1965

Problems of bridge supporting and expansion devices and an experimental comparison of the dynamic behavior of rigid and elastomeric bearings

Jack Howard Emanuel

Iowa State University

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PROBLEMS OF BRIDGE SUPPORTING AND EXPANSION DEVICES
AND AN EXPERIMENTAL COMPARISON OF THE
DYNAMIC BEHAVIOR OF RIGID AND ELASTOMERIC BEARINGS

by

Jack Howard Emanuel

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
DOCTOR OF PHILOSOPHY

Major Subject: Structural Engineering

Approved:

Signature was redacted for privacy.

In Charge of Major Work

Signature was redacted for privacy.

Head of Major Department

Signature was redacted for privacy.

Dean of Graduate College

Iowa State University
Of Science and Technology
Ames, Iowa

1965
CONTENTS

INTRODUCTION 1
Scope of the Investigation 2

REVIEW OF LITERATURE 4
Design Specifications 5
Thermal Expansion of Concrete and Steel 13
Concrete Expansion and Growth 16
Differential Temperature 17
Abutments, Piers, and Approach Slabs 19
Supporting and Expansion Devices 22
Supporting devices 22
Elastomeric pads 23
Other types of supporting devices 24
Expansion devices 25

SURVEY OF DESIGN PRACTICE 28

FIELD OBSERVATIONS 29

FACTORS WHICH INFLUENCE THE BEHAVIOR OF SUPPORTING AND EXPANSION DEVICES 34

EXPERIMENTAL INVESTIGATION 38
Laboratory Test Bridge 39
Instrumentation 42
Oscillator 49
Bridge Bearings 52
Curved steel sole plates 54
Neoprene pads 55
Testing Procedure 58
Forced vibration tests 59
Natural frequency 60
Static load tests 61

RESULTS OF EXPERIMENTAL INVESTIGATION 64
Static Load Tests 64
Natural Frequencies 67
Bridge Damping 74
Vibration Tests 75
Strain 82
Strain amplification factor 83
Deflection 84
Midspan deflection 84
End deflection 85
LIST OF TABLES

Table 1. Comparison of AASHO and German specifications 12
Table 2. Properties of test bridge 44
Table 3. End reactions--bridge only--and deviation of relative heights after shimming 55
Table 4. Properties of neoprene pads 58
Table 5. Typical calibration constants from static load tests 66
Table 6. Experimental natural frequencies 71
Table 7. Comparative natural frequencies 72
Table 8. Logarithmic decrements and viscous-damping factors 76
Table 9. Maximum reduced values from strain data 80
Table 10. Maximum reduced values from deflection data 81
Table 11. Tabulation of bridges and items observed 140
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fig. 1</td>
<td>Test bridge</td>
<td>41</td>
</tr>
<tr>
<td>Fig. 2</td>
<td>End supports</td>
<td>41</td>
</tr>
<tr>
<td>Fig. 3</td>
<td>General details of test bridge</td>
<td>43</td>
</tr>
<tr>
<td>Fig. 4</td>
<td>Midspan strain gages and deflectometers</td>
<td>46</td>
</tr>
<tr>
<td>Fig. 5</td>
<td>End deflectometer and curved steel sole plate</td>
<td>46</td>
</tr>
<tr>
<td>Fig. 6</td>
<td>End support with curved steel sole plate</td>
<td>47</td>
</tr>
<tr>
<td>Fig. 7</td>
<td>End support with neoprene pad</td>
<td>47</td>
</tr>
<tr>
<td>Fig. 8</td>
<td>Recording equipment</td>
<td>48</td>
</tr>
<tr>
<td>Fig. 9</td>
<td>Oscillator</td>
<td>50</td>
</tr>
<tr>
<td>Fig. 10</td>
<td>Oscillator</td>
<td>50</td>
</tr>
<tr>
<td>Fig. 11</td>
<td>Curved steel sole plate, bearing (masonry) plate, and neoprene pads</td>
<td>53</td>
</tr>
<tr>
<td>Fig. 12</td>
<td>Stress-deflection curves for neoprene pads</td>
<td>57</td>
</tr>
<tr>
<td>Fig. 13</td>
<td>Oscillator with concrete blocks</td>
<td>62</td>
</tr>
<tr>
<td>Fig. 14</td>
<td>Oscillator with concrete blocks</td>
<td>62</td>
</tr>
<tr>
<td>Fig. 15</td>
<td>Typical oscillograph chart--strain</td>
<td>77</td>
</tr>
<tr>
<td>Fig. 16</td>
<td>Typical oscillograph chart--deflection</td>
<td>78</td>
</tr>
<tr>
<td>Fig. 17</td>
<td>Typical supporting and expansion devices</td>
<td>137</td>
</tr>
<tr>
<td>Fig. 18</td>
<td>Typical combinations of supporting and expansion devices</td>
<td>138</td>
</tr>
<tr>
<td>Fig. 19</td>
<td>Typical irregularities observed</td>
<td>150</td>
</tr>
<tr>
<td>(a) Group 8, bridge (a)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b) Group 8, bridge (d)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fig. 20</td>
<td>Typical irregularities observed</td>
<td>151</td>
</tr>
<tr>
<td>(a) Group 9, bridge (a)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b) Group 10, bridge (c)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fig. 21</td>
<td>Typical irregularities observed</td>
<td>152</td>
</tr>
<tr>
<td>(a) Group 28, bridge (a)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b) Group 33, bridge (b)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fig. 22. Typical static load test calibration curve. Relationship between unit strain in load cell and attenuator-lines, beam A.

Fig. 23. Computer plot of strain-frequency relationship for beam A, curved steel sole plates, oscillator only, $W = 3.48$ lb, $e = 4.51$ in.

Fig. 24. Computer plot of strain-frequency relationship for beam C, 64 durometer neoprene pads, oscillator with concrete blocks, $W = 3.48$ lb, $e = 4.51$ in.

Fig. 25. Computer plot of strain amplification factor-frequency relationship for beam A, curved steel sole plates, oscillator only, $W = 3.48$ lb, $e = 4.51$ in.

Fig. 26. Computer plot of deflection-frequency relationship for beam 3-1, 64 durometer neoprene pads, oscillator with concrete blocks, $W = 3.48$ lb, $e = 4.51$ in.

Fig. 27. Computer plot of deflection amplification factor-frequency relationship for beam 3-1, 64 durometer neoprene pads, oscillator with concrete blocks, $W = 3.48$ lb, $e = 4.51$ in.

Fig. 28. Computer plot of deflection amplification factor-frequency relationship for north end of beam 3-1, 64 durometer neoprene pads, oscillator with concrete blocks, $W = 3.48$ lb, $e = 4.51$ in.

Fig. 29. Strain-frequency curves for beam A, oscillator only, $W = 3.48$ lb, $e = 4.51$ in.

Fig. 30. Strain-frequency curves for beam C, oscillator only, $W = 3.48$ lb, $e = 4.51$ in.

Fig. 31. Partial strain-frequency curves for beam A, oscillator only, $W = 3.48$ lb, $e = 4.51$ in.

Fig. 32. Partial strain-frequency curves for beam C, oscillator only, $W = 3.48$ lb, $e = 4.51$ in.

Fig. 33. Strain amplification factor-frequency curves for beam A, oscillator only, $W = 3.48$ lb, $e = 4.51$ in.

Fig. 34. Strain amplification factor-frequency curves for beam C, oscillator only, $W = 3.48$ lb, $e = 4.51$ in.

Fig. 35. Partial strain amplification factor-frequency curves for beam A, oscillator only, $W = 3.48$ lb, $e = 4.51$ in.
Fig. 36. Partial strain amplification factor-frequency curves for beam C, oscillator only, W = 3.48 lb, e = 4.51 in.

Fig. 37. Strain-frequency curves for beam A, oscillator with concrete blocks, W = 0.82 lb, e = 7.01 in.

Fig. 38. Strain-frequency curves for beam A, oscillator with concrete blocks, W = 3.48 lb, e = 3.26 in.

Fig. 39. Strain-frequency curves for beam A, oscillator with concrete blocks, W = 3.48 lb, e = 4.51 in.

Fig. 40. Strain-frequency curves for beam C, oscillator with concrete blocks, W = 0.82 lb, e = 7.01 in.

Fig. 41. Strain-frequency curves for beam C, oscillator with concrete blocks, W = 3.48 lb, e = 3.26 in.

Fig. 42. Strain-frequency curves for beam C, oscillator with concrete blocks, W = 3.48 lb, e = 4.51 in.

Fig. 43. Strain amplification factor-frequency curves for beam A, oscillator with concrete blocks, W = 0.82 lb, e = 7.01 in.

Fig. 44. Strain amplification factor-frequency curves for beam A, oscillator with concrete blocks, W = 3.48 lb, e = 4.51 in.

Fig. 45. Strain amplification factor-frequency curves for beam C, oscillator with concrete blocks, W = 0.82 lb, e = 7.01 in.

Fig. 46. Strain amplification factor-frequency curves for beam C, oscillator with concrete blocks, W = 3.48 lb, e = 4.51 in.

Fig. 47. Deflection-frequency curves for beam 1-1, oscillator only, W = 3.48 lb, e = 4.51 in.

Fig. 48. Deflection-frequency curves for beam 3-1, oscillator only, W = 3.48 lb, e = 4.51 in.

Fig. 49. Deflection amplification factor-frequency curves for beam 1-1, oscillator only, W = 3.48 lb, e = 4.51 in.

Fig. 50. Deflection amplification factor-frequency curves for beam 3-1, oscillator only, W = 3.48 lb, e = 4.51 in.
Fig. 51. Deflection amplification factor-frequency curves for north end of beam 1-1, oscillator only, \( W = 3.48 \) lb, \( e = 4.51 \) in.

Fig. 52. Deflection amplification factor-frequency curves for north end of beam 3-1, oscillator only, \( W = 3.48 \) lb, \( e = 4.51 \) in.

Fig. 53. Deflection amplification factor-frequency curves for south end of beam 1-1, oscillator only, \( W = 3.48 \) lb, \( e = 4.51 \) in.

Fig. 54. Deflection amplification factor-frequency curves for south end of beam 3-1, oscillator only, \( W = 3.48 \) lb, \( e = 4.51 \) in.

Fig. 55. Deflection-frequency curves for beam 1-1, oscillator with concrete blocks, \( W = 0.82 \) lb, \( e = 7.01 \) in.

Fig. 56. Deflection-frequency curves for beam 1-1, oscillator with concrete blocks, \( W = 3.48 \) lb, \( e = 3.26 \) in.

Fig. 57. Deflection-frequency curves for beam 1-1, oscillator with concrete blocks, \( W = 3.48 \) lb, \( e = 4.51 \) in.

Fig. 58. Deflection-frequency curves for beam 3-1, oscillator with concrete blocks, \( W = 0.82 \) lb, \( e = 7.01 \) in.

Fig. 59. Deflection-frequency curves for beam 3-1, oscillator with concrete blocks, \( W = 3.48 \) lb, \( e = 3.26 \) in.

Fig. 60. Deflection-frequency curves for beam 3-1, oscillator with concrete blocks, \( W = 3.48 \) lb, \( e = 4.51 \) in.

Fig. 61. Deflection amplification factor-frequency curves for beam 1-1, oscillator with concrete blocks, \( W = 0.82 \) lb, \( e = 7.01 \) in.

Fig. 62. Deflection amplification factor-frequency curves for beam 1-1, oscillator with concrete blocks, \( W = 3.48 \) lb, \( e = 4.51 \) in.

Fig. 63. Deflection amplification factor-frequency curves for beam 3-1, oscillator with concrete blocks, \( W = 0.82 \) lb, \( e = 7.01 \) in.

Fig. 64. Deflection amplification factor-frequency curves for beam 3-1, oscillator with concrete blocks, \( W = 3.48 \) lb, \( e = 4.51 \) in.
Fig. 65. Deflection amplification factor-frequency curves for north end of beam 1-1, oscillator with concrete blocks, $W = 0.82$ lb, $e = 7.01$ in.

Fig. 66. Deflection amplification factor-frequency curves for north end of beam 1-1, oscillator with concrete blocks, $W = 3.48$ lb, $e = 4.51$ in.

Fig. 67. Deflection amplification factor-frequency curves for north end of beam 3-1, oscillator with concrete blocks, $W = 0.82$ lb, $e = 7.01$ in.

Fig. 68. Deflection amplification factor-frequency curves for north end of beam 3-1, oscillator with concrete blocks, $W = 3.48$ lb, $e = 4.51$ in.

Fig. 69. Deflection amplification factor-frequency curves for south end of beam 1-1, oscillator with concrete blocks, $W = 3.48$ lb, $e = 4.51$ in.

Fig. 70. Deflection amplification factor-frequency curves for south end of beam 3-1, oscillator with concrete blocks, $W = 3.48$ lb, $e = 4.51$ in.
INTRODUCTION

The design of satisfactory supporting and expansion devices for highway bridges is a problem which has concerned bridge design engineers for many years. The problems associated with these devices have been emphasized by the large number of short span bridges required by the current expanded highway program of expressways and interstate highways.

Bridge supporting devices which have been and are being used vary from elaborate roller nests to cast or welded rockers, self lubricating bronze plates, curved steel plates, flat steel plates, and simple neoprene rubber, fabreeka or oil impregnated asbestos pads. Floor expansion devices may vary from finger joints to sliding plates, premolded joints elastomeric tubes, sponge rubber, cork, armored joints, or poured rubber or asphalt. Some bridges are constructed without any supporting or expansion devices whatsoever.

There appear to be incongruities in existing bridge design practice as to the type of expansion and supporting device required. For example, bridges of equal length, number of spans and general type of construction may show almost any of the devices listed above, or no devices may be used. Also, practice seems to dictate that steel bridges employ the more expensive supporting devices such as rockers or roller nests, whereas, concrete bridges made up of precast units tend to utilize relatively inexpensive bearing pads. The cost of each type of device varies over a wide range, approaching 10 to 15% of the cost of the fabricated steel, for steel bridges, and is often a factor in selection of design.

Field observations show that in many cases the supporting and expansion devices do not function as anticipated by the design engineer.
Common observations include "freezing" of supporting or expansion devices, abutments which have moved inward against the bridge superstructure, closed floor expansion devices, and inconsistencies of rocker movement.

Several engineers have suggested that it may be possible to utilize more extensively the simpler and less expensive expansion and supporting devices used primarily for precast concrete bridges. Lower fabrication costs and reduction of future maintenance, resulting from simplification of design, indicated that further information concerning design of such devices would be valuable to bridge design engineers. Thus a research project was initiated at the Iowa Engineering Experiment Station, Iowa State University, Ames, Iowa, to investigate the requirements for and the behavior of bridge supporting and expansion devices. Deck type highway bridges only were considered with emphasis on bridges of three or more spans, each 50 ft or longer.

Scope of the Investigation

The initial objectives of this investigation were: (1) To review and make a field study of devices used for the support of bridge superstructures and for provision of floor expansion; (2) To analyze the forces or factors which influence the design and behavior of supporting devices and floor expansion systems; and (3) To ascertain the need for future research particularly on the problems of obtaining more economical and efficient supporting and expansion devices, and determining maximum allowable distance between such devices. The experimental portion was conducted to evaluate one of the possible simple and economical solutions
to the problems observed in the initial portion. The investigation reported herein is divided into four major parts or phases as follows:

1. A review of literature;
2. A survey by questionnaire of design practice of a number of state highway departments and consulting firms;
3. Field observation of existing bridges; and,
4. An experimental comparison of the dynamic behavior of rigid and elastomeric bearings.
REVIEW OF LITERATURE

A review of available literature provides an insight to the many factors pertinent to the requirements for and the behavior of bridge supporting and expansion devices. Due to the diversity of these factors the references are grouped, as much as possible, for continuity and clarity.

The problem of expansion and contraction of bridge structures has been recognized for many years. Field observation or a study of bridge designs and specifications of bridges built during the last 30 or 40 years reveals a pattern or "evolution" of various devices in an attempt to successfully cope with this problem. The fact that no satisfactory and economical solution has been found is emphasized by the variety of devices still used.

The design engineer is responsible for the most suitable and economical design of a structure (24). However, economical design is dependent not only upon sound engineering theory but also upon the equally important and more elusive thing called engineering judgment or experience, especially in constructional procedure. For example, the cost of additional flange splices for some "least weight" designs for short span bridges has exceeded the amount "saved" by reduction in the amount of steel (36). One booklet reviews recent progress in the design of short span steel bridges which will permit bridge design engineers to achieve more economy in bridge construction. Possible weight savings are given for some of the suggestions (2).

Various articles have appeared in engineering publications which urge the consideration of more economical design practice successfully
used in other countries. One such article reports that in some foreign
countries bridges are designed for nonuniform temperature distribution,
which is associated with smaller movements than those resulting from an
assumption of constant temperature (51).

Design Specifications

A review of bridge design specifications shows differences and
changes of opinions for provision of contraction and expansion. It would
appear that perhaps many of the first highway bridge design engineers
were former railway engineers and that some of the early specifications
for highway bridges were a continuation of railway practice.

The oldest specifications found were those of the Iowa State Highway
Commission, Series of 1925, of which the following is pertinent to this
investigation (30):

28. Temperature. (Section 4 - Loads)
   All fixed masonry arch spans shall be designed for a
   maximum temperature range of forty (40) degrees F. above
   and forty (40) degrees F. below the temperature at time of
   construction. All fixed steel arches shall be designed for
   a temperature range of one hundred fifty (150) degrees F.

51. Expansion Joints. (Section 8 - Concrete Design)
   Provision for expansion and contraction to the extent
   of one-eighth (1/8) inch for each ten (10) feet of span,
   shall be made for all concrete bridges.
   (Note: The 1/8 inch provision is total, i.e.,
   + 1/16 inch.)

95. Expansion. (Section 9 - Structural Steel Design)
   Provision for expansion and contraction to the extent
   of one-eighth (1/8) inch for each ten (10) feet of span,
   shall be made for all bridges. Expansion ends shall be
   firmly secured against lifting or lateral movement.
   (Note: The 1/8 inch provision is total, i.e., 1/16
   inch expansion and 1/16 inch contraction.)
96. Expansion Bearings.

Spans of less than seventy (70) feet shall be arranged to slide upon metal plates with smooth surfaces, the coefficient of friction being assumed as one-fifth \((1/5)\). Spans of seventy (70) feet and over shall be provided with rollers or rockers, or with special sliding bearings as provided in Par. 100, Div. V. Neither rollers nor rockers shall be used for expansion bearings at the top of trestle posts.

The American Association of State Highway Officials (AASHO) began compiling standard bridge specifications in 1921. The specifications were available for several years in mimeograph form and the first printed edition was in 1931. Revisions were made in 1935, 1941, 1944, 1949, 1953, 1957, and 1961. The following specifications are from the 1961 edition (1).

1. 2. 15.—THERMAL FORCES.

Provision shall be made for stresses or movements resulting from variations in temperature. The rise and fall in temperature shall be fixed for the locality in which the structure is to be constructed and shall be figured from an assumed temperature at the time of erection. Due consideration shall be given to the lag between air temperature and the interior temperature of massive concrete members or structures.

The range of temperature shall generally be as follows:

<table>
<thead>
<tr>
<th>Metal Structures</th>
<th>Concrete Structures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moderate climate, from 0° to 120° F.</td>
<td>Temperature rise</td>
</tr>
<tr>
<td>Cold climate, from -30° to 120° F.</td>
<td>Temperature fall</td>
</tr>
<tr>
<td>Concrete Structures</td>
<td>Temperature</td>
</tr>
<tr>
<td>Moderate climate</td>
<td>30° F.</td>
</tr>
<tr>
<td>Cold climate</td>
<td>35° F.</td>
</tr>
</tbody>
</table>

1. 6. 46.—EXPANSION AND CONTRACTION. (Structural Steel Design.)

The design shall be such as to allow for total thermal movement at the rate of \(1\frac{1}{2}\) inches in 100 feet. Provision shall be made for changes in length of span resulting from live load stresses. In spans more than 300 feet long, allowance shall be made for expansion and contraction in the floor. The expansion end shall be secured against lateral movement.

1. 6. 47.—EXPANSION BEARINGS. (Structural Steel Design.)

Spans of less than 50 feet may be arranged to slide upon metal plates with smooth surfaces and no provisions for deflection of the spans need be made. Spans of 50 feet and greater shall be provided with rollers, rockers, or
sliding plates for expansion purposes and shall also be provided with a type of bearing employing a hinge, curved bearing plates, or pin arrangement for deflection purposes.

In lieu of the above requirements elastomeric bearing pads may be used for spans of 80 feet or less, subject to the following:

(a) The relationship between the loaded face and the side areas expressed as a "Shape Factor". For rectangular-shaped bearings with parallel (not over approximately 5° slope) loading surfaces

\[ S = \frac{ab}{2t(a+b)} \]

where

- \( S \) = shape factor
- \( a \) and \( b \) = length and width
- \( t \) = thickness

(b) The total of the positive and negative movements caused by anticipated temperature change shall not exceed one-half the thickness of the pad.

(c) Unit pressure on elastomeric bearing pads shall not exceed 500 psi under dead load nor 800 psi under a combination of dead load plus live load plus impact. The initial deflection under dead, live, and impact loads shall not exceed 15 percent of the thickness of the pad.

Elastomeric bearing pads shall be cast in a single integral layer except that multiple-layer pads, separated by nonelastic sheets to restrain deformation in thick pads, may be permitted. The variation in thickness in the longitudinal direction (taper) shall not exceed five percent of the length of the pad. The least horizontal dimension of the pad shall not be less than five times the thickness (shape factor 1.25 minimum).

(d) The physical properties of the pads shall conform to the following specifications:

- The pads shall be made of neoprene, shall be cast in molds under pressure and heat. Compositions for pads shall meet the requirements listed. Test specimens shall be in accordance with ASTM Method D 15, Part B.

**Physical Properties**

<table>
<thead>
<tr>
<th>Grade (Durometer)</th>
<th>60</th>
<th>70</th>
</tr>
</thead>
</table>

**Original Physical Properties**

<table>
<thead>
<tr>
<th>Hardness ASTM D 676.</th>
<th>60 ± 5</th>
<th>70 ± 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile strength, minimum psi ASTM D-412</td>
<td>2,500</td>
<td>2,500</td>
</tr>
<tr>
<td>Elongation at break, minimum percent</td>
<td>350</td>
<td>300</td>
</tr>
</tbody>
</table>

Accelerated Tests to Determine Long-Term Aging Characteristics
Oven Aged—70 Hrs./212° F., ASTM D-573

Hardness, points change, maximum . . . . . . . . 0 to + 15 0 to + 15
Tensile strength, % change, maximum . . . . + 15 + 15
Elongation at break, % change maximum . . . . = 40 = 40

Ozone—1 PPM in Air by Volume—20% Strain—100 + 2° F.—
ASTM D-1149

100 hours . . . . . . . . . . . . . . . No cracks No cracks

Compression Set—22 Hrs./158° F., ASTM D-395—Method B
% Maximum . . . . . . . . . . . . . . . . . . . 25 25

Low Temperature Stiffness—ASTM D-797
At—40° F., Young's Modulus, maximum psi. . 10,000 10,000

Tear Test—ASTM D-624—Die "C"
Pounds/lin. in., minimum . . . . . . . . 250 225

1.6.64.—EXPANSION JOINTS. (Floor System.)

