathcal{M}$, $s \in \mathcal{S}$, $t \in \mathcal{T}$

$y_i = 1$ if facility i is opened, 0 otherwise, $i \in \mathcal{F}$

e_t^{s+} , e_t^{s-} the number of carbon permits the firm purchases and sells in period t under scenario s , $s \in \mathcal{S}$, $t \in \mathcal{T}$

The following assumptions are used in the model.

Assumption 1. *In each period, the inventory is carried by retailers. Warehouse and collection center, which act as a break-bulk centers, do not accumulate stocks.*

This means the warehouse and collection center are used as cross docking facilities, which may not be usual in actual practice. But this is a common assumption made in previous research (Melo et al., 2009; Santoso et al., 2005; Listes, 2007).

Assumption 2. *The firm owns the transportation vehicles. Each transportation mode has unlimited capacity.*

In this paper, we do not consider the possibility of third-party logistics. To avoid the complication of routing and other operational decisions, we also assume that each transportation mode has unlimited capacity, which may not be realistic in practice. Considering capacitated transportation is a topic for additional research.

Assumption 3. *Under the carbon cap-and-trade system, carbon permits can be either purchased or sold at the same price in a given period.*

Speculative trading; i.e., the buying and selling of carbon permits to benefit from the price difference, would involve formulation of the firm's carbon permit trading strategy, which is beyond the scope of this paper. Similar assumptions are also made in Benjaafar et al. (2009); Hua et al. (2011).

Assumption 4. *Under the carbon cap-and-trade system, there is no banking or investment in financial derivatives of carbon allowances.*

This assumption permits a focus on the network design decision.

Assumption 5. *The returns in each period depend only on the sales volumes in that period; i.e., the random demands, \tilde{d}_{kt} , for retailer k in period t . The return rate $\tilde{\mu}_t$ is also a random variable. We further assume that for each retailer k , $\tilde{d}_{1t}, \dots, \tilde{d}_{kt}$ are mutually independent and independent of $\tilde{\mu}_t$. For each period t , $\{\tilde{d}_{k1}, \dots, \tilde{d}_{kt}\}$ and $\{\tilde{\mu}_1, \dots, \tilde{\mu}_t\}$ are also mutually independent.*

The retailer demands for new products and the return amounts in each period are the first source of uncertainty. The realizations of random variables \tilde{d}_{kt} and $\tilde{\mu}_t$ can be characterized by discrete scenarios. For each time period t , there are $|\mathcal{S}|$ discrete scenarios and ω_t^s is the probability of scenario s in period t . Thus, for each (k, t) , $\tilde{d}_{kt} = d_{kt}^s$ and $\tilde{\mu}_t = \mu_t^s$ with probability $w_t^s, s \in \mathcal{S}$. Once the y_i are fixed, we are actually solving $|\mathcal{S}| \times |\mathcal{T}|$ subproblems to determine the flows between different facilities. The extensive form of the two-stage multi-period stochastic programming model without carbon emission regulations (called the *baseline problem*) can be then formulated as follows:

$$\min \sum_{i \in \mathcal{F}} f_i y_i + \sum_{s \in \mathcal{S}} \sum_{t \in \mathcal{T}} \sum_{m \in \mathcal{M}} \sum_{ij \in \mathcal{A}} (\omega_t^s c_{ijm} x_{ijm}^s) \quad (3.1)$$

$$\text{s.t.} \quad \sum_{w \in \mathcal{W}} \sum_{m \in \mathcal{M}} x_{wktm}^s = d_{kt}^s, \forall k \in \mathcal{K}, s \in \mathcal{S}, \forall t \in \mathcal{T} \quad (3.2)$$

$$\sum_{l \in \mathcal{L}} \sum_{m \in \mathcal{M}} x_{klm}^s = r_{kt}^s, \forall k \in \mathcal{K}, s \in \mathcal{S}, \forall t \in \mathcal{T} \quad (3.3)$$

$$\sum_{i \in \mathcal{N}} \sum_{m \in \mathcal{M}} x_{ijm}^s - \sum_{j \in \mathcal{N}} \sum_{m \in \mathcal{M}} x_{jitm}^s = 0, \forall j \in \mathcal{W} \cup \mathcal{L}, s \in \mathcal{S}, \forall t \in \mathcal{T} \quad (3.4)$$

$$\sum_{j \in \mathcal{N}} \sum_{m \in \mathcal{M}} x_{ijm}^s - \Lambda_{it} y_i \leq 0, \forall i \in \mathcal{P}, s \in \mathcal{S}, \forall t \in \mathcal{T} \quad (3.5)$$

$$y \in \{0, 1\}^{|\mathcal{F}|}, x \in \mathbb{R}_+^{|\mathcal{A}| \times |\mathcal{M}| \times |\mathcal{T}| \times |\mathcal{S}|} \quad (3.6)$$

The objective is to minimize the long-run total cost of current investment and expected future operating costs. We do not include a discount factor for the cost over time, but the discounted model could be readily built. Constraints (3.2) and (3.3) ensure that retailer demands are met and returned products are collected. Constraints (3.4) ensure that the warehouse and collection facilities will not accumulate stocks. Constraints (3.5) enforce capacity constraints of the processing nodes. If facility i is not built ($y_i = 0$) the constraint will force all flows into the facility to zero.

3.3.1 Incorporating the Carbon Emission Regulation

When evaluating the firm's carbon emission intensity, we neglect those emissions resulting from the construction and maintenance of the facilities to focus our analysis on the logistic activities. The total carbon emissions Γ_t^s (tons per period) from transportation under scenario s in period t can be computed as:

$$\Gamma_t^s = w \sum_{ij \in \mathcal{A}} \beta_{ij} \sum_{m \in \mathcal{M}} \tau_m x_{ijm}^s, \forall t \in \mathcal{T}, s \in \mathcal{S} \quad (3.7)$$

We consider two possible regulatory policies. Under a linear carbon tax scheme, the regulatory party penalizes the units of carbon emitted in each period. For carbon tax rate α_t , the problem can be restated as:

$$\min \sum_{i \in \mathcal{F}} f_i y_i + \sum_{t \in \mathcal{T}} \sum_{s \in \mathcal{S}} \sum_{m \in \mathcal{M}} \sum_{ij \in \mathcal{A}} (\omega_t^s c_{ijm} x_{ijm}^s) + \sum_{t \in \mathcal{T}} \sum_{s \in \mathcal{S}} \alpha_t \omega_t^s \Gamma_t^s \quad (3.8)$$

subject to constraints (3.2) – (3.7). Here, the term $\sum_{t \in \mathcal{T}} \sum_{s \in \mathcal{S}} \alpha_t \omega_t^s \Gamma_t^s$ is the expected future cost of the carbon tax.

Under a cap-and-trade system, the firm will receive an allocation of carbon permits κ_t in each period (i.e., the “cap”). Every permit allows the firm to emit one ton of carbon. It may emit more than its cap if it buys additional permits from the market, and it can also sell excess permits. Under this setting, the problem can be reformulated as follows:

$$\min \sum_{i \in \mathcal{F}} f_i y_i + \sum_{t \in \mathcal{T}} \sum_{s \in \mathcal{S}} \sum_{m \in \mathcal{M}} \sum_{ij \in \mathcal{A}} (\omega_t^s c_{ijm} x_{ijm}^s) + \sum_{t \in \mathcal{T}} \sum_{s \in \mathcal{S}} \phi_t \omega_t^s (e_t^{s+} - e_t^{s-}) \quad (3.9)$$

$$\text{s.t. } \Gamma_t^s - e_t^{s+} + e_t^{s-} \leq \kappa_t, \forall s \in \mathcal{S}, t \in \mathcal{T} \quad (3.10)$$

$$e_t^{s+}, e_t^{s-} \geq 0 \quad (3.11)$$

in addition to constraints (3.2) – (3.7). Here, the last term in the objective function is the expected future cost or profit from the carbon trading market. Note that, although carbon emission permits are nondivisible, for simplicity we assume they can be traded in any continuous quantity.

There are some similarities between the regulation policies. In the cap-and-trade version of the model, which replaces (3.1) with (3.9) and includes constraints (3.10) and (3.11), for any $\phi_t \geq 0$ the net number of permits purchased, $e_t^{s+} - e_t^{s-}$, will be as small as possible at optimality. Therefore, constraint (3.10) will bind at optimality, so that $e_t^{s+} - e_t^{s-} = \Gamma_t^s - \kappa_t$ will hold for every scenario and every period. Thus, $\sum_{t \in \mathcal{T}} \sum_{s \in \mathcal{S}} \phi_t \omega_t^s (e_t^{s+} - e_t^{s-}) = \sum_{t \in \mathcal{T}} \sum_{s \in \mathcal{S}} \phi_t \omega_t^s (\Gamma_t^s - \kappa_t)$, where $\sum_{t \in \mathcal{T}} \sum_{s \in \mathcal{S}} \phi_t \omega_t^s \kappa_t = \sum_{t \in \mathcal{T}} \phi_t \kappa_t$. Because the

term $\sum_{t \in \mathcal{T}} \phi_t \kappa_t$ will affect only the objective value but not the optimal solution, it can be dropped from the objective function without loss of optimality. Then, the objective has the same form as that for the carbon tax (3.8). The two policies thus can be represented in a single model as follows:

$$\min \sum_{i \in \mathcal{F}} f_i y_i + \sum_{t \in \mathcal{T}} \sum_{m \in \mathcal{M}} \sum_{ij \in \mathcal{A}} (c_{ijm} \bar{x}_{ijtm}) + \sum_{t \in \mathcal{T}} (\alpha_t \bar{\Gamma}_t) \quad (3.12)$$

$$\text{s.t. } \Gamma_t^s - e_t^{s+} + e_t^{s-} = \kappa_t, \forall s \in \mathcal{S}, t \in \mathcal{T} \quad (3.13)$$

$$e_t^{s+}, e_t^{s-} \geq 0 \quad (3.14)$$

along with constraints (3.2) – (3.7). Here, $\bar{\Gamma}_t \equiv \omega_t^s \Gamma_t^s$ and $\bar{x}_{ijtm} \equiv \omega_t^s x_{ijtm}^s$ represent the expected amount of carbon emissions and the expected flows, respectively. A tax policy is represented by setting κ_t to a large enough value that it does not affect the optimization and setting α_t to the unit tax, while a cap-and-trade policy is represented by setting κ_t to a restrictive level and letting α_t represent the market price of carbon permits. In this paper, we consider the α_t and κ_t to be uncertain data, which vary within an uncertainty set (\mathcal{U}). The distributions of $\{\alpha_t\}$ and $\{\kappa_t\}$ are not known but the decision maker has the nominal data $\hat{\alpha}_t$ and $\hat{\kappa}_t$, which are estimates of α_t and κ_t .

There are several “easy” approximations for the problem. One is to replace the uncertain α_t with the nominal values but still retain the stochastic demands and returns in the formulation. This results in a *nominal stochastic* model.

$$\text{(Problem NS)} : \min \sum_{i \in \mathcal{F}} f_i y_i + \sum_{t \in \mathcal{T}} \sum_{m \in \mathcal{M}} \sum_{ij \in \mathcal{A}} (c_{ijm} \bar{x}_{ijtm}) + \sum_{s \in \mathcal{S}} \sum_{t \in \mathcal{T}} (\hat{\alpha}_t \bar{\Gamma}_t) \quad (3.15)$$

along with constraints (3.2) – (3.7) and (3.13) – (3.14). Another approximation of the problem is to replace the stochastic demands and returns in each period with their expected values. Under this approximation, the decision variables will be y_i and x_{ijtm} , as the second stage consists of a single scenario. The emission in each period Γ_t can be computed as $\Gamma_t = w \sum_{ij \in \mathcal{A}} \beta_{ij} \sum_{m \in \mathcal{M}} \tau_m x_{ijtm}, \forall t \in \mathcal{T}$. The *robust*

deterministic problem formulation can be stated as follows:

$$\text{(Problem RD)} : \min \sum_{i \in \mathcal{F}} f_i y_i + \sum_{t \in \mathcal{T}} \sum_{m \in \mathcal{M}} \sum_{ij \in \mathcal{A}} (c_{ijm} x_{ijtm}) + \sum_{t \in \mathcal{T}} (\alpha_t \Gamma_t) \quad (3.16)$$

$$\text{s.t. } \bar{\Gamma}_t - e_t^+ + e_t^- = \kappa_t, \forall t \in \mathcal{T} \quad (3.17)$$

$$\text{s.t. } \sum_{w \in \mathcal{W}} \sum_{m \in \mathcal{M}} x_{wktm} = \bar{d}_{kt}, \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \quad (3.18)$$

$$\sum_{l \in \mathcal{L}} \sum_{m \in \mathcal{M}} x_{kltm} = \bar{r}_{kt}, \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \quad (3.19)$$

$$\sum_{i \in \mathcal{N}} \sum_{m \in \mathcal{M}} x_{ijtm} - \sum_{j \in \mathcal{N}} \sum_{m \in \mathcal{M}} x_{jitm} = 0, \forall j \in \mathcal{W} \cup \mathcal{L}, \forall t \in \mathcal{T} \quad (3.20)$$

$$\sum_{j \in \mathcal{N}} \sum_{m \in \mathcal{M}} x_{ijtm} - \Lambda_{it} y_i \leq 0, \forall i \in \mathcal{P}, \forall t \in \mathcal{T} \quad (3.21)$$

$$y \in \{0, 1\}^{|\mathcal{F}|}, x \in \mathbb{R}_+^{|\mathcal{A}| \times |\mathcal{M}| \times |\mathcal{T}|}, e_t^+, e_t^- \geq 0, \alpha_t, \kappa_t \in \mathcal{U} \quad (3.22)$$

where \bar{d}_{kt} is the expected demand for the new products and $\bar{d}_{kt} = \sum_{s \in \mathcal{S}} \omega_t^s d_{kt}^s, t = 1, \dots, T$. Similarly, \bar{r}_{kt} is the expected amount of the returned products and $\bar{r}_{kt} = \sum_{s \in \mathcal{S}} \omega_t^s r_{kt}^s, t = 1, \dots, T$. If we replace the carbon permit price or tax rate in (3.16) with the estimated nominal values, we obtained the *nominal deterministic* problem.

$$\text{(Problem ND)} : \min \sum_{i \in \mathcal{F}} f_i y_i + \sum_{t \in \mathcal{T}} \sum_{m \in \mathcal{M}} \sum_{ij \in \mathcal{A}} (c_{ijm} x_{ijt}^m) + \sum_{t \in \mathcal{T}} (\hat{\alpha}_t \Gamma_t) \quad (3.23)$$

along with constraints (3.17) - (3.22).

3.4 Hybrid model for CLSC network design

3.4.1 Robust Optimization Methodology

The goal of robust optimization is to make decisions that are robust to any realization of the uncertain data. To illustrate the robust optimization methodology we will use in this study, consider a linear optimization problem with an objective function $c^T x$ to optimize, subject to constraints $Ax \leq b$ where uncertain parameters c, A, b vary in a given uncertainty set U . The general uncertain linear optimization problem can be stated as follows:

$$\{\min_x c^T x \text{ s.t. } Ax \leq b\} (c, A, b) \in U \quad (3.24)$$

Here the decision variables are x and the uncertain parameters c, A, b belong to a closed, bounded and convex uncertainty set U . A solution x is robust feasible if it satisfies constraint $Ax \leq b$ for all

realizations of A, b within U . Each robust feasible solution x is associated with a robust objective value $\hat{c}(x) = \sup_{c \in U} [c^T x]$. The purpose of robust optimization is to find a optimal solution x^* among robust feasible solutions x which will return the best robust objective value. Such x^* , called a robust optimal solution, is obtained by solving the following Robust Counterpart (RC) problem (Ben-Tal et al., 2009):

$$\min_x \{ \hat{c}(x) = \sup_{c \in U} [c^T x] : Ax \leq b, \forall (A, b) \in U \} \quad (3.25)$$

Ben-Tal and Nemirovski (1999, 2000) show that the RC of a linear optimization problem is tractable for most uncertainty sets. For the case of ellipsoidal uncertainty set, the RC is equivalent to a second-order cone program (SOCP). If U is polyhedral, the robust counterpart is equivalent to a linear optimization problem (Bertsimas et al., 2011).

3.4.2 Hybrid Model

The robust optimization approach is adopted here to address the policy uncertainty; i.e., we formulate an uncertainty set (\mathcal{U}) for the combination of α_t and κ_t , and then seek a solution to the robust counterpart of this problem. At the same time, we retain the stochastic demands and returns in the constraints. We introduce another variable z to represent objective function (3.12). The compact matrix form of this hybrid model can be stated as follows, and we denote this *robust stochastic* formulation as problem *RS*.

$$\text{(Problem RS) : } \min_{z, y, x, e^+, e^-} z \quad (3.26)$$

$$\text{s.t. } \forall (\alpha, \kappa) \in \mathcal{U} \begin{cases} \alpha^T \bar{\Gamma} \leq z - f^T y - c^T \bar{x} \\ \Gamma - e^+ + e^- = \kappa \end{cases} \quad (3.27)$$

$$Bx = \tilde{v} \quad (3.28)$$

$$-Fy + Gx \leq 0 \quad (3.29)$$

$$-w\tau\beta^T x + \Gamma = 0 \quad (3.30)$$

$$y \in \{0, 1\}^{|\mathcal{I}|}, x \in \mathbb{R}_+^{|\mathcal{A}| \times |\mathcal{M}| \times |\mathcal{T}| \times |\mathcal{S}|}, e^+, e^- \geq 0 \quad (3.31)$$

The vectors $\alpha, \bar{\Gamma}, f, \kappa$ and τ correspond to carbon prices or tax rates, expected carbon emissions, fixed opening costs, emission caps, and emission factors, respectively. Matrices c and β respectively

contain transportation costs and distances between different nodes. Define Γ^s, e^{s+}, e^{s-} as $|\mathcal{S}| \times 1$ vectors $\forall s \in \mathcal{S}$, respectively, that correspond to carbon emissions and emission credits purchased and sold under scenario s . Then, Γ is used to represent the vector that consists of $\Gamma^1, \dots, \Gamma^s$. Similarly, e^+ is used to represent the vector that consists of e^{1+}, \dots, e^{s+} and e^- is used to represent the vector that consists of e^{1-}, \dots, e^{s-} . Furthermore, κ consists of $|\mathcal{S}|$ vectors $\kappa_1, \dots, \kappa_t$. The matrices H, F and G contain coefficients of the constraints (3.5) and (3.7). Constraint (3.28) is the vector form for constraints (3.2)-(3.4), where B is the coefficient matrix and \tilde{v} is the matrix that consists of $\tilde{d}_{kt}, \tilde{r}_{kt}$ and zero. Also, all binary decision variables are included into the vector y , flow variables under different scenarios are included into the matrix x and the expected flows are included in the matrix \bar{x} . To obtain a tractable form of problem RS , let us first consider the following two LPs:

$$(P1) : \min_{u,s} c^T u \quad (3.32)$$

$$\text{s.t. } \forall (D, e) \in \mathcal{U} \begin{cases} Du \leq b \\ Hx + s = e \end{cases} \quad (3.33)$$

$$u \geq 0, \text{ sfree} \quad (3.34)$$

and

$$(P2) : \min_u c^T u \quad (3.35)$$

$$\text{s.t. } Du \leq b, \forall D \in \mathcal{U}_D \quad (3.36)$$

$$u \geq 0 \quad (3.37)$$

where the set \mathcal{U}_D is the projection of \mathcal{U} on the space of the data for constraint (3.36). To solve P1, we can apply the following:

Theorem 1. *If u^* is an optimal solution of P2 with objective value v , then $(u^*, e - Hu^*)$ is an optimal solution of P1 with the same objective value v .*

Proof. By contradiction. Assume u_2^* is an optimal solution for P2 but there is no optimal (u_1, s_1) for P1 with $u_1 = u_2^*$. This means we can find a solution u_1^* that satisfies $Du_1^* \leq b, Hu_1^* + s = e$ and $c^T u_1^* < c^T u_2^*$. Let Ω_1 and Ω_2 denote the feasible regions of P1 and P2, respectively, where $\Omega_1 = \{(u, s) : Du \leq b, Hu + s = e, \forall (D, e) \in \mathcal{U}, u \geq 0\}$ and $\Omega_2 = \{u : Du \leq b, \forall A \in \mathcal{U}_D, u \geq 0\}$. We can see that $\Omega_1 \subseteq \Omega_2$, which implies $c^T u_1^* \geq c^T u_2^*$. This contradicts our assumption and concludes the proof. \square

Based on Theorem 1, we can discard constraints $(\Gamma - e^+ + e^- = \kappa)$ and construct the following problem (RS') instead:

$$\text{(Problem } RS') : \min_{z,y,x} z \quad (3.38)$$

$$\text{s.t. } \alpha^T \bar{\Gamma} \leq z - f^T y - c^T \bar{x}, \forall \alpha \in \mathcal{U}_\alpha \quad (3.39)$$

along with constraints (3.28)-(3.31). The set \mathcal{U}_α is the projection of set \mathcal{U} on the space of the data for constraint (3.39), which describes only uncertainty in α .

