

RESIDUAL STRESS MEASUREMENTS FROM SURFACE WAVE VELOCITY DISPERSION

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I'm going to try to keep you awake and I hope to tie some of the subjects that were discussed earlier today in with this paper. You may be asking, "What do I mean by velocity dispersion?" and even more, "What do I mean by residual stress?"

You real world enthusiasts will appreciate the first figure (Fig. 1). These are plots of residual stress with depth in a metal sample of 4340 steel. These were destructive measurements made by Metcut Research Associates under a contract to the Air Force. Specifically, what I will be addressing is the kinds of residual stress gradients that are caused by different machine operations. As you can see, they vary a great deal depending on the kind of operation. The point is that we would like to be able to measure these nondestructively because we may not have complete confidence in the machinist. In particular, I'd like to point out that even though this abusively ground curve, which represents a rather large overall residual stress, seems to show very little stress at the surface, and most of the stress is hidden below the surface. You can see that typically the peak values can lie only a mil or two below the surface. So, this makes the measurement rather a challenge. Also, I would like to point out that for the gentle grind condition, the stress is compressive and it is very small. This is the kind of stress you would like to have, but it is important to know that any kind of machining operation does somehow alter the microstructure below the surface.

Figure 2 shows pictures made by Metcut of this material, and shows the microstructure below the surface for different milling conditions on flat and curved surfaces. We see that, when good machining practice is followed, there is a very thin layer on the top surface and that is barely detectable (Fig. 2a), but when a dull tool (Fig. 2c) is used, there is quite a large change in the microstructure below the surface. This is also the case using good machining practice on a curved surface (Fig. 2b) and bad machining practice (Fig. 2d). We think of these as trapped residual stresses below the surface. The problem is that, since these are such shallow gradients, it is very difficult using bulk waves to measure this sort of thing.

Figure 3 shows how we can approach this problem. We consider some sort of a microstructure change with depth in the material, and we call this $F(z)$, and we pass a surface acoustic wave along the top of the solid. The energy of the surface wave is roughly confined within a wavelength of the surface, so that by changing the frequency of the wave we can change its penetration depth and thus we will get an interaction between this energy and the change below the surface and velocity dispersion results. The problem is: what is the relationship between this gradient and the velocity dispersion?

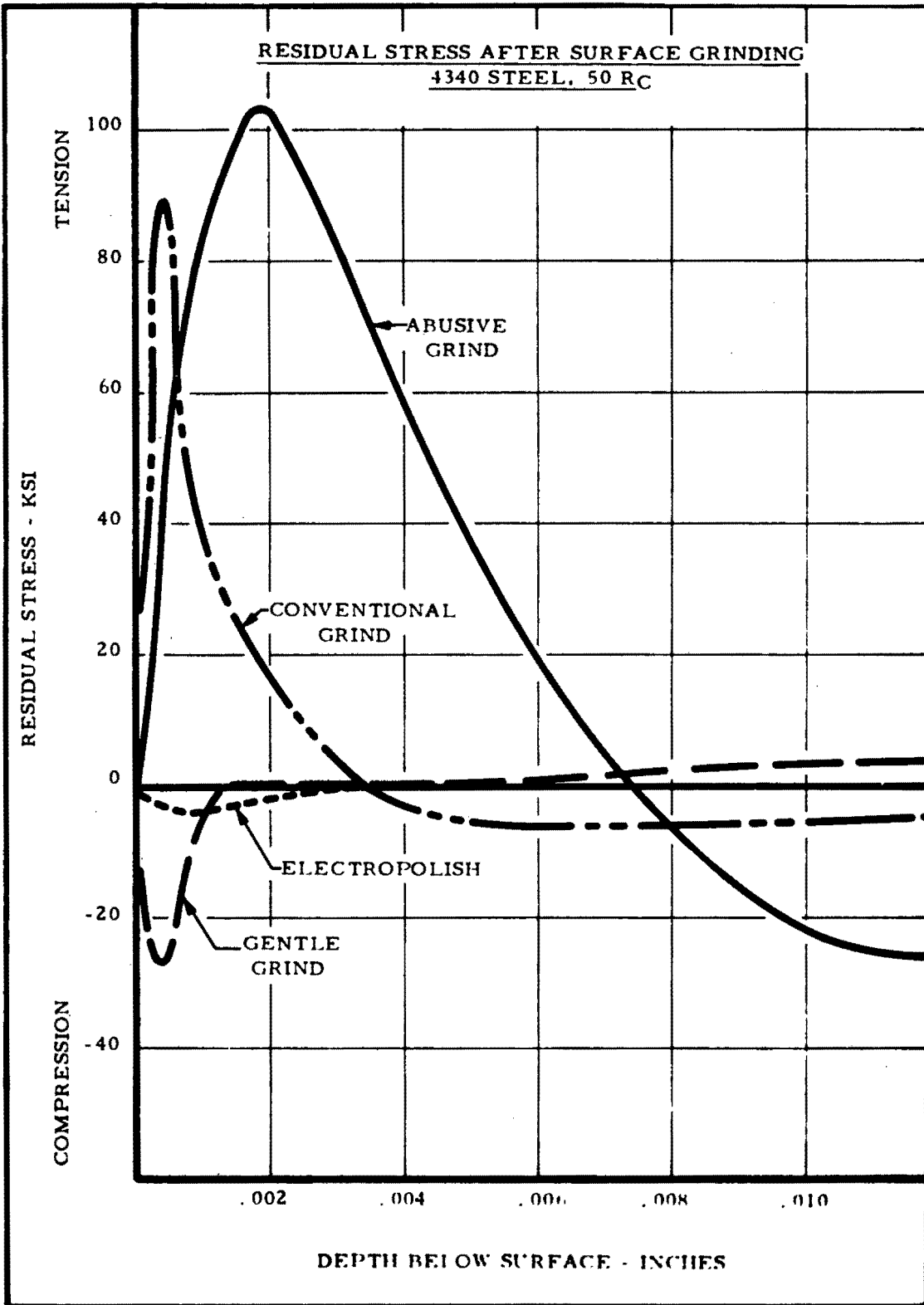
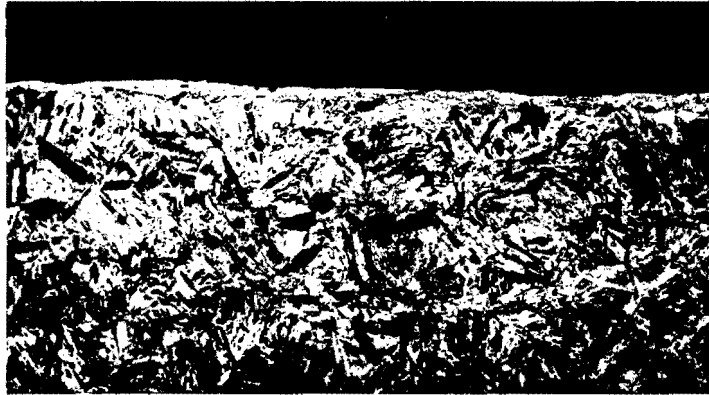


Fig. 1. Residual stress gradients for different grinding operations (from AFMDC 70-1).

