Economics of cooperative farming: objectives and optima in Hungary

Ferenc Fekete
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Economics of cooperative farming — objectives and optima
in Hungary

by

Fekete Ferenc

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Signature was redacted for privacy.

For the Graduate College

Iowa State University
Of Science and Technology
Ames, Iowa

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CHAPTER I. INTRODUCTION

Centuries ago scientists foresaw the age when man would free himself from his Promethean chains of Nature-bound constraints in pursuing his own and his fellowman's happiness. Our present scientific and technical revolution has brought science into the range of the most effective forces of production. The formula "science = production force" applies also to the social sciences whose explorations of human relationships and drives have reached previously unsuspected depths. Objectives, such as higher living standards and full employment, economic growth and stability, social equity and security, have both called for and provided a basis for the exploitation of possibilities offered by the natural and technical sciences.

In today's agriculture, age-old traditions are in the process of disintegration, but the heredity of a century (or that of even a millennium as in my country, Hungary) does not get dissolved without defending itself. Technical progress and social restructuration, the emergence of new scales of values and preferences, the adjustment of the rural communities to their new tasks and conditions — all these have transformed farm operations and farming techniques. But agriculture, even under its revolutionized surface, still hides deep, almost untouched layers. If economists and agriculturists are perplexed by the multitude and variety of the visible farm problems, there exist many others about which they can only guess, which they must follow up. In formulating and solving these problems, agricultural economists have professional
tasks, as pointed out by Heady (42, p. 10):

(1) helping farmers and farm people to attain their stated, socially feasible objectives, and

(2) facilitating the most efficient use of agricultural resources from the standpoint of the national economy.

Farmers' objectives originated in their motivations, preferences are embedded in the system of social and economic interests. In most modern societies, interests have a threefold vertical structure. There is individual, personal interest at the one end. The interest of the community as a whole which in practice appears as a state interest represented by the central political organisms (Parliament, i.e. the National Assembly in Hungary) is at other end. Between the two ends there exist the interests of groups or collectives such as interests of local communities and interests of firms owned by more than one household.¹

Today the middle of the interest hierarchy² is crucially important in Hungarian agriculture where large-scale cooperative farms represent the predominant organizational form and even larger-scale state farms are the second most important. The interests of collectives used to be considered obscure and shapeless, therefore insufficient for properly

¹In Holdren's (50) terminology, if any of the "multi-personality" groups behaved as though it were a "single-headed organism," then it shall be considered as a "basic decision unit" having "a single coherent structure" of objectives and interests. Holdren (50) also points out that the definition of "the household" as the individual consuming unit ignores its multi-person nature in almost all cases.

²If we call the two former zones of interests micro-interests and macro-interests, respectively, then meso-interest would be a practical term for the latter zone of interests.
transmitting society's interests to the individuals and vice versa. In
the new economic mechanism introduced in 1968, the vertical structure
has been modified to the extent that the interests of collectives (firms,
cooperatives, local communities, etc.) are far more apparent. Farmers'
interests, however, have remained heavily-weighted and tangible at the
other two levels as well.

In the light of Hungary's recent economic problems, many traditional
concepts appear obsolete; at the same time new ones are emerging. The
process of revaluating and re-ranking the tasks and methods of economic
science has begun but only with the theoretical preparation of the new
economic mechanism. However, one must not neglect to ask whether the old
barrels are still able to hold new wine. In quite a few fields it has
been found that the new wine of economic theory should not be poured
into the old goatskins or wooden barrels of methodology. Thus, special
attention and space have been devoted to utilizing economic functions
with the aid of mathematical methods and programming models.

Instead of futile ideological arguments, most of Hungary's compe-
tent economists fully agree with the first Nobel Laureate economist,
Ragnar Frisch (53, pp. 208, 210), who stated while visiting Hungary:

"It is my deep conviction that mathematical economics and,
particularly, the various programming procedures will be-
come absolutely indispensable tools of a modern economy ... 
especially in the socialist countries characterized by
management 'from above.' This obtains even greater empha-

1Hungarian economists' general response to the "capitalist" origin
of these techniques seems much like that of Anderson's "Ugly Duckling":
It does not matter where one is born as long as one comes from a swan's
egg.

This reaction is not equivalent to overlooking the places where
there is too much smoke for there to be no fire."
sis because of the reform of economic control and management since in almost all socialist countries the idea of management through incentives instead of control by commands is pushed into the fore. ... To control with the aid of incentives requires more mathematics than the more primitive system of commands.1

Instead of direct plan directives, the new economic mechanism operates by changing

(1) the budget constraints of firms and individual economic agents (consumers, managers, workers, cooperative members); and/or

(2) the weights attached to the activities of economic units; and/or

(3) the general environment for economic units in such a way that their decisions are affected in a manner consistent with social goals.2

As a result of numerous research projects during the late fifties, the mathematical methods of national economic planning were introduced in Hungary some years later. In the beginning, input-output models were in the foreground, but not long afterwards research studies based on mathematical programming started (among these, Kornai's two-level programming model was of considerable professional interest abroad).

In each part, this study applies the so-called quantitative analysis which — as demonstrated by Fox, Sengupta and Thorbecke (31, p. 11) — may be operationally divided still further into three interrelated parts:

(a) Characterizations of the economic problem: specifications of the objective (preference) function, the quantitative model and the

1 At the same time Frisch (53, pp. 209-210) repeatedly protested against abstract formal mathematization which he called "playometrics."

2 Similar phenomena are frequently called "spontaneous field control" in social psychology.
constraints or boundary conditions.

(b) The selection problem: classification of variables by their properties, such as direct or indirect (or lack of) controllability.

(c) The steering problem: derivation of optimum decision rules and optimal decision-making procedures under changing conditions, including new information and learning processes.  

The "box of analytical tools" in this study contains optimization (maximization or minimization) processes which belong to the domain of calculus or mathematical programming.  

With respect to optimization, Tinbergen (79, p. 260), the other co-winner of the first Nobel Prize in economics, made a relevant remark: "In essence any optimum problem is a problem of welfare economics." In such a problem an objective function is given which has to be maximized under a number of constraints. Among these, institutional constraints may also be applied.

As the title demonstrates, "raw materials" (statistical data, direct personal experience and "indirect learning processes") for "manufacturing" this study originated in Hungary while the tools and techniques were "made at Iowa State" in almost every case. In spite of the geographically limited scope of the analyzed data, this study definitely aims at a feasible level of theoretical generalization and professional synthesis.

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1 If a decision unit learns either new goals or new ways of achieving goals, its preferences expressed in its objective function change (50).

2 In the methodological aspects, the author of this study is a "tool-user" rather than a member of the distinguished group of "tool-makers."
Applying international comparisons in Part I and searching for formulations and conclusions of wider technical interest in all three parts hopefully will contribute to this effort. It is one of the deficiencies of economics in Hungary that up to the present no detailed description of the practical functioning of the national economy has been published such as that by Samuelson (70) on the US economy, or on the French economy by Pierre Bleton (5) and on the German economy by Heinz Pentzlin (67) or Walter Wannemacher (82).

Finally, the aspirant hopes that by the end of this paper the readers will at least partially agree with John Knapp (58, p. 95), who — when reviewing a book written by an American author of Hungarian origin about Hungary’s economic planning (4) — politely noted in the Journal of Political Economy: "It may be true that economic science has now reached a stage of development in which, in order to be an economist, it is no longer sufficient to be a Hungarian. But it is still a very great advantage."¹

Objectives

The major objectives of this study have been to analyse cooperative farming — its historical development, and socio-economic determinants — and to derive optimum solutions for this specific form of farm organization in harmony with cooperative economic goals and capacities. This study is concerned with the objectives and interests of the individual cooperative members, those of the collective enterprise, and the ob-

¹Knapp listed "originality of approach" ..., "the heroic ambition to be all-embracing" ..., "the superficially cool elegance of treatment" as "the hallmarks of the genuine Hungarian at work."
jectives of the cooperative organization (including members' homeplot farms and cooperative households).

Part I sets the stage by describing the transformation and the recent organization of Hungarian agriculture. Part II scrutinizes the socioeconomic characteristics of farmers' cooperatives and develops various models of cooperative farming. Part III applies the optimization technique of marginal analysis and mathematical programming in accordance with alternative objectives of the large-scale cooperative enterprises as a specific form of business firm. This part also views the economic objectives of the cooperative members as individuals and formulates a unified (integrated) objective function of the collective enterprise and its members in terms of a hypothesized cooperative welfare function. Finally, the major conclusions of the study are presented mostly from a methodological point of view.

1 Hungarian agricultural economists' sharpest dispute (see 1, p. 98), not yet concluded, centers on the collective interests of large cooperative enterprises. Some of the economists are of the opinion that the main interest of the cooperatives lies in maximizing gross income per member. Another group makes the purposeful distinction between a cooperative as a firm and its members as individuals. According to this viewpoint, a cooperative as a collective enterprise is interested in maximizing net income (profits); the cooperative members as individuals are, however, interested in remuneration (wages) for work performed on their own large-scale cooperative farms and in a direct share in the latter's profit.

Neither of the two groups has integrated into the framework of its argument the specific role of interests and objectives associated with the cooperative members' homeplot farming.

The author believes that the lack of mutual understanding is due mainly to the fact that mathematical models and optimization techniques had not been incorporated in this debate.
In this organization, the recent study tries to provide an answer for some economic questions which are current in Hungary. These questions have motivated this study.
PART I.

TRANSFORMATION AND RECENT ORGANIZATION OF HUNGARIAN AGRICULTURE
CHAPTER II. THE POSTWAR LAND REFORM

A quarter of a century ago the socio-economic map of agriculture was redrawn in Eastern Europe. The land of enormous latifundia was distributed among a large number of destitute peasants and owners of small holdings. As one of the United Nations' publications (62, p. 72) states: "The change has been most complete in Hungary, affecting nearly half the land."

What was the nature of this reform? What were its effects on the economic progress of the country? This chapter attempts to consider and answer these questions.

Historical and Economic Background

When World War II came to an end, Hungary was far below the European average level of economic development. Thus in 1945 the people and the government of the country were faced with the task of creating a society of greater equality and abundance. This they did by launching a vigorous program of economic development.

The economic development of the country necessarily started with a phase of reconstruction. In consequence of World War II, material losses amounted to a larger sum than seven years' aggregate national income. About 400,000 people had died. More than half of the livestock and more

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1 The original version of the first part of this chapter was written for Professor John F. Timmons' "Seminar on Agrarian Reform and Economic Development" at Iowa State University in 1969.
than one-third of the machine and tool equipment were destroyed by the war.

The basic requirement for economic development was the establishing of socio-economic conditions which would open the road to reconstruction and economic growth. As the very first step it was necessary to evolve new social and economic measures permitting the abolition of the feudal features of land ownership.

Up to 1945 Hungarian agriculture was, in the main, characterized by extensive farming on large estates. Technical backwardness, low livestock density and the deterioration of soil productivity caused by insufficient fertilization resulted in low yields. All this bore a close relationship to the large unemployment of the rural population and the extremely low level of labor efficiency.

About half the population made its living by agriculture, and only 10 per cent worked in the manufacturing industries prior to 1938. This year (which was the last normal prewar year in Eastern Europe) agriculture accounted for 69.9 per cent of Hungary’s exports (52, p. 353).

Data show the strategic importance of agriculture in the problematic situation of Hungary at the end of World War II.

Pre-Reform Conditions: Problematic Gaps between Norms and Existing Situations

The problematic situation of Hungarian agriculture had been reflected in a gap between the actual and the desired situation. Timmons (78, p. 9) states "The gap between consequences and goal is the problematic situation within which the problem for study is delimited."
The new government of the agrarian country could not accomplish political stability without the help of the peasantry. The key to the support of land-hungry peasants was the fulfilling of their desires which coincided with the norms and requirements of overall progress.

Thus we arrive at a quantified type of problematic gap between norm(s) and the actual situation in the field of land property distribution.

Table 2.1. Percentage distribution of land property in some Eastern European countries prior to World War II

<table>
<thead>
<tr>
<th>Country</th>
<th>Size of property in hectares</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>below 3</td>
</tr>
<tr>
<td>Rumania</td>
<td>12.8</td>
</tr>
<tr>
<td>Jugoslavia</td>
<td>6.5</td>
</tr>
<tr>
<td>Bulgaria (norm)</td>
<td>5.3</td>
</tr>
<tr>
<td>Hungary</td>
<td>10.9</td>
</tr>
<tr>
<td>Problematic gap</td>
<td>5.6</td>
</tr>
</tbody>
</table>

*Source: (75, p. 24).

The last row in Table 2.1 shows the problematic gap between Bulgaria and Hungary. In this comparison Bulgaria is taken as a norm of equitable land property distribution which also reflects wealth and income distribution. If equity, i.e. relatively even wealth and income distribution is considered as "the standard of the norm," then a quantified problematic gap will show up between Bulgaria (as a norm) and Hungary in this respect.

The definition of problematic gap does not restrain the applied norm
to a condition existing elsewhere — although the latter may be considered as a desired situation or norm. For this reason, the norm used in Table 2.1 and later on may be questioned on the valid ground that the chosen "bench mark" for comparison may itself not be optimal. By all means, Table 2.1 and the following tables express problematic gap of some sort which may serve well the given analysis.

In Hungary, 72.4 per cent of the total number of landowners, whose holdings all together made up only 10.1 per cent of the countryside, held very small plots (below 5 cadaster holds, i.e. 2.8 hectares); 21.4 per cent of the landowners, whose property made up 21.8 per cent of all land, owned small and middle-sized holdings. Large peasant farms (of 20 to 100 cad. holds, i.e. 11.4 to 57 hectares) were in the hands of 5.4 per cent of the landowners, and occupied about 20 per cent of the countryside. Middle and large estates (over 57 hectares), whose owners constituted only 0.8 per cent of the landowners, held 48.1 per cent of the land. In round figures, one per cent of the landowners held half of the land.

This extremely disproportionate distribution of land property was a consequence of the surviving system of large feudal estates, in spite of the fact that serfdom had been abolished a century before. At that time, the emancipation of serfs, however, had only resulted in the peasants’ acquiring about 30 per cent of the land, while 70 per cent remained in the possession of the big landowners. In the course of a whole century the latter proportion became only to 48.1 per cent of the total area.

Land ownership concentration was the strategic issue of the wide differences in social status. Farm workers with no land or with tiny parcels were more numerous in Hungary than in other near-by countries.
Another type of measurable problematic gap is given here.

Table 2.2. The percentage of farm workers in some Eastern European countries before World War II

<table>
<thead>
<tr>
<th>Country</th>
<th>The percentage of landless wage-earners in the whole farm labor force</th>
<th>The number of wage-earners with land below 0.57 ha. as percentage of the whole employment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Romania</td>
<td>8.9</td>
<td>7.6</td>
</tr>
<tr>
<td>Jugoslavia</td>
<td>9.3</td>
<td>6.6</td>
</tr>
<tr>
<td>Bulgaria (norm)</td>
<td>5.1</td>
<td>4.4</td>
</tr>
<tr>
<td>Hungary</td>
<td>38.5</td>
<td>27.4</td>
</tr>
<tr>
<td>Problematic gap</td>
<td>33.4</td>
<td>23.0</td>
</tr>
</tbody>
</table>

*Source: (75, p. 24).

Table 2.2 shows another important aspect of wealth and income distribution thus it quantifies a new problematic gap by using the same country in terms of standard as Table 2.1.

The problematic gap shown in Table 2.2 is the difference between the existing situation of Hungary and the Bulgarian data considered as norm.

In consequence of the serious inequalities of land distribution and chronic unemployment in agriculture, some 2.1 million Hungarian citizens emigrated overseas.

Another comparison is given if one looks at farm employment in proportion to the total labor force employed.
Table 2.3. Agricultural employment in some European countries as a percentage of total employment

<table>
<thead>
<tr>
<th>Country</th>
<th>Year(s)</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Switzerland</td>
<td>1930</td>
<td>21.2</td>
</tr>
<tr>
<td>France</td>
<td>1931</td>
<td>35.6</td>
</tr>
<tr>
<td>Czechoslovakia (norm)</td>
<td>1934–1938</td>
<td>37.9</td>
</tr>
<tr>
<td>Hungary</td>
<td>1934–1938</td>
<td>49.7</td>
</tr>
<tr>
<td>Problematic gap</td>
<td>--</td>
<td>11.8</td>
</tr>
</tbody>
</table>

\[a\text{Source: (25, p. 33 and 25, p. 24).}\]

Here the level of industrial development is considered as the standard of comparison. A lower proportion of farm employment with respect to the total labor force employed was the sign of a more developed national economy as well as that of better income-earning possibilities and living conditions in Czechoslovakia. Referring to this country as the norm a comparison with Hungary will show a problematic gap of 11.8 per cent point in Table 2.3.

In order to find still another type of quantified gap one compares a norm with the existing level of productivity. The problematic gap was wide and the productivity of Hungarian agriculture was very low mainly in the production of so-called intensive crops.

The "standard of the norm" applied in Table 2.4, i.e. average crop yields are of crucial nature in Hungary where the area of arable land cannot be expanded and population density is extremely high. Higher crop
yields were usually associated with larger farm income and with the larger "food-providing capacity" of a land unit. These phenomena stress the importance of the problematic gap between Hungary and Denmark or Czechoslovakia which is quantified in Table 2.4.

Table 2.4. Average yields of selected European countries in years 1934-1938 (tons per hectare)

<table>
<thead>
<tr>
<th>Country</th>
<th>Wheat</th>
<th>Corn</th>
<th>Sugar beet</th>
<th>Potatoes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Denmark</td>
<td>3.04</td>
<td></td>
<td>36.2</td>
<td>17.2</td>
</tr>
<tr>
<td>France</td>
<td>1.56</td>
<td>1.58</td>
<td>27.6</td>
<td>11.2</td>
</tr>
<tr>
<td>Czechoslovakia (norm)</td>
<td>1.71</td>
<td>2.11</td>
<td>28.6</td>
<td>13.5</td>
</tr>
<tr>
<td>Hungary</td>
<td>1.37</td>
<td>1.98</td>
<td>20.6</td>
<td>7.3</td>
</tr>
<tr>
<td>Problematic gap^b</td>
<td>1.67</td>
<td></td>
<td>15.6</td>
<td>9.9</td>
</tr>
<tr>
<td>Problematic gap^c</td>
<td>.34</td>
<td>.13</td>
<td>8.0</td>
<td>6.2</td>
</tr>
</tbody>
</table>

^a Source: (25, p. 31).
^b Between Denmark and Hungary.
^c Between Czechoslovakia and Hungary.

Hungary's agricultural economy was of an extensive nature. Cereals (excluding corn) were grown on 52.4 per cent of the entire ploughland. About 70 per cent of the agricultural exports was made up of commodities produced by extensive farming.

Finally one can find a problematic gap between the past agricultural policies of the neighboring countries and that of Hungary. After World War I in Czechoslovakia, Jugoslavia, and Rumania, a large number of latifundia was distributed among peasants. But the ruling classes of Hungary
did not follow this path. Looking at the wide problematic gaps one has to agree with the conclusion of the previously cited UN publication (62, p. 71): "The social effects of land reforms after World War I in Eastern Europe were undoubtedly beneficial ... by comparison with conditions in Hungary, where no significant reform took place till 1945."

The Diagnosis of Failure and Success Elements

In the diagnostic phase, first the failure elements are investigated which were responsible for gaps characterized in the previous section. Then the success elements are traced, both the actual and potential ones.

Large estates with extensive cultivation constituted the crucial failure element in Hungarian agriculture. The effect of this organizational form (called latifundia) distorted the agrarian structure.

In pre-war Hungary the existence of large feudal property ownership did not result in any of the advantages of large-scale operations or investments. The level of productive efficiency was very low. Agricultural production was not adjusted to the demand for food of high nutritional value. Half a million workers, i.e. one-fourth of the total agricultural labor force, were permanently unemployed.

In the latifundia agriculture of Hungary, absentee ownership was accompanied by the steady flight of capital out of farming, which further hindered technological progress. An enormous share of the landlords' income usually found its way into conspicuous consumption. The "objective functions" of the absentee owners of latifundia did not stimulate farm reinvestment out of the fabulous land rent since operating
capital and the low-level investment was provided by the tenant-entrepreneurs who also acted as managers hiring workers.

In the shade of large estates, land became relatively more expensive for small-scale farming. Mostly because of the different demand conditions, the land price per hectare was twice as much for land units up to 3 hectares as for estates over 570 hectares in years prior to World War II.

Large unemployment, low product prices and high land prices reduced the living standard of peasants, and denied them the opportunity to advance. At the same time, Hungary experienced the prevalence of farms which were too large to cultivate intensively and of farms which were too small to operate efficiently.

In pre-war Hungarian agriculture the only actual success element was the relatively intensive cultivation on small size farms. As Fazekas (25, p. 36) calculated, the agricultural value product of one hectare of cultivated land was 78 per cent greater on peasant owned land up to 15 hectares than on large farms whose land area surpassed 115 hectares. This great difference suggests that the level and composition of input, namely the capital-labor-land mix, as well as the output-mix were far from being optimal or efficient on most large farms. The much poorer performance of larger farms was also due to the conservative business-like management and feudalistic wage system.

In such a situation when the latifundia system was the major failure element and peasant farming was relatively more efficient, it seemed that the redistribution of land for smaller-scale operations would be beneficial to the farm population and beneficial also to the people in general. In other Eastern European countries (Czechoslovakia, Bulgaria,
Poland), employment conditions improved and agricultural production increased and grew more diversified after the land reforms following World War I.

The Land Reform Decree of 1945

Land reform appeared as a historical imperative for Hungary in the spring of 1945. Traditions, economic and social environment, conditioned, of course, its significance as well as its take-off. But as everywhere else, at the heart of the problems of land reform lay the question of who owned the land. In common with their fellows abroad, Hungarian peasants were emotionally tied to the land which they tilled. Doreen Warriner's metaphor (83, p. 238) describes Hungary's actual situation at that time: "the economic sails are filled by a political wind."

On March 18, 1945, the Land Reform Decree (64) came out with a modest title: No. 600/1945. Decree on the Abolition of the Latifundia System and on the Land Grant for Farm People.