Corollary 1. *If z^*, y^*, x^* are an optimal solution to RS' , and Γ^* is the corresponding total carbon emissions, then $z^*, y^*, x^*, e^{+*} = \max(\Gamma^* - \kappa, 0)$ and $e^{-*} = \max(\kappa - \Gamma^*, 0)$ is an optimal solution to problem RS .*

Proof. Theorem 1 remains valid after introducing binary variables y where $u = (x, y, z)$. Here the slack variable is $e^- e^+$. If no speculative trading or banking is considered, it is expected that the firm participates in buying or selling carbon permits only to handle the difference between their cap and their emissions. \square

From the decision maker's point of view, how should the uncertainty set \mathcal{U}_α be constructed? When carbon permit prices or tax rates are considered, some primary scenarios might be gained based on the experience of EU ETS. We could then construct the uncertainty set based on the available data and decision maker's attitude towards risk. Assume the actual carbon permit prices or tax rates, α_t , are unknown but bounded by a symmetric interval around an estimated nominal value. That is, $\alpha_t \in \Delta_t = [\hat{\alpha}_t - \delta_t, \hat{\alpha}_t + \delta_t]$, where $\hat{\alpha}_t$ is the nominal value and $\delta_t < \hat{\alpha}_t$ are the possible deviations in each period. We will then present two possible uncertainty sets, box and ellipsoidal that the decision maker could use.

3.4.2.1 Hybrid Model under Box Uncertainty Set

A box uncertainty set may be represented by $\mathcal{U}_{box} = \{\alpha \in \mathbb{R}^n : |\alpha_t - \hat{\alpha}_t| \leq \delta_t\}$, and $n = |\mathcal{T}|$. Define $W_t = \hat{\alpha}_t + \delta_t$, which is the worst case scenario of carbon prices or tax rates in each period. The problem

RS' under box uncertainty can be represented as follows:

$$\min_{z,y,x} z \quad (3.40)$$

$$\text{s.t. } \max_{\alpha \in \mathcal{U}_\alpha} \{\alpha^T \bar{\Gamma}\} \leq z - f^T y - c^T \bar{x} \quad (3.41)$$

along with constraints (3.28)-(3.31), and the problem RS' under box uncertainty set is further equivalent to the following *worst-case stochastic* problem:

$$\text{(Problem WS:)} \min_{z,y,x,\lambda} z \quad (3.42)$$

$$\text{s.t. } W^T \bar{\Gamma} \leq z - f^T y - c^T \bar{x} \quad (3.43)$$

along with constraints (3.28)-(3.31), where W is the vector of $\{W_t\}$. Considering $\bar{\Gamma} \geq 0$, it is straightforward that $\max\{\alpha^T \bar{\Gamma}\} = W^T \bar{\Gamma}$. This is the same approach proposed by Soyster (1973). The robust optimal solution would be obtained by solving the problem assuming the carbon permit price or tax rate in period t is W_t . Although the resulting problem WS is a MILP which could be solved effectively, choosing such an uncertainty set is very conservative.

3.4.2.2 Hybrid Model under Ellipsoidal Uncertainty Set

For the network design problem, the decision maker might be interested in a set of problem RS' solutions $(x, y, z) \in \Psi(\epsilon)$ such that (x, y, z) will violate the constraint (3.39) with probability at most ϵ . The set $\Psi(\epsilon)$ can be represented by following chance constraint (Miller and Wagner, 1965):

$$\Psi(\epsilon) = \{(x, y, z) : Pr(\alpha^T \bar{\Gamma} > z - f^T y - c^T \bar{x}) < \epsilon\} \quad (3.44)$$

where $\bar{\Gamma} = \omega^T \Gamma$. We want to design an uncertainty set such that the probability is guaranteed and the robust solution is feasible without being overly conservative. One way to design such an uncertainty set is to use an ellipsoidal set:

$$\mathcal{U}_{ellips} = \{\alpha \in \mathbb{R}^n : \sum_{t=1}^n \delta_t^{-2} (\alpha_t - \hat{\alpha}_t)^2 \leq \rho^2\} \quad (3.45)$$

Using P to denote the diagonal matrix with entries δ_t , an equivalent representation is $\mathcal{U}_{ellips} = \{\hat{\alpha} + Pu : \|u\|_2 \leq \rho\}$. The problem RS' under ellipsoidal uncertainty set can be represented as follows

(denoted as problem RS'_{ellips}):

$$\min_{z,y,x} z \quad (3.46)$$

$$s.t. \quad \max_{\|u\|_2 \leq \rho} \{(\hat{\alpha}^T + (Pu)^T)\bar{\Gamma}\} \leq z - f^T y - c^T \bar{x} \quad (3.47)$$

along with constraints (3.28)-(3.31).

Theorem 2. *The problem RS'_{ellips} is equivalent to the following problem:*

$$\min_{z,y,x,\lambda} z \quad (3.48)$$

$$s.t. \quad \hat{\alpha}^T \bar{\Gamma} + \rho \|P^T \bar{\Gamma}\|_2 \leq z - f^T y - c^T \bar{x} \quad (3.49)$$

along with constraints (3.28)-(3.31).

Proof. The left-hand side in constraint (3.47) is $\hat{\alpha}^T \bar{\Gamma} + \max_{\|u\|_2 \leq \rho} (Pu)^T \bar{\Gamma}$, where $\max_{\|u\|_2 \leq \rho} (Pu)^T \bar{\Gamma} = \max_{\|u\|_2 \leq \rho} \sqrt{((Pu)^T \bar{\Gamma})^2}$. According to the Cauchy–Schwarz inequality, $((Pu)^T \bar{\Gamma})^2 \leq (P^T \bar{\Gamma})^2 (u^T)^2 \leq (P^T \bar{\Gamma})^2 \rho^2$. Thus, $\max_{\|u\|_2 \leq \rho} (Pu)^T \bar{\Gamma} \leq \rho \|P^T \bar{\Gamma}\|_2$, which concludes the proof. \square

We can get different sets by varying the value of the uncertainty budget ρ . For $\rho = 0$, \mathcal{U}_{ellips} shrinks to the nominal data $\hat{\alpha}_t$. For $\rho = 1$, \mathcal{U}_{ellips} is the largest ellipsoid contained in \mathcal{U}_{box} . For $\rho = \sqrt{n}$, which is the worst case uncertainty budget \mathcal{U}_{ellips} is the smallest volume ellipsoid containing the \mathcal{U}_{box} . Ben-Tal and Nemirovski (2000) have proved that the feasible solutions will violate constraint (3.39) with probability at most $\exp(-\rho^2/2)$. For example, $\rho = 3.0349$ will guarantee at least 0.99 feasibility. But for problems with a small number of uncertain data, this bound is not particularly attractive, and we can obtain a tighter bound based on Dufour and Hallin's work.

Theorem 3. *If the uncertainty intervals are given by $\Delta_t = [\hat{\alpha}_t - \delta_t, \hat{\alpha}_t + \delta_t]$ and (x, y, z) is a feasible solution of problem, then $Pr\{\alpha^T \bar{\Gamma} > z - f^T y - c^T \bar{x}\} < B(\rho, t)$ as tabulated in Table 3 of Dufour and Hallin (1993).*

Proof. Dufour and Hallin derive the probability bound for $|\sum_{i=1}^n a_i Y_i| \geq y$ where $\sum_{i=1}^n a_i^2 = 1$ and random variables $|Y_i| \leq 1, \forall i = 1, \dots, n$. The uncertainty in carbon permit price in each period t can be represented by $\tilde{\alpha}_t = \hat{\alpha}_t + \eta_t \delta_t$, where the random variable η_t obeys an unknown but symmetric distribution on $[-1, 1]$. Then $Pr\{\alpha^T \bar{\Gamma} > z - f^T y - c^T \bar{x}\} = Pr\{\sum_{i=1}^n (\tilde{\alpha}_t + \eta_t \delta_t) \bar{\Gamma}_t > z - f^T y - c^T \bar{x}\}$, and $z - f^T y - c^T \bar{x} \geq$

$\sum_{i=1}^n \tilde{\alpha}_t \bar{\Gamma}_t + \rho \sqrt{\sum_{i=1}^n (\delta_t \bar{\Gamma}_t)^2}$ from constraint (3.49). So we have $Pr\{\alpha^T \bar{\Gamma} > z - f^T y - c^T \bar{x}\} < Pr\{\sum_{i=1}^n \eta_i \delta_t \bar{\Gamma}_t > \rho \sqrt{\sum_{i=1}^n (\delta_t \bar{\Gamma}_t)^2}\} < Pr\{\sum_{i=1}^n \eta_i \delta_t \bar{\Gamma}_t \geq \rho \sqrt{\sum_{i=1}^n (\delta_t \bar{\Gamma}_t)^2}\}$. Let $p_t = \delta_t \bar{\Gamma}_t / \sqrt{\sum_{i=1}^n (\delta_t \bar{\Gamma}_t)^2}$, and $\sum_{i=1}^n p_t^2 = 1$. Then we have $Pr\{\alpha^T \bar{\Gamma} > z - f^T y - c^T \bar{x}\} < Pr\{\sum_{i=1}^n \eta_i p_t \geq \rho\} \leq Pr\{|\sum_{i=1}^n \eta_i p_t| \geq \rho\}$. This probability bound can be derived based on the Proposition 1 of Dufour and Hallin (1993). \square

We should note that bound in Dufour and Hallin (1993) is tighter than $\exp(-\rho^2/2)$. For example, based on Dufour and Hallin's calculation, 0.99 feasibility will be guaranteed at $\rho = 2.686$. Thus, the decision maker can easily balance the trade-off between robustness and performance by changing the size of ellipsoidal set. In addition, even though the problem RS'_{ellips} is a mixed-integer second order cone program (MISOCP), it can be solved efficiently by some commercial solvers, e.g., the ILOG CPLEX Optimizer.

3.5 Computational Experiments

In this section, we describe numerical experiments to understand the impact of uncertainties on the CLSC network configuration. Specifically, we investigate the impacts of carbon emission regulation uncertainty and product flow variability on the number of facilities opened, transportation mode selection, total carbon emissions and total cost. Before presenting the results, we first describe the detailed method to generate the parameters.

3.5.1 Parameter Generation

All the parameters are randomly generated according to uniform distributions. The candidate facility locations are randomly generated in a $[0, 5000] \times [0, 5000]$ square. The fixed cost f_i (\$M) of opening a factory, warehouse, or collection center are randomly generated according to uniform distributions on $[5, 8]$, $[0.5, 1.5]$, and $[0.125, 0.5]$, respectively. Capacities of the factory, warehouse and collection center in each period Λ_{it} are randomly generated according to uniform distributions on $[2.5, 4]$, $[0.25, 0.75]$, $[0.062, 0.25]$ (Million units), respectively. We assume that only the road transport options are available and do not consider rail, water and air transport. For the transportation modes, the calculations of carbon emission factors are based on data from Pirog et al. (2001). The cost per km per ton is calculated based

on the data from Byrne et al. (2006), which is calculated based on fuel costs, capital costs, operation and maintenance cost over the fleet's life cycle and adjusted by incorporating the weight consideration. The distance between any two locations is considered to be the Euclidean distance (with kilometer as distance unit). We further assume that the weight of 1,000 units is one ton; i.e., $w=1000g$.

In each period, three scenarios for new product demand are considered, namely low (L), medium (M) and High (H). The following steps are used to generate demands and returns of each retailer in each period:

1. For $t = 1$ to T :
2. The probability ω_t^L is randomly generated between $[0.3, 0.35]$, ω_t^M is randomly generated between $[0.3, 0.35]$ and $\omega_t^H = 1 - (\omega_t^L + \omega_t^M)$.
3. For $k = 1$ to K :
4. Low, medium and high demand scenarios d_{kt}^L , d_{kt}^M and d_{kt}^H are randomly generated in $[800, 2000]$, $[3000, 6000]$, and $[8000, 10000]$, respectively. The return rates under the low, medium and high demand scenarios, μ_t^L , μ_t^M , and μ_t^H are randomly generated in $[0.05, 0.08]$, $[0.07, 0.1]$ and $[0.08, 0.12]$, respectively.
5. Next k
6. Next t .

Table 3.1 Characteristics of road transport options

Transport Mode	Fuel Type	CO_2 Emissions Factor (g/ton-km)	Cost per Km per Ton
1.Heavy-duty Truck	Diesel	62	0.47
2.Mid-size Truck	Diesel	122	0.32
3.Light Truck	Gasoline	459	0.19

In the computational experiments, the estimated carbon price $\hat{\alpha}_t$ is randomly generated in $[5, 30]$ while the δ_t is randomly generated in $[4, 10]$ to satisfy $\hat{\alpha}_t - \delta_t \geq 0$. The randomly generated $\hat{\alpha}_t$ and δ_t will be discarded if $\delta_t > \alpha_t$. Based on the discussion of the uncertainty set, only the ellipsoidal uncertainty set are considered in this experiment so we abbreviate RS'_{ellips} as RS . The proposed problems RS and RD are implemented in GAMS and solved by CPLEX 11.0 MIQCP solver. The baseline problem and

problem NS are solved by CPLEX 11.0 MIP solver. The data are manipulated by GDXMRW utilities with Matlab (Michael C. Ferris, 2011). All computations are carried out on an Intel Core(TM)2 Quad CPU 3.00 GHz, 3.25 GB RAM computer.

3.5.2 Network Configuration under Different Problem Formulation

To investigate how the optimal network configurations are affected by different model formulations, we first generate a test problem with 10 potential factories, 15 potential warehouse locations, 10 potential collection centers and 30 retailers with $n = 6$ planning periods and uncertainty budget $\rho = 2.45$. The locations for retailers and potential facilities are shown in Figure 3.2. We then solve the baseline case, problem ND , problem RS , problem NS and problem RD . Figure 3.3 shows the network configurations under different demands and returns scenarios from the baseline problem. The arcs represent the product flows between different nodes, where an arc between two nodes is displayed if a flow occurs of any period between the two nodes. Different line widths are used to represent different amounts (average over time periods) of product flow between two nodes. For the forward flow between the factory and warehouse, the thickest line represents the product flows greater than 1 million units and the medium thick line represent product flows greater than 0.5 million units. Between the warehouse and retailers, the line thicknesses represent product flows greater than 100000 units and 50000 units, respectively. The reverse flows are shown by dashed lines. The line thickness between the retailer and the collection center represent product flows greater than 10000 units and 5000 units, respectively. Similarly, the line thickness from the collection center to the factory represent product flow greater than 100,000 units and 50,000 units, respectively.

The main issue is to determine an appropriate network design that simultaneously optimizes both forward and reverse network flows on average. The optimal solution for the baseline problem reflects the trade-offs between the facility investment costs, transportation costs and satisfaction of the capacity constraints. Generally speaking, a higher fixed cost will result in fewer facilities while a higher transportation cost will favor more facilities. Under the baseline case, 1 factory, 3 warehouses and 1 collection center will be opened. As there are no costs other than for investment and transportation in the baseline formulation, it is not surprising that a relatively “centralized” network configuration is obtained where a few facilities serve different markets and each facility serves a large subregion. As the

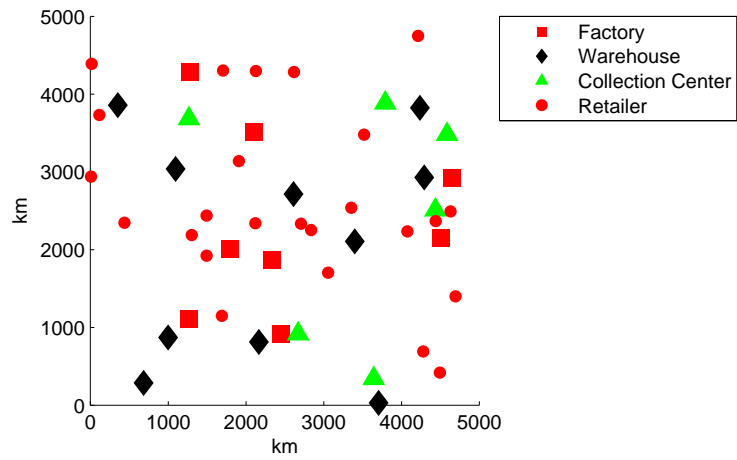


Figure 3.2 Customer locations and potential locations for facilities

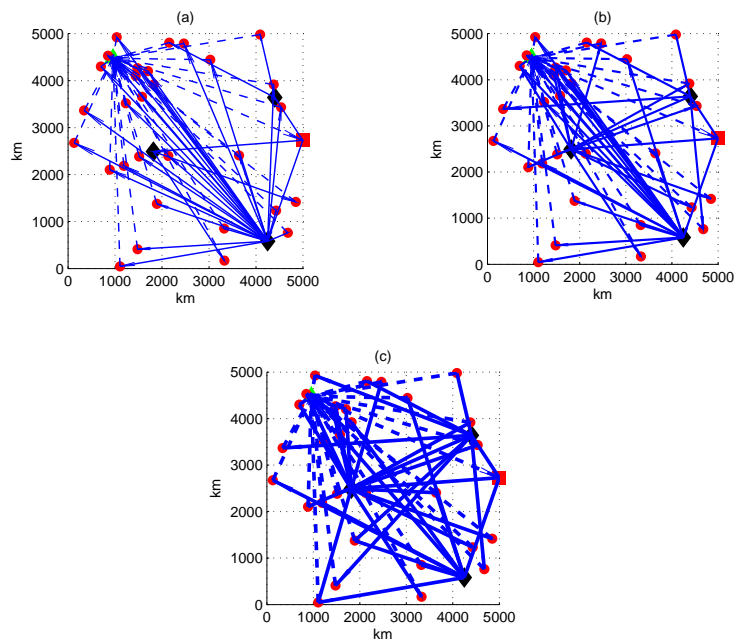


Figure 3.3 Optimal network configuration for baseline problem under different scenarios (a) Low; (b) Medium; (c) High

capacities of different transportation modes are unlimited, the light truck is favored exclusively because it has the lowest unit transportation cost.