To provide for expansion and contraction movement, floor expansion joints shall be provided at the expansion ends of all spans and at other points where they may be necessary.

Apron plates, when used, shall be designed to bridge the joint and to prevent, so far as practicable, the accumulation of roadway debris upon the bridge seats. Preferably, they shall be connected rigidly to the end floor beam.

1.7.3.—EXPANSION. (Concrete Design.)

In general, provision for temperature changes shall be made in all simple spans having a clear length in excess of 40 feet.

In continuous bridges, provision shall be made in the design to resist thermal stresses induced or means shall be provided for movement caused by temperature changes.

Expansion not otherwise provided for shall be provided by means of hinged columns, rockers, sliding plates or other devices.

The German Federal Specifications include the following:

DIN 1073 Jan. 1941 - Computations for Steel Highway Bridges.

For determination of stresses in statically indeterminate bridge systems:

(a) Uniform increase of temperature \( \Delta t = \pm 35° C. \) (\( \pm 63° F. \)) with an initial temperature of \( 10° C. \) (\( 50° F. \)).

---

1 Samples to be solvent wiped before test to remove any traces of surface impurities.

(b) Non-uniform increase of temperature across the height of the cross-section $\Delta t = \pm 15^\circ$ C. ($\pm 27^\circ$ F.).
(Under special conditions the stress due to non-uniform increase in temperature may be ignored. The change in length of the bridge, for determination of expansion bearings, for uniform increase in temperature, $\Delta t = \pm 35^\circ$ C.).

PROPOSED REVISION TO DIN 1073 (refer to items (a) and (b) above)

(a) $\Delta t = \pm 20^\circ$ C. ($\pm 36^\circ$ F.) (steel uniform)

(b) $\Delta t = \pm 5^\circ$ C. ($\pm 9^\circ$ F.) (steel non-uniform)

DIN 1072, 1952 - Loading Assumptions for Concrete Highway and Road Bridges

(a) $\Delta t = \pm 15$-20 C. ($\pm 27$-36 F.) from initial temperature of $+10^\circ$ C. ($+50^\circ$ F.). (From initial temperature of $+10^\circ$ C. for structures with a least dimension of 70 cm. (27.6 in.). If sheltered from excessive temperature variations, the temperature may be decreased by $5^\circ$ C.)

(b) Non-uniform increase of temperature is only considered for special conditions, such as the tie rod in a 2-hinged arch. In this case $\Delta t = \pm 5^\circ$ C.

Standard Specifications for Steel Highway Bridges as published by the Canadian Engineering Standards Association (CESA), 1938, require the following (10):

Temperature 58.

Provision shall be made for temperature stresses due to an extreme variation of 160 degrees Fahrenheit, with a normal temperature of 60 degrees and a range from 40 degrees below zero to 120 degrees above.

Expansion 163.

Provision for expansion, to the extent of one inch for all bridges. Spans of less than 100 feet may be arranged to slide upon steel plates with smooth surfaces; but spans of 100 feet and over shall be provided with turned rollers or rockers, or with special sliding bearings, as described below.

Bridge Manual, 1956 edition, includes the following specifications for New Zealand (42).

3.2.18.- Temperature and Shrinkage Loads. (Loads)

Provision shall be made for stresses or movements
resulting from variations in temperature. The range of temperature to be considered should be:—

For metal structures $\pm 30^\circ$ F. (For special cases a larger range may be advisable.)

For concrete structures $\pm 20^\circ$ F.
For concrete decks on steel girders $\pm 30^\circ$ F.

The coefficient of expansion of concrete may be taken as $0.000006$.

Shrinkage stresses in concrete shall be calculated as the equivalent temperature stresses resulting from a temperature change of $-30^\circ$ F. (For simplicity an overall coefficient of $0.0002$ may be used.)

3.6.46. -- Base Plates (or Sole Plates). (Structural Steel Design)

Base plates on plate girders should have a thickness not less than $3/4$ inch and not less than the thickness of the flange angles plus $1/8$ inch. Base plates should be so designed that under extreme earthquake there are preventers to restrict relative movement between base plate and bed plate.

3.7.1. -- General Assumptions. (Concrete Design)

(13) Piers of bents constructed integrally with footings placed on a skew exceeding $10^\circ$ should be considered fixed at the top of the footing.

In assessing shrinkage and temperature effects and in designing girder sections for negative moment for structures having heavy skew, the increased stiffness of skewed piers should be taken into account. A check calculation should be run using the moments of inertia for piers about axes square to the roadway. The conditions of fixity resulting from this check should be used for assessment of stress for shrinkage, temperature or negative moment in girders.

3.7.5. -- Expansion. (Concrete Design)

In general, provision for temperature changes shall be made in all simple spans having a clear length in excess of 40 feet. These provisions may either be by providing freedom to move or by providing strength which can safely resist induced stress.

In continuous bridges, provision shall be made in the design to resist thermal stresses induced or means shall be provided for movement caused by temperature changes.

Expansion not otherwise provided for shall be provided by means of hinged columns, rockers, sliding plates or other devices. The simpler these mechanisms can be made the better.

3.10.8. -- Stresses Due to Shrinkage of Slab. (Composite Beams)

These are additive to normal stresses due to vertical loadings and must be taken into account. Plastic flow may be
considered to relieve these stresses a certain amount; this may be taken into account by taking the coefficient of contraction as \(2 \times 10^{-4}\).

Although railway bridges are generally of different type deck construction, the American Railway Engineering Association General Specifications for Steel Railway Bridges are of interest in this investigation. The 1931 and 1947 revisions were available. The 1931 revision (4) is for fixed spans less than 300 feet in length and the 1947 edition (5) applies to fixed spans less than 400 feet in length.

From the 1947 edition:

**Types of Bridges (I General Features of Design)**

102. The preferred types of bridges are as follows:
- Rolled beams for spans up to 50 ft.
- Plate girders for spans up to 125 ft.
- Riveted trusses for spans 100 ft or longer.
- Pin-connected trusses for spans 300 ft or longer.

**Expansion (IV Details of Design)**

442. The design shall be such as to allow for the changes in length of the span, resulting from changes in temperature, at the rate of 1 in. in 100 ft. Provision shall be made for changes in length of the span resulting from live load stresses. In spans more than 300 ft long, allowance shall be made for expansion in the floor.

**End Bearings**

443. In spans more than 70 ft long, there shall be hinged bearings at both ends and rollers or rockers at the expansion end. Shorter spans shall be designed to slide on bearing with smooth surfaces.

Bearing and ends of spans shall be secured against lateral movement.

End bearings on masonry preferably shall be raised above the bridge seat by metal pedestals or bolsters.

A comparison of AASHO and German Federal Specifications for temperature range and provision for total expansion are shown in Table 1.
Table 1. Comparison of AASHO and German specifications

<table>
<thead>
<tr>
<th>Specification</th>
<th>Special Conditions</th>
<th>Total Temperature Range, °F</th>
<th>Total Expansion (in.) per 100 ft</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Steel</td>
<td>Concrete</td>
</tr>
<tr>
<td>AASHO 1. 2. 15</td>
<td>Moderate Climate</td>
<td>120</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td>Cold Climate</td>
<td>150</td>
<td>80</td>
</tr>
<tr>
<td>AASHO 1. 6. 46</td>
<td></td>
<td>(160)</td>
<td></td>
</tr>
<tr>
<td>German Existing</td>
<td>Exposed</td>
<td>126</td>
<td>72</td>
</tr>
<tr>
<td></td>
<td>Sheltered</td>
<td>126</td>
<td>54</td>
</tr>
<tr>
<td>German Proposed</td>
<td></td>
<td>72</td>
<td></td>
</tr>
</tbody>
</table>

Expansion Range: Steel 0.56 to 1.25
Concrete 0.42 to 0.62

No basis for the lower temperature range in the German Proposed Specifications has been found. One investigator stated that the seasonal variations for Central European conditions would probably have a maximum of about ± 15° C (± 27° F) if uniform temperature distribution is assumed within each substance, steel and concrete, of a composite beam (48).
Thermal Expansion of Concrete and Steel

Standard values for the coefficient of thermal expansion of concrete and steel are generally accepted as $6.0 \times 10^{-6}$ and $6.5 \times 10^{-6}$ per degree F respectively. This leads to the conclusion that the possibility of stresses due to difference between the thermal expansion of the two materials may be ignored.

However, numerous tests have shown a wide range of values for concrete with different aggregates and proportions. Further, the coefficient of thermal expansion of concrete varies with temperature, humidity, moisture, method of curing, and amount of reinforcing steel.

For temperatures of 200° F to 1100° F the coefficient of expansion for steel may be determined by means of a formula (3, p. 6-9).

Among values given for the coefficient of thermal expansion of concrete with different aggregates are those by David and Meyerhof (13) for temperatures above 32° F. For gravel aggregates, coefficients of $7.3 \times 10^{-6}$ and $6.8 \times 10^{-6}$ were found for air storage and wet storage respectively. For the nine types of aggregates tested the coefficients varied from $4.1 \times 10^{-6}$ to $7.3 \times 10^{-6}$ and $3.4 \times 10^{-6}$ to $6.8 \times 10^{-6}$ for air and wet storage respectively.

Coefficients lower than the accepted value were found for both saturated and oven dry conditions during the Michigan Road Test (39):

Coefficients derived from these tests with specimens in a saturated condition for the temperature range between 32° and 130° F. averaged 0.0000053. In a saturated condition the specimens contained approximately 4.1 percent of absorbed moisture. The concrete in an oven-dry state gave a lower value of 0.0000049 for the coefficient of expansion.
On the basis of these data it was determined that for a temperature of 72° F, the average change in length of the specimens from a dry to a saturated state was 0.000246 inch per inch of length. This value is equivalent to a change in temperature of about 46° F. Assuming the same relative linear contraction of the specimen in all directions, the change in volume from saturated to dry state was 0.075 per cent which agrees closely with the results of Davis (2)\(^*\) for concrete with gravel aggregate. According to Davis, subsequent saturation produces an increase in volume of only about one-tenth of the original contraction for this type of concrete.

Measurements of the coefficient of thermal expansion of different aggregates (British rocks) and of concretes prepared from them with different cements were made over the temperature range 32° F to 104° F by Bonnel and Harper (7). The resultant values show general agreement with other investigators, but disagreement as to the effect of curing and water content. Their research indicates:

The age of concrete has little effect on its thermal expansion.

The thermal expansion of desiccated and water-saturated concretes are identical, and concretes in conditions between these two extremes have higher expansions.

For structural members not exposed to the weather... drying-shrinkage will normally be greater in magnitude than any subsequent thermal movement. Exceptions to this are to be found in flat concrete roof-slabs covered by dark bituminous roofing materials where, because of the high absorption of heat from the sun's rays, the rise within the slab can be considerable.

In structures exposed to the weather, wetting and drying movements will be superimposed on the thermal movements but, whereas the latter are both diurnal and seasonal, the former are mainly seasonal. Since, however, an increase in temperature will usually be associated with some degree of drying, the net movement will be less than that calculated from the thermal expansion.

Walker, Bloem, and Mullen (64) also found that coefficients of expansion for saturated and oven-dry conditions are similar while those for a partially dry condition are considerably higher.

\(^*\)Davis (14, p. 677).
Tests of thermal contraction and expansion of concrete show a variation with temperature. Values for 8 in. by 10 in. by 3 ft specimens with gravel aggregate are given by Chow (12). Some of his general conclusions were:

The thermal coefficient of concrete during a freezing cycle is less than that observed during a thawing cycle.

The coefficient of thermal contraction and expansion varies with the temperature. The coefficients have smaller values than the one commonly accepted. The average coefficient of thermal contraction of concrete made of limestone or gravel aggregate is equal to 55% or 70% respectively of the value for carbon steel which is 0.0000065 per degree F.

A certain amount of "residual expansion" appears at the end of a complete cycle of freezing and thawing.

Valore (63) reported that residual expansion for repeated cycles of freezing and thawing are cumulative.

The thermal coefficient of expansion of concrete was reported by Hatt (27) to vary from 4.0(10)^{-6} at 65° F to 6.0(10)^{-6} at 140° F.

Strom (60) reports that when a pavement is covered with asphalt the expansion of the pavement is cut down to a great degree. He refers to limited tests made on two 7-in. pavement slabs, one exposed and the other having a 3-in. bituminous concrete. The maximum temperature difference between thermocouples set at the top surface of the concrete and 1 in. above the bottom surface was 22° F for the exposed slab and 14° F for the surfaced slab. The temperature of the surfaced slab was about 64% of that of the exposed slab and would reduce the expansion of the slab to a considerable extent.

Numerous articles such as one by Mercer (37) report concrete failures resulting from a difference in thermal expansion of the aggregate and
Concrete Expansion and Growth

Concrete growth may be due to either chemical or physical causes. The problem of concrete growth has been the subject of extensive field and laboratory work. Chemical causes, especially those due to alkali reaction, were not considered in this investigation as many states have discontinued the use of high alkali cements and deleterious aggregates.

Physical causes of concrete growth include freezing-thawing, wetting-drying, and heating-cooling. The variation of the thermal coefficient of expansion of concrete with temperature was found by Chow (12) to result in growth at the end of a cycle of cooling and warming. The same investigation showed a definite growth due to alternate wetting and drying. Old specimens of concrete which have been exposed to conditions of freezing and thawing were found to have the tendency to grow more than newly prepared specimens. Specimens with reinforcing steel showed a smaller change in length due to a change in moisture conditions of the concrete than those without steel.

Growth due to moisture does not require visible wetting, but is also influenced by humidity of the air. David and Meyerhof (13) report that concrete growth will produce stresses in composite bridges similar to those produced by differences in thermal expansion coefficients of steel and concrete at low temperatures. Davis (15) reported that for thoroughly dried concrete bars, air of high relative humidity has been found to be
nearly as effective in producing expansions as is the immersion in water. Lea and Davey (33) state that the drying shrinkage and reversible wetting movement of concrete are of the same order as the seasonal thermal movement, approximately $\frac{1}{4}$ in. in 100 ft. Mercer (38) reported that a paper presented by Professor A. H. White in 1915 contained the information that a small bar cut from a sidewalk after 20 years' service elongated 0.175 per cent by successive immersions at room temperature, and successive long immersions with intermediate dry periods caused progressive expansion far greater than changes due to temperature. Mercer also warned against consideration of moisture change as an equivalent thermal change since the influence upon the structure through the thermal coefficient can be vastly different from the effects of permeability and moisture movement.

Differential Temperature

Although the thermal coefficients of expansion of concrete and steel are considered approximately equal, other factors are involved in the calculation of thermal stresses and movement.

The horizontal deck of the bridge is exposed to solar radiation to a different degree than a vertical member, say a pier or abutment. Also, portions of the bridge are exposed to direct sunlight while others are entirely shaded. Further, the superstructure and substructure of a bridge over a stream are subjected to different air and solar effects than those of a bridge over a highway.

Movement and stresses are also influenced by such factors as time-lag between the change in temperature and flow of heat, thermal resistance and capacitance and color. Article 1. 2. 15 of the 1961 AASHO
Specifications (1) states "Due consideration shall be given to the lag between air temperature and the interior temperature of massive concrete members or structures." No indication was found as to the background of the statement or what is meant by "massive." It also requires provision for a larger thermal movement for steel bridges than for concrete bridges. No background for this requirement could be found during this investigation other than, perhaps, temperature lag, since steel should reach a uniform temperature change more rapidly than a mass of concrete.

Naruoka, Hirai and Yamaguti (41) measured the temperature distribution of an asphalt pavement covered concrete bridge slab with steel girders. The asphalt pavement was 50 mm thick and the concrete slab 150 mm thick. One steel girder was 900 mm deep, the other 1120 mm deep. The temperature was measured at 10 to 20 minute intervals from about 9 a.m. to 7 p.m. during two days in July. Highest temperatures in the concrete slab were reached at 4 p.m. Prior to that time, when apparently water was scattered on the asphalt, the surface temperature of the asphalt was higher than that of the concrete. Interior temperatures of the concrete slab were not uniform and, in general, were higher than for the steel.

From the above investigation, it would appear that a concrete bridge deck would undergo larger daily thermal movement than the steel girders. Also, for prolonged periods of extreme heat or cold it would seem reasonable that concrete and steel girders would experience similar magnitudes of seasonal movement, assuming nearly equal coefficients of thermal expansion.

Changes in strain and stress directly proportional to the air temperature (only) were observed by Reinitzhuber (48).
The maximum radiant heat per hour on a structure for any hour of the day may be obtained from handbooks. The diurnal variation is generally sinusoidal, or nearly so, the period of oscillation depending upon several variables. Stephenson (59) reported that theoretical values of curvature and stress due to total restraint of tall slender columns subjected to solar radiation can be made if such factors as density, specific heat, thermal conductivity, absorptivity factor, surface resistance, coefficient of thermal expansion, modulus of elasticity of the concrete and latitude of the structure are known.

Abutments, Piers, and Approach Slabs

Among the forces acting upon piers and abutments are those generated by solar radiation, vertical and longitudinal traffic loads transmitted through the superstructure, and surcharge load of embankments and wheel loads. Many cases of abutment movement have resulted from growth of approach slabs, compaction and/or settlement of approach fill, or "freezing" of support or expansion devices.

Longitudinal traffic loads are discussed by Erickson and Van Eenam (19) in an article on the application and development of AASHO Specifications to bridge design.

Settlement of approach slabs and movement of abutments is accepted as commonplace. According to Peck and Ireland (47) these are due in many cases to improper compaction or use of improper material or constructional procedure. Approaches of a number of bridges in California have been studied by Jones (32). The investigation revealed that more approach patching was required for closed-abutment bridges than for open-end
structures. This was believed due largely to better compaction of the
approach fill for open-end structures by construction equipment and
consolidation of the underlying ground by the weight of the approach fill,
usually completed before the bridges are built. Little difference was
found in the amount of approach surface patching for structures on piles
as compared with those on spread footings. Although some settlement was
often noted during the first year, approach patching generally was not
needed until 2 or 3 years after the road was opened to traffic. Jones
proposes several measures for reducing surface irregularities on bridge
approaches.

Examples of typical approach slab action are given by Strom (60):

(1) One Highway Department investigated the expansion of concrete
pavements and concluded that due to lack of reinforcing,
the concrete pavement never came back to its original
length after expanding. Hair cracks, formed during
expansion, filled with dirt during rain, and the pave-
ment became longer and longer.

(2) In Arkansas, 3-in. slots were cut in the pavement about
50 ft from the end of various bridges. The slot was fill-
ed with a mixture of sawdust and asphalt and covered with
3/4 in. of joint filler. Approximately three years later,
the pavement had moved 2 or more inches and the sawdust and
asphalt mixture was compressed so that sections could be
lifted out like a board. Five-inch slots were then cut in
these pavements. Abutment backwalls of bridges throughout
the state have been broken by pavement movement.
(3) In another instance the pavement pushed the approach
spans so that the bank pier of a 130-ft span was 2-3/8 in.
out of plumb.

Strom reported that the expansion of the pavement is greatly reduced
when the pavement is covered with asphalt.

In an effort to eliminate settlement of approach slabs and excessive
movement of abutments, which frequently results in broken abutment back-
walls or closing of expansion devices, many states are experimenting with
granular backfills, predrilling for piling, and stub type abutments.
Another apparent method would be the provision for a greater amount of
movement in the expansion devices. However, it is also apparent that this
method would only tend to delay rather than eliminate this source of
trouble and could, in fact, result in an increase of abutment backwall
failures. A better method proposed by Girten (24) would be to anticipate
and forestall this type of failure by fixing the superstructure to
flexible piers and stub abutments. In this case the girders are tied
to the piers and abutments and only a single row of vertical piles is
used in the abutments to support the vertical loads. This type of design
has been used on bridges with a total length between abutments of 300 ft.
Also, since there is no relative movement between the abutment and the road-
way slab, no roadway expansion is provided. Instead, the roadway
slab is carried across the top of the abutment to a mastic joint at the
back face of the abutment.

One of the bridges observed during the course of this investigation
was constructed similar to the method outlined above. Constructed in 1934,
the three span, 190 ft continuous steel beam bridge was fixed at all
points of support. A single row of vertical piling with concrete cap was used at the ends of the bridge with a separate retaining wall spaced 3/4 in. at 80° F from the end of the bridge. The concrete center piers rest upon two vertical rows of piling. No roadway expansion device other than mastic was used. The bridge was in very good condition and the designer* reported:

The bridge was the first we had designed, whereby the piers and columns were deflected to take care of the expansion of the bridge. It has proven very satisfactory, since we have had no maintenance except an occasional painting of structural members. The bridge has been subject to the ravages of the Floyd River and the great increase of traffic over the past many years with no appreciable effect.

One State Highway Department reported the use of this type of construction over a period of the past 15 years.

Supporting and Expansion Devices

**Supporting devices**

Rockers and roller nests are among the oldest supporting devices used for provision of expansion of the superstructure. Originally used for truss bridges, their present use on deck type bridges is, to some extent, a "carry-over" from former usage. Corrosion and "freezing" of these devices is not unusual, especially for expansion lengths of less than 200 to 300 ft.

Other types of supporting devices currently used include lubricated bronze plates, curved and flat steel plates, lubricated asbestos pads,

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rubber, neoprene, Fabreeka pads, and Teflon-surfaced neoprene and Fabreeka pads. The cost of the various types of supporting devices varies over a considerably large range. Because of the variation in cost and the large number of devices sometimes used, the relative cost of supporting devices to the overall structure is worthy of consideration. One 3 span bridge observed during this investigation had 36 rockers.

**Elastomeric pads**

The use of elastomeric pads is relatively new in the United States and has been used more extensively for prestressed concrete bridges than for steel bridges. However, elastomeric pads have been used in Europe for some time. The La Bourget bridge in France was constructed with some of the structural members resting on rubber pads. After 20 years the steelwork had nearly corroded away and extensive repairs were made. However, according to Bolton (6), the rubber pads were reported to be in such good condition that they were merely turned over and put back in place for another 20 years.

The Pelham Bridge, Lincoln, England, has an overall length of 1470 ft, a width of 68 ft, and was built on a horizontal curve (53). Reisser, Wright, and Bolton (49) describe the design and construction of this bridge which had 24 by 16 by 7-1/8-in. laminated rubber bearings over abutments and intermediate flexible steel portals.

The use of elastomeric pads for highway bridges is discussed by Paxson (46); details concerning the use of neoprene bearing pads, curved steel plates, and lubricated bronze plates are given by Dean (16); and Graves (26) reported the first usage of a neoprene plate developed by the Texas
Highway Department Bridge Division to be used as a bearing material for precast, prestressed concrete beams.