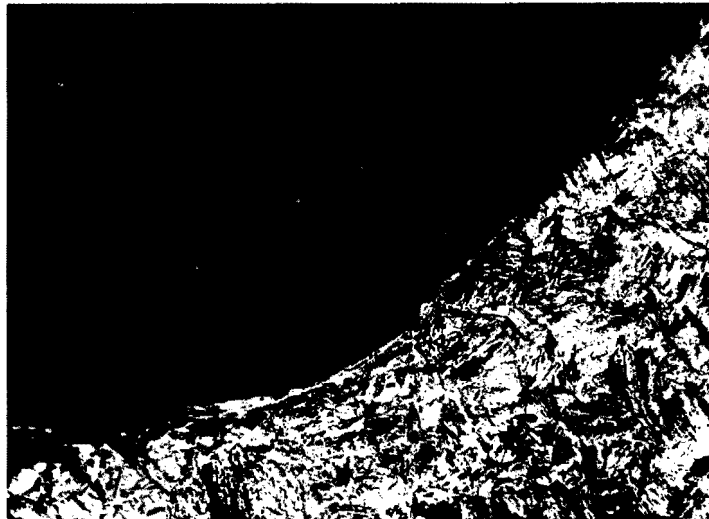


(A) FLAT SURFACE, SHARP TOOL 750X
 THIN LAYER OF PLASTICALLY DEFORMED
 METAL APPROXIMATELY .0002 IN. DEEP

.001 in.



(C) FLAT SURFACE, DULL TOOL 750X
 FAIRLY HEAVY BANDS OF UTM (65 R_c)
 AND DARK ETCHED OTM (52 R_c)



(B) TRANSIENT SURFACE, SHARP TOOL 750X
 SAME EFFECT AS FIG. A ABOVE



(D) TRANSIENT SURFACE, DULL TOOL 750X
 SAME EFFECT AS FIG. C ABOVE

Fig. 2. Microstructure of flat and curved milled surfaces (from AFMDC 70-1);
 and carbide face milling of 4340 steel quenched and tempered to 54 R_c.

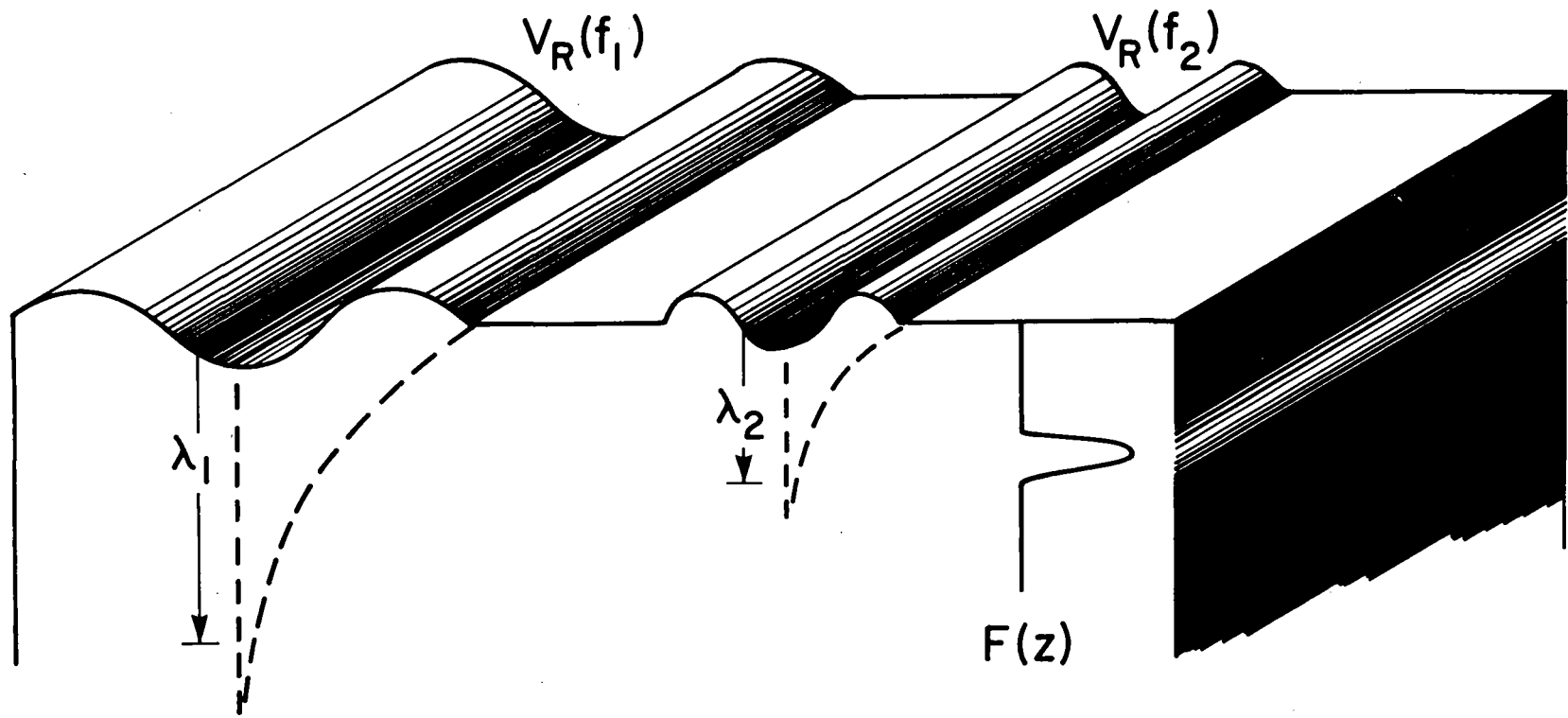


Fig. 3. Surface acoustic waves interacting with subsurface gradient.

