LONG WAVE SCATTERING OF ELASTIC WAVES FROM VOLUMETRIC AND CRACK-LIKE DEFECTS OF SIMPLE SHAPES

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ABSTRACT

The development of several approximations appears to permit accurate and practical calculations of the scattering of elastic waves from volumetric and crack-like defects of simple shapes if the wavelength of the incident wave is larger than the characteristic length of the shape. These approximations, which I call the quasi-static and extended quasi-static, use static solutions of defects in uniform strains to predict scattered (dynamic) fields. Since static solutions for several simple defect shapes (oblate and prolate spheroid, ellipsoid, and circular and elliptical cracks) are available, scattering predictions are possible, and the results of such calculations are presented.

Introduction

I was asked to preface my presentation with a few words about the current state of ultrasonic defect characterization studies in the ARPA/AFML program compared to their state at its start. Consequently, my presentation has two distinct parts: One part I call "Past to Present" in which, emphasizing the role played by theoretical studies, I try to assess what I regard as benchmarks in ultrasonic flaw characterization studies. The other part I call "Long Wave Scattering from Simple Shapes" in which I discuss my recent work.

Past to Present*

A Chronology

Zeroth Year (Krumhansl, Gubernatis)

At the start of the ARPA/AFML program, the existing literature on the scattering of elastic waves from defects was not properly oriented to the problem of flaw characterization. No systematic way existed to study the scattering from shapes more complicated than a sphere (or an infinite cylinder). Since the sphere is the only shape of finite volume solvable in closed form, a need for the development of efficient numerical techniques or approximations existed.

First Year (Krumhansl, Gubernatis, Domany, and Huberman)

As a first step, a formal theory of the scattering of elastic waves was developed.[1,2] This theory has a strong analogy to scattering theories used in quantum mechanics and thus has the potential susceptibility to a variety of numerical and approximate techniques. A decision was made to avoid costly numerical solutions and concentrate on less costly and simple approximate techniques, which would perhaps reveal some physical insight. An approximation, called the Born Approximation in quantum mechanics, was studied and was compared to exact scattering results from a sphere. With some limitations, the approximation appeared quite useful.[3] A special usefulness was the fact that the shape of a defect enters through easily calculable factors involving the Fourier transform of the volume V occupied by the defect (the shape factor)[2,3], i.e.,

$$\int_V d\underline{r}\, e^{i\underline{q}\cdot\underline{r}}$$

where $\underline{q}$ is the difference between the scattered and incident wave vectors.

Tittmann[4] meanwhile, measured the scattering of elastic waves from a sphere and found a good comparison between measured values and results calculated by exact theory. I regard the agreement as significant because I personally find as unconvincing explanations why what experimentalists measure is what theorists calculate.

In short, a useful and simple approximation now existed.

Second Year (Krumhansl, Domany, Teitel, Muzikar, and Wood)

The Born Approximation was now applied to a spheroid[5], and the scattering was measured by Tittmann[6] and Adler and Lewis[7]. Although the theory worked better for an oblate than for a prolate spheroid, when theory and Tittmann's experiment were compared, a correlation between the aspect ratio of spheroidal shapes and measured results was found; a definite relationship was established between scattering data and an identifying geometrical feature of a defect (a non-spherical one). In part, a flaw was characterized.

The theorists began to examine other approximations, and the scattering from crack-like flaws was computed.[5]

Third Year (Krumhansl, Domany, Rose, Teitel)

The purpose of this meeting is to report the results for the third year; thus, I will conclude my view of the past after making several remarks that are difficult to time-sequence.

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Comments and Opinions

The program has benefited greatly from a strong interaction between theory and experiment. This interaction is not accidental, but is indebted to the foresight shown in establishing the program when diffusion bonded samples containing identifiable defects of simple shapes (spheres and spheroids) were constructed. If the theorist had to work with the "flat bottom hole", I doubt as much progress would have occurred.

Theory needs experiment, but can (and in some cases it has) progressed to the point where it is considerably cheaper to do the calculations than it is to make the measurements. For example, the Born Approximation, and to some extent another approximation I discuss below, produce a data point for a fraction of a penny and a data base (about 1000 points) for several dollars. A comparably sized data base with experimental uncertainty currently takes several days to measure which is several hundred dollars in salaries. This cost-effectiveness is germane to the work of Tony Mucciardi[8] and of Eytan Domany.[9] Tony needs a large data base on which to train his computer. Eytan, by being able to compute cheaply and examine a variety of scattering cases, is seeing systematics translatable to experimental procedures for identifying flaw shapes. The cost-effectiveness of theoretical studies cannot be emphasized strongly enough.