Evaluation tests have been made on elastomeric bridge bearings (35, 44, 45); publications on the design of neoprene pads are available (17, 23); and the use of elastomeric bearings for steel bridges is permitted by the 1961 AASHO Standard Specifications (1).

Investigations have also been made on the use of elastomeric pads as bearings for steel beams. Fairbanks (21) conducted tests using 18WF50, 24WF76, and 30WF108 steel beams and found a lower allowable value for shearing strain of neoprene pads than previous investigators.

Other types of supporting devices

Lubrite expansion plates were used to replace worn expansion roller nests under 15 pin connected railroad trusses with spans of approximately 170 ft (56). It was reported that the use of slide plates at expansion ends of bridges is not unusual, but that their installation under steel spans more than 70 ft long is distinctly so.

Some engineers believe that the use of less corrosive alloy steel will provide satisfactory behavior. Tests have been made of bridge bearing plates clad with a 0.0625 and 0.1250 in. thickness of stainless steel. It is reported that there was no damage to bond and shear strength of bonds was not lowered after 2,000,000 passes, which is supposedly equal to 50 years' service (58).

Interest is growing in the search for more economical and efficient supporting devices. One new material under investigation is Teflon. Molded Teflon is reported by Ricklin and Miller (50) to have a dry
coefficient of friction with steel as low as 0.04. Rulon, a filled Teflon material, is being investigated as possible shaft bearings for some items of machinery. Teflon used with neoprene pads is under study for bridge bearings (43). Epoxide resin has been used for bridge support bearings (18).

Fabreeka-Teflon pads are being used on several test bridges (20). These pads consist of 2 Fabreeka pads with filled Teflon—a filled Tetrafluorethylene Fluorocarbon resin, commonly called filled TFE—1/32 in. thick bonded to the bearing surface of each pad so that Teflon slides on Teflon.

The coefficient of friction is reported to be 0.075 at 100 psi decreasing progressively to 0.047 at 1000 psi for sliding speed of 1 in. per min.

No doubt other new materials will be advocated for possible use as bridge bearings.

**Expansion devices**

To allow movement of the deck slab some type of floor expansion device is used in conjunction with the supporting devices. Floor expansion devices vary from the complicated finger joints to sliding plates, some type of asphaltic fiberboard, asphaltic mastic, expanded neoprene fillers, sponge rubber, or open armored joints. As with supporting devices, the cost of these devices varies over an extremely wide range. In some cases suspended hanger arrangements have been used in the bridge beams in an effort to provide for possible longitudinal and vertical movement of the bridge, piers and abutments. Suspended
hanger arrangements usually use a larger number of floor expansion devices than other arrangements.

In general, it seems to be desirable that the floor expansion device prevent the passage of water and debris which might fall upon the bearing seats. Various arrangements of water stops and gutters have been tried for this purpose.

Finger joints, the most expensive of the steel devices currently in use, are generally used to accommodate relatively large horizontal movement. While they tend to provide a smooth, uninterrupted traffic surface, they usually do not prevent the passage of water and debris. Some gutters under finger joints were observed during the period of this research. The gutters observed were filled and debris had accumulated around the supporting devices.

Most of the floor expansion devices currently used function unsatisfactorily in some respect. This is not unusual because floor expansion devices must accommodate not only the predicted movement of the superstructure but also the unpredicted movement of piers and abutments as well as growth of approach and bridge slabs.

Infiltration of foreign matter in expansion joints is given by Russell (54) and Strom (60) as one cause of trouble in concrete bridge slabs that requires constant maintenance.

In some cases, open expansion joints have proven to be the most satisfactory. Wright (65) reported the use of open expansion joints—only a gap left, no plates or other form of filler across the joint—since the design engineers had experienced trouble with plates. The maximum gap was 3/4 in. and a gutter was provided.
Since some types of filled joints do not return to their original dimensions after initial compaction, neoprene devices have been developed. One neoprene and steel device is 13\(\frac{1}{2}\) in. wide and is reported to accept a movement of 3 in. (52). Another neoprene and steel load carrying type of bridge expansion device which accommodates a 3-in. travel has been used on a trial bridge*.

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SURVEY OF DESIGN PRACTICE

The second phase of the project was a survey by questionnaire of current bridge design practice. The questionnaire and letter shown in Appendix A were mailed to ten Bridge Engineers of State Highway Departments and 13 consulting firms and engineers of the mid-central portion of the United States. Replies were received from nine of the Bridge Engineers and seven consulting firms or engineers.

The response to the questionnaire indicated that bridge design engineers are concerned with and interested in the problems associated with bridge supporting and expansion devices. Many of the respondents also sent copies of standard details and plans of bridges.

Due to the diversity of types of devices used and requirements for usage, no concise, graphical tabulation of current practice could be made. A summary tabulated in two groups, according to whether the reply was received from State Highway Departments or from consulting firms, is presented in Appendix A. Each group is tabulated by question number for easy comparison and replies are identified alphabetically to provide some insight and continuity as to the overall design practice of each department or organization. Answers to Questions 1 and 2 are grouped according to type of supporting device and type of expansion device.

Three of the consulting firms simply stated that their organization followed the design standards of the particular state involved, and, thus, there were no specific answers for tabulation.

The answers to the questionnaire show interesting variations in present design procedure—especially since, as one Bridge Engineer indicated, all respondents are designing according to the AASHO Specifications.

Typical devices used are shown in Figs. 17 and 18, Appendix B.
FIELD OBSERVATIONS

The third phase of the research project consisted of field observation of the behavior of bridge supporting and expansion devices. Both steel and concrete deck type bridges were observed. Steel bridges included plate girder, I-beam, and WF-beam bridges. Concrete bridges included reinforced concrete girder, precast prestressed beam, and concrete box girder bridges. All bridges had a concrete roadway slab, and some had an asphalt wearing surface. The design of the bridges may or may not have been based on composite action. General limitations for observation were that the bridges should be of three or more spans, either simply supported or continuous. Span lengths were usually 50 ft or more.

The following basic procedure was used to select at random bridges of various types and ages. A general area, usually within one day's auto travel of Ames, Iowa, was selected and all concrete and steel deck type bridges within the span limitation were observed in sequence. For areas removed from Ames, the same general procedure was followed. For example, bridges were observed in sequence on U. S. Highway 20 between Sioux City and Sac City, Iowa, on Iowa Highway 92 between Interstate Highway 35 and Carson, Iowa, and representative bridges were observed on the Kansas Turnpike between Wichita and Kansas City, Kansas. Because of time limitations, only three bridges were carefully observed in Nebraska. Thus, the greater number of steel than concrete bridges observed was a natural rather than planned occurrence. For instance, virtually all bridges on the Kansas Turnpike are steel bridges.

During observation the following items were noted: date; temperature; location and type of bridge; whether or not the bridge was open to traffic;
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During observation the following items were noted: date; temperature; location and type of bridge; whether or not the bridge was open to traffic;
year of construction, if known or could be determined; type of abutments; types of supporting devices at abutments and piers; type of floor expansion device; design opening of expansion device (if determined from design drawings); and opening of expansion device observed. Also a sketch was made showing span lengths, position and type of supporting and expansion devices, and, if applicable, relative movement of supporting devices. Irregularities in the functioning of supporting and expansion devices or other component parts of the bridge and the cause, if possible to determine, were likewise noted. For this investigation the term "irregularity" denotes any behavior of the bridge (superstructure or substructure) not anticipated or desired in the design of the bridge. Thus irregularities include such items as "freezing" of supporting or floor expansion devices, shifting of abutments, spalling or cracking of abutments, and inconsistencies of rocker movement. An irregularity does not necessarily reflect the capacity of the bridge. However, many irregularities would be repaired if they were known to exist. As field observations progressed, other factors of possible influence became evident. For example, it became apparent that the amount of expansion still available between the approach slab and abutment backwall or floor slab could influence abutment movement and on subsequent observations this also was noted where possible.

The bridges observed which fall within the limitations of the project are grouped and tabulated in Table 11, Appendix C. For ease of possible comparison, the bridges were basically grouped by type, number of spans, approximate length of structure, and type of supporting device. The irregularities observed are listed separately and follow Table 11, Appendix C. Typical irregularities observed are shown in Figs. 19-24,
Appendix C. Typical supporting and expansion devices, and combinations of devices observed are shown in Figs. 17 and 18, Appendix B.

Of the 83 bridges tabulated, 39 show irregularities. It will be noted that many of the bridges had floor expansion devices which either were closed "tight" or had much less provision for further expansion than designed for at the observed temperature. Shifting of abutments, spalling and cracking of abutment backwalls, inconsistent rocker movement, cracked concrete bearing seats, and extrusion of asphaltic expansion joints were common observations.

The field observations show a high frequency of irregularities of bridge behavior and emphasize the probability of their continued occurrence under design methods now used. However, from the bridges observed, no definite trend or regularity of pattern can be isolated nor can a prediction be made as to which irregularity will occur or when it will occur. For example, both closed and open floor expansion devices were observed on bridges with concrete, asphalt, and gravel approaches. Similarly, evidence of abutment movement and irrational rocker movement were observed for all types of approaches and heights of approach fill. Irregularities were also observed in both old and relatively new bridges.

During the field observations different eras of design procedure became apparent. In many cases the approximate age of the bridge can be estimated within reasonable limits by the type of abutments, piers, and supporting devices used. However, one bridge observed was constructed in 1934 and utilized the essential characteristics of a currently proposed method of using flexible abutments and piers with no provision for expansion of supporting devices or floor slab. The bridge, located on U.S.
Highway 75 at Sioux City, Iowa, has a single row of vertical piling with concrete cap at the ends of the bridge with a separate retaining wall spaced 3/4 in. at 80° F from the end of the bridge. The concrete center piers rest upon two vertical rows of piling. No roadway expansion device was used other than mastic at the approach slab. The bridge was in very good condition, and the design engineer reported that this bridge was the first so designed that the piers and columns were deflected to take care of the expansion of the bridge, that the bridge had no maintenance other than an occasional painting of structural members, and further that the bridge has been subject to the ravages of the Floyd River and the great increase of traffic over the past many years with no appreciable effect.

Several bridges were observed on the Kansas Turnpike with flexible stub abutments and flexible concrete column piers. The steel beams, or girders, were anchored at all points. These bridges had been in service five or six years and showed no apparent irregularities.

Several interesting bridges were observed which were not within the limitations of this project and thus were not included in the tabulation. These include portions of freeways in Kansas City, Kansas. One freeway had plate girders resting on concrete columns with approximately 1000 ft between expansion devices. Another used curved steel girders with field welded splices. Single steel columns were used, and the girders were fixed to the columns except for the last two columns before the expansion joints. There was approximately 1000 ft between expansion joints, and spans were approximately 90 ft. One 1500 ft steel bridge on the Kansas Turnpike was constructed on flexible steel pile bents with 500 ft
between expansion joints. Another bridge, a 450 ft concrete box girder, used single column concrete piers and no expansion device.

It is unfortunate that many bridge design engineers cannot make periodic field investigations of their respective bridges. Such investigations are extremely interesting and provide a better insight to "engineering judgment".
FACTORS WHICH INFLUENCE THE BEHAVIOR OF SUPPORTING AND EXPANSION DEVICES

During the course of the first three phases of the investigation the multiplicity of factors which may influence the behavior of supporting and expansion devices became more and more apparent. Some of these factors are recognized and partially taken into consideration during the design. Others are either not recognized or they are neglected because of the difficulty of calculation or lack of present analytical methods. Fortunately, many of the factors are accommodated by the inherent factor of safety of the structure.

The proper consideration of these factors is made more difficult by their interdependence and interaction. Often a greater allowance for one factor merely delays rather than prevents the action due to that factor or magnifies the action of another. Other variables of behavior are introduced by the fact that in many cases the Design Engineer has no control over constructional procedure. Behavior of a component part, not normally anticipated in design, may influence the behavior of other members of the structure.

The reaction capacity of the supporting devices and the provision for thermal expansion are considered in calculating the final sizes and allowance for expansion of supporting and expansion devices required. Design values for total range of expansion vary between 0.56 in. per 100 ft (German Proposed Specifications) and 1.25 in. per 100 ft (AASHO Specifications) for steel or between 0.42 in. per 100 ft (German Specifications) and 0.70 in. per 100 ft (AASHO Specifications) for concrete.
The use of the above values for thermal expansion assumes the coefficients of thermal expansion for a steel bridge and a concrete bridge to be $6.5 \times 10^{-6}$ and $6.0 \times 10^{-6}$ per °F respectively. As shown in the Review of Literature, the coefficient of thermal expansion of concrete varies within a wide range depending on method of curing, humidity, precipitation, temperature, and other variables. The coefficient for concrete is different from both that of the aggregate or the cement. Values of the coefficient for concrete have been found from $3.4$ to $7.3 \times 10^{-6}$ per °F. It is logical that the coefficient of thermal expansion for a steel bridge with a concrete deck will be different from that of the steel or concrete and will be influenced by the factors which vary the coefficient of the concrete.

Other factors also influence the thermal expansion of the superstructure. It is accepted that the absorption of solar radiation is influenced by the color and orientation of the surface. Thus, a light colored steel beam shaded by the bridge deck and exposed to the cooler air below the bridge will not expand in the same manner as an exposed truss painted black.

In calculating the provision for expansion, it is natural to assume that the abutments and piers have been designed so no shifting will occur. Field observations show that this assumption is very often incorrect. Abutment movement may be caused by compaction, settling, or shifting of approach fill; growth or expansion of approach slabs; "freezing" of supporting and expansion devices; and many other factors. Pier movement may be caused by many of the same factors.
Regardless of the cause, if abutment or pier movement occurs and the abutment comes in contact with the end of the bridge additional stresses are induced in the bridge. Thermal expansion must then overcome not only frictional forces due to the supporting and expansion devices, but also the passive soil pressure behind the abutment and, perhaps, the friction between the soil and an unknown length of approach and pavement slab. Spalled and cracked abutments often result from pier and abutment movement.

The above examples point out only a few of the factors involved. However, they illustrate the fact that the behavior of bridge supporting and expansion devices may be influenced by any one of an extremely large number of variables.

As a result of this investigation, the following factors are considered to be some of the variables which influence the behavior of supporting and expansion devices:

A. Properties of component materials and structural elements.

1. Coefficient of thermal expansion
2. Thermal diffusivity
3. Coefficient of thermal conductivity
4. Stiffness or flexibility of superstructure, piers and abutments
5. Porosity and moisture absorption
6. Ductility
7. Tensile strength
8. Resistance to chemical action
9. Corrosion resistance

B. Environmental influence (atmospheric conditions).

1. Fluctuation and range of ambient temperature
2. Solar radiation
3. Precipitation
4. Humidity
C. Geometry.

1. Orientation of bridge
2. Degree of skew
3. Allowance for expansion
4. Physical arrangement of bridge deck, abutments and roadway approach
5. Amount and arrangement of reinforcing steel
6. Composite action
7. Span lengths
8. Overall length of structure

D. Other.

1. Type of bridge
2. Type and arrangement of devices used
3. Relationship between dead and live loads
4. Traction of live loads
5. Frequency of traffic flow
6. Direction of traffic flow
7. Speed of vehicles
8. Type and condition of wearing surface
9. Type and condition of roadway and approaches
10. Height of abutment fill
11. Type of abutment fill
12. Method and order of abutment fill compaction
13. Stability of soil
14. Types of soil strata
15. Fluctuation of water table
16. Active and passive soil pressures
17. Horizontal and vertical movements of piers and abutments
18. Accumulation of debris
19. Maintenance

Any attempt to predict the future behavior of a bridge, taking into account all of the above factors, is virtually impossible. A prediction based upon an assumption that only a small isolated group is pertinent would be erroneous. A quick, easily calculated solution cannot be anticipated.
EXPERIMENTAL INVESTIGATION

The results of the first three phases of the investigation indicated that the use of elastomeric bearings should eliminate many of the problems associated with supporting and expansion devices. Elastomeric bearings are being used but their use has been limited to prestressed concrete bridges by many engineers. Several engineers have expressed an interest in the use of elastomeric bearings for steel bridges but feel that more information is needed.

A theoretical analysis by Zuk (66) indicated possible beneficial changes in vibration characteristics of highway bridges by the use of elastomeric bearing pads. Comparing the behavior of bridges (at the fundamental frequency) with elastomeric bearings and with conventional rigid bearings, Zuk obtained the following general conclusions:

(a) the dynamic bridge deflections are increased;
(b) the frequency of vibration is reduced;
(c) the elastomeric bearings add damping to the system; and
(d) the dynamic stresses in the bridge are significantly reduced.

If, as concluded by Zuk, dynamic stresses are reduced by the use of elastomeric bearings, benefits would include: either an increase in factor of safety and fatigue life, which is desirable since traffic loads and volume continue to increase; or a possible reduction of load factor for impact with a resultant saving in material.

A model test bridge which could be adapted to vibration tests for comparison of the behavior of rigid and elastomeric bearings was available for this investigation. It was felt that the results of such tests would be of value to bridge design engineers.
Laboratory Test Bridge

Two model bridges are located in the basement of the Iowa Engineering Experiment Station. Both have a roadway of 10 ft and spans of 10 and 25 ft each. Constructed during the period from the summer of 1952 to the summer of 1953, they were originally used by Holcomb (29) for static and dynamic tests of load distribution. The research was sponsored by the Iowa State Highway Commission as Research Project HR-12. Dynamic loads were applied by the use of model "trucks." Both bridges were subsequently used by Senne (55) for static concentrated load distribution tests, HR-61, and the 25-ft bridge was used by Smith (57), HR-67, for impact factor studies using stationary dynamic and moving load tests to correlate field tests by Linger (34).

The 25-ft bridge (Fig. 1) was used for this investigation and is approximately one-third the size of a standard highway bridge; however, it cannot be considered an exact model of a prototype highway bridge due to changes in some of the dimensions for test purposes. It does, however, represent a 75-ft span highway bridge with somewhat thinner slabs and lighter beams than those used in actual design.

The deck is a 2\(\frac{1}{16}\)-in. concrete slab. Primary reinforcement consists of No. 5 smooth wires (0.207 in. diameter) spaced on 2 in. centers. Two of every three rods are bent up over the supports for negative reinforcing and an additional straight rod is located near the top of the slab above the third rod. Longitudinal reinforcement consists of No. 5 wires spaced 7.7 in. on center, or six rods per panel, all near the bottom. Coverage is 7/16 in. to the center of the primary reinforcement at both faces.
This arrangement of reinforcing uses about one-half the weight of steel which would be required for a one-third scale ratio between model and prototype.

Each beam has a constant cross section with the composite moment of inertia of the interior beams approximately 1\(\frac{1}{2}\) times that of the exterior beams. The relative size of the interior and exterior beams was intended to be about the same as for highway bridges, but the beams were made smaller than would be obtained by scale reduction. This was done in anticipation of a possible future change in AASHO Specifications and to increase the strains and deflections measured. Shear lugs were welded to the top of the beams for composite action. The 5/8-in. diameter reaction rods were not adaptable to this investigation and were replaced with a "stub-column" type of support (Fig. 2) made from a 6WF section.

The weight of a one-third-scale model is reduced to 1/27 of that of a prototype made of the same materials. To obtain the same dead load strains and to obtain dead load deflections reduced by the scale factor, the weight of the model should be 1/9 that of the prototype. The deficiency in weight of the laboratory bridge was made up by hanging concrete blocks from the slab for the tests made by Holcomb and Senne. However, this was not deemed advisable for this investigation due to the probability of undesirable vibrational effects.

Senne reported that the surface of the concrete deck had many hair line cracks which were considered usual for such slabs. During this investigation, transverse cracks extending across the width of the bridge, including curbs, were observed in the deck slab at approximately 5, 9,
Fig. 1 Test bridge

Fig. 2 End supports
12½ and 17 ft from the south end of the bridge. The width of the crack at midspan was less than the others. Another transverse crack, about 20 ft from the south end, extended through the east curb to the center-line of the bridge. Intermittent longitudinal cracks were observed approximately above the interior girders and penetrating the depth of the end diaphragms. No appreciable increase in width of crack openings was observed during the period of testing.

General details of the bridge are shown in Fig. 3 and properties are shown in Table 2.

Instrumentation

The instrumentation was designed to determine vertical displacements and strains in the steel girders for dynamic loading at various frequencies. The tests were duplicated for three bearing conditions.

Strains and deflections were measured by means of Type A-1, SR-4 electric wire resistance gages. For strain measurement, two gages were mounted on the lower surface of the bottom flange at the midspan of each beam and located symmetrically with respect to the web. Compensating gages were mounted on a steel plate (Fig. 4) attached to angles which were fastened with epoxy cement to the bottom of the flange and adjacent to the active gages.

Deflections were measured at midspan and at the supports. Gages were mounted on the top and bottom of small aluminum cantilever beams used for deflectometers. These beams were 0.090-in. thick, 1.0-in. wide, 10-3/4-in. long at midspan and 7-in. long at the supports. Picture-wire cables were connected 3/4 in. from the end of the midspan deflectometers.
Fig. 3. General details of test bridge.
Table 2. Properties of test bridge

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Span (L), ft</td>
<td>25</td>
</tr>
<tr>
<td>Roadway width, ft</td>
<td>10</td>
</tr>
<tr>
<td>Beam spacing (S), ft</td>
<td>3.22</td>
</tr>
<tr>
<td>Slab thickness, in.</td>
<td>2.25</td>
</tr>
<tr>
<td>Ratio $I_{int.}/I_{ext.}$ at midspan</td>
<td>1.48</td>
</tr>
<tr>
<td>EI of total section $^c$ at midspan, $(10)^9$ lb-in.$^2$</td>
<td>37.43</td>
</tr>
<tr>
<td>Total weight of bridge $^d$, lb</td>
<td>11,031</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Interior Beam</th>
<th>Exterior Beam</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I$ of beam $^c$ at midspan, in.$^4$</td>
<td>379</td>
<td>256</td>
</tr>
<tr>
<td>EI of beam $^c$ at midspan, $(10)^9$ lb-in.$^2$</td>
<td>11.14</td>
<td>7.52</td>
</tr>
<tr>
<td>$I_{st}/c_{st}$ at midspan, in.$^3$</td>
<td>35.8</td>
<td>25.9</td>
</tr>
<tr>
<td>$I$ of beam $^e$ at midspan, in.$^4$</td>
<td>3030</td>
<td>2047</td>
</tr>
<tr>
<td>$c_{slab}$, in.</td>
<td>3.87</td>
<td>3.25</td>
</tr>
<tr>
<td>$c_{st}$, in.</td>
<td>10.57</td>
<td>9.88</td>
</tr>
<tr>
<td>Area of beam, sq in.</td>
<td>6.75</td>
<td>5.01</td>
</tr>
<tr>
<td>Area of concrete, sq in.</td>
<td>86.9</td>
<td>67.6</td>
</tr>
<tr>
<td>Weight of beam, plf $^e$</td>
<td>22.0</td>
<td>16.5</td>
</tr>
<tr>
<td>Weight of concrete, plf</td>
<td>90.5</td>
<td>70.5</td>
</tr>
</tbody>
</table>

$^a$ In part from Caughey and Senne (11, p. 11) and Holcomb (29, p. 76)

$^b$ Composite interior and exterior beams

$^c$ Equivalent all-steel section, n = 8

$^d$ Experimentally determined, this research

$^e$ Equivalent all-concrete section
and at the end of the support deflectometers. The center of the strain gages was 3/4 in. from the fixed end of the deflectometers. The midspan deflectometers were initially deflected 1 in. and tied to brackets mounted with epoxy cement at the midspan of the beam (Fig. 4). The end deflectometers were deflected 1/2 in. and tied to the junction of the web and bottom flange as close as possible to the reaction points (Figs. 5, 6, 7).

Precision resistors were used for dummy gages for the deflectometers. Four strand, shielded cable, Belden No. 8723 was used with an individual pair of wires for each gage. The circuitry provided double sensitivity and temperature compensation for all points of measurement.

Available equipment for recording strain and deflection data included eight Model BL-520 Brush universal amplifiers with Model BL-350 strain gage input boxes; two Model BL-274 Brush four channel oscillographs; and three 20 channel and one 6 channel Baldwin SR-4 bridge balancing units (Fig. 8). One Baldwin-Lima-Hamilton Type N, SR-4 strain indicator; one Baldwin SR-4 load cell, Type C, 20,000 lb capacity; and one Blackhawk 10-ton hydraulic Porto-Power unit were also used for static load tests.

Four of the amplifiers and one oscillograph were used for recording strains at midspan of each beam. The other four amplifiers and oscillograph were used with the four bridge balancing units to record deflections. The deflectometers were connected to the bridge balancing units so that either midspan deflections or end deflections at either end of all four beams or the midspan and end deflections of individual beams could be recorded at one time. To avoid internal adjustments of the strain gage
Fig. 4. Midspan strain gages and deflectometers

Fig. 5. End deflectometer and curved steel sole plate
Fig. 6. End support with curved steel sole plate

Fig. 7. End support with neoprene pad
Fig. 8. Recording equipment
input boxes it was necessary to add variable capacitors at the bridge balancing units for some of the deflectometers.

Oscillator

Due to lack of room for end approaches, Smith (57) found that it was impossible to obtain satisfactory results by using model "trucks" for dynamic loading. Therefore, the oscillator used by Smith was remodeled and used to provide the vertical driving force for this investigation. This provided a steady state forced vibration with harmonic excitation and assured repetitive load conditions for the various tests.

As shown in Figs. 9 and 10, the oscillator is a counterrotating eccentric weight oscillator, or exciter. Thus, the horizontal components of force cancel each other and the vertical components are additive. For equal weights and eccentricities, the vertical driving force produced is given by the equation (40, p. 51):

\[ F = \frac{2We}{g} \omega^2 \]  

where,

- \( F \) = vertical driving force, lb
- \( W \) = weight of each rotating mass, lb
- \( e \) = eccentricity of the mass center of each weight, in.
- \( g \) = acceleration of gravity, in./sec\(^2\)
- \( \omega \) = rotational velocity, rad/sec

The weights were mounted on threaded shafts so the eccentricity could be varied. Of course, the weight of the shafts must also be considered in the above equation.
Fig. 9. Oscillator

Fig. 10. Oscillator
The oscillator was powered by a one-horsepower variable speed motor. The variable speed lever of the motor was connected to a gear and train attached to a small motor controlled by a Powerstat (transformer). Thus, by remote control, the frequency of oscillation could be changed from one steady state to another or varied with time. The necessity for increasing or decreasing the frequency of the forcing function with time became apparent early in the investigation. It was found that as the forcing frequency approached the natural frequency of the bridge, an attempt to slightly increase, or decrease as the case might be, the speed of the motor resulted in a larger amplitude of deflection rather than an increase in forcing frequency. At a certain point the forcing frequency "slipped" past the natural frequency and leveled off approximately 1 1/2 to 2 cycles per second beyond the natural frequency. This occurred for both increasing and decreasing changes of motor speed and frequency and made it impossible to obtain maximum amplitudes of strain and deflection at the natural frequency by means of constant frequency tests. Timoshenko (62, p. 48) states that other investigators have observed this increase of amplitude for forced vibrations without damping.

Due to the bridge camber, the oscillator was supported at three points similar to the points of an isosceles triangle. The oscillator was placed upon three scales and 50 lb weights were used to balance the reactions longitudinally and transversely. Total weight of the oscillator and weights was 644 lb. The supports consisted of small swivel type circular plates attached to 3/4-in. leveling bolts. The circular plates rested in holes milled in 5 by 5 by 1-in. steel plates which were fastened to the bridge slab with epoxy cement. No noticeable uplift was
encountered but the oscillator was anchored adjacent to the support points by 5/16-in. bolts threaded into the steel plates and also by a bolt passing through the slab at the midpoint to reduce a slight torsional tendency of the oscillator.

Contact points activated by a cam on one of the shafts of the oscillator were used to operate an event marker on the oscillograph for determination of the frequency of the oscillator. A tachometer was mounted on the instrument panel and attached to the oscillator to provide an approximation of operating frequency during the tests.

Bridge Bearings

Three bearing conditions were used: curved steel sole plates; 64 durometer hardness neoprene pads; and 49 durometer neoprene pads (Figs. 6, 7, 11). The model bridge had been constructed with 5/8-in. diameter cold-rolled reaction rods which were not adaptable to this investigation. Also, observations indicated that the reactions were no longer the desired values. Type A-1, SR-4 strain gages diametrically attached to the reaction rods were in good condition. The bridge was jacked up and the reaction rods removed and recalibrated using a 60,000 lb capacity Baldwin-Southwark universal hydraulic testing machine. The reaction rods were replaced with the nuts for the abutment bolts below rather than above the reaction rod brackets. The desired reaction values were calculated assuming the reactions of interior girders to be the weight of the girder plus one panel of deck. The reactions of exterior girders were then assumed to be the difference between the total weight of the bridge and the reactions of the interior girders. Thus, the reactions
Fig. 11. Curved steel sole plate, bearing (masonry) plate, and neoprene pads
of the interior girders (only) were determined by accepted design practice (9, p. 7-4). The desired reactions were obtained in each reaction rod by raising or lowering the nuts on the abutment bolts and determining the reactions by means of an SR-4 Indicator and switch balancing unit. This was a very sensitive, tedious, and time-consuming operation due to the stiffness of the end diaphragm.

Small aluminum T-sections had been fastened with epoxy cement to the inner face at the top of the abutment for reference points prior to obtaining the desired reactions. The distance from the T-sections to the bottom flange was measured with inside micrometers. The relative difference in heights to the bottom flange of each beam was used to assure the same reactions when changing from one type of bearing to another. That is, shims were used to maintain the same relative heights and, thus, the same reactions.

Reaction values and heights after shimming for each bearing condition are shown in Table 3.

**Curved steel sole plates**

The curved steel sole plates (Fig. 11) were made of 7/8 by 2½ by 4 in. cold-rolled steel turned to a 5½-in. radius in the 2½-in. dimension and welded to ½ by 5½ by 7-in. blocking plates. Bearing (masonry) plates were ½ by 6 by 10 in. hot-rolled steel and contact surfaces were milled. Both the curved steel sole plates and the bearing plates had a finish finer than ASA-125 required by some State Highway Departments. Two ½-in. tapered pintles were mounted in each bearing plate. Two 37/64-in. holes were drilled in the curved steel sole plates for the fixed end and slotted holes 19/32 by 3/4-in. were provided for the expansion end.
Table 3. End reactions—bridge only—and deviation of relative heights after shimming

<table>
<thead>
<tr>
<th></th>
<th>Exterior</th>
<th>Interior</th>
<th>Interior</th>
<th>Exterior</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>End reactions, lb</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>North end</td>
<td>1188</td>
<td>1593</td>
<td>1593</td>
<td>1192</td>
</tr>
<tr>
<td>South end</td>
<td>1165</td>
<td>1565</td>
<td>1579</td>
<td>1156</td>
</tr>
<tr>
<td><strong>Deviation of relative heights after shimming, in.</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Curved steel sole plates</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>North end</td>
<td>+0.0003</td>
<td>0.0000</td>
<td>+0.0001</td>
<td>0.0000</td>
</tr>
<tr>
<td>South end</td>
<td>-0.0004</td>
<td>0.0000</td>
<td>-0.0004</td>
<td>-0.0004</td>
</tr>
<tr>
<td>64 durometer pads</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>North end</td>
<td>-0.0020</td>
<td>0.0000</td>
<td>+0.0001</td>
<td>-0.0004</td>
</tr>
<tr>
<td>South end</td>
<td>+0.0015</td>
<td>0.0000</td>
<td>-0.0011</td>
<td>-0.0025</td>
</tr>
<tr>
<td>49 durometer pads</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>North end</td>
<td>-0.0054</td>
<td>0.0000</td>
<td>-0.0041</td>
<td>+0.0015</td>
</tr>
<tr>
<td>South end</td>
<td>+0.0041</td>
<td>0.0000</td>
<td>-0.0012</td>
<td>-0.0039</td>
</tr>
</tbody>
</table>

**Neoprene pads**

The neoprene pads (Fig. 11) were 4 by 3\(\frac{1}{2}\) by 0.660 in. (average) and were furnished by The General Tire and Rubber Co. One set of pads averaged 64 durometer hardness, the other 49 durometer. The 3\(\frac{1}{2}\)-in. dimension was placed in the longitudinal direction of the girders. Shape factor (average) was 1.41; see p. 7.
Stress-deflection curves are shown in Fig. 12.

The natural frequency of undamped free vibration of the pads may be calculated from (25, p. 95):

\[ f_n = \frac{188}{\sqrt{\frac{K_1}{W}}} \]  

(2)

where,

\( f_n \) = undamped natural frequency, cpm
\( K_1 \) = spring rate, lb/in.
\( W \) = weight of the mass, lb

This reduces to:

\[ f_n = \frac{188}{\sqrt{d}} \]  

(3)

where,

\( d \) = static deflection, in.

Properties of the neoprene pads are shown in Table 4.

The compressive strain is less, approximately 20% harder, than that predicted by accepted graphs for rubber (25), which is not surprising since these pads were made from especially compounded formulas for bridge bearing use.

The size of the neoprene pads used was not selected on the basis of a one-third scale reduction, but the pads are felt to be representative. For example, no consideration was given to size required by possible slippage due to thermal movement since the tests were conducted in a relatively stable temperature environment. Also, some design pamphlets (17) indicate that, as long as compressive stresses are within reasonable limits, size of pads may be governed somewhat by esthetic appeal.
Fig. 12. Stress-deflection curves for neoprene pads
Table 4. Properties of neoprene pads

<table>
<thead>
<tr>
<th></th>
<th>64 durometer</th>
<th>49 durometer</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Spring modulus</strong>*, lb/in.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>at 715 psi</td>
<td>69,000</td>
<td>57,100</td>
</tr>
<tr>
<td>at 250 psi</td>
<td>64,800</td>
<td>41,400</td>
</tr>
<tr>
<td>at 100 psi</td>
<td>64,800</td>
<td>35,000</td>
</tr>
<tr>
<td><strong>Natural frequency</strong>, cps</td>
<td></td>
<td></td>
</tr>
<tr>
<td>using d at 715 psi</td>
<td>8.3</td>
<td>7.5</td>
</tr>
<tr>
<td>using d at 250 psi</td>
<td>13.5</td>
<td>10.8</td>
</tr>
<tr>
<td>using d at 100 psi</td>
<td>21.3</td>
<td>15.7</td>
</tr>
<tr>
<td><strong>Compressive strain</strong>, per cent</td>
<td></td>
<td></td>
</tr>
<tr>
<td>at 715 psi</td>
<td>21.9</td>
<td>26.8</td>
</tr>
<tr>
<td>at 250 psi</td>
<td>8.2</td>
<td>12.9</td>
</tr>
<tr>
<td>at 100 psi</td>
<td>3.3</td>
<td>6.1</td>
</tr>
</tbody>
</table>

*a* From Fig. 12  
*b* From Equation 3

**Testing Procedure**

The same basic testing procedure was used for the three bearing conditions. A complete series of forced vibration, natural frequency and static load tests was completed for each type of bearing in the following order: (1) curved steel sole plates; (2) 49 durometer neoprene pads; and (3) 64 durometer neoprene pads. As may be seen in Fig. 6, the bearing (masonry) plate and one blocking plate were removable. This was necessary...
due to the pintles used for the curved steel sole plates and the fact that these sole plates were welded to blocking plates. For neoprene pads, these plates were replaced with smooth plates. No surface machining was done to the plates used for neoprene pads other than removal of mill scale with a belt type sander. The neoprene pads and plates were thoroughly cleaned with toluene at the time of placement.

To interchange bearings, the bridge was jacked up, bearings changed and shims inserted between the two blocking plates to give the same relative distance in each case from the bottom flange to the reference points. This assured the same reactions for each type of bearing. Some creep was encountered when shimming for the neoprene pads.

Considerable difficulty was experienced with capacitance change due to amplifier transformers and amplifier drift between the series of tests made with the curved steel sole plates and the 49 durometer pads. These problems were resolved by replacing all amplifier transformers; mounting cooling fans on each amplifier; letting the amplifiers "run" continuously during the remainder of the testing period; and enclosing the basement stairwell during extremely cold weather.

**Forced vibration tests**

For one group of tests, the oscillator only was placed at the midpoint of the bridge as shown in Figs. 9 and 10. A series of constant frequency and variable frequency tests were made, with the frequency varying from about 4 cps to the natural frequency and from about 12 cps to the natural frequency. Two different pairs of eccentric weights were used. The weights in one pair weighed 0.82 lb each; the other 3.48 lb each.
The small set of weights was placed at an eccentricity of 7.01 in. Two eccentricities were used for the larger set; 3.26 in. and 4.51 in. One-half-in. diameter rods were machined to the desired length and used as inside micrometers to assure repetitive eccentricities. Thus, three tests were conducted with different forcing functions for each bearing condition for a total of nine tests for the oscillator only at the mid-point.

Like tests were also conducted with 1576 lb of concrete blocks placed alongside the oscillator as shown in Figs. 13 and 14. This gave an additional nine tests or a total of 18 tests for this investigation.

Tests were also conducted with the oscillator placed at the midspan and near the edge of the bridge, with and without concrete blocks, for the small set of eccentric weights and the large set at the smaller eccentricity. For reasons of time and expense, the data from these tests have not been evaluated.

Strains in the bottom flange at midspan of each beam were recorded for all types of bearings. Deflections at midspan of each beam were recorded for the curved steel sole plates. End deflections for the sole plates were found to be negligible, if any. For the neoprene pads, deflections at the midspan and ends of one girder (exterior, interior) and at midspan of the adjacent girder (interior, exterior) were recorded. This procedure for deflections (neoprene pads) was repeated for all four girders.

Natural frequency

The natural frequency of the bridge was obtained for the three bearing and two dead load conditions from oscillograph recordings of strain and
deflection at midspan. Deflections at the ends of the bridge were recorded but were of such small magnitude, even at full gain of the amplifiers, that no reliable data could be reduced.

The method used to obtain free vibrations was to have a person set the bridge in oscillation by dropping onto and instantaneously lifting himself off the bridge.

Like tests were also made for the vibrator placed near one side of the bridge, but as previously stated, this data was not reduced.

Natural frequencies were also obtained for the deflectometers at midspan and at the ends by disconnecting the anchoring cables and releasing the deflectometers from a displaced position.

**Static load tests**

Static load tests were conducted for the three bearing conditions to check the linearity of the amplifiers and provide calibration curves for reduction of data. A hydraulic jack and Baldwin SR-4 load cell were placed between the oscillator and the floor above for downward loading. Upward loading was accomplished by placing the jack and load cell between the floor below and a steel plate mounted to the bottom of the deck slab at midpoint. Maximum loading was approximately 4000 lb downward and 3000 lb upward.

Strain oscillograph recordings were taken at the midspan of the girders and deflection oscillograph recordings (of the deflectometers) were taken at midspan and the ends of the bridge. Also, dial gages with a least count of 0.001 in. per division were placed below the girders at midspan and values recorded for each load increment.
Fig. 13. Oscillator with concrete blocks

Fig. 14. Oscillator with concrete blocks
An additional calibration check was made by connecting the deflectometers to a turnbuckle and placing a dial gage below (or above) the deflectometers. The deflectometers were displaced, above and below the initial displaced position, by tightening or loosening the turnbuckle. Dial readings and pen movement of the oscillograph were recorded for correlation.
RESULTS OF EXPERIMENTAL INVESTIGATION

Static Load Tests

The data obtained from the static load tests were reduced and used to plot calibration curves for such relationships as load-strain, deflection-attenuator lines, and load-deflection. Approximately 110 calibration curves were plotted using a Mosely Model 2D-2 X-Y Recorder with Model 40D Keyboard. These curves showed a linear relationship between the two variables; thus, the slope of the various curves was used for calibration constants.

Values of strain at midspan of the girders were obtained from the equation (8):

\[ S = \frac{R}{NK_c F_m (R_c + R)} \]  

where,

- \( S \) = sensitivity, strain/attenuator-line
- \( N \) = number of active strain gages (2, this case)
- \( K_c \) = calibration point, attenuator-lines (75, this case, or 15 lines at attenuator 5)
- \( F_m \) = gage factor of strain gage (2.06, this case)
- \( R \) = resistance of strain gage (120.0 ohms, this case)
- \( R_c \) = resistance of calibrator resistor (390 K, this case)

This reduces to (for static load tests):

\[ S = 1 \text{ micro-in.}/\text{in. of strain/attenuator-line} \]  

(4-a)

For vibration tests, since double amplitudes were measured:

\[ \text{Strain, micro-in.}/\text{in.} = \frac{1}{2} \text{ (number of attenuator-lines)} \]  

(4-b)
Attenuator-line values were obtained by multiplying the oscillograph pen deflection—number of lines on the chart paper at 1 mm per line—by the attenuator—amplifier sensitivity—setting.

A typical calibration curve is shown in Fig. 22, Appendix D. Typical calibration constants obtained from the calibration curves are shown in Table 5.

Contrary to theory, which would indicate the same strain for a given load regardless of type of supports, the calibration curves show a greater static load required for the curved steel sole plates than for the neoprene pads to produce the same strain. This behavior becomes compatible when considering the fact that when the bridge deflects the bottom flange, located below the neutral surface, tends to "elongate" at the supports. This movement is opposed by friction between the curved steel sole plate and bearing (masonry) plate. This opposing force, also located below the neutral surface, produces the same effect as a negative moment and an axial force acting at the neutral axis. The strain at midspan resulting from the negative moment is of opposite sign to that caused by the static load. Under similar longitudinal movement at the support, the neoprene pads deflect longitudinally producing shear strain in the pads and, thus, some force opposing movement of the flange. The relative magnitude of negative moments produced would basically be dependent upon the coefficient of friction for the steel on steel and shear modulus (17) for the neoprene.
Table 5. Typical calibration constants from static load tests

$\mu^a$, micro-in./in. = $C_1(\Delta$, dial divisions)

<table>
<thead>
<tr>
<th>Bearings</th>
<th>$C_1$, exterior beam 1-1</th>
<th>$C_1$, interior beam 3-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curved steel</td>
<td>10.10</td>
<td>7.00</td>
</tr>
<tr>
<td>sole plates</td>
<td></td>
<td></td>
</tr>
<tr>
<td>64 durometer pads</td>
<td>9.30</td>
<td>6.25</td>
</tr>
<tr>
<td>49 durometer pads</td>
<td>8.70</td>
<td>6.05</td>
</tr>
</tbody>
</table>

$\mu^a$, micro-in./in. = $C_2(\Delta^b$, deflection attenuator-lines, single amplitude)

<table>
<thead>
<tr>
<th>Bearings</th>
<th>$C_2$, exterior beam 1-1</th>
<th>$C_2$, interior beam 3-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curved steel</td>
<td>8.50</td>
<td>6.30</td>
</tr>
<tr>
<td>sole plates</td>
<td></td>
<td></td>
</tr>
<tr>
<td>64 durometer pads</td>
<td>7.85</td>
<td>5.58</td>
</tr>
<tr>
<td>49 durometer pads</td>
<td>7.40</td>
<td>5.20</td>
</tr>
</tbody>
</table>

$\mu^a$, micro-in./in. = $C_3(\Delta^b$, strain attenuator-lines, single amplitude)

<table>
<thead>
<tr>
<th>Bearings</th>
<th>$C_3$, exterior beam A</th>
<th>$C_3$, interior beam C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curved steel</td>
<td>14.70</td>
<td>7.20</td>
</tr>
<tr>
<td>sole plates</td>
<td></td>
<td></td>
</tr>
<tr>
<td>64 durometer pads</td>
<td>13.60</td>
<td>6.65</td>
</tr>
<tr>
<td>49 durometer pads</td>
<td>13.50</td>
<td>6.60</td>
</tr>
</tbody>
</table>

$\Delta^b$, deflection attenuator-lines = $ES_1^c(\mu^a$, micro-in./in.)

<table>
<thead>
<tr>
<th>Bearings</th>
<th>$ES_1$, point 4-2</th>
<th>$ES_1$, point 2-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>64 durometer pads</td>
<td>0.0070</td>
<td>0.0097</td>
</tr>
<tr>
<td>49 durometer pads</td>
<td>0.0145</td>
<td>0.0179</td>
</tr>
</tbody>
</table>

$^a$Unit strain reading for load cell. Load, $P$, lb = 4.95$\mu$

$^b$Oscillograph pen movement

$^c$South end of bridge
Natural Frequencies

An extensive theoretical study of the dynamic behavior of the test bridge is beyond the scope of this investigation. This is due partly to the large number of variables involved -- such as cracks in the deck slab, number and arrangement of girders, and type and degree of damping--and the lack of methods for predicting the influence of many of the variables on bridge behavior. Even a simplified analysis based on simplifying assumptions requires further assumptions such as type of damping and magnitude of damping coefficient.

However, natural frequencies of the test bridge, based on the accepted assumption that the bridge acts as a single beam, are easily handled and provide a means of comparison with experimental results.

Assuming the bridge to behave as a single beam, the natural frequency of the unloaded bridge only with steel (rigid) supports may be computed from the equation (28, p. 25):

\[ f_b = \frac{\pi}{2L^2} \sqrt{\frac{EIg}{w_b}} \]  \hspace{1cm} (5)

where,

- \( f_b \) = natural frequency of the unloaded bridge, cps
- \( L \) = length of bridge, in.
- \( EI \) = flexural rigidity of the bridge, lb-in.\(^2\)
- \( g \) = acceleration of gravity, in./sec\(^2\)
- \( w_b \) = weight of the bridge per unit length, lb/in.

