AN IMPROVED TECHNIQUE FOR DETERMINING THE EQUATION OF STATE OF CONCRETE AND GEOLOGICAL MATERIALS

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INTRODUCTION

Concrete is an extremely versatile building material. It is being used extensively as a building material for defense and civilian structures and infrastructure. In defense applications, concrete is often used as the primary structural component in facilities that are hardened against enemy attack, especially projectiles that can impact the structure with a high rate of speed and a large explosive force. The high strain and strain rate of such an event make it imperative to know the mechanical behavior of concrete at these elevated loads in order to properly design the appropriate weapons that can penetrate such structures, or, conversely for defensive purposes, design the structure to withstand and survive such an event. Similar conditions can occur in the civilian sector. Depending on the geographical location of these structures, they can be exposed to similar conditions as some of the defense facilities. For example, an earthquake is typically composed of several different types of shock waves [1]. The exact nature of the shock waves is dependent on the nature of the earthquake source.

Currently, there are insufficient data to develop a model that will predict the mechanical performance of concrete and geological materials at these high strains and strain rates. The information that is available is not sufficient to meet the requirements of developing a verifiable model to predict the mechanical performance of concrete.

A realistic solution to obtaining a significant database of the behavior of concrete and similar materials at high strain levels uses ultrasonic measurements at elevated pressures. From ultrasonic longitudinal and shear wave measurements, the elastic constants of concrete can be determined as the sample is being subjected to hydrostatic pressure. By taking the pressure derivatives of the elastic constants, semi-empirical equations of state can be derived [2]. The equations of state of metallic materials obtained by this technique have been compared to equations of state generated by other techniques, such as shock wave experiments. The results have shown an excellent correlation between the ultrasonic equations of state and those derived shock wave techniques.

The ultrasonic approach provides several advantages over the traditional method of using shock waves to determine the equation of state of materials. The ultrasonic approach is not restricted to the requirement that the materials being tested support large shear stresses. This is a consequence of the ultrasonic measurement technique being performed under true hydrostatic pressures [2]. As a result, the material being evaluated is not placed under any shear stresses. This is an ideal technique to use with brittle materials. Also, this eliminates phase transformations driven by shear stresses. The material parameters obtained from this testing method are true thermodynamic variables that can be directly substituted into the hydrocode computer models used to predict the penetration capability of munitions. Therefore, no additional interpretation of the data is required. In addition, the ultrasonic methodology allows a broad range of concrete and geological materials to be evaluated in a rapid, reliable and cost effective manner.

THEORETICAL BACKGROUND

For this program, the sample was placed inside a hydrostatic chamber, allowing uniform pressure to be applied to all sides. As the sample is subjected to elevated pressures, the ultrasonic longitudinal and shear wave velocities are measured in the sample. From these ultrasonic measurements, the adiabatic elastic constants can be determined. The bulk modulus, B_{so}, can be expressed in terms of these velocities as follows [3]:

$$B_{so} = \rho_o(C_l^2 - 4/3(C_t^2)) \tag{1}$$

where: B_{so} = adiabatic bulk modulus
V_l = ultrasonic longitudinal wave velocity,
V_t = ultrasonic shear wave velocity
and ρ_o = density.

By performing the ultrasonic measurements as a function of pressure, the pressure derivative of the adiabatic bulk modulus can also be obtained from plotting the pressure vs. the bulk modulus and determining the nature of the curve fit between the two parameters.

The bulk moduli that are obtained from the ultrasonic measurements cannot be directly inserted into the semi-empirical equations of state of Murnaghan [4] and Birch [5]. The Murnaghan equation is based on the assumption that the adiabatic bulk modulus is linear with pressure and is written as:

$$\frac{V}{V_0} = \left(1 + \frac{B'_{To}}{B_{To}}\right)^{-1/B'_{To}} \tag{2}$$

where the prime indicates the pressure derivative of the isothermal bulk modulus at atmospheric pressures, and V and V_0 are the volume of the solid at pressure P and at atmospheric pressure, respectively. The Birch equation assumes that the strain energy can be expanded in terms of linear strains. Expanded to the third power, it is commonly written as:

$$P = \frac{3}{2}B_{To}\left[\left(\frac{V_0}{V}\right)^{7/3} - \left(\frac{V_0}{V}\right)^{5/3}\right]\left[1 - \left(3 - \frac{3}{4}B_{To}\right)\left(\left(\frac{V_0}{V}\right)^{2/3} - 1\right)\right] \tag{3}$$

These equations incorporate the isothermal bulk modulus at atmospheric pressure and its pressure derivative. This requires a conversion from the adiabatic bulk modulus and pressure derivative of the bulk modulus, which were obtained from the ultrasonic measurements. For the bulk modulus, the following conversion can be used [6]:

$$B_{so} = \frac{B_{To}}{1+\Delta} \tag{4}$$

where:

$$\Delta = \beta^2 T B_{so} / C_p \rho_o$$

and:

Δ = adiabatic to isothermal conversion factor
B_{To} = isothermal bulk modulus
β = volume coefficient of thermal expansion,
T = temperature (K),
C_p = heat capacity at constant pressure,
and ρ_o = density at atmospheric pressure.

The isothermal pressure derivative, B'_{To}, of the bulk modulus is obtained from the adiabatic pressure derivative by a conversion derived by Overton [6]:

$$\left(\frac{\partial B_{To}}{\partial P}\right)_T = \left(\frac{\partial B_{so}}{\partial P}\right)_T + \frac{\Delta}{1+\Delta}\left[1 - \frac{2}{\beta B_{To}}\left(\frac{\partial B_T}{\partial T}\right)_P - 2\left(\frac{\partial B_{so}}{\partial P}\right)_T\right] + \left(\frac{\Delta}{1+\Delta}\right)^2\left[\left(\frac{\partial B_{so}}{\partial P}\right)_T - 1 - \frac{1}{\beta^2}\left(\frac{\partial \beta}{\partial T}\right)_P\right] \tag{5}$$

As the pressure measurements are performed at approximately room temperature, the coefficient of thermal expansion, β, can be assumed to be constant. Therefore, its temperature derivative in equation (5) vanishes. The pressure derivative of the adiabatic bulk modulus is obtained from the ultrasonic measurements. The remaining term in the relationship from Overton is the temperature derivative of the isothermal bulk modulus, which is shown by Overton to deviate less than 0.2% from the adiabatic temperature derivative of the bulk modulus. This is true if the Grüneisen parameter is assumed not to change with temperature in the vicinity of room temperature [6]. Therefore, the adiabatic temperature derivative of the bulk modulus can be used. The temperature derivative of the adiabatic bulk modulus can be obtained by measuring the ultrasonic longitudinal and shear velocities of the concrete as a function of temperature, using a temperature bath at constant pressure. Thus, the ultrasonic approach provides a simple and highly accurate technique to determine the equation of state.

EXPRIMENTAL APPROACH

The concrete samples that were examined were prepared by the Building Materials Division of the National Institute of Standards and Technology (NIST). The difference in the constituents of the samples is demonstrated in Table I. The porosity was controlled by varying the types of sand and the water/cement weight ratio. Typical sample dimensions were 2 inches in diameter and 0.6 inches in length for examinations at elevated pressures. The cylindrical samples were cut to obtain two flat and parallel surfaces. The sample geometry simplified measuring the travel path of the ultrasonic wave. The samples listed in Table I were kept in lime water until they were encapsulated for evaluation at elevated pressure. This experimental procedure was followed to maintain consistency with the procedure used at NIST to measure air content in the concrete.

Table I. Constituents of the samples that were evaluated

Sample Number	Water/Cement Weight Ratio	Sand/Cement Weight Ratio	Admixture: Plasticizer	Types of Sand (*)	Prepared in	Air Content (%): Porosity
SP-72W	0.29	1.7	0.50%	4 Types	Vacuum	>0.5
SP-72D	0.29	1.7	0.50%	4 Types	Vacuum	>0.5
SP-45	0.29	1.411	0.50%	4 Types	Air	2.1
SP-49	0.29	1.411	0.50%	C109	Air	4.5

* The four types of sands have the following size distribution: F95 = 75 - 300 μm, 20-30 = 600 - 850 μm, C109 = 212 - 600 μm and S15 = 850 - 2360 μm.

The high pressure cell had a cell size of 2.5 inches in diameter and 12 inches in length. The pressure is applied to the cell by an air driven pump. For the ultrasonic measurements to be performed at the elevated pressures, an electrical feed-through was incorporated into the cap of the pressure cell. With the feed-through, a maximum pressure to 35 ksi (0.23 GPa) was achieved.

A method to encapsulate the concrete was developed to prevent the infiltration of the high pressure transfer fluid into the pores of the concrete. The concept for the encapsulation system is based on vacuum bagging techniques developed for manufacturing composites. Such a system consists of heavy polymer sheeting that is wrapped around the composite part and sealed at the three open edges with a thick, pliable polymer sealant. The sealant allows feed-throughs to be placed into the vacuum bag. With this technique, the concrete was isolated from the pressure transfer fluid to prevent infiltration of the fluid into the porous concrete structure. Also, the transducer elements were isolated from the same fluid, preventing electrical shorts between the two electrical poles of the transducer element. In addition, a vacuum was required inside the encapsulation bag to prevent it from bursting under the applied hydrostatic pressure. All of these requirements were met by using the vacuum bag technique to encapsulate the concrete samples.

The operating pressures that were used inside the pressure cell require that bare-element transducers be used instead of commercial transducers. The structure of the commercial transducers cannot support the pressures that are reached inside the pressure cell. However, the bare piezoelectric element can function under such pressures. The bare piezoelectric transducers were coupled to the sample using a thin plastic film. The transducers were fastened to the sample and insulated from other electrical signals. With this system, the concrete samples were tested up to a pressure of 35 ksi (~2.5 kbar) in 2.5 ksi steps..

The ultrasonic measurements were performed using longitudinal and shear waves at a frequency of 2.25 MHz. The measurements were first performed on the samples prior to their encapsulation. Once the sample and transducers were encapsulated and placed in the pressure cell, the ultrasonic measurements were repeated at atmospheric pressure. The pressure was then increased by 2.5 ksi increments, with a dwell period of a minimum of 3 minutes at each new pressure before the ultrasonic measurement was performed. After the sample reached a maximum pressure of 35 ksi, the pressure was released and the sample was removed from the pressure cell and a final ultrasonic measurement was performed. The time-of-flight of the ultrasonic signal was determined by overlapping the first peak of the received sinusoidal ultrasonic signal and the leading edge of the generation pulse. These measurements were multiplied by a correction factor to eliminate the delay caused by using the first peak instead of the leading edge of the received ultrasonic signal. The correction factor was obtained by comparing the results from the measurement performed outside the pressure cell, where the decrease in the noise level allowed leading edge overlaps to be performed, and the measurement performed inside the pressure cell at zero applied pressure.

To determine the velocities as a function of pressure, the change in the time-of-flight of the ultrasonic wave was measured as a function of increasing pressure. The velocity was calculated from the time-of-flight and the length of the travel path of the concrete prior to placement inside the pressure cell. The length of the travel path changes due to compression at pressure, but was found to vary by less than 0.2%, according to Cook's relation [2], a method to correct for changes in the travel path due to applied hydrostatic pressure:

$$\frac{l_o}{l_p} = 1 + \frac{1+\Delta}{\rho_o}\int_0^p \frac{dp}{3(V_{lo}^p)^2 - 4(V_{to}^p)^2} \tag{9}$$

where: l_o = acoustic wave path length at atmospheric pressure,
l_p = acoustic wave path length at applied pressure,
ρ_o = density at atmospheric pressure,
p = pressure,
V_{lo}^p = approximate longitudinal wave velocity calculated with the acoustic wave path length at atmospheric pressure,
and V_{tp}^p = approximate shear wave velocity calculated with the acoustic wave path length at atmospheric pressure.