Longwave Approximations for Simple Shapes**

Introduction

Completed studies used integral equation methods to describe the scattering. Recently, the theory of the scattering of elastic waves from flaws by use of integral equations was developed systematically.[1,2] Previously, nearly all theoretical studies of the scattering of elastic waves have used partial differential equations. With this approach the exact equations for scattering from a spherical flaw were found; however, the methods of partial differential equations have not been successful for shapes of finite volume other than the sphere: Boundary conditions are very difficult to apply, and a systematic development of a perturbation theory for general finite volumes is equally difficult. By contrast, the boundary conditions, at least at the start, are automatically built into the integral equation for all volumes, and a variety of systematically developed approximations is possible.

Some Technical Details

The basic scattering equation is[1,2]

$$u_i(\underline{r}) = u_i^0(\underline{r}) + \int d\underline{r}' g_{ij}(\underline{r},\underline{r}')\, v_{jk}(\underline{r}')\, u_k(\underline{r}') \quad (1)$$

where repeated subscripts imply summation. The vector field $u_i(\underline{r})$ represents the displacement field, and the vector u_i^0 is the incident displacement field. The flaw is hosted in an infinite elastic medium assumed isotropic and described by the Lamé parameters λ and μ. The Green's function for this medium $g_{ij}(\underline{r},\underline{r}')$ equals

$$g_{ij} = \frac{1}{4\pi\rho\omega^2}\left[\beta^2\delta_{ij}\frac{e^{i\beta R}}{r} - \frac{\partial}{\partial x_i}\frac{\partial}{\partial x_j}\left(\frac{e^{i\alpha R}}{R} - \frac{e^{i\beta R}}{R}\right)\right] \quad (2)$$

where ω is the frequency of the incident wave, ρ the density of the host material, α the wave number of the longitudinal mode (P wave), β the wave number of the transverse mode (S wave), and $R = |\underline{r}-\underline{r}'|$. The tensor field operator $v_{ij}(\underline{r})$ represents the flaw and is equal to

$$v_{ij} = \delta\rho\omega^2\delta_{ij} + \frac{\partial}{\partial x_k}\,\delta C_{ikj\ell}\,\frac{\partial}{\partial x_\ell}\,. \quad (3)$$

The quantity $\delta\rho$ is the difference between the density of the flaw and its host; correspondingly, $\delta C_{ijk\ell}$ is the difference between the elastic constants tensor of flaw and its host. Although the flaw is hosted in an isotropic medium, the flaw can have an arbitrary density and elastic properties; however, the case of particular interest is the scattering from a void ($\rho = C_{ijk\ell} = 0$).

Equation (1) describes the scattering of vector fields by a tensor "potential", and the scattered fields propagate from $\underline{r}$ to $\underline{r}'$ by a tensor Green's function (2). Additionally, because an isotropic elastic medium has two modes of propagation, the far field displacement is of the form

$$u_i \sim u_i^0 + A_i\frac{e^{i\alpha r}}{r} + B_i\frac{e^{i\beta r}}{r}$$

where the vector A_i represents the amplitude of the longitudinal scattered wave and B_i the transverse. As a consequence, for a given frequency, the differential cross-section is

$$\frac{dP}{d\Omega} = \frac{\alpha(\lambda+2\mu)\,|A_i|^2 + \beta\mu|B_i|^2}{\alpha(\lambda+2\mu)\,|a_i|^2 + \beta\mu|b_i|^2} \quad (5)$$

where a_i and b_i are the vector amplitudes of an incident longitudinal and a transverse displacement field. From Equation (5) it is apparent that even if $|a_i|^2$ or $|b_i|^2$ equals zero, the cross-section always involves contributions from two modes of scattering associated with the same frequency (mode conversion).

Because of the differential operators appearing in the "potential", the scattered vector amplitudes can be regarded as functionals both of the displacement and strain fields internal to the flaw:

$$A_i = A_i[u_i;\epsilon_{ij}]$$

$$B_i = B_i[u_i;\epsilon_{ij}]$$

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The Born Approximation corresponds to replacing the displacement field by the incident displacement and the strain field by the strain field ϵ^0_{ij} associated with the incident displacement field:

$$A_i = A_i[u^0_i;\ \epsilon^0_{ij}]$$

$$B_i = B_i[u^0_i;\ \epsilon^0_{ij}]$$

The results of this approximation have been computed for a variety of spherical flaws and compared to the exact solution.[3] This approximation is found to describe backscattering well when the wavelengths are larger than the radius of a sphere. This observation was verified experimentally.[6]