The First Article (64, p. 9) summarizes the main objectives as follows: "To abolish the feudal system of latifundia in order to guarantee the democratic transformation and future development of the country." In words of the First Article: The Land Reform opened the road to the political, social, economic, and intellectual advance for the Hungarian peasantry oppressed for centuries.

In its Articles 4-24 the Land Reform Decree fixed the maximum size of holdings which varied basically according to their owners' occupations. An area of land equivalent to the maximum-size of holding was
kept for the owner.  

The maximum size of holdings became 114 hectares for farmers and 57 hectares for persons having non-agricultural occupations. The Decree also fixed the maximum size of granted land units to 15 cadaster holds (9.55 hectares).

The Decree defined the scope of eligibility in its Articles 35-38. Farm hands (permanent workers on latifundia), agricultural workers, and dwarf holders (with land property smaller than 2.85 hectares) became eligible for land grants. They were declared the owners of their granted lands.

According to Article 36, preference rights for obtaining land grants shall accrue to those: (1) who gained distinction or died in the fight against German fascism and its Hungarian supporters; (2) who performed 1945 farm work, setting a good example; (3) who supported three or more children in his household.

The Decree states that trained specialists like agronomists and managers who lost their jobs on the expropriated large estates were also eligible for land grants. Where sufficient land was available, forest workers and rural craftsmen also obtained eligibility.

The established maximum size of a land grant was applied in those few areas where there was enough land. Each family was allotted 9.55 hectares creating farming units equivalent to the size of a contemporary

_1In this respect the Decree made two exceptions: 1. The land property of "an active supporter of German fascism" was confiscated regardless of its size. 2. The whole area of large estates surpassing 570 hectares was expropriated with compensation._
small holding. Where land did not suffice to grant this unit to each family, the size of the grants was reduced. Eligible peasants insufficiently provided for preserved their rights for new grants in other regions where available lands remained.

The Decree decided on compensation for expropriated lands and also on payments for land grants. Article 39 (64, p. 97) states: "The owners of expropriated lands will be compensated. The compensation will be paid by the government in harmony with its financial capacity."

According to the Decree, the benefited peasants would have to pay an amount — either in cash or in kind — determined by the cadastral value of the granted land (Article 40). The declared price of granted lands was only a small fraction of its actual value. In the author's calculation, the price as required payment for 1 hectare of granted land was even less than the value of a crop for an average year on the same land.¹

The 1945 Land Reform expropriated altogether 3.2 million hectares of land, i.e. roughly one-third (34.6 per cent) of the territory of the

¹According to Article 40 an amount which is twenty times as much as the net value in the cadaster records had to be paid as the price of granted lands. The net value originally expressed in Crowns (former monetary unit) is to be transferred by multiplier 1.16 in terms of Pengoes (the contemporary monetary unit). The same Article states that payments in kind (wheat) will be counted at a price of 400.00 Pengoes per tons.

The average net value of Hungarian lands is 20 Crowns per hectare in the cadaster. Following the calculation schedule of the preceding paragraph one arrives at a "real price" of 1.16 tons of wheat (20X20 = 400; 400 X 1.16 = 464; 464; 400 = 1.16). Table 2.4 shows that average yield of wheat amounted to 1.37 tons in Hungary during the pre-war period.
country; 75,500 holdings, among them 1,069 "mammoth" latifundia, were expropriated. Of the expropriated land, roughly 60 per cent (3.2 million cad. holds) was granted to the applicants. The remaining area consisted of state-owned forests and uncultivable land. In short, the 1945 Land Reform Decree resulted in new ownership for one of every three hectares of land in Hungary.

Granting land to 642,342 applicants, the land reform gave rise to 400,000 new peasant holdings. The average size of land grants was 2.8 hectares (per benefited family).  

The 1945 land reform meant much more than a simple shift in land tenure rights. It meant the revolutionary abolition of feudal relationships by way of radical changes in the structure of land title and in the status of the working peasantry.

The 1945 Land Reform as Remedial Action

In practice, the 1945 land reform was a remedial action. An analytical evaluation can point out its consequences in terms of new success and failure elements.

The 1945 land reform involved the actual division of large estates into small holdings. Together with the transfer of land ownership it made a substantial change in the scale of farming operations.

Very few communities were not involved in the land reform, and none existed in which inhabitants were not given land. The radical change in

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1 The whole payment was due in equal annual installments during a period of 10 or 20 years. Landless workers and farm hands were given twice as long terms as the benefited dwarf holders and other persons. Only 10 per cent of the declared "price" was required as down payment in 1945.
the distribution of land property is reflected in Table 2.5.

Table 2.5. Distribution of land property before and after the reform

<table>
<thead>
<tr>
<th>Size of property in hectares</th>
<th>Percentage distribution of number of all farms</th>
<th>ploughland</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1935 1949</td>
<td>1935 1949</td>
</tr>
<tr>
<td>Below 0.57</td>
<td>17.6 13.2</td>
<td>1.2 1.4</td>
</tr>
<tr>
<td>0.57 - 2.87</td>
<td>45.6 40.1</td>
<td>11.1 17.9</td>
</tr>
<tr>
<td>2.87 - 5.75</td>
<td>16.7 27.6</td>
<td>12.0 30.2</td>
</tr>
<tr>
<td>5.75 - 11.50</td>
<td>11.8 14.5</td>
<td>16.6 29.3</td>
</tr>
<tr>
<td>11.50 - 57.50</td>
<td>7.3  4.4</td>
<td>24.4 16.7</td>
</tr>
<tr>
<td>57.50 - 575</td>
<td>0.9  0.2</td>
<td>14.5 4.5</td>
</tr>
<tr>
<td>Above 575</td>
<td>0.1  0</td>
<td>20.2 0</td>
</tr>
<tr>
<td>Total</td>
<td>100.0 100.0</td>
<td>100.0 100.0</td>
</tr>
</tbody>
</table>

^Source: (52, p. 413).

The dominant, almost exclusive, type of farm became the small and medium-sized traditional peasant farm based on family labor. In spite of the somewhat different scales, comparison between Tables 2.5 and 2.1 bears witness to the filling of the most important problematic gap.

The implementation of the land reform lowered the proportion of landless workers within the whole farm labor force from 38.5 per cent (in 1930) to 13.2 per cent (in 1949). In comparison with Table 2.2 the latter data give evidence of the substantial reduction of problematic gap.

The four years after 1945 saw the development of a rhythm never
before experienced in Hungary. By 1949 livestock generally had reached 
the prewar level. The cattle stock of the country had deteriorated in 
quality and diminished in numbers to less than half its former size 
during World War II. By 1949, however, it had regained the peace-time 
level. During the war, over 80 per cent of the sheep stock was destroyed, 
and Hungarian sheep breeding thus had virtually to be started anew.

The structure of field crop production has undergone significant 
changes. The acreage of bread grains (previously the main export crops) 
has considerably diminished. In addition to corn, the area under sugar 
beet and oil plants has been expanded many times. In 1949 the territorial 
ratio of vegetables and legumes as well as the whole irrigated area was 
about double the pre-war figure. Thus the structure of field crop pro-
duction has developed in an essentially more intensive direction. Idle 
land was reduced to 1.6 per cent of the whole ploughland in 1949. This 
percentage had been 2.5 in years 1931-1940, and 13.9 in 1945.

As a result of the land reform the gap regards lower land use and 
the so-called intensity gap disappeared in most fields. (The level of 
productivity will be dealt with shortly.)

In real life the elimination or reduction of problematic gaps 
reflects the positive consequences of the 1945 land reform on economic 
growth.

Signals of a New Problematic Situation

The land reform changed the type of farm organization and created 
new incentives in the form of peasant ownership. The extent to which 
this incentive could be effective was, however, limited by the general
economic conditions.

In most parts of the country there was insufficient land. Thus peasant farms generally remained small. The production means were also insufficient, and technology remained primitive. The scope for possible advance through the incentive of peasant ownership in these conditions is necessarily narrow. Many disabilities of too small-scale production, "the ill effects of minifundia" (62, p. 10) started to show up a couple of years after the land reform.

On most farms the land base was very small (as shown in Table 2.5). The abolition of large estates provided only little capital for the newly created farms. The equipment granted per farm had an equivalent value of 3.5 tons of wheat (25, p. 44).

Although to have approached the pre-war level might be considered a significant success, the technological backwardness of Hungarian agriculture nevertheless persisted, the basic obstacle to further development being precisely the small-scale character of production. Over one million of Hungary's two million farmers still had fewer than three hectares of land.

The Hungarian land reform affected directly the input combination within farms and the resource allocation among farms. The creation of small farms resulted in multiple needs for capital equipment. Most forms of this equipment were used only within the boundary of a single farm. As Heady (42, p. 163 and p. 149) demonstrated, areas of irrational resource combinations are inevitable under the conditions of small-scale production.
Fig. 2.1. Isoquant exhibiting some areas of irrational resource combination of small farms

In ranges Ia and bQ of the above isoquant, both capital and labor could be used in smaller quantities without a decrease in the aggregate output. Small farms usually apply either more labor or more capital, or more of both, than the upper limit of the rational use (OC or OL respectively) in order to produce a given output. This point was very relevant in Hungary, where capital was the limiting factor of production, and at that time quite a few people believed that the land reform had created almost unlimited substitutibility between labor and capital. Another form of isoquant depicting small-scale production may provide a direct answer to the latter misbelief.

In Figure 2.2 Q_1Q'_1 and Q_2Q'_2 are isoquants indicating combinations of labor and capital (and all nonlabor) inputs which produce the specified levels of output (Q_2 > Q_1).
Fig. 2.2. Isoquant with limited substitutibility between labor and capital

If the amount of capital available is OC\textsuperscript{1}, then at least OL\textsubscript{1} amount of labor is necessary to produce Q\textsubscript{1}. Given the form of isoquant, neither the capital required will decrease nor the level of output will increase as labor input rises from OL\textsubscript{1} to OL\textsubscript{2} or even to OL\textsubscript{3}. Labor input must be less than OL\textsubscript{1} (like OL\textsuperscript{*}) and at the same time capital input shall be greater than OC\textsubscript{1} (like OC\textsuperscript{*}) when substitutibility between labor and capital exists along isoquant Q\textsubscript{1}Q\textsubscript{2}.

Labor input OL\textsubscript{1} is sufficient, on the other hand, for producing a larger output (Q\textsubscript{2}) if capital input is at least OC\textsubscript{2}. The marginal product of labor (MPP\textsubscript{L}) is zero in ranges aQ\textsubscript{1} and bQ\textsubscript{2}, and it indicates the lack of substitutibility, i.e. zero value for the rate of substitution (RTS\textsubscript{L} for C = rate of technical substitution of labor for capital = MPP\textsubscript{L}/MPP\textsubscript{C} = 0).

The intensity of production and average crop yields are of great importance in Hungary where population density is very high (107 persons
The density of the agrarian population is also high: 4.6 hectares of cultivated land per person engaged in agriculture. But average crop yields (per hectare) increased too slowly after the 1945 land reform. In 1948 the average crop yields were still lower than a decade before.

Table 2.6. Average crop yields in Hungary before and after World War II (tons per hectare)

<table>
<thead>
<tr>
<th>Crop</th>
<th>1938</th>
<th>1948</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheat</td>
<td>1.62</td>
<td>1.12</td>
</tr>
<tr>
<td>Corn</td>
<td>2.21</td>
<td>2.09</td>
</tr>
<tr>
<td>Sugar Beet</td>
<td>21.42</td>
<td>15.30</td>
</tr>
<tr>
<td>Potatoes</td>
<td>7.19</td>
<td>7.48</td>
</tr>
</tbody>
</table>

*Source: (25, p. 46).*

Comparing Tables 2.6 and 2.4 shows that the problematic gap did not cease; on the contrary, it widened concerning the most important crops. The uneconomic small size of holdings did not permit improvement in methods of cultivation.

The final economic lesson of the 1945 Hungarian land reform was much like the evaluation of the United Nations' publication (62, pp. 71-72) about the former land reforms in this geographical area: "The Eastern European experience in this period (after World War I) showed that a change in the scale of farm operation, if it is to be successful in raising the standard of living and promoting development, must be followed by a change in the type of production; and this change will not take
place simply as a result of peasant ownership ... It showed also that land reform ... is not a sufficient remedy of rural overpopulation."

The lesson of the 1945 remedial action is that land reform cannot be considered separately either from the whole farm organization or from the overall economic development. In the light of the Hungarian experience, Gittinger (35, p. 241) is inevitably right in saying "To be most effective, agrarian reform must be continuous; one major reform ... may only succeed in starting a cycle that leads to future need for another major reform."

The Hungarian peasantry had not reached port and had not arrived at the Promised Land by the sailboat of the 1945 land reform.
CHAPTER III. THE SOCIALIST TRANSFORMATION OF HUNGARIAN AGRICULTURE

During the past twenty-five years two radical changes shaped the structure of Hungarian agriculture: the land reform in 1945 and the socialist transformation between 1949 and 1961, largely completed in 1960-1961. In 1945, large estates were liquidated and the agricultural structure changed over to small-scale farming. As a result of the socialist transformation in 1961, about three thousand cooperative and state farms were in existence to carry on large-scale agricultural production; in addition, almost a million of household plots remained or were brought into being.

The transformation\(^1\) of agriculture along large-scale collective lines took place about sixteen years after the land reform. Its political and economic purpose was to raise the output and efficiency standards of Hungarian agriculture, sadly deficient in many respects, to a higher, more up-to-date level through the establishment of large-scale socialist farms.

Periods of the Socialist Transformation

Following the land reform, Hungarian agriculture went through a period of reconstruction before the large-scale socialist transformation definitely started.

In the period of reconstruction (1945-1949) characterized in the

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\(^1\) The term "reorganization" was used to denote this transformation process in Hungary and in most other socialist countries.
last section of Chapter II, the traditional small-scale individual farms made up the country's farm sector in which most work was manual and there were not even enough draught animals. Consumers' cooperatives, marketing cooperatives and cooperatives for savings and loans were active in the countryside but only a couple of farmers' cooperatives existed in the sphere of production prior to 1948.

In 1948 by means of an association of individual tenants and peasant farming without draught animals, cooperative farms came into being in larger numbers. At the same time a network of state machine stations was created as a technical basis for large-scale farming. The number of cooperative farmers increased from 1948 to 1953 when some of the cooperatives were dissolved; in 1955 there was an increase again. In 1956 many farmers withdrew from the cooperatives; in 1957, however, the consolidation and expansion of farmers' cooperatives began again, although still at a moderate pace. In 1959 the organization of cooperative farms made a new start and extended a great numbers of farmers who were still operating individually. This process continued in 1960 and also in 1961 when socialist large-scale farms became predominant in Hungarian agriculture.

The figures of Table 3.1 pinpoint the changes in the number of farmers' cooperatives and their members.  

\[ ^1 \]

\[ ^1 \]The significant temporary decrease in numbers of cooperative farms and cooperative members was due mainly to political reasons both in 1953 and 1956. After 1961 some of the smaller cooperatives were united and the decrease in the number of cooperative members was mostly due to their retirement.
Table 3.1. Number of farmers' cooperatives and their members\(^a\)

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of cooperative farms</th>
<th>Number of cooperative members</th>
</tr>
</thead>
<tbody>
<tr>
<td>1950</td>
<td>2,185</td>
<td>119,527</td>
</tr>
<tr>
<td>1952</td>
<td>5,110</td>
<td>369,203</td>
</tr>
<tr>
<td>1953</td>
<td>4,536</td>
<td>250,009</td>
</tr>
<tr>
<td>1954</td>
<td>4,381</td>
<td>229,952</td>
</tr>
<tr>
<td>1955</td>
<td>4,816</td>
<td>305,501</td>
</tr>
<tr>
<td>1956</td>
<td>2,089</td>
<td>119,315</td>
</tr>
<tr>
<td>1957</td>
<td>3,394</td>
<td>155,794</td>
</tr>
<tr>
<td>1959</td>
<td>4,489</td>
<td>564,568</td>
</tr>
<tr>
<td>1960</td>
<td>4,576</td>
<td>960,311</td>
</tr>
<tr>
<td>1961</td>
<td>4,523</td>
<td>1,182,894</td>
</tr>
<tr>
<td>1962</td>
<td>4,019</td>
<td>1,164,340</td>
</tr>
<tr>
<td>1965</td>
<td>3,524</td>
<td>1,098,301</td>
</tr>
<tr>
<td>1969</td>
<td>2,840</td>
<td>1,023,937</td>
</tr>
<tr>
<td>1970</td>
<td>2,676</td>
<td>1,035,019</td>
</tr>
</tbody>
</table>

\(^a\)Sources: (52, p. 415; 77, pp. 173,176).

During the first part of the socialist transformation, from 1949 to 1956, an attempt was made to apply a uniform cooperative model largely resembling the structure of the kolkhoz in the Soviet Union. An attempt was also made to increase production in the new cooperatives through propaganda campaigns and the rigid enforcement of large-scale farming patterns. Such methods were unlikely to lead to success, since a number of methods unsuitable to agricultural technology were imposed by central government bodies, and because organizational patterns alone could not make up for the lack of capital and professional skill.

Beginning with 1957, in the concluding period of the socialist transformation, a substantial change occurred, not without producing its results. The increase of large-scale socialist enterprises was fortunately no longer coupled with the enforcement of rigid cooperative and farming patterns: the up-to-date methods and the experience of socialist and
capitalist countries were critically considered and adapted to Hungarian conditions, and the experience at home served as a basis for improvement. There were, of course, unsuccessful experiments in this period as well, but they were corrected through practice. And the useful small-farm traditions came into their own again, especially on the household plots of cooperative members, and due to initial shortages in modern large-scale capital equipment — in many fields even within the framework of large-scale farming.¹

Changes in Social Stratification

As a result of socialist transformation, there was a complete change in social stratification and in the whole production pattern of agriculture. In the years prior to the land reform, 6 per cent of the farm families belonged to the upper social group of the peasantry², 25 per cent were middle peasants, and 69 per cent were poor peasants. After the land reform the number of landless farm laborers decreased by half a million, whereas the number of independent farmers considerably increased.

Following the socialist transformation, the largest group of the farm population was made up of cooperative members (without assisting family members) approximately two-thirds of the whole; state-farm workers formed the second largest group, with nearly one-fifth of the 1962 agricultural employment.

¹The mechanics and procedures of developing from the small-scale private farms to large-scale socialist farms as well as the major social and economic features of cooperative farming will be discussed in Chapter VI.

²Members of this group are known as kulaks in Russian history.
An occupational re-stratification of the whole population also took place after 1949, particularly at the beginning of the fifties. The most characteristic feature of this process was the considerable decrease in the size and proportion of the agricultural population. The number of the latter reached its maximum in 1949; since then it has decreased. Up to 1949 around 50 per cent of all those employed worked in agriculture, today (1970) only about 28 per cent. Those who left agriculture at first found jobs mainly in industry and later in the field of services.

The socialist transformation has hastened the disappearance of the traditional way of peasant life, a process which is going on everywhere in the world.

In Hungary the urban population went up by 17.8 per cent from 1949 to 1960\(^1\), the village population rose by only 3 per cent, and the number of villages with decreasing populations was larger than that of villages where there was a small increase (23).

Another change in the Hungarian countryside is that the population is no longer a homogeneous occupational group of peasants working on farms. Many of the villagers no longer work in agriculture, though a significant proportion of them continue to make their permanent homes in the villages.

Technical Progress in the New Farm Organization

The first stages of modern technical progress in Hungary coincided with the socialist transformation. The transformation of agriculture was initiated with a greater supply of tractors which quickly replaced

\(^1\) A census was taken in 1949, 1960 and in 1970.
draught animals. The tractors and other modern equipment were not divisible according to the extremely small size of actual peasant holdings. However, Hungarian experience has shown that machine indivisibilities can be overcome through cooperative ownership and cooperative use of equipment with large capacity and high fixed costs. This point is clearly made by Heady (42, p. 378) in the context of family farms. But in the first part of the socialist transformation (1949-1956) neither sufficient machines nor modern means of production and materials were as numerous as required. In the second part (1957-1961) large quantities of new machines, equipment, and chemicals were made available, certain up-to-date technical elements were introduced, but really modern technological systems did not replace the old ones until the end of the socialist transformation (1961).

Technical development in Hungarian agriculture may be characterized by the following comprehensive indices (23):

— the manpower per unit (100 hold) of agricultural land increased from 12.7 in 1930 to 13.9 by 1949, but dropped to 9.4 in 1963;

— the index of machines and materials of industrial origin increased from 100 in 1939 and 1949, to 293.4 by 1963.

Technical progress, in its widest form, the substitution of farm labor by capital equipment, was mainly due to the following (23):

— the tractor power used in agriculture greatly increased (the traction power ratio to one laborer was .54 in 1938; after its drop to .44 in 1950, it increased to .57 by 1963); the ratio of mechanical to animal traction changed entirely in the same period (expressed by round
figures this ratio was 10:90 in 1938, 20:80 in 1950, 78:22 in 1963;  
— the use of fertilizers increased from a very low level per hold, from 79.5 kilograms in 1959 to 142.2 in 1963;  
— the area set for reclamation of soil in a year increased more than thirty times (from 5,000 hold in 1947 to 155,600 hold in 1963);  
— the use of chemical weed-killers more than doubled (these were used on 379,000 holds in 1960, increasing to 814,000 in 1963).

Rapid technical progress started to develop a few years after 1961 following the socialist transformation. But even this progress has not been spreading evenly. In some branches (broilers, wheat, corn) it has achieved high speed. In others (hogs, vegetables, fruit, wine), it is developing, but some areas (dairy, pasture) it has made little advance. Looking at the present situation, one might say that the scientific-technical revolution of our age has been conquering Hungarian agriculture largely since 1957. At first state farms took the lead in technical development; recently, however, the cooperative farms have been closing the gap.

Some Organizational Aspects of the Socialist Transformation

Cooperative groups and farmers' cooperative

The socialist transformation was finally carried out through the principle of voluntary enrollment in cooperatives. The peasantry of each village was free to organize more than one cooperative, and the small holdings could be united in agricultural or farmers' cooperatives of different types. Cooperatives of Types I and II, the agricultural cooperative groups, include only crop production carried out collectively.
Cooperatives of Type III, agricultural cooperative farms or agricultural producers' cooperatives, also produce livestock jointly and hold it in common (cooperative) property, except for that kept on the household plots. This third type of cooperative has come to predominate, but some cooperative groups of the other two types continue to exist.