More pertinent to this investigation is the natural frequency of the test bridge with the oscillator at midpoint, with and without concrete
blocks. For this case, it is customary to use the following equation as explained by Timoshenko (62, p. 27) and Myklestad (40, p. 43):

$$ f_{bo} = \frac{1}{2\pi} \sqrt{\frac{48EI}{L \left( \frac{mL}{2} + M \right)}} = \frac{1}{2\pi} \sqrt{\frac{48EIg}{L \left( \frac{wL}{2} + W \right)}} $$

(6)

where,

- $f_{bo}$ = natural frequency of the loaded bridge, cps
- $m$ = mass of the bridge per unit length, lb-sec$^2$/in.
- $M$ = mass of the load, lb-sec$^2$/in.
- $W$ = weight of the load, lb

and all other terms are as given in Equation 5, above.

A solution for the natural frequency of the loaded or unloaded bridge with elastic supports is given by Zuk (66, p. 28) assuming, for simplicity of analysis, the oscillations of the load (vehicle) equal to that of the bridge. This condition is similar to that of the test bridge with oscillator. In part, Professor Zuk's analysis is as follows:

Using Rayleigh's energy method of analysis (2)*, the total bridge deflection will be considered as

$$ y = (a + b \sin \frac{\pi x}{L}) \cos pt = X \cos pt $$

(7)

where $p$ is the circular frequency, $t$ is time, $a$ is the maximum support deflection, and $b$ is the maximum beam deflection.

In Rayleigh's method, the maximum potential energy, $V$, is equated to the maximum kinetic energy, $T$.

*Timoshenko (62).
From Article 4 in Timoshenko (2)

\[ V = \frac{EI}{2} \int_0^L \left( \frac{d^2X}{dx^2} \right)^2 dx + 2 \frac{K}{2} a^2 \]  

(8)

where \( EI \) is the flexural rigidity of the beam, and \( k \) is the spring constant of the flexible support.

\[ T = \frac{mp^2}{2} \int_0^L x^2 dx + \frac{1}{2} Mv_1^2 \]  

(9)

where \( m \) is the mass of the bridge beam per unit length, \( M \) is the mass of the vehicle applied to the beam, and \( v_1 \) is the maximum vertical velocity of the vehicle, taken as \( p(a + b) \) for harmonic motion.

After equating \( V \) to \( T \), regrouping, reducing, and minimizing to obtain the fundamental mode, Professor Zuk obtains the following:

\[ a = nb \]  

(10)

where

\[ n = \pi^5 EI(Lm+M) - 2\pi KL^3(Lm+2M) + \left[ 2\pi KL^3(Lm+2M) - \pi^5 EI(Lm+M) \right]^2 \]

\[ + 16\pi^4 EIKL^3(2Lm+\pi M)^2 \]  

\[ / 16KL^3(2Lm+\pi M) \]  

(11)

[In the above equation for \( n \), all terms to the left of the indicated division are included in the dividend.]

The circular frequency, \( \omega \), is then determined by Professor Zuk from:

\[ \omega = \sqrt{4\pi kn^2 L^3 + \pi^5 EI \over 2\pi mn^2 L^4 + 8mnL^4 + \pi mL^4 + 2\pi Mn^2 L^3 + 4\pi MnL^3 + 2\pi ML^3} \]  

(12)

The fundamental frequency in cycles per unit of time is given by:

\[ f = \frac{\omega}{2\pi} \]  

(13)
Another method for determination of the natural frequency of a beam system on elastic supports which takes into account the mass of the beam as well as the two equivalent masses of the supports is given by Jacobsen and Ayre (31, p. 86). Using the static-deflection method, a special case of Rayleigh's method, they obtain:

\[ p^2 = g \frac{\sum W_i \Delta_i}{\sum W_i \Delta_i^2} \]  \hspace{1cm} (14)

where,

- \( p \) = natural frequency of the system, radians/sec
- \( g \) = acceleration of gravity, in./sec^2
- \( W_i \) = weight of the mass of individual concentrated loads, equivalent mass of beam, and equivalent mass of supports, lb
- \( \Delta_i \) = deflection of respective \( W_i \) weights, in.

An empirical formula is given for determining the weight, \( W_{leq} \), of the equivalent mass of the beam as:

\[ W_{leq} = \left[ \frac{17}{35} + \frac{18}{35} \tanh \frac{\delta_{av}}{\delta_o} \right] W_{1} \]  \hspace{1cm} (15)

where,

- \( \delta_{av} \) = midspan deflection due to supports, in.
- \( \delta_o \) = midspan deflection due to dead load of beam plus concentrated loads.

The natural frequency in cycles per sec is given by Equation 13.

Myklestad (40, p. 42) states that undamped frequencies found by the equation

\[ f_n = \frac{1}{2\pi} \sqrt{\frac{g^2 W}{\sum M_y^2}} \]  \hspace{1cm} (16)
are higher than actual and discusses a refinement of the equation in the form of

$$f_n = \frac{1}{2\pi} \sqrt{\frac{g \sum M_y}{\Sigma y^2}}$$  \hspace{1cm} (17)$$

Experimental natural frequencies are shown in Table 6 and comparative theoretical and experimental values of the natural frequency of the test bridge are shown in Table 7.

Table 6. Experimental natural frequencies

<table>
<thead>
<tr>
<th>Supports</th>
<th>Frequency, cps</th>
<th>Number of values averaged</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Avg.</td>
<td>Max.</td>
</tr>
<tr>
<td>Oscillator only</td>
<td>Steel</td>
<td>8.26</td>
</tr>
<tr>
<td></td>
<td>64 durom.</td>
<td>7.83</td>
</tr>
<tr>
<td></td>
<td>49 durom.</td>
<td>7.64</td>
</tr>
<tr>
<td>Oscillator with conc. block</td>
<td>Steel</td>
<td>7.45</td>
</tr>
<tr>
<td></td>
<td>64 durom.</td>
<td>7.01</td>
</tr>
<tr>
<td></td>
<td>49 durom.</td>
<td>6.90</td>
</tr>
</tbody>
</table>
The experimental and theoretical values of natural frequency show relatively poor correlation. This may be partially accounted for by one or more of the following: (1) theoretical methods for determination of natural frequency usually give values that are slightly too high, especially for the fundamental frequency; (2) the aforementioned cracks in the deck slab may have partially destroyed the integrity of the bridge; (3) the assumption that the bridge acts as a single beam is, no doubt, in error; and (4) the assumed value for n, and ultimately EI, is incorrect.

From Table 5 it is evident that under static load there is considerable difference in the deflection of the exterior and interior beams and
that the bridge does not behave as a single beam. Using Table 5 to
determine an experimental value of $EI$, assuming an average of the beam
deflections for curved steel sole plates may be used with the load-
deflection relationship for a single beam, a value of approximately 63% of
the theoretical value is obtained. However, this reduced value
results in theoretical natural frequencies generally lower than the
experimental values.

More important to this investigation is the observation that both
theoretical and observed frequencies for the model bridge seem lower than
would be expected. From an assumed one-third model to prototype relation­
ship and dimensional analysis, it may be shown, utilizing Equation 5,
that the natural frequency of the model bridge (only) should be three
times that of its prototype. The calculated and observed values for the
test bridge would result in lower natural frequencies than observed by
other investigators for approximate prototypes and bridges in general
(22, 28). However, due to modifications discussed previously, the test
bridge was not originally intended to be an exact model.

Since this investigation is concerned with the relative behavior
of the three types of support bearings tested, it is believed that the
discrepancies between theoretical and observed natural frequencies have
little influence on the basic objectives of the research.
Bridge Damping

It is generally accepted that damping of bridges is neither viscous nor frictional. However, logarithmic decrements, based on the assumption of viscous damping, have been used for comparison (22, p. 94; 28, p. 39).

Logarithmic decrements were determined from oscillograph recordings of natural frequency tests and based, in general, on the first two cycles of maximum readable oscillation.

The logarithmic decrement may be determined from experimental data by the equation (61, p. 46):

$$\delta = \frac{1}{n} \ln \frac{x_0}{x_n}$$  \hspace{1cm} (18)

where,

$\delta$ = logarithmic decrement

$x_o$ = amplitude of initial vibration

$x_n$ = amplitude of nth vibration

$n$ = number of cycles.

The decrement can also be expressed as:

$$\delta = \frac{2\pi \zeta}{\sqrt{1-\zeta^2}}$$  \hspace{1cm} (19)

where,

$\zeta$ = damping factor, or damping ratio of the coefficient of viscous damping to the critical damping.

For small values of $\zeta$ the denominator is assumed as unity and the following relationship used (61, p. 44):

$$\delta \approx 2\pi \zeta$$  \hspace{1cm} (20)
Experimental logarithmic decrements and damping factors are shown in Table 8. As would be expected, the observed damping factors may be considered negligible. However, it should be noted that these values were for small amplitudes and based on assumed viscous damping.

Structural damping (61, p. 72) involves energy dissipation when structural materials are cyclically stressed. The energy dissipated per cycle of stress is independent of the frequency and proportional to the square of the strain amplitude for most materials.

Thus for forced vibration near the natural frequency, or perhaps under impact loading, damping may become more significant.

In a comparison of structural and viscous damping for a steady state near resonance, Thomson (61, p. 72) arrives at a value for the structural damping factor (as defined by Thomson) of twice the viscous damping factor.

Vibration Tests

As shown in Figs. 15 and 16, each oscillograph provided four channels of strain or deflection data plus a 1 sec tic mark on one side and a revolution tic mark on the other side of the chart paper. Chart speeds used were either 25 mm per sec or 50 mm per sec. Over 3 miles of chart paper was used in recording the various data of the investigation. In general, the method of data reduction was as follows: (a) by visual observation, from 50 to 125 selected points were marked on the chart paper for each test-run; (b) the total, or double, amplitude of each channel was measured to 0.1 mm and the distance of one cycle of oscillation, or
Table 8. Logarithmic decrements and viscous-damping factors

<table>
<thead>
<tr>
<th>Supports</th>
<th>Logarithmic decrement</th>
<th>Number of values averaged</th>
<th>Average damping factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Avg.</td>
<td>Max.</td>
<td>Min.</td>
</tr>
<tr>
<td>Oscillator only</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steel</td>
<td>0.1524</td>
<td>0.2060</td>
<td>0.1150</td>
</tr>
<tr>
<td>64 durom.</td>
<td>0.1410</td>
<td>0.2560</td>
<td>0.0592</td>
</tr>
<tr>
<td>49 durom.</td>
<td>0.1222</td>
<td>0.1915</td>
<td>0.0535</td>
</tr>
<tr>
<td>Oscillator with</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>conc. block</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steel</td>
<td>0.1599</td>
<td>0.2760</td>
<td>0.0761</td>
</tr>
<tr>
<td>64 durom.</td>
<td>0.1594</td>
<td>0.2230</td>
<td>0.0970</td>
</tr>
<tr>
<td>49 durom.</td>
<td>0.1582</td>
<td>0.2320</td>
<td>0.1080</td>
</tr>
</tbody>
</table>

average of 2 cycles, was measured with a magnifying comparator to 0.001 in. and reconciled with the distance between revolution tic marks (these were usually in close agreement, although slight discrepancies were sometimes noted at the peak, or maximum, amplitudes at the natural frequency); (c) for tests with neoprene pads the phase relationship between deflectometers at midspan and the ends was checked and found to be in phase for all cases; (d) the double amplitude, in millimeters, was multiplied by the attenuator, or sensitivity, setting to obtain values of attenuator-lines.

These values of chart speed, distance per cycle of oscillation, and magnitude of double oscillation were used with the calibration constants determined from the static load tests for reduction by an IBM 7074 computer. The computer program was written in Fortran.
Fig. 15. Typical oscillograph chart—strain
Fig. 16. Typical oscillograph chart—deflection
The strain data were reduced for values of frequency, cycles per second; dynamic strain in the bottom flange of each girder at midspan, inch per inch; driving force applied, pound; strain amplification factor \( (\varepsilon/\varepsilon_0) \), dimensionless, i.e., the ratio of dynamic strain to the strain which would be produced by the dynamic force statically applied; and ratio of frequency to natural frequency, dimensionless.

Deflection data for the steel supports were reduced for values of frequency, cycles per second; deflection of each girder at midspan, inches; forcing function, pound; deflection amplification factor for each girder \( (\Delta/\Delta_0) \), dimensionless, i.e., the ratio of dynamic deflection to the deflection which would be produced by the dynamic force statically applied; and ratio of frequency to natural frequency.

Deflection data for neoprene pads were reduced to give values for exterior girder 1-1 (called A for strain) and non-adjacent girder 3-1 (called C for strain) of frequency, cycles per second; total, or gross, deflection at midspan and each end, inches; forcing function, pound; average end deflection, inches; net deflection at midspan, inches; gross deflection amplification factors \( (\Delta/\Delta_0) \) at midspan and ends, dimensionless; average end deflection expressed as per cent of gross midspan deflection; and ratio of frequency to natural frequency.

Graphs of various combinations of variables were plotted using an IBM 7074 computer and a Cal-Comp incremental plotter.

Some representative maximum values obtained are shown in Tables 9 and 10. Typical computer plots and composite graphs are shown in Appendix D.
Table 9. Maximum reduced values from strain data

<table>
<thead>
<tr>
<th>Supports</th>
<th>STA&lt;sup&gt;a&lt;/sup&gt;</th>
<th>STB&lt;sup&gt;b&lt;/sup&gt;</th>
<th>STC</th>
<th>STD</th>
<th>AFSTA&lt;sup&gt;c&lt;/sup&gt;</th>
<th>AFSTB&lt;sup&gt;b&lt;/sup&gt;</th>
<th>AFSTC</th>
<th>AFSTD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oscillator only at midpoint, 7.01 in. eccentricity, small weights</td>
<td>Steel</td>
<td>105.0</td>
<td>102.8</td>
<td>97.0</td>
<td>91.3</td>
<td>72.0</td>
<td>30.6</td>
<td>32.8</td>
</tr>
<tr>
<td></td>
<td>64 durometer</td>
<td>62.0</td>
<td>64.5</td>
<td>62.3</td>
<td>56.3</td>
<td>49.3</td>
<td>24.9</td>
<td>24.3</td>
</tr>
<tr>
<td></td>
<td>49 durometer</td>
<td>65.0</td>
<td>65.0</td>
<td>63.8</td>
<td>57.5</td>
<td>51.5</td>
<td>25.6</td>
<td>24.7</td>
</tr>
<tr>
<td>Oscillator only at midpoint, 3.26 in. eccentricity, big weights</td>
<td>Steel</td>
<td>190.0</td>
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</tbody>
</table>

<sup>a</sup> Strain (micro-in./in.), girder A

<sup>b</sup> Some values believed in error due to malfunctioning of amplifier

<sup>c</sup> Amplification factor ($\varepsilon/\varepsilon_0$) for strain, girder A
Table 10. Maximum reduced values from deflection data

<table>
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<tr>
<th>Supports</th>
<th>Y1-1</th>
<th>Y2-1</th>
<th>Y3-1</th>
<th>Y4-1</th>
<th>AFY1-1</th>
<th>AFY2-1</th>
<th>AFY3-1</th>
<th>AFY4-1</th>
<th>YS</th>
<th>YN</th>
<th>YEA</th>
<th>YMN</th>
<th>EFCYM</th>
<th>AFYS</th>
<th>AFYN</th>
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</thead>
<tbody>
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<td>Oscillator only, small weights at 7.01 in.</td>
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<td>118.3</td>
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<td>34.8</td>
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<tr>
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<tr>
<td>Oscillator with conc. blocks, small weights at 7.01 in.</td>
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<td>76.4</td>
<td>76.5</td>
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<td>Oscillator with conc. blocks, big weights at 3.26 in.</td>
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<td>177.8</td>
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<td>Oscillator with conc. blocks, big weights at 4.51 in.</td>
<td>Steel</td>
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<td>34.4</td>
<td></td>
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</tbody>
</table>

\*Y1-1 is total dynamic deflection—downward or upward—at midspan of girder 1-1; AFY1-1 is deflection amplification factor \((A/A_N)\) for midspan of girder 1-1; YS is end deflection at south end; YN is end deflection at north end; YEA is average end deflection; YMN is net deflection at midspan of girder; EFCYM is average end deflection expressed as per cent of gross midspan deflection; and AFYS is deflection amplification factor for south end. All deflections are in units of in. \((10)^{3}\)
Strain

Typical midspan strain-frequency curves are shown in Figs. 29-32 and 37-42, Appendix D. The vertical driving force as determined from Equation 1 is also shown on these graphs. Since the driving force is a function of the weight of the rotating mass, the eccentricity of the mass and the rotational velocity, comparisons should only be made between those tests where the same weights and eccentricities were used.

From the strain-frequency curves and Table 9 the following may be seen:

(1) The maximum strain, and thus the stress, in each girder at the respective natural frequency for each bearing condition was less for the elastomeric pads than for the curved steel sole plates, varying from 1.7 to 41.3% less. Strains were usually, although not always, slightly less for the 49 durometer pads than for the 64 durometer pads.

(2) At the natural frequency maximum strains at midspan of the four girders were approximately equal for a given bearing condition and in general decreased slightly from the west to the east side of the bridge when loaded with the oscillator only. When loaded with the oscillator and concrete blocks, strains at midspan of the interior girders were larger than for the exterior girders.

(3) The shape of the strain-forcing frequency curves for each bearing condition are generally similar but displaced to the left for the elastomeric pads. Thus at frequencies below the natural frequency for curved steel sole plates
slightly larger strains (and stresses) were observed for the elastomeric pads than for the curved steel sole plates. In general slightly higher strains were observed for the 49 durometer pads.

(4) Differences in strain become negligible in the region of the lower and higher frequencies tested—at approximately 4 to 5½ and 9½ to 11½ cps.

It should be remembered that, while differences in strain were observed, the magnitudes of strain were of low order.

**Strain amplification factor**

As used herein, strain amplification factor \( \varepsilon / \varepsilon_0 \) denotes the ratio of dynamic strain to static strain, or the strain which would result if the driving force was statically applied. Typical strain amplification factor-frequency curves are shown in Figs. 33-36 and 43-46, Appendix D. A comparison of these curves and Table 9 shows relationships similar to those for strain with reference to type of bearing condition used. The following observations should be noted:

(1) Whereas the maximum strains in the exterior and interior girders were approximately of equal value—when loaded with the oscillator only—the strain amplification factors for the exterior girders were much greater than for the interior girders, usually about twice as large.

(2) The maximum magnitude of strain amplification did not necessarily occur at the maximum strain.
Deflection

Midspan deflection  As may be seen from Table 10 and the typical midspan deflection-frequency curves shown in Figs. 47-48 and 55-60, Appendix D, the deflection relationships between the three bearing conditions are similar to those for strain. The deflection relationships may be summarized as follows:

(1) At the respective natural frequencies, the total, and thus also the net, deflection at midspan was less—except for one case—for the neoprene pads than for the curved steel sole plates, varying from 1.4% more to 34.2% less. Differences between the total deflection for 49 and 64 durometer pad supports were small.

(2) At the natural frequency maximum total deflections of the four girders were approximately equal for a given bearing condition when the test bridge was loaded with the oscillator only. When loaded with the oscillator and concrete blocks, deflections of the interior girders were larger than those of the exterior girders.

(3) The shape of the midspan deflection-frequency curves for each type of bearing condition are generally similar but displaced to the left for the elastomeric pads. At frequencies below the natural frequency for curved steel sole plates slightly larger deflections were observed for the elastomeric pads than for the curved steel sole plates.

(4) The curves show nearly equal magnitudes of deflection for each type of support condition at the lower and higher frequencies tested.
The maximum total deflection observed was approximately ½ in., downward or upward, and occurred for both cases of loading (oscillator only and oscillator with concrete blocks) at the maximum driving force condition of big weights at 4.51 in.

Since end deflections for the neoprene pads were of such small magnitude, and then only for the case of resonance for the test bridge, only typical curves for total midspan deflection-frequency are shown.

**End deflection**  
As stated previously, end deflections for the neoprene pads were of small magnitude and pronounced only within the region of the natural frequency. As shown in Table 10, deflections at the north end of the test bridge were larger than those at the south end. This is believed due in part to: (1) the total end reaction at the north end was slightly larger than at the south end; and (2) the larger number of transverse cracks in the deck slab of the south portion of the test bridge possibly reduced the energy transmitted to the south supports.

The maximum average end deflections occurred in the region of the natural frequency and varied between 2.8 and 8.8% of the total dynamic midspan deflection for the respective type of neoprene bearing.

In addition to vertical deflection, some longitudinal deflection of the neoprene pads was observed during vibration, but there was no discernible transverse deflection.

**Deflection amplification factor**

Midspan deflection  
Deflection amplification factor \( (\Delta/\Delta_0) \) as used herein denotes the ratio of dynamic deflection to static deflection, or the deflection which would result if the driving force was statically
applied. From Table 10 and the typical deflection amplification factor-frequency curves shown in Figs. 49-50 and 61-64, Appendix D, the same general relationships are observed as for the strain amplification factor. Although the amplification factors for total deflection at midspan of the exterior girders are larger than for the interior girders, the ratio is not as large as for strain.

**End deflection**

Typical deflection amplification factor-frequency curves for end deflection are shown in Figs. 51-54 and 65-70, Appendix D. From these curves and Table 10 the following general observations may be made:

1. As would be expected from the end deflections, the amplification factors for deflection at the north end of the bridge are generally larger than for the south end.
2. Amplification factors for end deflection of exterior girders are larger than for interior girders.
3. The amplification factors for maximum end deflections are generally of about the same magnitude as for maximum total mid-span deflections.
This investigation was initiated because some engineers were concerned about (1) the multiplicity of bridge supporting and expansion devices used by design engineers, (2) the wide range of cost of the various types of devices used which, frequently, influences the type of bridge design, (3) the fact that many types of devices do not function as anticipated, (4) incongruities which exist in design practice as to usage of the different kinds of devices, and (5) the apparent lack of information concerning behavior of bridge supporting and expansion devices.

The initial objectives of the investigation were: (1) to review and make a field study of devices used for the support of bridge superstructures and for provision of floor expansion; (2) to analyze the forces or factors which influence the design and behavior of floor expansion devices and floor expansion systems; and (3) to ascertain the need for future research, particularly on the problems of obtaining more economical and efficient supporting and expansion devices, and determining maximum allowable distance between such devices. As the research progressed it became apparent that additional items should be considered and the initial objectives were altered slightly. The experimental portion of the investigation was conducted to provide more information about one of the possible solutions to the problems observed during the initial portion.

The investigation was divided into four parts or phases as follows:

(1) A review of literature;

(2) A survey by questionnaire of design practice of a number of state highway departments and consulting firms;
Field observation of bridges; and,
(4) An experimental comparison of the dynamic behavior of rigid and elastomeric bearings.

Since the initial and experimental portions of the research are basically two separate investigations, the conclusions will be divided into two parts: (1) bridge supporting and expansion devices; and (2) dynamic behavior of rigid and elastomeric bearings.