Figure 4 shows a theoretical approach which first attracted my attention because it was done here at the Science Center, and it was used to measure the gradient on quench hardened steel. Bruce Thompson and Bernie Tittmann used a perturbation theory which was started by Auld and which relates changes in velocity to property gradient, as shown by this integral equation. Here it is assumed that the changes in density (ρ) and in elastic constants (C_{ij}) are small and vary with depth in the same functional form as shown at the top of the figure. The integral expression giving the velocity dispersion is shown below, where V_R is the surface wave velocity, U is the displacement field of the unperturbed surface wave solution, ΔC is the perturbed elastic stiffness tensor, $\Delta\rho$ is the perturbed density, and P_R is the power flow per unit width of the unperturbed solution. The integral equation can also be written in the form shown at the bottom of the figure. Here f is the frequency and M_j and q_j are constants determined by the mathematical operations implicit in the previous formulae and include both the detailed form of the surface wave solution and the changes in density and elastic constants. What I hope to show is the relationship between these two sets of expressions and also a way of getting an equation for $F(z)$ in terms of the velocity change.

As we examine Fig. 5, note the last equation of Fig. 4. It looked familiar, and, in fact, it was a scaled Laplace transform. We are familiar with this operation and also the mathematical operation of obtaining the inverse transform (top two equations). We have written an expression for the dispersion in terms of a sum of scaled Laplace transforms of the gradient function (third equation). By taking the inverse transform of this we can get the gradient (fourth equation). So, now we have two equations that in closed analytic form give us the relationship³ between the dispersion and gradient.

Here (Fig. 6) we see that there are different weighting functions (equal to the expression in parenthesis in the last equation of Fig. 4) that appear in the integral. They are all different depending on whether the density or one of the elastic constants is involved. This accounts for the difference in dispersion curves that you will see. In practice it is necessary to isolate which one of these variables is operating in the experiment. These are basically determined by external means other than the velocity dispersion measurements.

Now, I'd like to show you what some of the gradient and dispersion curves look like (Fig. 7). On the right are different kinds of property changes with depth. We have something like a thick layer or a very thin layer. These gradients include an impulse function, a Gaussian type of layer below the surface, and other kinds that vary smoothly from the surface, a complimentary error function, an exponential decay and a linear change. Now, the different dispersion curves are shown here on the left. They are similar. In each set of dispersion curves there is one for a change in the density or in the shear or longitudinal elastic constants. It turns out what is really important here is the area under the gradient curve; this can give us a great deal of information about the type of function. In fact, you can see that the impulse function which could be possibly used to measure a change caused by a thin bond layer actually gives a dispersion over a rather broad frequency range.

SMALL PERTURBATIONS

$$\rho(z) = \rho_0 + \Delta\rho F(z), \quad C_{ij}(z) = (C_{ij})_0 + \Delta C_{ij} F(z)$$

VELOCITY DISPERSION RELATION:

$$\frac{\Delta V_R}{V_R} = \frac{V_R}{4P_R} \int_0^{\infty} \left[-\Delta\rho\omega^2 \underline{U}^* \cdot \underline{U} + \nabla_S \underline{U}^* : \Delta \underline{C} : \nabla_S \underline{U} \right] F(z) dz$$

$$\frac{\Delta V_R(f)}{f} = \int_0^{\infty} \left(\sum_{i=1}^3 M_i e^{-q_i f z} \right) F(z) dz$$

Fig. 4. Perturbation theory equations for surface wave velocity dispersion.

INVERSE SOLUTION

$$\bar{F}(q_i f) = \int_0^{\infty} e^{-(q_i f)z} F(z) dz \quad \text{LAPLACE TRANSFORM}$$

$$F(z) = \mathcal{L}^{-1}[\bar{F}(s)] = \frac{1}{j2\pi} \int_{\sigma-j\infty}^{\sigma+j\infty} e^{sz} \bar{F}(s) ds \quad \text{INVERSE TRANSFORM}$$

$$\frac{\Delta V_R(f)}{f} = \sum_{i=1}^3 M_i \bar{F}(q_i f)$$

$$\mathcal{L}^{-1}\left[\frac{\Delta V_R(f)}{f}\right] = \sum_{i=1}^3 \frac{M_i}{q_i} F(z/q_i)$$

Fig. 5. Laplace transform relation between gradient and dispersion.