Cooperatives of Type III have also become the "principal form" of farming in Hungary. As producers' cooperatives they are collectively owned by the members who are both the partners and workers in a large-scale farm operation. The cooperative farm is managed on the basis of collective responsibility and for their own account. Each cooperative elects its own executive organs and management personnel.

It was an essential characteristic of the socialist transformation that the peasants joining the cooperatives might keep as their household plot a maximum of 1 cado (0.57 ha) of land, with one cow and its progeny, and pigs and poultry without limit. Thus, side by side with the large-scale collective-cooperative farms, a part of agricultural production has remained in the sphere of household farming. The household farming of cooperative members, as later discussed in more detail from different aspects, is connected in various forms with the collective large-scale farming.

Later this limit set by the centrally approved Cooperative Statute was abandoned and the right of limitation was placed under the authority of General Assembly in each cooperative.
State farms

During the socialist transformation, state farms were also organized as publicly owned large-scale socialist firms for the provision of agricultural products. They resemble in organizational character the state-owned industrial and business firms, their direction, inner organization, and management being similar. At the end of the socialist transformation, state farms occupied roughly one-tenth of the ploughland. Their actual role, however, has been substantially greater in technical progress and in modern capital formation than would correspond to their territorial proportion. They are not simply farms conducting market production on a large-scale but were designated to provide the other farming sectors with improved seeds and breeding stock of outstanding quality, at the same time setting an example in the application of scientifically-founded, up-to-date methods of production and management. For many years state farms were given priorities in the distribution of the very limited capital supply and investment funds available for agriculture. (This is the reason why a substantial difference of fixed assets still exist between an average state farm and an average cooperative farm. Table 4.2 will show this difference.)

If one adds to the state and cooperative sector the land allotted to workers on state farms as part of their pay in kind, and one also adds the group of so-called subsidiary farms consisting of plots less than 0.57 ha which belong to those who employed in socialist industry, mining, communication, etc., one can see the complex structure of the socialist sector in Hungarian agriculture. As the data show, the share of the
individual peasant farms has generally become rather insignificant.

The main characteristics of the extensive and multilateral process of socialist transformation were, however, the emergence of cooperative farming and the development of the farmers' cooperative as a predominant cooperative form of agricultural organization.

Socialist Transformation with Private Land Ownership

When, on the one hand, land reform put an end to the feudal distribution of land ownership in Hungary, on the other, it no doubt strengthened an attitude of private ownership. Just as it was socially necessary to carry out the "democratic" transformation of 1945 by passing the land into small-scale ownership, it was socially necessary during the socialist transformation not to nationalize the land of the peasant farmers who joined the cooperative. As Fekete (26, Chapter VII) analyzed it from the viewpoint of the theory of land rent, the collective form of land ownership was not considered an end in itself but rather as a means of developing socialist agriculture.

The government had to reckon with the strong — and between 1945 and 1948 even strengthening — attitude among the Hungarian peasantry in favor of private land ownership. It is relevant here that the Law-Decree No. 7 of 1959 (the first comprehensive high-level statute concerning the formation of producers' cooperatives and their operation) regulated the acquisition of land and the obligation to pay land rent in a very full and differentiated way. At that time there were voices in favor of transferring the land ownership at once to the producers' cooperatives as organizational units. Undoubtedly the forced intro-
duction of cooperative land ownership when the cooperatives were first established would have delayed the whole process of the socialist transformation.

A workable solution was found, whereby only the use of the land entered into the cooperative was socialized. But because the three parts of ownership concept (right of use, right of possession, right of disposition) are closely interconnected, the socialization of the right of use necessarily involved a redefinition of the right of ownership. The socialization of the right of use also practically meant the socialization of the right of possession.

The right of disposition was treated differently. Part of this right ceased; lands in cooperative use could no longer be bought or sold or mortgaged. Another part of the right of disposition was transferred to the cooperative; for example, the voluntary exchange of land between the cooperative and its member in allocating household plots. At the same time the cooperative member can still exercise definite rights important to the individual: e.g., he may dispose of his land by will.

Law Decree No. 7 of 1959 also regulated in detail the inheritance of lands which were in the use of the large-scale enterprises of producers' cooperatives. The essence of this regulation was that the general rules govern such inheritance, but if the inheritor was not a member of the given producers' cooperatives — although he still inherited the title to the land — he had to lease the land to the cooperative for an unlimited term.¹

¹In 1968 this regulation changed, particularly in favor of the inheritors as discussed in Chapter VI.
Under these legal conditions the joining member was obliged to hand over to the producers' cooperative for common use all land owned, leased or otherwise used by him and by members of his households. Of course, this did not include the household plot since its land is in the personal ownership of the cooperative member.

Leaving the land title in the register intact and the careful regulation of inheritance, basically respect the attitude and attachment of the cooperative member to his private land ownership. At the same time, the cooperative form of the right of use and possession has removed a part of internal content of private land ownership and has brought it closer to social (cooperative) ownership. The member can exercise only part of his ownership rights; this form of private land ownership is called by jurisprudence "partial private ownership."

New Developments in International Comparison

Apart from land ownership, the socialist transformation of Hungarian agriculture came to an end as a practical application of the Marxist theory. The major relevant theses of the classical Marxist theorists may be summed up as follows:

— the social ownership of land;

— the development of large-scale collective firms with the form of farmers' cooperatives or state farms;

— the system of farming in large-scale socialist enterprises to be carried out on a collective plan;

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1 In my book (26, pp. 272-279) written before the law came out, I used the term "restricted private land ownership."
— the progress of the so-called productive forces achieved by applying advanced technology and scientific farming methods in the large, collectively owned enterprises.

At the beginning, the socialist countries established in Europe after the Second World War simply took over the Soviet model unchanged. With different conditions prevailing in these countries, this approach became an obstacle to the socialist transformation of agriculture. In two countries — Jugoslavia and Poland — it led to an evident break in the progress of socialist transformation, with the result that although a certain cooperative framework has developed in both countries, this differs considerably from that followed in the rest of the socialist countries. In Poland, as in the Soviet Union, state farms have significantly grown in scopes; but small individual farms dominate in number.

In the Soviet Union the central decisions adopted in September 1953 started to put an end to the formerly rigid structure and uniform development of cooperatives. Major socio-economic changes followed (27):

— New methods of macroeconomic planning were introduced giving the collective farms more scope for independent decisions.

— The role of the agricultural machine stations (which formerly acted as servicing firms as well as control of the government at the same time) changed, and large-scale agricultural machinery passed into the possession of the collective farms.

— Compulsory deliveries of produce to government agencies were abolished and a new price system gradually appeared.

Basically the same steps were undertaken in Hungary as well as in the other socialist countries, but not in a rigid pattern — rather in accordance with the special conditions of the countries concerned. In Hungary the socialist transformation was completed by 1961; in Bulgaria earlier; in Czechoslovakia at the same time; in the German
Democratic Republic and in Rumania shortly afterwards.

A number of further significant changes occurred in the organizational structure of cooperative farming. The most decisive of these are the following:

— With the representative social organs of the cooperatives established at both a nation-wide and regional level, the scope of authority of the central and local government administration has been restricted to the functions of legal state control.

— Highly diversified forms of management and work organization are emerging in the cooperatives, with the principles of cooperative democracy and autonomy asserting themselves in an increasingly consistent pattern.

— The right to household plots of individual members has been firmly recognized, including the right to make use of cooperative equipment in the cultivation of individual plots.

In reviewing the socialist transformation the organic progress which modified the organizational relationships within the cooperatives deserves special attention. The transformation itself was mainly the result of political measures which could not fulfill the complex task of creating large-scale collective enterprises in full operation. It, however, could create a new organizational framework, expressing the social values of the working people and the political priorities of the governing organs.

In the period of consolidation (1962-1967), the organizational framework and the internal cooperative relations were stabilized, and well-proved forms of cooperative democracy and efficient management were widely consolidated. This was the phase when various local solutions had to be found within the internal farm organization.

The recent period started in 1968; its beginning is marked by the reform in the direction of the economy and by the codification of the
new cooperative law.1

State, Cooperative and Private Sectors in Hungary's Economy

The 1969 data of national income clearly show the organizational structure of Hungary's economy. The proportion of the three major sectors has been essentially the same since the organization of large-scale cooperatives and state farms took place.

Table 3.2. The sources of Hungary's national income by economic sectors in 1969a

<table>
<thead>
<tr>
<th>Sector</th>
<th>Per cent</th>
</tr>
</thead>
<tbody>
<tr>
<td>State sector</td>
<td>77</td>
</tr>
<tr>
<td>Cooperative sector</td>
<td>20</td>
</tr>
<tr>
<td>Private sector</td>
<td>3</td>
</tr>
<tr>
<td>National income, total</td>
<td>100.0</td>
</tr>
</tbody>
</table>

aSource: (77, p. 5).

Table 3.3 Employment in the major sectors of Hungary's national economy (in percentages)a

<table>
<thead>
<tr>
<th>Sector</th>
<th>1959</th>
<th>1968</th>
</tr>
</thead>
<tbody>
<tr>
<td>State sector</td>
<td>54.1</td>
<td>65.0</td>
</tr>
<tr>
<td>Cooperative sector</td>
<td>8.8</td>
<td>30.9</td>
</tr>
<tr>
<td>Private sector</td>
<td>37.1</td>
<td>4.1</td>
</tr>
<tr>
<td>Active earners, total</td>
<td>100.0</td>
<td>100.0</td>
</tr>
</tbody>
</table>

aSource: (77, pp. 8-9).

1Both will be discussed in Chapter VI.
Distribution of employment in the national economy also demonstrates the changes in the proportions of the three sectors. The breakdown of those in employment by social sectors at the beginning of 1959 and 1968 is shown by Table 3.3.

By the middle of 1969 — when the total population amounted to 10.25 million — the number of those employed was exactly 5 million (76).

The sectorial distribution of active earners of agriculture is shown in Table 3.4.

Table 3.4. Sectorial distribution of active earners in agriculture, 1969.

<table>
<thead>
<tr>
<th>Sector</th>
<th>Persons (1,000)</th>
<th>Per cent</th>
</tr>
</thead>
<tbody>
<tr>
<td>State sector</td>
<td>176.8</td>
<td>12.8</td>
</tr>
<tr>
<td>Cooperative sector</td>
<td>1,146.4</td>
<td>83.2</td>
</tr>
<tr>
<td>Private sector</td>
<td>55.6</td>
<td>4.0</td>
</tr>
<tr>
<td><strong>Total in agriculture</strong></td>
<td><strong>1,378.8</strong></td>
<td><strong>100.0</strong></td>
</tr>
</tbody>
</table>

*Source: (1, p. 204).

A breakdown by numbers of those employed in various economic branches also shows the process of industrialization. In 1950, the total number of those employed in agriculture amounted to 52 per cent, with only 19 per cent employed in industry; by 1969, however, this proportion changed to 35 per cent in industry and 30 per cent in agriculture. In 1969, active agricultural earners made up 27.6 per cent of the total number.
of active earners (76, 77).\(^1\)

The sectorial division of cultivated land also developed in line with the socialist transformation.

Table 3.5. The distribution of total arable land among state, cooperative and private farms in 1969\(^a\)

<table>
<thead>
<tr>
<th>Sector</th>
<th>Per cent</th>
</tr>
</thead>
<tbody>
<tr>
<td>State farms</td>
<td>14.1</td>
</tr>
<tr>
<td>Cooperative farms of all types</td>
<td>80.3(^b)</td>
</tr>
<tr>
<td>Private farms</td>
<td>5.6(^c)</td>
</tr>
<tr>
<td>Total</td>
<td>100.0</td>
</tr>
</tbody>
</table>

\(^a\)Source: (77, p. 177).

\(^b\)This figure includes the land area of household plots which made up 9.8 per cent. This figure also includes the land area of cooperatives other than producers' cooperatives which made up 3.7 per cent.

\(^c\)Including the auxiliary farming of people with non-agricultural occupations.

From different aspects, the proportion of major agricultural sectors may be shown in details.

\(^1\)Fazekas (25) calculated that in 1964 half of the people engaged in productive activities — roughly two million "breadwinners" — were employed in the so-called agribusiness sector, in which he included the marketing and processing of agricultural products, and manufacturing of production means for agriculture. Fazekas also included forestry in this sector but excluded the agricultural sphere of banking and other financial services (e.g. local and regional savings and loan associations).
Table 3.6. The share of state farms, cooperatives and homeplots in land use and in the output, 1969 (in percentages)\(^a\)

<table>
<thead>
<tr>
<th></th>
<th>State farms</th>
<th>Cooperatives(^b)</th>
<th>Household plots</th>
<th>Others(^c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agricultural area total</td>
<td>15.5</td>
<td>77.8</td>
<td>9.5</td>
<td>6.7</td>
</tr>
<tr>
<td>Arable land</td>
<td>14.1</td>
<td>80.3</td>
<td>9.8</td>
<td>5.6</td>
</tr>
<tr>
<td>Gardens, orchards, vineyards</td>
<td>14.4</td>
<td>61.0</td>
<td>25.7</td>
<td>24.6</td>
</tr>
<tr>
<td>Cattle stock</td>
<td>12.0</td>
<td>78.0</td>
<td>56.5</td>
<td>10.0</td>
</tr>
<tr>
<td>Pigs</td>
<td>17.6</td>
<td>63.6</td>
<td>33.3</td>
<td>18.8</td>
</tr>
<tr>
<td>Poultry stock</td>
<td>9.0</td>
<td>62.0</td>
<td>54.0</td>
<td>29.0</td>
</tr>
<tr>
<td>Gross production of agriculture</td>
<td>16.0</td>
<td>70.0</td>
<td>24.0</td>
<td>14.0</td>
</tr>
<tr>
<td>Procurement</td>
<td>21.5</td>
<td>71.4</td>
<td>15.0</td>
<td>7.1</td>
</tr>
</tbody>
</table>

\(^a\)Source: (77, pp. 177, 184, 185).

\(^b\)Including household plots.

\(^c\)Including private farms.

The dimensions of enterprises and the size of farms have also changed. A new, basically large-scale organization has emerged in the place of almost two million "minifundia." Before the start of the socialist transformation over one million of these small farms had fewer than three hectares of land, and only a few were of the maximum size of 14.5 hectares.
Growth and the Level of Economic Development

The damage of the Second World War to Hungary's national economy amounted to $4,000 - 5,000 million. Due to the losses in productive capacity, inventories and manpower, the national income in 1945 - 1946 consisted of only 45 per cent of that in 1938 - 1939. Ten per cent of the national income was spent on reparations and another 6 per cent on the most urgent reconstruction work. After the war the country went through a severe inflation.

Economic reconstruction was successful so that the pre-war level of national income was reached in general by 1949. At that time, per capita national income was about $240.

Since the end of the reconstruction period (1949) Hungary's national income has grown as shown in Fig. 3.1.

Fig. 3.1. The growth of Hungary's national income (1949 = 100)
Fig. 3.1 depicts an annual increase of 5.7 per cent. The graph clearly shows the trend: Hungary's national income grew from $2,100 million in 1949 to $7,400 million in 1969. The latter sum corresponds to a gross national product of $8,700 million.

Per capita national income in Hungary was some $720 in 1969. Thus, as generally regarded by the level of economic development, Hungary's place is at the lower limit of that group of European countries to which Austria, Czechoslovakia, and Italy belong at a somewhat higher level. At the same time, Hungary's place is above the higher limit of that group to which Spain, Portugal and Greece belong. Therefore, Hungary is considered as a member of the group of medium-developed European countries.
CHAPTER IV. SCALE RETURNS, FARM SIZE AND CHANGES IN FARM ORGANIZATION

The transformation of Hungary's agriculture was definitely associated with radical changes in the number and size of farms. For this reason, it is worthwhile to deal with some aspects of scale and size relationships in this chapter. After brief theoretical qualifications, the scale-size implications of both the land reform and the socialist transformation will be discussed. Finally the farm size and organizational units of the Hungarian agriculture will be described.

Returns to Scale and Farm Size

The conceptual problems of scale and size are thoroughly analyzed by authors of agricultural economics and microeconomic theory (41, 42, 44, 49, 50).

Pure scale relationships are considered only when all inputs are increased by the same proportion. This qualification implies that any pure scale relationship is conceived as a long-run concept. If output increases by the same proportion as each input, then there are constant returns to scale. If output expands by a greater (smaller) proportion than inputs, then increasing (decreasing) returns to scale prevail in production.

Scale relationships are based on long-run production functions, no factor of which is fixed in quantity and all factors of which represent one single aggregate resource in effect. Heady (42) conducted

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1 Chapters IX and X will discuss certain implications of optimization to farm size under alternative objective functions of the large cooperative enterprise and under those of the members' households.
research on scale relationships using the technique of elasticity of production, considering different factors separately, applying cost functions as well as analyzing external and internal economies. Holdren (50) emphasizes that the microeconomic theory of scale relationships has been worked out thoroughly only for an individual activity or process of production. Farms and management units within large farms (the latter correspond to the plants of a business firm) are, of course, operating with multiprocess. A complex production function for a product and for a farm can be derived by applying intermediate processes and by the application of the theory of joint production.

In order to give returns to scale a quantified meaning, the following single-product, multi-input production function is applied

\[ q = f \left( x_1, \ldots, x_i, \ldots, x_m \right) \]  

(4.1)

where \( q \) is output level, the \( x_i \) are resource inputs (\( i = 1, \ldots, m \)).

After multiplying each independent variable by some positive number, \( c \), Equation 4.1 becomes

\[ bq = f \left( cx_1, \ldots, cx_i, \ldots, cx_m \right) \]  

(4.2)

The new product output equals \( b \) times the previous one. If

\[ b = c \quad \text{or} \quad \frac{b}{c} = 1 \]  

(4.3)

constant returns are the case. If:

\[ b > c \quad \text{or} \quad \frac{b}{c} > 1 \]  

(4.4)

increasing returns to scale prevail, and if

\[ b < c \quad \text{or} \quad \frac{b}{c} < 1 \]  

(4.5)

decreasing returns to scale are in existence. The returns to scale are given the same precise meaning by using the concept of elasticity of
production which is the ratio of the marginal product (MPP) and the average product (AP). In this case b stands for MPP and c represents AP in Expressions 4.3 - 4.5.

The meaning of scale returns may be further clarified by the concept of homogeneity of production functions. In all regions of a homogeneous production function, the returns to scale are the same: either constant, increasing at a particular rate, or decreasing at a particular rate. As a rule, nonhomogeneous production functions possess at least two of the possible three cases and may also have regions of constant, increasing, and decreasing returns to scale.

The scale returns implications of a generalized-type nonhomogeneous production function are shown on the following diagram.

![Diagram showing scale returns implications](image)

**Fig. 4.1.** Nonhomogeneous production function maintaining increasing, constant, and decreasing returns to scale
Between points 0 and A the above nonhomogeneous production function has increasing returns at a decreasing rate to scale. In the region between points A and B, this function has constant returns to scale. In the section following point B, the given function exhibits decreasing returns at an increasing rate to scale. It may be noted that a non-homogeneous production will not give rise to a straight-line expansion path on the map of its isoquants.

Distinction between scale relationships and proportionality is made by referring to the time dimension and its characteristic production function. In contrast with pure scale relationships and their production function — defined as having no fixed factor — proportionality relationships involve the short-run production functions of which one or more factors are fixed. In the long-run planning period, time is long enough to establish the minimum cost process such that the decision maker may consider all the relevant areas and forms of the production function. The above short-run implies that the decision maker cannot freely vary the rate at which he makes use of the services flowing from the fixed factor(s).\footnote{Holdren (50) realizes also another kind of short-run in which time is not sufficiently long "to construct the relevant minimum cost process, but other process with higher unit costs may be constructed."}

In the economic literature the concept of size is more frequently encountered, and this concept may be conceived of as a sort of extension of the concept of pure scale relationships. In Heady’s definition (42, p. 361), the term of size refers to "different firm or plant capacities which are possible through expansion by either the scale or
The farm size is denoted by different levels of fixed factors in this vein.

The concept of size expresses the fact that a farm is a firm having one or more geographically fixed plants, and in the long-run the relative quantities of production factors change since land available is limited.

In connection with farm sizes Heady (44) argues that scale returns and efficient resource use are not tied directly to any form of land ownership. Under the conditions of private land ownership, major inefficiencies have arisen historically when privately-owned land is considered as the basis for:

- judging equity in case of borrowing capital;
- providing social security, especially concerning farmers who reach retirement age.

It may be stated in general that returns and efficiency depend mostly on resources applied.

The Implications of Scale Returns to the Hungarian Land Reform

Introducing the land reform of 1945, Hungarian policy makers might have possessed certain ideas about the prevailing scale returns and an implied objective concerning efficiency in order to relate returns to farm size.

Representing the efficiency criterion by the output-input ratio (O/I) and considering the number of farms as the inverse of farm size, the

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1 Heady deals with the implications of scale returns to land reform programs in his courses at ISU.
three possible implications of scale returns to land (or, in general, to farm size) reform programs may be depicted by diagrams.

The three curvatures of Fig. 4.2 represent alternative trade-offs between farm size and efficiency. Scale returns are the decisive factor that determines the nature of the three alternatives. In Fig. 4.2-A the slope of the curve expresses a trade-off which relies on decreasing returns to scale. As farm size becomes smaller and the number of farms increases (i.e., as one proceeds along the horizontal axis to the right), efficiency improves in terms of output per input. In Fig. 4.2-B the trade-off curve is horizontal and its zero slope (i.e., the lack of trade-off) is based on constant scale returns. In this case efficiency is not related to farm size and any number of farms results in the same output per input. Figure 4.2-C depicts the results of production possibilities which are characterized by increasing scale returns. Here the curvature of the trade-off curve shows that efficiency improves (incidentally rather rapidly) when farm size increases, (i.e., as one moves along the size axis to the left).

![Fig. 4.2. Efficiency under decreasing, constant, and increasing returns to scale](image-url)
In the case depicted by Figure 4.2-C, there is clearly some sacrifice of efficiency if the number of farms is increased and average farm size is to be smaller. Exactly the opposite is the case if the actual production possibilities are of such nature that they are reflected in the slope of the trade-off curve in Fig. 4.2-A.

