DETERMINATION OF THE EQUATION OF STATE OF CONCRETE BY

ULTRASONIC METHODS

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INTRODUCTION

The development of modeling efforts to simulate the high strain, high strain rate events that occur as a structure is impacted by a projectile requires a large number of variables to accurately determine the behavior of a material under such physical conditions. Amongst the parameters that must be included in such a model is the equation of state, which describes the change in material properties as a function of pressure. For many materials, the equation of state can be obtained by using a shock wave method which requires that the material be subjected to an impact by a metallic plate traveling at very high velocities by means of a gas gun [1]. To obtain meaningful information from such a test, the material that is impacted must plastically deform. Therefore, the testing of brittle materials, such as concrete, yields limited amounts of meaningful data as the samples fail before the completion of the experiment.

An ultrasonic based approach provides an alternative method that can be used for brittle materials, such as concrete and other ceramics. Measurements of the ultrasonic shear and longitudinal wave velocities, in combination with density values, allows the determination of the elastic moduli. From the variation of the elastic moduli with pressure and their pressure and temperature derivatives, a semi-empirical equation of state can be determined [2]. This approach has been verified for metallic materials by comparison with results obtained from the shock wave method.

The ultrasonic approach has several distinct advantages when compared to traditional shock wave methods to obtain the equation of state. Since the measurements are performed under true hydrostatic conditions, no shear stresses are generated over the exterior of the sample. The absence of shear stresses prevents fracture of brittle material, such as concrete, and eliminates potential phase transitions that are driven by shear

stresses. With this approach, a broad range of brittle materials with any value of porosity can be readily evaluated without custom sample constraints. The result of this method is a direct and accurate determination of the equation of state that yields true thermodynamic variables for the test material. These, in turn, can be directly substituted into computer models that examine the behavior of concrete and other brittle materials as they are subjected to high strain, high strain rate events.

THEORETICAL BACKGROUND

The elastic moduli of concrete, mortar and cement samples were obtained by measurement of the ultrasonic velocities for the samples as a function of hydrostatic pressure. The bulk modulus was determined from the measured ultrasonic velocities by using the following well known equation [3]:

$$B_{so} = \rho_o (C_l^2 - 4/3(C_t^2)) \tag{1}$$

where B_{so} is the adiabatic bulk modulus, C_l is the ultrasonic longitudinal wave velocity, C_t is the ultrasonic shear wave velocity, and ρ_o is the density. The bulk modulus was determined as a function of pressure, enabling the pressure derivative of the bulk modulus to be calculated. With these two parameters it is possible to obtain the equation of state by means of the following expression [4]:

$$\frac{V}{V_0} = \left(1 + P\frac{B_{To}'}{B_{To}}\right)^{-1/B_{To}'} \tag{2}$$

where B_{To}' indicates the pressure derivative of the isothermal bulk modulus (B_{To}) at atmospheric pressures, and V and V_0 are the volume of the solid at pressure P and at atmospheric pressure, respectively. This equation is referred to as the Murnaghan Equation of State.

It is important to note that the ultrasonic method measures the adiabatic bulk modulus, whereas calculation of the equation of state requires the isothermal bulk modulus. However, the two different thermodynamic variables can be related using the following conversion [5]:

$$B_{so} = \frac{B_{To}}{1 + \Delta} \tag{3}$$

where: $$\Delta = \beta^2 T B_{so} / C_p \rho_o$$

and Δ is the adiabatic to isothermal conversion factor, β is the volume coefficient of thermal expansion, T is the temperature (K), C_p is the heat capacity at constant pressure, and ρ_o is the density at atmospheric pressure. The isothermal pressure derivative, B'_{To}, of the bulk modulus is obtained from the adiabatic pressure derivative by a conversion derived by Overton [5]:

$$\left(\frac{\partial B_{To}}{\partial P}\right)_T = \left(\frac{\partial B_{so}}{\partial P}\right)_T + \frac{\Delta}{1+\Delta}\left[1 - \frac{2}{\beta B_{To}}\left(\frac{\partial B_T}{\partial T}\right)_P - 2\left(\frac{\partial B_{so}}{\partial P}\right)_T\right]$$
$$+ \left(\frac{\Delta}{\Delta+1}\right)^2\left[\left(\frac{\partial B_{so}}{\partial P}\right)_T - 1 - \frac{1}{\beta^2}\left(\frac{\partial \beta}{\partial T}\right)_P\right] \tag{4}$$

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As the pressure measurements are performed at approximately room temperature, the coefficient of thermal expansion, β, can be assumed to be independent of temperature, eliminating the temperature derivative in equation (4). By applying the Overton relationship, the temperature derivative of the isothermal bulk modulus for concrete was found to deviate less than 0.2% from the temperature derivative of the adiabatic bulk modulus determined from the ultrasonic data. Therefore, the temperature derivative of the adiabatic bulk modulus was used in the calculation of the equation of state. It is possible to obtain the temperature derivative of the adiabatic bulk modulus by measuring the ultrasonic longitudinal and shear wave velocities of concrete as a function of temperature. However, evaluation of these parameters has demonstrated that they do not vary as a function of temperature for the materials that have been characterized over the temperature ranges in question [6]. Thus, the ultrasonic approach provides a simple and highly accurate technique to determine the equation of state.