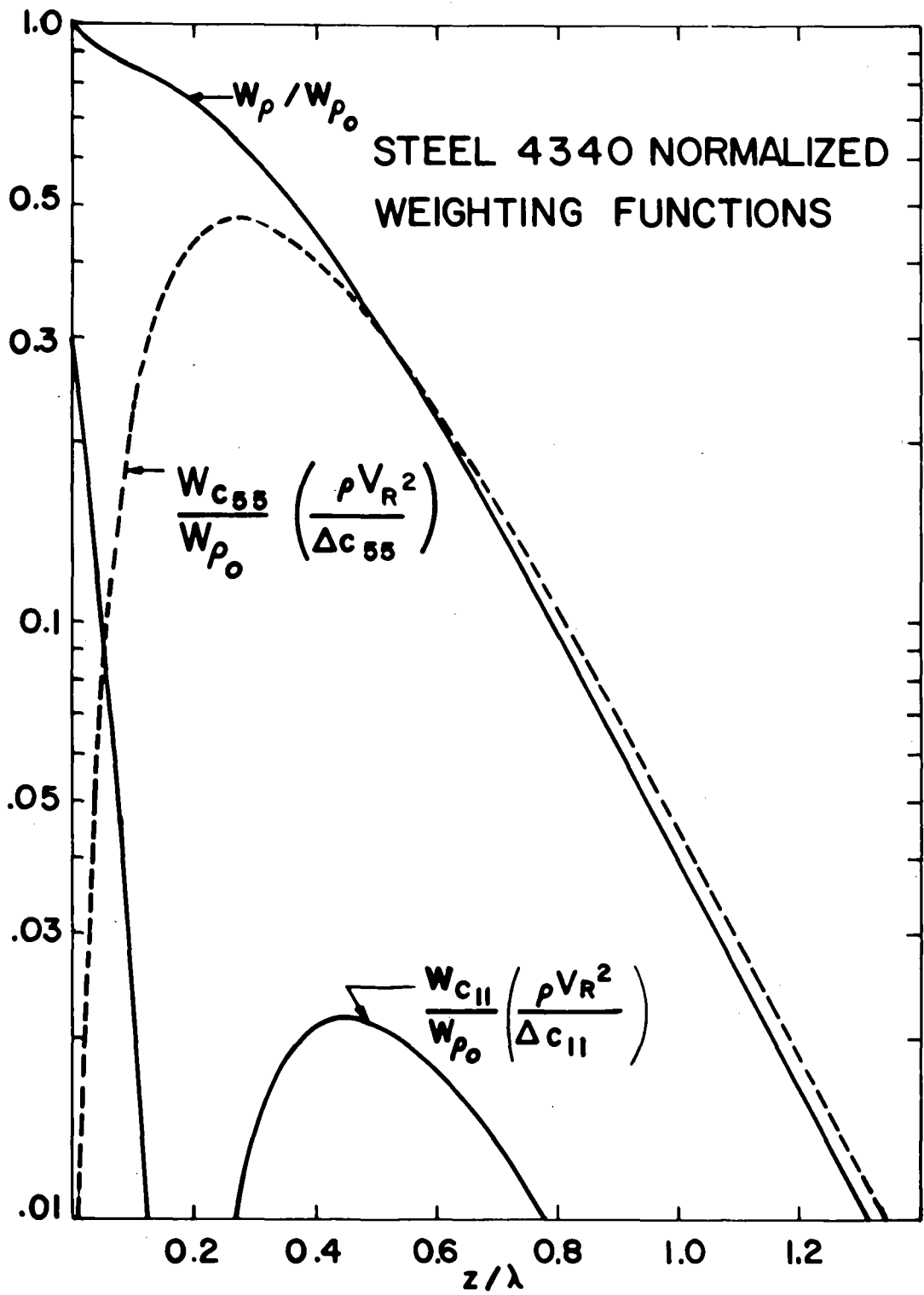


Fig. 6. Surface acoustic wave integral weighting functions.

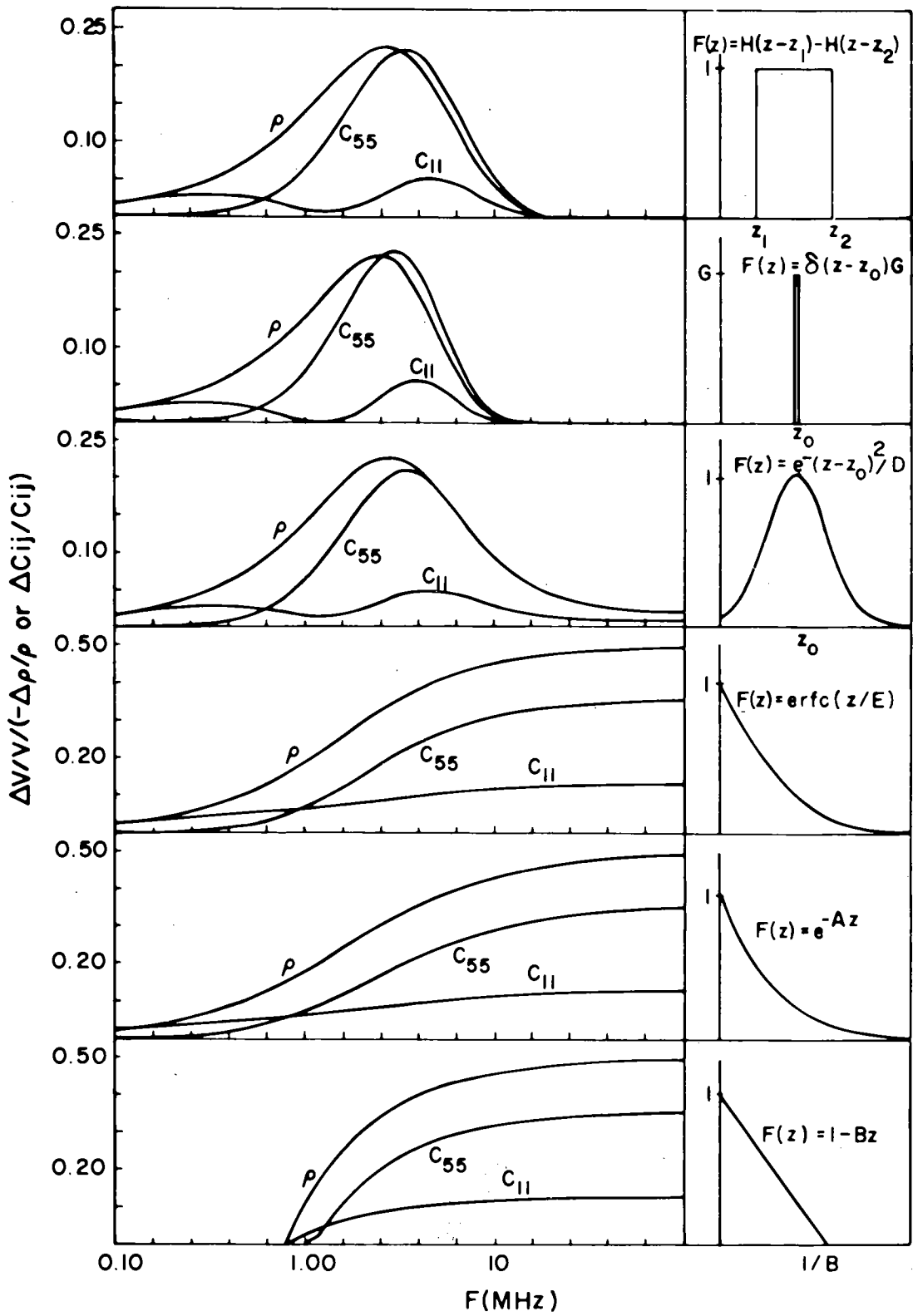


Fig. 7. Normalized velocity dispersion curves for six gradient functions.

Now, let us look at some of these curves in detail (Fig. 8). This is the kind of curve we would be most interested in for a residual stress gradient. We can compare all these functions directly by requiring $F(z)$ to have an equivalent area. These curves show that for low frequencies the surface wave is insensitive to the actual shape of the gradient. You can't distinguish between the different gradient functions. You need higher frequencies to resolve these different functions.

In the real world, unfortunately, you don't always have a nice analytic expression for the velocity dispersion. In this case you are usually stuck with some data points. Now, the problem is how do you get to the gradient from that kind of information. Well, this turns out to be a fallout of this Laplace transform approach. If you express the gradient as a power series of this form (Fig. 9, top equation) and if you make a least squares fit of your data points as a function of wavelength (second equation), then by going through these operations (third and fourth equations) you can come out with an expression which gives you the coefficient b_n directly in terms of the least squares fit coefficients a_n (final equation). All the other quantities are known constants. Now, usually to do this you do need several data points to work with, and the more the better. The wider the frequency range the better.

In the next paper Dr. Richardson will discuss a different approach to getting the gradient from data using estimation theory.