Which of the three relationships may have influenced the decision of policy makers when they voted for the Land Reform Decree? The case of increasing returns to scale (depicted in Fig. 4.2-C) was clearly ruled out by action and by the reverse historical evidence of large feudal estates. Both the other two alternatives, however, might well have been in operation. Indifferences between various sizes and numbers of farms, i.e. the constant return case (shown in Fig. 4.2-B) may have been considered but were dominated by the political and social priorities at the same time. The version of decreasing returns to scale (characterized by Fig. 4.2-A) unequivocally may have served as an economic argument in favor of the land reform. In this respect, the implied objective function of the land reform was to improve efficiency by increasing the number of owner-operated farms. This objective naturally involved a decrease in the average size (scale) of farms. Smaller farms operating in the first phase of the classical production function were assumed to be more efficient.

Scale and Size Aspects of the Socialist Transformation

Decreasing returns to scale was evidently ruled out as the decisive relationship during the socialist transformation. From an economic point of view, the organization of large farms may explicitly have been sup-
ported by the notion of increasing returns to scale (Fig. 4.2 C).

In Hungary there has been no definite tendency of either increasing or decreasing the cooperative farm size (and of changing the number of cooperative farms respectively) for the last year or so. From a theoretical viewpoint, this fact may be considered as practical evidence that constant returns to scale is the relevant "size-determining" economic concept for policy makers and for cooperative decision-makers these days.

Scale returns are directly related to farm size in a planned economy via a certain group of size-determining socio-economic factors. As is thoroughly analyzed by Heady (41)\(^1\), farm size \(F_S\) is determined by three groups of relevant variables

\[ F_S = f (SAC, LAC; K, M, S; P_{ij}, P_{iq}) \]  \hspace{1cm} (4.6)

The first group involves short-run (average) costs, SAC, and long-run (average) costs, LAC, derived from the production function exhibiting constant and/or increasing and/or decreasing returns to scale expressed by Equations 4.2 and 4.3, 4.4, 4.5. The concept of scale returns is closely connected with this group of size-determining factors.\(^2\)

In the second group, \(K\) stands for capital supplied, \(M\) represents managerial abilities, and \(S\) symbolizes sociological factors, such as family size, age of farm operators, properties of rural communities, etc.

\(^1\)In his recent lectures at ISU, too.

\(^2\)The classical type of production function exhibits all the three variations of scale returns.
Capital availability is crucial in each form of farm organization. Sociological factors are of special interest in cooperative farming.

The components of the third size-determining group (input prices, product prices, and interest rates, i.e. the prices of borrowed capital) are becoming more and more important under the conditions of the new economic mechanism.¹

Some aspects implicitly or explicitly considered by the policies of socialist transformation may be discussed from the viewpoint of external economies.

The major types of external economies are mostly connected with pricing and economic policy:

1. Farm output prices may be constructed as an increasing function of quantity sold. Such was the case during the early 1960's when cooperative farms were paid a higher output price than small individual farms. This fact was explained by two basic reasons:

a. newly formed cooperative enterprises required state price support in order to accumulate necessary funds for up-to-date large-scale operation within a relatively short time;

b. trading companies could save expenses on transportation and handling if they brought large quantities from a farmers' cooperative instead of much smaller quantities from private farm units.

¹Most economists relate the impacts of technological variables and managerial factors to farm size. In addition to these, Heady (42) stresses the role of capital supply and uncertainty as the framework of farm decision-making. If uncertainty were not in existence, then the second group of size-determining variables would lose its great significance.
In this system a cooperative farm realized increasing prices if it sold a larger quantity. Categories were set for quantities marketed and each category was given a separate price.

The price break between cooperative and private farms as well as the existence of a certain stepped-price function for farmers' cooperatives may be expressed in terms of a pseudo-scale relationship of some sort.

\[ P_q = f(q), \quad (4.7) \]

and

\[ \frac{dP_q}{dq} > 0 . \quad (4.7a) \]

This relationship supported the organization and initial activities of large farms.

2. Input prices may be set in such a way that these are a decreasing function of quantities purchased; in other words, input prices are a decreasing function of output produced

\[ P_x = h(q), \quad (4.8) \]

and

\[ \frac{dP_x}{dq} < 0 . \quad (4.8a) \]

The economic reasoning for state support provided to cooperative farms was the same behind the lower input price and the higher output price mentioned before. Price policies and their pseudo-scale aspects reflected by Expressions 4.8 and 4.8a also contributed to the relatively fast development of Hungarian cooperative farming in the decade of 1957-1967.
Financial, mainly tax policy and credit supply policy, as well as legislative actions, also promoted the operations of large state-owned and cooperative farms in certain periods of the socialist transformation.

All three arguments raised before have shown aspects of the external economies operating in favor of larger farm size. In practice, however, internal (intrafarm) economies are likely to be more decisive in the advocacy of larger farm decision units. Internal economic forces are really responsible for why the long-run average cost curves decline in the range of relatively large outputs. Intrafarm (internal) economic forces promote the development of larger farms:

a. output grows faster if more of some input is applied (in other words the index of production elasticity has a value which is larger than unity);

b. the previous case is extended to the whole aggregate of resources used in farming;

c. divisibility of capital exists in the sense that larger farm equipment may be more efficient than smaller equipment;

d. discontinuity or "lumpiness" of capital (and investment) prevails—all the perceivable sizes of various farm equipment are generally unavailable;

e. some other aspects of capital and technology are encountered, e.g. doubling the outside dimensions of a farm building may increase its inner (storage) capacity more than twice;

f. divisibility and efficient distribution of labor (i.e. Adam Smith's famous argument in advocacy of larger scale operations) are in
existence.

All the factors listed are of a technological nature and have important implications on production costs in exactly the same manner as buying inputs in larger quantities may be associated with lower or discount prices. Thus numerous internal economies may result in decreasing costs and in declining portions the long-run average cost curve over quite a wide range of larger farm size and greater farm output.

Farm Size and Firm Decision Units

During the years of socialist transformation (1949-1961) large-scale cooperative farms and state farms were organized. Following this transformation period, a concentration process took place in both the state and the cooperative sectors of agriculture, as a result of which farm sizes were increased further. The number of decision units below indicates both the result of the socialist transformation and the extent of the concentration process afterwards.

1There existed a few state farms before World War II, mainly to provide horses for the army. During the socialist transformation the major resource of expansion of state farms was the land of "rich peasants" and others who left farming mainly in the period of 1950-1952.

2This paper adopts Holdren's (50) term of "decision unit" instead of "unit of management" used in Hungarian statistics published in English.
Table 4.1. Decision units of the Hungarian agriculture in 1960 and 1968*

<table>
<thead>
<tr>
<th></th>
<th>1960</th>
<th>1968</th>
</tr>
</thead>
<tbody>
<tr>
<td>State farms</td>
<td>333</td>
<td>208</td>
</tr>
<tr>
<td>Agricultural producers' cooperatives</td>
<td>4,507</td>
<td>2,840</td>
</tr>
<tr>
<td>Producers' cooperative groups</td>
<td>69</td>
<td>210</td>
</tr>
<tr>
<td>Producers' cooperatives of fishers</td>
<td>21</td>
<td>22</td>
</tr>
<tr>
<td>Cooperative associations</td>
<td>127</td>
<td>215</td>
</tr>
<tr>
<td>Private farms with land area bigger than 0.57 hectare (number in thousands)</td>
<td>471</td>
<td>100</td>
</tr>
</tbody>
</table>

*Source: (77, p. 173).

**For the details of this change see Table 3.1.**

***Specialized producers' groups and cooperatives producing grapes and fruit.***

In the state and cooperative sectors the average size of farms may be characterized by the fairly complex set of data of Table 4.2, 4.3, and 4.4.

An average state farm is generally much larger than an average cooperative farm and it is also reflected in the distribution of farms according to size. At the same time, the capital-labor ratio is considerably different, mostly because of the capital supply system which was already referred to.

Table 4.2 contains the most characteristic data with respect to the size of a typical state and cooperative farm.
Table 4.2. Average size of state and cooperative farms in 1968

<table>
<thead>
<tr>
<th></th>
<th>State farms</th>
<th>Agricultural producers' cooperatives</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cultivated land, hectare</td>
<td>4,623</td>
<td>1,672</td>
</tr>
<tr>
<td>Value of fixed assets, million forints</td>
<td>115.9</td>
<td>16.2</td>
</tr>
<tr>
<td>Cumulated value of production, million forints</td>
<td>74.6</td>
<td>18.4</td>
</tr>
<tr>
<td>Gross revenue, million forints</td>
<td>58.6</td>
<td>14.0</td>
</tr>
<tr>
<td>Labor force (number of employees and members respectively)</td>
<td>756</td>
<td>379</td>
</tr>
</tbody>
</table>

^Source: (77, pp. 175-176).

A different grouping will be applied to give the characteristic distribution of state and cooperative farms according to size.

Table 4.3. The distribution of state farms according to size in 1966

<table>
<thead>
<tr>
<th>The size of holding (in hectares)</th>
<th>Share of farms in all state holdings number</th>
<th>Share of farms in all state holdings land area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Below 1,150</td>
<td>7.9</td>
<td>1.3</td>
</tr>
<tr>
<td>Between 1,151 - 2,300</td>
<td>18.6</td>
<td>6.9</td>
</tr>
<tr>
<td>2,301 - 3,450</td>
<td>19.5</td>
<td>12.2</td>
</tr>
<tr>
<td>3,451 - 4,600</td>
<td>15.8</td>
<td>13.3</td>
</tr>
<tr>
<td>4,601 - 5,750</td>
<td>12.6</td>
<td>13.4</td>
</tr>
<tr>
<td>5,751 - 6,900</td>
<td>7.0</td>
<td>9.2</td>
</tr>
<tr>
<td>6,901 - 8,050</td>
<td>5.1</td>
<td>8.1</td>
</tr>
<tr>
<td>Above 8,050</td>
<td>13.5</td>
<td>35.6</td>
</tr>
<tr>
<td>Total</td>
<td>100.0</td>
<td>100.0</td>
</tr>
</tbody>
</table>

^Source: (52, p. 414).
Table 4.4 shows the distribution of cooperative farms according to size in a relevant grouping:

Table 4.4. The distribution of cooperative farms according to size in 1966

<table>
<thead>
<tr>
<th>The size of landholdings (in hectares)</th>
<th>Share of farms in all cooperative holdings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>number</td>
</tr>
<tr>
<td>Below 290</td>
<td>0.7</td>
</tr>
<tr>
<td>Between 291 - 575</td>
<td>10.3</td>
</tr>
<tr>
<td>576 - 1,150</td>
<td>32.3</td>
</tr>
<tr>
<td>1,151 - 1,725</td>
<td>24.9</td>
</tr>
<tr>
<td>1,726 - 2,300</td>
<td>14.6</td>
</tr>
<tr>
<td>2,301 - 2,875</td>
<td>8.0</td>
</tr>
<tr>
<td>2,876 - 3,450</td>
<td>4.6</td>
</tr>
<tr>
<td>Above 3,450</td>
<td>4.6</td>
</tr>
<tr>
<td>Land of intercooperative enterprises</td>
<td></td>
</tr>
</tbody>
</table>

Total 100.0 100.0

\( ^a \) Source: (52, p. 414).

\( ^b \) Enterprises (e.g. broiler production, hog feeding) organized and managed by two or more cooperatives.

The new framework of decision making and the recent form of farm organization in Hungary is perhaps best characterized by the data of Table 4.1.

Output Performance and Commercialization

About 60 per cent of the increase in agricultural output since 1945 took place during the period following the end of the socialist transformation in 1961. The increase in agricultural output taking place in
the socialist transformation period was due to deliberate efforts since this period did not enjoy above-average weather conditions and other benevolent natural factors (25).

The gross cumulated output of Hungary's agriculture grew by 24 per cent between 1938 and 1966, "noncumulated" production by 33 per cent at constant prices of 1959.\(^1\) The 2.1 per cent increase per annum between 1957 - 1967 is the fastest growth rate of Hungarian agriculture output in our century.

The yield of wheat, the country's most important cereal, reached 2.71 tons per hectares in 1969 and averaged 2.51 tons over the period 1966 - 1969. This compares with 1.86 tons in the average years of 1961-1965 and 1.34 tons in the period of 1931 - 1940. The average yield of corn was 3.8 tons per ha in 1969, and 3.2 and 1.84 in the above periods (1, 76, 77).

The speed-up growth is due, among others, to the fact that the capital supply, especially investments in mechanization and chemization, increased considerably since the beginning of the sixties.

As a result of increasing output, Hungary has a good standing among the Eastern European countries concerning the magnitude of per-capita agricultural products (exc. milk).

\(^1\)In Hungarian statistics, cumulated output involves double (or even multiple) counting of materials (seeds, feeds) produced and used within the same farm (decision) unit. Noncumulated production excludes double counting of this sort.
Table 4.5. Farm products per capita in the Eastern European countries

<table>
<thead>
<tr>
<th>Countries</th>
<th>Cereals kilogram</th>
<th>Meat kilogram</th>
<th>Milk liter</th>
<th>Eggs dozen</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulgaria</td>
<td>643</td>
<td>59</td>
<td>190</td>
<td>16.25</td>
</tr>
<tr>
<td>Czechoslovakia</td>
<td>712</td>
<td>70</td>
<td>331</td>
<td>21.0</td>
</tr>
<tr>
<td>German Democratic Rep.</td>
<td>463</td>
<td>71.5</td>
<td>407</td>
<td>19.5</td>
</tr>
<tr>
<td>Hungary</td>
<td>830</td>
<td>91.1</td>
<td>175</td>
<td>22.0</td>
</tr>
<tr>
<td>Poland</td>
<td>575</td>
<td>66.6</td>
<td>456</td>
<td>16.5</td>
</tr>
<tr>
<td>Rumania</td>
<td>654</td>
<td>40</td>
<td>200</td>
<td>13.0</td>
</tr>
<tr>
<td>Soviet Union</td>
<td>521</td>
<td>49</td>
<td>345</td>
<td>12.0</td>
</tr>
<tr>
<td>Jugoslavia</td>
<td>613</td>
<td>39.5</td>
<td>138^</td>
<td>9.0</td>
</tr>
</tbody>
</table>

^Source: (36, p. 1231).

^In 1968.

^In 1967.

^In 1970.

^In 1966.

Hungarian agriculture has become more commercialized as a result of increasing farm size. Market production (net commodity turnover) of agriculture increased by 55 per cent between 1938 and 1966 (calculated at constant prices of 1959). The increased level of commercialization is well reflected by the share of marketing in gross agricultural production.
Table 4.6. Commodity turnover as percentage of gross agricultural production* 

<table>
<thead>
<tr>
<th>Form of turnover</th>
<th>1960</th>
<th>1968</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purchase by state and cooperative trade companies*</td>
<td>38.5</td>
<td>46.0</td>
</tr>
<tr>
<td>Trade on free market</td>
<td>9.1</td>
<td>10.3</td>
</tr>
<tr>
<td>Global turnover (sum)</td>
<td>47.6</td>
<td>56.3</td>
</tr>
<tr>
<td>Repurchased by agriculture</td>
<td>8.3</td>
<td>7.5</td>
</tr>
<tr>
<td>Net commodity turnover (as a part of agricultural output)</td>
<td>39.3</td>
<td>48.8</td>
</tr>
</tbody>
</table>

*Source: (77, p. 174). 

Generally based on contract and/or on forward pricing.

Large farms are rapidly increasing their purchase of inputs of industrial origin.
CHAPTER V. CHANGES IN HUNGARY'S FARM ORGANIZATION: A MATHEMATICAL RESTATMENT

In this chapter simple models are presented in order to quantify the objectives and constraints which might implicitly have existed at the time of the land reform and during the socialist transformation.

The Land Reform Implications

It is assumed in this paper that the policy makers' economic objective was the maximization of the value of total agricultural output when they decided in favor of the land reform. This assumption has some logical basis in the great post-war food shortage and agriculture's role in export and employment, both shown in Chapter II.

The implicit objective function of the land reform is written as

$$\text{Max } \sum_{i=1}^{s} \sum_{j=1}^{n} p_j q_{ji}$$

where

- $i = 1, \ldots, s$ the number of landowners,
- $j = 1, \ldots, n$ different agricultural products,
- $p_j$ = the price of product $j$,
- $q_{ji}$ = the quantity of product $j$ produced on the land of person $i$.

The upper limit placed by the Land Reform Decree on the size of landholding may be expressed as

$$114 L_F + 57 L_{NF} \leq H,$$

where

- $L_F$ = number of landowners with agricultural occupations,
\[ L_{NF} = \text{number of landowners with nonagricultural occupations}, \]
\[ H = \text{total agricultural land of Hungary}, \]
\[ 114 \text{ ha} = \text{maximum size of holding for } L_F, \]
\[ 57 \text{ ha} = \text{maximum size of holding for } L_{NF}. \]

The maximum limits were far from being typical in practice. The maximum size of granted land units was constrained as
\[ r_i \leq 9.55 \text{ ha}, \quad (5.3) \]
where
\[ r_i = \text{the size of land granted to person } i. \]

The land area subject to expropriation restricted the number of persons actually benefited and mainly the size of their granted land.

\[ \sum_{i=1}^{s^0} r_i \leq 3.2 \text{ million ha}, \quad (5.4) \]
where
\[ s^0 = \text{the number of persons benefited by the reform}. \]

The inequality sign in Expression 5.4 states the fact that state-owned forests and uncultivable land were also included in the expropriated area.

The production matrix was thoroughly affected by the reform. The input mix changed. The small holdings used more labor relative to capital and land in comparison with the former large holdings. The crop mix also changed. Capital, both in its financial and physical form, became even more scarce. Labor still remained the only surplus factor.
A Note on the Feasibility of Alternative Objective Functions

At the time of the land reform of 1945 as well as somewhat later on, one often encountered opinions and hints that the direct objective of this reform was to maximize the output per land unit. This belief might well be supported by the fact that the only "success element" of the pre-reform agriculture was the relatively higher output performance per land unit on small-scale farms. Instead of Expression 5.1, the alternative objective function should have assumed the form:

$$\max \frac{Q}{H},$$  \hspace{1cm} (5.1a)

where

$$Q = \text{the value of agricultural output.}$$

The existence of Expression 5.1a as an objective function, even in a slightly implicit form, may be rejected on the basis that the reform did not aim at any reallocation of resources after its completion. The maximization of per-acre-output apparently should have required the reallocation of resources, not only between the latifundia and peasant holdings but also among the "minifundia."

In the view of Expression 5.1a, the direction of resource allocation would be to channel all capital and labor away from enough hectares of land so that $\frac{Q}{H}$ could be maximized on the remaining land area.

Even a more relevant objective function could have been formulated during the land reform or shortly after:

$$\max \frac{Q}{CL},$$  \hspace{1cm} (5.1b)

where
Since labor was abundant and capital was scarce at that time, it would have required to satisfy the following conditions:

\[
\frac{\partial Q_1}{\partial C_1} = k, \quad (5.5a)
\]

and

\[
\frac{\partial Q_2}{\partial C_2} = k, \quad (5.5b)
\]

\[
\vdots
\]

and

\[
\frac{\partial Q_s}{\partial C_s} = k. \quad (5.5c)
\]

Equations 5.5a - 5.5c denote that the MPP per capital unit should have been equal on all farms.

In short, both objectives expressed by Expressions 5.1a and 5.1b should have presumed the reallocation of resources among small-scale private farm decision units. This reallocation had become possible when the socialist transformation created large cooperative enterprises as resource pools by amalgamating the small peasant farms incapable of prosperous and efficient operation.

Implicit Objective Functions in the Socialist Transformation

The socialist transformation of Hungarian agriculture took place in favor of the working peasantry and reflected the vital interests of farm workers and small farmers. They were directly interested in finding a new organizational form of farming which would provide a higher
living standard and greater social security. The interests and objectives of the newly established cooperative farms as decision units are located at the medium level of an interest hierarchy discussed in the Introduction. This section is concerned with the mathematical forms of social interests associated with the socialist transformation. When the socialist transformation was initiated, the objective function of policy makers may have been the maximization of total agricultural value product, but this maximization was subject to the minimum-output-per-farm restraints expressing political priorities and high social values attached to large-scale farming at that time. Considering the new socialist organization, decision units took the place of landowners.

The objectives function which may implicitly have existed during the socialist transformation is written as:

$$\text{Max} \sum_{i=1}^{s} \sum_{j=1}^{n} p_j q_{ji}$$

subject to

$$q_{ji} \geq q_{ji}$$

where

- $i = 1, \ldots, s$ the number of decision units in socialist agriculture,
- $j = 1, \ldots, n$ different agricultural products,
- $p_j$ = the price of product $j$,

\[ \text{The old-age pension offered by the cooperative system proved to be a very strong stimulus for older farmers in joining the cooperatives. Chapter X will investigate how other priorities and individual interests of farmers appear within cooperative farming.}

\[ \text{The relevant meso-zone of interests, i.e., the objectives and interests of cooperative firms as business enterprises are focused on in Chapter IX.} \]
\[ q_{ji} = \text{the quantity of product j produced on the farm of decision unit i,} \]
\[ \bar{q}_{ji} = \text{minimum restraint on output of the j-th product for the i-th farm.} \]

This minimum constraint is also of implicit nature. (If possible, the constraint is even more implicit than the objective function here.) It reflects the consideration that policy makers and cooperative decision makers might have an idea of minimum farm size as a target variable of some sort. This minimally achievable farm size is denoted by a fixed level of output (which is \( \bar{q}_{ji} \) for output j and farm i).

During the socialist transformation, Hungarian agriculture made its contribution to the country's economic development in the following forms:

a. producing a larger amount of food for the increasing non-farm population;

b. supplying raw materials for the corresponding industrial branches;

c. providing export commodities in order to extend the import material base of developing industries;

d. producing "surplus" a part of which was taxed away in the forms of direct and indirect taxes;

e. releasing labor force in large (in years 1951-1952 larger than optimal) quantities to support the economic development outside agriculture.

