Stationary-Phase Method vs. Pencil Method for the Modal Radiation of Guided Waves by Finite-Sized Sources in a Semi-Infinite Isotropic Plate

Mathilde Stévenin and Alain Lhémery, CEA LIST, Gif-sur-Yvette, F-91191, France

Modal decomposition of guided waves (GW) in isotropic plates over Lamb and shear horizontal modes is helpful to interpret signals measured in GW NDE methods. Simulation tools of GW NDE examinations are developed notably to provide help interpretation. Thus, models on which they rely can provide an even greater help if they predict the various phenomena involved in terms of modal amplitudes. Most GW sources being of finite size, diffraction effects occur. Plate-like structures to be tested being of finite size, edge reflection with mode-conversion occurs too. Two models are proposed for the fast prediction of how the two phenomena combine.

For both models, the field in a semi-infinite plate is given by the sum of the direct field and that resulting from reflections on plate edge. The expression [1] of modal 3D Green’s function is used for the direct field; in isotropic plate, this results in a modal series of cylindrical waves of amplitude decreasing as $1/\sqrt{r}$, ($r$, distance of a source point to a field point).

In the first model, each cylindrical wave is decomposed in the spatial Fourier domain as a spectrum of infinite plane GW. The reflection of a plane GW on the straight free edge of the plate is computed as in [2]. The inverse Fourier transform to get back to the spatial domain is calculated analytically by means of the stationary phase method, stationary phase path of a given mode with reflection being easily obtained. The total field is expressed by a convolution over the source surface of the Green’s function (direct and reflected terms) with source terms. Finally, this surface integral is calculated analytically thanks to a Fraunhofer-like approximation, shown in [3] to lead to accurate results for the direct field at a very low computational cost (analytic expressions for sources of standard geometry exerting either normal or tangential traction).

In the second model, expression of the direct field is reinterpreted in terms of propagation of modal infinitesimal pencils propagating along energy paths with a spreading factor deduced from the principle of energy conservation. Evolution of the pencil of a given mode is calculated by chaining propagation matrices and boundary interaction (reflection) matrices, the latter involving the same reflection coefficients as for the first model. From this, the pencil spreading at the calculation point is obtained. Applying it to the case of a straight edge, one obtains the same final expression for the whole field as that obtained with the first modeling approach.

In the first approach, assumption and approximation made are rigorously mastered but the final result is of restricted applicability (straight edge). The second modeling approach easily applies to more complex configurations, in particular, to configurations implying several reflections on possibly curved boundaries. Both models can include local calculation of reflection coefficients for different boundary conditions.

References: