# ON THE RELATIONSHIP BETWEEN ULTRASONIC AND MICRO-STRUCTURAL PROPERTIES OF IMPERFECT INTERFACES IN LAYERED SOLIDS

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## INTRODUCTION

The interaction of ultrasonic waves with interfaces formed by two non-conforming, rough surfaces in contact has been the subject of numerous investigations [1-10]. The motivations behind these studies have been various: from the assessment of the real area of contact between two rough surfaces [1], to the modeling of crack closure near the tip of a fatigue crack [4]; from the identification of the nature of interfacial imperfections in kissing and partial bonds [6], to the generation of ultrasonic waves [8]. In most of these studies, the characterization of the interfacial properties has been attempted by studying the reflection of longitudinal and shears waves at normal incidence. Only recently, the problem concerning the interaction of ultrasonic waves with realistic complex systems such as that formed by two neighboring imperfect interfaces has been addressed. Lavrentyev and Rokhlin [9, 10] used ultrasonic spectroscopy to evaluate the interfacial conditions from the spectra of longitudinal and shear waves reflected normally from the interfaces.

In most of the studies cited above, the low frequency response of two contacting surfaces has been described within the framework of the quasi-static approximation (QSA) [5]. According to QSA, the interfacial imperfections cause a discontinuity in the displacement components that is proportional to the stress field at the interface. The components of the stress fields are assumed to be continuous everywhere. Thus, the boundary conditions enforced on the plane of an imperfect interface, z = 0, are

$$\sigma_{13}(0^+) = \sigma_{13}(0^-)\ ,\ \sigma_{33}(0^+) = \sigma_{33}(0^-)\ , \tag{1.a}$$
$$\sigma_{13}(0) = K_T[u_1(0^+) - u_1(0^-)]\ ,\ \sigma_{33}(0) = K_N[u_3(0^+) - u_3(0^-)]\ . \tag{1.b}$$

In eq. (1.b), the quantities $K_N$ and $K_T$ are the normal and transverse stiffness constants of the imperfect interface, respectively.

The QSA does not provide any way to evaluate $K_N$ and $K_T$, and, therefore, additional micro-mechanical modeling is needed to properly account for the effect of the interfacial imperfections on ultrasonic scattering. Quite different approaches have been developed to model the elasto-plastic behavior of rough surfaces in contact [1, 2, 5, 11, 12, 13, 14, 15, 16]. In the most sophisticated models the contacts between the rough surfaces are assumed to occur at the surface asperities, and in many cases their interaction is assumed to be purely elastic [1, 11, 12, 13]. Attempts to describe plastic deformation at the asperities can be found in several articles [2, 14, 15, 16].

In the present work reflectivity of longitudinal and shear waves at both normal and oblique incidence from single and double imperfect interfaces is studied. It is shown that a unique set of the interfacial stiffness constants, $K_N$ and $K_T$, is sufficient to characterize the macroscopic elastic response of the interfaces regardless of the direction of incidence of the ultrasonic wave. Micromechanical models of two rough surfaces in contact are also used to predict the interfacial stiffness constants. The purpose of this effort is to link the experimental values of $K_N$ and $K_T$ to the micromechanics and topography of the contacting surfaces.

## SURFACES IN CONTACT: A MICROMECHANICAL MODEL

In this section a brief description of the micromechanical models predicting the interfacial stiffness constants is presented. The models were developed by Brown and Scholz [11] and by Boitnott et al. [12] to describe the closure, δ, and the sliding of two rough surfaces against each other when they are subjected to an external normal or transverse load, respectively. The models account for the topography of the two rough surfaces through a fictitious composite surface. By defining the composite surface by means of an appropriate algebraic sum of the two contacting surfaces, the contacts of the real interface are transformed into the peaks of the composite surface. The heights of the peaks of the composite surface are distributed according to a probability density function, φ(z). Brown and Scholz [11] used the probability density function for the peak height found by Adler and Firman [17]. In this work, a simpler expression for φ(z), that is approached by the exact one when the composite surface contains high frequency components, is used. The function φ(z) used here depends on two parameters: the number of degrees of freedom, and the surface r.m.s. roughness of the composite surface [17]. According to Brown and Scholz's model, the dependence of applied normal pressure, P, on the interface closure, δ, of the two surfaces is given by the following relationship,

$$P = \frac{4}{3} n \langle E' \rangle \langle \sqrt{\beta} \rangle \langle \psi \rangle \int_{d_o - \delta}^{d_o} (z - d_o + \delta)^{3/2} \varphi(z) dz \,. \qquad (2)$$

In eq. (2) the interaction at each contact is assumed to be elastic and to follow Hertz's theory of contacting spheres. The symbol 'n' is the peak density of the composite surface,

$$K_N = \frac{\partial P}{\partial \delta}. \qquad (3)$$

⟨E'⟩ is the elastic modulus of two contacting spheres, ⟨ψ⟩ is the tangential stress correction factor, β is the curvature of a composite peak, and, finally, $d_o$ is the distance between the mean planes of the surfaces at P = 0. The quantity $d_o$ is related to the maximum height of the surface, $Z_{max}$, and depends on the degree of conformity of the two surfaces. The normal stiffness constant, $K_N$, is found by differentiating P with respect to δ, Adopting a similar approach, Boitnott et al. [12] derived the following relationship between the applied shear stress, $\tau$, and the relative displacement between the two rough surfaces, w,

$$\tau = n \int_{d_O-\delta}^{d_O} f(w/z, \xi_i, P)\varphi(z)dz. \qquad (4)$$

In eq. (4) the function $f(w/z, \xi_I, P)$ represents a constitutive relation between contacts. The shear interaction depends on the tangential relative displacement, w, on the composite surface height, z, on other micro-mechanical and topographical parameters of the interface, $\xi_i$, and on the normal pressure, P, applied to the interface. The transverse interfacial stiffness, $K_T$, can be found by differentiating the applied stress, $\tau$, with respect to the relative displacement, w,

$$K_T = \left(\frac{\partial \tau}{\partial w}\right). \qquad (5)$$

As pointed out by Boitnott et al. [12], the initial transverse stiffness, that is, the value of $K_T$ at w = 0, is independent of the constitutive law used to describe the shear interaction between the contacts. This observation is relevant to this work insofar as it implies that the model's ability to reproduce the experimental results depends strongly on the assumptions made about the probability density function $\varphi(z)$.

## EXPERIMENTAL RESULTS

### Experimental Setup

Figure 1 illustrates the main features of the experimental setup used in this work. The system consists of a hemispherical aluminum block that is loaded against a second block of the same material. An aluminum layer having a thickness of the order of 0.5 mm is interposed between the two, so to form the double imperfect interface system. The hemispherical block hosts the emitting and receiving transducers whose signals are recorded via an oscilloscope and further processed by a personal computer. The surfaces of the aluminum components are treated to have the suitable roughness for the investigation.

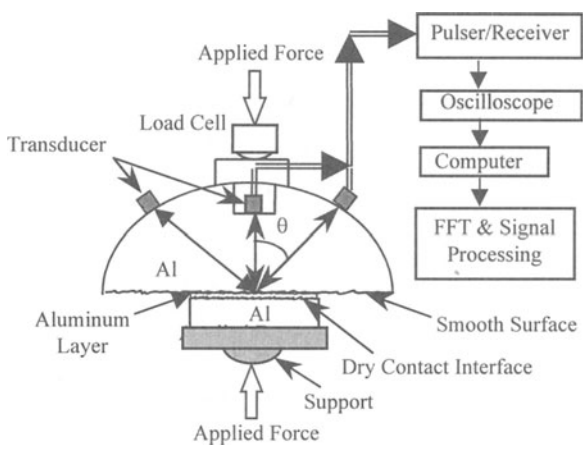


Figure 1. Experimental setup for ultrasonic measurements of the double interface stiffness.

Single Imperfect Interface

To test the validity of the models, preliminary measurements were performed on a single interface between two surfaces having an r.m.s. roughness $\sigma = 0.68$ μm. The value of the r.m.s. roughness was measured before and after the ultrasonic measurements were performed, and no appreciable variation due to surface loading was recorded. Both longitudinal and shear waves were used at normal incidence. The applied pressure was varied up to about 80 MPa. Figure 2 shows the experimental data as well as the models' predictions for both the normal, $K_N$, and the transverse, $K_T$, stiffness constants. The theoretical curves were obtained using the roughness of the surfaces as an input parameter. The values of the remaining parameters were chosen to fit the experimental data. Table 1 reports the values of the model parameters used to produce the plots of this and of the following figures. The agreement between the measured and predicted values of $K_N$ and $K_T$ in Fig. 2 shows that the models incorporate the essential physical elements of the interface interaction. In particular, for a small roughness the assumption that the interaction between the surface asperities is elastic seems to be validated.

Figure 3 reports the values of the normal and transverse stiffness constants measured on an interface between two surfaces with an initial r.m.s. roughness of 2.4 μm. Two sets of data (open and solid squares) for the normal stiffness are shown. They were obtained during the first and second loading cycle. The data for the transverse stiffness (solid circles) were obtained during subsequent loading cycles. The stiffening of the interface illustrated by these results is a typical consequence of the occurrence of plastic deformation at the interface. It is of interest to observe the poor agreement between the best fitting curves the models were able to yield and this set of experimental data. A close examination of the parameters used in this simulation also indicates that the value of the

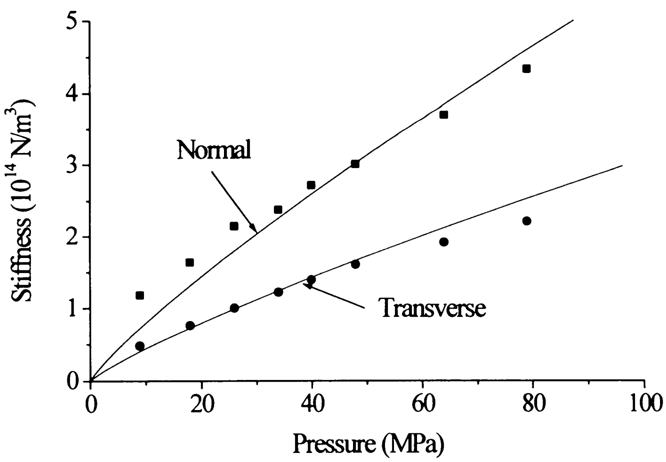


Figure 2. Normal and transverse stiffness constants of a single interface. The surface r.m.s. roughness is $\sigma = 0.68$ μm. The data were acquired at normal incidence.

Table 1. Model parameters.

| | $\sigma$ (μm) | DoF | n (μm$^{-2}$) | $Z_{max}$ | $\beta$ | $d_o$ |
|---|---|---|---|---|---|---|
| Figure 2 | 0.68 | 16 | 2 | 3 $\sigma$ | 2 $\sigma$ | 0.9 $Z_{max}$ |
| Figure 3 | 2.4 | 15 | 5 | 4 $\sigma$ | 2 $\sigma$ | 0.88 $Z_{max}$ |
| Figure 5 | 0.2 | 15 | 24 | 3 $\sigma$ | 2 $\sigma$ | 0.8 $Z_{max}$ |