EXPERIMENTAL APPROACH

The samples that were examined were prepared at the Building Materials Division of the National Institute of Standards and Technology (NIST) according to ASTM Standard C 305-91. Table 1 provides a list of the constituent materials for each sample. Note that the ratio of water to cement and sand to cement were used to control the porosity for each sample. The porosity for each sample was determined using ASTM Standard C 642 – 90. The samples were cut into rectangular blocks that measured no more than 0.6 x 0.6 x 1.5 inches, enabling them to fit inside the 1.0 inch diameter, 6 inch long high pressure cell. Prior to placing the sample into the cell, quartz transducers were dry coupled to the sample. Once attached, the entire sample was encapsulated in a water proof membrane to prevent impregnation of the porous samples by the pressure transfer fluid. A simplified schematic of the experimental configuration is shown in Figure 1.
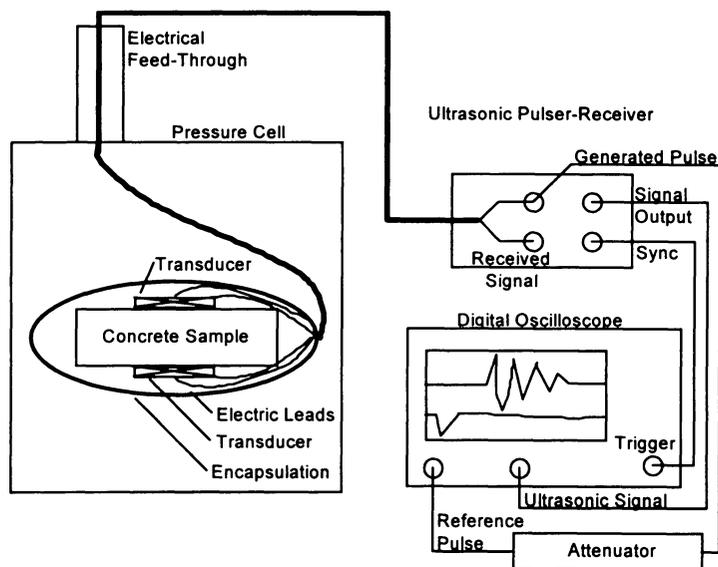

Figure 1. Simplified schematic diagram of the experimental configuration used for evaluating the elastic properties of brittle materials as a function of pressure.

Table 1. Sample porosity and composition.

Porosity	Cement (g)	Water (g)	W/C Ratio	Sand (g)	S/C Ratio	Plasticizer
24%	250	90	0.36	0	0	0
17%	250	72.5	0.29	350	1.4	0.50%
13.6%	250	75	0.30	450	1.8	0.50%
12.7%	250	85	0.34	350	1.4	0.50%

The ultrasonic measurements were performed at a frequency of 2.25 MHz. A velocity measurement was made for each sample prior to encapsulation. Once in the pressure cell, a second measurement was performed at ambient pressure. The pressure was increased in approximately 5 to 10 ksi increments to a total pressure of 105 ksi. At each increment, the pressure was held for a period of over 5 minutes to allow the temperature inside the pressure cell to equilibrate with room temperature. After the dwell period, the ultrasonic measurement was performed. The waveform was averaged for 100 pulses and the time-of-flight was recorded to the first negative peak of the wave. A sample waveform from a longitudinal signal at 105 ksi is shown in Figure 2. Note that the leading edge was not used for time-of-flight measurements due to the low signal-to-noise-ratios (SNR) at pressures below 15 ksi. This SNR level was attributed to the dry acousto-coupling between the transducer and sample which is less effective at low hydrostatic pressures.