Bridge Supporting and Expansion Devices

The following conclusions are based upon information obtained from the first three phases of the investigation: (1) review of literature; (2) survey of design practice; and (3) field observations. They are believed valid for short span, deck type highway bridges.

1. There are extremely wide variations in present bridge design practice of state highway departments and consulting firms or engineers in regard to the type of bridge supporting and expansion devices used and the limitations for each type of device. These incongruities exist especially with respect to steel versus concrete bridges, but also for similar bridges of like material.

2. The necessity of providing for an expansion of steel bridges equal to almost twice that of concrete bridges is questionable.

3. In design of main supporting members, the use of the same temperature range for both exposed and sheltered conditions is questionable.

4. There is a minimum expansion length, perhaps 200 to 250 ft,
below which many of the currently used supporting devices, especially rockers, do not experience sufficient thermal movement to insure continued free movement.

5. A large percentage of bridges do not retain the designed provision for thermal movement. In many cases the bridges experience closure of floor expansion devices and excessive movement of supporting devices due to abutment movement. This movement is often caused by approach slab action.

6. There appears to be no direct relationship between abutment movement and age of structure, type of approaches, or type of supporting or expansion devices.

7. The influence of residual expansion (growth) of concrete under varying environmental conditions on the superstructure of concrete and steel bridges has apparently not received the attention warranted.

8. The use of flexible abutments and piers tied directly to the superstructure has been demonstrated to be feasible and satisfactory for both concrete and steel deck type highway bridges.

9. Where applicable, open armored joints have operational and economic advantages over the "conventional" sliding plate and finger joint types of floor expansion devices.

10. Elastomeric bearings need not be used more extensively for concrete bridges than for steel bridges.
Dynamic Behavior of Rigid and Elastomeric Bearings

This portion of the investigation was limited to the testing of one model bridge only with one set each of curved steel sole plate, 64 durometer and 49 durometer bearings. It is highly probable that somewhat different results would be obtained from tests using a different test bridge, method of inducing vibration, or neoprene pads with a different shape factor or spring modulus. Many of the results obtained differ from those anticipated by theoretical considerations.

Therefore, the following conclusions, or observations, resulting from the experimental portion of the research should be considered indicative, rather than conclusive, of the dynamic behavior of a bridge with rigid and elastomeric bearings subjected to a steady state forced vibration with harmonic excitation until more fully substantiated by additional tests with a variety of parameters.

1. For a single concentrated load applied at the center of the bridge, both static load strains and deflections at midspan are less for curved steel sole plate than for neoprene bearings. 
2. The natural frequency of a given bridge superstructure is less for elastomeric bearings than for rigid bearings and elastomeric support bearings having the smallest spring modulus will have the lowest natural frequency. 
3. The maximum strain (and thus the stress) and deflection, total and net, at midspan, evaluated at the respective natural frequency for each type of bearing condition, is less when elastomeric, rather than rigid, bearings are used. In general,
"softer" pads reduce the strain and deflection more than "harder" pads.

4. For relatively small dead loads the maximum strains (and deflections) at midspan of the interior and exterior girders are approximately equal. An increase of dead load tends to increase the interior/exterior girder ratio of strain and deflection.

5. At low and high frequencies (approximately 4 to 5\(\frac{1}{2}\) and 9\(\frac{1}{2}\) to 11\(\frac{1}{2}\) cps for this investigation) the magnitude of strain, and deflection, is nearly equal for elastomeric and rigid bearings. At intermediate frequencies below the natural frequency with curved steel sole plates the magnitude of strain, and deflection, is greater for elastomeric than rigid bearings. This is a result of displacement of similar shaped curves due to the lower natural frequency when elastomeric pads are used.

6. Amplification factors for strain are much larger for exterior than interior girders. The same relationship exists for deflection amplification factors, but to a lesser degree.
COMMENTS AND RECOMMENDATIONS FOR FURTHER STUDY

During the course of this investigation, many interesting problems and questions arose. Most of these are still unanswered. Some came as an outgrowth of the research and others were suggested by individuals. Many are inferred in the list of factors which influence the behavior of bridge supporting and expansion devices. The multiplicity of these factors and the dependence of the ultimate behavior of the structure upon almost any one of these factors make a single solution for the behavior of the devices investigated--considering all possible factors--a horrendous task, if at all possible.

For example, the function of the floor expansion device and supporting devices at non-fixed points is to provide for the relative movement between the superstructure and the substructure. The cause of excessive movement is not important--to the devices. The observation of excessive movement of supporting devices, closed expansion devices, differences in relative rocker movement, and cracked or spalled abutments is not, in itself, especially disquieting. However, the consideration of the amount of movement due to the superstructure or substructure and the factors which cause and influence the movement intrigue the imagination. Some possible answers can be found in the literature of prior research; some by discussion and correspondence with design engineers who have observed these conditions and their repair; and some by field observations--if a trend or pattern of behavior can be found.

The problem of closure of expansion devices and excessive movement of supporting devices seems to be solved by the use of continuous concrete
and steel bridges which have no movable supporting or expansion devices. This raises the question as to what additional forces—and their effects—may be induced. This is similar to additional forces caused by abutments which move and rest against the ends of the beams or diaphragms.

Common to bridges with and without expansion devices is the thermal force and/or movement due to expansion resulting from temperature differentials, concrete growth, moisture, humidity, solar radiation and other causes. It is possible that under the influence of environmental variables concrete and composite steel structures could have a thermal coefficient different from the accepted values of either of the component materials.

The use of elastomeric, Teflon-surfaced, and other proposed types of bearings introduces other factors for consideration. For example, in this investigation static load stresses were not the same when steel and elastomeric bearings were used. This is due, possibly, to differences in restraint of longitudinal movement by friction (steel) and shear modulus (elastomeric) of the respective bearings. However, the influence of friction would be different for static and impact loading.

Other items relative to the use of elastomeric bearings requiring additional study are the relationship between the reduced natural frequency and vehicular induced vibrations and the possible slightly increased stresses—indicated in this investigation—at a given frequency.

Some of the investigations which should provide information of value to bridge design engineers are the following:

(1) A study of temperature distribution and the actual movement and stresses experienced by steel and concrete bridges under environmental
conditions. This should include a comparison of such variables as type of bearing, location—such as over a highway or a stream—and prolonged periods of extreme high and low temperatures.

(2) Long-term field observations of the behavior and maintenance required by a large number of bridges of different age and type. This could produce trends of behavior, but probably little information as to the cause or prevention. An exchange of such information by a number of State Highway Departments would provide useful information of interest to many bridge design engineers.

(3) A review of the relative cost of the various types of devices used, including cost of installation. This information, with that concerning behavior, would be helpful to bridge design engineers in the selection of dependable and economical devices.

(4) A study of the additional forces involved by tying the superstructure to flexible piers and abutments and those forces introduced when abutments move inward and rest against the beams or end diaphragms.

(5) A study of the minimum expansion length required for sufficient thermal movement to assure continued free movement of such devices as rockers.

(6) A study of approach slab movement and an experimental investigation of methods for prevention or reduction of movement and growth—such as "keying" the slab to the approach fill.

(7) The development of a usable method for design purposes for determining or assuming the coefficient of damping for bridge structures other than experimentation on the completed bridge.
(8) An investigation of the response of actual highway bridges with both conventional rigid and elastomeric bearings. Such an investigation is being conducted under the direction of Professor H. L. Kinnier, University of Virginia, Charlottesville, Virginia.

(9) Further studies of dynamic load distribution, especially with respect to rigid and elastomeric bearings and the torsional effect of off-center loading.

(10) Further investigation of the correlation of behavior as determined by a stationary oscillating load and moving vehicle loads on highway bridges.
REFERENCES CITED


5. _______ Specifications for steel railway bridges for fixed spans not exceeding 400 feet in length. Chicago, Ill., author. 1947.


15. ________ A summary of the results of investigations having to do with volumetric changes in cements, mortars and concretes, due to causes other than stress. American Concrete Institute Proceedings 26: 407-443. 1930.


ACKNOWLEDGMENTS

The author wishes to express appreciation to the members of his Graduate Committee, Professors C. E. Ekberg, Jr. (Chairman), R. A. Caughey, S. J. Chamberlin, Glenn Murphy, M. G. Spangler, and R. E. Untrauer, for their patience and instruction throughout the writer's graduate program. He is especially grateful to Dr. Ekberg for his guidance and counsel in this project, and to Dr. Murphy for his encouragement and consideration, without which the joys and pleasures of teaching might never have been experienced by the author.

Acknowledgment is made to the American Iron and Steel Institute, major sponsor of the experimental portion of the investigation; Paxton and Vierling Steel Co., Omaha, Nebraska, sponsor of the first three phases of the investigation and co-sponsor of the experimental portion; the Iowa Highway Research Board, co-sponsor of the experimental portion; the Pittsburgh-Des Moines Steel Co., Des Moines, Iowa for providing the structural steel used in this investigation; and The General Tire and Rubber Co., Industrial Products Div., Wabash, Indiana for providing the neoprene support bearings.
APPENDIX A:

QUESTIONNAIRE AND REPLIES TO QUESTIONNAIRE
A research project involving a detailed study of bridge supporting and expansion devices is being conducted by the Iowa Engineering Experiment Station. The purpose of this project is (1) to study the factors which influence the requirements for and the design of supporting devices and floor expansion systems, (2) to review the existing devices currently in use, and (3) to develop, if possible, more efficient and economical supporting and expansion devices.

The project concerns both concrete and steel bridges of deck girder type construction with individual spans of 150 feet or less. A search of the available literature has revealed that the cost of existing bridge supporting and expansion devices varies over an extremely large range. However, there is little or no information concerning the numerous and varied types of bridge supporting and expansion devices currently utilized by bridge design engineers, the changes under consideration in the design of these devices, and the factors which govern the preference and selection of the type of devices to be used for a given design.

Reliable information required for the project can be obtained only from engineers engaged in bridge design. Therefore, we are sending this letter and attached questionnaire to a selected group of highway commissions and consulting firms.

We shall appreciate any information you can supply us and are especially interested in copies of bridge plans or standard design details in use by your organization. If you would like to have it, we shall be glad to send you a copy of the report on this survey when it is completed.

Sincerely yours,

Jack H. Emanuel, Asst. Prof.
Engineering Mechanics

JHE/ph
Enc.
QUESTIONNAIRE

A Study of Supporting and Expansion Devices for Bridges with Spans up to 150 Ft.
(Includes both steel and reinforced or prestressed concrete bridges.)

1. (a) What are the types of supporting devices currently used by your organization; e.g., rockers, simple steel sole plates, curved steel plates, bronze plates, elastomeric bearing pads, or other?

(b) What are the limitations such as span lengths for each of the supporting devices used?

2. (a) What are the types of floor expansion devices currently used by your organization; e.g., finger joint, joint sealer only, sliding plates, armor joints, elastomeric tubes, sponge fillers, or other?

(b) What are the limitations such as span lengths for each of these expansion systems?

3. (a) Has your organization utilized, or considered the use of, flexible stub abutments without expansion or rocker type supporting devices; e.g., tying the girder directly to the abutment?

(b) What are the limitations such as span length for type of design described above?

4. (a) Has your organization experienced cases of movement of the abutments inward toward the bridge such that the abutments lean against the ends of the girders or end diaphragms and movement of the supporting devices has thus become "frozen?"

(b) If so, could you please advise the locations of the bridges involved?

5. (a) Has your organization experienced cases of "freezing" due to rust or corrosion, of the devices currently used?

(b) If so, could you please advise the approximate location of one or two of the bridges involved?
State Highway Departments

Question 1.

(a) What are the types of supporting devices currently used by your organization; e.g., rockers, simple steel sole plates, curved steel plates, bronze plates, elastomeric bearing pads, or other?

(b) What are the limitations such as span lengths for each of the supporting devices used?

Answers based on type of supporting device:

<table>
<thead>
<tr>
<th>Highway Department</th>
<th>Conditions for Use</th>
</tr>
</thead>
</table>

**Cast or Welded Steel Rockers.**

- **A** Used for spans from 100 to 150 ft.
- **B** Used almost entirely on steel superstructures; suitable for any span length.
- **D** The limitations are determined by the loads carried. The following uses are shown on Standard Details of this Department:
  - Steel beam bridges--at ends of all continuous units and at expansion end of simple spans of 45 ft and longer.
  - I-beam bridges--for abutment expansion bearing and interior expansion bearings.
  - R.C. deck girder bridges--for abutment expansion bearing and interior expansion bearings.
  - (Bolster used for interior fixed bearings for R.C. deck girder and I-beam bridges.)
Answers to Question 1, (Continued)

<table>
<thead>
<tr>
<th>Highway Department</th>
<th>Conditions for Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>Used for spans with more than 80 ft of expansion length* and/or heavier loadings; continuous WF-beam bridges.</td>
</tr>
<tr>
<td>F</td>
<td>Used for I-beam spans from 40 to 100 ft.</td>
</tr>
<tr>
<td>G</td>
<td>Cast steel bearings are most commonly used on cantilever steel girder bridges having spans of 80 ft or more. A structural steel device similar to the casting is made as an alternate to the casting. This type of expansion bearing does offer some resistance to movement and is most commonly used in continuous steel girder bridges. Welded rockers and rollers are used on steel girder bridges where reduction of frictional resistance in the roller is desired.</td>
</tr>
</tbody>
</table>
| H                  | For the following typical cases:  
1. Steel beam bridge with 2 spans, each 90 ft or longer. Cast steel rockers used at abutments. (Curved steel plates used for fixed points at center pier.)  
2. Steel beam bridge with 3 spans, each 85 ft or longer. Cast steel rockers with lubricated bronze bearings. |

*The term expansion length is defined as a longitudinal segment of bridge one end of which is free to displace.*
Answers to Question 1, (Continued)

Highway Conditions for Use
Department

bushing for pin type bearing used at expansion points. (Cast steel with lubricated bronze bushing for pin type bearing used for fixed point at one pier only.)

I  No limitation on span length.

Simple Steel Sole Plates.

D  The limitations are determined by the loads carried. Standard details show flat plates used for single span I-beam bridges less than 45 ft.

E  Simple WF-beam bridges: Used with slotted holes for expansion on spans to 35 ft. Capacity (of device) is about 50 K.

G  Used very seldom and when used are confined to short span secondary structures.

Curved Steel Plates.

C  Steel beam spans under 150 ft; in general do not use beam spans in excess of 100 ft.

D  The limitations are determined by the loads carried. Standard Details show the use of curved plates for fixed bearing at abutment for I-beam bridges, and for fixed end at abutment of R.C. deck girder bridges.
Answers to Question 1, (Continued)

<table>
<thead>
<tr>
<th>Highway Department</th>
<th>Conditions for Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>Simple WF-beam bridges: Used with spans 35 to 80 ft. Continuous WF-beam bridges: Used with no more than 80 ft of expansion (length).</td>
</tr>
<tr>
<td>G</td>
<td>Used on prestressed concrete girder bridges, and steel girder bridges of single span and cantilever type where spans are generally less than 80 ft. Have been used on prestressed spans up to 110 ft and steel girder spans up to 100 ft (simple spans).</td>
</tr>
</tbody>
</table>
| H                  | For the following typical cases:  
  1. Continuous concrete slab bridge with 5 spans, each varying between 30 and 40 ft. Curved steel plate used for fixed points at the two center piers. (Curved steel plate with lubricated bronze plate type bearings used for expansion points at abutments and end piers.)  
  2. Used at fixed end of prestressed concrete girders for span lengths between 30 and 90 ft. (Curved steel plate with lubricated bronze type bearings used at expansion ends.)  
  3. Steel beam bridges with span lengths varying between 35 and 100 ft. Curved steel plate used at fixed points and expansion hinge. (Curved steel |
Answers to Question 1, (Continued)

Highway Department Conditions for Use

plate with lubricated bronze type bearing used at expansion points.) One 3 span continuous welded steel beam bridge was fixed at one pier. One 4 span continuous and cantilever steel beam bridge was fixed at center pier and abutments.

4. Refer to Cast or Welded Steel Rockers on previous page, Highway Dept. H, paragraph 1.

I Used for spans up to about 75 ft.

Bronze Plates.

A For spans less than 100 ft, lubricated bronze plates for expansion placed above curved steel plates to provide for rotation.

B Lubricated bronze bearings are occasionally used where very little bearing height is available.

H For the following typical cases:

1. Refer to Curved Steel Plates on previous page, Highway Dept. H, paragraphs 1, 2, and 3.

2. Concrete deck girder bridge with 3 spans varying between 30 and 60 ft. Flat steel plate with lubricated bronze plate type bearings used for expansion end at abutments. (Concrete keyway and dowels used for fixed ends at both ends of center
Answers to Question 1, (Continued)

<table>
<thead>
<tr>
<th>Highway Department</th>
<th>Conditions for Use</th>
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<tbody>
<tr>
<td></td>
<td>span and at piers for interior beams of end spans.</td>
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</table>

**Elastomeric Bearing Pads.**

<table>
<thead>
<tr>
<th>Letter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>Not used.</td>
</tr>
<tr>
<td>C</td>
<td>Used for all prestressed concrete girders; maximum length 70 ft at present.</td>
</tr>
<tr>
<td>E</td>
<td>Neoprene used for concrete deck girder bridges with simple spans to 60 ft.</td>
</tr>
<tr>
<td>F</td>
<td>Plain neoprene pads used for prestressed concrete spans from 40 to 100 ft. Laminated neoprene pads have been used for floor-beams and simple I-beam spans. Investigations have been made on use of laminated neoprene pads for simple and continuous I-beam spans, but no reports are available at this time.</td>
</tr>
<tr>
<td>G</td>
<td>Have been used for prestressed concrete bridges only.</td>
</tr>
<tr>
<td>I</td>
<td>Used for spans up to about 80 ft.</td>
</tr>
</tbody>
</table>

**Other.**

<table>
<thead>
<tr>
<th>Letter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>A 1 x 1-in. bar with curved top and welded to the bearing plate is occasionally used for very light steel superstructures. The flange of the I-beam bears directly upon the curved surface of the bar.</td>
</tr>
</tbody>
</table>
Answers to Question 1, (Continued)

<table>
<thead>
<tr>
<th>Highway Department</th>
<th>Conditions for Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Normally no bearing device used for concrete slabs and T-beam spans; concrete is poured directly upon the pier or abutment.</td>
</tr>
<tr>
<td>D</td>
<td>Limitations are determined by the loads carried. Standard Details show the following: Graphited asbestos bearing pads at expansion bearings for precast prestressed concrete bridge deck. Fabric bearing pads at fixed bearings for precast prestressed concrete bridge deck. Abutment curtain wall monolithic with deck slab for steel beam and reinforced concrete girder bridges with single spans under 45 ft. A six-in. roller for abutment expansion bearings of reinforced concrete slab bridges.</td>
</tr>
<tr>
<td>E</td>
<td>Two layers of ordinary roofing felt between superstructure and supporting concrete surface used on continuous concrete slabs and box girder spans up to about 40 ft. Longer spans vary with conditions but are generally similar to &quot;6-in. roller bearing&quot; (with pintles).</td>
</tr>
<tr>
<td>F</td>
<td>Expansion joint material or roofing felt usually used for concrete slab or girder spans varying from 20 to 40 ft.</td>
</tr>
</tbody>
</table>
Answers to Question 1, (Continued)

<table>
<thead>
<tr>
<th>Highway Department</th>
<th>Conditions for Use</th>
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<tbody>
<tr>
<td>H</td>
<td>For the following typical cases:</td>
</tr>
</tbody>
</table>

1. Precast concrete channel bridges with span lengths from 18 to 30 ft have both ends set on masonry mortar and fixed at both ends with a steel dowel.

2. Prestressed precast concrete slab with span lengths from 30 to 47 ft. Slab set on bituminous felt and fixed with a steel dowel at fixed end. Slab set on bituminous felt and steel dowel allowed to flex at expansion end. Both ends of some are fixed.

3. Continuous concrete slab bridges with 3 spans varying between 30 and 46 ft. For fixed points at piers, the roadway slab and pier cap are integral. For expansion ends at abutments, the slab is set on troweled seat painted with asphalt and shear lug with cork used to allow for expansion.

4. Refer to Bronze Plates on previous page, Highway Dept. H, paragraph 2.

5. Concrete box girder bridge with 5 spans varying between 75 and 125 ft. For fixed points: at single column pier, pier cap and box girder were integral;
Answers to Question 1, (Continued)

<table>
<thead>
<tr>
<th>Highway Department</th>
<th>Conditions for Use</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>at other fixed points, curved steel plate type</td>
</tr>
<tr>
<td></td>
<td>bearings were used. For expansion points, curved</td>
</tr>
<tr>
<td></td>
<td>steel plate with lubricated bronze plate type bear-</td>
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<tr>
<td></td>
<td>ings were used. (A simple span, cantilever, and</td>
</tr>
<tr>
<td></td>
<td>suspended spans were used on this bridge.)</td>
</tr>
<tr>
<td>6. Refer to Cast or Welded Rockers on previous</td>
<td></td>
</tr>
<tr>
<td>page, Highway Dept. H, paragraph 2.</td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>Fabreeka (hard)--used for spans up to about 40 ft.</td>
</tr>
</tbody>
</table>
Question 2.
(a) What are the types of floor expansion devices currently used by your organization; e.g., finger joint, joint sealer only, sliding plates, armored joints, elastomeric tubes, sponge fillers, or other?
(b) What are the limitations such as span lengths for each of these expansion systems?