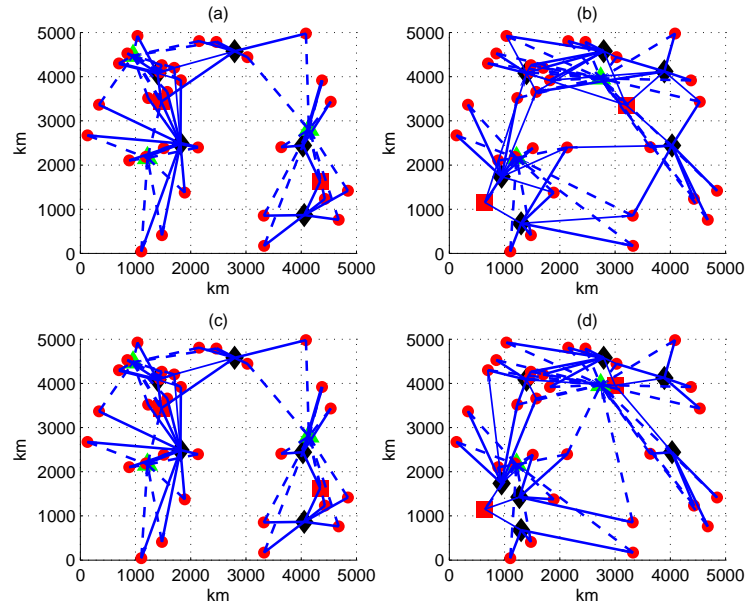


Figure 3.4 Optimal network configuration and expected flows for different formulations (a) ND; (b) NS ; (c) RD; (d) RS

The network configurations obtained when carbon regulation is considered show quite different characteristics compared to the baseline case. If the carbon regulations are incorporated, the variable costs include both transportation and carbon emission costs. As the carbon emission cost is proportion to the distance traveled, more facilities will be opened to mitigate the carbon cost and we will get a relatively “decentralized network” for problems *RS*, *NS* and *RD*. For problems *ND* and *RD*, 2 factories, 5 warehouse and 3 collection centers will be opened. For problem *NS*, 2 factories, 6 warehouse and 3 collection centers will be used. For problems *RS*, 2 factories, 7 warehouse and 2 collection centers will be opened. Comparing to the baseline solution, facilities are located closer to markets.

To study the usage of transportation modes, we also calculate the shares of different transportation modes over all scenarios (see Table 3.2) for the five different formulations. The portions of total flows carried by heavy-duty truck, mid-size truck and light truck are denoted as MR_1 , MR_2 and MR_3 . The

MR_m is computed as follows:

$$MR_m = \frac{|\sum_{s \in \mathcal{S}} \sum_{t \in \mathcal{T}} \sum_{ij \in \mathcal{A}} x_{ijtm}^s > 0|}{|\sum_{s \in \mathcal{S}} \sum_{t \in \mathcal{T}} \sum_{m \in \mathcal{M}} \sum_{ij \in \mathcal{A}} x_{ijtm}^s > 0|} \times 100\% \quad (3.50)$$

Transportation modes balance the trade-offs between transportation and carbon costs. The heavy-duty truck has a lower emission factor but higher unit cost while the light truck will lead to higher carbon cost but lower unit transportation costs. For problems RS and RD , transportation mode 1 and 2 will be favored as they have lower emissions, while for problem NS and ND , transportation mode 2 will be favored exclusively as the mid-size truck has the medium transportation cost and carbon emission rate.

Table 3.2 Shares of transportation by mode

	MR_1	MR_2	MR_3
Baseline	0	0	100%
ND	0	100 %	0
NS	0	100 %	0
RD	52.88%	46.85%	0.27%
RS	49.08%	48.61%	2.31%

Figure 3.4 shows the difference in network configurations between the deterministic and the stochastic problems. The network configurations under deterministic demands and returns are relatively “centralized” while the stochastic versions are more “decentralized”. This is because the total transportation cost is lower under the deterministic settings compared to the stochastic settings. Thus, under the deterministic settings, fewer facilities will be utilized. Also, from Figure 3.4 (a) and (c) we can observe that problems ND and RD have the same selection of facilities. This is because the optimal network configuration is obtained by balancing the trade-off between fixed cost, transportation cost and carbon cost. If the carbon cost under problems ND and RD lacks much impact on the total cost, then we will get a rather similar configuration. Otherwise, the network configurations will be different between ND and RD . For comparison purposes, we reduce the transportation cost to 0.047, 0.032, 0.019 for the corresponding transportation modes. The results from resolving all the five problems are shown in figures 3.5 and 3.6. Under this new settings, the transportation cost will be reduced, and problem RD

have more facilities than problem *ND*. This means more facilities will be opened under problem *RD* to further reduce the carbon costs. Problem *NS* and *RS* have different configurations under both settings. This means if the decision maker solves problem *NS* rather than problem *RS*, the optimal network configurations will be quite different. This is because under the stochastic settings, the problem will have a higher expected carbon cost, which might have more impact on the total cost. In the next section, we will show that problems *NS* and *RS* have different cost implications. Generally speaking, the optimal configuration obtained under problem *NS* will result in a higher cost than problem *RS* under the worst case scenario.

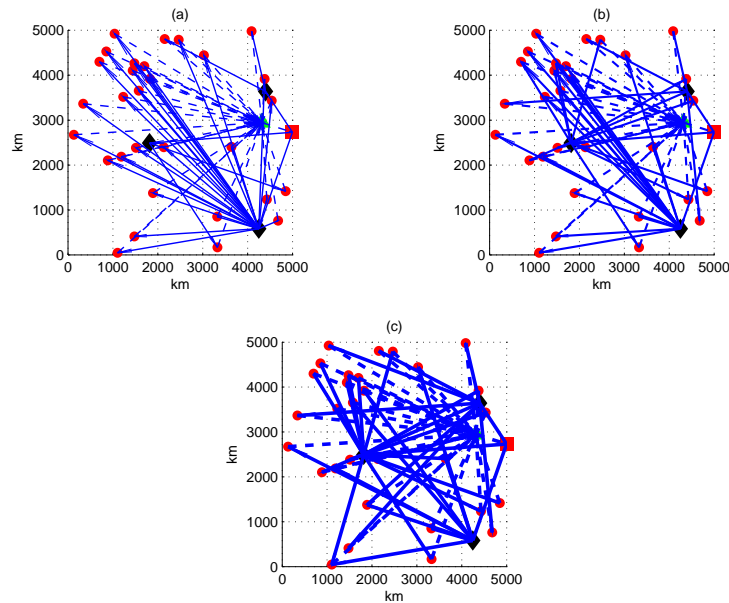


Figure 3.5 Optimal network configuration for low transportation cost baseline problem under different scenarios (a) Low; (b) Medium; (c) High

3.5.3 Impact of Carbon Emission Regulation Uncertainty

To study the impact of carbon emission regulation uncertainty on CLSC network, we perform a computational experiment by varying the uncertainty budget. A larger ρ will result in the carbon prices or tax rates varying within a larger ellipsoidal uncertainty set and, thus, the degree of uncertainty faced by the decision maker. Thus, by purposely changing the value of ρ , we can change the volatility of the emission regulation uncertainty. We then design 6 levels of uncertainty from $\rho = 0$ to $\rho = 20$. For

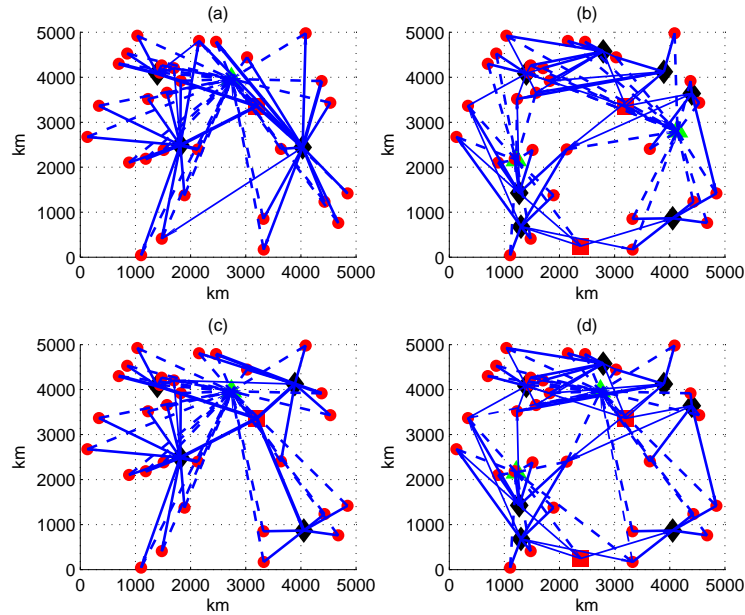


Figure 3.6 Optimal network configuration and expected flows for different formulations under low transportation cost (a) ND; (b) NS ; (c) RD; (d) RS

each level of uncertainty, we randomly generate 10 instances. We then compute the average number of each facility type, expected total emission and total cost. We also compute the share of transportation by each mode following equation (3.50). We use N_F , N_W , N_C to denote the average number of factory, warehouse and collection center that are used. Finally, we use TE and TC to denote the total expected carbon emission and total expected cost for all periods. The complete results of the experiment are shown in Table 3.3.

Table 3.3 Impact of carbon emission regulation uncertainty

Uncertainty Level	N_F	N_W	N_C	MR_1 (%)	MR_2 (%)	MR_3 (%)	TE (M ton)	TC (M \$)
0	3	7.1	5	34.14	52.69	13.17	1.52	111.29
4	3.5	7.3	5.7	58.61	29.50	11.89	1.08	127.97
8	3.6	7.6	6.1	70.97	19.91	9.13	0.93	140.88
12	3.9	7.4	5.9	89.14	7.78	3.09	0.89	152.82
16	4	7.6	6.4	94.71	3.72	1.58	0.87	164.68
20	4.2	7.7	6.3	100	0	0	0.86	176.43

Observe that more facilities will be opened as the uncertainty level increases. As the policy uncer-

tainty increases, it is possible that the permit will end up with a relatively high price. To counter the emissions from transportation, more facilities will be opened to lower the distance traveled. Second, total expected emissions will decrease as the uncertainty level increases while the total expected cost will increase as the regulation uncertainty level increases. Again, the total expected emissions will decrease because of the possible high permit price as the policy uncertainty level increases. The total expected cost will increase due to the construction of more facilities and the employment of transportation mode 1. Third, the share of transportation mode 1, with the lowest emission rate, will increase as the uncertainty level increases.

3.5.4 Impact of Random Product Flow Variability

We considered three different levels of the variability, with the scenario distributions generated above considered as medium. Under the low variability level, the probabilities of high and low demand in each period are generated in $[0.05, 0.1]$, while under the high variability level, the probabilities of high and low demand in each period are generated in $[0.4, 0.45]$. We generated 20 instances for each variability level. The complete results of the experiment are shown in Table 3.4. We observe that the product flow variability has rather limited impact on the number of facilities opened and transportation mode selection, but the total expected emission and total cost will increase as the variability level increases.

Table 3.4 Impact of random product flow variability

Variability Level	Average SD (M)	N_F	N_W	N_C	MR_1 (%)	MR_2 (%)	MR_3 (%)	TE (M ton)	TC (M \$)
Low	0.064	2.8	6.3	4.8	47.34	36.99	15.67	1.19	112.43
Medium	0.117	2.9	6.8	5.2	48.83	38.37	12.80	1.30	121.33
High	0.151	3.2	7.4	4.9	47.52	37.26	15.23	1.32	125.23

3.6 Performance of the Robust Optimization Solution

Because the benefits of results solved by stochastic demands and returns have been discussed in Birge and Louveaux (1997) and Hingle (2005), we will focus our discussion on the case of stochastic

demands and returns. To study the performance of the robust optimization solution, we first solve problems NS , WS , and RS . Then, we compute the following values:

Z_R The optimal value of problem RS

Z_N The optimal value of problem NS

Z_W The optimal value of problem WS

Z_{NW} The objective value of NS solution under the worst case scenario

Z_{RW} The objective value of RS solution under the worst case scenario

Z_{NW} and Z_{RW} are obtained by evaluating the optimal solutions of problem NS and RS under the problem WS objective function, respectively. To see the relative gap between those values, we also compute $R_{WR} = \frac{Z_W - Z_R}{Z_R}$, $R_{NR} = \frac{Z_{RS} - Z_{NS}}{Z_{NS}}$ and $R_{NRW} = \frac{Z_{NW} - Z_{RW}}{Z_{RW}}$. The quantity R_{NR} is the relative loss of optimality of the robust solution compared results from the nominal data of carbon prices or tax rates. The ratio R_{WR} is the relative improvement of the robust solution on the results with the worst case of carbon prices or tax rates. The quantity R_{NRW} is the relative improvement of the robust solution compared with the results from the nominal data of carbon prices or tax rates if the worst case carbon prices or tax rates were to occur.

For the experiment, we considered four different protection levels and generated 30 random instances for network configuration with 6 potential plant locations and 10 potential locations for warehouse, 7 potential locations for collection center and 20 retailer locations, respectively. Those random instances are solved with different ρ . The length of the horizon is set to be 12. The other parameters are generated according the aforementioned method. The computation time for problems NS and WS are less than 1 minutes while the solver took 10-15 minutes to solve problem RS . We report the mean and standard deviation for the results in Table 3.5.

As we observe from Table 3.5, $Z_W \geq Z_R \geq Z_N$ and $Z_{NW} \geq Z_{RW} \geq Z_W$. R_{WR} decreases, R_{NR} increases and NRW increases as the protection level increases. Problem NS provides an unrealistically optimistic approximation to the true problem while problem WS offers a conservative strategy to solve the true problem. The optimal solution under proposed problem RS lies between the “best solution” obtained by solving problem NS and the “worst solution” obtained by solving problem WS . Formulating the

Table 3.5 Results under different protection level (Mean \pm standard deviation) | \mathcal{T} | = 12

Protection level	ρ	Z_N (M \$)	Z_R (M \$)	Z_W (M \$)	Z_{NW} (M \$)	Z_{RW} (M \$)	R_{NR} (%)	R_{WR} (%)	R_{NRW} (%)
80%	1.575	163.29 \pm 19.06	172.9 \pm 20.17	181.59 \pm 21.41	183.73 \pm 21.71	182.27 \pm 21.33	5.89 \pm 0.75	5.01 \pm 0.71	0.79 \pm 0.4
90%	1.901	163.29 \pm 19.06	174.73 \pm 20.39	181.59 \pm 21.41	183.73 \pm 21.71	182.11 \pm 21.36	7.02 \pm 0.88	3.91 \pm 0.6	0.88 \pm 0.42
95%	2.174	163.29 \pm 19.06	176.23 \pm 20.53	181.59 \pm 21.41	183.73 \pm 21.71	182.04 \pm 21.33	7.94 \pm 1	3.02 \pm 0.48	0.92 \pm 0.49
99.9%	2.686	163.29 \pm 19.06	178.85 \pm 20.96	181.59 \pm 21.41	183.73 \pm 21.71	181.93 \pm 21.42	9.53 \pm 1.19	1.52 \pm 0.35	0.99 \pm 0.48

problem as *WS* might be too conservative to be of real interest. Formulating the problem as *NS* seems promising considering the cost savings between the solutions of problem *NS* and *RS*. But problem *NS* can only account for an “average” situation of the carbon prices or tax rates, not the variability in carbon prices or tax rates. Problem *RS*, on the other hand, allows the decision maker to choose between robustness and performance. For example, setting $\rho = 1.575$, the problem *RS* does not immunize much against uncertainty, but the solutions perform close to the problem *NS*. Setting $\rho = 2.686$, the performance of problem *RS* decreases as the model provides higher protection against the uncertainties in carbon prices or tax rates. In addition, under the worst case scenario, solutions of problem *RS* always provide a lower cost than solutions of problem *NS*, the percentage of cost saving varying from 0.79% – 0.99%. Moreover, as the protection level increases, the solution of problem *RS* will perform better than the solution of problem *NS* under the worst case scenario.

For this experiment, the number of uncertain coefficients of problem *RS* is only 12. To see that the attractiveness of formulating the problem as *RS* will increase as the number of uncertain data increases, we perform another experiment with the length of horizon | \mathcal{T} | = 20. As we can observe from Table 3.6, the gap between Z_R and Z_N decreases and the gap between Z_R and Z_W increases as the horizon increases for different level of protections. This result is similar to that of Bertsimas and Thiele (2006).

Table 3.6 Results under different protection level (Mean \pm standard deviation) | $\mathcal{T} = 20$

Protection level	ρ	Z_N (M \$)	Z_R (M \$)	Z_W (M \$)	Z_{NW} (M \$)	Z_{RW} (M \$)	R_{NR} (%)	R_{WR} (%)	R_{NRW} (%)
80%	1.580	240.16 ± 40.13	252.63 ± 42.32	268.87 ± 45.51	272.65 ± 45.88	270.97 ± 45.88	5.18 ± 0.54	6.41 ± 0.92	0.64 ± 0.59
90%	1.911	240.16 ± 40.13	255.33 ± 42.94	268.87 ± 45.51	272.65 ± 45.88	270.82 ± 46.03	6.3 ± 0.67	5.29 ± 0.75	0.7 ± 0.59
95%	2.194	240.16 ± 40.13	257.28 ± 43.36	268.87 ± 45.51	272.65 ± 45.88	270.7 ± 46.01	7.1 ± 0.78	4.5 ± 0.68	0.74 ± 0.63
99.9%	2.736	240.16 ± 40.13	261 ± 43.97	268.87 ± 45.51	272.65 ± 45.88	270.46 ± 45.82	8.66 ± 0.86	3.01 ± 0.56	0.83 ± 0.52

3.7 Conclusions

In this paper, we consider a closed-loop supply chain network design problem where the demands and returns of products are stochastic variables. To cope with the uncertainty in carbon emission regulations, two regulatory policies are considered and a robust extension of a stochastic program is proposed. Further, tractable robust counterparts of the proposed hybrid model are developed to find the robust solutions for box and ellipsoidal uncertainty sets. A case study illustrates how optimal network configuration balances the trade-offs between investment, transportation and carbon emission costs if the carbon regulation is incorporated. More facilities will be opened and the total expected cost will increase as the uncertainty level increases. Moreover, the share of transportation by the low-emitting modes will also increase as the regulation policy uncertainty level increases. The problem formulation with nominal carbon prices or tax rates provides an unrealistically optimistic estimation of the real problem while the worst case scenario problem provides a conservative solution. The problem formulation with ellipsoidal uncertainty set allows the decision maker to balance the trade-off between robustness and performance. In addition, the proposed model can provide certain protection under the worst case scenario.

The methodology presented in this paper can be applied to the planning of other systems as well. The integration of discrete optimization with robust methods that address policy uncertainty and stochastic formulations that use available probabilistic information will enable the decision maker to reduce different types of risk and derive better managerial insights. Many possible extensions could be

made on this topic. For example, addressing the problem in a multi-product setting, considering operational issues such as inventory management or routing, of combining logistics outsourcing decisions together with the network design problem would be interesting topics for future research. It would also be interesting to study how the impact of policy uncertainty would affect firm's participation in CLSC activities. Other extensions could be made to relax the assumptions on speculative trading and banking, and to study how the CLSC network design will be affected when firm's carbon permit trading strategy is taken into condition.

**CHAPTER 4. AN INTEGRATED MODEL FOR PRODUCT RETURN
MANAGEMENT AND CLOSED-LOOP SUPPLY CHAIN NETWORK DESIGN
UNDER UNCERTAIN CARBON COST**

4.1 Introduction

Product returns are an essential part of the exchange process between firms and consumers. Large retailers can have return rates in excess of 10% of their sales, and the total costs of commercial returns exceed \$100 billion per year (Atasu et al., 2008). The most common mechanism of mitigating consumers' regret from purchasing is to refund some portion of the price. Both empirical and theoretical study has proven the significance of the return policy. For example, marketing and economics research has confirmed that the return policy reduces consumer risk and provides a better match between consumers' needs and product quality (Mann and Wissink, 1988, 1990; Heiman et al., 2001). Also, a lenient return policy will increase purchase rates and product return rates for customers in remote purchase environments (Wood, 2001). Thus, a critical challenge for the manager is to understand how a return policy, that allows refunds for any reason, affects the consumer purchase and return decisions.