To obtain the velocity of the ultrasonic wave in the mortar and cement samples, the compression of the sample due to the elevated pressure must be included in determining the travel path of the ultrasonic wave. The length of the travel path was found not to significantly change as a function of pressure by using Cook's Relation [2]. This relationship provides method to correct for sample compression due to applied hydrostatic pressure:

$$\frac{l_o}{l_p} = 1 + \frac{1+\Delta}{\rho_o} \int_b^p \frac{dp}{3(V_{lo}^p)^2 - 4(V_{lo}^p)^2} \qquad (5)$$
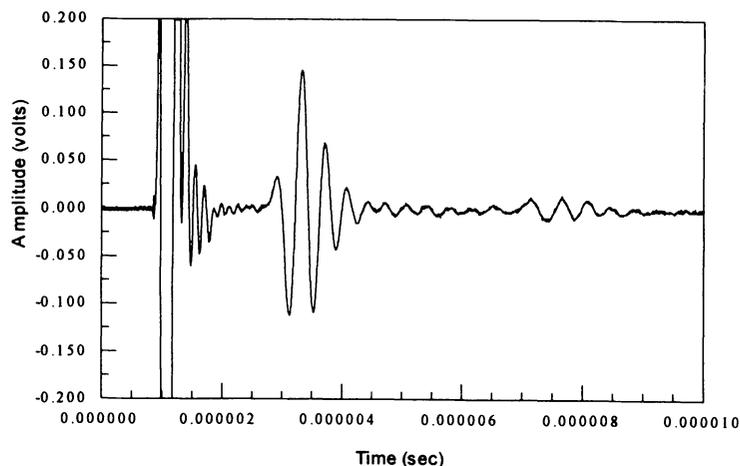

Figure 2. Received ultrasonic waveform for mortar sample with 13.6 percent porosity at a hydrostatic pressure of 105 ksi.
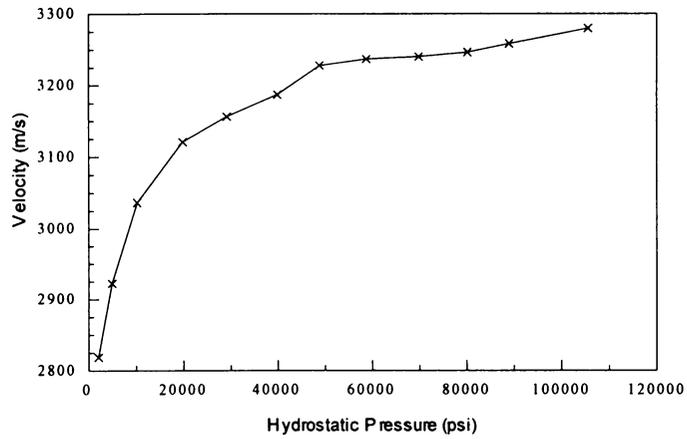
Figure 3. Shear wave velocity as a function of hydrostatic pressure for the mortar sample with 13.6% porosity.
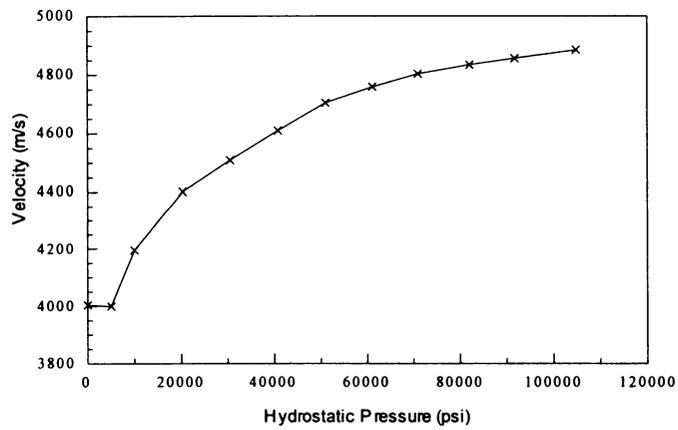