Answers based on type of expansion device:

<table>
<thead>
<tr>
<th>Highway Department Conditions for Use</th>
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<tbody>
<tr>
<td><strong>Finger Joint.</strong></td>
</tr>
<tr>
<td>A Used for expansion lengths above 300 ft.</td>
</tr>
<tr>
<td>B Used for indicated joint movements in excess of 3 in. total movement.</td>
</tr>
<tr>
<td>C Used for longer bridges (seldom used for steel beam bridges).</td>
</tr>
<tr>
<td>D Used for $L &gt; 250$ ft (for skew angle = 0 and $L =$ length of structure from fixed support; limiting value of $L$ varies with skew angle).</td>
</tr>
<tr>
<td>E Used for over 250 ft of expansion (length).</td>
</tr>
<tr>
<td>F Used where expansion exceeds $2\frac{1}{2}$ in. at the coldest design temperature.</td>
</tr>
<tr>
<td>G Open finger joints are commonly used on primary structures over stream crossings (steel girder bridges only). Closed finger type of devices have been used where a waterproof device is desired. This device tends to freeze up and its use has been discontinued. (This is a finger</td>
</tr>
</tbody>
</table>
Answers to Question 2, (Continued)

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<tr>
<th>Highway Department</th>
<th>Conditions for Use</th>
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<tbody>
<tr>
<td></td>
<td>device with solid plate in contact with bottom surface.)</td>
</tr>
<tr>
<td></td>
<td>Gap in the device is designed for the length of structure for which the device functions.</td>
</tr>
<tr>
<td>H</td>
<td>For the following typical cases:</td>
</tr>
<tr>
<td></td>
<td>1. Concrete box girder bridge with 5 spans: Finger joint used between a cantilever and end span; length of expansion 67 ft. A 1-in. cork expansion joint used for 68 ft length of expansion at abutments; joint over pier; and between suspended span and cantilever spans.</td>
</tr>
<tr>
<td></td>
<td>2. Deck plate girder bridge with 3 spans; length of expansion 100 ft. Finger joints used over hangers of suspended center section. Sliding plates, used at abutments, slide directly on concrete of abutment. (Bridge fixed at piers; center section suspended; rockers used at abutments.)</td>
</tr>
<tr>
<td>I</td>
<td>No limitation on expansion length.</td>
</tr>
<tr>
<td></td>
<td><strong>Joint Sealer.</strong></td>
</tr>
<tr>
<td>F</td>
<td>Currently used in construction joints and also on pan form structures (30 to 40 ft spans).</td>
</tr>
<tr>
<td></td>
<td><strong>Sliding Plates.</strong></td>
</tr>
<tr>
<td>A</td>
<td>Used for expansion lengths from 60 to 300 ft.</td>
</tr>
</tbody>
</table>
Answers to Question 2, (Continued)

<table>
<thead>
<tr>
<th>Highway Department</th>
<th>Conditions for Use</th>
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</thead>
<tbody>
<tr>
<td>B</td>
<td>Used for joints having indicated movement in both directions of less than 3 in.</td>
</tr>
<tr>
<td>C</td>
<td>Used for longer bridges (used generally for steel beam bridges).</td>
</tr>
<tr>
<td>D</td>
<td>Used for $200 &lt; L &lt; 250$ ft (for skew angle = 0 and $L =$ length of structure from fixed support; limiting values of $L$ vary with skew angle).</td>
</tr>
<tr>
<td>E</td>
<td>Used for 80 to 250 ft of expansion (length).</td>
</tr>
<tr>
<td>F</td>
<td>Have used.</td>
</tr>
<tr>
<td>G</td>
<td>Have been used extensively on steel and prestressed bridges; currently using this basic device with short fingers on the apron and stop plates to improve the riding quality; gap in the device is designed for the length of structure for which the device functions.</td>
</tr>
</tbody>
</table>
| H                   | For the following typical cases:  
1. Continuous concrete slab bridge with 5 spans. Sliding plate type expansion device used at abutments; length of expansion 98 ft.  
2. Steel beam bridge with 3 spans. Sliding plate type expansion device used at abutments; sliding plate slides directly on concrete of front face of abutment parapet; length of expansion 183 ft. |
Answers to Question 2, (Continued)

Highway Department Conditions for Use

3. Steel beam bridge with 2 spans and fixed at center pier. Sliding plate type expansion device used for length of expansion of 90 ft and over. Plate slides directly on concrete of front face of abutment parapet.

I Expansion length limitation approximately 375 ft.

Armored Joints--Open.

D Used for L<200 ft (for skew angle = 0 and L = length of structure from fixed support; limiting value of L varies with skew angle).

F Used where expansion does not exceed 2\(\frac{1}{2}\) in. at the coldest design temperature.

H For typical case of 4 span continuous and cantilever steel beam bridge. A 1\(\frac{1}{2}\)-in. open joint with steel plates to protect edges of concrete roadway slab used over hinge connections; length of expansion 75 ft.

Elastomeric Tubes.

B Elastomeric tubes are being tried on some existing joints where the original filler was asphalt impregnated felt. They do not consider that any joint using bituminous felt will prove to be satisfactory for very long but hope that they may be able to maintain this type of joint with
Answers to Question 2, (Continued)

<table>
<thead>
<tr>
<th>Highway Department</th>
<th>Conditions for Use</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>elastomeric tubes and special joint sealer although they have no experience to date with this type.</td>
</tr>
<tr>
<td>F</td>
<td>Have used.</td>
</tr>
</tbody>
</table>

**Sponge Fillers.**

| E | Sponge rubber used for up to 80 ft of expansion (length) on high type construction. |

**Other.**

| A | For short expansion lengths up to 60 ft, a one-in. thickness of joint filler sealed with hot poured rubber is used. |
| C | For concrete T-beam slab or prestressed beam bridges do not, at present, provide expansion devices, but provide a slender abutment designed such that the abutment can flex and take the necessary movement. Have constructed bridges up to about 200 ft total length, and they work very satisfactorily. |
| E | Bituminous preformed material used on the secondary system for up to 80 ft of expansion (length). |
| G | Preformed expansion joint fillers have been used for concrete slab bridges and are sealed with joint sealer. Most current design is based on continuous structures,
Answers to Question 2, (Continued)

<table>
<thead>
<tr>
<th>Highway Department</th>
<th>Conditions for Use</th>
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<tbody>
<tr>
<td></td>
<td>thereby eliminating expansion device in the deck except at bridge ends.</td>
</tr>
<tr>
<td></td>
<td>For the following typical cases:</td>
</tr>
<tr>
<td></td>
<td>1. Precast concrete channels: No expansion device used.</td>
</tr>
<tr>
<td></td>
<td>2. Prestressed precast concrete slab bridge with 3 simple spans and length of expansion varying from 30 to 47 ft. A 1-in. cork expansion joint was used.</td>
</tr>
<tr>
<td></td>
<td>3. Continuous concrete slab bridge with 3 spans and length of expansion of 60 ft. Flexible stub abutments used.</td>
</tr>
<tr>
<td></td>
<td>4. Concrete deck girder bridge with 3 simple spans. Expansion joint of 1-in. cork used for length of expansion from 30 to 60 ft.</td>
</tr>
<tr>
<td></td>
<td>5. Refer to Finger Joint on previous page, Highway Dept. H, paragraph 1.</td>
</tr>
<tr>
<td></td>
<td>6. Prestressed concrete girder bridge with 4 spans. No expansion device used; end of roadway slab at abutments extends over abutment parapet wall. Length of expansion 108 ft.</td>
</tr>
<tr>
<td></td>
<td>7. Prestressed concrete girder bridge with 4 simple spans. A 1-in. cork joint used for length of expansion of 30 to 90 ft.</td>
</tr>
</tbody>
</table>
8. Steel beam bridge with 3 simple spans. A 1-in. cork joint used for length of expansion of 30 to 70 ft.

<table>
<thead>
<tr>
<th>Highway Department</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>8. Steel beam bridge with 3 simple spans. A 1-in. cork joint used for length of expansion of 30 to 70 ft.</td>
</tr>
<tr>
<td></td>
<td>I Mastic joint filler--expansion length limitation approximately 75 ft.</td>
</tr>
<tr>
<td></td>
<td>Cord, dehydrated--expansion length limitation approximately 100 ft.</td>
</tr>
</tbody>
</table>
Question 3.
(a) Has your organization utilized, or considered the use of, flexible stub abutments without expansion or rocker type supporting devices; tying the girder directly to the abutment?
(b) What are the limitations such as span length for type of design described above?

Answers:

<table>
<thead>
<tr>
<th>Highway Department</th>
<th>Conditions for Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Yes. Have used for concrete structures, both cast in place and prestressed, up to bridge lengths of about 100 ft.</td>
</tr>
<tr>
<td>B</td>
<td>Yes. Maximum length of structure constructed about 400 ft; spans vary from 30 to 125 ft; have been using continuous T-girder, box girder, reinforced concrete slabs and reinforced concrete voided slabs with flexible stub abutments without expansion devices or expansion joints for a number of years; the pier columns are made monolithic with the superstructure and made sufficiently flexible so that they can accommodate the expansion; probably have over 100 structures of this type.</td>
</tr>
<tr>
<td>C</td>
<td>Yes. Have used flexible abutments tied directly into the superstructure for nearly 15 years; numerous cases where this has been done for continuous slab bridges to 160 ft, continuous T-beam bridges to 240 ft and prestressed bridges with floor slabs continuous as long as 240 ft; have used</td>
</tr>
</tbody>
</table>
Answers to Question 3, (Continued)

<table>
<thead>
<tr>
<th>Highway Department</th>
<th>Conditions for Use</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>for I-beam bridges in a few cases, but provide slotted holes and sliding plates in case the longitudinal thrust becomes excessive.</td>
</tr>
<tr>
<td>D</td>
<td>Use fixed-top type of abutments on single spans less than 45 ft.</td>
</tr>
<tr>
<td>E</td>
<td>Yes. Continuous concrete slabs and box girders:</td>
</tr>
<tr>
<td></td>
<td>Steel or concrete bearing pile integral with superstructure; has been used on a structure 400 ft long and greater lengths believed possible.</td>
</tr>
<tr>
<td></td>
<td>Small diameter concrete columns integral with superstructure; present maximum about 150 ft from center of structure.</td>
</tr>
<tr>
<td>F</td>
<td>Discontinued. Have used in the past on prestressed and short pan form structures; have discontinued the use of this type of abutments in their new standards; no limitations due to span.</td>
</tr>
<tr>
<td>G</td>
<td>Yes. Have recently designed some grade separation structures (county roads over interstate) using flexible stub abutments. These structures are four span continuous structures, either steel or prestressed</td>
</tr>
</tbody>
</table>
Answers to Question 3, (Continued)

<table>
<thead>
<tr>
<th>Department</th>
<th>Conditions for Use</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>concrete. This type of design is confined to square structures carrying low traffic volumes of approximately 230 ft in length.</td>
</tr>
<tr>
<td>H</td>
<td>Yes. Used on:</td>
</tr>
<tr>
<td></td>
<td>Continuous steel beam bridge with 3 spans and length of expansion 66 ft.</td>
</tr>
<tr>
<td></td>
<td>Continuous concrete slab bridge with 3 spans and length of expansion 57 ft.</td>
</tr>
<tr>
<td></td>
<td>Prestressed concrete slab bridge with 3 spans and length of expansion 55 ft.</td>
</tr>
<tr>
<td>I</td>
<td>No. Have not utilized flexible stub abutments without allowances made for relative horizontal movement between superstructure and abutments.</td>
</tr>
</tbody>
</table>
Question 4.

(a) Has your organization experienced cases of movement of the abutments inward toward the bridge such that the abutments lean against the ends of the girders or end diaphragms and movement of the supporting devices has thus become "frozen"?

(b) If so, could you please advise the locations of the bridges involved?

Answers:

<table>
<thead>
<tr>
<th>Highway Department</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Yes. Have experienced abutment crowding in some cases to the extent that the expansion joints are completely closed.</td>
</tr>
<tr>
<td>B</td>
<td>Yes. Have experienced many cases in almost every part of the State.</td>
</tr>
<tr>
<td>C</td>
<td>Yes. Have a large number of cases where the abutments have moved inward until they lean against the end steel girders and the expansion device, as a result, is completely closed; also have a large number of cases of steel bridges built with sliding plates where the plates apparently have never moved and have become &quot;frozen&quot;; &quot;in fact, the failure of our expansion joints to move is what prompted us in the first place to start constructing bridges without expansion joints&quot;.</td>
</tr>
<tr>
<td>D</td>
<td>Yes. Have had, in the past, cases where the abutments moved forward, so as to block expansion; now allow an extra space between the abutments and beams for this</td>
</tr>
</tbody>
</table>
Answers to Question 4, (Continued)

<table>
<thead>
<tr>
<th>Highway Department</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>purpose; have also tightened up on the specifications for backfilling.</td>
</tr>
<tr>
<td>E</td>
<td>No. In general, have experienced no extensive movement of bridge abutments involving contact with the superstructure.</td>
</tr>
<tr>
<td>F</td>
<td>Yes. (Gave location of two Expressways.)</td>
</tr>
<tr>
<td>G</td>
<td>Have observed some structures that have been in use some 20 or 30 years; however, no apparent damage has taken place. The expansion bearing plates become closed and the abutments apparently expand with the bridge. Newer structures are functioning satisfactorily with expansion devices.</td>
</tr>
<tr>
<td>H</td>
<td>Yes. All known cases have been repaired or corrected.</td>
</tr>
<tr>
<td>I</td>
<td>Yes. Various locations. Five abutment repairs made so far this year because of this condition.</td>
</tr>
</tbody>
</table>
Question 5.

(a) Has your organization experienced cases of "freezing" due to rust or corrosion of the devices currently used?

(b) If so, could you please advise the approximate location of one or two of the bridges involved?

Answers:

<table>
<thead>
<tr>
<th>Highway Department</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>No. No such cases for devices currently used; to their knowledge.</td>
</tr>
<tr>
<td>B</td>
<td>Yes. Have experienced a great deal of this on deck girder structures constructed during the 1930 decade when sliding plates were freely used; almost every simple span bridge constructed during this period will show at least one failure of this type; typical failure is fracture of ends of concrete deck girders at abutments and pier caps and/or fracture of abutment at bridge seat which probably could have been avoided by use of anchor rods rather than the short studs frequently used.</td>
</tr>
<tr>
<td>C</td>
<td>Yes. Have many cases where the expansion devices consisting of sliding plates have become &quot;frozen&quot; due to rusting or corrosion.</td>
</tr>
<tr>
<td>D</td>
<td>No. No records of &quot;freezing&quot; due to rust or corrosion of devices currently used; has occurred on some old structures with sliding plate type bearings.</td>
</tr>
</tbody>
</table>
Answers to Question 5, (Continued)

<table>
<thead>
<tr>
<th>Highway Department</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>No. Have had no reports of &quot;freezing&quot; of bearing and expansion devices currently in use.</td>
</tr>
<tr>
<td>F</td>
<td>No. Maintenance on bridges is extensive and therefore eliminates much of this trouble.</td>
</tr>
<tr>
<td>G</td>
<td>Some freezing due to rust or corrosion has been experienced especially on older structures. Old truss spans require cleaning. Have not experienced too much difficulty with this problem.</td>
</tr>
<tr>
<td>H</td>
<td>Yes. All known cases have been repaired or corrected.</td>
</tr>
<tr>
<td>I</td>
<td>Yes. Various locations. Two repairs made so far this year because of this condition.</td>
</tr>
</tbody>
</table>
Consulting Firms and Engineers

Question 1.

(a) What are the types of supporting devices currently used by your organization; e.g., rockers, simple steel sole plates, curved steel plates, bronze plates, elastomeric bearing pads, or other?

(b) What are the limitations such as span lengths for each of the supporting devices used?

Answers based on type of supporting device:

<table>
<thead>
<tr>
<th>Consulting Firm</th>
<th>Conditions for Use</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cast or Welded Steel Rockers.</strong></td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>Used for maximum total movements of more than 3 in.</td>
</tr>
<tr>
<td>I.</td>
<td>If the amount of the reaction is such as to require a large plate that would not distribute the load well, a built-up rocker or shoe is desirable; also, when the amount of expansion is more than about ( \frac{1}{2} ) in., and where the abutment or pier is rigid, a rocker is generally used.</td>
</tr>
<tr>
<td>M</td>
<td>Used for both rolled beam and concrete girder bridges for all span lengths practical for these types of structures.</td>
</tr>
<tr>
<td>N</td>
<td>No span limitations (expanding length).</td>
</tr>
</tbody>
</table>

**Simple Steel Sole Plates.**

(Apparently not used by any of the respondents.)
Answers to Question 1, (Continued)

Consulting Firm Conditions for Use

<table>
<thead>
<tr>
<th>Curved Steel Plates.</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>Used for maximum total movement of 3 in.</td>
</tr>
<tr>
<td>L</td>
<td>Used for expansion of about ½ in. or less; also used for &quot;fixed&quot; bearings on short span bridges.</td>
</tr>
<tr>
<td>N</td>
<td>Used for 50 to 60 ft expanding length.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bronze Plates.</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>Have not used.</td>
</tr>
<tr>
<td>N</td>
<td>Used for 100 ft maximum expanding length.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Elastomeric Bearing Pads.</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>Have not used.</td>
</tr>
<tr>
<td>N</td>
<td>Neoprene used for 60 ft maximum expanding length; have used only in Florida on prestressed concrete beam spans.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Other.</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>Solid cylindrical steel rollers with top and bottom plates have been used for short span continuous concrete slab bridges. For fixed bearings usually use a bolster fabricated from WF section with top flange curved.</td>
</tr>
<tr>
<td>N</td>
<td>Rollers: no span limitations (expanding length).</td>
</tr>
</tbody>
</table>
Question 2.

(a) What are the types of floor expansion devices currently used by your organization; e.g., finger joint, joint sealer only, sliding plates, armored joints, elastomeric tubes, sponge fillers, or other?

(b) What are the limitations such as span lengths for each of these expansion systems?

Answers based on type of expansion device:

<table>
<thead>
<tr>
<th>Consulting Firm</th>
<th>Conditions for Use</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Finger Joint.</strong></td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>Used for movement in excess of 7 in.</td>
</tr>
<tr>
<td>M</td>
<td>Used for expansion lengths in excess of 250 ft for straight bridges.</td>
</tr>
<tr>
<td>N</td>
<td>Used when expanding length is over 350 ft.</td>
</tr>
<tr>
<td><strong>Joint Sealer.</strong></td>
<td></td>
</tr>
<tr>
<td>(Apparently not used by any of the respondents.)</td>
<td></td>
</tr>
<tr>
<td><strong>Sliding Plates.</strong></td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>Used for 7 in. total movement.</td>
</tr>
<tr>
<td>L</td>
<td>Currently in use.</td>
</tr>
<tr>
<td>M</td>
<td>Used for expansion lengths not exceeding 250 ft for straight bridges; up to about 350 ft for skewed bridges.</td>
</tr>
<tr>
<td>N</td>
<td>Used for expanding length of 50 ft to 350 ft.</td>
</tr>
<tr>
<td><strong>Armored Joints--Open.</strong></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>On shorter span bridges with paved roadway, usually</td>
</tr>
</tbody>
</table>
Answers to Question 2, (Continued)

Consulting Firm

<table>
<thead>
<tr>
<th>Conditions for Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>specify a 1½-in. open joint (50⁰ F) at ends of the super-structure slab; used for length from fixed support not greater than 200 ft on straight bridges or up to 280 ft for skewed bridges.</td>
</tr>
<tr>
<td>N Used for expanding length up to 100 ft.</td>
</tr>
</tbody>
</table>

Elastomeric Tubes.

(Apparently not used by any of the respondents.)

Sponge Fillers.

(Apparently not used by any of the respondents.)

Other.

K Bituminous expansion joint fillers used for 1 in. total movement.

L A 1½-in. non-extruding joint sealer above 1½ in. preformed expansion joint is used between bridge and approach slab of steel bridges with girders fixed at both abutments.

M If approach slabs are not paved, sometimes cantilever the end of bridge slab over part of the abutment back-wall with a ⅝-in. premolded joint between bottom of slab and top of abutment wall. On projects for State Highway Commissions their standard practices and details are followed.
Answers to Question 2, (Continued)

<table>
<thead>
<tr>
<th>Consulting Firm</th>
<th>Conditions for Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>Closed joints with premolded joint filler used for expanding length of 50 ft or less.</td>
</tr>
</tbody>
</table>
Question 3.

(a) Has your organization utilized, or considered the use of, flexible stub abutments without expansion or rocker type supporting devices; tying the girder directly to the abutment?

(b) What are the limitations such as span length for type of design described above?

Answers:

<table>
<thead>
<tr>
<th>Consulting Firm</th>
<th>Conditions for Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>Yes. Total length of 180 ft.</td>
</tr>
<tr>
<td>L</td>
<td>Yes. Have recently designed several steel bridges with total lengths up to 300 ft between abutments and fixed the girders at both abutments; abutment piles were driven vertical.</td>
</tr>
<tr>
<td>M</td>
<td>No.</td>
</tr>
<tr>
<td>N</td>
<td>Yes. Have used monolithic abutments for continuous concrete slab bridges and continuous box girder bridges up to 350 ft of structure length. Have not used for steel structures.</td>
</tr>
</tbody>
</table>
Question 4.

(a) Has your organization experienced cases of movement of the abutments inward toward the bridge such that the abutments lean against the ends of the girders or end diaphragms and movement of the supporting devices has thus become "frozen"?

(b) If so, could you please advise the locations of the bridges involved?

Answers:

<table>
<thead>
<tr>
<th>Consulting Firm</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>No. Have noticed rotational movement of pile bent type abutments; do not know of any structures designed by this firm where the rotation has been of such magnitude that expansion of the structure has been restricted.</td>
</tr>
<tr>
<td>L</td>
<td>Yes. (Gave location of several bridges, but they were not designed by this firm.)</td>
</tr>
<tr>
<td>M</td>
<td>No. Not experienced to the best of their knowledge.</td>
</tr>
<tr>
<td>N</td>
<td>Yes. Stub abutments, on . . . Turnpike and . . . Turnpike. Stub abutments and full-depth abutments, on . . . Expressway.</td>
</tr>
</tbody>
</table>
Question 5.

(a) Has your organization experienced cases of "freezing" due to rust or corrosion of the devices currently used?

(b) If so, could you please advise the approximate location of one or two of the bridges involved?

Answers:

<table>
<thead>
<tr>
<th>Consulting Firm</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>No. Corrosion resistant materials used where rust and corrosion might be excessive under normal maintenance.</td>
</tr>
<tr>
<td>L</td>
<td>Yes. Almost any older truss span will show this condition, particularly the simple span trusses less than about 300 ft.</td>
</tr>
<tr>
<td>M</td>
<td>No. Not experienced to the best of their knowledge.</td>
</tr>
<tr>
<td>N</td>
<td>No. No such cases known for devices currently being used.</td>
</tr>
</tbody>
</table>
APPENDIX B:

TYPICAL SUPPORTING AND EXPANSION DEVICES

AND COMBINATIONS OF DEVICES
Note: Only basic components of devices are shown. Pintles, bolts, keepers, or slotted holes may be required for specific usage.