From the manufacturer's perspective, it is their responsibility to manage the forward product flows to retailers and the reverse flows of the returned products. Guide and Van Wassenhove (2009) defined closed-loop supply chain (CLSC) management as: "The design, control, and operation of a system to maximize value creation over the entire life cycle of a product with dynamic recovery of value from different types and volumes of returns over time" (p.10). One important strategic decision in a firm's planning and operation is the design of such a CLSC network. This process involves determining the locations of the factories, warehouses and collection centers. The firm also must establish the capacity for each facility to deliver the product to consumers and collect the returned products. In a general sense, those facilities are expected to be in use for a relatively long time; thus, the manufacturer must

consider all the factors that might affect the network design decisions.

Besides the cost factors and the demand and return quantities, the manufacturer must also anticipate the impacts of possible environmental regulations on its operations. In recent years, concerns over global warming caused by the atmospheric concentration of carbon dioxide have increased. As the transportation sector is a main source of carbon emissions, it is not surprising that several countries are considering including the transportation sector in carbon emission regulations. For example, the United States, Japan and Canada are discussing cap-and-trade systems that would include the transportation sector. A carbon tax system is proposed by the Australia government. The EU Emissions Trading System (EU ETS) does not include road transport but is adding the aviation sector in 2012 (Flachsland et al., 2011). For a manufacturer, the actual costs that will result from carbon regulations are not clear because of the uncertainty about the type of policy that will be administered (carbon tax or cap-and-trade system), and its stringency.

Thus, a manufacturer that needs to design a CLSC network with heavy logistic activities should anticipate the possible cost due to the carbon regulations. Moreover, the retailer should understand the role of its return policy in consumers' purchasing and return behavior. Frequently, those two decisions are examined separately because manufacturers and retailers are responsible for the network design decision and the refund for the new product, respectively. However, if the manufacturer can control the return process via the return policy design, then integrating the return policy decision into the design process of the CLSC network might improve the overall performance. This situation occurs if the manufacturer owns retail stores (Wang et al., 2009). Besides the traditional retail channel, some manufacturers like Apple, Bally, Nike and Ralph Lauren sell the product to end consumers through company-owned stores. Moreover, outlet malls in the US consist of a substantial number of manufacturer-owned retail stores.

The purpose of this paper is to formulate and solve an integrated model for product return management and CLSC network design considering uncertain carbon costs. We first derive consumers' expected demands and returns as a result of the firm's return policy, the probability of receiving an unsatisfactory product and the consumers' product valuation. The network design decisions, including facility capacities and locations, occur in the initial period. In the following periods, the firm observes the demand and return quantities, and then determines the allocation of shipments to facilities to sat-

isfy demands and collect the returns. After this basic CLSC network design model is established, it is then extended to include uncertain carbon emission costs from potential regulations (e.g., carbon tax or cap-and-trade system). We assume that the firm has only estimates and ranges of the possible carbon costs in each period, and no other probability distribution information is available. We adopt the robust optimization (RO) approach developed by Ben-Tal and Nemirovski (1998, 1999, 2000), and represent the carbon cost uncertainty in each period by an ellipsoidal uncertainty set. The nonlinearity in the profit introduced by the refund decision is approximated as piecewise linear for tractable and accurate solution.

We compare the robust optimization model with a basic model, where no carbon cost is considered, and a nominal data model in terms of the solutions they provide. In a numerical case study with carbon costs estimated from the EU ETS, the solutions of the basic model and the nominal data model are very similar because emissions account for only a small portion of the total cost. However, the RO solutions feature more forward supply chain facilities, smaller optimal refunds, and fewer collection centers for returns. Extensive parametric experiments are performed to study the impact of different parameters on the profit, the refund provided and the network topology. The optimal solutions must balance the tradeoffs between revenues, transportation costs, and emission-related operational costs. We find that introducing carbon regulation might result in an “environmental paradox”. Uncertainty in the carbon cost reduces the optimal value of the refund, which means more unwanted products might go to landfill instead of being recycled or reused.

This paper contributes to the literature by formulating an integrated model for product return management and CLSC network design subject to uncertain carbon cost. The extensive numerical results improve our understanding of the effects of various parameters on both problem aspects. In addition, it contributes to a solution method designed for models with quadratic terms as well as ellipsoidal uncertainty.

The remainder of the paper is organized as follows. In the next section, we review the relationship between our work and the existing literature. In section 3, expected demand and return quantities are derived as functions of the refund and a basic CLSC network design model is proposed. This basic model is then extended to include the uncertain carbon cost. The computational issues and piecewise linear approximation are discussed in section 3. We conduct extensive parametric analysis to illustrate

the impact of different parameters in section 4 and conclude the paper in section 5.

4.2 Literature Review

In this section, we briefly review some of the related streams of studies in product return management, supply network design and robust optimization. Marketing and economics research have confirmed that a return policy is very important in the product exchange process. Davis et al. (1995) determine conditions when the return policy will enhance the retailer's profit. They use an analytical model to help identify potential causes for variation among retailers' return policies (Davis et al., 1998). Wood (2001) finds that the return policy is very important in e-commerce and a lenient return policy will increase consumers' purchases as well as returns in that context. Anderson et al. (2009) quantify the value of returns by developing a model that incorporates consumers' order and return decisions. They illustrate that the firm must balance both demand and cost to identify the optimal return policy. A review paper that summarizes consumers' return behavior is provided by Petersen and Kumar (2009). They argue that return policy should be an important aspect of product management as it affects consumers' future purchase behavior. The role of a return policy is also studied extensively in the field of operations management, where scholars have investigated how to use a return policy to improve operations efficiency (Liu and Xiao, 2008; Shulman et al., 2009; Su, 2009; Ketzenberg and Zuidwijk, 2009). Among them, Mukhopadhyay and Setoputro (2004) examine the case where the price and return policy are decision variables. They use linear functions for the demand and return quantities as a result of the two decisions. Mukhopadhyay and Setoputro (2005) extend their analysis to explore the value of offering a refund for build-to-order products, which is enabled through the modular design.

There is a considerable amount of literature on network design problems. In most CLSC network design studies, the demands and returns are either deterministic or characterized by discrete scenarios that are independent of firm's decisions (Faccio et al., 2011; Easwaran and Üster, 2010). In contrast, we assume the firm can affect the expectations consumer demands and returns by designing the return policy. Our work is also related to studies (Chaabane et al., 2008; Diabat and Simchi-Levi, 2009; Ramudhin et al., 2008), in which the environmental parameters are taken into consideration. But those studies focus on the contract and subcontract activities within a forward-only supply chain network.

The robust optimization methodology was developed by El Ghaoui and Lebret (1997), El Ghaoui et al. (1998), and Ben-Tal and Nemirovski (1998, 1999, 2000). The goal of this approach is to provide solutions that are less conservative than the earlier worst-case solutions developed by Soyster (1973). We refer the reader to a textbook for comprehensively understanding the subject (Ben-Tal et al., 2009). In the area of network design, Pishvaei et al. (2011) proposed a robust optimization model for handling the inherent uncertainty of input data (customer demands and transportation costs) in a CLSC. Their model is a single stage robust optimization problem with box uncertainty, which can be converted to a equivalent mixed-integer linear program. Baron et al. (2011) consider a multi-period fixed-charge network location problem using robust optimization. They use box and ellipsoidal uncertainty sets to characterize the uncertainty in consumers' demand.

In a similar context, Gao and Ryan (2011b) address a multi-period capacitated CLSC network design problem subject to uncertainties in the demands and returns as well as the potential carbon emission regulations, in a similar setting to this paper. But here we focus on the situation where the manufacturer has the control over the return process and incorporates the refund decision into the network design problem. Thus, this paper combines product return management and supply chain network design, which are usually examined separately. Consumers' expected demands and returns are derived as a function of the firm's return policy, and then incorporated into the CLSC network design problem. In addition, we also consider the uncertain carbon cost caused by the possible carbon emission regulation, and formulate a RO model. To the best of our knowledge, this is the first research that simultaneously addresses all these aspects in the context of network design.

4.3 Problem Statement

We consider an organization that is responsible for designing a single product CLSC network to serve retailers at various locations, and is also responsible for designing the return policy for the retailers. The retailer must satisfy consumers' demand and accept returns in its geographic region. The locations of facilities for manufacturing new and recovering returned products are to be established. We assume that the firm opens separate warehouse and collection facilities for distributing new products and collecting returned products, respectively. The model can be extended easily to handle integrated

warehouse and collection/testing centers. In each period, the retailers' demands are satisfied by the warehouses, and returns arise due to buyer remorse, product malfunction and other reasons. The collection centers collect the returned products and then ship them to a factory for inspection and recovery. Furthermore, several transportation modes are available for the firm to accommodate the flows between facilities. Each mode has different cost and emission implications. We further assume that each mode has unlimited capacity. The generic network topology is illustrated in Figure 4.1.

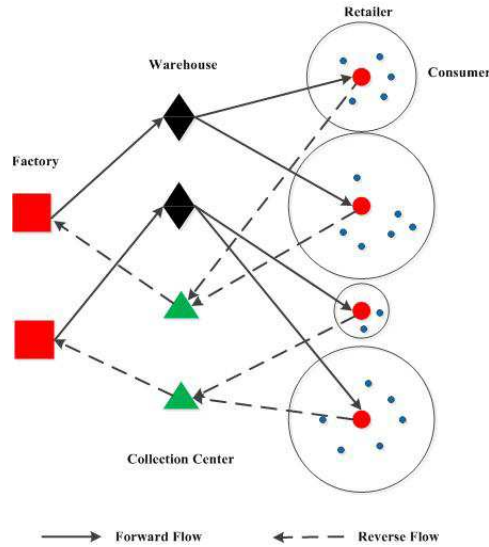


Figure 4.1 Closed-loop supply chain network structure

We will first derive consumers' expected demand and returns based on the firm's return policy, consumer valuations and the probability of receiving an unsatisfactory product. The return policy here is defined as the amount of refund provided to a consumer if he/she returns the product. We assume that the return policy will be fixed over the multi-period horizon, and there are no changes in the structure and capacities of the facilities after the first period. We initially formulate the problem without carbon emission costs as a mixed-integer quadratic program. We then extend this basic model to incorporate the uncertainty of carbon cost arising from either a tax or a cap-and-trade system. The following notations will be used throughout this paper.

Sets and Indices

\mathcal{E} set of potential factories for manufacturing new and recovering used products, $e \in \mathcal{E}$

\mathcal{W} set of potential warehouses for new product distribution, $w \in \mathcal{W}$

\mathcal{L} set of potential collection centers for returned products, $l \in \mathcal{L}$

\mathcal{H} set of retailer locations, $k \in \mathcal{H}$

\mathcal{M} set of transportation modes, $m \in \mathcal{M}$

\mathcal{T} set of time periods, $t \in \mathcal{T}$, $n = |\mathcal{T}|$

\mathcal{A} set of all the arcs in the network $\mathcal{A} \equiv \{ij : (i \in \mathcal{E}, j \in \mathcal{W}) \text{ or } (i \in \mathcal{W}, j \in \mathcal{H}) \text{ or } (i \in \mathcal{H}, j \in \mathcal{L}) \text{ or } (i \in \mathcal{L}, j \in \mathcal{E})\}$

\mathcal{F} set of potential facilities, $\mathcal{F} = \mathcal{E} \cup \mathcal{W} \cup \mathcal{L}$

\mathcal{N} set of all the nodes in the network, $\mathcal{N} = \mathcal{F} \cup \mathcal{H}$

General Parameters

v salvage value for the returned product (\$/unit)

p sale price of the new product (\$/unit)

β_{ij} the distance from node i to node j (km)

τ_e^m carbon emission factor for transportation mode m , $m \in \mathcal{M}$ (ton/unit-km)

τ_p the production emission intensity (ton/unit)

Cost Parameters

τ_c^m transportation cost factor for mode m , $m \in \mathcal{M}$ (\$/unit-km)

c_{ij}^m unit transportation cost from node i to node j using transportation mode m , $c_{ij}^m = \tau_c^m \beta_{ij}$, $ij \in \mathcal{A}$, $m \in \mathcal{M}$ (\$/unit)

f_i fixed investment cost for building facility, $i \in \mathcal{F}$ (\$)

K_0 cost per unit of capacity (\$/unit)

g unit production cost (\$/unit)

α_t carbon cost in period t , $t \in \mathcal{T}$ (\$/unit)

Consumer Profile Parameters

μ_{kt} total population of potential consumers in location k at period t , $k \in \mathcal{H}$, $t \in \mathcal{T}$

ϕ_k probability of “peach” product

s_k discounted value value of “lemon” product relative to value of “peach” product

Random Variables

Θ consumer’s valuation of the product, uniformly distributed on [0,1]

$$I_k \text{ product acceptability, } I_k = \begin{cases} 1 & \text{with probability } \phi_k \text{ ("peach")} \\ 0 & \text{with probability } 1 - \phi_k \text{ ("lemon")} \end{cases}$$

Decision Variables

x_{ijt}^m the amount of product transported from node i to node j using transportation mode m in period t , $ij \in \mathcal{A}, m \in \mathcal{M}, t \in \mathcal{T}$

$y_i = 1$ if facility i is built, 0 otherwise, $i \in \mathcal{F}$

Λ_i capacity of facility i , $i \in \mathcal{F}$ (units/period)

r the amount of refund for a returned product (\$/unit)

4.3.1 Consumer Demand and Return behavior

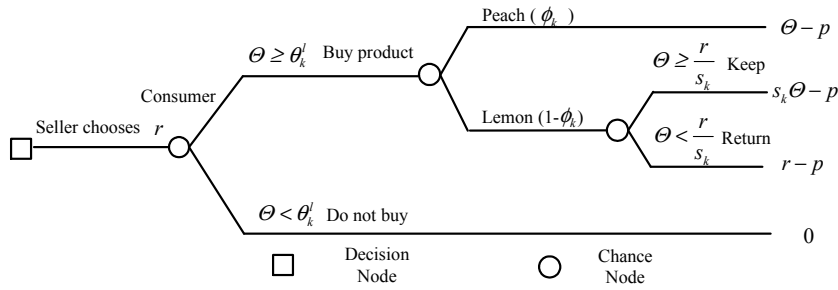


Figure 4.2 Sequence of events for individual consumers

In this paper, we formulate consumer demand and return behavior as follows. The firm decides on the refund for the product at the beginning of the study horizon. Then in each period, potential consumers at each location enter the market and decide whether to purchase the product based on maximizing their own surplus according to their realization of the valuation random variable, Θ . A binary random variable I_k represents whether the product turns out to be a good match (“peach”) or bad match (“lemon”) to the consumers’ expectations. The sequence of events is illustrated in Figure 4.2. Each consumer is interested in consuming at most one unit of the product and consumer valuations of the product Θ are uniformly distributed on $[0,1]$. The price for the product is p , which is exogenously determined. A consumer with valuation θ decides whether to return the product or not after receiving and trying it. If it is a peach, they will keep it and obtain utility $\theta - p$. If the product is a lemon, and the consumer keeps the it, they will obtain utility $s_k\theta - p$, where $s_k < 1, \forall k \in \mathcal{K}$. If they decide to return

the product, they will obtain utility $r - p$, where the r is the refund provided by the firm. Assuming I_k and Θ are independent, the utility a consumer at location k will derive from purchasing the product can be stated as follows:

$$U_k(r; \Theta, I_k) = \begin{cases} \Theta - p, & \text{if } I_k = 1 \\ -p + \max[s_k \Theta, r], & \text{if } I_k = 0 \end{cases} \quad \forall k \in \mathcal{K} \quad (4.1)$$

The expected utility for consumers at location k before purchasing the product can be stated as follows:

$$\hat{U}_k^b(r; \Theta)_k = \phi_k(\Theta - p) + (1 - \phi_k)(-p + \max[s_k \Theta, r]), \quad \forall k \in \mathcal{K} \quad (4.2)$$

Consumers with valuation equal to $\hat{\theta}_k = \frac{r}{s_k}$ will be indifferent between returning or keeping the lemon products. Consumers with valuation higher than $\hat{\theta}_k$ will keep the lemon products while consumers with valuation lower than $\hat{\theta}_k$ will return them. Thus, $\hat{U}_k^b(r; \Theta)$ can be further stated as follows:

$$\hat{U}_k^b(r; \Theta) = \begin{cases} \phi_k(\Theta - p) + (1 - \phi_k)(s_k \Theta - p), & \Theta \geq \hat{\theta}_k \\ \phi_k(\Theta - p) + (1 - \phi_k)(r - p), & \Theta < \hat{\theta}_k \end{cases} \quad (4.3)$$

A consumer with $\Theta = \theta$ buys and keeps the lemon product if $\phi_k(\theta - p) + (1 - \phi_k)(s_k \theta - p) \geq 0$ and $\theta \geq \hat{\theta}_k$. By simplifying the first inequality, a consumer with valuation greater than $\max[\hat{\theta}_k, \frac{p}{\phi_k + (1 - \phi_k)s_k}]$ will buy and keep the lemon products.

Similarly, the consumer will buy and then return the lemon products only when $\phi_k(\theta - p) + (1 - \phi_k)(r - p) \geq 0$ and $\theta < \hat{\theta}_k$. By simplifying the first inequality, consumers with valuation between $\theta_k^l = \frac{p - (1 - \phi_k)r}{\phi_k}$ and $\hat{\theta}_k$ will buy the product and return lemons.

To ensure the possibility of product returns, we assume that the firm sets r so that $\theta_k^l \leq \hat{\theta}_k$; thus, $r \geq \frac{ps_k}{\phi_k + s_k(1 - \phi_k)}, \forall k \in \mathcal{K}$. This also implies that $\frac{p}{\phi_k + s_k(1 - \phi_k)} \leq \hat{\theta}_k$, so that a consumer with valuation $\theta \geq \hat{\theta}_k$ will buy and keep the lemon product. Thus, there will be only three types of consumers in the market: (1) consumers with valuation $\theta \leq \theta_k^l$ will not buy any products; (2) consumers with valuation $\theta_k^l \leq \theta \leq \hat{\theta}_k$ will buy and return the product if it is a lemon; (3) consumers with valuation $\theta \geq \hat{\theta}_k$ will buy and keep the product, whether it is lemon or not. We also constraint $r \leq s_k, \forall k$ which implies that $\hat{\theta}_k \leq 1$. Two extreme cases will be: (1) $\theta_k^l = \hat{\theta}_k$, where all the consumers who receive lemons keep them;

(2) $\hat{\theta}_k = 1$, where all lemons are returned. The larger the r , the more lenient the return policy is. Also, we should note that $\theta_k^l \geq 0$ is implied by $r \leq p$ and $0 \leq \phi_k \leq 1$.