In some periods of the socialist transformation, releasing labor force could be so important as to require a separate expression in the objective function. Labor transfer may assume the completely separate
form of an objective function without simultaneous maximization of agricultural output. Historical experience shows that appropriate constraints must take care of the target agricultural output (if such a target really exists in practice, as it should). In this case Expression 5.6 has become

$$\text{Max } \Delta F$$

subject to

$$\sum_{s} \sum_{j=1}^{n} p_j q_{ji} \geq Q_F,$$

where

$$\Delta F = \text{the number of units of labor transferred from agriculture to other sectors},$$

$$Q_F = \text{value of the target total farm output}.$$

Recalling some shorter periods of the socialist transformation in Hungary and in the Soviet Union, one may remember that Expression 5.6 appeared in a much narrower form. The system of compulsory deliveries of farm products indicated that the "marketed" (released) part of output gained first-rate importance in opposition to the total output, including farmers' self-supplying production and farm inputs. This narrower objective function is given by

$$\text{Max } \sum_{s} \sum_{j=1}^{n} p_j^T q_{ji}^{NF},$$

where

$$q_{ji}^{NF} = \text{quantity of product } j \text{ produced on the farm } i \text{ and leaving agriculture through different channels},$$

$$p_j^T = \text{purposefully depressed price of agricultural product } j.$$
In Expression 5.6b the quantity superscript (NF) indicates that the first three forms stated previously were emphasized (probably over-emphasized) among the five possible alternatives of agriculture's contribution. The superscript (T) of prices resembles the fact that extremely low levels of producers' prices were also applied as a means of taxation in channeling agricultural "surplus" (occasionally even more than that) to subsidize nonagricultural sectors. (This was listed in form d. before.)

New Sets of Constraints

Originating from the Socialist Transformation

The socialist transformation created a completely new organization and resulted in a new set of restrictions.

In contrast with the land reform of 1945, the social transformation of the Hungarian agriculture did not impose a general maximum restriction on the size of landholding of farms; rather it implicitly contained minimum-output-per-farm restraints as stated in Expression 5.6. At the same time an upper limit was placed on the size of land belonging to the household plots of individual cooperative members. This maximum size of homeplot land was restricted:

\[ a_m \leq 0.57 \text{ ha}, \]  

(5.7)

where

\[ a_m = \text{the size of homeplot land of cooperative member } m. \]

\[ \text{It was already mentioned that in most socialist countries as well as in the Soviet Union the cooperative farms allot a unit of homeplot land to every household and not to each cooperative member individually.} \]
As a function of the actual number of cooperative members, the total land area used separately by cooperative households can be defined at any point of time as

\[
\sum_{m=1}^{s'} a_m = H_h , \quad (5.8)
\]

where

\[m = 1, \ldots, s'\text{ the number of cooperative members,}\]

\[H_h = \text{the country's land area used as homeplots by cooperative members.}\]

As a result of the socialist transformation, the pattern of landholding has resumed a mathematical form expressed by

\[
\sum_{v=1}^{s^n} a_v + \sum_{u=1}^{s^u} a_u + \sum_{m=1}^{s'} a_m + \sum_{z=1}^{s^z} a_z \leq H , \quad (5.9)
\]

where

\[v = 1, \ldots, s^n \text{ the number of cooperative farms,}\]

\[a_v = \text{the landholding of the } v\text{-th cooperative farm considering only the land area of the large-scale cooperative farm (i.e., excluding the land area of the homeplots of cooperative members)},\]

\[u = 1, \ldots, s^u \text{ the number of state farms,}\]

\[a_u = \text{the landholding of the } u\text{-th state farm,}\]

\[z = 1, \ldots, s^z \text{ the number of agricultural decision units (land users) other than those mentioned above (this category is called "other farms"),}\]

\[a_z = \text{the landholding of the } z\text{-th units in the group of other farms.}\]

The expression 5.9 states the limited availability of farm land and it also allows the nonfarm uses of some arable land, which is increasing in the process of urbanization, road construction, etc.
The labor constraints on production are given in decomposed form by

$$\sum_{v=1}^{s_v} b_v + \sum_{u=1}^{s_u} b_u + \sum_{m=1}^{s_m} b_m + \sum_{z=1}^{s_z} b_z \leq F, \quad (5.10)$$

where

- $b_v = \text{the number of units of labor used by the } v\text{-th cooperative enterprise},$
- $b_u = \text{the number of units of labor used by } u\text{-th state farm},$
- $b_m = \text{the number of units of labor used by the homeplot farm of the } m\text{-th cooperative member},$
- $b_z = \text{the number of units of labor used by the } z\text{-th units of category of other farms},$
- $F = \text{total labor amount available for agriculture}.$

The functional form of the capital constraints reflects the fact that one part of agricultural capital is specifically adapted to the conditions of large-scale enterprises, and another part of the capital can satisfy only the needs of small-scale farming because of its physical form.

The first part of capital constraint is given by

$$\sum_{v=1}^{s_v} e_v k_v + \sum_{u=1}^{s_u} e_u k_u \leq K_{v+u}, \quad (5.11a)$$

where

- $e_v, e_u = \text{adaptation (a kind of production) coefficient},$
- $k_v = \text{capital used by the } v\text{-th cooperative enterprise},$
- $k_u = \text{capital used by the } u\text{-th state farm},$
- $K_{v+u} = \text{total capital available on large-scale (state and cooperative) farms}.$
The capital constraint of the small-scale farm operations is expressed as

\[
\sum_{m=1}^{s^m} e_m k_m + \sum_{z=1}^{s^z} e_z k_z \leq K_{m+z},
\]

(5.11b)

where

- \( e_m, e_z \) = adaptation coefficients,
- \( k_m \) = capital used by the \( m \)-th cooperative member's homeplot farm,
- \( k_z \) = capital used by the \( z \)-th units belonging to the category of other farms,
- \( K_{m+z} \) = total capital available for homeplot and other small-scale farm operations.\(^1\)

A smaller than unity value of coefficient \( e_{m} \) expressed, for example, the fact that some production means (small plows, hoes, etc.) in the possession of cooperative farms given by their joining members were not adaptable to large-scale operations. This was a characteristic during the first period of the socialist transformation. A value which is greater than unity for coefficient \( e_{m} \) may reflect the situation discussed later where the homeplot farming of cooperative members is making temporary use of large-scale equipment owned by the large cooperative enterprise.

One may apply here also a constraint standing for non-agricultural inputs which support farm production from outside the agricultural sector. Such an input is, for example, the capital embodied in the

---

\(^1\)Since \( e_{m} \) and \( e_{z} \) as well as \( e_{y} \) and \( e_{u} \) may theoretically be defined as constant coefficients in any time period and \( k_{y}, k_{u}, k_{m}, k_{z} \) are given in money terms, the value of \( K_{y+u} \) and \( K_{m+z} \) can be computed.
country's education system which provides necessary conditions for agricultural production.

\[
\sum v + \sum u + \sum m + \sum z \leq \theta
\]

(5.12)

where

\[ \theta_v, \theta_u, \theta_m, \text{and } \theta_z = \text{a specified input (as education) other than land, capital, and farm labor required by the previously described four sectors of Hungarian agriculture.} \]

The socialist transformation resulted in previously unknown values for coefficients \( a, b, c \) and \( k \). Consequently, the production matrix of agriculture was changed almost completely. The input mix of small-scale farm operations was also affected. As a result of extensive industrial development, labor ceased to be a surplus factor in the agriculture of most regions. In spite of its rapid growth, capital still remained a relatively scarce factor of farming. The new social and political structure of the country, as well as the general economic and cultural development, made a substantial change of positive character in \( \theta \). (For roads and communication facilities, etc. the same type of relationships are derivable as Expression 5.12 for education. Since each term is expressed in money form, an aggregate requirement of farming may also be computed with respect to the whole "infrastructure" of the country in value terms.)

By the time of the socialist transformation, national economic planning was introduced and the state sector became decisive or even exclusive in manufacturing, in home trade as well as in foreign trade, and in the financial sector. Central planning created new opportunities,
especially for price policy and capital supply concerning large-scale farming.

Using Expression 5.1 as an implicit objective function, a relatively simple model of the land reform may be set up. A model of the socialist transformation may be constructed by using Expression 5.6 as an objective function and minimum constraints together with Expressions 5.9, 5.10, 5.11a, 5.11b, and 5.12 as maximum constraints. It is obvious that the foregoing models present optimization problems which are naturally adapted to treatment by mathematical programming. There exists in each case an objective function whose variables are (implicit) functions of factors of production; i.e. the decision variables and the objective function variables can be connected by a production function. Finally, the problem is constrained by a set of inequalities involving the decision variables.

The restatement in this chapter may also serve as a mathematical summary of the whole Part I of this study. Hungarian cooperative farming, its major socio-economic characteristics and its economic model will be presented in Part II.
PART II.

COOPERATIVE MODELS
CHAPTER VI. THE MAJOR SOCIAL AND ECONOMIC FEATURES OF COOPERATIVE FARMING

The location of cooperative farms has been shown on the map of Hungary's farm organization. Now, a systematic procedure is presented for analyzing ideas and observations with respect to the cooperative form of farm organization. Chapters VI and VII provide a comprehensive survey and mathematical models of the major social and economic characteristics of Hungarian farmers' cooperatives.

The Dual Character of Farmers' Cooperatives

Socialist cooperative farming has thrown a new light on traditional farm organization. The United Nations' publication (62, p. 80) previously cited states:

"The term 'cooperative farm' covers several different kinds of farm organization, in which the principles of individual and joint ownership and operation are combined in differing degrees."

In practice there is much variety within the above general concept of the cooperative farm in which individual and collective objectives appear jointly and simultaneously.¹ Within this unified general conception of the cooperative farm, farmers' cooperatives have developed as the chief form of decision units in socialist agriculture.

¹In the United States, different kinds of farmers' cooperatives are in operation, for example, the marketing cooperatives of the Associated Milk Producers, Inc. The cited definition is also applicable to farm organizations such as family corporations, partnerships, and trusts.
Today Hungarian farmers' cooperative has a clear-cut dual character. On one side it is a social organization, and on the other, an economic organization, primarily a business firm (enterprise). As a social organization it undoubtedly belongs to a wider category involving all cooperatives organized in the fields of marketing, retailing, services, housing, savings, and manufacturing. In this one-sided aspect there is no substantial difference between cooperatives and other social institutions (for example, trade unions) according to the Marxist theory of political organization. The real difference is the double character of the cooperative as a social organization and as an economic organization.

Under the conditions of a socialist society, all types of cooperatives are of socialist character. Cooperative ownership as a form of socialist ownership, though different from state ownership which is the property of the nation, is by no means a lower-grade type. The cooperative organization is no less consistently socialist in its social character. It is, however, obviously a narrower form of ownership in comparison with state ownership since the property of state-owned factories and farms are conceived of as the property of the whole nation. At the same time, cooperative farms represent a much broader type of ownership in comparison with individually owned small farms.¹

¹It had been one of Stalin's theoretical misconceptions that farmers' cooperatives (kolkhoz) represented a second-rate, non-consequent form of socialist ownership in contrast with a state-owned business firm or farm (sovkhoz). The linkage is not weak and not imaginary between this concept and the actual policies symbolized by Expression 5.5b as an objective function.

At the same time, Marxist theory validly emphasizes that cooperative ownership qualifies as socialist ownership only under the conditions of a socialist political system and worker-peasant state government.
Agricultural economists in the socialist countries had to recognize that it was not possible to understand the complex nature of cooperative farming, and even much less possible to solve its important problems if the farmers' cooperative is not conceived of as a defined economic organization. Sticking to such basically non-economic concepts as "farmers' association," "farmers' union," "social mass organization," or "cooperative membership," neither the objectives nor the material interests associated with cooperative farming, with its individual members and managers can be clearly shown. Not even a comprehensive unified model of farmers' cooperative can be unambiguously formulated. The conclusion is drawn that the farmers' cooperative must be conceived of as an economic organization embodied in the cooperative farm.

The cooperative farm consists of two constituent parts: the large-scale cooperative firm and the small-scale homeplot farms of cooperative members. The large-scale cooperative farm is considered as a business organization and, because of collective-cooperative ownership, as a special type of business firm characterized by collective management and decision-making at the top level; and by its members as being both partners (co-owners) and laborers (employees). The members' homeplot farms must also be conceived of as an economic, partially business-type organization.

In the Soviet Union the first cooperatives of agricultural producers came into being 50 years ago and in the Eastern European countries some 20 years ago. The established system of cooperative farming can look back on a past of four decades in the Soviet Union and only a decade in the
rest of the socialist countries. In the beginning, farmers' cooperatives appeared on the stage of history as socio-economic organizations of such a special character that even the first statutes did not define what kind of organization they were in an adequate economic sense. Beyond the initial stage of their historical development, it has become evident that farmers' cooperatives are, on the one hand, social organizations operating in socialist cooperative form and, on the other, economic organizations which are centered on large-scale collectively-owned enterprises pursuing economic objectives.

An important step in the strengthening of their socialist cooperative character was made by the Land Act passed by Parliament in 1967. This Act has opened the road to the transfer of land into cooperative ownership. In settling the question of ownership, a distinction was made between the members' land ownership and the ownership of outsiders living and/or working in cities. The Act decreed the compulsory purchase of

1The first so-called Standard Statute of Kolkhozes in the Soviet Union was adopted in 1933 and the first cooperative statutes of all other socialist countries are the rules of socio-political association rather than directives on the economic organization of business enterprises. Surprisingly it is also true that at the 6th Congress of Soviets in 1931 the kolkhoz was referred to as "an enterprise founded by peasants voluntarily collectivizing their own productive resources, and managed by themselves." Yet later on, as Erdei (20, p. 16) notes, "its character as enterprise became blurred, and several decades had to pass before these aspects came again to the fore."

2Act IV of 1967 on "Certain questions of land ownership and land use" is analyzed by Nagy (66).

3It has been estimated that in 1967 about 18-20 per cent of cooperative land was owned by such "third" persons and this percentage was estimated to increase by 1-2 per cent every year.
the lands of the latter, and their compulsory acquisition by the farmers' cooperatives to come into force by January 1, 1969. Thus landowners had to decide themselves whether to accept membership in the cooperative and to continue to be owners of their land and receive the land rent; or not to join the cooperative and to let the Act's provisions for compulsory purchase be applied. The Act also decreed that the cooperative was obliged to indemnify the owners for the land whose title it acquired.

The Land Act of 1967 set out to have the land freely offered for sale by the cooperative member in a mutually satisfactory free agreement between the cooperative and its member. In this category the indemnity is twice that which applied to outsiders. The new regulation enabled the cooperatives to buy land from private persons and from the state.

As a first step of extending the sphere of cooperative ownership, farmers' cooperatives became eligible to buy large-scale machines in 1957. As a second step, they bought, gradually, all the machinery from state-owned machine and tractor stations (MTS). As a third step, since 1967, the cooperatives could become owners of the land they use. As a fourth step, farmers' cooperatives may now also run auxiliary enterprises (e.g. food processing plants, smaller-scale mines) which were monopolized by state-owned firms prior to the reform of the economic mechanism.

1 The Act allowed the owner to request that his land be exempt from compulsory purchase. There were two causes for exemption: (a) 0.57 hectare land might be left to each owner; (b) all land of the owner might be exempted if the owner was an elderly person whose only livelihood was the land rent that he received from the farmers' cooperative.
Large Cooperative Enterprise and Small Homeplot Farms

At present, cooperative farming is a peculiar and complex form of agricultural organization in which the new, large-scale cooperative pattern co-exists with the still remaining (and adjusting) elements of the earlier system of individual family farms.

The development and expansion of large-scale cooperative farms took place in Hungary simultaneously with the economic progress of individual peasant farms rising to a higher technical and organizational level. The collective cooperative farms did not confront the small-scale individual farms as a hostile power, but they encouraged and organized their gradual union into large-scale farming. By this transformation the small peasant farms did not use up all their available material resources but transferred them into the framework of cooperative farming.

When a cooperative farm was established, only a part of the productive resources (land, livestock, machinery and equipment) of the farmers joining the cooperative were taken into collective use; the rest remained under the individual management of the owner-farmer (now a cooperative member) within the frame of his small-scale homeplot farm. The united, collectively used means of production (which are also collectively to be replaced and increased in the future) constitute the technical base of a large-scale cooperative firm (enterprise). The means of production re-

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1 In this paper, the following terms are used as synonyms: large cooperative enterprise, large-scale collective enterprise, cooperative enterprise, cooperative firm. All these terms denote the large-scale portion of the farmers' cooperative. The expressions farmers' cooperative, agricultural producers' cooperative, cooperative farm and collective farm are also used synonymously. The terms homeplot farm, household plot farm or household farm also have the same meaning.
remaining in individual use serve as a foundation for the homeplot farms of the cooperative members. Thus, homeplot farms come into existence simultaneously with a peculiar separation of ownership of productive resources, earlier owned privately.

During the period when the early cooperatives were organized, one of the most critical problems was to decide which were the productive resources necessarily, feasibly, and justifiably to be taken into the large-scale collective farming. From a purely organizational point of view, scattered buildings suitable only for a few animals and their fodder, or certain simpler working tools useless for large-scale operation, could not be taken into the collective farming. At the same time, in the newly established collective enterprises there were no appropriate buildings even for the livestock at hand; therefore, a part of the cooperative-owned livestock was temporarily left out of the large-scale firms. Also when the land use was united, a number of technical problems arose — e.g., scattered vineyards and orchards, gardens round the farmhouses, dispersed dwelling houses in "tanya" areas hemmed in between ploughlands — all complicated the cooperative development and inhibited the rapid formation of the large-scale collective firm.

However, economic and organizational factors constituted only a part of the difficulties; another part is of a subjective and political character. A complete elimination of private resource ownership and individual resource use would have meant a disregarding of all these relevant personal preferences, and it could have led to disastrous

1The Hungarian name of isolated farmsteads.
consequences. Thus, the establishment of household plots and homeplot farming, as well as their maintenance at present and in the future is justified by various factors and multiple reasons (29, 30).

The characteristic form of farmers' cooperatives so far developed is such that a proportion (although not a fundamental one) of productive resources is individual personal property in the household plots of the members. The existence of homeplot farming does not affect the social character of producers' cooperatives; moreover, at present it is a conspicuous feature — though not indispensable from an academic point of view — of the socialist cooperative character.

As regards resource ownership relations, homeplot farms are built on a double fundamental: 1. on the extending collectively owned resources of increasing importance; i.e., on the partial use of modern large-scale forces of production in cooperative property; 2. on the individual private possession and use of means of production (of decreasing importance, except for some livestock) inherited from former family farms. This double fundamental may be theoretically antagonistic in itself, still it does not necessarily cause unmanageable conflicts between the large-scale cooperative enterprise and homeplot farms or between the interests of different groups of cooperative members. Contradictions and conflicting

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1 The analyzed complexity scores to explain the fact that in official statistics (as well as in Table 3.6 of this study) large-scale cooperative enterprises and small-scale homeplot farms are treated under the common heading of "farmers' cooperative sector," but the data on homeplot farms are also given separately. Rightly so, since the cooperative enterprise and homeplot farms form a closely coordinated decision unit and yet homeplot farms as disintegrated subunits have to be taken into account separately at the same time.
economic interests arise as differences and conflicts within the same comprehensive decision unit — the socialist cooperative farm.

The homeplot farm is finally, a socialist economic formation but, at the same time, it is more detached and more disintegrated than other sectors of the socialist economy. Homeplot farming is a small-scale operation based on a special kind of socialist ownership of productive resources. Close cooperation with the large-scale cooperative enterprise and the favorable conditions created by national economic planning may moderate but cannot entirely eliminate the various inner economic limitations of the small size of homeplot farming.

The large-scale cooperative enterprise and its members' homeplot farms are thus not independent units but essential parts of one single economic entity: the cooperative farm. Members and managers of the large-scale enterprise are guided in their decisions at an increasing rate by the recognition of the complementary and competitive needs of both the large-scale farm and the small-scale homeplot operations.

As a rule the large-scale collective firm contributes to the efficient and profitable utilization of the productive capacity of homeplots by mechanical soil cultivation, and transportation; by supplying fodder in the framework of contract and by different organizational

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1 In socialist and in Western countries one hears more than once the opinion that homeplot farming represents a definite private sector and it is simply a manifestation of peasant individuality, a compromise with the peasants' psychology. This idea is expressed in its crudest versions as "homeplot farming carries the germ of the revival of capitalism." As Fekete and Varga (29) argue, these opinions reflect a great misjudgment of the basic social and economic character of the homeplot farms of cooperative members.
and advisory (extension services) activities in the field of plant protection, disease control, marketing, etc. The larger the scale of home-plot farming and the more diversified its production, usually the more it requires the various services of the large-scale cooperative enterprise.

As at many places, homeplot lands are involved in the crop rotation of the large cooperative enterprise in the same way homeplot farming is everywhere included in the "cooperative entity," i.e., in the economic union of the large-scale collective firm and of small-scale homeplot operations. The food supply of the family household and many other commodities are produced by homeplot farming, but all these originate largely from the fodder supply and cornfield utilization furnished by the large collective enterprise.  

The market production of homeplot farms clearly bears the character of an income-earning enterprise. At the same time, the self-supplying (producing for subsistence) character of homeplot farming is also evident. Thus the functions of homeplot farming are determined simultaneously by the household, which is the decision unit of consumption and of labor supply, and by the profit incentives of market production. In addition to supplementary income, the household plot also provides a place and opportunity for recreation and leisure activities, such as gardening, bee-keeping, and some livestock (and pigeon) raising.

1Generally corn is planted on 80 per cent of the homeplot land of cooperative members, and soil preparation, fertilization, planting, and transportation of crops are completed with the equipment of the large-scale cooperative enterprise.
Usually the older farmers are willingly occupied with these activities because they are used to them and, furthermore, they like them. As a new generation substitutes the old, the recreation function of homeplot activities may take first place.

As an interim summary, the dual character of the farmers' cooperative as well as the bilateral relationships between the large-scale collective enterprise, the members' homeplot farm and the members' household is sketched in Figure 6.1.

The cooperative farm, the large-scale collective enterprise, as well as the homeplot farm and the cooperative household are decision units in microeconomics. Their multilateral relationships and connections with the economic life outside the cooperative demonstrate that micro-units are "drops in the ocean" of the national economy.¹

Before turning attention to the large-scale enterprise, a closing remark is due on the perspective of homeplot farming.

