ABSTRACT

This report describes an imaging system employing the digital reconstruction of the near fields of an object using the amplitude and phase of the scattered fields measured on a remote plane in the Fresnel zone. This imaging technique is shown to be significantly different from holographic and phased array systems and basically computationally implements a classical lens system having the reconstruction plane parallel or perpendicular to the measurement plane. The theory of the imaging approach is detailed and the method of obtaining a real-time implementation is described. Test data for the first 64x64 element array using PVDF elements and electronic scanning is presented.

I Introduction

In this report we describe the theory and experimental results for a technique of acoustic imaging using wavefront reconstruction to obtain the source or scatter near-in fields from a measurement of the scattered fields in the Fresnel zone of the object or source. The reconstruction technique has its roots in physical optics wherein the operation of lens are described by mathematical operations and visa versa [1]. Thus the procedure is to perform the image formation through a series of mathematical operations rather than via a set of physical objects such as lens. The burden of the image quality is thus placed upon the initial data collection and finally the precision of the mathematical transformation process. There is of course considerable flexibility offered by this approach because, as will be shown, one set of two dimensional data can yield an infinity of reconstruction planes, whereas variable focal length acoustic lenses are much more difficult to implement and still require field measurements in their focal planes.

This paper discusses the formalism of the imaging technique, progress obtained on the implementation of the imaging system using two dimensional acoustic arrays, and a brief comparison with holographic [2] and phased array systems.

II Theoretical Basis of the Imaging Approach

The basis of the imaging system, Figure 1, starts from Huygens theory describing the superposition of a radiation field from many point sources of the source or scattered field,[1]

\[ U(u,v) = \frac{2\pi}{ik}\int_{-\infty}^{\infty} U(x,y) e^{ikr} \frac{dx}{r} \delta_{x-u} \delta_{y-v} \]  

where \( U(u,v) \) is the measured field in the \( u,v \) plane located a distance \( R \) from an assumed scattering plane, \( k \) is the wavenumber, \( U(x,y) \), the source or scattered field and \( r \) is the distance between a scattering element and field position,

\[ r = R \left[ 1 + \frac{1}{R} ((x-u)^2 + (y-v)^2) \right]^{\frac{1}{2}}. \]  

Using the small angle approximation we essentially restrict the transverse dimensions of the scattering region and measurement plane such that they are much less than \( R \) the distance between planes. In practice this para-axial approximation restricts the maximum angle between \( r \) and \( R \) to approximately 30 degrees. However, for the usual cases of water-solid interfaces this represents a more than adequate angle of incidence due to longitudinal wave cutoff conditions.

Using the paraxial ray approximation, the distance \( r \) is approximated by

\[ r \approx R + \frac{1}{2R}((x-u)^2 + (y-v)^2) \]  

and (1) becomes

\[ U(u,v) = 2\pi e^{ikR} \int_{x}^{y} U(x,y) \exp \left[ i \frac{k}{2R} ((x-u)^2 + (y-v)^2) \right] dx \]  

The quadratic phase factor may now be expanded and a normalized relationship is obtained

\[ \tilde{U}(u',v') = \mathcal{C} \tilde{U}(x',y') \exp \left[ -ik' (u'x' + v'y') \right] dx' dy' \]  

where

\[ \tilde{U}(u',v') = U(u',v') \exp \left[ -ik' (u'^2 + v'^2) \right] / 2 \]

\[ U(x',y') = \tilde{U}(x',y') \exp \left[ -ik' (x'^2 + y'^2) \right] / 2 \]

\[ \mathcal{C} = 2\pi e^{ikR} \]

\[ y' = y/a, \quad u' = u/a, \quad v' = v/a \]

\[ k' = \frac{ka}{R} = \frac{ka}{R} \]
or where \( \Delta s = \frac{R}{2a} \) maximum sampling internal allowed without considerable information loss. At the widest angle of operation, \( \frac{a}{R} \) would have a minimum value of \( \sqrt{3} \) and the maximum allowed sampling interval would be \( \Delta s = \frac{\sqrt{3}}{2} \).

