NONDESTRUCTIVE DETECTION OF VOIDS.
BY A HIGH FREQUENCY INVERSION TECHNIQUE

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ABSTRACT

A direct solution of the ultrasonic inverse scattering problem, known by the acronym POFFIS, which stands for Physical Optics Far Field Inverse Scattering, has been developed. This technique has been used to reconstruct the shape and size of both simulated and real void flaws in materials.

When a wave (e.g., acoustic, elastic, electromagnetic) propagates through a medium, the wave is deformed by irregularities in that medium. The deformation of the probing wave is characteristic of the irregularity. This is the basis of a method for nondestructive evaluation of solid materials. A known high frequency probing wave is introduced into the medium. The wave scattered by the irregularity is observed. The nature of the irregularity is to be inferred from the nature of the scattered wave.

The type of problem we have described here is called an inverse problem. The corresponding direct problem would be to find the scattered wave given the "incident wave" (probe) and the irregularities of the medium. Inverse problems are often attacked indirectly; i.e., one solves the direct problems for prescribed irregularities, seeking an irregularity for which the scattered wave most nearly approximates the data.

In contrast, we are engaged in research on direct solutions of the inverse problem. An inverse method for nondestructive detection of scatterers of high contrast, such as voids or strongly reflecting inclusions will be described here. This method is known by the acronym POFFIS, which stands for Physical Optics Far Field Inverse Scattering. In this method, the phase and range normalized far field scattering amplitude is shown to be directly proportional to the Fourier transform of the characteristic function of the scatterer. The characteristic function is equal to unity inside the region occupied by the scatterer and is zero outside. Thus, knowledge of this function provides a description of the scatterer. However, for bandlimited data, it is easier to reproduce a directional derivative of the characteristic function. These ideas were developed in a series of papers.

The specific application here is to describe flaws contained in a sphere. Observations of backscatter acoustic signals are made on the surface of the sphere. The following formula may be derived for the radial derivative of the characteristic function (see 2):

\[ R \cdot \gamma (\mathbf{x}) = \frac{A}{c} \int_{\Omega} d \mathbf{\Omega} \cdot \mathbf{x} \cdot \mathbf{\hat{x}} \frac{\partial V}{\partial t} \mathbf{\hat{x}}, \text{ } 2(x_0 - \mathbf{x} \cdot \mathbf{\hat{x}}_0)/c. \tag{1} \]

Here, \( R \) is the unknown reflection coefficient, \( \gamma \) is the unknown characteristic function, \( \mathbf{\hat{x}} \) denotes unit vector, \( A \) is a known constant which depends on the source configuration, \( V \) is the range normalized backscattered ("pulse-echo") impulse response in the direction \( \mathbf{x}_0 \) at a time \( t = 2(x_0 - \mathbf{x} \cdot \mathbf{\hat{x}}_0)/c \), \( x_0 \) is the radius of the observation sphere and the integration domain is the unit sphere with variable \( \mathbf{\hat{x}}_0 \). Thus implementation of the POFFIS result requires an integration over the directions of observation of the scattered data.

We now present the results obtained from a computer implementation of this method. The computer program uses either simulated flaws or experimental data from real flaws and incorporates the following "real world" constraints: (1) bandlimiting: only a limited range of frequencies are present in an ultrasonic pulse, (2) aperture limiting: a flaw can usually be viewed from only a small range of angles due to the geometry of the part in which it is located.

Several types of graphical display of the results are presented. Consider a longitudinal "slice" through the flaw, as depicted in Fig. 1. In order to see the radial derivative function which results from the integration, its value can be displayed along a number of radii of the flaw. Two such radii are shown in Fig. 1. This type of display is used in Figs. 2-7. The estimated position of the surface of the flaw is at the peak of the radial derivative function along each radius. A simple display of the estimated shape and size of the flaw is obtained by plotting the locus of these peak positions in a variety of radial directions. This type of display is labeled "flaw" in Fig. 1, and is used in Figs. 8-13.
The first tests of the method involved the use of synthetic backscattered data from a spherical void of 400 microns radius. The value of the dimensionless parameter "ka" was about seven. Tests were run with ninety-eight observations and two hundred observations around the surface of the observation sphere. Output of the type in Fig. 1 is plotted for a slice at longitude 0° for several latitudes.

Figure 2 shows the result for ninety-eight observations. Figure 3 shows the result for two hundred observations. In each case, the maximum error in location of the boundary is less than 5%.

The above cited papers also develop a theory for dealing with data given over less than the full 4π steradians of the observation sphere. For such aperture limited data, the function 3\sqrt{t} in the integrand above is taken to be zero outside the aperture of observation and the same integration formula is applied. The theory predicts that only a portion of the boundary will be produced, namely that section for which the normal to the boundary is in the aperture of the non-zero data.

This theory was tested on a series of synthetic examples. Figure 4 shows the output when the aperture is taken as a pair of opposite octants in the lower hemisphere. This requires fifty observations at 25° aperture. Here the output is a line approximately through the aperture. Figure 4 demonstrates the validity of this limited aperture theory.

Figure 5 depicts the output when the aperture consists of one octant - 25 observations or 12.5° aperture - in the lower hemisphere. Again the output slice cuts the aperture symmetrically.