Figure 10 shows an experiment⁴ that was done measuring the velocity dispersion on a very finely polished piece of lithium niobate, polished to optical finish. In this case the grinding process introduced a very thin layer of changed material near the surface. This layer can be represented by an impulse function in space. In this case the theory predicts linear dispersion. In other words, even though the frequency of velocity measurement was in the giga-hertz region, the wavelength was much larger than the layer, and thus for this experiment, the layer appeared as an impulse function. I won't go into this, but this is actually an anisotropic material. There are ways of arranging the elastic constants to make equivalents to the isotropic situation and a more complicated anisotropic theory was used to check this result. It turns out this dispersion is equivalent to a negative change in density of about 20 percent, and the thickness of this layer is only about 40Å. This layer is similar to what Dr. Ushioda described previously on these kinds of effects. This result, we will see later, is important in the residual stress measurement that I will be describing.

Figure 11 shows a velocity change that is caused by the average stress that Otto Buck discussed previously. In this case⁵, an aluminum block of 2014 alloy was placed in a bending moment. This is a well-known classic case in which the stress varies linearly with depth, going to 0 in the middle of the material. Here the change is in the longitudinal elastic constant, and we can see that our theory predicts the experimental measurements of delay time well. Also shown as a dotted line is the theory that was used by Noronha, assuming that the surface wave samples an average of its wavelength below the surface.

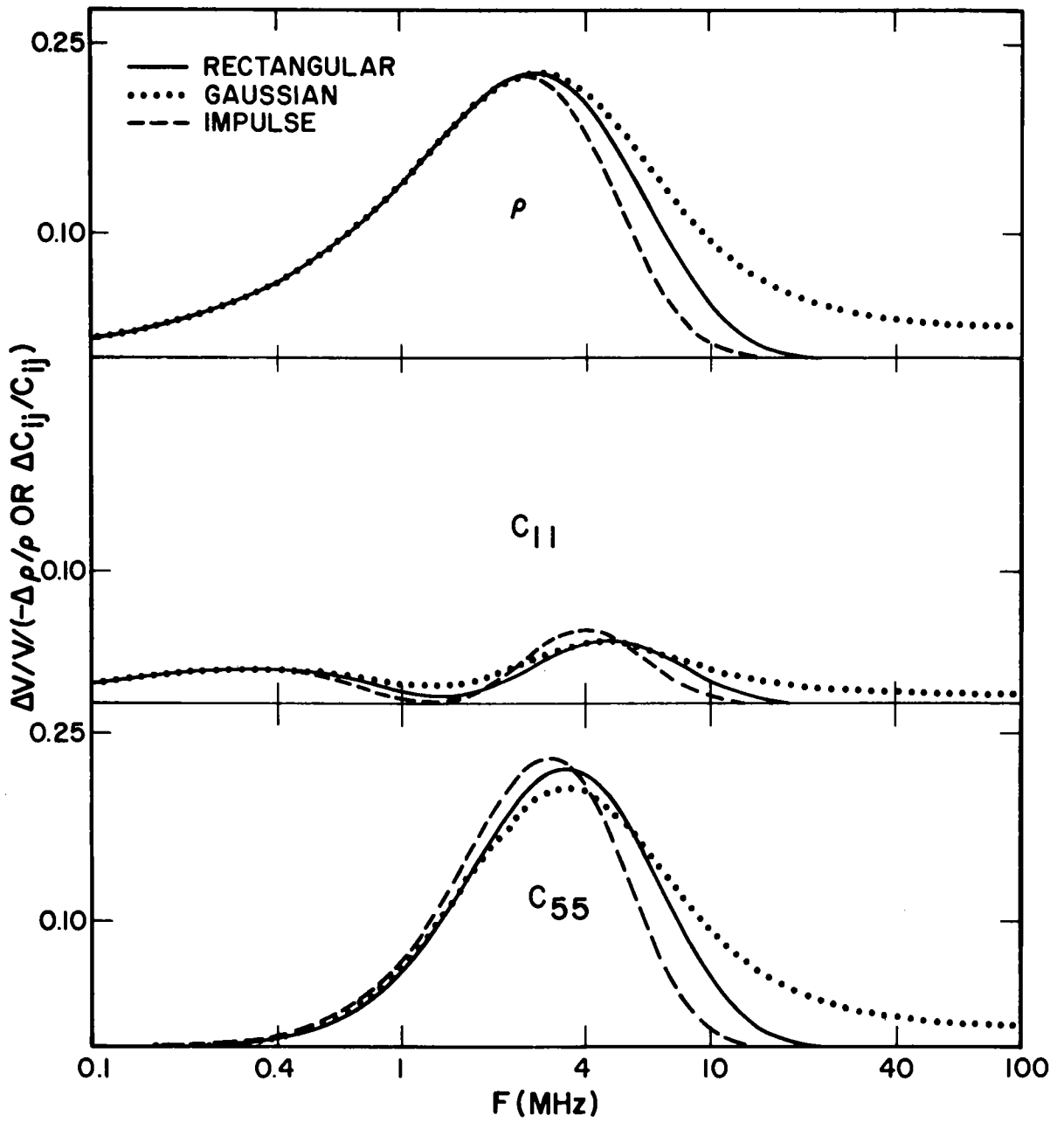


Fig. 8. Enlarged view of normalized dispersion curves for three gradients.

$$F(z) = \sum_{n=0}^N b_n z^n$$

$$\Delta V_R(\lambda) = \sum_{n=0}^N a_n \lambda^n$$

$$\mathcal{L}^{-1} \left[\frac{\Delta V_R(f)}{f} \right] = \sum_{n=0}^N \frac{a_n v_R^n z^n}{n!}$$

$$\mathcal{L}^{-1} \left[\frac{\Delta V_R(f)}{f} \right] = \sum_{i=1}^3 \frac{M_i}{q_i} F\left(\frac{z}{q_i}\right)$$

$$b_n = \frac{a_n v_R^n}{n!} \left(\frac{M_1}{q_1^{n+1}} + \frac{M_2}{q_2^{n+1}} + \frac{M_3}{q_3^{n+1}} \right)$$

Fig. 9. Obtaining the gradient from raw dispersion data.