The image is formed by a sequence of mathematical operations. First, the actual field at the measurement plane must be obtained by taking the measured data and multiplying by a correction factor that accounts for variations within the measurement system. Next the data is multiplied by a correction factor, an operation equivalent to lens \( L_1 \) in Fig. 2. Next the data is corrected by a smoothing function in order to reduce ringing in the final image due to the finite spatial sampling that is necessarily employed in the measurement process. Clearly, small defects or sharp edges of defects, such as crack tips, scatter widely, and a truncation of the fields due to finite spatial sampling causes a smoothing in the image. The weighting function is used to eliminate the Gibb's ringing created by this artificial truncation.

By taking the inverse transform of (5) we obtain

\[
\tilde{U}(x', y') = C \int \hat{U}(u', v') \exp(ik') \exp(ik'x' + ik'y') \, du'dv'
\]

which is the normalized image field. If phase information is desired then the parabolic phase un-normalization is carried out, (7). The Fourier transform has the same effect as lens \( L_2 \) in Fig.2. Note that the plane of reconstruction is determined by the parabolic phase normalization and that many images may be formed from one set of measured data. However, only one image would be in focus. The operation is like that of an optical microscope wherein the operator adjusts the focus for the desired image except that here the focus is obtained electronically. If the distortion caused by the detector or array element spatial response requires correction, then the final image is divided by the known transform of the detector field pattern.

Since the imaging technique has its roots in conventional optics, many processing techniques employed there may be usefully employed with the added advantage of having phase and amplitude as variables and not just intensity.

The imaging process may now be summarized: 1) measure the amplitude and phase \( \tilde{U} \) in the measurement plane, 2) weight the fields and correct for known measurement errors such as array non-uniformities, 3) choose a reconstruction plane and phase normalize the measured fields, 4) do a two-dimensional Fourier transform to obtain the normalized scattered fields, 5) and phase un-normalize to obtain the complete fields. This data is now ready for display and possible enhancement.

III Side Looking Imaging

In NDE applications there are situations where the parallel plane reconstruction technique, currently employed in our work, is not suited to the physical test configuration. Such a case might be the inspection of vertical weld joints where the top surface is generally non-planer due to the crown of the weld. In other cases it might be of interest to have the depth information available in real time rather than in near-real time through successive parallel plane transformations.

The fundamental requirement of the parallel plane reconstruction algorithm is the small angle approximation. It is this approximation that allows the problem to be cast into the Fourier transform format. Once in the Fourier transform format the inversion is accomplished by the inverse Fourier transform. Finally, an unnormalization of the fields gives the desired result. With that procedure in mind the reconstruction algorithm can be derived for the case of side looking perpendicular plane imaging similar to that described above for parallel plane imaging.

The geometry of the side looking imaging problem is shown in Figure 3. Here the \((x,y)\) plane is taken as the source plane or origin of scattered fields and \((u,v)\) the measurement plane just as for the previous case. The scattering region is centered a distance \( S \) to the side and a distance \( D \) in depth from the measurement plane. The planes are taken to be perpendicular. Here \( r \) is given exactly by

\[
R = \left[ (x-u)^2 + (s+y)^2 + (D+y)^2 \right]^{\frac{1}{2}}
\]

and may be factored to obtain,

\[
r = R_0 \left[ 1 + \frac{1}{R_0} (vS+yD) + \frac{v^2 + u^2 + u^2y^2 - 2uvy}{R_0^2} \right]^{\frac{1}{2}}
\]

where \( R_0 = (S^2 + D^2)^{\frac{1}{2}} \) is the distance between the centers of the two regions of interest. If the \( x,y \) and \( u,v \) regions are of limited extent and much smaller than the total distance \( r \) then,

\[
R = R_0 \left[ 1 + \frac{1}{R_0} (vS+yD) + \frac{v^2 + u^2 + u^2y^2 - 2uvy}{R_0^2} \right] + \frac{1}{2R_0^2} (vS+yD)^2 + \ldots
\]
This may now be written in the form

\[ R = R_0 + R_{\text{m}}(x,y) + R_{\text{sm}}(x,u,y,v) \]  
(14)

where

\[ R_{\text{m}}(x,y) = \frac{(x^2+y^2(1-D^2/R_0^2)+2yD)}{2R_0} \]  
(15)

\[ R_{\text{sm}}(x,u,y,v) = -(x+yvR/D/R_0^2) \]  
(16)