Therefore, the expected demands of consumers at location k in period t can be computed as follows:

$$D_{kt}(r) = (1 - \theta_k^l)\mu_{kt} = \left(1 - \frac{p}{\phi_k} + \frac{(1 - \phi_k)r}{\phi_k}\right)\mu_{kt}, \forall k \in \mathcal{K}, t \in \mathcal{T}. \quad (4.4)$$

The expected amount of returns from location k in period t is:

$$R_{kt}(r) = (1 - \phi_k)(\hat{\theta}_k - \theta_k^l)\mu_{kt} = (1 - \phi_k)\left(\frac{(\phi_k + s_k(1 - \phi_k))r}{\phi_k s_k} - \frac{p}{\phi_k}\right)\mu_{kt}, \forall k \in \mathcal{K}, t \in \mathcal{T}. \quad (4.5)$$

4.3.2 Basic Model Formulation

The mixed-integer quadratic programming model without carbon cost can be formulated as follows:

$$\begin{aligned} \text{Basic model: } \max_{x, y, r, \Lambda} & (p - g) \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} D_{kt}(r) + (v - r) \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} R_{kt}(r) \\ & - \sum_{i \in \mathcal{F}} (f_i y_i + C_0 \Lambda_i) - \sum_{i \in \mathcal{T}} \sum_{m \in \mathcal{M}} \sum_{j \in \mathcal{A}} c_{ij}^m x_{ijt}^m \end{aligned} \quad (4.6)$$

$$\text{s.t. } \sum_{w \in \mathcal{W}} \sum_{m \in \mathcal{M}} x_{wkt}^m - D_{kt}(r) = 0, \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \quad (4.7)$$

$$\sum_{l \in \mathcal{L}} \sum_{m \in \mathcal{M}} x_{klt}^m - R_{kt}(r) = 0, \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \quad (4.8)$$

$$\sum_{i \in \mathcal{N}} \sum_{m \in \mathcal{M}} x_{ijt}^m - \sum_{j \in \mathcal{N}} \sum_{m \in \mathcal{M}} x_{jit}^m = 0, \forall j \in \mathcal{W} \cup \mathcal{L}, \forall t \in \mathcal{T} \quad (4.9)$$

$$\sum_{j \in \mathcal{N}} \sum_{m \in \mathcal{M}} x_{ijt}^m - B y_i \leq 0, \forall i \in \mathcal{E}, \forall t \in \mathcal{T} \quad (4.10)$$

$$\sum_{j \in \mathcal{N}} \sum_{m \in \mathcal{M}} x_{ijt}^m - \Lambda_i \leq 0, \forall i \in \mathcal{E}, \forall t \in \mathcal{T} \quad (4.11)$$

$$r \geq \frac{ps_k}{\phi_k + s_k(1 - \phi_k)}, \forall k \in \mathcal{K} \quad (4.12)$$

$$r \leq s_k, \forall k \in \mathcal{K} \quad (4.13)$$

$$y \in \{0, 1\}^{|\mathcal{F}|}, x \in \mathbb{R}_+^{|\mathcal{A}| \times |\mathcal{M}|} \quad (4.14)$$

The objective is to maximize the long-run profit after subtracting the total cost of current investment as well as expected future production and operating costs. The first term in the objective function is the revenue from selling new products while the second term is the loss from consumer returns. Constraints (4.7) and (4.8) ensure that expected customer demands are met and returned products are collected. Constraints (4.9) ensure that the warehouse and collection centers will not accumulate stocks. Constraints (4.10) enforce capacity limits of the processing nodes, where B is a sufficiently large number.

The capacity constraint (4.11) requires that the total quantity of all products flowing from a processing node i is smaller than the capacity Λ_i of facility i if it is built ($y_i = 1$). If facility i is not built ($y_i = 0$) the constraint will force all flow variables $x_{ijt}^m = 0$ for all $j \in \mathcal{N}, t \in \mathcal{T}$. Constraints (4.12) and (4.13) are equivalent to $\theta_k^l \leq \hat{\theta}_k$ and $\hat{\theta}_k \leq 1$, respectively.

4.3.3 Carbon Cost Model

We will consider the carbon emissions from production as well as transportation. The total carbon emissions Γ_t (ton) from transportation and production in period t can be computed as:

$$\Gamma_t(x) = \sum_{ij \in \mathcal{A}} \beta_{ij} \sum_{m \in \mathcal{M}} \tau^m x_{ijt}^m + \tau_p \sum_{i \in \mathcal{E}} \sum_{j \in \mathcal{W}} \sum_{m \in \mathcal{M}} x_{ijt}^m, \forall t \in \mathcal{T} \quad (4.15)$$

Assume the firm has to pay α_t for each ton of carbon emitted in period t . Then the basic model can be extended to incorporate the resulting carbon cost, which is stated as follows.

$$\begin{aligned} \text{Carbon cost model: } \max_{x,y,r,\Lambda} & (p-g) \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} D_{kt}(r) + (v-r) \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} R_{kt}(r) - \sum_{i \in \mathcal{F}} f_i y_i \\ & - \sum_{t \in \mathcal{T}} \sum_{m \in \mathcal{M}} \sum_{ij \in \mathcal{A}} c_{ij}^m x_{ijt}^m - \sum_{t \in \mathcal{T}} \alpha_t \Gamma_t(x) \end{aligned} \quad (4.16)$$

subject to constraints (4.7) – (4.15).

The essence of the market mechanisms for regulating the carbon emissions is to put a price on emission. This model can accommodate certain features of proposed market mechanisms of carbon emission regulation. For a carbon tax regime, the α_t could represent the carbon tax imposed by the regulatory party. For a carbon cap-and-trade system, Gao and Ryan (2011b) demonstrate that the model is also applicable to the firm who only sells and buys the permits (i.e., no banking of permits is involved) to maintain the firm's actual emissions below the imposed cap.

4.3.4 Carbon Cost Model under Ellipsoidal Uncertainty Set

The objective function (4.16) is actually a concave quadratic function of r , as seen below:

$$\begin{aligned} \max & r \left((p-g) \sum_{k,t} \frac{(1-\phi_k)\mu_{kt}}{\phi_k} + v \sum_{k,t} (1-\phi_k) \frac{(\phi_k+s_k(1-\phi_k))\mu_{kt}}{\phi_k s_k} \right) \\ & + r \left(p \sum_{k,t} (1-\phi_k) \frac{\mu_{kt}}{\phi_k} \right) + (p-g) \sum_{k,t} \left(1 - \frac{p}{\phi_k} \right) \mu_{kt} - v \sum_{k,t} (1-\phi_k) \frac{p\mu_{kt}}{\phi_k} \\ & - r^2 \sum_{k,t} (1-\phi_k) \frac{(\phi_k+s_k(1-\phi_k))\mu_{kt}}{\phi_k s_k} - \sum_{i \in \mathcal{F}} f_i y_i - \sum_{t \in \mathcal{T}} \sum_{m \in \mathcal{M}} \sum_{ij \in \mathcal{A}} x_{ijt}^m - \sum_{t \in \mathcal{T}} (\alpha_t \Gamma_t(x)) \end{aligned} \quad (4.17)$$

For the sake of simplicity, we introduce constants Δ , Ω and Φ where:

$$\Delta = \sum_{k,t} \frac{(\phi_k + s_k(1-\phi_k))\mu_{kt}}{\phi_k s_k} (1 - \phi_k) \quad (4.18)$$

$$\Omega = (p - g) \sum_{k,t} \frac{(1-\phi_k)\mu_{kt}}{\phi_k} + v \sum_{k,t} (1 - \phi_k) \frac{(\phi_k + s_k(1-\phi_k))\mu_{kt}}{\phi_k s_k} + p \sum_{k,t} (1 - \phi_k) \frac{\mu_{kt}}{\phi_k} \quad (4.19)$$

$$\Phi = (p - g) \sum_{k,t} (1 - \frac{p}{\phi_k}) \mu_{kt} - v \sum_{k,t} (1 - \phi_k) \frac{p\mu_{kt}}{\phi_k} \quad (4.20)$$

We introduce another variable z to represent objective function (4.16). The carbon cost model can be stated as follows.

$$\max_{x,y,r,\Lambda} z \quad (4.21)$$

$$\text{s.t. } \alpha^T \Gamma(x) \leq -z - \Delta r^2 + \Omega r - f^T y - C_0 \Lambda - c^T x + \Phi, \forall \alpha \in \mathcal{U} \quad (4.22)$$

along with constraints (4.7) – (4.15). The vectors α , f , and c correspond to carbon costs, fixed opening costs and transportation costs, respectively. Also, the vector y represents all binary decision variables, and the vector x represents all flow variables. $\Gamma(x)$ represents the vector of carbon emissions in each period.

The parameter α is highly uncertain. We assume that the only carbon cost data available to the decision maker are: (1) the nominal values $\bar{\alpha}_t$; and (2) the possible deviations δ_t where $\delta_t < \bar{\alpha}_t$. This means that α_t may vary within the range $[\bar{\alpha}_t \pm \delta_t]$. Define X as a set of (x, y, r, Λ) that satisfy constraints (4.7) – (4.15). As the carbon cost parameter α is uncertain, the decision maker might be interested in a set of solutions $X(\epsilon) \in X$ that satisfies the constraint (4.22) with probability at least $1 - \epsilon$. We can define the set $X(\epsilon)$ by using the following chance constraint.

$$X(\epsilon) = \{(x, y, r, \Lambda) \in X : Pr[\alpha^T \Gamma(x) \leq -z - \Delta r^2 + \Omega r - f^T y - C_0 \Lambda - c^T x + \Phi] \geq 1 - \epsilon\} \quad (4.23)$$

Instead of solving in terms of the chance constraint directly, robust optimization presents a different approach. Given a set of solutions X , we define an uncertainty set \mathcal{U}_ρ and a set of robust feasible solutions as follows.

$$X_r(\rho) = \{(x, y, r, \Lambda) \in X : \alpha^T \Gamma(x) \leq -z - \Delta r^2 + \Omega r - f^T y - C_0 \Lambda - c^T x + \Phi, \forall \alpha \in \mathcal{U}_\rho\} \quad (4.24)$$

The parameter ρ , referred to as the budget of uncertainty, controls the size of the uncertainty set \mathcal{U}_ρ . The key point of robust optimization is designing \mathcal{U}_ρ such that $X_r(\rho) \subseteq X(\epsilon)$. The first requirement for

the resulting uncertainty set is to preserve the computational tractability. Moreover, the choice of the parameter ρ should not be too conservative; i.e., we want to find the minimum value of ρ such that the robust solution satisfies the chance constraint (4.23). One way to design such an uncertainty set is to use an ellipsoid:

$$\mathcal{U}_{ellips} = \{\alpha \in \mathbb{R}^n : \sum_{t=1}^n \delta_t^{-2} (\alpha_t - \bar{\alpha}_t)^2 \leq \rho^2\} \quad (4.25)$$

where ρ controls the size of the ellipsoidal set. Applying the Cauchy–Schwarz inequality as in Gao and Ryan (2011b), the equivalent robust counterpart of the above problem can be stated as follows:

$$\text{Robust carbon cost model: } \max_{z,y,x,\Lambda} z \quad (4.26)$$

$$\text{s.t. } \bar{\alpha}^T \Gamma(x) + \rho \sqrt{\sum_{t=1}^n (\delta_t \Gamma_t(x))^2} \leq -z - \Delta r^2 + \Omega r - f^T y - C_0 \Lambda - c^T x + \Phi \quad (4.27)$$

along with constraints (4.7) – (4.15).

The value of ρ depends on the decision maker’s conservatism. As shown in Gao and Ryan (2011b), when $\rho = 2.686$ and $n = 6$, a feasible solution to constraint (4.27) satisfies constraint (4.23) with 99% feasibility ($\epsilon = 0.01$). If $\rho = 0$, we have the *nominal carbon cost model*, which can be formulated as follows.

$$\text{Nominal carbon cost model: } \max_{r,y,x,\Lambda} -\Delta r^2 + \Omega r + \Phi - f^T y - C_0 \Lambda - c^T x - \bar{\alpha}^T \Gamma(x) \quad (4.28)$$

subject to constraints (4.7) – (4.15).

The major computational difficulty lies in dealing with constraint (4.27), which has both the l^2 -norm of the decision variable vector and a quadratic term in r . The available solvers for Second-Order Cone Problems (SOCP) require the right-hand side of constraint (4.27) to be linear. In this paper, we will use a piecewise linear function to approximate the quadratic function $Q(r) \equiv -\Delta r^2 + \Omega r + \Phi$. Before we proceed to the piecewise linear approximation, we conjecture that the optimal refunds for the three models have the following relationship.

Conjecture 1. *Under the same settings for cost, revenue and consumer profile parameters, the optimal refund of the basic model, nominal data model and robust optimization model r_B^* , r_N^* and r_R^* satisfy:*

$$r_B^* \geq r_N^* \geq r_R^*$$

This is matched by the following observations. The objective function of the basic model is $\max_{r,x,y,\Lambda} Z(x,y,r,\Lambda)$, where $Z(x,y,r,\Lambda) = -\Delta r^2 + \Omega r + \Phi - f^T y - C_0 \Lambda - c^T x$. The objective function

of the nominal data model is $\max_{r,x,y,\Lambda} Z(x,y,r,\Lambda) - \bar{\alpha}^T \Gamma(x)$. The objective function of the RO model is $\max_{r,x,y,\Lambda} Z(x,y,r,\Lambda) - \bar{\alpha}^T \Gamma(x) - \rho \sqrt{\sum_{t=1}^n (\delta_t \Gamma_t(x))^2}$. The expected demand and returns function are increasing in r . Thus, if a “no return” policy is optimal for the basic model, then providing a generous refund in the nominal and RO model will not increase the profit but incur an additional cost. Thus, the ‘no return’ policy will be optimal for the nominal data model and robust optimization model. Likewise, if a “no return” policy is optimal for the nominal data model, then it should be optimal for the robust optimization model. This conjecture is also supported by the results of the computational experiments shown in section 4.4.

Conjecture 1 suggests possible unintended side effect of carbon emission regulation. Introducing regulation to limit carbon emissions will reduce the firm’s willingness to provide as high a level of product refund as it does when no carbon regulation is implemented. Potentially, it might result in less recycling/remanufacturing and more landfilling of products that do not meet consumers’ expectations. Moreover, uncertainties in the carbon regulations exacerbate this effect.

4.3.5 Piecewise Linear Approximation for the Second Order Cone Constraint

Define $L_r = \max_{\forall k \in \mathcal{K}} (\frac{ps_k}{\phi_k + s_k})$ and $U_r = \min_{\forall k \in \mathcal{K}} (s_k)$. The concave function $Q(r)$ is approximated by first introducing a number N of sampling coordinates r_1, \dots, r_N with $r_1 = L_r$ and $r_N = U_r$. The function is then approximated by an $N - 1$ segment continuous piecewise linear function $h_i(r)$ defined over the range $[r_i, r_{i+1}]$, $i = 1, \dots, N - 1$, where $Q(r) \approx \sum_{i=1}^{N-1} h_i(r)$ (see Figure 4.3). For any r with $r_i \leq r \leq r_{i+1}$, $h_i(r)$ can be defined by the convex combination of $g(r_i)$ and $g(r_{i+1})$. Let $\lambda \in [0, 1]$ and $r = \lambda r_i + (1 - \lambda)r_{i+1}$. The $h_i(r)$ can be evaluated by :

$$h_i(r) = \begin{cases} \lambda g(r_i) + (1 - \lambda)g(r_{i+1}), & \text{for } r_i \leq r \leq r_{i+1} \\ 0, & \text{otherwise} \end{cases} \quad (4.29)$$

where $\lambda = \frac{r_{i+1} - r}{r_{i+1} - r_i}$. To obtain a tractable form of $h_i(r)$, we introduce a binary variable b_i associated with the i th interval where $b_i = 1$ if $r_i \leq r \leq r_{i+1}$ and $b_i = 0$ otherwise ($i = 1, \dots, N$), with $b_0 = b_N = 0$. We also introduce another continuous variable a_i for each coordinate where $a_i \in [0, 1]$, $i = 1, \dots, N$.

Then the $h_i(r)$ can be evaluated by following constraints:

$$\sum_{i=1}^{N-1} b_i = 1 \quad (4.30)$$

$$a_i \leq b_{i-1} + b_i, i = 1, \dots, N \quad (4.31)$$

$$\sum_{i=1}^N a_i = 1 \quad (4.32)$$

$$r = \sum_{i=1}^N a_i r_i \quad (4.33)$$

$$h_i(r) = \sum_{i=1}^N a_i g(r_i) \quad (4.34)$$

$$0 \leq a_i \leq 1, b_i = 0 \text{ or } 1, i = 1, \dots, n \quad (4.35)$$

By replacing $Q(r)$ with $\sum_{i=0}^{N-1} h_i(r)$ and appending constraints (4.30) - (4.35) to the other constraints in the robust model, we obtain a tractable robust counterpart of the problem.

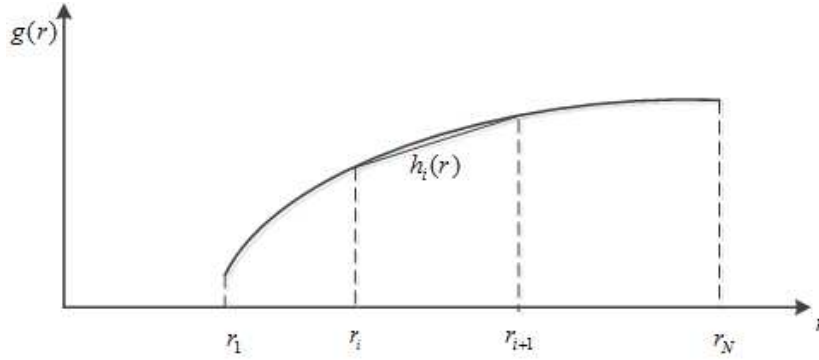


Figure 4.3 Piecewise linear approximations for $Q(r)$

4.4 Numerical Study

The purpose of this numerical study is threefold: first, to test the effectiveness of the piecewise linear function approximation method; second, to investigate how solutions provided by the basic model, the nominal data model and the RO model differ in the network topology, the refund provided and the profit; and third, to study the impact of parameters on solutions provided by the RO model. Before the presentation of results, we describe the details of the experiments.

4.4.1 Description of Base-case

For the base-case, there are 25 retail stores in the network. We consider 8, 10 and 6 potential locations for factories, warehouses and collection centers, respectively. Locations for retail stores and candidate facilities are randomly generated in a 5000×5000 kilometer square. There are 6 time periods. We use the cost, revenue and consumer profile parameters summarized in Tables 4.1 and 4.2. The fixed opening costs are based on data from Fleischmann et al. (2001) and Salema et al. (2007). The carbon emission factors are based on data from Pirog et al. (2001) and the cost factor is based on the data from Byrne et al. (2006). We let $\bar{\alpha}_t = \$0.15$ and $\delta_t = \$0.05$ ($\forall t$), which are based on the data from Benz and Trück (2009). As consumer valuation is normalized between 0 and 1, the values for other parameters are normalized as well. The production emission intensity $\tau^p = 0.0001$ (ton/unit). We set $\rho = 1.564$ for the ellipsoidal uncertainty set (0.8 feasibility guarantee). The basic model, the nominal data model and the RO model are then solved by the GAMS CPLEX MIQCP solver. All the experiments were solved by a PC with an Intel Core Quad 3.00 GHz processor. For each model, the optimal solution of one instance can be obtained within 30 seconds. Thus, computation time is not an issue.

Table 4.1 Parameter values in the base-case

Parameter	Value
$v, p, g, C_0(\$)$	0.6, 0.8, 0.3, 0.01
$f_p, f_w, f_c(\$)$	37500, 4500, 2250
μ_{kt}	15000 ($\forall k, t$)
ϕ_k, s_k	0.8, 0.7 ($\forall k$)

Table 4.2 Emission and cost factors of transportation modes in the base-case

Transport Mode	CO_2 Emissions Factor (τ_e^m) (ton/unit-km)	Cost (τ_c^m) (\$/unit-km)
H	0.60×10^{-6}	1.50×10^{-4}
M	1.20×10^{-6}	1.13×10^{-4}
L	2.40×10^{-6}	3.75×10^{-5}

4.4.2 Effectiveness of the Piecewise Linear Function Approximation

We assume for the piecewise linear function all segments have an equal width. The number of segments $N - 1$ depends on the desired approximation error η , where η can be stated as follows:

$$\eta = \max\{ \max_{r_i \leq r \leq r_{i+1}} |h_i(r) - Q(r)| \} \quad (4.36)$$

Based on theorem 2 in Frenzen et al. (2010), the number of segments needed for the approximation error η can be estimated by:

$$N \sim \frac{(U_r - L_r) \sqrt{2\Delta}}{4\sqrt{\eta}} + 1 \quad (4.37)$$

We calculate the maximum N needed for all parametric experiments. As $\Delta > 0$ implies $Q(r)$ is a concave function, the minimum value of $Q(r)$ will be the value at either U_r or L_r . Given the approximation error $\eta = 10$, which is 0.000612% of the minimum value of $Q(r)$ under the base-case, 56 sampling coordinates will be needed. We also compare the results with approximation error $\eta = 4$ with 88 sampling coordinates. To do so, 30 random location profiles are generated. We use N_F , N_W , N_C , respectively, to denote the average numbers of factories, warehouses and collection centers opened and $\bar{\Lambda}_F$, $\bar{\Lambda}_W$, $\bar{\Lambda}_C$, respectively (in units of 1000), to denote the average capacities. The symbols of $\#_{FW}$ and $\#_{WR}$ represent the average number of arcs per factory with positive flow between the factory and the warehouse, and the average number of arcs per warehouse with positive flow between the warehouse and the retail store, respectively. We use $\#_{RCF}$ to represent the average number of arcs per collection center with positive flow between either the collection center and a retail store or the collection center and a factory. The values of $\#_{FW}$ and $\#_{RCF}$ are computed as follows, with $\#_{WR}$ calculated in a similar manner as $\#_{FW}$.

$$\#_{FW} = \frac{\sum_{i \in \mathcal{E}} \sum_{j \in \mathcal{W}} \sum_{m \in \mathcal{M}} \mathbf{1}_{[\sum_{i \in \mathcal{T}} x_{ij}^m > 0]}}{\sum_{i \in \mathcal{E}} y_i}$$

$$\#_{RCF} = \frac{\sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{L}} \sum_{m \in \mathcal{M}} \mathbf{1}_{[\sum_{i \in \mathcal{T}} x_{ij}^m > 0]} + \sum_{i \in \mathcal{L}} \sum_{j \in \mathcal{E}} \sum_{m \in \mathcal{M}} \mathbf{1}_{[\sum_{i \in \mathcal{T}} x_{ij}^m > 0]}}{\sum_{i \in \mathcal{L}} y_i}$$

We also calculate the average profit (Π^* , in \$1000) and the average optimal refund (r^*) for the solutions provided by three different models. The 95% confidence interval for the two different ap-

proximation errors are reported in Table 4.3. Because the results were nearly identical, 56 sampling coordinates were used for all the computational experiments.