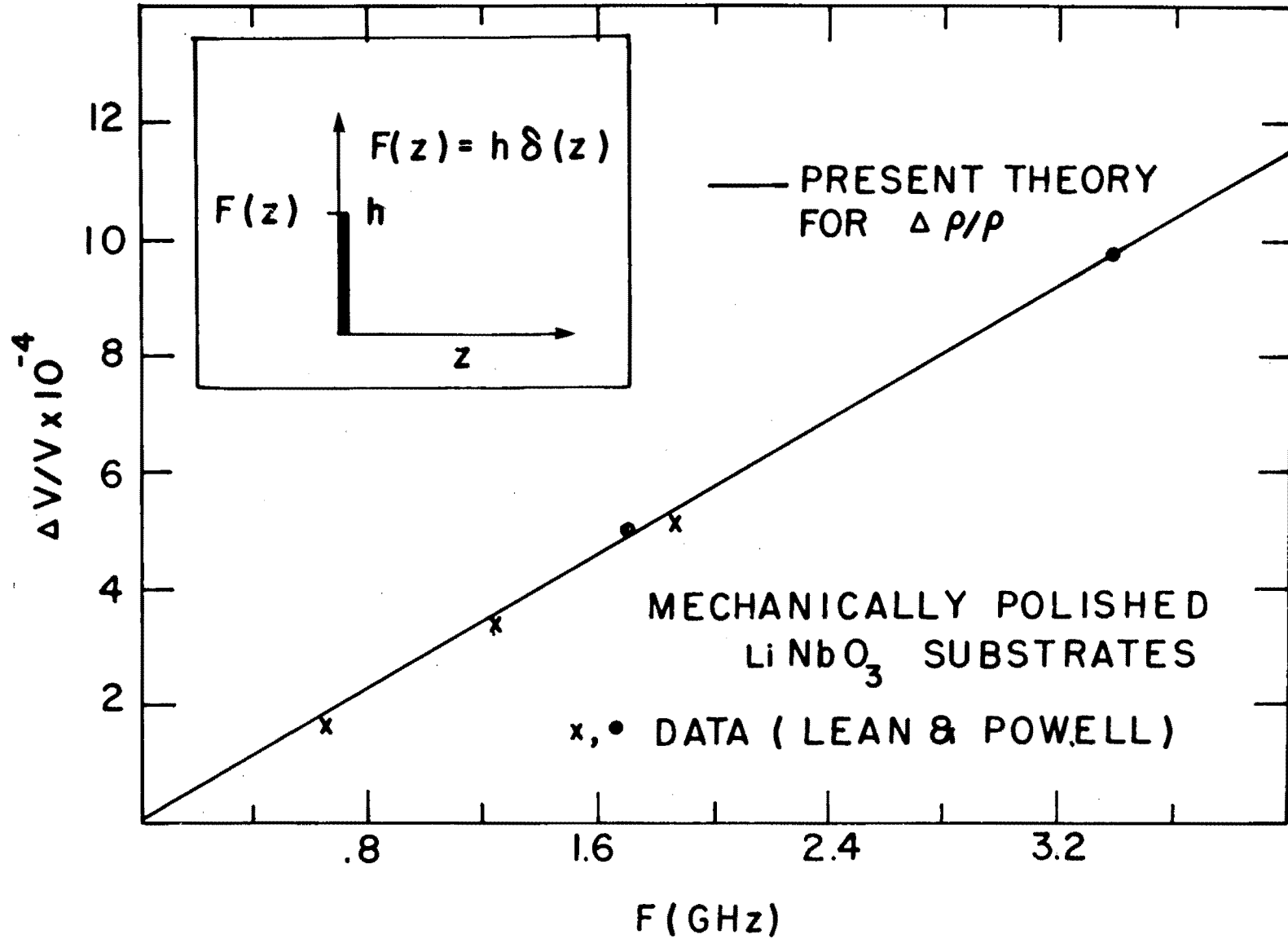


Fig. 10. Dispersion caused by mechanical polishing.