This factoring of \( r \) is most essential to the formulation because it allows the field quantities to be normalized by phase terms and then cast into the Fourier transform format. The \( 1/r \) dependence in (1) may be removed from the integral under the small angle approximation which essentially assumes all sources are equally distant from the measurement plane for the purposes of amplitude determination. Using (14) for \( R \), the field expression (1) becomes

\[ U'(u,v) = C \int U'(x,y)e^{ikR_{\text{sm}}} \, dx \, dy \]  
(18)

where \( C = e^{ikR_0/R_0} \)  
(19)

\[ U'_m(x,y) = U'_s(x,y)e^{ikR_s(x,y)} \]  
(20)

normalized source field

\[ U'_m(u,v) = U'_m(u,v)e^{-ikR_m(u,v)} \]  
(21)

normalized measured field.

In (18) the normalized fields are clearly in the Fourier transform format through the definition of \( r \) in (17). Taking the inverse transform of (18) yields

\[ U'_s(x,y) = C \int U'_m(u,v)e^{ikR_{\text{sm}}} \, du \, dv \]  
(22)

which completes the major part of the field inversion process.

There is a distortion in the relationship between the \( y \) and \( v \) coordinates due to the scale factor, \( SD/R_0 \), in (17). The effect of this is most apparent when (22) is cast in the form of a discrete Fourier transform, DFT. In transition to the DFT formulation the coordinates are given by

\[ x = Idx \]  
(23a)

\[ u = Jdu \]  
(23b)

\[ y = Kdy \]  
(23c)

\[ v = Ldv \]  
(23d)

where \( I,J,K,L = 1,2, \ldots N \).  
(23e)

The phase factor in (22) is now written as

\[ e^{-ikR_{\text{sm}}} = e^{i\Phi(x,u)}i\Phi(y,v) \]  
(24)

where \( \Phi(x,u) = 2\pi x/R \)  
(25a)

and \( \Phi(y,v) = 2\pi y/v \)  
(25b)

Using (23) the phase factors become,

\[ \Phi(x,u) \rightarrow \Phi_{IJ} = \frac{2\pi I}{N} \frac{NdS}{\lambda R_0^3} \]  
(26a)

and \( \Phi(y,v) \rightarrow \Phi_{KL} = \frac{2\pi K}{N} \frac{NdS}{\lambda R_0^3} \)  
(26b)

where

\[ \frac{NdS}{\lambda R_0^3} = 1 \]  
(27a)

and

\[ \frac{NdS}{\lambda R_0^3} = 1 \]  
(27b)

These relations are necessary to make \( \Phi_{IJ} \) \( \Phi_{KL} \) cyclic of period \( 2\pi \) as required by the FFT algorithms. Thus from (27) we find

\[ dx = \frac{\lambda R_0^3}{NdS} \]  
(28a)

or

\[ dx' = \frac{\lambda R_0^3}{NdS} \]  
(28b)

and

\[ dy = \frac{\lambda R_0^3}{NdS} \]  
(28c)

or

\[ dy' = \frac{\lambda R_0^3}{NdS} \]  
(28d)

In (28) the primed quantities are the coordinates normalized in terms of the wavelength. For parallel plane imaging both \( x \) and \( y \) are related to \( u \) and \( v \) by the same reduction factor, in the perpendicular plane case the reduction factor may be different. In practice this linear distortion would cause no problems because there is always the option of compensating the distortion in the display device. Also, note that if \( S = 0 \) there is no distortion.

IV Two Dimensional Acoustic Arrays

From the standpoint of this particular imaging technique there is no real size limitation on the array other than the small angle requirement suggesting some bounds on the ratio of array area to object distance from the array. Thus a small array could be located close to the scattering fields and a large array farther away assuming both had the same number of array elements. The number of array elements is largely determined by the resolution and number of pixels desired. Our previous work on transducer
characterization suggested that an array of size 64 x 64 is satisfactory from the reconstruction standpoint and that 32 x 32 would give somewhat less favorable images for these confined fields. The rate of sampling is determined by the characteristics of the image and how the scattered fields interfere at the remote sampling plane. The mathematical requirements on the number of sampling points is determined by sampling theory. If the image were known to have a simple symmetry then a two dimensional array would not be required or at most a simple array would give all the necessary information required.

