Several factors affecting the ultrasonic determination of stress in bolts are examined which help to clarify existing problems with the interpretation of certain experiments. To begin with, the use of ultrasonic waves to determine stress inverts the results of experiments used to evaluate third-order elastic constants. Thus, the unambiguous determination of stress must be subject to the same conditions as the experimental measurement of higher-order moduli. For example, Thurston and Brugger's expression for the transit time in terms of the natural velocity and the unstressed length provides an alternative to the usual practice of relating the transit time to the stress dependent true velocity and the stress induced change in length. Using the natural velocity emphasizes the explicit stress dependence of the velocity and avoids the unnecessary corrections for changes in path length and density. Although temperature and dislocation mobility are closely controlled in third-order elastic constant determinations, these factors are rarely considered in ultrasonic stress measurements. It is shown that in steel, a stress of \( \sim 1 \text{OMP}a \) (\( \sim 1.5 \text{Ksi} \)) is equivalent to a \( 1^\circ \text{C} \) change in temperature. The possible role of defects or temperature in the relaxation phenomena observed in ultrasonic stress determinations is also examined. The effects of constrained thermal stress and unloaded bolt length on the so-called stress-acoustic constant are presented. The correct functional form of the stress and strain dependence of the sound velocity is shown to be crucial to the problem of thermally modified bolt load. The effect of unloaded bolt length is considered for the cases where the nut is stationary during loading and where it is tightened to produce the load. Finally, the difficulties caused by longitudinal wave mode conversion upon reflection off the bolt sides are examined.

THIRD-ORDER ELASTIC CONSTANTS

The variation in the elastic modulus with stress is very small. It is not measurable in a standard tensile test. The velocity of ultrasonic waves is coupled to the stress through the higher-order elastic coefficients. In part, the resonant frequency of a solid is changed by the stress the same way a violin string frequency is changed by stretching. Determining the stress from the change in resonant frequency or transit time and the known third-order moduli is the inverse of the usual procedure for measuring third-order moduli. For this reason, the technique is applicable only for elastic deformations.

NATURAL VELOCITY

The repetition frequency can be expressed as either the true velocity over instantaneous length or the natural velocity over original length. This is a direct analogy to expressing the load in a standard tensile test. The velocity of ultrasonic waves is coupled to the stress through the higher-order elastic coefficients. In part, the resonant frequency of a solid is changed by the stress the same way a violin string frequency is changed by stretching. Determining the stress from the change in resonant frequency or transit time and the known third-order moduli is the inverse of the usual procedure for measuring third-order moduli. For this reason, the technique is applicable only for elastic deformations.

STRESS ACOUSTIC CONSTANT

In resonant techniques, the sections of unstressed length in bolts must be treated as parts of a composite resonator. The resultant modifications to the stress acoustic constant are presented for the case of the nut stationary during loading and the case where the nut is turned to produce the load. In addition to a factor which represents the percent of the length that is stressed, there is a term which depends on the harmonic number. An expression for the stress acoustic constant for thermally induced loads in constrained bolts has yet to be developed.

SIDE WALL REFLECTIONS

The frequency spectra of bolts in tension contain nulls which do not track with the change in stress. It is shown that upon reflection off the bolt sides a mode converted transverse wave generates a series of longitudinal pulses. The nulls in the spectrum correspond to the time separation between the pulses, i.e., the time it takes the transverse wave to traverse the bolt from side to side. The spectrum for an initial transverse wave does not contain these nulls. This is because a longitudinal wave is not produced by the reflection at grazing incidence of a transverse wave.

ACKNOWLEDGEMENTS

Much of the work reported on here was done while the author was with the Instrument Research Division of the NASA Langley Research Center, Hampton, VA on an IPA Agreement from Pennsylvania State Univ.
Third-Order Elastic Constant, $C_{ijk}$, Experiments → Ultrasonic Stress Measurements

The change in the transit time, $\Delta t$, or its inverse, the repetition frequency, $\Delta f$, is recorded versus an applied stress, $\sigma$.

![Graph showing change in transit time and repetition frequency versus stress](image)

The third-order elastic constants are found by measuring the stress and the repetition frequency. By the inverse operation the stress is determined from the known third-order moduli and measurements of the repetition frequency. Although care is taken to eliminate the contributions from temperature changes and defect motion in finding $C_{ijk}$, this is not the case for ultrasonic stress measurements.

After rapid loading the repetition frequency is often found to relax asymptotically to a value which depends on the initial load. It has yet to be determined whether this is due to the frictional heat of loading or a thermoelastic effect or stress-induced interstitial ordering.