The small change in the dimensions of the concrete were confirmed by the measurements of thickness and diameter performed before and after each characterization at pressure. The change in dimensions was found to be typically less than 0.2%. Therefore, the ultrasonic velocity was calculated by dividing the original thickness of the sample by the time-of-flight of the ultrasonic wave through the concrete.

Each characterization procedure at elevated pressure was performed on a concrete sample that had not been previously exposed to elevated pressures. This procedure was used in case the exposure to the elevated pressures caused an irreversible change in the elastic properties of the concrete. Therefore, a minimum of four samples of each type of concrete was obtained for evaluation at pressure. Once the sample had been characterized at pressure, it was not subjected to at pressure testing again.

RESULTS AND DISCUSSION

Using the measurement technique described in the previous section, the longitudinal and shear wave ultrasonic velocity for the four samples were measured as a function of pressure to 35 ksi. The results from the longitudinal and shear measurements of a representative sample are shown in Figure 1 and 2, respectively.

The bulk modulus of the four concrete samples for each pressure value can be calculated using equation 1, as the small aggregate size allows the concrete samples used in this program to be considered as homogeneous and isotropic. The results in Figure 3 show the bulk modulus as a function of pressure for the representative sample. These results show that the bulk modulus appears to be linear as a function of pressure for all of the samples. The slope of the linear dependence may change as a function of porosity. Using this linear relationship, the pressure derivative of the adiabatic bulk modulus can be determined.

To determine the equation of state of the concrete sample, the adiabatic bulk modulus and its pressure derivative must be converted to the isothermal bulk modulus and its pressure derivative.

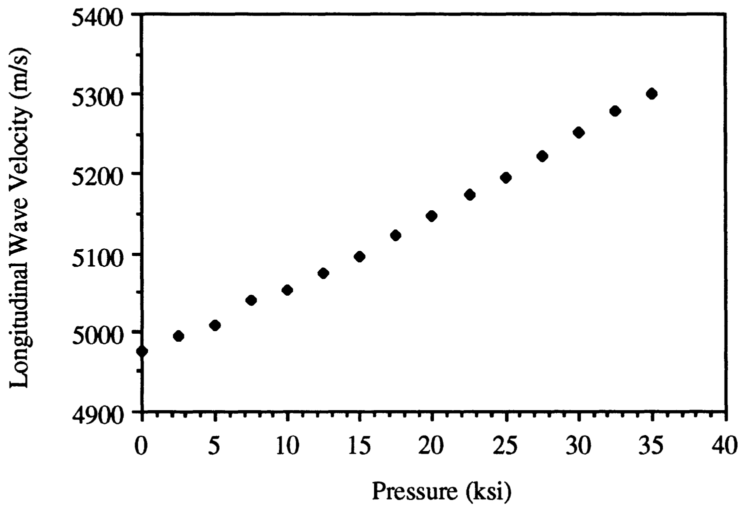

Figure 1. Longitudinal wave velocity vs. pressure for concrete sample SP 49.