In a two dimensional array one major technological problem is that of accessing the elements of the array. In the imaging system described above, the actual image is formed by computation using a matrix of data. Accordingly there is some flexibility in the manner in which the data is gathered and sequential acquisition seems most appropriate. In addition, the array may be operated in either the send only, send and receive, or receive only mode.

There are acoustical constraints placed on the array as well in terms of transducer element bandwidth, efficiency, and radiation pattern.

Below is discussed the electronic and acoustic problems in more detail.

Electronic Scanning

In an array of size 64 x 64 there are 4096 transducer elements that must be sequentially or randomly accessed. Such a large number of elements implies that the cost per element must be small if the overall array is to be practical. A further constraint was the decision to use 254 microns (100 mil) spacing of the array elements. This was a reasonable choice based upon the finite amount of piezoelectric available and the desire to keep the overall array and associated electronics of finite size. Thus with elements on a 254 micron (100 mil) grid the available cross-sectional area for the electronic switching is somewhat limited whereas the depth may be anything reasonable.

The electronic switching is accomplished using CMOS 8-1 multiplexer hybrid integrated circuits. These eight channel switches are enclosed in a standard sixteen pin DIP ceramic package. A group of eight such switches form a 64 element cell, Figure 4. Here eight MUX chips are stacked side by side and share common power and row address lines. The eight analog I/O lines are further multiplexed to produce a single I/O line and resultant column address lines. This 8 x 8 cell was used for prototype array designs before going to the full sized array. Accordingly various combinations of such cells can be used to form a larger array although any subset of the 8 x 8 cell would work as well. In a 64 x 64 array, 64 such cells are combined in a manner indicated schematically in Figure 5. Here representative signal paths are shown in order to illustrate the decoding technique and to facilitate the computation of the signal attenuation and spurious feedthrough within the array. Also shown schematically is a 64 x 64 array composed of 64 8 x 8 cells having one cell (shaded) with one chip active (solid line) and within that chip one channel is active. The dashed line denotes the selected 8 x 8 cell and within that cell the active chip. The inhibited chip, denoted by INH, represents the remaining seven inactive chips within that cell. The two chips below the active cell represent the remaining seven 8 x 8 inactive cells in the selected row of cells common to the active cell. The bottom most chips represent all those other cells which are inactive because they occupy unselected rows of cells. Such a picture is necessary for computing the attenuation because the attenuation through a chip is greater for the unselected state than for the selected state. In an active chip the adjacent channel isolation is from 25 dB to 35 dB and all other channels within the chip show approximately 40 dB isolation. An inactive chip shows a much larger attenuation of 45 to 50 dB. If we let A represent the fractional isolation of unselected channels within an active chip, and AI the fractional isolation in an inactive chip, then, assuming unit amplitude at all 4096 channels, and no attenuation through the ON switches, the output signal is given by,

\[ A_{\text{out}} = 1 + 7A[1 + 8A1 + (8A1)^2 + (8A1)^3] \]

and results in a worst case condition. Clearly the 7A term dominates and indicates that the given switch isolation are adequate.

For the 64 x 64 array, there are 512 MUX chips at the first level nearest the array which are in turn multiplexed by 64 MUX chips which in turn are multiplexed by 8 chips and finally one chip. The result is 585 analog MUX chips and some low power TTL decoder chips. The output wires from the immediate vicinity of the array consist of 12 address lines 6 for row and 6 for column, two power supply lines, one analog I/O line and ground for a total of only 16 wires. The digital lines plug into an IEEE 488 interface which provides sequential scanning via external or computer trigger or complete random access of the array elements.

Transduction Elements

In the large two dimensional array it is necessary to have 4096 transduction elements. Because of the number of elements involved, the construction of an individual element must be simple if the overall array fabrication is to be practical. The physical constraints
imposed upon the individual elements by the imaging process are somewhat flexible. Plainly, all the bandwidth of the transducers need only be sufficient to pass a 5 MHz tone burst of duration greater than approximately 5 microseconds since the imaging system employs coherent waves. The lower limit of the pulse duration is determined by the physical extent of the object since all of the object must be illuminated. The upper limit is set by stray echo considerations since a modest amount of pulse delay discrimination is very desirable.

The beam angle of the array element must be large enough to sample the object area. A practical upper bound is set by the small angle requirement of the imaging process and is approximately 30 degrees for the half angle. It is only necessary that the element transducer sample the entire image area preferably without any nulls, since the detector response can always be compensated for in the final image. It is more important to have uniformity in the beam pattern from one element to the next than to have a higher pattern but with greater variance between elements.