In the coming years homeplot farming will play an important role in the market production of agriculture as a whole, apart from the direct food supply to the owners' household. By the same token, one must take into consideration the facts that homeplot farming will some day become extinct. More specifically, the homeplot farms have to lose their economic importance both in food supply and as a source of personal income. The main preconditions are, on the one hand, a consid-

¹As regards cooperative farming, Shubik's remark (74, p. 345) is evidently relevant: "The development of micro-economic theory has been entering a stage similar to that experienced by twentieth century physics in its break away from the simple comfortable closed system of its precursor."
Fig. 6.1 Dual character and structure of a farmers' cooperative
erable increase in the technical level and the performance of large-scale production; and, on the other hand, the substantial increase in the cooperative members' personal income derived from the large cooperative enterprise — an increase to the general wage level of alternative (non-farm) employment. As these conditions are gradually met, the living conditions and the work-leisure preferences of members of farmers' cooperatives will approach those of other persons who have non-farm occupations and reside in the countryside. On homeplots, future activities will be in the categories of recreation and sport; production itself will always be of secondary importance and market production will be entirely lacking.

Membership, Management and Decision-making

The socio-economic (cooperative) character of the cooperative farms gives the members the co-owners' standing of the large-scale enterprise with the right to decide collectively on all the principal questions of cooperative activities. At the same time, the large collective enterprise is based on the work of the cooperative members who may be considered as an employee of their own collective enterprise in this respect.¹ This dual (co-owner and labor supplier) role of cooperative members is crucial when one views the human and social relations and decision-making processes in cooperative farms, especially within

¹"The owner as employee" case is discussed by Harl and Timmons (40, p. 59) concerning corporations where a partner in a partnership becomes an employee of the corporation upon incorporation of the business and when "a new relationship is created by law between the corporate entity and the people whom the corporation employs to conduct its business."
large-scale collective enterprises.

Erdei (24) puts a high emphasis on the fact that cooperative membership itself is not sufficient for the optimal organization of a large-scale collective cooperative enterprise. For this, the precise regulation of the members' employment relations is also required. The theoretical problem involved in all this is: how the co-owner and employee characters of the cooperative members can be reconciled in such a way as to create consistency between the objectives of the large cooperative enterprise management and the objectives of the individual member-employees.

Economic and organizational relations within a cooperative are regulated by its statutes. These statutes lay down the rights and responsibilities of members, the method of election to the leading organs of the cooperative, the principles of work organization and income distribution. Certain binding regulations are, of course, laid down by law but within these rather broad limits each cooperative farm itself determines its own detailed statutes. The statutes discussed and passed by the assembly of the cooperative's members must be approved by the appropriate, regional state authority in order to make sure compliance with the legal rules and regulations has taken place.

Once its statutes have been approved, every cooperative is entered on the official register of cooperative farms — somewhat similarly to the incorporation of a firm in the United States.

The microeconomic and legal concept of the firm is so broad that it is applicable to the cooperative farm without any theoretical diffi-
In spite of this fact, it was widely debated for a long time whether an emphasis on the enterprise character of the large-scale cooperative firm was justified and whether categories of wages, net revenue, profits, capital consumption allowance (amortization), etc. customary in firms (enterprises) as such should be applied to farmers' cooperatives. The debate has already been decided since "civil rights" of these categories had been "declared" by the practices of cooperative farming itself, and the same stand was unequivocally taken by the reform of the economic mechanism and by legislation.

The statutes provide the right of each cooperative member to participate equally in making collective decisions on resource allocation and income distribution of the large-scale cooperative enterprise (firm).

In detail, the members' most important rights are the following:

- right to take part in the General Assembly and equal right to vote;
- right to be elected to any office in the farmers' cooperative;
- right to share in the cooperative income according to labor performance;
- right to the rent for land owned transferred to the cooperative;
- right to retain or claim the individually used (household) plot in the conditions specified by the law and the statutes;
- right to old-age pension.2

In microeconomic theory, the firm is uniformly qualified as an autonomous decision (administrative) unit which transforms inputs into outputs in seeking some entrepreneurial objective function consistently with a technical production function (38,49,50).1

The recent pension system for members of farmers' cooperatives, introduced in 1968, places the retired cooperative members in pension categories on the basis of their personal income earned in the large cooperative enterprise. There are, altogether, 21 pension-categories. Duration of cooperative membership is considerably emphasized in determining the sum of the pension.
The most important duties of the cooperative members are:

- the transfer of land and equipment (in excess of the approved needs of homeplot farming) to the large collective enterprise at the time of joining the cooperative;
- participation in the activity of the collective enterprise by supplying labor.

The inner organization of a large-scale cooperative enterprise is given in Figure 6.2.

The highest authority in the farmers' cooperative is the General Assembly. (In cooperatives of a very large size it is replaced by the Assembly of Delegates.) The major responsibilities and functions of the General Assembly are:

- election of the leading bodies;
- approval of the long-run and annual farming plans determining the allocation of collective resources;
- approval of the yearly final accounts determining income distribution;
- contracting credits above specified limits;
- admission and exclusion of members.

The General Assembly elects the management responsible for the operative direction of the farmers' cooperative as an agricultural firm (enterprise). The number of administrative board members is generally between seven and seventeen. As a rule, the General Assembly also elects a control commission and a disciplinary commission, and various committees concerned with homeplot farming, cooperative welfare, social security, etc.

The president is the responsible leader and legal representative of the cooperative, who directs the farming and administration in the name of, and relying upon, the management. The president is elected for a four-year period. A vice-president is also elected. Very often the chief
Fig. 6.2. Scheme of decision-levels and management in large cooperative firms
agronomist is elected to be vice-president if he is a member of the cooperative. (If it is necessary, managers may also be employed by the cooperative from outside the cooperative members.) The vice-president may be elected with a specified responsibility, e.g., the direction of all the subsidiary enterprises (machine repair shop, construction work, food processing, etc.) and/or marketing activities. The other leaders of the cooperative are the chief agronomist, the chief zootechnician, the chief accountant, the heads of subsidiary enterprises, etc., as is shown on Fig. 6.2.

Fig. 6.2 also illustrates the structure of production and labor organization. The basic production units of the large cooperative firm are generally the brigades, which at the same time constitute units of labor organization; their responsible head is the brigade chief (foreman) who is entrusted by the management with this function. Within the brigades and other productive units (such as dairy farm and pig-feeding operation) working groups, generally of 5-15 persons, are organized if a good deal of manual work is to be done because of the low level of mechanization.

The machine tools are generally kept in a centralized organization on smaller collective farms but the crews working with tractor-drawn machines are assigned to the brigades. On larger collective farms the so-called complex production brigades (units) are equipped with the machines used throughout the year.

In the case of row crops requiring much manual labor, most collective enterprises distribute among the members the planted areas, to be
individually cultivated. The essence of this family system of cultivation, regarded as a Hungarian "innovation," is that side-by-side with the mechanized and team work carried out by large-scale operation, the typically manual work is undertaken by the individual members in order to cultivate a given land area by small-scale family operation. In the future when these processes will be mechanized in an efficient and profitable manner, no such organization of production will be necessary.

In each cooperative the distribution of the income of the collective enterprise is planned ahead in the framework of the yearly working plan, and it is incorporated in the annual final accounts, both of which are prepared by the management and have to be discussed and adopted by the General Assembly.

Income is distributed partly in accordance with the land transferred to the cooperative enterprise (land rent), and, for the most part, according to the labor input performed; and it is paid partly in money and partly in kind. Share in the income was exclusively, and still is for a large but rapidly decreasing part, based on what are called "work units."\(^1\) The work unit stands for the average work (concerning intensity, physical and skill requirement) performed during a typical working day, and the different labor operations performed are converted into this equivalent. The work unit system determines only the relative labor incomes of members (both a daily and yearly sense) since the level of income

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\(^1\)The work unit is an intra-cooperative category adapted from the practice of the Soviet kolkhoz system. Authors also use the term labor unit.
due to a work unit depends on the total income to be distributed. The money equivalent ("value") of a work unit labor performance varies from year to year within the same cooperative and from cooperative to cooperative in the same year (11).

Some cooperatives apply the system of remuneration or share in income, whichever it may be, reckoned in percentages of the output (mainly crop yield). An ever-increasing number of farmers' cooperatives provide fixed labor allowances (wages) combined with profit sharing. ¹

As Erdei (23, p. 36) pointed out, in many Hungarian cooperatives "the leadership is called the management," and this is neither a problem of use of the language nor a matter of nomenclature. It rather expresses the general practice that the elected cooperative body is de facto managing the large-scale cooperative operations with full responsibility for the operative farming decisions. At the same time, the top-level decisions, among these decisions on farm plans, resource allocation and income distribution, are still made by the Assembly.

Within the frameworks of cooperative farming (depicted in Fig. 6.1) and of large-scale collective organizations, cooperative families make decisions as (a) consumers and (b) suppliers of production factors (mostly labor) in their households associated with their homeplot farming, at the same time the cooperative members of families make decisions as

¹As Fekete (28, Chapter IV) argued from the points of view of societal, economic and business organizations, wages fixed at the level of income from alternative (non-cooperative) employment and profit sharing are most consistent with the future model of cooperative farming.
(c) entrepreneurs and (d) co-owners by voting in the General Assembly of the cooperative. If cooperative family members are also considered as (e) voters in political decision making, the multiple role of cooperative families appears in a comprehensive pattern which is similar to the decision-making functions of private farm families analyzed by Heady (42, p. 496).

In their economic decisions concerning homeplot farming, cooperative families are generally characterized by a limited planning horizon and by a sort of "family farm cycle" investigated in the research of Heady, Black and Peterson (46). In their business decisions on the large cooperative enterprises, members may be faced with an extended, almost unlimited planning horizon which is known as a characteristic of incorporated firms (38).

Cooperative Democracy and State Guidance

The principle of cooperative democracy requires that farmers' cooperatives are functioning as democratically led communities and the state exercises its role of national economic guidance toward them in a specific manner.

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1 This study is concerned with the different types of decisions listed above in its corresponding sections; Chapter IX deals with decisions belonging to categories c and d, Chapter X with decisions in category b; Chapter XI focuses on decision categories a and e.

2 Traditional family farming follows the pattern of a "family farm cycle" because the farm business parallels closely the personal life cycle of the sole proprietor who is the head of the family. Since family farm organization units are "born" and also "die" within or with each generation, inefficiencies due to shortages of management ability and capital occur in the early years as well as in the declining period of farming (38, 46).
In the contemporary system of economic mechanism, the state guidance of the cooperative farms is, besides legal supervision, of entirely economic-policy character. State guidance respects fully the principle that the cooperative farm is the collective, cooperative property of the members, and thus the guidance of the state does not mean the right to give orders. It endeavours to achieve its economic targets by influencing the cooperative farms in the framework of the national economic planning, by economic incentives such as granting favorable-term credits, fixing and varying prices, production and marketing contracts, technical advice, information, recommendations, etc.

The annual farming plan and the yearly final accounts are the two main links with which farmers' cooperatives are fitted into the socialist planned economy as a whole.

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As far as state firms are concerned the State is the owner and, moreover, the manager (director) appointed by a government body is the one-person representative of the State in the management of state firms. At the same time, the principle of self-management concerning cooperatives, is recognized and applied; in their operation the State does not "infer qua owner" (18).
CHAPTER VII. MATHEMATICAL MODELS OF COOPERATIVE FARMING

In order to construct mathematical models of farmers' cooperatives, one finds algebraic expressions for relationships and interdependencies as analyzed in the last chapter and partially depicted by Figures 6.1 and 6.2. At first the model-builder may formulate a general behavioral model of cooperative farming. Then as the central construction, a linear programming model of farmers' cooperatives can be set up which is followed by a simple cooperative model placing emphasis on investment and technology. Finally, a separate section provides a relatively large set of equalities that specify the real content of economic categories prevailing in socialist cooperative farming.¹

In this chapter the constructed models are of a generalized type to serve as a basis for various optimization procedures and for further research orientation in this fairly unexplored but rapidly changing field.

The Behavioral Model

The first model presented may be called the "behavioral model of cooperative farming." This model consists of functions and equations specifying the manner in which a variable behaves or responds to changes in other variables. Human behavior (such as cooperative members' labor supply pattern in relation to alternative wage rates) as well as

¹In this section an attempt has also been made to give mathematical forms corresponding to the two different economic terminologies commonly used in the United States and in Hungary.
nonhuman "behavior" (such as how the total revenue of a cooperative enterprise reacts to output changes) will be treated in this model. Behavioral relationships expressed in mathematical form will be used to describe the general organizational setting of a cooperative model, including the technological (for example, production function) and institutional (for example, homeplot farming structure) aspects.

The model construction starts with the more complex yet smaller microeconomic unit of the cooperative farm organization, with a cooperative household and with its homeplot farm. The **production function for homeplot farming**

Each cooperative household is furnished with a homeplot. The production function for the homeplot farm of a cooperative household is denoted by

\[ f_i (L_{ui}, h_i, K_i, X_i, q_{i1}, \ldots, q_{ji}, \ldots, q_{ni}) \geq 0. \quad (7.1) \]

In Expression 7.1 the letter \( i \) refers to the \( i \)-th cooperative household and \( i = 1, \ldots, M \). Each homeplot is potentially a producer of any usual farm output; \( q_{ji} \) denotes the output of commodity \( j \) produced on the \( i \)-th homeplot and \( j = 1, \ldots, n \). The quantity of the output is determined by the first four components of the production function:

- \( L_{ui} \) is the vector of the \( u \)-th type of labor inputs used on the \( i \)-th household plot from members of this household and in general \( u = 1, \ldots, U \) but \( u = 1, \ldots, s \), if the cooperative household includes \( s \) persons and each person represents only one kind of labor;

- \( h_i \) is the quantity of land available in the household plot;

- \( K_i \) is the capital used on the \( i \)-th homeplot;
\( X_i \) = the vector of processing services supplied to the \( i \)-th household plot by the large cooperative firm.

The production function has been expressed implicitly. The first four variables are inputs and the last \( n \) variables are outputs. It may arbitrarily be specified that the partial derivatives are positive with respect to the inputs and the derivatives are negative with respect to the outputs. If capacity output (relative to the inputs) is being produced, the function equals zero. The function is greater than zero, if excess capacity exists (that is, if the outputs are less than obtainable from the given inputs). The marginal products of the various kinds of labor are written as the matrix

\[ f' \]

A single entry in Expression 7.2 serves as an example

\[ \frac{\partial q_{ij}}{\partial l_{ui}} \]  

(7.2a)

which is understood as the marginal product of the \( u \)-th type of labor in the production of commodity \( j \). In the matrix denoted by Expression 7.2 for each commodity there is a column containing the marginal products of various types of labor; for each type of labor there is a row of marginal products.

The income function of cooperative household

A cooperative household has one or more persons possessing membership in a farmers' cooperative. It is not required by this term that the head of the household be a cooperative member. Full cooperative membership and the right to a homeplot land area is an essential criterion of such a household. According to recent practice, a cooperative
member has the right to a homeplot individually; therefore, if the cooperative household includes two or three cooperative members, it may have twice or three times as large a homeplot land area as another cooperative household which only one cooperative member belongs to.

The income function of a typical cooperative household is given by

\[ Y_i = \sum_{u=1}^{\ell_i} (v_{eu_1}e_1, v_eL_{e1}, V^*L^*_1, I^*_1) \]

where

- \( Y_i \) = the flow of spending power or its real good equivalent to the i-th cooperative household and \( i = 1, \ldots, M \) as before;

- \( v_{eu} \) = the wage rate paid by the large cooperative enterprise for the u-th type of labor to the person in this household who is a member of the cooperative in question;

- \( l_{e1} \) = the labor supplied by the cooperative member of the i-th household to the large cooperative enterprise (the quantity of labor is expressed in work hours);

- \( V_e \) = a vector of the wage rates paid by the large cooperative enterprise for various kinds of labor inputs of persons of the i-th cooperative household who are not members of the cooperative;

- \( L_{e1} \) = the vector of the hours of various kinds of labor supplied by the members of the i-th household who are not cooperative members but supply labor to the large cooperative enterprise;

- \( V^* \) = the vector of wage rates obtainable in non-cooperative employment;

---

1. It should be clear from the previous chapters up to the present that the larger part of Hungarian farmers' cooperatives has not adopted a direct wage system. However, a considerable (and rapidly increasing group of cooperatives) applies this system in various forms.

2. These persons are called "helping family members."
\[ L_i = \text{the vector of hours worked by the members of household } i \text{ in various non-cooperative employment}; \]

\[ L_i = \text{the vector of hours of different kinds of labor supplied by the } i-\text{th household to its own homeplot farm}; \]

\[ f_i = \text{the matrix of marginal products of } L_i \text{ on the homeplot farm (see Expression 7.2)}; \]

\[ P = \text{the vector of prices of the outputs produced on the homeplot farm; some of these may be the accounting prices of products for which there is no equivalent commodity on the market.} \]

In Equation 7.3, and everywhere later on, subscript \( e \) indicates the large collective enterprise; its "range" is \( e = 1 \) in this model which deals with one cooperative farm incorporating a large-scale portion and all its members' homeplot farms. Each of the first four terms of Equation 7.3 includes a labor supply variable. Thus, each of these terms contains a household decision variable. (If Expression 7.3 did not contain decision variables, it would reflect an accounting relationship rather than a behavioral function.) Only one of these variables has a formal constraint. It is a minimum type constraint expressed by

\[ L_{ekt} \geq a^0_{kit}, \quad (7.4) \]

where \( a^0_{kit} \) is the number of hours of work in the large collective enterprise (as determined by the General Assembly) required from the \( k-\text{th} \) member of the \( i-\text{th} \) cooperative household in time period \( t \).

Expression 7.4 is a product of both institutional and technological considerations. Membership in a farmers' cooperative is essentially based on a regular contribution of labor to collective farming. The large cooperative firm is basically dependent upon the members' labor supplied in harmony with the seasonal nature of agricultural production. When the General Assembly of the cooperative enterprise decides upon a
minimum labor quantity (for example 1,200 work hours per calendar year) it also specifies that a part of the labor has to be supplied in certain peak labor periods.

The remaining terms in Expression 7.3 are not decision variables for the cooperative household but they affect the total flow of spending power to this household:

\[ P_{ei} = \text{the household's share of the cooperative’s dividend (profit) including land rent;} \]

\[ W_{ei} = \text{payments from the cooperative welfare fund to the household;} \]

\[ T_{i} = \text{non-wage income of the i-th household from non-cooperative firms;} \]

\[ W_{i}^{*} = \text{payments to this household from the welfare fund of non-cooperative employers;} \]

\[ W_{i}^{**} = \text{payments to the household from the centralized, national welfare fund administered by a state agency;} \]

\[ I_{e} = \text{the household’s implicit share in the net investment of the collective cooperative enterprise.}\]

Later, particularly in Chapters X and XI, different objective (welfare) functions will be set up which will contain also this latter group of variables.\(^3\)

---

\(^1\)This model does not consider land rent as a separate item of members’ income mostly because of the perspective of collective-cooperative land ownership discussed in Chapter VI.

\(^2\)This share is implicit in the very sense of the word since the net investments make up the so-called "Indivisible Fund" of the large cooperative firm. As the name shows, this fund cannot be divided among the cooperative members in the event of cessation of membership.

\(^3\)See especially Equations 10.2 through 10.4, and Expressions 10.29 and 10.32.
minimum labor quantity (for example 1,200 work hours per calendar year) it also specifies that a part of the labor has to be supplied in certain peak labor periods.

The remaining terms in Expression 7,3 are not decision variables for the cooperative household but they affect the total flow of spending power to this household:

$$\Pi_{ei} = \text{the household's share of the cooperative's dividend (profit) including land rent;}$$

$$W_{ei} = \text{payments from the cooperative welfare fund to the household;}$$

$$\Pi_i^* = \text{nonwage income of the i-th household from non-cooperative firms;}$$

$$W_i^* = \text{payments to this household from the welfare fund of non-cooperative employers;}$$

$$W_i^{**} = \text{payments to the household from the centralized, national welfare fund administered by a state agency;}$$

$$I_e = \text{the household's implicit share in the net investment of the collective cooperative enterprise.}$$

Later, particularly in Chapters X and XI, different objective (welfare) functions will be set up which will contain also this latter group of variables. 3

---

1 This model does not consider land rent as a separate item of members' income mostly because of the perspective of collective-cooperative land ownership discussed in Chapter VI.

2 This share is implicit in the very sense of the word since the net investments make up the so-called "Indivisible Fund" of the large cooperative firm. As the name shows, this fund cannot be divided among the cooperative members in the event of cessation of membership.

3 See especially Equations 10.2 through 10.4, and Expressions 10.29 and 10.32.
The production function of the large cooperative enterprise

Large-scale cooperative farming enterprises are multiproduct and multiprocess in nature. Many of the processes are such that a part or all of their output is an intermediate good applied either to other processes in the large-scale enterprise or to the homeplots. Some processes produce final outputs, intermediate products, and outputs that are inputs to homeplot farming. \(^1\) It is useful to write to production function in such a fashion as to view the large cooperative enterprise as utilizing factors to create processes and using these processes to produce final outputs. In its simplified form this production function is given by

\[
f_e (y_1, \ldots, y_r; x_1, \ldots, x_m; q_1, \ldots, q_n) \geq 0, \quad (7.5)
\]

where \(q_1\) to \(q_n\) are final outputs, \(y_1\) to \(y_r\) are factors of production (any of which may be fixed in quantity). The \(x_1\) to \(x_m\) are processes or activities, the concept made familiar in linear programming. If relevant, different \(y\)'s can be considered as labor supplied in different seasons or labor classified by whether or not it is supplied by a member, by a member's household or by an employee not belonging to a cooperative household. The \(x\)'s may be classified with respect to whether they provide final outputs or inputs to other processes or inputs to homeplots or some combination of the above. This approach yields operational flexibility for the model. The production function of the collective firm,

\(^1\)In this respect, the feed and fodder supply to homeplot livestock is a well-definable task of the large cooperative firm. The lack of an appropriate-size feed market explains why the extremely dense livestock population of homeplot farms (indicated by Table 3.6) depends heavily on the feed and fodder production of the large cooperative enterprise.
$f_e$, is an increasing function of $y_1$ to $y_r$ and a decreasing function of $x_1$ to $x_m$. An increase in a supply of factors increases production possibilities. An increase in the utilization of any of processes decreases residual capacity. Constraints can be applied as by their limitations on the $y$'s or $x$'s. In fact, both cases will appear and provide great flexibility in treating various types of short-runs and institutional situations. Of course, this approach is only one of the possible alternatives.