For long wavelengths, the system is in a quasi-static condition. An alternative approximation, called the quasi-static approximation,[10] is to replace the displacement fields by the amplitude u^σ_i of the incident mode and to replace the strain field by the associated static strain field ϵ^σ_{ij}.

$$A_i = A_i[u^\sigma_i;\ \epsilon^\sigma_{ij}]$$
$$B_i = B_i[u^\sigma_i;\ \epsilon^\sigma_{ij}] \qquad (6)$$

The results of this approximation are, for long wavelengths, identical to the exact results for a sphere,[11] i.e., it is not limited to backscattering. By a systematic study of the iterative solution of the integral equation, it was recently shown that the approximation represented by Equation (6) is exact for any finite shape at long wavelengths.[12] This result permits the exact calculation of the scattering of elastic waves from a flaw other than a sphere, albiet at long wavelengths. The approximation determines exactly the ω^4 contributions (the Rayleigh limit) to the scattering cross-section. Solutions of ϵ^σ_{ij} are available for a number of geometries. The most famous and the most convenient is Eshelby's solution[13] for a spheroid and ellipsoid. These shapes are extremely important, for by varying aspect ratios of the shapes one can distort them to resemble needle and disc crack-like geometries, types of flaws that one is most eager to detect.

A more powerful approximation is the extended quasi-static approximation.[14] In this approximation

$$A_i = A_i[u^\sigma_i\, e^{i\underline{k}_0\cdot\underline{R}};\ \epsilon^\sigma_{ij}\, e^{i\underline{k}_0\cdot\underline{R}}]$$
$$B_i = B_i[u^\sigma_i\, e^{i\underline{k}_0\cdot\underline{R}};\ \epsilon^\sigma_{ij}\, e^{i\underline{k}_0\cdot\underline{R}}] \qquad (7)$$

where $\underline{k}_0$ is the wave vector of the incident wave. An important point is that for ellipsoids and spheroids this approximation also uses Eshelby's solutions. This approximation, in contrast, to the Born and the quasi-static, is not well-defined in terms of a perturbation expansion.

A description of the full details of these approximations is in preparation.[12] What is more useful to present interests are calculated results for the variously shaped defects.

Results

Figure 1 illustrates the manner in which the scattering angles θ and ϕ are defined with respect to the direction of the incident wave $\underline{k}_0$. The incident direction is always chosen along the z-axis, and the defects are always hosted in Ti-6Aℓ-4V. The differential cross-section is in decibels, and zero is equated to -100 db.

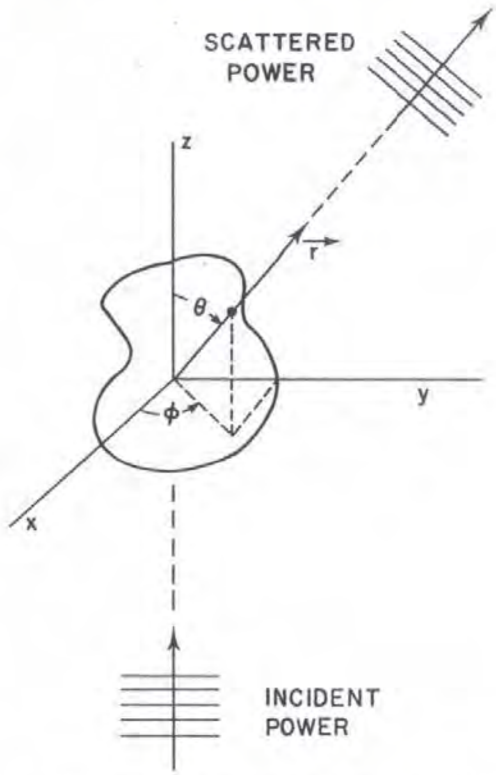

Figure 1. The basic scattering picture. Incident power scatters from a flaw into a receiver in the direction $\underline{r}$.

In Fig. 2 is a comparison of three approximations, the Born, quasi-static, and extended quasi-static, with exact results for a sphere. The incident wave is a longitudinally polarized plane wave, the defect is a spherical void, k is the wavenumber of a longitudinal wave, and a is the radius of the sphere. The range of ka is to 2. For a void, the extended quasi-static approximation appears to be quite useful up to ka $\simeq$ 1.5.