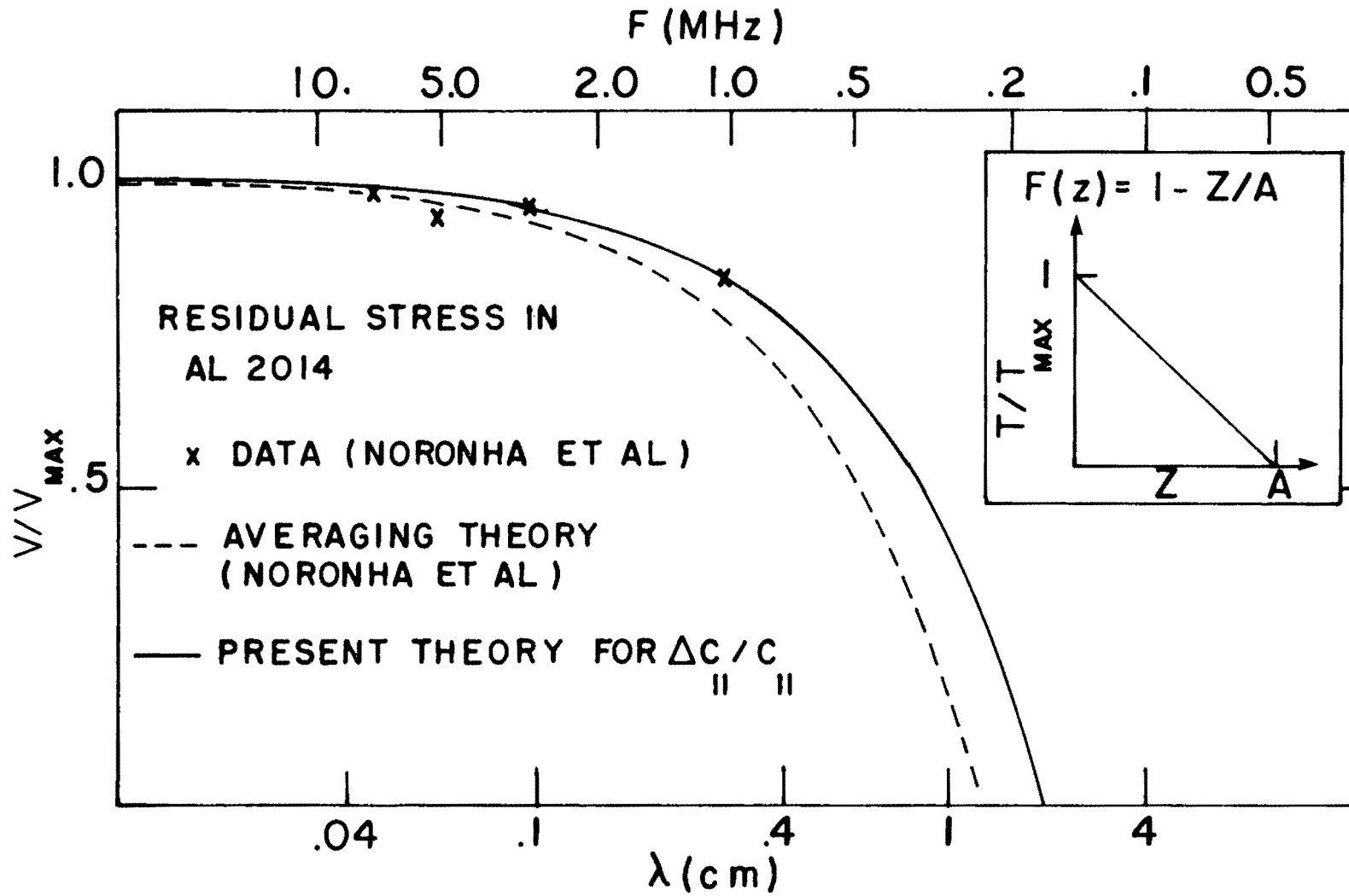


Fig. 11. Velocity change for a bent 2014 Al bar; corresponding stress profile is indicated in the inset.

In Fig. 12 there are two residual stress gradients very much like those in Fig. 1. The lower curve is for 4340 steel again, and the upper curve is for an abusively ground sample. It has a peak residual measured stress of 300 ksi. The bottom curve is for a gently ground specimen with a peak at 75 ksi. The abscissa is depth into the sample from the surface. Now, besides those analytic functions that I presented before, you can synthesize other kinds of functions by piecewise continuous functions. In the abusively ground case you can represent this curve very well by a linear section on the left, a flat section in the middle, and an exponential section on the right. Similarly, you can get a result for gently ground case. Once you have analytic expressions for the gradients, you can find out what the expected velocity dispersions would be for these two conditions.

Figure 13 shows the dispersion curves for those gradients. The abusively ground residual stress gradient is much broader, and it is farther down in the bulk of the material, so you would expect that it would require a lower frequency to measure the dispersion, and that's what we see here. The greatly ground case is much closer to the surface, and it is shifted up in frequency in this manner. All these curves are normalized, and it is kind of misleading, because the peak dispersion at around 4 MHz would be expected to be on the order of 1 percent. The C_{55} dispersion would be somewhere around six times 10^{-3} , and the C_{11} dispersion would be only about three or four parts in 10^{-4} .

So, the question is now: How can you distinguish between these different components of the dispersion in the experiment? This is a large problem, because if it is the C_{11} curve and the change is very small, it would be very difficult to measure experimentally. However, fortunately, I have made some preliminary measurements of the dispersion at two frequencies.

Now the phase this work is going into is an attempt to measure this velocity dispersion, and to get some correlation with the gradients measured by destructive means. The velocity changes that I have measured for the abusively ground sample at 9 MHz and 15 MHz have been between 0.5 and 0.7 percent. So, this does look like the main effect is caused by the density change. That is very encouraging. Also, I made measurements on the gently ground sample. This maximum is expected to be something like seven or eight parts in 10^{-4} for the density curve. This is lower than the value for abusive grinding because what is important in determining the magnitude of the velocity dispersion and its distribution is the area under the gradient curve. In this case I measured a change of four parts in 10^{-4} for the same frequencies. It looks like this case would be more difficult to characterize experimentally, but hopefully with proper experimental techniques, residual stress gradients can be measured in this way.

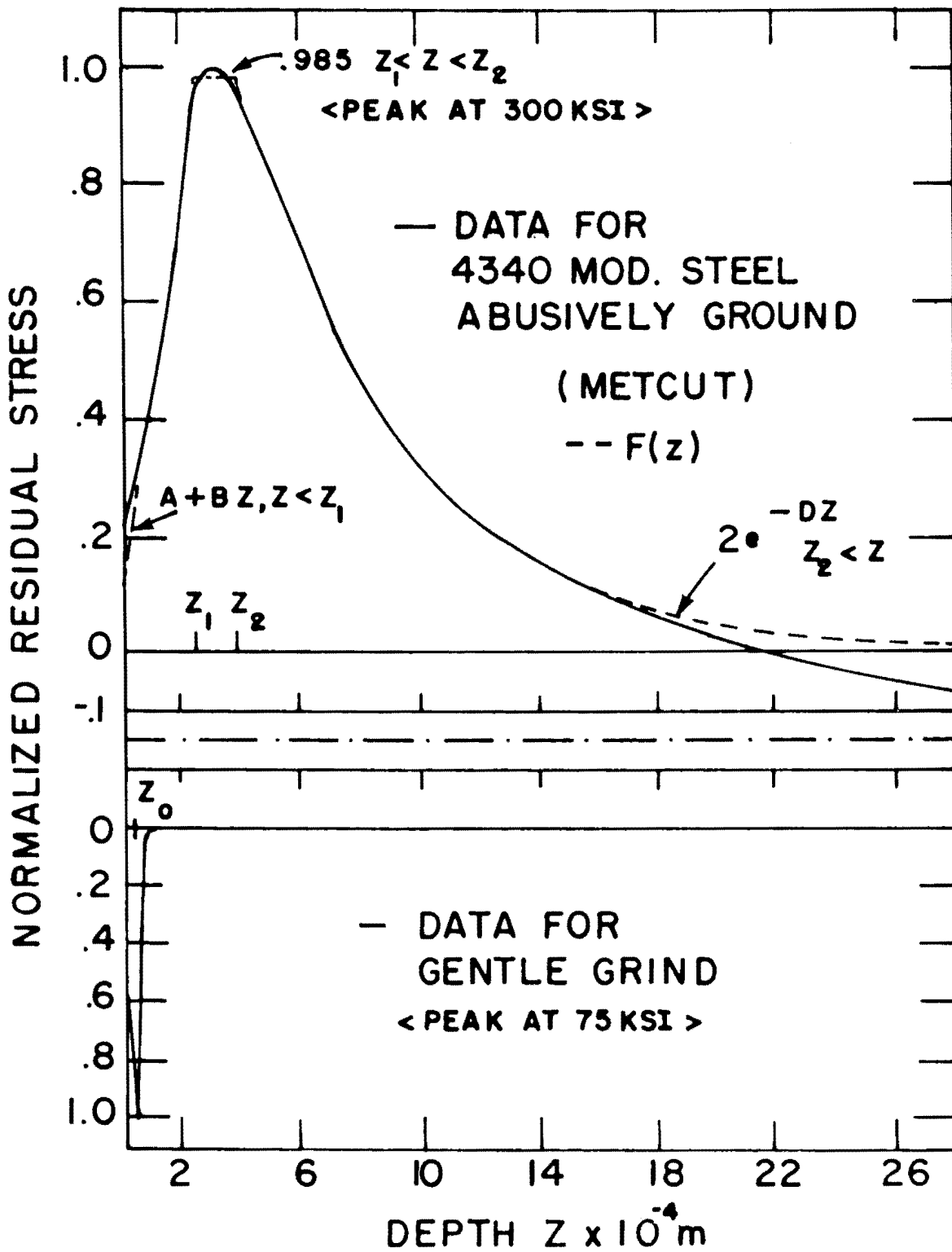


Fig. 12. Measured stress gradients represented by piecewise continuous functions.