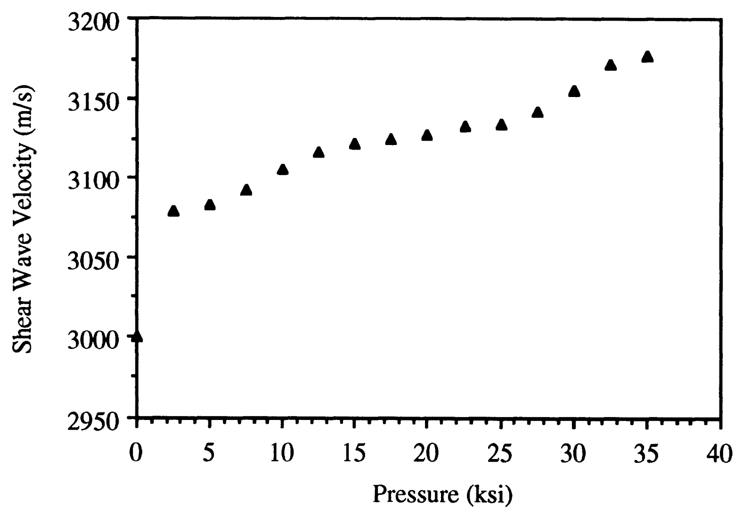

Figure 2. Shear wave velocity vs. pressure for concrete sample SP 49.

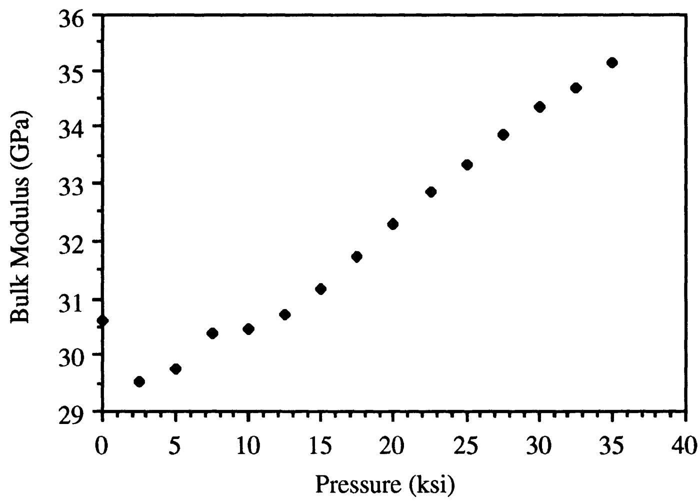

Figure 3. Bulk modulus vs. pressure for concrete sample SP 49.

Using the relations shown in equation 4, the adiabatic to isothermal conversion factor can be calculated to be approximately 2.8 x 10^{-8}. With such a small value, there is a negligible difference between the adiabatic and isothermal bulk moduli. Also, the conversion factor for the pressure derivative of the bulk modulus effectively goes to zero, indicating that the adiabatic and isothermal pressure derivatives are approximately the same for the pressure range evaluated.

With the values for the bulk modulus and its pressure dependence obtained from Figure 3, it is possible to determine the equation of state for the concrete samples using the Murnaghan equation of state (equation 2). For sample SP-49, the bulk modulus at ambient pressure is approximately 29 GPa (extrapolating the linear relationship) and at 1 kbar, it is 32 GPa. Therefore, the pressure derivative can be calculated as:

[(32-29)/0.1] = 30.

Substituting in the values for the bulk modulus and its pressure derivative into equation 2, the following is obtained:

$$\frac{V}{V_o} = \left(\left(1+\frac{3.0}{0.1}\right)\left(\frac{1}{29}\right)\right)^{-1/30}$$

Completing the calculation, V is found to equal $0.998V_0$ for pressure values between 0 and 1 kbar. Similar values are obtained for the pressure range that was evaluated, which is between 0 and 2.5 kbar. The linear relation is expected at these relatively low pressure values as the Murnaghan equation of state is based on the assumption that the adiabatic bulk modulus is a linear function of pressure.

CONCLUSIONS

The presented work demonstrates the feasibility of using ultrasonic measurements as a function of hydrostatic pressure to obtain the bulk modulus and its pressure derivative for concrete.

From these two parameters, the equation of state of concrete has been calculated. This approach allows the true thermodynamic variables to be obtained. These variables can be substituted into hydrocode modeling of the high strain and high strain rate properties of concrete. From these models, the mechanical behavior of concrete subjected to such conditions can be predicted. This information will lead to enhanced munitions design and improved performance of structures designed to withstand high strain events.

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