The transduction efficiency of the element should be as high as possible as in any acoustic array. However, in this case the element transducer sample the entire image area preferably without any nulls, since the detector response can always be compensated for in the final image. It is more important to have uniformity in the beam pattern from one element to the next than to have a higher pattern but with greater variance between elements.

The configuration of the elements is shown in Figure 6. Here two possible configurations are shown, the left one uses a spherical surface radiator and shows the PVDF film pressed over the electrode posts. The posts are in turn connected to the MUX IC through a socket, left, or directly as shown on the right. The posts are made of brass and act as high impedance acoustic backings for the low impedance PVDF films. Although several such configurations were evaluated as 8 x 8 cells, the final 64 x 64 array was constructed using flat posts and IC sockets in order to simplify the PVDF bonding and to allow the IC's to be moved to other arrays. The 64 x 64 array was constructed using four blocks of 32 x 32 since 32 x 32 was the largest physical size that could be fabricated using available PVDF sheet and hydraulic press areas. The final array is shown in Figures 7, 8.

Comparison to Other Imaging Systems

The image reconstruction system described above has some similarity to acoustic holography [2] in that the fields are measured in a two dimensional plane. However, in this case the phase information acquired in the data collection is used directly in the reconstruction process. This system does not require an acoustic reference beam during the measuring process and consequently the high spatial frequency introduced by the reference beam is absent. Thus the data may be collected with more widely spaced transducers significantly reducing the physical implementation. In optical holography the high spatial frequency caused by the interference of the angled reference beam with the scattered waves does not cause any difficulty because the recording media, high resolution film, has a spatial resolution much greater than the spatial frequency of the interference pattern. In implementing an acoustical holographic system, the high spatial frequency must be sampled by a mechanically spaced transducer or finely spaced transducer array whose implementation is much more difficult except at very low frequencies (long wavelengths). In addition, the number of sampling points is determined by the extent of the scattered field divided by the sampling interval. Thus a holographic system requires a significantly larger number of sampling points due to the smaller sampling interval required by the high spatial frequencies created by the reference beam.

In phased array systems a number of array elements are excited in such a manner that signals arrive at a common focal point. In order to have only one focal point the array elements are closely spaced (of order \( \lambda / 2 \)) in order to eliminate grating lobes caused by the finite element spacing. In such systems spatial resolution is obtained by the use of short pulses and also by the size of the focal spot as determined by the number of array elements. In the reconstruction system described here the object is illuminated with a tone burst of sufficient duration to encompass the entire object but not so long as to introduce stray reflections from regions that are not of interest. The spacing of the transducer is determined by the spatial frequency of fields created by the interference of waves coming from different regions of the object. Since in our case the image is formed by a Fourier transform process the nature of the reconstruction is only dependent upon the sampling interval and number of sampling points chosen just as in any other Fourier transform operation. Since the sampling is discrete, the transforms become periodic and therefore the image is periodic and unless the sampling is done carefully the images will run together creating the well known aliasing effect. Thus sampling more closely in the measurement plane causes the periodicity of the image to decrease and increasing the number of sampling points better defines the image.

VI Experimental Results

A two dimensional array was fabricated using the switching and transduction system described above. Measured data for the array are shown in Figs. 9-11. In Fig. 6 the amplitude data for each element is shown in a grey scale display in order to test the element fallout pattern. Here each element was mechanically positioned
over a 0.5 inch diameter 5 MHz transducer located 18 inches away. The system was operated in the pulsed CW mode, the signal envelope digitized by an A/D converter and then sent to the display memory. In the display, the black regions represent zero signal conditions which are due to faulty MUX chips or wiring errors. The 1x8 drop-out regions are due to errors within the 8x8 cells and the 8x8 drop-outs are due to errors in wiring at the next higher (64 chip) decoding level. Since the chips are replaceable, the 1x8 and 8x8 drop-out regions may be repaired. The 1x1 or single element regions are either due to short-circuited transducer elements or bent pins where the IC's plug into the transducer socket. Single element drop-outs occurring in a random manner do not degrade image quality because of the nature of the Fourier transform process.