True Velocity, $V$ → Natural Velocity, $W$

The repetition frequency, $f$, can be expressed equivalently in terms of either the true velocity, $V$, over deformed length, $L$, or the natural velocity, $W$, over the unstressed length, $L_0$.

\[ f = \frac{V}{L} = \frac{W}{L_0} = \tilde{f} \]

\[ \Delta f = \Delta \left( \frac{V}{L} \right) = \frac{\Delta V}{L_0} + \frac{V_0}{L_0} \Delta \left( \frac{1}{L} \right) = \frac{V_0}{L_0} \frac{\Delta V}{L_0} + \frac{W_0}{W_0} \frac{\Delta W}{W_0} = \Delta \left( \frac{W}{L_0} \right) = \Delta f \]

\[ \Delta f = \frac{\Delta V}{V_0} \frac{\Delta L}{L_0} = \frac{\Delta W}{W_0} \]

The expression for the change in true velocity contains a term which exactly cancels the strain term. Expressing the change in repetition frequency in terms of the natural velocity, $W$, avoids the unnecessary corrections for changes in path length and density.

Stress Acoustic Constant, $K_\sigma$ / Temperature Acoustic Constant, $K_T$

The repetition frequency, $f$, is a relatively stronger function of temperature than it is of stress.

431 Stainless Steel*

\[
\begin{align*}
\Delta f & = f_0 \frac{40^\circ - 20^\circ}{42\text{MPa} - 21\text{MPa}} \\
& = 3 \times 10^{-3} \\
3 \times 10^{-3} & = \text{Temperature} \\
42\text{MPa} & = \text{Stress, slope -5 \times 10^{-4}/\text{C}} \\
6 \times 10^{-6} & = \text{Slope -1.4 \times 10^{-5}/\text{MPa}} (-1.4 \times 10^{-5}/\text{Ksi}) \\
\end{align*}
\]

The change in repetition frequency produced by a 1°C change in temperature is roughly the same as that produced by a 10MPa (1.5 Ksi) change in stress.


Effect of Unloaded Bolt Length on the Stress Acoustic Constant

In resonant techniques the stress acoustic constant is expressed in terms of the $m$th harmonic of the bolt, $\omega_m$.

\[
K_\sigma = \frac{\omega_m}{\sigma} \frac{d\omega}{d\sigma} = \frac{1}{V_0} \frac{dV}{d\sigma} - \frac{1}{E} = \frac{1}{W_0} \frac{dW}{d\sigma}
\]

If only a portion of the length is stressed, then the bolt must be treated as a composite resonator. The resonance condition on the system for the case where the nut is stationary during loading yields the effective stress acoustic constant, $K_\sigma$.
When the nut is tightened to produce the load, the velocity changes in the loaded section whereas the length change occurs in the unloaded section. Shown below is the resulting expression for the effective stress acoustic constant, \( K_\sigma \):

\[
K_\sigma = \frac{1}{\omega m} \frac{d\omega m}{d\omega} = \frac{1}{\omega m} \frac{d\omega m}{d\omega} \ln \left( \frac{\omega m + \omega m}{\omega m} \right)
\]

On the Stress Acoustic Constant for Thermal Loading

For thermal loading under constraint, the repetition frequency can be expressed as the stress and temperature dependent velocity over the constant length, \( L_0 \):

\[
f = \frac{V(\sigma, T)}{L_0}, \quad df = \frac{1}{L_0} \left[ \frac{\delta V}{\delta T} dT + \frac{\delta V}{\delta \sigma} d\sigma \right]
\]

The \( \frac{\delta V}{\delta T} \) term is straightforward. A difficulty arises with the \( \frac{\delta V}{\delta \sigma} \) term. Most researchers use the expressions of Hughes and Kelly\(^*\) which unfortunately incorporate the constitutive relation for unconstrained deformation and are therefore not applicable here. To date a self-consistent analysis for constrained thermal loading has not been published.

Effect of Side Wall Reflections in Measuring Bolt Tension

Reflections off the bolt sides can lead to inaccuracies in determining bolt tension.

Because of the conditions a longitudinal plane wave, $\ell_1$, must satisfy upon reflection off a boundary, a mode converted transverse wave, $t$, is generated in the bolt. Upon each reflection a portion of the transverse wave is reconverted back to a longitudinal wave giving rise to a series of longitudinal pulses traveling down the bolt. In low attenuation materials a way around this problem is to start with a transverse wave which upon reflection yields only the transverse wave.