Table 4.3 95% confidence intervals of results for the robust carbon cost model under different approximation errors

N	N_F	N_W	N_C	$\bar{\Lambda}_F$	$\bar{\Lambda}_W$	$\bar{\Lambda}_C$	$\#_{FW}$	$\#_{WR}$	$\#_{RCF}$	r^*	Π^*
88	2.73 ± 0.23	4.03 ± 0.36	1.40 ± 0.51	226.50 ± 27.33	153.00 ± 15.87	9.90 ± 3.67	1.54 ± 0.15	6.70 ± 0.71	6.16 ± 2.38	0.62 ± 0.01	1184.80 ± 22.45
56	2.73 ± 0.23	4.03 ± 0.36	1.40 ± 0.51	226.55 ± 27.33	153.03 ± 15.86	9.99 ± 3.72	1.54 ± 0.15	6.75 ± 0.71	6.16 ± 2.38	0.62 ± 0.01	1184.80 ± 22.45

4.4.3 Topology Comparison

In this section, we compare how the number of facilities opened, the average capacity of facilities, and transportation links between nodes differ in the solutions provided by the three different models. We report the 95% confidence intervals for the number of facilities, the capacities and the number of arcs between facilities in Table 4.4 for the basic and nominal models. In addition, the utilization rate of each transportation mode MR_m is calculated. For example, the utilization rate of transportation mode L can be computed as follows. MR_M and MR_H are defined in a similar way.

$$MR_L = \frac{\sum_{ij \in \mathcal{A}} \sum_{t \in \mathcal{T}} \mathbf{1}_{[x_{ijt}^L > 0]}}{\sum_{ij \in \mathcal{A}} \sum_{t \in \mathcal{T}} \sum_{m \in \mathcal{M}} \mathbf{1}_{[x_{ijt}^m > 0]}}$$

Figure 4.4 shows potential locations of one instance. Solutions of different models for this instance are presented in Figures 4.5 to 4.7. In the figure, an arc indicates that in at least one period products are shipped between two nodes. The forward flow is represented by a solid line while the reverse flow is represented by a dashed line. For the base-case, solutions provided by the nominal data model and the basic model are very similar, and they have the nearly same network configuration for all 30 instances. Figures 4.5 and 4.6 present one instance that has the same configurations. This is because the carbon cost accounts for a small portion of the total costs. The only differences are in the total profit, which decreases by 3417.92 ± 118.28 .

By making paired comparisons with the basic model and calculating the confidence interval, results

obtained via the RO model open more facilities for the forward flow but fewer facilities for the reverse flow. Compared to Table 4.3 with $N = 56$, the average number of factories opened increases by 0.43 ± 0.24 in the robust model. The average number of warehouses opened increases by 0.9 ± 0.43 , but the average number of collection centers opened decreases by 0.97 ± 0.50 . The average facility capacities is smaller and the total profit is lower under the RO model. The average capacities of the factories, warehouses and collection centers decrease by 86.53 ± 37.18 , 80.41 ± 24.87 and 50.70 ± 7.20 thousand units, respectively. The total profit decreases by 16.94% comparing to the basic model. Under the RO model, the firm wants to reduce the carbon cost. Because the carbon emissions are proportional to the distance traveled and the quantities of demands and return, the total carbon emissions will be reduced by building more facilities and providing a lower refund (which also results in smaller capacities).

Transportation mode L is favored almost exclusively (100% for the basic and the nominal data model and 98.88% for the RO model) because of its lowest cost. This is also a common occurrence in most of the following experiments; thus, we only report the transportation utilization rate when the pattern is significantly different.

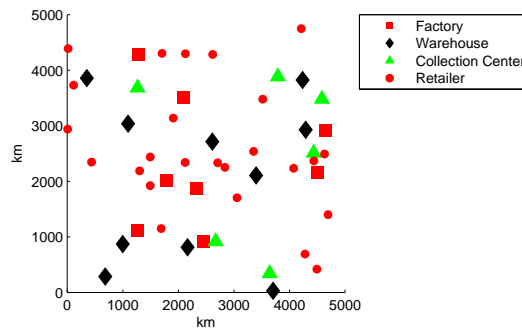


Figure 4.4 Potential locations of facilities

4.4.4 The Parametric Experiments

The computational experiments are extended to provide a better understanding of the solutions found by the robust optimization model. In this section, we explore how changes in the parameters can affect the optimal refund offered, profit and network topology. As our experiments involve solving a

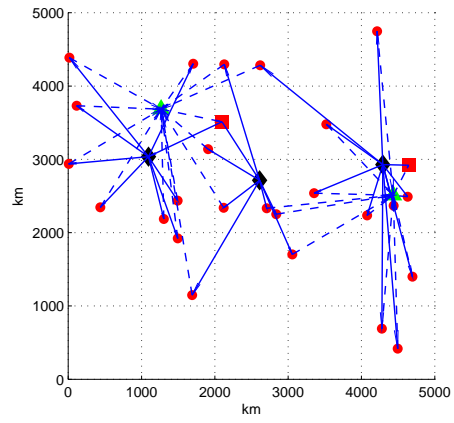


Figure 4.5 Network configuration of the solution of the basic model

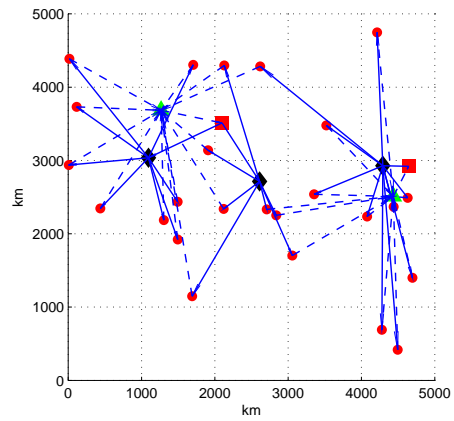


Figure 4.6 Network configuration of the solution of the nominal data model

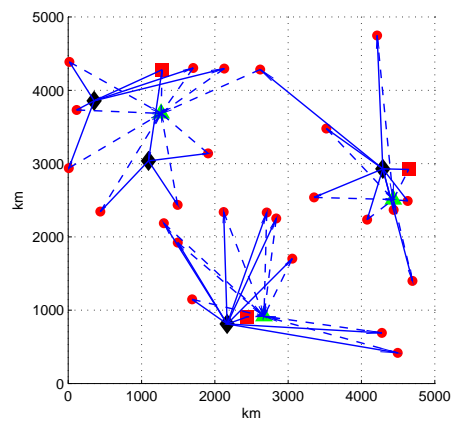


Figure 4.7 Network configuration of the solution of the RO model

Table 4.4 95% confidence intervals of results for the base-case from the basic and nominal models

Model	N_F	N_W	N_C	$\bar{\Lambda}_F$	$\bar{\Lambda}_W$	$\bar{\Lambda}_C$	$\#_{FW}$	$\#_{WR}$	$\#_{RCF}$	r^*	Π^*
Basic	2.30	3.13	2.37	313.03	233.40	60.60	1.39	8.89	12.60	0.70	1426.50
	± 0.23	± 0.41	± 0.24	± 38.88	± 26.61	± 7.93	± 0.18	± 1.01	± 1.51	± 0.00	± 11.67
Nominal	2.30	3.13	2.37	313.00	231.56	60.56	1.39	8.83	12.60	0.70	1423.10
	± 0.23	± 0.41	± 0.24	± 38.85	± 26.99	± 7.93	± 0.18	± 1.03	± 1.51	± 0.00	± 11.77

MIQCP model with many parameters, a full factorial experiment would involve an unrealistic number of cases. Thus, we design parametric experiments in which the RO model is solved for three parameter levels (low, medium and high) for each generated location profile. For each location profile, we also make paired comparisons and use Δ_{ML} to denote the difference between the solutions from the low to the medium parameter levels, and Δ_{HM} to denote the difference between the solutions from the medium to the high parameter levels. The 95% confidence intervals for Δ_{ML} and Δ_{HM} are summarized in Table 4.5.

Experiment 1: impact of transportation emission factors. For the low level, we reduce the transportation emission factors for the three modes by 25% from the base case (medium) level and increase them by 25% for the high level. As a generous refund will create more demand and returns for the product, more carbon will be emitted from the transportation and the production. Therefore, smaller refunds will be provided as the emission factor level increases, and the firm tends to discourage both demand and returns to reduce the carbon cost. Shrinkage in demand and returns also leads to smaller average capacities for all the facilities, fewer collection centers and the lower profit. But the number of factories opened increases from the low to medium level and the number of warehouses opened increases from the medium to high level. Under the high emission factor level, mode L accounts for 95.31% of the total transportation modes. The portion of shipments by transportation mode L slightly decreases due to its high carbon emission rate.

Experiment 2: impact of transportation cost factors. For the low level, we reduce the transportation cost factors for the three modes by 25% from the base case (medium) level and increase them by 25% for the high level. A lower refund will be provided to discourage both demand and returns as the transportation cost factors increases. The reason is quite intuitive, as the transportation cost are pro-

portional to the quantity of products and the distance traveled. Thus, under the high transportation cost factor, the optimal solution will discourage returns and fewer collection centers with smaller average capacities are opened. The average capacities of the factories decrease from the low to medium level, and the average capacities of the warehouses decrease from the medium to high level.

Experiment 3: impact of fixed opening costs. For the low level, we reduce the fixed opening costs for the three facilities by 25% from the base case (medium) level and increase them by 25% for the high level. We find that the increase in the fixed opening costs has no impact on the optimal refund. Moreover, as fixed opening costs increase, fewer facilities will be opened, and the average capacities of the factories and the warehouses increase. The total profit decreases as the fixed opening costs increase.

Experiment 4: impact of the probability of receiving the peach product. For the low level, we reduce the the probability of receiving the peach product ($\forall k \in \mathcal{K}$) by 0.1 from the base case (medium) level and increase it by 0.1 for the high level. When ϕ_k is low, the product is less “attractive”. So providing a large amount of refund will be favorable. As ϕ_k increases, consumers are more likely to get a “peach” product and are less likely to return the product, which results in a higher profit and lower refund. At the same time, more warehouses are opened to handle the forward flow, and more factories are opened as the probability of receiving the peach product increases from the medium to the high level. The average capacities for the factories and warehouses also increase as the probability increases. The average capacity of the collection centers decreases from the low to medium level.

Experiment 5: impact of the salvage value. For the low level, we reduce the salvage value of the lemon product by 0.1 from the base case (medium) level and increase it by 0.1 for the high level. As the salvage value of the returned product increases, it is more favorable for the firm to receive a returned product. Thus, the amount of the optimal refund will be larger and the profit will be higher. Meanwhile, the average capacity, and the number of collection centers opened will increase to handle the surge in the reverse flow. The average capacities of the factories also increase.

Experiment 6: impact of the discounted value. For the low level, we reduce the discounted value ($\forall k \in \mathcal{K}$) by 0.1 from the base case (medium) level and increase them by 0.1 for the high level. As the discounted value increases, the total profit and the optimal refund increase. In addition, consumers are less likely to return the product, which results in fewer collection centers with smaller average capacities. The lower the valuation for the “lemon” product, the less likely it is that the consumer will

purchase the products. When $s_k = 0.6$, a full refund will be provided to increase the demand. Thus, more factories and warehouses are opened under the low s_k level.

Experiment 7: impact of the consumer population. For the low level, we reduce the consumer population for all locations and all periods ($\forall k \in \mathcal{K}, \forall t \in \mathcal{T}$) by 25% from the base case (medium) level and increase it by 25% for the high level. As the consumer population increases, the amounts of demand and returns also increase. As a result, more facilities for the forward flow will be opened, and the average facilities capacity as well as the total profit will increase. But a lower refund will be provided to reduce the potential loss from the returns. As a result, fewer collection centers will be utilized and their average capacities decrease.

Experiment 8: impact of the production cost. For the low level, we reduce the production cost by 0.1 from the base case (medium) level and increase it by 0.1 for the high level. As the production cost increases, the unit revenue from selling and the total profit both decrease. Thus, a lower refund will be provided to prevent excessive loss from consumer returns. At the same time, fewer collection centers will be utilized and the average capacities for the collection centers will decrease as the production cost increases. The average capacities of factories decrease from the medium to high level.

Table 4.5 95 % confidence interval of Δ_{ML} and Δ_{HM} for different parameters

Parameter	N_F	N_W	N_C	$\bar{\Lambda}_F$	$\bar{\Lambda}_W$	$\bar{\Lambda}_C$	r^*	Π^*
τ_e^m								
Δ_{ML}	0.13±0.12	0.13±0.23	-0.97±0.42	-22.48±9.95	-14.78±7.37	-19.33±4.11	-0.04±0.00	-81.42±4.00
Δ_{HM}	0.10±0.11	0.23±0.22	-1.30±0.51	-21.43±19.21	-14.95±9.86	-9.53±3.57	-0.02±0.01	-80.34±6.18
τ_c^m								
Δ_{ML}	0.03±0.07	-0.03±0.15	-0.60±0.35	-8.30±5.86	-3.62±3.20	-9.25±2.75	-0.02±0.00	-61.60±1.64
Δ_{HM}	0.03±0.07	0.03±0.07	-0.83±0.46	-13.42±18.05	-5.83±6.00	-6.77±3.05	-0.01±0.00	-58.18±2.00
f_i								
Δ_{ML}	-0.20±0.15	-0.43±0.26	-0.27±0.24	21.11±20.32	16.09±10.42	-1.11±1.30	0.00±0.00	-32.30±2.18
Δ_{HM}	-0.20±0.15	-0.27±0.21	-0.33±0.29	14.83±11.44	8.43±6.66	-0.42±1.44	0.00±0.00	-29.62±2.18
ϕ_k								
Δ_{ML}	0.16±0.13	0.20±0.26	-0.63±0.36	23.47±11.19	16.19±7.40	-10.41±3.21	-0.02±0.00	214.44±5.05
Δ_{HM}	0.1±0.11	0.30±0.25	0.23±0.43	18.26±18.78	8.78±12.09	-0.43±3.50	-0.01±0.00	194.27±6.75
ν								
Δ_{ML}	0.03±0.06	0.07±0.10	1.40±0.51	4.64±3.57	2.50±2.92	9.90±3.77	0.02±0.01	4.21±2.07
Δ_{HM}	0.10±0.11	0.10±0.14	1.43±0.44	5.73±0.88	4.82±6.01	16.32±3.38	0.04±0.01	31.10±6.02
s_k								
Δ_{ML}	-0.20±0.17	-0.23±0.22	-1.77±0.45	28.60±24.01	15.59±9.80	-26.06±5.70	0.02±0.01	142.58±7.38
Δ_{HM}	0.10±0.11	0.17±0.21	-1.17±0.50	3.51±18.90	3.59±9.68	-8.58±3.27	0.05±0.00	132.22±3.73
μ_{kt}								
Δ_{ML}	0.30±0.17	0.43±0.24	-0.37±0.36	28.59±17.84	22.75±9.17	-6.00±2.85	-0.02±0.00	292.03±7.68
Δ_{HM}	0.20±0.15	0.47±0.28	-0.53±0.45	25.10±19.78	13.83±11.00	-4.56±2.93	-0.01±0.00	279.94±9.01
g								
Δ_{ML}	-0.10±0.11	-0.07±0.13	-1.20±0.43	-2.49±8.74	-3.67±6.36	-13.14±3.16	-0.03±0.00	-355.39±4.36
Δ_{HM}	-0.03±0.07	-0.07±0.09	-1.40±0.51	-4.64±3.57	-2.50±2.92	-9.90±3.67	-0.02±0.01	-339.31±2.07

4.4.4.1 Summary of Parametric Experiment Results

Parameters vary in their impacts on the network topology, the refund provided and the profit. The impact of parameters on the RO model is summarized in Table 4.6. We use the signs + and – to represent significant increase and decrease in the value of both Δ_{ML} and Δ_{HM} , in response to a increase of the corresponding parameter respectively. The sign \pm indicates that there is no significant increase or decrease in both. We obtain several interesting observations. First, transportation emission factors are the only parameter that will affect the utilization rate of transportation modes. If the policy maker wants to change the utilization pattern of transportation modes under the incoming carbon emission regulation, it might achieve this goal only if the emission factor of current transportation modes is sufficiently high. Second, consumer population is the only factor that has significant impacts on the network topology, the refund provided and the profit. Surprisingly, the firm will provide a less generous refund as the overall consumer population increases. Third, lowering cost is always a wise way to improve profit, but improving product quality, represented by the portion of “peach” products, and the attractiveness as well as the salvage value of the “lemon” products can also help to improve the overall profit. Finally, some parameters have opposite effects on the forward and reverse channel of the CLSC. For example, a larger consumer population will result in fewer collection centers with small average capacities. But at the same time, more forward facilities are opened, and their average capacities are larger.

Table 4.6 Parameter impact on the RO model

Parameter	N_F	N_W	N_C	$\bar{\Lambda}_F$	$\bar{\Lambda}_W$	$\bar{\Lambda}_C$	r^*	Π^*
Transportation carbon emission factor	\pm	\pm	–	–	–	–	–	–
Transportation cost factor	\pm	\pm	–	\pm	\pm	–	–	–
Fixed opening cost	–	–	–	+	+	\pm	\pm	–
Probability of peach products	\pm	\pm	\pm	\pm	\pm	\pm	–	+
Salvage value of returned products	\pm	\pm	+	+	\pm	+	+	+
Consumer discounted valuation	\pm	\pm	–	\pm	\pm	–	+	+
Consumer population	+	+	–	+	+	–	–	+
Production cost	\pm	\pm	–	\pm	\pm	–	–	–

4.5 Conclusion

In this paper, we propose an integrated model for product return management and CLSC network design problem considering uncertain carbon cost. This integrated model has the potential to benefit the manufacturer who has control over the return process or owns the retailer stores. The proposed model can accommodate several features of practical relevance. For example, consumers have demand for new products and some of the unsatisfactory products will be returned, where both consumers' purchasing and return behavior will be affected by the refund offered. Meanwhile, the firm must design the network for distributing the new products and collecting the returned products. The decisions regarding the location of the factory, warehouse and collection center, capacity of each facility and flow routing through the network must be made within a multi-period framework.