Fig. 17. Typical supporting and expansion devices
Fig. 18. Typical combinations of supporting and expansion devices
APPENDIX C:

FIELD OBSERVATIONS AND IRREGULARITIES OBSERVED
Table 11. Tabulation of bridges and items observed

<table>
<thead>
<tr>
<th>Group</th>
<th>Type</th>
<th>No. Spans</th>
<th>No. Bridges observed irregularity</th>
<th>Length (ft)</th>
<th>Abutments</th>
<th>Expansion Joint</th>
<th>Supporting Device</th>
<th>Fixed Abut.</th>
<th>Piers</th>
<th>Gap in Floor Expansion Device at Expansion Joint</th>
<th>Approach Slab Opening</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1-plate girder, 1-steel beam</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>60</td>
<td>1-med</td>
<td>curved plate (unknown)</td>
<td>curved plate flat plate</td>
<td>(no pier)</td>
<td>mastic</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>1-plate girder, 1-steel beam</td>
<td>2</td>
<td>2</td>
<td>64-90</td>
<td>1</td>
<td>1-high</td>
<td>curved plate flat plate</td>
<td>curved plate flat plate</td>
<td>(no pier)</td>
<td>mastic</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>Concrete girder</td>
<td>2</td>
<td>2</td>
<td>68-60</td>
<td>1</td>
<td>2-high</td>
<td>old portion, flat plates; new portion, fa-brasca</td>
<td>old portion, flat plates; new portion, fa-brasca</td>
<td>old portion, flat plates; new portion, fa-brasca</td>
<td>mastic or tar paper</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>Steel beam (simple span)</td>
<td>3</td>
<td>3</td>
<td>120-150</td>
<td>5</td>
<td>flat plate</td>
<td>flat plate</td>
<td>flat plates</td>
<td>mastic or tar paper</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Steel beam</td>
<td>3</td>
<td>3</td>
<td>120-150</td>
<td>5</td>
<td>2-curved plate</td>
<td>2-curved plate</td>
<td>2-curved plate</td>
<td>1-mastic</td>
<td>2-sliding plate</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>Steel beam</td>
<td>3</td>
<td>5</td>
<td>35-55</td>
<td>150-180</td>
<td>1</td>
<td>2-curved plate</td>
<td>2-curved plate</td>
<td>2-curved plate</td>
<td>1-mastic</td>
<td>2-sliding plate</td>
</tr>
<tr>
<td>7</td>
<td>Steel beam</td>
<td>3</td>
<td>1</td>
<td>'34</td>
<td>190</td>
<td>flex pier</td>
<td>flat plate</td>
<td>flat plate</td>
<td>flex plate</td>
<td>sliding plate</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>3-plate girder, 6-steel beam</td>
<td>3</td>
<td>5</td>
<td>33-89</td>
<td>200-300</td>
<td>9</td>
<td>8-curved plate</td>
<td>8-curved plate</td>
<td>8-curved plate</td>
<td>sliding plate</td>
<td>5</td>
</tr>
<tr>
<td>9</td>
<td>Steel beam</td>
<td>3</td>
<td>1</td>
<td>38</td>
<td>95</td>
<td>high</td>
<td>curved plate</td>
<td>curved plate</td>
<td>rocker</td>
<td>sliding plate</td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td>Steel cantilever (over R.R.I)</td>
<td>3</td>
<td>4</td>
<td>34-36</td>
<td>130-140</td>
<td>4</td>
<td>2-curved plate</td>
<td>2-curved plate</td>
<td>2-curved plate</td>
<td>sliding plate</td>
<td>1</td>
</tr>
<tr>
<td>11</td>
<td>Pre-stressed concrete (simple span)</td>
<td>3</td>
<td>3</td>
<td>50-99</td>
<td>120-210</td>
<td>2</td>
<td>1-med</td>
<td>curved plates</td>
<td>curved plates</td>
<td>mastic</td>
<td>3</td>
</tr>
<tr>
<td>12</td>
<td>Concrete continuous girders</td>
<td>3</td>
<td>4</td>
<td>93-59</td>
<td>155-105</td>
<td>4</td>
<td>curved plates</td>
<td>curved plates</td>
<td>rockers</td>
<td>sliding plate</td>
<td>2</td>
</tr>
<tr>
<td>13</td>
<td>Concrete cantilever</td>
<td>3</td>
<td>1</td>
<td>17 &amp; 35</td>
<td>154</td>
<td>high</td>
<td>Pre-formed expansion joint (flexible)</td>
<td>asphaltic &quot;fibro-board&quot;</td>
<td>asphaltic &quot;fibro-board&quot;</td>
<td>none</td>
<td>3</td>
</tr>
<tr>
<td>14</td>
<td>Concrete box girder with one column (span)</td>
<td>3</td>
<td>1</td>
<td>50</td>
<td>450</td>
<td>built integral</td>
<td>built integral</td>
<td>built integral</td>
<td>none</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Steel beam or plate girder</td>
<td>3</td>
<td>8</td>
<td>55-56</td>
<td>120-350</td>
<td>8</td>
<td>rocker</td>
<td>rocker</td>
<td>1-pier rocker, 1-pier fixed</td>
<td>1-sliding plate both ends, 6-sliding plate one end</td>
<td>3</td>
</tr>
<tr>
<td>16</td>
<td>Steel beam (single span)</td>
<td>3</td>
<td>5</td>
<td>150</td>
<td>approx. 150</td>
<td>1</td>
<td>curved plate</td>
<td>curved plate</td>
<td>curved plate</td>
<td>asphalt filled finger plate over pier; filled asphalt filled mastic</td>
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*One bridge expands both ends.

1Due only to spalling of abutment backwall.
Table II (Continued).

<table>
<thead>
<tr>
<th>Group</th>
<th>Type</th>
<th>Span(s)</th>
<th>Length ft</th>
<th>Abutments</th>
<th>Supporting Device</th>
<th>Gap at Expansion Ends of Slab Opening</th>
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<tbody>
<tr>
<td>17</td>
<td>1-steel beam</td>
<td>4-2</td>
<td>358</td>
<td>158</td>
<td>2</td>
<td>curved plate rocker</td>
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<tr>
<td></td>
<td>(1-plate)</td>
<td></td>
<td>157</td>
<td>570</td>
<td>2</td>
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<tr>
<td>18</td>
<td>pre-stressed conc. beam</td>
<td>4-2</td>
<td>358</td>
<td>220</td>
<td>2</td>
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<td></td>
<td>(simple span)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>curved plate</td>
</tr>
<tr>
<td>19</td>
<td>combination steel beam &amp; plate girder</td>
<td>4-2</td>
<td>both '55 &amp; '57</td>
<td>316</td>
<td>2</td>
<td>old portion, flat plates; new portion, curved plate</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>old portion, flat plates; new portion, curved plate</td>
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<tr>
<td>20</td>
<td>steel beam</td>
<td>4-1</td>
<td>'55-'56</td>
<td>approx. 220</td>
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<td>curved bar</td>
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<td>steel beam (with Gus. conc. const.)</td>
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<td>'55-'56</td>
<td>approx. 220</td>
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<td>rocker</td>
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<td></td>
<td></td>
<td>rocker</td>
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<td>'55-'56</td>
<td>approx. 220</td>
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<td>rocker</td>
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<tr>
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<td>1-plate girder</td>
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<td>rocker</td>
</tr>
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<td>'55-'56</td>
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<td>rocker</td>
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<td>rocker</td>
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<tr>
<td>24</td>
<td>plate girder</td>
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<td>'55-'56</td>
<td>approx. 240</td>
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<td>fixed</td>
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<td>'56-'57</td>
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<td>(simple span)</td>
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<td>flat plates</td>
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<td>steel beam</td>
<td>5-3</td>
<td>'56-'57</td>
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<td>2-curved plate 1-bronze plate</td>
</tr>
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<td>2-curved plate 1-bronze plate</td>
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<td>plate girder</td>
<td>5-2</td>
<td>'55-'56 in 575</td>
<td>2</td>
<td>rocker</td>
<td>1-steel beam (simple span)</td>
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<td></td>
<td></td>
<td></td>
<td>rocker</td>
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<tr>
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<tr>
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<td>curved plates</td>
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<tr>
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<td>steel beam (1-plate)</td>
<td>6-1</td>
<td>'57</td>
<td>454</td>
<td>1</td>
<td>curved plates on simple span; rockers on 5 span continu.</td>
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<tr>
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<td>curved plates on simple span; rockers on 5 span continu.</td>
</tr>
<tr>
<td>No. bridge observed</td>
<td>No. bridges showing irregularity</td>
<td>Approximate year built (if known)</td>
<td>Floor expansion device at expansion joint (if present)</td>
<td>Group Type A, B, or C, according to AASHTO Bridge Design Specifications</td>
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<td></td>
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</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1959</td>
<td></td>
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<tr>
<td>2</td>
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<td></td>
<td></td>
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<tr>
<td>3</td>
<td>3</td>
<td>1961</td>
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</table>

TABLE II (continued)
Irregularities Observed

Group 1*--One Span Steel Bridges, 60 ft long.
Two observed, one irregularity: Severe "squeezing" and extrusion of mastic expansion joint at 90° F temperature; (high abutments).

Group 3--Two Span Concrete Deck Girder Bridges, 68 to 80 ft long.
Two observed, one irregularity: One abutment settled resulting in cracking and severe spalling of pier under flat bearing plate. Approximately 40 years old; high abutments.

Group 4--Three Span Steel Beam Bridges, simple spans, 120 to 130 ft long.
Three observed, one irregularity: One pier cracked near end; one abutment spalled under one beam.

Group 5--Three Span Steel Bridges, 120 to 130 ft long.
Three observed, two irregularities:

a. Rockers on both piers cocked opposite to direction of thermal movement.

b. Rockers on both piers not in agreement; on north side the rockers are cocked in direction of thermal movement and on south side the rockers are cocked opposite to direction of thermal movement; inner rockers show various stages of disagreement with less magnitude. Expansion abutment has moved inward (toward center of bridge); sliding plate expansion joint is tight.

* Group numbers refer to Table 11.
Group 6--Three Span Steel Beam Bridges, 150 to 180 ft long.

Seven observed, four irregularities:

a. Both abutments moved inward (toward center of bridge); top of backwall of expansion abutment spalled and cracked at approach slab near curb.

b. Expansion abutment moved inward and backwall spalled; sliding plate tight; (high fill approaches).

c. Abutments have moved inward; sliding plate tight; (on gravel road).

d. Backwall of abutment cracked inward; no visible movement of expansion end for some time.

Group 8--Three Span Plate Girder and Steel Beam Bridges, 200 to 300 ft long.

Nine observed, five irregularities:

a. Very severe spalling of backwall of expansion abutment and reinforcing steel exposed; sliding plate tight (Fig. 19-a).

b. Both abutments moved inward; fixed abutment undercut at one end, piling exposed and abutment cracked at two places; sliding plate tight.

c. Sliding plate appears to have "frozen" and pulled backwall of abutment inward; sliding plate tight and abutment backwall tight against diaphragm.

d. Fixed abutment cracked and spalled at three of the four masonry bearing plates (1-in. square bars under bearing plates); no apparent movement of expansion support for some time; sliding plate tight (Fig. 19-b).
e. Both abutments moved inward; expansion slot in curved plate tight against bolts at each abutment; sliding plate tight and has broken loose from approach slab at one point.

Group 9—Three Span Steel Beam Bridge, 95 ft long.
One observed, one irregularity: Both high abutments cracked diagonally outward (away from center of bridge) at junction of backwall and bearing seat; stalactites up to 1-3/4 in. long at fixed abutment and other intermittent points along the north side (Fig. 20-a).

Group 10—Three Span Cantilever Type Steel Bridges, 110 to 140 ft long.
Four observed, four irregularities:
   a. Backwall of expansion abutment moved inward and spalled; fixed abutment spalled; very little, if any, movement of rockers; (bridge and road blacktop over concrete).
   b. Backwall of abutment badly spalled; sliding plate would be tight except for spalling of backwall and fact that approximately 1\(\frac{1}{2}\) in. of sliding plate on one side of bridge has been cut away; (high fill approaches).
   c. Expansion abutment moved inward and is badly spalled; however, there is over 1-3/4 in. between backwall and approach slab; sliding plate tight; fixed abutment appears to have moved (Fig. 20-b).
   d. Spalling at one end of expansion abutment; sliding plate tight.

Group 12—Three Span Concrete Continuous Girder Bridges, 155 and 185 ft long.
Four observed, four irregularities:
Note: These are two sets of twin bridges. The old bridges were built in 1953 and the new twin bridges were built in 1959. The grading is completed for the new twin road, but paving has not been placed.

a. Expansion abutment moved inward; sliding plate tight; (old twin).

b. Expansion abutment moved outward; 1 1/2 in. of shop paint on sliding plate exposed; (new twin).

c. Both abutments moved inward; sliding plate tight; rockers on pier next to fixed end cocked approximately three times as much as other pier; (old twin).

d. Expansion abutment moved outward; 3/4 in. of shop paint exposed on sliding plate; (new twin).

Group 15--Three Span Steel Beam or Plate Girder Bridges, Approximately 120 to 350 ft long.

Eight observed, six irregularities:

a. Some evidence of abutment shifting.

b. Rockers practically vertical.

c. Rocker on pier adjacent to fixed pier cocked opposite to direction of thermal movement.

d. One abutment tilted inward (this bridge observed by a bridge design engineer to have no apparent movement winter or summer).

e. Rockers at abutment adjacent to fixed pier practically vertical (expansion strip approximately 1/2 in. wide; crack in asphalt approach).
f. Rockers at abutment cocked opposite to direction of thermal movement. Some opening of expansion strip to a total of approximately 3/4 in.

Group 19--Four Span Combination Steel Beam and Plate Girder Bridges, 176 ft long.
Two observed, 1 irregularity: Concrete diaphragms tight against abutment backwall; at one abutment this appears to be due to approach slab; (bridge originally built in 1930 and widened in 1956 when concrete highway was widened; now covered with asphalt).

Group 23--Four Span Steel Beam Bridge, approximately 240 ft long.
One observed, one irregularity: Expansion device for longer length of expansion closed and abutment cracked. Rocker at other abutment cocked opposite to direction of thermal movement. Entire bridge has apparently shifted.

Group 25--Five Span Steel Beam Bridge, simple spans, 132 ft long.
One observed, one irregularity: Number four pier moved inward; crack at top of number 3 pier; (high approach fill).

Group 27--Five Span Plate Girder Bridges, less than 575 ft long.
Two observed, one irregularity: Abutment rockers cocked excess amount in proportion to intermediate rockers (apparently abutments have moved).

Group 28--Five Span Combination Concrete Box Girder and Concrete Girder Bridge, 136 ft long.
One observed, one irregularity: Expansion strip severely extruded from
sides at junctions of girder and box girder; cracking of girders over piers; (girders are recessed to receive box girder) (Fig. 21-a).

Group 29--Five Span Prestressed Concrete Bridges, simple spans, 337 ft long. Two observed, one irregularity: One abutment rotated outward and moved inward.

Group 32--Six Span Steel Beam Bridges with Cantilevered End Spans, approximately 330 ft long. Two observed, one irregularity: These are twin bridges on an Interstate Highway. Rockers on one bridge at pier adjacent to abutment not in agreement; on north side the rocker is cocked opposite to direction of thermal movement and on south side the rocker is cocked in direction of thermal movement. Also, at the opposite end of the bridges the rockers at the piers adjacent to the abutment do not show the same relative movement of the two bridges.

Group 33--Six and Seven Span Combination Steel Beam and Plate Girder Bridges 275 and 332 ft long. Two observed, two irregularities:

a. Spalling of concrete at expansion end of plate girder. Rockers under plate girder not in agreement; rockers on south side cocked in direction of thermal expansion; rockers on north side cocked opposite to direction of thermal movement. Approach slabs "all cracked up".

b. Spalling of concrete at ends of plate girder; cracking of raised beam seat on piers at junction of beams and plate girders (Fig. 21-b).
Group 36—Steel Viaduct, 4559 ft long (maximum span 362 ft).

One observed, one irregularity: Several instances observed of rocker movement opposite to direction of thermal movement. In some instances this occurred over a range of 4 adjacent piers.
Fig. 19. Typical irregularities observed.
Fig. 20. Typical irregularities observed
Fig. 21. Typical irregularities observed
APPENDIX D:

TYPICAL GRAPHS OF REDUCED DATA FROM EXPERIMENTAL INVESTIGATION
Fig. 22. Typical static load test calibration curve. Relationship between unit strain in load cell and attenuator-lines, beam A.
Fig. 23. Computer plot of strain-frequency relationship for beam A, curved steel sole plates, oscillator only, \( W = 3.48 \text{ lb}, e = 4.51 \text{ in.} \)
Fig. 24. Computer plot of strain-frequency relationship for beam C, 64 durometer neoprene pads, oscillator with concrete blocks, $W = 3.48$ lb, $e = 4.51$ in.
Fig. 25. Computer plot of strain amplification factor-frequency relationship for beam A, curved steel sole plates, oscillator only, $W = 3.48$ lb, $e = 4.51$ in.
Fig. 26. Computer plot of deflection-frequency relationship for beam 3-1, 64 durometer neoprene pads, oscillator with concrete blocks, 
W = 3.48 lb, e = 4.51 in.
Fig. 27. Computer plot of deflection amplification factor-frequency relationship for beam 3-1, 64 durometer neoprene pads, oscillator with concrete blocks, $W = 3.48$ lb, $e = 4.51$ in.
Fig. 28. Computer plot of deflection amplification factor-frequency relationship for north end of beam 3-1, 64 durometer neoprene pads, oscillator with concrete blocks, W = 3.48 lb, e = 4.51 in.
Fig. 29. Strain-frequency curves for beam A, oscillator only, $W = 3.48$ lb, $e = 4.51$ in.
Fig. 30. Strain-frequency curves for beam C, oscillator only, W = 3.48 lb, e = 4.51 in.
Fig. 31. Partial strain-frequency curves for beam A, oscillator only, \( W = 3.48 \text{ lb}, \ e = 4.51 \text{ in.} \)
Fig. 32. Partial strain-frequency curves for beam C, oscillator only, $W = 3.48$ lb, $e = 4.51$ in.
Fig. 33. Strain amplification factor-frequency curves for beam A, oscillator only, W = 3.48 lb, e = 4.51 in.
Fig. 34. Strain amplification factor-frequency curves for beam C, oscillator only, \( W = 3.48 \) lb, \( e = 4.51 \) in.
Fig. 35. Partial strain amplification factor-frequency curves for beam A, oscillator only, $W = 3.48$ lb, $e = 4.51$ in.
Fig. 36. Partial strain amplification factor-frequency curves for beam C, oscillator only, $W = 3.48$ lb, $e = 4.51$ in.
Fig. 37. Strain-frequency curves for beam A, oscillator with concrete blocks, $W = 0.82$ lb, $e = 7.01$ in.
Fig. 38. Strain-frequency curves for beam A, oscillator with concrete blocks, \( W = 3.48 \text{ lb}, e = 3.26 \text{ in.} \)
Fig. 39. Strain-frequency curves for beam A, oscillator with concrete blocks, \( W = 3.48 \) lb, \( e = 4.51 \) in.
Fig. 40. Strain-frequency curves for beam C, oscillator with concrete blocks, $W = 0.82$ lb, $e = 7.01$ in.
Fig. 41. Strain-frequency curves for beam C, oscillator with concrete blocks, \( W = 3.48 \) lb, \( e = 3.26 \) in.
Fig. 42. Strain-frequency curves for beam C, oscillator with concrete blocks, \( W = 3.48 \text{ lb} \), \( e = 4.51 \text{ in.} \).
Fig. 43. Strain amplification factor-frequency curves for beam A, oscillator with concrete blocks, \( W = 0.82 \text{ lb}, e = 7.01 \text{ in.} \)
Fig. 44. Strain amplification factor-frequency curves for beam A, oscillator with concrete blocks, $W = 3.48 \text{ lb}$, $e = 4.51 \text{ in.}$
Fig. 45. Strain amplification factor-frequency curves for beam C, oscillator with concrete blocks, $W = 0.82$ lb, $e = 7.01$ in.
Fig. 46. Strain amplification factor-frequency curves for beam C, oscillator with concrete blocks, $W = 3.48$ lb, $e = 4.51$ in.
Fig. 47. Deflection-frequency curves for beam 1-1, oscillator only, $W = 3.48$ lb, $e = 4.51$ in.
Fig. 48. Deflection-frequency curves for beam 3-1, oscillator only, \( W = 3.48 \) lb, \( e = 4.51 \) in.
Fig. 49. Deflection amplification factor-frequency curves for beam 1-1, oscillator only, 
$W = 3.48 \text{ lb}, e = 4.51 \text{ in.}$
Fig. 50. Deflection amplification factor-frequency curves for beam 3-1, oscillator only, \( W = 3.48 \text{ lb}, e = 4.51 \text{ in.} \)
Fig. 51. Deflection amplification factor-frequency curves for north end of beam 1-1, oscillator only, $W = 3.48$ lb, $e = 4.51$ in.
Fig. 52. Deflection amplification factor-frequency curves for north end of beam 3-1, oscillator only, $W = 3.48$ lb, $e = 4.51$ in.
Fig. 53. Deflection amplification factor-frequency curves for south end of beam 1-1, oscillator only, $W = 3.48$ lb, $e = 4.51$ in.
Fig. 54. Deflection amplification factor-frequency curves for south end of beam 3-1, oscillator only, \( W = 3.48 \text{ lb} \), \( e = 4.51 \text{ in.} \).
Fig. 55. Deflection-frequency curves for beam 1-1, oscillator with concrete blocks, $W = 0.82\ \text{lb}$, $e = 7.01\ \text{in.}$
Fig. 56. Deflection-frequency curves for beam 1-1, oscillator with concrete blocks, $W = 3.48$ lb, $e = 3.26$ in.
Fig. 57. Deflection-frequency curves for beam 1-1, oscillator with concrete blocks, \( W = 3.48 \text{ lb} \), \( e = 4.51 \text{ in.} \).
Fig. 58. Deflection-frequency curves for beam 3-1, oscillator with concrete blocks, $W = 0.82$ lb, $e = 7.01$ in.
Fig. 59. Deflection-frequency curves for beam 3-1, oscillator with concrete blocks, W = 3.48 lb, e = 3.26 in.
Fig. 60. Deflection-frequency curves for beam 3-1, oscillator with concrete blocks, $W = 3.48$ lb, $e = 4.51$ in.
Fig. 61. Deflection amplification factor-frequency curves for beam 1-1, oscillator with concrete blocks, $W = 0.82$ lb, $e = 7.01$ in.
Fig. 62. Deflection amplification factor-frequency curves for beam 1-1, oscillator with concrete blocks, \( W = 3.48 \text{ lb} \), \( e = 4.51 \text{ in.} \).
Fig. 63. Deflection amplification factor-frequency curves for beam 3-1, oscillator with concrete blocks, $W = 0.82 \text{ lb}, e = 7.01 \text{ in.}$
Fig. 64. Deflection amplification factor-frequency curves for beam 3-1, oscillator with concrete blocks, \( W = 3.48 \text{ lb}, e = 4.51 \text{ in.} \)
Fig. 65. Deflection amplification factor-frequency curves for north end of beam 1-1, oscillator with concrete blocks, $W = 0.92$ lb, $e = 7.01$ in.
Fig. 66. Deflection amplification factor-frequency curves for north end of beam 1-1, oscillator with concrete blocks, $W = 3.48$ lb, $e = 4.51$ in.
Fig. 67. Deflection amplification factor–frequency curves for north end of beam 3-1, oscillator with concrete blocks, $W = 0.82$ lb, $e = 7.01$ in.
Fig. 68. Deflection amplification factor-frequency curves for north end of beam 3-1, oscillator with concrete blocks, $W = 3.48$ lb, $e = 4.51$ in.
Fig. 69. Deflection amplification factor-frequency curves for south end of beam 1-1, oscillator with concrete blocks, $W = 3.48$ lb, $e = 4.51$ in.
Fig. 70. Deflection amplification factor-frequency curves for south end of beam 3-1, oscillator with concrete blocks, W = 3.48 lb, e = 4.51 in.