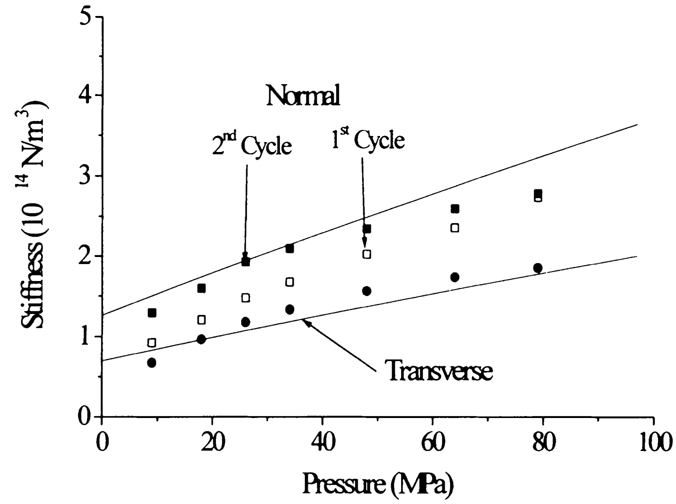


Figure 3. Normal and transverse stiffness constants of a single interface. The surface r.m.s. roughness is $\sigma = 2.4$ μm. The data were acquired at normal incidence.

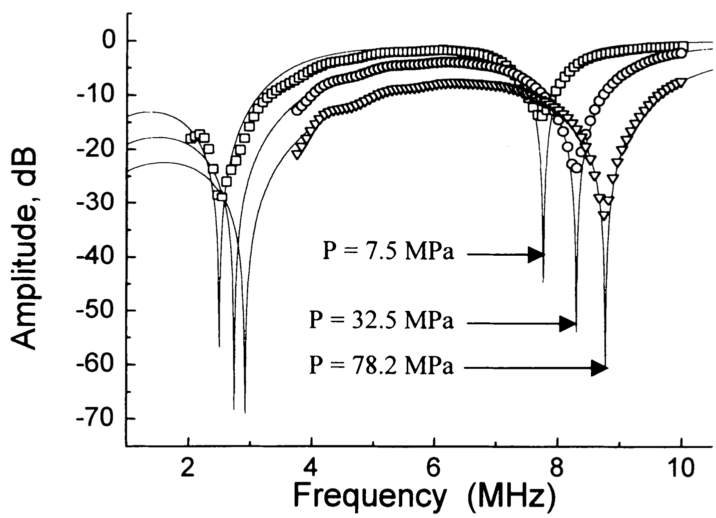


Figure 4. Longitudinal wave spectra reflected at normal incidence from a double imperfect interface. The solid lines are the best fitting curves according to the QSA.

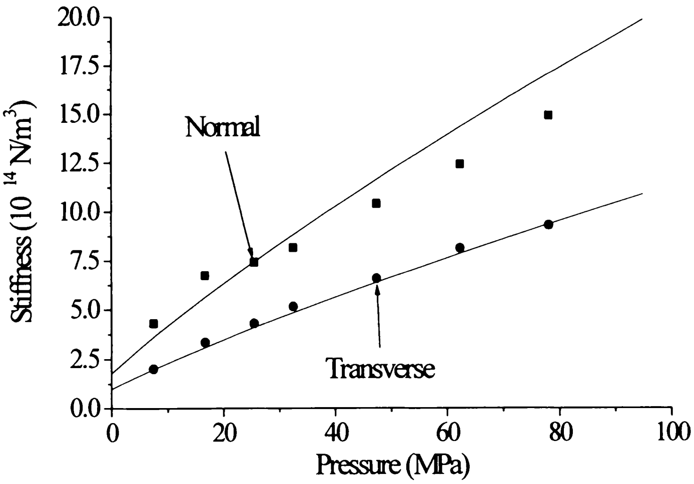


Figure 5. Comparison between experimental and theoretical values of $K_N$ and $K_T$ for a double interface system.

peak density is unreasonably high. In conclusion, these results quite clearly illustrate that the occurrence of plastic deformation poses a limit to the validity of the models used here.