In addition, the explicit consideration of carbon cost within the network design is crucial under the emergence of emission trading or carbon tax schemes. The proposed model also accommodates this feature by considering uncertainty in carbon cost. Utilizing the robust optimization approach to characterize such uncertainty, a model with both the l^2 -norm of the decision variable vector and a quadratic term is derived. We then propose a piecewise linear function approximation method to solve the model. Computational results illustrate the trade-offs in revenues, transportation cost and carbon cost in designing the network and the return policy. Different parameters vary in their impact on the refund provided, the profit and the network topology. For policy implications, we find that uncertainty in carbon regulation has the effect of reducing the optimal refund and thus, results in fewer returns. This might not benefit the environment as the products might go to landfill instead of being recycled or reused.

An important extension to the setting considered in this paper would be to include the price of the product as a decision variable. The pricing and refund decisions are naturally related in many practical planning decisions, as the decision maker has to balance the trade-offs in revenue and costs when the pricing decision is taken into consideration. However, the inclusion of price as a decision variable in the robust optimization framework will generate significant computational difficulties, as an approximation algorithm for a two-dimensional surface will be needed. Nevertheless, this is certainly an interesting and challenging research direction. In this paper, we also assume that the price and/or the refund is fixed,

it might be worthwhile to relax this assumption and allow the price and/or the refund to be changed with the study horizon. A second way to extend the current research is to consider alternative structures for the network. An integrated warehouse/collection center model might worth investigating. This is also a feature that exists in many practical reverse logistics planning contexts. It might be interesting to see how this integrated structure combined with uncertainty in carbon regulation would influence the network design. In this paper, only the expected value of demand and returns are used in the optimization model. An interesting way to extend this setting is to construct scenarios to approximate their distributions and formulate a stochastic program for the basic model. It is also interesting to study the expected demand and returns under other distributions for Θ . In addition, we consider carbon emission only from transportation and production in this paper, and it might be worthwhile to consider the carbon emissions from building and operating facilities as well.

CHAPTER 5. GENERAL CONCLUSION

The unifying theme of this dissertation is to provide insights in understanding product return management and CLSC network design in a comprehensive manner. It utilizes both empirical and quantitative study, and adopts both stochastic programming and robust optimization to handle the different types of uncertainties in the model. The managerial results derived from this dissertation will benefit different firms to gain a deeper understanding in product return management, CLSC network design under carbon policy uncertainty or the integration issues of both.

In this dissertation, we start with product return management. We focus on the case when a seller offers both new and refurbished products. Our empirical study indicates that there exist consumers who will never consider purchasing refurbished products and consumers who will only consider purchasing the refurbished version of a given model. But the majority of consumers will consider purchasing both products and have uncertainty concerning their valuation of refurbished electronics. Those tactical consumers want enough cost savings to compensate for the valuation uncertainty and seek various mechanisms to mitigate such unpredictability. We analyzed three different strategies for the seller and explored the optimal price and restocking fees in a numerical study. Optimality conditions and numerical analysis show that different combinations of price and restocking fees are favored under different conditions. We also show that the combination of low price and strict return policy might be favored under very special conditions. Moreover, parameters vary in their impacts on the variety decision and restocking fee decision. We show that by protecting the consumer against a lemon product, allowing returns enables the seller to serve the tactical consumers with refurbished products and charge a relatively high price even when their expected valuation is low. In this way, both the seller and the consumer benefit from allowing returns. The results also suggest that sellers should deliberately consider the market segment conditions, consumer valuations, and cost factors to choose the appropriate price and restocking fees for refurbished products.

Next, we study a closed-loop supply chain network design problem where the demands and returns of products are stochastic variables. We consider two regulatory policies to cope with the uncertainty in carbon emission regulations, and we propose a robust extension of a stochastic program. Further, we develop tractable robust counterparts of the proposed hybrid model to find the robust solutions for box and ellipsoidal uncertainty sets. In general, the optimal network configuration must balance the trade-offs between investment, transportation and carbon emission costs if the carbon regulation is incorporated. More facilities will be opened and the total expected cost will increase as the uncertainty level increases. Moreover, the share of transportation by the low-emitting modes will also increase as the regulation policy uncertainty level increases. The problem formulation with nominal carbon prices or tax rates provides an unrealistically optimistic estimation of the real problem while the worst case scenario problem provides a conservative solution. The problem formulation with ellipsoidal uncertainty set allows the decision maker to balance the trade-off between robustness and performance. In addition, the proposed model can provide certain protection under the worst case scenario.

For the last part of this dissertation, we propose an integrated model for product return management and CLSC network design considering uncertain carbon cost. The proposed model can accommodate several features of practical relevance. For example, consumers have demand for new products and some of the unsatisfactory products will be returned, where both consumers' purchasing and return behavior will be affected by the refund offered. Meanwhile, the firm must design the network for distributing the new products and collecting the returned products. The decisions regarding the location of the factories, warehouses and collection centers, capacities of each facility and flow routing through the network must be made over multiple periods. We explicitly consider the uncertainty in the carbon cost. Utilizing the robust optimization approach to characterize such uncertainty, a model with both the l^2 -norm of the decision variable vector and a quadratic term is derived. We then propose a piecewise linear function approximation method to solve the model. Computational results illustrate trade-offs in revenues, transportation cost and carbon cost in designing the network and the return policy. Different parameters vary their impact on the refund provided, the profit and the network topology. For policy implications, we find that uncertainty in carbon regulation has the effect of reducing the refund and thus, result in less returns. This might not benefit environment as the products might go to landfill instead of being recycled and reused in the future.

In conclusion, we believe that building a comprehensive framework for product return management and CLSC network design is crucial under the emergence of emission trading or carbon tax schemes. It opens many research directions. For the future work, it is important to incorporate the inventory and routing issues into this framework. Thus, it would not only have the strategic decisions such as the locations of the facilities, but it would also include operational decisions such as routine and inventory policy. In addition, what other consumer behaviors the firm should take into consideration when facing the design of the network is also worth investigating. Finally, the integration of robust optimization and stochastic programming holds great promise as an optimization tool for practical problems. Stochastic programming can take advantage of the probability distributions of the data which are known or can be estimated. Robust optimization, on the other hand, is rather deterministic and set-based, which doesn't depend on the probabilistic description. Thus, when the optimization model has uncertainties that can be described by probability distributions and other types of uncertainties that can be characterized by certain uncertainty sets, the integration of the two approaches might worth consideration. Virtually any optimization problem that involves two different types of uncertainties can be revisited by the judicious combination of the two approaches.

APPENDIX A. ADDITIONAL MATERIAL

Netnography Quotations

Q1: *“It is a 50/50, toss of the coin issue when it comes to buying refurbished anything. Sometimes you make out OK, sometimes you don’t.”* (http://forums.cnet.com/7723-10157_102-525329.html)

Q2: *“‘Refurbished’ is a crap shoot. Refurbished products come from a variety of sources. There may have been nothing at all wrong with it. It may have an intermittent problem that does not immediately make its presence known. It may have an overheating problem and work fine for a few minutes, then completely quit working and is returned for that reason... After saying all this, in my experience, the odds are with you in the long run. Not every device that you buy ‘refurbished’ will be a defective device. You’re going to get several good ones for every bad one you get, but expect a bad one every now and then.”* (http://forums.cnet.com/7726-10157_102-5124344.html)

Q3: *“Apple brings the products back to as-new specifications. They even replace the outer cases on products like the iPods, so you don’t even have to worry about getting one with cosmetic damage.”* (http://forums.cnet.com/7723-10157_102-525329.html)

Q4: *“Refurbished computers – I wouldn’t do it again.”* (http://forums.cnet.com/7723-7586_102-278416.html)

Q5: *“I have always stayed away from refurbished products they weren’t as good.”* (http://forums.cnet.com/7723-7595_102-222574.html)

Q6: *“I almost always buy refurbished rather than brand new.”* (http://forums.cnet.com/7726-10157_102-5124372.html)

Q7: *“High-volume production lines typically omit 100% testing of all production items, opting instead to bear the cost of a small percentage being returned to make them serviceable. Thus, such*

'refurbished' items are more thoroughly tested than a new off-the-shelf unit at a dealer, boding well for buying such a unit... I'm a proponent of refurbished electronics and have great success with them while saving money!" (http://forums.cnet.com/7726-10157_102-5124325.html)

Q8: "The camcorder (Canon Vixia HF200) is \$749 new but the refurbished one is \$399! Is the price worth the camcorder being refurbished?" (http://forums.cnet.com/7723-7594_102-388858.html)

Q9: "The key is making sure you're comfortable with where you're getting the item from and making sure they have some kind of warranty (whether it's exchange or return)." (http://forums.cnet.com/7726-10157_102-5125014.html)

Q10: "I would not hesitate to buy a factory refurbished product, especially if I'm buying it from the manufacturer." (http://forums.cnet.com/7723-7595_102-304633.html)

Q11: "I think I'm going to buy a refurbished Macbook (Aluminium or Pro), but what is the return policy? If I get it, and I'm not happy can I return it? What if it has a scratch or dead pixels? Can I return it for a replacement?" (<http://forums.macrumors.com/showthread.php?t=814600>)

Q12: "I recently bought a refurbished 24" ACD and it had a big piece of dust between the glass and the screen. It was very noticeable to me, so I returned it immediately for a full refund. I tried for a second refurbished 24" ACD and it had a smaller piece of dust, but it also had a dead pixel (stuck on red), so I returned it as well. I just couldn't justify paying \$599 for a display that wasn't perfect." (<http://forums.macrumors.com/archive/index.php/t-782725.html>)

Appendix A2

Strict Return Policy

The Karush-Kuhn-Tucker (KKT) conditions are:

$$\frac{\partial \hat{L}}{\partial p_r} = \frac{\partial \hat{\Pi}}{\partial p_r} - \hat{\lambda}_1 - \hat{\lambda}_2(1 - \delta)\phi + \hat{\lambda}_3 + \hat{\lambda}_4 = 0 \quad (\text{A.1})$$

$$\frac{\partial \hat{L}}{\partial f} = (1 - \delta\phi)\hat{\lambda}_2 - \hat{\lambda}_4 = 0 \quad (\text{A.2})$$

$$\hat{\lambda}_1((1 - \delta\phi)p_n - p_r) = 0 \quad (\text{A.3})$$

$$\hat{\lambda}_2((1 - \delta\phi)f - (1 - \delta)\phi p_r) = 0 \quad (\text{A.4})$$

$$\hat{\lambda}_3(p_r - p_n + \delta\phi) = 0 \quad (\text{A.5})$$

$$\hat{\lambda}_4(p_r - f) = 0 \quad (\text{A.6})$$

$$\hat{\lambda}_1 \geq 0 \quad \hat{\lambda}_2 \geq 0 \quad \hat{\lambda}_3 \geq 0 \quad \hat{\lambda}_4 \geq 0 \text{ and Constraints (??) - (3.3) hold} \quad (\text{A.7})$$

Because f is not involved in the objective function, its value will not affect the result. Without loss of optimality, then, we assume that constraint (3.2) holds as an equality; i.e., $\hat{\theta}_l = \bar{\theta}$. This implies $\hat{\lambda}_4 = 0$ and from equation (A.2), $\hat{\lambda}_2 = 0$. Note that this also implies $f \leq p_r$.

Table A.1 Critical Values in Strict Return Policy

Point	T-cons. Buy	Expression
A	New	$p_r(A) = (1 - \delta\phi)p_n$ $f(A) = (1 - \delta)\phi p_n$
B	Both	$p_r(B) = \frac{\beta^R \delta\phi(1 - \delta\phi)(1 + c_r) + \beta^T (c_r + (1 - \delta\phi)(2p_n - (1 - c_n)))}{2\beta^T + 2\delta\phi(1 - \delta\phi)\beta^R}$ $f(B) = \frac{\delta\phi^2(1 - \delta)\beta^R(1 - \delta\phi)(1 + c_r) - \phi(1 - \delta)\beta^T (c_r + (1 - \delta\phi)(2p_n - c_n))}{2(1 - \delta\phi)(\beta^T + (1 + \beta^R)(1 - \delta\phi) - 1)}$
C	Refurb.	$p_r(C) = p_n - \delta\phi$ $f(C) = \frac{(1 - \delta)\phi(p_n - \delta\phi)}{1 - \delta\phi}$

Conditions under which points A, B and C are optimal under the strict return policy.

Point A is defined by $\hat{\lambda}_3 = 0$ and constraint (??) holds as an equality. Option A must satisfy $\hat{\lambda}_1(A) \geq 0$ to become optimal.

$\hat{\lambda}_1(A)$ is determined by the following equation:

$$\hat{\lambda}_1(A) = \frac{\beta^T (c_r - (1 - \delta\phi)c_n)}{\delta\phi(1 - \delta\phi)} + \beta^R (1 + c_r - 2(1 - \delta\phi)p_n)$$

Point B is defined by $\hat{\lambda}_1 = \hat{\lambda}_3 = 0$. The values of $p_r(B)$ and $f(B)$ are shown in table [A.1](#).

Point C is defined by $\hat{\lambda}_1 = 0$ and constraint (3.3) holds as an equality. Option C must satisfy $\hat{\lambda}_3(C) \geq 0$ to become optimal.

We can solve for $\hat{\lambda}_3(C)$ as follows:

$$\hat{\lambda}_3(C) = -\frac{\beta^T (2\delta\phi + c_r - (1 - \delta\phi)c_n - 2\delta\phi p_n)}{\delta\phi(1 - \delta\phi)} - \beta^R (1 + 2\delta\phi + c_r - 2p_n)$$

Appendix A3

Lenient Return Policy

The Hessian matrix for the profit function is positive definite. Thus, the function $\check{\Pi}(p_r, f)$ is concave and the following KKT conditions are sufficient for optimality.

$$\frac{\partial \check{L}}{\partial p_r} = \frac{\partial \check{\Pi}}{\partial p_r} + (1 - (1 - \delta)\phi)\check{\lambda}_1 - (1 - \delta)\check{\lambda}_2 + (1 - \delta)\check{\lambda}_3 = 0 \quad (\text{A.8})$$

$$\frac{\partial \check{L}}{\partial f} = \frac{\partial \check{\Pi}}{\partial f} - \delta\phi\check{\lambda}_1 - \delta\check{\lambda}_2 + \delta\check{\lambda}_3 + \check{\lambda}_4 = 0 \quad (\text{A.9})$$

$$\check{\lambda}_1(1 - (1 - \delta)\phi p_r - \delta\phi f - (1 - \phi)p_n) = 0 \quad (\text{A.10})$$

$$\check{\lambda}_2((1 - \delta)p_n - \delta f - (1 - \delta)p_r) = 0 \quad (\text{A.11})$$

$$\check{\lambda}_3(\delta f + (1 - \delta)p_r - p_n + \delta) = 0 \quad (\text{A.12})$$

$$\check{\lambda}_4 f = 0 \quad (\text{A.13})$$

$$\check{\lambda}_1 \geq 0 \quad \check{\lambda}_2 \geq 0 \quad \check{\lambda}_3 \geq 0 \quad \check{\lambda}_4 \geq 0 \text{ and Constraints (2.10) – (2.13) hold} \quad (\text{A.14})$$

Table A.2 Critical Values under Lenient Return Policy

Point	T-cons. Buy	Expression
D	Both	$p_r(D) = \frac{\phi^2 \delta (1 - \delta) \beta^R (1 + c_r) + \beta^T (\phi (c_r - (1 - \delta) c_n) + 2(1 - \delta) p_n - v(1 - \delta) \phi)}{2(\beta^T + \phi^2 \delta (1 - \delta) \beta^R)}$ $f(D) = \frac{(1 - \delta) \beta^R \delta \phi ((1 - (1 - \delta) \phi)(1 + c_r) - 2(1 - \phi) p_n) - \beta^T ((1 - (1 - \delta) \phi)((1 - \delta) c_n + \delta v - c_r) - 2\delta \phi (1 - \delta) p_n)}{2\delta(\beta^T + \phi^2 \delta (1 - \delta) \beta^R)}$
E	Refurb.	$p_r(E) = p_n - \delta \phi$ $f(E) = p_n + (1 - \delta)\phi - 1$
D'	Both	$p_r(D') = \frac{(1 - \phi) p_n}{1 - (1 - \delta)\phi}$ $f(D') = 0$
F	New	$p_r(F) = \frac{1 + c_r}{2}$ $f(F) = \frac{(1 - \delta)(2p_n - 1 - c_r)}{2\delta}$
G	Both	$p_r(G) = \frac{1 + c_r}{2}$ $f(G) = \frac{\delta(c_r - v) + (1 - \delta)(2p_n - c_n - 1)}{2\delta}$
H	Refurb.	$p_r(H) = \frac{1 + c_r}{2}$ $f(H) = \frac{2p_n - 2 + (1 - \delta) - (1 - \delta)c_r}{2\delta}$
I	Refurb.	$p_r(I) = \frac{p_n - \delta}{(1 - \delta)}$ $f(I) = 0$
J	Both	$p_r(J) = \frac{\beta^R \delta (1 + c_r) - \beta^T ((1 - \delta)(2p_n - c_n) - v\delta + c_r)}{2((1 - \delta)\beta^T + \beta^R \delta)}$ $f(J) = 0$
K	New	$p_r(K) = p_n$ $f(K) = 0$

Conditions under which points A and D (D') through K are optimal under the lenient return policy.

Point A is defined by $\check{\lambda}_3 = 0$ and $\check{\lambda}_4 = 0$. Constraints (2.10) and (2.11) hold as equalities. Option A must satisfy $\check{\lambda}_1(A) \geq 0$ and $\check{\lambda}_2(A) \geq 0$ to become optimal under strategy 2.

$\check{\lambda}_1(A)$ and $\check{\lambda}_2(A)$ are determined by the following equations.

$$\check{\lambda}_1(A) = -\beta^R (1 + c_r - 2(1 - \delta\phi)p_n)$$

$$\check{\lambda}_2(A) = \frac{(1 - \delta)\beta^R \delta\phi(1 - \delta)(1 + c_r - 2(1 - \delta\phi)p_n) + \beta^T (c_r - (1 - \delta)c_n - \delta v)}{\delta(1 - \delta)}$$

Point D is defined by $\check{\lambda}_2 = 0$, $\check{\lambda}_3 = 0$ and $\check{\lambda}_4 = 0$ and constraint (2.10) holds as an equality. Option D must satisfy $\check{\lambda}_1 \geq 0$ to become optimal.

Solving equations (A.8) and (A.9), we can obtain the expression for $\check{\lambda}_1(D)$ as follows.

$$\check{\lambda}_1(D) = -\frac{\beta^R \beta^T (1 + (1 - \phi)c_r + \phi(\delta v + (1 - \delta)c_n) - 2p_n(1 - \delta\phi))}{\beta^R \phi^2 \delta(1 - \delta) + \beta^T}$$

Point D' is defined by $\check{\lambda}_2 = 0$, $\check{\lambda}_3 = 0$ and constraints (2.10) and (2.13) holds as an equality. Option D' must satisfy $\check{\lambda}_1 \geq 0$ and $\check{\lambda}_4 \geq 0$ to become optimal.

Solving equations (A.8) and (A.9), we can obtain the expression for $\check{\lambda}_1(D')$ and $\check{\lambda}_4(D')$ as follows, where $\tau = 1 - \phi + \delta\phi$.

$$\check{\lambda}_1(D') = \frac{\beta^T (\delta - 1)\tau c_n + \tau c_r + \beta^R \delta (\tau + 2p\phi + \tau c_r - 2p_n) - \delta (v\tau - 2(1 - \delta)\phi p_n)}{\delta\tau^2}$$

$$\check{\lambda}_4(D') = \frac{\beta^R (1 - \delta)\delta\phi (\tau (1 + c_r) - 2(1 - \phi)p_n) + \beta^T (\tau (c_r - v\delta - (1 - \delta)c_n) + 2(1 - \delta)\delta\phi p_n)}{(1 - \delta)\tau^2}$$

Point E is defined by $\check{\lambda}_2 = 0$ and $\check{\lambda}_4 = 0$. Constraints (2.10) and (2.12) hold as equalities. Option E must satisfy $\check{\lambda}_1(E) \geq 0$ and $\check{\lambda}_3(E) \geq 0$ to become optimal.