In every strictly short-run situation where all processes are fixed and can only be varied with respect to the intensity of application, one encounters a typical linear programming problem. The objective function is linear and separable within some domain of relevance. Without loss of realism one may also view the variable costs per unit of output to be constant.

The Linear Programming Model

The large-scale enterprise

At the start, a "direct requirements matrix" for final outputs is set up in the construction of the linear programming model of a large cooperative enterprise. A final output may be sold on the market or may be "marketed" within the given cooperative organization. The latter refers to the feed and fodder output of the large collective enterprise which is sold to its members as inputs to homeplot production. The system of production coefficients will be set up in a somewhat different manner as it is usually encountered in linear programming models. The first matrix of this model is analogous to the direct requirement matrix
of a Leontieff-type input-output model and it is not the coefficients of the matrix of production processes. The input-output or the direct requirement matrix is given by

\[ q_1 = \alpha_{11}x_1 + \cdots + \alpha_{1m}x_m \]

\[ \vdots \]

\[ q_n = \alpha_{n1}x_1 + \cdots + \alpha_{nm}x_m. \]

Equation 7.6 contains all the final outputs (q's) that could be produced as well as all the conceivable processes or activities (x's) that could be utilized by the large collective enterprise. In other words, n is total number of final outputs, no matter whether they are sold on a usual market or on the "internal cooperative market." In similar manner, m is always the number of activities no matter where they are used. Thus, some of the \( \alpha \)'s may be zero in Equation 7.6.

The next group of equations is associated with activities which are not assignable directly to any final output. Applying the term used in bookkeeping and budgeting, this subset of activities results in output of an over-head type. Top-level management always, road construction and other building processes as well as complex investments and repair shop works in many cases, make up the over-head services as outputs, the requirement matrix of which is given by

\[ z_1 = \delta_{11}x_1 + \cdots + \delta_{1m}x_m \]

\[ \vdots \]

\[ z_n = \delta_{n1}x_1 + \cdots + \delta_{nm}x_m. \]

The outputs (z's) are essential, especially for large-scale farming
but they are generally not final outputs on the part of an agricultural firm. The number of these outputs goes up to \( n' \), and the \( z \)'s may use scarce inputs, scarce processes as expressed by Equation 7.7 and thus can be competitive with final outputs.\(^1\)

The third type of the large enterprise output is represented by community services which are not assignable as production costs. Cooperatively financed whole-year local cultural and cooperative welfare institutions together with seasonal lunch programs and daily care centers (the latter for members' children, the former for members and family helpers and/or for their children) may promote large-scale farming operations in several ways, but their cost cannot be added to the direct cost of a unit of labor input. Many items in this group may partake of some aspects of public goods. They may constitute fringe benefits to a single cooperative household. The outputs of cooperative public good character are expressed by

\[
\begin{align*}
\theta_1 &= \theta_{11} x_1 + \cdots + \theta_{1m} x_m \\
& \vdots \\
\theta_n &= \theta_{n1} x_1 + \cdots + \theta_{nm} x_m.
\end{align*}
\]

The number of cooperative public goods (\( \theta \)'s) goes up to \( n'' \) and they cover human needs which may be defined as having both non-production

\[1\text{In an alternative way, the } z \text{'s could be expressed as } q \text{'s with zero prices but minimum constraints assigned to them. Any } z \text{ could be treated as a specified supply from which the processes use and thus it serves in terms of a restraint}\]

\[z_i \geq \sum a_{ij} q_j,
\]

where \( q_j \) is the amount of the \( j \)-th output and \( a_{ij} \) is the amount of the \( i \)-th overhead services used by a unit of the output \( j \).
(consumption, health, etc.) aspects and a collectively (community-wise) satisfiable nature.

The next set of expressions designates the requirement matrix of homeplot farms for all the processes produced by the large-scale enterprise.

\[ H_j = h_{j1}x_1 + \cdots + h_{jm}x_m \]  \hspace{1cm} (7.10)

Any column on the right-hand side of Expression 7.10 is the vector for a given process (for example, the first column is for activity 1) and contains the quantities of this process assigned to all the homeplot farms. (The total number of household farms is assumed to be \( M \).) Any row denotes the assignment of all the processes to a particular homeplot farm (for example, the last row stands for the \( M \)-th homeplot).

Some processes may be inputs in parts to other processes, such as feed to cattle. Therefore, an intermediate product matrix is also required

\[ x_i = \beta_{i1}x_1 + \cdots + \beta_{im}x_m \]  \hspace{1cm} (7.11)

\[ x_m = \beta_{m1}x_1 + \cdots + \beta_{mm}x_m \]

where \( \beta_{ij} = 0 \) for \( i = j \).

In Equation 7.11 each process is expressed as a linear combination of all processes or activities. The principal diagonal is zero since all activities are defined "net." (In an alternative approach, total outputs
can be divided into final outputs, $q_1, ..., q_g$; intermediate products, $q_h, ..., q_m$; overhead productive services, $q_n, ..., q_p$; cooperative public foods, $q_r, ..., q_t$; and services to homeplot farms, $q_u, ..., q_z$."

Now the usual programming constraint set will be brought into the model of the large collective enterprise. It becomes clear why every process had to be included in each row of Expressions 7.6 and 7.7 and in Equations 7.9 through 7.11. As a result, one set of constraints will do the job. The constraining inequalities are

$$a_{11}x_1 + \cdots + a_{1m}x_m \leq b_1$$

$$\vdots$$

$$a_{r1}x_1 + \cdots + a_{rm}x_m \leq b_r.$$

The model has $r$ constraints. Each $b$ may represent a determined quality of land, a form of capital, a certain kind of labor available at a particular time or some combination of labor and capital embodied in a given production process (for example, wheat harvesting by combine). They have the usual meaning one finds in programming models.

The model building reaches the stage where it is required to have value weights for the objective function. At first a vector of variable costs of each process is needed to determine these value weights (net prices) in the model of the large cooperative enterprise. This vector is given by

$$\mathbf{T}_1 \cdots \mathbf{T}_i \cdots \mathbf{T}_m.$$  

(7.13)

However, the $A$ matrix is derivable from Expression 7.12, the latter is not the one that goes into the simplex tableau of the cooperative model. Consequently, this vector is not directly applicable for evaluating
the final outputs. This model will obtain both the direct and indirect variable costs assignable to one unit of each final output \( (q) \)

\[
VC_{q_1} = \alpha_{11} \tau_{11} + \cdots + \alpha_{1m} \tau_{1m} + \alpha_{11} \sum_{i=1}^{m} \beta_{1i} \tau_{1i} + \cdots + \alpha_{1m} \sum_{i=1}^{m} \beta_{mi} \tau_{1i} \\
\vdots \\
VC_{q_n} = \alpha_{n1} \tau_{n1} + \cdots + \alpha_{nm} \tau_{nm} + \alpha_{n1} \sum_{i=1}^{m} \beta_{1i} \tau_{1i} + \cdots + \alpha_{nm} \sum_{i=1}^{m} \beta_{mi} \tau_{1i} 
\]

Equation 7.14 is expressed in matrix notation

\[
VC_{q_1} = A_1 T + \alpha_{11} B_1 T + \cdots + \alpha_{1m} B_m T \\
= A_1 T + T B A_1 \\
\vdots \\
VC_{q_n} = A_n T + \alpha_{n1} B_1 T + \cdots + \alpha_{nm} B_m T \\
= A_n T + T B A_n
\]

where \( B \) denotes a matrix and \( A, B, T \) indicates vectors.

Expression 7.14 demonstrated that other processes also served as requirements of any single process. Now Equations 7.14 and 7.14a also reflect the requirements for intermediate outputs serving as inputs; for example, the second term of Expression 7.14a gives the product of
and of a complex term which was previously obtained from multiplying the intermediate output requirements of one unit of each process ($B_1$) by its variable costs ($T$). Equation 7.14 has $m$ terms of this type for each final product ($q_j$). After global horizontal summation, every row of these equations will provide the total variable costs associated with the production of one unit of a particular final output. Here some entries may of course be zero.

The same procedure gives rise to the total variable costs of other outputs treated by Expressions 7.7 and 7.9. The variable costs of outputs denoted by Equations 7.10 and 7.11 may also be attained by similar means.

Keeping in mind that a price vector ($P$) is given exogenously, the entries of the usual $c$ (the net prices or profits associated with unit activity) vector can be calculated easily. If the sum of the right-hand-side terms of Equation 7.14 is subtracted from the corresponding price, then the required $c$ value will be provided for the simplex tableau.\(^1\)

Market prices do not exist for outputs of $z$ and $\Theta$ type but they are provided with shadow prices by the linear programming dual. Therefore, implicit values will be placed on them. The same is to be said about the services supplied by the large collective enterprise to homeplot farms. By this approach even the profitability of the outputs under discussion may be evaluated since their variable costs can be compared with their implicit values.

---

1 In a standard linear programming model, some $c$ values can of course be negative and hence behave also as a variable cost.
As an example, the evaluation of one unit of output $\theta_1$ is shown here. At the beginning, the vector of shadow prices is written, which is obtained for the scarce resources from the optimal solution, in the form of

$$u_1^0 \ldots u_r^0.$$  

(7.15)

In this expression the shadow prices of constraints (b's) are presented. To evaluate one unit of $\theta_1$, it is possible to proceed deriving terms such as

$$\theta_1 A_1 U^0 + \theta_1 A_{11} U_{11}^0$$  

(7.16)

where $A_1$, $U^0$, and $U_{11}^0$ are vectors, and $A_{11}$ is a matrix.

In Expression 7.16 the unit usage of the scarce factors are calculated at their shadow price. It should be stressed that this expression gives only the first terms the evaluation.

Outputs of $z$ and $\theta$ type do not appear in the objective function, rather they are used as constraints; in other words, the $z$'s and $\theta$'s represent predetermined output-levels from the viewpoint of the present model. The $H$'s (Expression 7.10) will receive the same treatment.

One must also calculate the variable costs of the activities utilized as well as their implicit costs from the shadow prices. Thus the implied value can be derived for a unit level of output $\theta_1$ and the implied value is its alternative cost. Finally, the valuation of $q$'s is given by

$$P_{q_1} - VC_{q_1} = c_1$$

$$P_{q_2} - VC_{q_2} = c_2$$

$$\vdots$$

$$P_{q_n} - VC_{q_n} = c_n.$$  

(7.17)
These $c$ values serve as weight and net profit associated with one unit of final outputs in the objective function.\footnote{The arithmetic burden is much smaller in the standard linear programming model which applies positive $c_j$ values for all final outputs and negative $c_j$ values for the intermediate products.}

To construct a simplex tableau for this model of the large collective enterprise, all the $z$'s, the $\theta$'s, and $H$'s should be considered as they represent predetermined demands on the processes or activities denoted by the $x$'s. One may proceed by calculating the impingement of these required assignments of the $x$'s on the original constraints. This will result in calculating a new vector of constraints $(b'_1, \ldots, b'_r)$ which is the original constraint vector $(b)$ with the impingement of $z$'s, $\theta$'s and $H$'s subtracted. Thus from the $A$ and $B$ matrices it is possible to calculate a new $A$ matrix denoted as $A'$ which is the matrix of coefficients applying to final outputs ($q$'s). Then the simplex procedure is applicable in the usual fashion.

**The homeplot farms**

For each homeplot farm, it is convenient to set up a simple linear programming matrix without any of the intervening matrices of the model of the large-scale enterprise as

$$q_i A_i \leq B_i \quad (7.18)$$

In Expression 7.18, $q_i$ is the vector of all final outputs produced by the $i$-th homeplot farm, $A_i$ stands for the production coefficient matrix of this homeplot farm and $B_i$ is the vector of scarce resources available for the given unit.
In some linear programming models which could be formulated, one may consider the labor input by the household in its own homeplot farm as a scarce (fixed) input in order to place a shadow price on this labor.\(^1\) At other times the family labor need not enter the vector of b's and may be considered as a variable input priced at "opportunity wages" attainable from alternative employment either in the large-scale enterprise or in non-cooperative firms. But in this latter case an equation is still needed for the different kinds of family labor, and these labor supplies are denoted as a variable. It should also be noted that any alternative wage of labor would contain all fringe benefits.

Similarly, \(T\) may express the vector of variable costs associated with the final outputs of the i-th homeplot farm. In the same vein, \(P\) denotes the vector of prices for outputs sold on the market by the i-th household from its own homeplot; and \(P\) may stand for the vector of implicit prices for the homeplot products which are consumed by the owner's household. In the second case the retail price that the household would otherwise pay for the product seems to be the relevant one.

In various income accounting procedures, one may decide to utilize the equivalent wholesale prices rather than the retail prices depending upon the purpose of the analysis in hand. The work-leisure choice of the cooperative household will be dealt with in Chapter X.

\(^1\) Actually when one lets the model "decide" where the cooperative household should use a unit of labor, the household objective function is in operation and the opportunity cost is the shadow price, but optimization also provides shadow prices for the household as a "final value."
The unified cooperative model

By now there are at hand all the relevant ingredients for a unified linear programming model of the whole cooperative farm organization where certain collective decisions concerning member's rights and duties, and members' individual decision concerning the division of their labor supply among various pursuits, have already been determined. With these elements predetermined, it is possible to perform sub-optimization, i.e. maximizing the objective function for the large-scale enterprise separately and maximizing the objective function for each of the homeplot farms. The total output and total profits of a farmers' cooperative organization then would simply be a sum of those of the large collective enterprise and of those of each of the homeplot farms.

The whole problem of integration could have been approached in a rather different way but it would have come to the same answer. One can think of each of the previous matrices, $A$ and $B$, as being a partition of a prior-existing matrix which would include the production matrices of all the individual homeplot farms. Intrinsically, the constraints for the individual homeplots could then have included in an overall constraint vector. The production of any commodity on the $i$-th homeplot would then have been considered as a final output activity which is different from the production of the same commodity on another homeplot. The production of the same commodity by the large collective enterprise would have been yet another activity. The essential point is that so long as one is talking about a single period, short-run situation this is a simple partitioning problem for the matrices. Since developing the $A$ and $B$ matrices
for the homeplot farms would not have cast light on any new theoretical problem, this model has chosen to utilize the "pre-partitioned matrices," so to speak. It would not be a too complicated matter to expand the model as noted above, but it is far from being essential. Implicitly, when more general, nonlinear optimizing models are considered, the previously mentioned decisions will not have already been undertaken and one will be faced with the allocation problems between homeplots and the large-scale enterprise and also among the homeplots themselves in order to derive equilibrium positions or to analyze the consequences of alternative cooperative organizations, including doing away with the homeplot farms entirely.

**Underlying assumptions and further considerations**

To establish the linear programming model the following underlying assumptions have been used:

First assumption is the basic short-run assumption. The model deals with a short-run situation in which each production process (activity) is determined or fixed both with respect to the technological form and the total available quantity or intensity per annum at which the process can operate.

Second is the standard linear programming assumption that all variable inputs or all variable processes are linearly related to the process intensity.

The third assumption is that all processes that are limited with respect to the intensity or quantity available per unit of time are linearly related to some constraining element. This gives a usual $A$ matrix of linear programming. There are only constant scale returns both
to the fixed factors and to the variable factors in the definition of process.

The fourth assumption states that all processes are independent or separable, thus the level of utilization of any one process does not affect the variable inputs or fixed inputs to another process and neither does it change the transformation coefficients (the $a_{ij}$ values).

Since optimization has not been carried out with this model so far, no additional assumptions are needed explicitly; otherwise an assumption would be that the objective function is both linear and separable in all its variables.

It has been noted above that certain decisions must be predetermined in order to give useful application to a model of linear programming. In the following chapters optimization will be applied with respect to the linear programming model and then various relevant objective functions will be explored in the context of the general behavioral model, with nonlinear relationships within the production sector of the model. This will demonstrate the relevance of various linear programming models which may be more useful in operational application than the expanded nonlinear model. Thus, the non-behavioral linear model will serve as a basis for analyzing the consequences of various objective functions and for building various relevant linear programming optimization models.
A Simple Model with Emphasis on Investment and Technology

The cooperative model presented so far is of short-run nature; in other words, this model refers to a single production period in which technology is considered only implicitly and unchanged. In this one-period model a part of the proceeds is separated as investment funds but none of the effects of this investment is accounted for. The following model will contain investment and technology. As a rule, technical progress has an influence on technology through the application of improved means of production and through the development of more advanced management, enterprise organization and information. The latter is the case of disembodied technical progress, and the former refers to embodied changes.

The model treats the cooperative farm and its two portions in a parallel fashion. The model defines outputs as

\[ Q_e = F (K_e, D_e, T_e, L_e | X_H) \]  \hspace{1cm} (7.19a)

\[ Q_i = f (L_i, K_i, T_i | D_i, X_i) \]  \hspace{1cm} (7.19b)

\[ Q_c = Q_e + \sum_{i=1}^{M} Q_i \]  \hspace{1cm} (7.19c)

Equation 7.19a gives the output of the large cooperative enterprise as a function of capital \( (K_e) \), land \( (D_e) \), technology \( (T_e) \) and labor \( (L_e) \) available for this enterprise while the previously defined requirements of all members' homeplot farms \( (X_H) \) appear in terms of a fixed factor (a given magnitude). In the production function of a member's homeplot farm denoted by Equation 7.19b, there are two fixed
factors: the size of the homeplot land area and the productive services available to this household from the large cooperative firm (see Equation 7.1). Equation 7.19c sums up the outputs of the two portions of a farmer's cooperative.

The following expressions state that technology is a function of investment.

\[ T_e = T(I_e); \quad (7.20a) \]

\[ T_i = t(I_i); \quad (7.20b) \]

\[ T_c = T_e \cup T_i \cup T_M - T_e \cap T_i - T_e \cap T_M - T_M \cap T_e \cap T_i. \quad (7.20c) \]

Technologies available for the large cooperative enterprise and for the homeplot farms may, of course, not be added or multiplied but they may be treated by set operations. The cooperative technology set \( T_c \) is given by Expressions 7.20c in a simplified form, selecting only the \( i \)-th and the \( M \)-th homeplots. The collective technology set \( T_e \) and the homeplot technology sets usually have intersections because productive services are provided by the large collective enterprise for the homeplot farms. These processes are denoted in algebraic form as

\[ X_H = \sum_{i=1}^{M} X_i, \quad (7.21) \]

where

\[ X_H = \text{the vector of the productive processes that the large cooperative firm supplies to the homeplot farms of all its members in a given production period.} \]

Some elements of \( X_i \) are typical productive processes and the provision
of any homeplot excludes the other homeplots from simultaneous utilization. Other forms of \( X_1 \) may result in a sort of public good provided by the large enterprise; for example, flood and disease control, market information or managerial services can be given free of charge to any individual cooperative member. The intersection of the technology sets will be different in these two cases. Both cases are shown in Fig. 7.1 depicting only two of the homeplot farms. (The relative magnitude of homeplot sets is nonproportionally increased for illustrative purposes.)

Each of the next three equations gives a definition for investment and considers investment as a function of enterprise profit and total surplus value respectively.

\[
I_e = dK_e = I(E_e) \quad (7.22a)
\]

\[
I_i = dK_i = g(m_i) \quad (7.22b)
\]

\[
I_c = I_e + \sum_{i=1}^{M} I_i \quad (7.22c)
\]

Expression 7.22c aggregates the investments of the large-scale collective enterprise and the investments of all the homeplots which belong to the \( c \)-th cooperative farms.

---

1. The properties and significance of the so-called cooperative public goods are discussed extensively in Chapter X.

The prospects of homeplot farming and its alternative technologies will be examined largely in Chapter XI.
= The technology set of the large cooperative enterprise.

= The technology set of the i-th homeplot farm.

= The technology set of the M-th homeplot farm.

= The intersection of $T_e$ and $T_M$ ($T_e \cap T_M$).

= The intersection of $T_e$ and $T_i$ ($T_e \cap T_i$).

= The intersection of $T_M$, $T_e$ and $T_i$ ($T_M \cap T_e \cap T_i$).

Fig. 7.1. Intersecting technology sets of cooperative farming
Profits and total surplus value are considered as a function of technology and labor input.

\[ \mathcal{P}_{e} = \mathcal{P} \left( T_{e}, L_{e} \right) ; \]  
\[ (7.23a) \]

\[ m_{1} = \mu \left( T_{1}, L_{1}, D_{1}, X_{1} \right) ; \]  
\[ (7.23b) \]

\[ m_{c} = \mathcal{P}_{e} + \sum_{i=1}^{N} m_{i} , \]  
\[ (7.23c) \]

where \( m_{c} \) is the total surplus value produced by the \( c \)-th farmers' cooperative.

Labor supply functions are given by a similar set of equations as

\[ L_{e} = L \left( V_{e}, N \right) ; \]  
\[ (7.24a) \]

\[ L_{1} = L \left( \Theta Q_{1}, L_{1}, V_{1} \right) ; \]  
\[ (7.24b) \]

\[ L_{c} = L_{e} + \sum_{i=1}^{N} L_{1} . \]  
\[ (7.24c) \]

In Expression 7.24a \( V_{e} \) denotes the total wage bill of the large collective enterprise, the symbol \( N \) stands for the institutional requirement decided by the General Assembly. Expression 7.24b includes the marginal product of labor and here \( V_{1} \) is the calculated (normative) wage bill of the \( i \)-th homeplot farm.

Both the total wage bill of the large cooperative enterprise and the computed wage bill of homeplot farming are assumed to be a function of investment in order to determine their values within the given system.

\[ V_{e} = V \left( I_{e} \right) ; \]  
\[ (7.25a) \]

\[ V_{1} = v \left( I_{1} \right) ; \]  
\[ (7.25b) \]
Finally, the model is closed by three definitional equations of net output:

\[ q_e = \prod_e + v_e \quad (7.26a) \]

\[ q_i = m_i + v_i \quad (7.26b) \]

\[ q_c = \prod_e + \sum_{i=1}^{M} m_i + v_e + \sum_{i=1}^{M} v_i \quad (7.26c) \]

The net outputs defined by Equations 7.26a through 7.26c are identical with the concept of gross income quantified by Equations 7.28a through 7.28c in the next section.