In Figs. 10 and 11 are shown the measured radiation pattern of the source transducer. Clearly visible is the side lobe structure in the linear amplitude plots. No correction was made for variations in element transfer functions although this would be required in an eventual imaging system.

VII Conclusions

An acoustic imaging system has been detailed and the means to implement the system using two dimensional arrays has been described. The two dimensional array has been constructed using PVDF piezoelectric films and CMOS analog switches for sequential accessing. The experimental results obtained to date on the 64x64 array clearly demonstrate the feasibility of the imaging implementation using two dimensional arrays. The imaging technique was previously demonstrated in relation to transducer characterization using a single mechanically scanned transducer rather than an array.

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IX References

Fig. 5. Schematic of overall multiplexing system. Dashed lines outline the 8 x 8 cell, next lower row of chips, the bottom 8 x 64 row, and finally the bottom row represents all other inactive chips.

Fig. 6. Array element construction showing high impedance electrode posts connected to IC socket or the IC directly as possible choices.

Fig. 7. View of array on acoustic side showing 64 x 64 elements in groups of 32 x 32. Imperfections are only in the regions between the electrode posts.

Fig. 8. Electronics side of the array showing the MUX integrated circuits.

Fig. 9. Fallout pattern of array, grey scale amplitude display. Black represents zero signal level.

Fig. 10. Radiation pattern of 0.5" dia. transducer; grey scale display.

Fig. 11. Plotted data of 32 x 32 region of radiation pattern of Fig.10.
Jerry Posakony (Session Chairman-Battelle Northwest): We have time for a couple of questions.

Gordon Kino (Stanford University): This is basically a holographic reconstruction system, as I understand it.

Ken Lakin: I don't use that terminology because it gets confused with holographic systems where you use an optical wave. I prefer to speak of it as a digital lens by implementing the acoustic lens mathematically.

Gordon Kino: The question I'm asking: doesn't it have the same problems in that you're using a single frequency. You're going to have the speckle problems, aren't you? You're going to have, essentially, range definition problems, too.

Ken Lakin: It has no more problems than a lens. It is a lens.

Gordon Kino: Fair enough.

Ken Lakin: What you get in the way of aliasing depends on how you do the reconstruction. I have been a little hesitant to charge off on the array processor because my current feeling is I don't want to do a 64 by 64, two-dimensional transforms. Instead, I want to take my 64 by 54 data and mesh it in a 128 square matrix. That helps reduce the alias problem.

Dale Collins (Holosonics, Inc.): I think what we have seen is the longitudinal resolution problem due to using coherent sound. When you use a lot of range and use coherent sound, your range resolution is very poor in a coherent system. I'm just saying, like Kino says, it's very difficult for range resolution in a coherent system.

Ken Lakin: Range resolution is dependent on how much angle you get in the lens, and right now I feel that the reconstruction algorithm is good for plus or minus 30 degrees.

Dale Collins: But you do compute your image in a certain plane?

Ken Lakin: Yes, we just take slices.

Dale Collins: If you had your range resolution (inaudible) you could make your algorithm.

Ken Lakin: The question of range resolution is one of the reasons I looked at the side looking imaging approach. There I could make out a vertical slice because I could get transverse interference patterns.

Jerry Posakony: One last question up here, please.

Paul Gammell (Jet Propulsion Lab): Do I understand what you're bringing out on the range resolution was the fact that the length of the burst which you're using to get the azimuthal resolution is sacrificing your time of flight information?

Dale Collins: That's true usually in the pulse echo system. We use a broadband system to obtain range resolution.

Paul Gammell: You can get a data trade-off -- there's mathematical limitations still. If you were to use something where there is some time coding. I'm thinking of specifically phase coded or pseudo-random techniques.

Dale Collins: Such coding will bring back broadband situations.

Ken Lakin: The spatial resolution here comes from the interference phenomenon and therefore I do not want the shortest possible pulse. If you have two objects, one behind each other, then you can see that if you're going to measure any sort of reasonable interference from that configuration, you have to be sufficiently off to the side in order to get a large enough angle to see where the interference starts. If the objects are side-by-side there is no problem and you get nice transverse resolution. That's why I was looking at an algorithm for side-looking imaging. Because, if you get off to the side, then you can see objects behind one another, and the interference pattern from those two sources can be easily measured off to the side.