Figure 6 shows the output from a 7.5% aperture. The longitudinal slice is taken through the center of the aperture and the appropriate quarter circle is detected.

Figure 7 tests the constraint that the dimensionless parameter of the theory used to predict the peak across the boundary really remains large. Here the bandlimited Fourier transform of the characteristic function of an ellipsoid is processed. The radial derivative still demonstrates a detectable peak, even though the so-called large parameter is as small as three. In the second quadrant the true boundary points on each ray are connected and the computed boundary on each ray is shown by dots. In the third quadrant the computed boundary points are connected to yield the approximate ellipse.

In order to explore the robustness of the POFFIS algorithm with respect to having only a limited amount of data available in the temporal domain (i.e., bandlimited), a set of calculations was performed using a "simulated ultrasonic transducer." This transducer's output is a maximum at a center frequency f_0 and drops smoothly to zero by zero frequency and by 2 f_0. An appropriate measure of the "highness" of the frequencies being used is

\[ ka = 2\pi f_0 \frac{a}{v} \]  

which is numerically equal to about 1/2 f_0 in MHz in this case. Because it is a high frequency technique, POFFIS would be expected to perform less well as f_0 is reduced.

Figure 8 shows the estimated flaw outline for a 450 μm sphere inspected by an 8 MHz transducer (ka = 4). The size of the flaw is well reconstructed and, as the statistics accompanying the figure show, the size is only slightly underestimated. As the transducer frequency is reduced, the shape is still well reconstructed, but the size underestimation becomes more serious. Figure 9 shows the case of ka = 1.5. Thus, POFFIS works relatively well even at frequencies as low as ka~2.

Another important question is: how coarse can the spatial sampling of the scattered sound field be without leading to incorrect results. In other words, how few transducer positions are needed?

Figure 8 was obtained using 100 observations around the full 4π steradian solid angle. As the number of observations is decreased, the size of the flaw remains unchanged but its outline begins to become slightly irregular. Eventually, the algorithm begins to make errors. Figure 10 shows the case of using only 8 observations spread over the entire angular range. At the north and south poles, substantial errors occurred.

The reason for these errors is that at a given frequency, the spatial sampling must be fine enough to adequately sample the rate at which the scattered sound field varies with angle. A corollary is that if high frequencies are used, more observation positions are needed. Also, more observations are needed for irregularly shaped flaws. A rule of thumb for sphere-like flaws is that the angular separation between observations obey

\[ \theta < \frac{2}{ka} \text{ radians} \]

This equation is an angular equivalent of the Nyquist sampling theorem.

In Figs. 4-7, it was shown that when the flaw is observed through a limited aperture, the part of the flaw facing the aperture is accurately reconstructed. In Fig. 11, the calculation of the surface of the flaw was automatically terminated when the amplitude of the radial derivative function dropped below a specified level. In practice, this level could be the noise level of the measurements. In this case, the calculation was allowed to continue beyond the edges of the aperture and the characteristic asymptotic behavior is seen. A more conservative use of the algorithm might involve also terminating the calculation at the edges of the aperture.

What pattern of transducer positions gives the best reconstruction? Because the reconstruction works only within the aperture of available observations, it follows that the reconstruction at a given point is determined primarily by the observations at nearby angles. Therefore, by observing the quality of reconstruction at various points on the angular sampling grid, one can judge what local arrangement of transducers does the best job. The following conclusions have been drawn. A grid which
approximates a square array (checkerboard) does better than one having a measurement point in the middle of a circle of other measurement points. This, in turn, does better than a grid consisting of the ring of points with no center point.

For a nonspherical flaw, the POFFIS technique reproduces the shape well. Figure 12 is the reconstruction of a simulated oblate spheroid of semi-axes 200 \( \mu \text{m} \times 400 \ \mu \text{m} \).

Finally, POFFIS has been applied to experimental data. Figure 13 is a reconstruction of a 400 \( \mu \text{m} \) radius spherical void built into a block of Ti-6Al-4V by the diffusion bonding process. The size and shape are well reproduced. The "roughness" of the shape is due primarily to the metallurgical noise of the titanium.

Note that all of the above reconstructions have been done with only the limited range of frequencies present in a commercial ultrasonic transducer. No attempt has been made to deconvolve the transducer out of the data. The results were seen to be fairly insensitive to the type of transducer. This means that there may be no need to remove the transducer properties.

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REFERENCES

OPPOSITE OCTANTS IN LOWER HEMISPHERE
25 PERCENT APERTURE (50 OBSERVATIONS)
Fig. 4

OCTANT IN LOWER HEMISPHERE
7.5 PERCENT APERTURE (15 OBSERVATIONS)
Fig. 6

OCTANT IN LOWER HEMISPHERE
12.5 PERCENT APERTURE (25 OBSERVATIONS)
Fig. 5

400 X 200 MICRON ELLIPSE
Fig. 7
Fig. 8 450 μm sphere, 8 MHz transducer, \( ka = 4 \).

Fig. 9 450 μm sphere, 3 MHz transducer, \( ka = 1.5 \).

Fig. 10 Insufficiently fine angular sampling.

Fig. 11 Limited aperture of observations.

Fig. 12 200 μm x 400 μm oblate spheroid.

Fig. 13 400 μm radius spherical void.