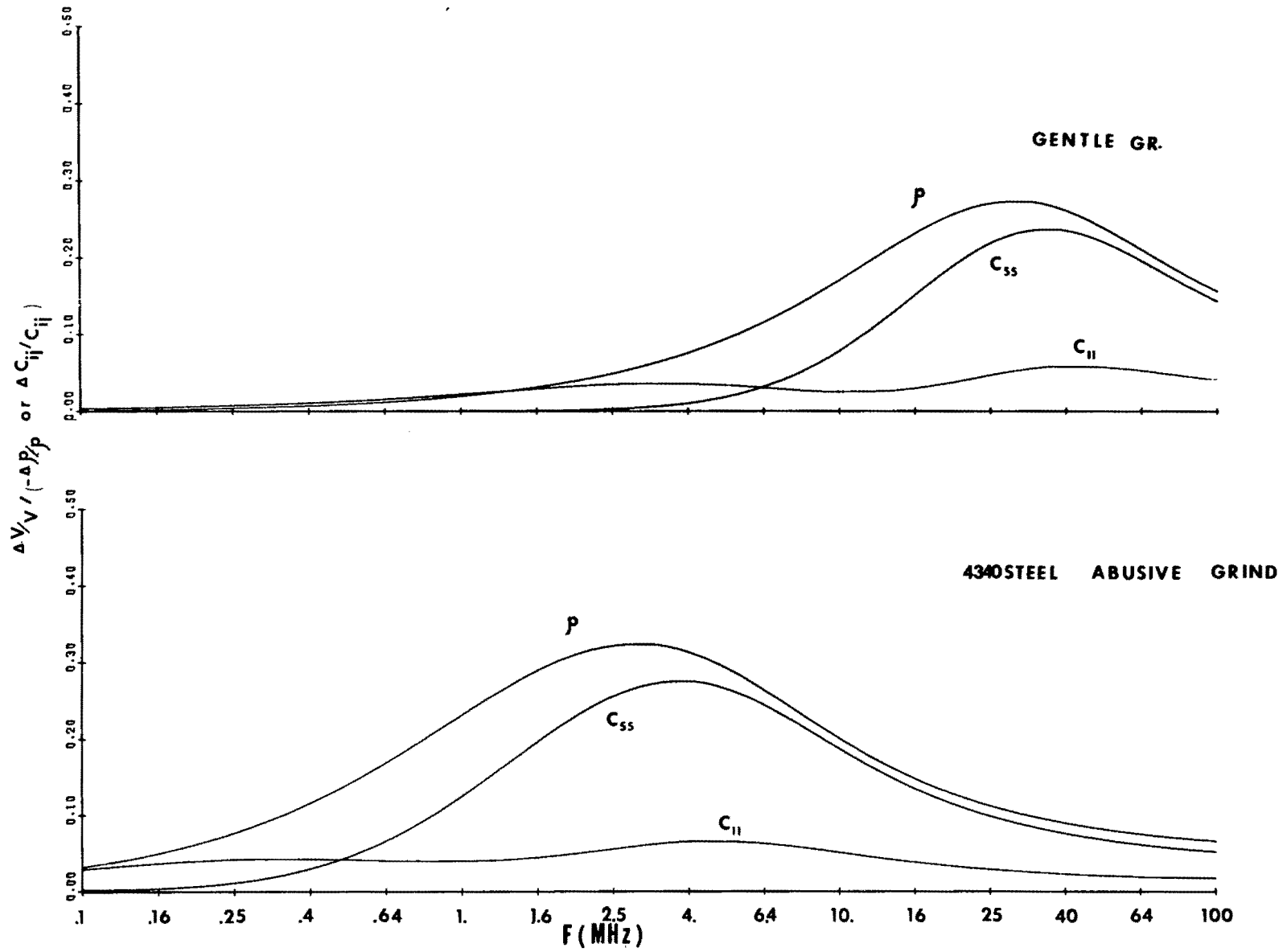


Fig. 13. Dispersion curves for gradients in Figure 12.

References

1. Machining of High Strength Steels with Emphasis on Surface Integrity, AFMDC-70-1, Air Force Machinability Data Center, Metcut Research Associates, Inc.
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DISCUSSION

DR. WALKER: Are there any questions? The doctor from RPI.

PROF. HARRY TIERSTEN (Rensselaer Polytechnical Institute): I see that you have found perturbations in elastic constants and in density, but I don't see what right you have to say that that has anything directly to do with residual stress. Could you explain that to me?

DR. SZABO: Certainly I can explain that. There is really no correlation per se in the theory, but this is something to be determined experimentally. There is a change below the surface of a certain functional kind and, hopefully, it will be the $F(z)$ that I am measuring.

PROF. TIERSTEN: No, Bruce Thompson and I think Bernie Tittmann did work where they had a perturbation of the elastic constant and you are using the same theory. But in order to include residual stress, you have to start off with a properly invariant nonlinear theory of elasticity. The linear theory works because it is the linear limit of the proper nonlinear theory. Now, if you put a stress on it, you have to linearize on the stress bias and you'd better linearize from the correct equations because you can't square infinitesimal strain because strain itself is quadratic in displacement, so I don't particularly approve of the fact that you used the word "residual stress" at all.

DR. SZABO: Well, we will see how it turns out experimentally.

DR. WALKER: Any more questions?

DR. BRUCE THOMPSON (Rockwell Science Center): I think you may have said this and it went by. In your last figure you show the dispersions which would be expected for a given change in density or a given change in C_{11} or a given change in C_{55} for an abusive hardening. How did you decide what the relative weights of those changes would be, to draw that figure?

DR. SZABO: If you will recall, one of the slides was a weighting function, and in that slide I had the three weighting functions normalized and I used an equivalent normalization to get the velocity dispersion curves. In other words, I knew what the stresses were, and I worked backward from that.