Figure 4. Longitudinal wave velocity as a function of hydrostatic pressure for the mortar sample with 13.6% porosity.
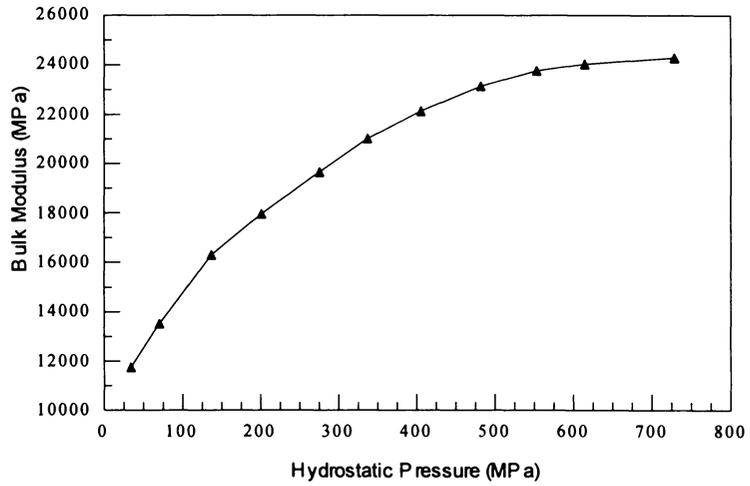
Figure 5. Bulk modulus as a function of hydrostatic pressure for the mortar sample with 13.6% porosity.
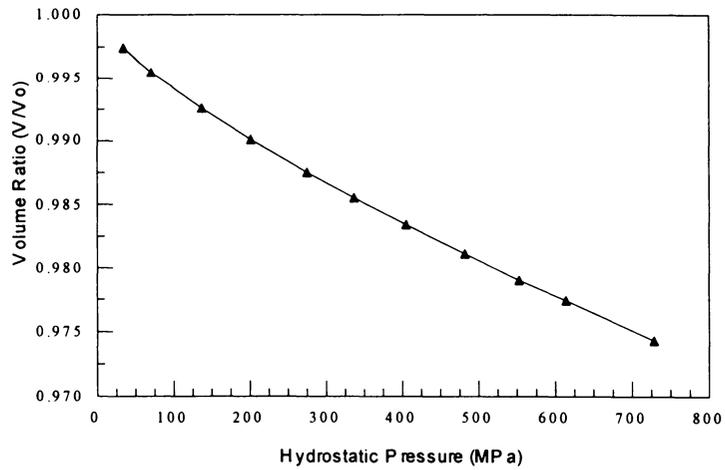

Figure 6. The equation of state for the mortar sample with 13.6% porosity.

where l_o is the acoustic wave path length at atmospheric pressure, l_p is the acoustic wave path length at applied pressure, ρ_o is the density at atmospheric pressure, p is the pressure, V_{lo}^p is the approximate longitudinal wave velocity calculated with the acoustic wave path length at atmospheric pressure, and V_{tp}^p is the approximate shear wave velocity calculated with the acoustic wave path length at atmospheric pressure. Using this relation, the change in travel path due to pressure was calculated to be less than 0.1%. The small change in dimension was confirmed by measuring the thickness of the sample prior to each characterization at pressure and subsequent to the completion of such a pressure run. It is important to note that there is an initial compaction of the sample when it is first exposed to elevated pressures. This causes an approximate change of 1% in the dimensions of the sample. This initial compaction is irreversible and is performed prior to the experimental procedure that leads to the calculation of the equation of state.

RESULTS AND DISCUSSION

The ability to determine the equation of state from the ultrasonic velocities will be demonstrated for the mortar sample with 13.6% porosity. The shear and longitudinal velocities as a function of pressure are shown in Figures 3 and 4, respectively. Using the data from Figures 3 and 4, in combination with the known density of this sample, it is possible to determine the bulk modulus as a function of hydrostatic pressure, shown in Figure 5. Using a curve fitting routine, an approximate equation for the curve shown in Figure 5 was determined. By taking the derivative of this equation and substituting the values of pressure into the derivation, the values of the pressure derivative of the bulk modulus have been determined. These values of the bulk modulus and the pressure derivative of the bulk modulus were substituted into Murnaghan's Equation of State (Eqn. 2) to yield the equation of state for the mortar sample with 13.6% porosity. The equation of state, as fully determined, is given in Figure 6. Performing similar analysis will give the equation of state for the three other porosity values were being evaluated.

There is a noteworthy feature of the data shown in Figure 6. The behavior of the curve at higher pressures appears to approach an asymptotic value with respect to increasing pressure. This trend is even more apparent in Figure 5, where the data points indicate a definite exponential behavior that eventually will cause the bulk modulus to attain a fixed value at increasing pressures. This corresponds with the expected behavior for concrete and similar materials. As they approach this asymptotic value at approximately 150 ksi, all the porosity is squeezed from the sample, rendering it completely devoid of any open cell porosity. Once this occurs, the trend in the data can be extrapolated to much higher pressures. The only limit on the extrapolation is the occurrence of a phase transition in the material being evaluated [2]. Therefore, it is expected that the equation of state that is determined by this method can be extended up to pressures where there may be a phase transition.

SUMMARY

The present work has demonstrated the ability to determine the equation of state for brittle materials by means of ultrasonic characterization of a sample as a function of hydrostatic pressure. The true thermodynamic variables obtained by this methodology can be substituted into models the enable the determination of the behavior of concrete and similar materials subjected to high strain, high strain rate events. In addition, the results from these measurement presented here can be used to extrapolate the behavior of brittle materials to much higher pressures.

ACKNOWLEDGMENTS

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