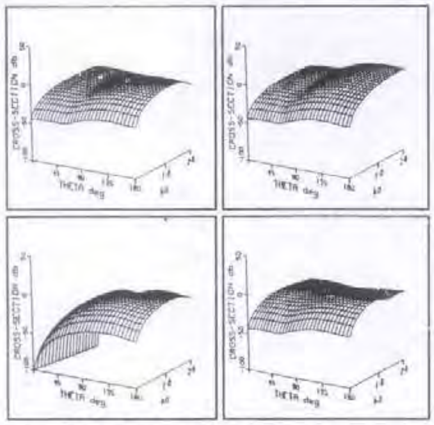

Figure 2. Incident longitudinal plane wave scattering from a spherical void. The differential cross-section is calculated four ways: clockwise from lower left-hand corner, the Born Approximation, the extended quasi-static approximation, the quasi-static approximation, and the exact calculation.

The remaining figures, because of the absence of exact results, were computed with the extended quasi-static approximation.

The scattering of a longitudinally polarized plane wave from a stainless steel prolate spheroid is shown in Fig. 3. k is still the longitudinal wave number, but a refers to the semi-major axis. The axis of symmetry is perpendicular to the incident direction and along the y-direction. The major axes, along x and y, are 4 times the minor axis.

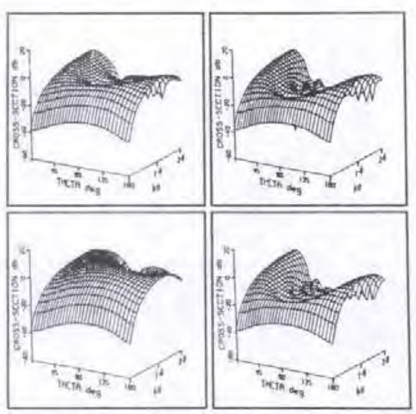

Figure 3. Incident longitudinal plane wave scattering from a prolate spheroidal void calculated from the extended quasi-static approximation. The incident direction, along the z-axis, and the axis of symmetry are perpendicular, along the y-axis. The ratio of the z-axis to the y-axis is 1/4. Clockwise from lower left, θ equals 0, 30, 60, 90 degrees.

By letting the short axis of an ellipsoid or oblate spheroid go to zero, one produces elliptical and circular disc-like cracks.[16] Figure 4 shows the scattering of a longitudinal plane wave incident normal to the face of a crack lying in the xy-plane. k is the longitudinal wave number, and a is the radius of the crack. On the left is the longitudinal-to-longitudinal scattering; on the right, longitudinal-to-transverse (mode converted) scattering.

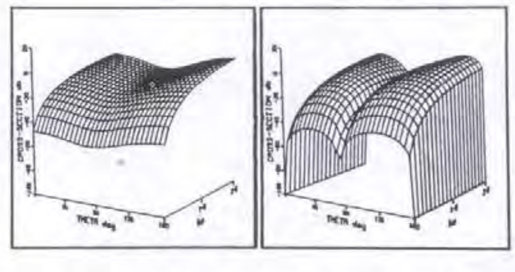

Figure 4. Incident longitudinal plane wave scattering from a circular crack. The incident direction, along the z-axis, is normal to the crack plane. On the left is longitudinal-to-longitudinal scattering; on the right, longitudinal-to-transverse scattering.

In Fig. 5, a transversely polarized plane wave is scattered off the edge of an elliptical crack. The cracks lie in the xz-plane the semi-major axis, along x, is twice the semi-minor axis. The angle $\phi = 90^\circ$. On the left is the transverse-to-longitudinal scattering (mode converted); on the right the transverse-to-transverse scattering.

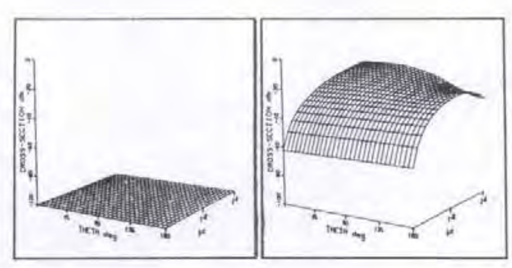

Figure 5. Incident transverse plane wave scattering from an elliptical crack. The incident direction is along the z-axis, and the polarization is along the x-axis. The crack is in the xz-plane; its major axis, along x, is twice its minor axis. $\theta = 90^\circ$. On the left is transverse-to-longitudinal scattering; on the right, transverse-to-transverse scattering.

Several conclusions are evident. The scattering is not isotropic. Different shapes produce different angular distributions. For acoustic and many quantum mechanical problems the scattering at long wavelengths is isotropic. This illustrates the fact that the elastic wave scattering problem has distinguishing features which can prevent the blind adoption of techniques and concepts successful in these other areas. Scattering signatures exist, but there is a need for systematic study to exploit them fully. The amount of data that can be easily generated is enormous, but it can be done cheaply.

Time and space permit the showing of only a small sample of nearly 100 calculations. The complete set of calculations is being prepared as a Los Alamos Report.[15]

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