Solving equations (A.8) and (A.9) along with equations (2.10) and (2.12), we can obtain the expressions for $\check{\lambda}_1(E)$ and $\check{\lambda}_3(E)$ as the following.

$$\check{\lambda}_1(E) = -\beta^R (1 + c_r - 2(p_n - \delta\phi))$$

$$\check{\lambda}_3(E) = -\frac{(1 - \delta)\beta^R ((1 + c_r - 2(p_n - \delta\phi)) + \beta^T (c_r - (1 - \delta)c_n - \delta(2(p_n - 1) + v)))}{\delta(1 - \delta)}$$

Point F is defined by $\check{\lambda}_1 = 0$, $\check{\lambda}_3 = 0$, $\check{\lambda}_4 = 0$ and constraint (2.11) holds as an equality. Option F must satisfy $\check{\lambda}_2(F) \geq 0$ to become optimal.

We can obtain the expression for $\check{\lambda}_2(F)$ as follows.

$$\check{\lambda}_2(F) = -\frac{\beta^T (c_r - v\delta - (1 - \delta)c_n)}{\delta(1 - \delta)}$$

Point G is defined by $\check{\lambda}_1 = 0$, $\check{\lambda}_2 = 0$, $\check{\lambda}_3 = 0$ and $\check{\lambda}_4 = 0$. Solving equations (A.8) and (A.9), we can obtain the expression for p_r^G and f^G .

Point H is defined by $\check{\lambda}_1 = 0$, $\check{\lambda}_2 = 0$ and $\check{\lambda}_4 = 0$. Constraint (2.12) holds as an equality. Option H must satisfy $\check{\lambda}_3(H) \geq 0$ to become optimal.

We can obtain the expression for $\check{\lambda}_3(H)$ as follows.

$$\check{\lambda}_3(H) = -\frac{\beta^T (c_r - (1 - \delta)c_n - \delta(2(p_n - 1) + v))}{\delta(1 - \delta)}$$

Point I is defined by $\check{\lambda}_1 = 0$ and $\check{\lambda}_2 = 0$. Constraints (2.12) and (2.13) hold as equalities. Option I must satisfy $\check{\lambda}_3(I) \geq 0$ and $\check{\lambda}_4(I) \geq 0$ to become optimal.

The expressions for $\check{\lambda}_3(I)$ and $\check{\lambda}_4(I)$ are as follows.

$$\check{\lambda}_3(I) = -\frac{\delta\beta^R (2(1 - p_n) - (1 - \delta)(1 - c_r)) + (1 - \delta)\beta^T (c_r - (1 - \delta)c_n - \delta(2(p_n - 1) + v))}{\delta(1 - \delta)^2}$$

$$\check{\lambda}_4(I) = \frac{\delta\beta^R (2(1 - p_n) - (1 - \delta)(1 - c_r))}{(1 - \delta)^2}$$

Point J is defined by $\check{\lambda}_1 = 0$, $\check{\lambda}_2 = 0$ and $\check{\lambda}_3 = 0$. Constraint (2.13) holds as an equality. Option J must satisfy $\check{\lambda}_4(J) \geq 0$ to become optimal.

We can obtain the expression for $\check{\lambda}_4(J)$ as follows.

$$\check{\lambda}_4(J) = -\frac{\delta\beta^T \beta^R (\delta(c_r - v) + (1 - \delta)(2p_n - c_n - 1))}{(1 - \delta)((1 - \delta)\beta^T + \delta\beta^R)}$$

Point K is defined by $\check{\lambda}_1 = 0$ and $\check{\lambda}_3 = 0$. Constraints (2.11) and (2.13) hold as equalities. Option K must satisfy $\check{\lambda}_2(K) \geq 0$ and $\check{\lambda}_4(K) \geq 0$ to become optimal.

We can obtain the expressions for $\check{\lambda}_2(M)$ and $\check{\lambda}_4(K)$ as follows.

$$\check{\lambda}_2(K) = -\frac{\delta\beta^R (2p_n - c_n - 1)}{(1 - \delta)}$$

$$\check{\lambda}_4(K) = \frac{(1 - \delta)\beta^T c_n - (\beta^T + \beta^R \delta)c_r + \delta(v\beta^F + \beta^R(2p_n - 1))}{\delta(1 - \delta)}$$

Appendix A4

Intermediate Return Policy

The KKT conditions are:

$$\frac{\partial \tilde{L}}{\partial p_r} = \frac{\partial \tilde{\Pi}}{\partial p_r} + (1 - \delta)\phi\tilde{\lambda}_1 - (1 - (1 - \delta)\phi)\tilde{\lambda}_2 + \tilde{\lambda}_3 = 0 \quad (\text{A.15})$$

$$\frac{\partial \tilde{L}}{\partial f} = \frac{\partial \tilde{\Pi}}{\partial f} - (1 - \delta\phi)\tilde{\lambda}_1 + (\delta\phi)\tilde{\lambda}_2 + \lambda_4^3 = 0 \quad (\text{A.16})$$

$$\tilde{\lambda}_1((1 - \delta)\phi p_r - (1 - \delta\phi)f) = 0 \quad (\text{A.17})$$

$$\tilde{\lambda}_2((1 - \phi)p_n - (1 - (1 - \delta)\phi)p_r + \delta\phi f) = 0 \quad (\text{A.18})$$

$$\tilde{\lambda}_3(p_r - p_n + \delta\phi) = 0 \quad (\text{A.19})$$

$$\tilde{\lambda}_4 f = 0 \quad (\text{A.20})$$

$$\tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\lambda}_3, \tilde{\lambda}_4 \geq 0 \text{ and Constraints (2.15) – (2.17) hold} \quad (\text{A.21})$$

Table A.3 Critical Values under Intermediate Return Policy

Point	T-cons. Buy	Expression
<i>L</i>	Refurb.	$p_r(L) = p_n - \delta\phi$ $f(L) = \frac{p_r(L) - p_n + \delta\phi}{2(1-\phi)\delta + 2(1-\delta)}$
<i>M</i>	Both	$p_r(M) = \frac{\beta^R \delta\phi(1-\delta\phi)(1+c_r) + \beta^T (2(1-\delta\phi)p_n - (1-\delta\phi)c_n + c_r)}{2(\beta^T + \beta^R \delta\phi(1-\delta\phi))}$ $f(M) = \frac{\beta^R (\delta\phi(\phi(1-\delta) - \nu(1-\delta\phi)) + \delta\phi(1-\delta\phi)c_r) + \beta^T (\phi(1-\delta)(2p_n - c_n) - \nu + c_r)}{2(\beta^T + \beta^R \delta\phi(1-\delta\phi))}$
<i>M'</i>	Both	$p_r(M') = \frac{\beta^R \delta(1-\phi)\phi(1+c_r) + \beta^T (\nu\delta^2\phi^2 - (1-\phi)c_n + (1-\phi)(1+\delta\phi)c_r + 2(1-\phi)p_n)}{2\beta^R \delta(1-\phi)\phi + 2\beta^T (1-\phi + \delta\phi - \delta\phi^2 + \delta^2\phi^2)}$ $f(M') = 0$
<i>E'</i>	Refurb.	$p_r(E') = p_n - \delta\phi$ $f(E') = 0$

Conditions under which option L, M, A through E are optimal under the intermediate return policy.

Point A is defined by $\tilde{\lambda}_3 = 0$ and $\tilde{\lambda}_4 = 0$. Constraints (2.15) and (2.16) hold as equalities. Option A must satisfy $\tilde{\lambda}_1(A) \geq 0$ and $\tilde{\lambda}_2(A) \geq 0$ to become optimal.

$\tilde{\lambda}_1(A)$ and $\tilde{\lambda}_2(A)$ are determined by following expressions:

$$\tilde{\lambda}_1(A) = \frac{(1 - \delta)\beta^R \delta\phi(1 - \delta)(1 + c_r - 2(1 - \delta\phi)p_n) + \beta^T (c_r - (1 - \delta)c_n - \delta\nu)}{(1 - \phi)(1 - \delta)}$$

$$\tilde{\lambda}_2(A) = \frac{\beta^T (c_r - (1 - \delta\phi)c_n)}{\delta\phi(1 - \phi)} + \frac{\beta^R (1 - \delta\phi)(1 + c_r - 2(1 - \delta\phi)p_n)}{1 - \phi}$$

Point B is defined by $\tilde{\lambda}_2 = 0$, $\tilde{\lambda}_3 = 0$ and $\tilde{\lambda}_4 = 0$. Constraint (2.15) holds as an equality. Option B must satisfy $\tilde{\lambda}_1(B) \geq 0$ to become optimal.

$\tilde{\lambda}_1(B)$ is determined by the following equation:

$$\tilde{\lambda}_1(B) = \frac{\delta\beta^T ((1-\phi)c_r - (1-\delta\phi)v)}{(1-\delta)(1-\phi)(1-\delta\phi)}$$

Point C is defined by $\tilde{\lambda}_2 = 0$ and $\tilde{\lambda}_4 = 0$. Constraints (2.15) and (2.17) hold as equalities. Option C must satisfy $\tilde{\lambda}_1(C) \geq 0$ and $\tilde{\lambda}_3(C) \geq 0$ to become optimal.

$\tilde{\lambda}_1(C)$ and $\tilde{\lambda}_3(C)$ are determined by the following expressions:

$$\tilde{\lambda}_1(C) = \frac{\delta\beta^T ((1-\phi)c_r - (1-\delta\phi)v)}{(1-\delta)(1-\phi)(1-\delta\phi)}$$

$$\tilde{\lambda}_3(C) = -\frac{\beta^T (2\delta\phi(1-p_n) - (1-\delta\phi)c_n + c_r) + \beta^R \delta\phi(1-\delta\phi)(1+c_r - 2(p_n - \delta\phi))}{\delta\phi(1-\delta\phi)}$$

Point D is defined by $\tilde{\lambda}_1 = 0$, $\tilde{\lambda}_3 = 0$ and $\tilde{\lambda}_4 = 0$. Constraint (2.16) holds as an equality. Option D must satisfy $\tilde{\lambda}_2(D) \geq 0$ to become optimal.

$\tilde{\lambda}_2(D)$ is given by:

$$\tilde{\lambda}_2(D) = \frac{\beta^{F^2} (v\delta\phi + (1-\phi)(c_r - c_n)) + \beta^T \beta^R \delta\phi ((1-\delta\phi)((1-\phi)(c_r - 2p_n) - \delta\phi v) - 1 + \phi)}{(1-\phi)(1-\delta)\delta\phi(\beta^T + \beta^R \phi^2 \delta)}$$

Point E is defined by $\tilde{\lambda}_1 = 0$ and $\tilde{\lambda}_4 = 0$. Constraints (2.16) and (2.17) hold as equalities. Option E must satisfy $\tilde{\lambda}_2(E) \geq 0$ and $\tilde{\lambda}_3(E) \geq 0$ to become optimal.

$\tilde{\lambda}_2(E)$ and $\tilde{\lambda}_3(E)$ are given by:

$$\tilde{\lambda}_2(E) = \frac{\beta^T ((1-\phi)((2(p_n - 1) - c_r)) + (1-\delta\phi)v)}{\phi(1-\phi)(1-\delta)}$$

$$\tilde{\lambda}_3(E) = \frac{(1-\delta)\beta^R \delta\phi (2(p_n - \delta\phi) - (1+c_r)) + \beta^T ((1-\delta)c_n - c_r - \delta(2-v - 2p_n))}{\phi\delta(1-\delta)}$$

Point L is defined by $\tilde{\lambda}_1 = 0$, $\tilde{\lambda}_2 = 0$ and $\tilde{\lambda}_4 = 0$. Constraint (2.17) holds as an equality. Option L must satisfy $\tilde{\lambda}_3(L) \geq 0$ to become optimal.

Solving equations (A.15) and (A.16), we can obtain the expression for $\tilde{\lambda}_3(L)$.

$$\tilde{\lambda}_3(L) = -\frac{\beta^T (c_r - (1-\delta\phi)c_n + 2\delta\phi(1-p_n)) + \beta^R \delta\phi(1-\delta\phi)(1+c_r - 2(p_n - \delta\phi))}{\delta\phi(1-\delta\phi)}$$

Point M is defined by $\tilde{\lambda}_1 = 0$, $\tilde{\lambda}_2 = 0$, $\tilde{\lambda}_3 = 0$ and $\tilde{\lambda}_4 = 0$. Solving equations (A.15) and (A.16), we can obtain the expressions for p_r^M and f^M , which are given in Table 4.

Point D' is defined by $\tilde{\lambda}_1 = 0$, $\tilde{\lambda}_3 = 0$ and constraints (2.16) and (2.18) hold as an equality. Option D' must satisfy $\tilde{\lambda}_2 \geq 0$ and $\tilde{\lambda}_4 \geq 0$ to become optimal.

Solving equations (A.15) and (A.16), we can obtain the expression for $\tilde{\lambda}_2(D')$ and $\lambda_4^3(D')$ as follows, where $\tau = 1 - \phi + \delta\phi$ and $\kappa = \delta\phi(\tau + \tau c_r - 2(1 - \phi)p_n)$.

$$\tilde{\lambda}_2(D') = \frac{\beta^T(\tau(1 - \phi)(c_n - (1 + \delta\phi)c_r) - \delta\phi^2(v\delta\tau - 2(1 - \delta)(1 - \phi)p_n)) - \beta^R(1 - \phi)\kappa}{-\delta\phi(1 - \phi)\tau^2}$$

$$\tilde{\lambda}_4(D') = -\frac{\beta^R(1 - \delta)\kappa - \beta^T((1 - \delta)\tau c_n - \tau c_r + \delta(v\tau - 2(1 - \delta)\phi p_n))}{(1 - \delta)(1 - \phi)\tau^2}$$

Point M' is defined by $\tilde{\lambda}_1 = 0$, $\tilde{\lambda}_2 = 0$ and $\tilde{\lambda}_3 = 0$. Constraint(2.18) holds as an equality. Option M' must satisfy $\tilde{\lambda}_4 \geq 0$ to become optimal.

Solving equations (A.15) and (A.16), we can obtain the expression for $\lambda_4^3(M')$ as follows.

$$\tilde{\lambda}_4(D') = -\frac{\beta^F\beta^R\delta^2\phi((1 - \delta)\phi - v(1 - \delta\phi) + (1 - \delta\phi)c_r) - \beta^{F^2}\delta(v + (1 - \delta)\phi c_n - c_r - 2(1 - \delta)\phi p_n)}{-\beta^T(1 - \delta)(1 - \phi + \delta\phi - \delta\phi^2 + \delta^2\phi^2) - (1 - \delta)\delta(1 - \phi)\phi\beta^R}$$

Point E' is defined by $\tilde{\lambda}_1 = 0$, $\tilde{\lambda}_2 = 0$. Constraints (2.18) and (2.17) hold as an equality. Option E' must satisfy $\tilde{\lambda}_3 \geq 0$, $\tilde{\lambda}_4 \geq 0$ to become optimal.

Solving equations (A.15) and (A.16), we can obtain the expression for $\tilde{\lambda}_3(E')$ and $\lambda_4^3(E')$ as follows, where $\rho = 2\delta\phi + \delta\phi^2(-2 + \delta(2 + v - 2\phi) + 2\delta^2\phi)$.

$$\tilde{\lambda}_3(E') = -\frac{\beta^T\delta(-v - 2\delta\phi^2 + v\delta\phi + 2\delta^2\phi^2 + (1 - \phi)c_r + 2(1 - \delta)\phi p_n)}{(1 - \delta)(1 - \phi)}$$

$$\tilde{\lambda}_4(E') = -\frac{\beta^R(\delta(1 - \phi)\phi(1 + 2\delta\phi + c_r - 2p_n)) - \beta^T(\rho - (1 - \phi)c_n + (1 - \phi)(1 + \delta\phi)c_r - 2\delta\phi(1 - 2\phi + \phi\delta)p_n)}{-\delta(1 - \phi)\phi}$$

Appendix A5

Proof of Theorem 1

PROOF. (i) If $c_r < \min [2(1 - \delta\phi)p_n - 1, (1 - \delta\phi)c_n]$, then $\hat{\lambda}_1(A) < 0$.

(ii) If $c_r > \max [(1 - \delta\phi)c_n + 2\delta\phi(p_n - 1), 2p_n - 2\delta\phi - 1]$, then $\hat{\lambda}_3(C) < 0$.

Proof of Theorem 2

PROOF. (i). If $c_r < \min [2(1 - \delta\phi)p_n - 1, \delta v + (1 - \delta)c_n]$, then $\check{\lambda}_2(A) < 0$. As $2(1 - \delta\phi)p_n - 1 > 2(p_n - \delta\phi) - 1$, if $c_r < \min [2(1 - \delta\phi)p_n - 1, \delta v + (1 - \delta)c_n]$, then point G will violate the lower bound $p_n - \delta\phi$ for $p_r^G = \frac{1 + c_r}{2}$.

(ii). If $c_r > \max \left[v\delta + (1 - \delta)c_n, 1 - \frac{2(1 - p_n)}{1 - \delta} \right]$, then $v\delta + (1 - \delta)c_n > (v - 2(1 - p_n))\delta + (1 - \delta)c_n$, so $\check{\lambda}_3(H) < 0$ and $\check{\lambda}_3(I) < 0$.

(iii). If $c_r > 2p_n - 1$ and $c_r > 2(p_n - \delta\phi) - 1$, then $\check{\lambda}_2(E) < 0$ and $f(F) < 0$. If $p_n < 1 - (1 - \delta)\phi$, then $f(E) < 0$.

(iv). From Assumption 5, if $2p_n - c_n - 1 > 0$, then $\check{\lambda}_4(J) < 0$ and $\check{\lambda}_2(K) < 0$.

Proof of Theorem 3

PROOF. (i). If $c_r < \min [2(1 - \delta\phi)p_n - 1, \delta v + (1 - \delta)c_n, 1 - \delta\phi c_n]$, then $\tilde{\lambda}_1(A) < 0$ and $\tilde{\lambda}_2(A) < 0$.

(ii). If $c_r > \max [(1 - \delta\phi)c_n - 2\delta\phi, 2(p_n - \delta\phi) - 1]$, then $\tilde{\lambda}_3(L) < 0$ and $\tilde{\lambda}_3(C) < 0$.

(iii). If $\frac{v}{c_r} > \frac{1 - \phi}{1 - \delta\phi}$, $(1 - \phi)c_r < (1 - \delta\phi)v$, then $\tilde{\lambda}_1(B) < 0$ and $\tilde{\lambda}_1(C) < 0$.

(iv). If $(1 - \phi)c_r - (1 - \delta\phi)v > 2(1 - \phi)(1 - p_n)$, then $\tilde{\lambda}_2(E) < 0$.

(v). As $(1 - \delta\phi)((1 - \phi)(c_r - 2p_n) - \delta\phi v) - (1 - \phi) < 0$, if $v\delta\phi + (1 - \phi)c_r < (1 - \phi)c_n$, then $\tilde{\lambda}_2(D) < 0$.

Proof of Corollary 1

PROOF. (i). If $c_r < \min [2(1 - \delta\phi)p_n - 1, \delta v + (1 - \delta)c_n, (1 - \delta\phi)c_n]$, the Lagrangian multipliers associated with point A will be negative under any return policy.

(ii) If $c_r > \max \left[(1 - \delta\phi)c_n - 2\delta\phi(1 - p_n), 2p_n - 1, \frac{(1 - \delta\phi)v}{1 - \phi} + 2(1 - p_n) \right]$, the Lagrangian multipliers associated with points C, L and E will be negative under any return policy.

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