Double Imperfect Interface

Following the approach already used by Lavrentyev and Rokhlin [9] on a similar system, ultrasonic spectroscopy was used to evaluate the interfacial stiffness of a double imperfect interface from the normally reflected power spectra of both longitudinal and shear waves. The roughness of the four surfaces was estimated to be about 0.2 μm. Figure 4 shows three examples of typical spectra of reflected longitudinal waves for increasing values of the applied pressure. The solid lines are the best fitting curves according to the QSA. Both the values of the stiffness constant and of the thickness layer were used as fitting parameters. Similar results were obtained by using shear waves at normal incidence.

The values of $K_N$ and $K_T$ obtained by best fitting the experimental spectra were compared with the models' predictions for a single imperfect interface. Figure 5 illustrates the comparison between the estimated and the theoretical values of $K_N$ and $K_T$. The satisfactory agreement shows that the values of the stiffness constants of the double interface are not affected by the structure of the system, and thus the single interface model provides a sufficiently accurate description of the double imperfect interface system under examination here.

The values of $K_N$ and $K_T$ obtained at normal incidence were used to predict the response of the double imperfect interface at oblique incidence. Figure 6 shows an example of the measured and predicted spectra of a longitudinal wave reflected at 40 degrees from the double interface. The theoretical results reproduce the main features of the experimental spectrum, and, in particular, correctly place the minimum of the spectrum. Similar results were obtained for other values of the angle of incidence.

CONCLUSIONS

The main conclusion of this investigation is that the elastic response of a double imperfect interface formed by rough surfaces in contact can be described by a single pair of

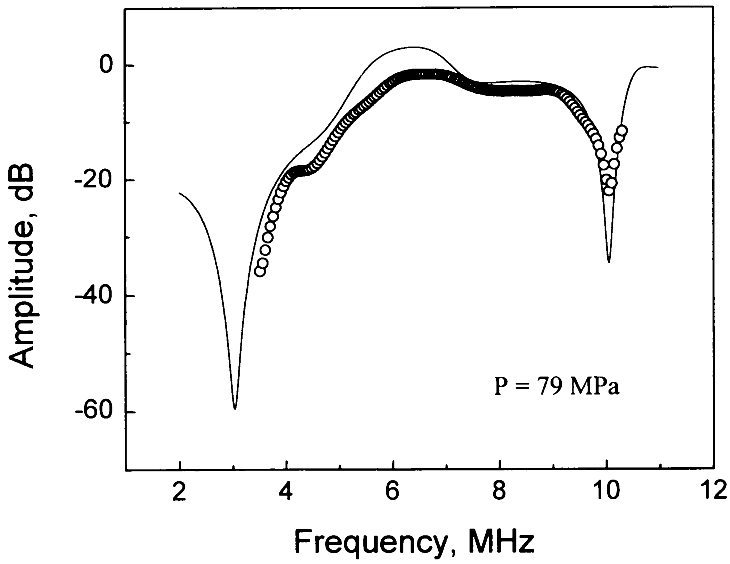


Figure 6. Experimental and theoretical spectra of a longitudinal wave at 40 degree of incidence. The stiffness constants are those obtained at normal incidence.

stiffness constants whose values are independent of the angle of incidence of the interrogating wave. This conclusion could not have been assumed to be true *a priori*. In fact, the macroscopic elastic response of the interface is determined by the scattering properties of the interfacial defects, and the scattering cross-section of the latter depends on the angle of incidence. Therefore, the possibility that the interface as a whole would have macroscopic properties dependent on the direction of incidence of the interrogating wave could not be excluded prior to this investigation.

Finally, the micromechanical models used in this work to predict the dependence of the interfacial stiffness constants on the applied normal pressure have been shown to provide a sufficiently accurate description of the elastic behavior of the system. However, as discussed by Brown and Scholz [11], since several model parameters are grouped in clusters, the determination of their values based on macroscopic measurements is affected by a certain degree of uncertainty. Furthermore, upon the occurrence of plastic deformation at the contacts between the surfaces the models have been shown not to predict the values of the interfacial stiffness constants correctly. Additional work is therefore required to extend the validity of these micromechanical models.

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