Revenue, Income and Profit Qualifications

This section provides a complete set of equations and identities in order to give the socialist cooperative categories precise, quantifiable meaning both in the Marxian terminology used in Hungary and in economic terms applied in the United States.\(^1\)

At first, attention will be focused on the large-scale cooperative enterprise.

The total gross revenue of the large collective enterprise may be denoted by

\[ R_e = PQ_e \quad (7.27a) \]

\(^1\) In this section, capital letter A attached to the identifying number of an equation denotes the American concepts and capital letter B denotes the Hungarian versions. The lower-case letters are included in the alternative formulae of a given category.
In Equation 7.27A \( R_e \) is the total gross revenue, \( P \) is a vector of commodity prices and \( Q_e \) is a vector of final outputs of the large cooperative enterprise.

Viewing the total gross revenue from the Marxian standpoint it becomes

\[
GU_e = C_{NL} + V_e + m_e , \quad (7.27B)
\]

where

\( GU_e \) = gross value product of a large cooperative enterprise,

\( C_{NL} \) = the vector of nonlabor costs, the latter includes capital consumption allowance (amortization costs) and the costs of all material inputs,

\( V_e \) = the vector of wages or labor costs (remuneration paid to members and nonmembers for their labor input),

\( m_e \) = total surplus value embodied in the products of the large cooperative enterprise.

In the practice of socialist planning and statistics, the concept of national (or gross) income is of central interest. It corresponds to the Western concept of net national product. The subtraction of nonlabor costs from Equation 7.27B will give the gross income of a microeconomic producing unit which makes up the national income in final aggregation; in symbols

\[
GY_e = GU_e - C_{NL} , \quad (7.28a)
\]

or

\[
GY_e = V_e + m_e , \quad (7.28b)
\]

or

\[
GY_e = NU_e , \quad (7.28c)
\]

where

\( GY_e \) = the gross income of a large cooperative firm,
\( \text{NU}_e = \) the net value product of a large cooperative enterprise.

It is customary, although it may be rather vague, to speak of the net income of a socialist firm as a remainder of its gross income after having wage costs subtracted from it. According to this practice, net enterprise income would be equal to total surplus value in the Marxian sense. With respect to a large cooperative enterprise, net firm income or total surplus is defined in this vein as

\[
\text{NY}_e = \text{GU}_e - V_e - C_{\text{NL}}, \tag{7.29a}
\]

or

\[
\text{NY}_e = \text{NU}_e - V_e, \tag{7.29b}
\]

or

\[
\text{NY}_e \equiv m_e, \tag{7.29c}
\]

where

\( \text{NY}_e = \) the "net income" of large cooperative firm.

To introduce the concept of cooperative enterprise profits, total surplus as well as the above net income may be equated with this profit

\[
\Pi_e \equiv m_e \equiv \text{NY}_e, \tag{7.30}
\]

where

\( \Pi_e = \) the profits of the large cooperative enterprise.

In Expression 7.30 the profit includes taxes. Taxes can be conceived of as social surplus

\[
m_{es} = X_e, \tag{7.31}
\]

where

\( m_{es} = \) social surplus (a part of total value of the large collective enterprise which is devoted to cover the needs of society as a whole),
$X_e =$ taxes paid by the large cooperative firm.

A third kind of surplus can be distinguished as the term of the cooperative surplus with respect to the large cooperative enterprise. This surplus equals the enterprise gross income and, curiously enough, it is larger than the total surplus defined by the classical Marxian economics. Cooperative surplus ($S_e$) is given in this sense by

$$S_e = V_e + m_e , \quad (7.32a)$$

or

$$S_e = GY_e . \quad (7.32b)$$

The composition of the profits of a large cooperative enterprise is characterized by

$$\Pi_e = X_e + I_e + W_e + \sum_{i=1}^{M} \Pi_{ei} . \quad (7.33)$$

Profits are divided into taxes ($X_e$), expenditures on investments ($I_e$), contributions to the cooperative welfare funds ($W_e$) and payments (of dividends) to members of the cooperative (last term in Equation 7.33).

At this stage 7.278 will be redefined and decomposed as

$$GU_e = C_{NL} + V_e + m_{es} + m_{ec} + \sum_{i=1}^{M} m_{ei} + \sum_{i=1}^{M} W_{ei} , \quad (7.34B)$$

where

$m_{ec} =$ profits (surplus) retained by the large cooperative firm,

$m_{ei} =$ dividend of the $i$-th cooperative member (household),

$W_{ei} =$ benefits of the $i$-th member (household) from the cooperative welfare fund.

An attempt may also be made to redefine and decompose Equation 7.27A in an equivalent form of Equation 7.34B.
\[ R_e = FC + VC + T_e + \Pi_{ec} + \sum_{i=1}^{M} \Pi_{ei}, \]

where

- \( FC \) = fixed costs,
- \( VC \) = variable costs,
- \( \Pi_{ec} \) = profits retained by the incorporated enterprise,
- \( \Pi_{ei} \) = dividend of the \( i \)-th stockholder.

With respect to Equation 7.34A it should be noted that workers' fringe benefits and employers' contributions to the social welfare funds (social security payments, pay-roll taxes, unemployment insurance premium) are involved in the wage fund and in the variable costs according to the calculation practice of business firms in the United States.

Using normative magnitudes as wage costs and accounting prices where these are necessary, equations may be set up to characterize the relevant categories of the homeplot farm portion of a farmers' cooperative. The homeplot versions of Equations 7.27A and 7.27B are especially important. This American version is given by

\[ R_i = P Q_i, \]

where

- \( R_i \) = the gross revenue of the \( i \)-th cooperative household's homeplot farm,
- \( Q_i \) = the total output of the homeplot farm of the \( i \)-th cooperative household (including self-supplying production).

An Hungarian version can be formulated as

\[ GU_i = C_{NI} + V_i + m_i, \]

where
GU\_i = the gross value product of the i-th homeplot farm,
V\_i = the calculated wage costs of the i-th homeplot farm as labor input,
m\_i = the total surplus value as a part of GU\_i.

The homeplot versions of Equations 7.28 and 7.29 are derived in a similar manner.

The entirety of a farmers' cooperative may be shown by equations integrating the large collective enterprise and all its members' individually operated homeplot farms

\[ R = R_c + \sum_{i=1}^{M} R_i, \quad (7.36A) \]

where

\[ R_c = \text{the gross revenue of the c-th cooperative farm.} \]

The extension of Equation 7.27B is given by the same route

\[ GU_c = GU_e + \sum_{i=1}^{M} GU_i, \quad (7.36B) \]

where

\[ GU_c = \text{the gross value product of farmers' cooperative c.} \]

There is involved no novelty in developing a unified expression for gross income or net value product which may well be a good measure of the economic performance of the complex cooperative farm organization.

In a new set of equations, the personal income of a cooperative household may be derived from Expression 7.3 by the means of the following procedures:

- exclusion of I_e;
- replacement of L\_i f\_i P by R\_i;
--- summation of all the variables on the right-hand side.

Thus the personal income of the i-th cooperative household \( \bar{Y}_i \) is given by

\[
\bar{Y}_i = v_{ei} + v_{e1} + v_{L} + R_1 + \Pi_{1i} + W_1 + W^*_1 + W^*_1,
\]

(7.37)

where the symbols (except \( R_1 \)) are defined in connection with Equation 7.3. Some terms may be missing (have zero value) while some terms may appear more than once (for example, more than one cooperative member or non-cooperative employee or welfare-benefited person may belong to a given cooperative household).

Completing the set of equations, one may formulate the property of equilibrium prices on a perfectly competitive market as

\[
P^*_q = MC_q,
\]

(7.38A)

where

\[
P^*_q = \text{the market equilibrium price of product } q,
\]

\[
MC_q = \text{the marginal cost of producing the } q\text{-th output.}
\]

The Marxian non-marginal concept of the "value-price" which is a sort of ideal, and the equilibrium quantity can be expressed by the help of alteration of Equation 7.27B

\[
P^*_q = \frac{C^o_{NL}}{Q^o} + \frac{V^o}{Q^o} + \frac{m^o}{Q^o},
\]

(7.38B)

where

\[
P^*_q = \text{the "value price" of a unit of product } q,
\]

\[
Q^o = \text{the socially desirable ("market-clearing") quantity of product } q,
\]
\( C^0_{NL}, v^0, \pi^0 \) = the socially necessary (accepted) level of nonwage costs, wages and surplus value (generated in the production of output \( q \) respectively.

At first glance it is evident that Equation 7.38B reflects unit (average) cost pricing in contrast to the marginal cost pricing principle expressed in its twin equation.
PART III.

COOPERATIVE OPTIMIZATION
CHAPTER VIII. THEORETICAL AND METHODOLOGICAL ASPECTS OF
COOPERATIVE OPTIMIZATION

Short Review of Literature

It would take a separate study to show the similarities and dis­similarities among cooperative theories developed so far and the co­operative models presented in this paper. In Hungary, Erdei (22) wrote a comprehensive survey of the history of cooperative concepts. This book, published more than a decade ago, analyzed the role and capac­ities of cooperative organizations in social progress and in economic development. In Erdei's study there may be revealed both the cognizance of the dual character of cooperative organizations and the conception of cooperatives as business enterprises. These concepts were canonized by the new Hungarian Cooperative Law in 1967.

For a long time cooperative theories did not view the cooperative association as a firm. The generally accepted theories were restricted to marketing cooperatives and these cooperatives were considered merely as an extension of their patrons' economic activity. In total accordance with the concept of Emelianoff (19), Robotka (69, p. 113) formulated:

"The cooperative association, as such ... is a sovereign unit only with respect to its external relationships. Internally, the participants act in their individual capacities in a mutually agreed upon manner, hence the acts of the cooperative represent the sum of the act of the participants. Functioning cooperatively thus represent ... an extension of their entre­preneurial functioning."

In this theoretical framework the cooperative, as such, does not have its own entrepreneurial organs and the cooperative association is shown as an aggregate of economic units, each fully retaining its independ­ence in seeking profits.
Following the track of Emelianoff (19) and Robotka (69), Phillips (68, p. 75) states:

"As an economic institution, the cooperative association ... is an organization ... of sovereign economic units — firms or households."

Phillips argues that a member firm should be treated as a multi-plant, vertically integrated firm and the entrepreneurs of each associated firm must allocate resources to the common plant in the same manner as a multi-plant firm allocates productive resources to each of its plants.

In the light of organization theory, Helmberger and Hoos (48) conceive the cooperative as a firm defined in a more complex (organizational) sense. From the authors' viewpoint, organization theory provides a broader interpretation of the firm that is useful for empirical research on cooperative decision making. The study of Helmberger and Hoos (48, p. 275) suggests:

"... by making certain assumptions within an organizational framework, the marginal analysis can be used in deriving hypotheses about cooperative performance in much the same way as it has been used in traditional (microeconomic) theory."

In the vein of the conceptual approach of Helmberger and Hoos, one may view Domar's (16) purely methodological article on the Soviet collective farms from wider, more operational angles.

A most recent publication of Heady (44) has shown that behavioral relations and equilibrium positions can be derived through traditional marginal analysis after assuming optimizing behavior on the part of the cooperative enterprise.

The models introduced previously and optimizing procedures shortly to be discussed are based on the firm notion that a cooperative farm,
its objectives, and its environment are all multivariate systems.  

General Theoretical Considerations

To develop a theory of cooperative farms, the foregoing part of this study involved the construction of several models, each of which purports to be a compact description of some relevant aspects of the real cooperative farm organization. Even the most casual observations on socialist cooperative farming indicate that its goals, structure and behavior may be sufficiently different from those of other types of farm organization. This part of the study will predict rational behavior which aims at optimization, i.e. at maximizing goal achievement on the part of the decision makers in the given, somewhat differentiating organizational, institutional framework.

For prediction purposes, an adequate model of the behavior of the cooperative farm must at least implicitly reflect considerations associated with:

a. the objectives of individual cooperative members as decision makers;

b. the large-scale cooperative enterprise as the basic economic and organization unit;

c. the special features of small-scale homeplot farming as a complementary and subsidiary activity;

d. the interactions within the cooperative entirety as well as between the cooperative community and its socio-politico-economic environment.

The cooperative farms are essential components of the agricultural

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1 Shubik (74, p. 365) made a similar statement concerning corporate organizations.
sector in a socialist economy; at the same time the cooperative organization consists of many people embedded in local communities and finally in the whole society.

Being interested in predicting the behavior of the large cooperative enterprise owned by the members' collective or having interest in predicting the behavior of individual cooperative member, the economist must thoroughly analyze the requirements for optimizing behavior and develop models which foresee behavior as well as makes distinction between optimal behavior and actual behavior. An essential part of the theory of cooperative farms should be concerned with characterizing the optimal or equilibrium situation for decision units involved. By assuming that these units behave optimally, the relevant models of cooperatives predict with appropriate accuracy in a wide range of circumstances. Thus great operational value is attached to working out optimal decision rules for individuals as cooperative members and for cooperative collectives as groups of individuals.

Throughout Part Three the separate chapters will be dealing with predictions derivable from optimizing behavior of cooperative decision units. As Holdren (50) stresses, optimal behavior always depends upon the restraints on the decision maker, the choice variables available to him and his knowledge of them. The models of optimizing cooperative behavior themselves incorporate the chosen set of variables which are relevant to economic decisions and embody the construct in such a manner that predictions of considerable welfare significance are made workable. Thus cooperative economics concentrates on variables, models,
predictions, and outcomes that bear direct relationships to the "economic welfare" of individual members and to that of collective (cooperative) groups. Thus, given the value system of any particular society, cooperative economic theory attempts to define states which, relative to those values, make cooperative members best off as a societal stratum in some economic sense.

As has been shown, in a cooperative farm organization, the business firm is not distinct from the members' governing collective represented by the General Assembly, and the large-scale collective enterprise is not separated entirely from its members' individual homeplot farms. The economic spheres of consumption, market production and labor supply—all are interrelated by the decisions of a typical cooperative household. Both theoretical considerations and practical evidence indicate that the economic structure and behavior of the small-scale individually managed homeplot farms are rather dissimilar from those of the large collective firm. Within the large-scale collective enterprise, the distinctions among manager, administrator, entrepreneur and leader are not always clear. This fact and the multi-person nature of the large cooperative decision unit may give rise to behavior which would be contradictory or even irrational for an individual but which is not necessarily so for a multi-personality group such as the General Assembly

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The reasoning of this paragraph, and some others later on, is parallel to the general approach of Holdren (50) connecting economic models and the welfare concept. Holdren also draws attention to the fact that economics can be thought of as a parent discipline for operation research since the latter utilizes very much the same types of models as economics but it is primarily concerned with what one might call "private optimization."
of a farmers' cooperative. Even if this decision unit behaves as a single-headed organism, it is impossible to predict its behavior without a good understanding of the social environment. Cooperative theories and models confined to microeconomics and "simple" constructs could be severely limited in the comprehensive description of a complex, changeable environment.

The Set of Alternative Objective Functions

After all the foregoing, it may not seem unrealistic to hypothesize that the relevant set of objectives of a large collective-cooperative enterprise might include:

1. maximization of firm net profits,
2. maximization of members' benefits in terms of a residual income on their labor input,
3. maximization of output per land unit,
4. maximization of total output of the enterprise,
5. minimization of the production costs of a given level of output,
6. maximization of the firm's net worth (capital value) over time.

According to these separable objectives, the optimizing behavior may be expressed by six consecutive models of the large cooperative firm such as:

1. the pure enterprise model
2. the cooperative model of the family-farm type,
3. the intensity model,
4. the output-maximizing model,
5. the efficiency models,
6. the growth models.
All the six basic versions of optimization of the large-scale collective firm will be elaborated in Chapter IX. The maximization of the individual "economic welfare" (objective) function of cooperative households (members) will be dealt with in Chapter X. Following these two suboptimization procedures, a unified, long-run cooperative optimization model is presented in Chapter XI. These chapters may also be a contribution to the debate in Hungary and in other socialist countries on the economic objectives and material incentives of the farmers' cooperative. For a long time it has been debated whether the system of cooperative incentives should be connected with the gross income of the cooperative or with the net income of the cooperative enterprise. Both approaches arrive at their conclusions by largely logical arguments concerning optimal output levels, optimum production patterns and efficient resource utilization. The quantitative analysis, especially mathematical programming can provide optimal solutions and optimality criteria relative to an objective function and by means of a specified production function.

The Concept of a Cooperative Entrepreneur

In most of the following models, each cooperative member will be considered as an entrepreneur. But how should one define a cooperative entrepreneur? This very proper question in a general (non-cooperative) sense was raised by Haavelmo (37, pp. 209-210) and his unusual way of answering is challenging to follow. It may be argued that a cooperative member as an entrepreneur is not just a capital (and land)-owner, or
one who has the collectively-exercised right of disposal over capital. Even in a managerial position, the cooperative member is not simply a manager because cooperative managers could be hired as employees. The best way out offered is to define the entrepreneur as the "owner of a production function." In this vein each cooperative member possesses some sort of exclusive right which is exercised by determining production plans in the General Assembly. Still remaining on Haavelmo's track, there may exist other production functions that will influence very strongly what this exclusive right belonging to a cooperative member as "owner" of a particular production function is worth.¹

The type of production function really lies at the heart of optimization. It is crucial to select a functional form which characterizes the farm organization under scrutiny, and also fits in the optimization technique applied. If a linear programming model is in application, then production processes (activities) exhibiting constant returns to scale are exclusively workable. If constant returns to scale are the case, then optimal farm size cannot be determined by marginal analysis. Applying Heady's (44) approach, a simplified example will clarify this point by entering the field of optimization in the large-scale collective cooperative enterprise.

¹Haavelmo (37, p. 210) also notes that the deeper understanding of why and how a new enterprise is established is probably the task of social science in a much wider sense than of economics.
The Implications of Constant Returns to Scale

A production function exhibiting constant returns to scale is given by

$$q = a + bM,$$  \hspace{1cm} (8.1)

where $q$ stands for output, $M$ represents one resource category in terms of a certain mix or some unified package of different inputs including labor; $b$ expresses the constant marginal (and average) productivity of $M$; and $a$ is a particular portion of $q$ conceivable as "gift from nature" or output without scarce inputs. Equation 8.1 is a long-run production function in which all inputs are variable and changed in the same proportion (as was discussed in Chapter IV).

Given Equation 8.1, the profit function for the cooperative decision unit assumes the form

$$\Pi = P_q (a + bM) - (V_M + rV_x) M,$$  \hspace{1cm} (8.2)

where:

- $\Pi$ = enterprise profit,
- $P_q$ = price of output,
- $V_M$ = price of labor input,
- $V_x$ = average price of aggregate nonlabor input,
- $M$ = labor (cooperative members') input,
- $r$ = the ratio in which the aggregate resource is applied together with $M$.

The objective function of a cooperative decision unit is given by the maximizing Equation 8.2, i.e.

$$\max_{M} \Pi = P_q (a + bM) - (V_M + rV_x) M.$$  \hspace{1cm} (8.3)
The letter M placed under the profit symbol draws the attention to the fact that labor input or the number of cooperative members is the decision variable in the objective function.

To optimize farm size expressed by the value of M (i.e. by the number of cooperative members), the derivative of Equation 8.3 is to be calculated with respect to the decision variable

$$\frac{\partial \Pi}{\partial M} = bPq - V M - rV_x.$$  \hfill (8.4)

Since every term, thus the derivative itself, is a constant which indicates that if profit is positive in Equation 8.2, there is no limit to the optimal size of the enterprise.

Constant returns to scale would suggest that cooperative decision units could be as large as possible and any farm size might be consistent since production functions with first degree homogeneity leave no room for enterprise size optimization. The production function of this nature excludes profit-maximizing behavior based on marginal analysis. Neither technology nor input prices influence perceivably farm size when constant returns to scale dominate.

It is instructive to turn to the cooperative version of Haavelmo's entrepreneur. If an entrepreneur's production function were homogeneous of first degree, he would never integrate his smaller-scale individual farm into the larger-scale cooperative unit. If it has already been done, our homo oeconomicus (the cooperative member possessing this particular production function) would show complete indifference in respect to the division of his labor and other resources between the collective enterprise and his homeplot farm.
Selecting an Appropriate Production Function

Leaving out the constant term \( a \), Equation 8.1 becomes a special case of the power or Cobb-Douglas production with exponent (production elasticity) unity. A C-D function, even in its generalized form which does not have first degree of homogeneity, cannot be characteristic for agricultural production in the long-run as has been verified by Heady and Dillon (47).

The C-D function exhibits too many constancies, e.g. constant marginal and average productivity, constant elasticity of production, and constant resource mix (when input price ratio is unchanged). This function does not fit when both increasing and decreasing MPP prevails; the function does not "accept" both negative MPP and positive MPP for the whole production process, therefore, does not "allow" increasing total product in some region and diminishing total product in other. The C-D function also does not define maximum output.

To overcome most of the previous unrealities, a cooperative optimizer may select from the set of other production functions such as quadratic, functions named after Spillman or Heady and Pesek, square root, etc (47). The last one has been chosen in the following chapter to characterize large-scale cooperative operations.

A production function of the square root type is given in its one-input and long-run form by

\[
q = a - bM + cM^{5/2}.
\]

(8.5)

The major characteristics of the square root function may be summed up as (47):

The production function of the square root type allows decreasing...
and negative marginal productivity while its marginal products decline at a diminishing rate. The square root equation also accepts a diminishing total product and defines a maximum output level; its elasticity of production decreases when input and output magnitudes lessen.

When two resources are applied and their positive interaction (substitutibility) is assumed, the square root function gives rise to curved isoclines passing through the origin of the input space. These isoclines converge at a particular input combination which corresponds to the maximum output denoted by the peak of the production surface. The curved isoclines and expansion path indicate that the least cost path to larger outputs marks an altering input mix.

Programming Approach to Cooperative Optimization

A non-linear programming model

The production function as constraint  To set up a programming model for cooperative optimization, first the production function will be given for the large-scale cooperative enterprise in terms of a constraint as

$$ g_m = f_e (y_1, \ldots, y_r; x_1, \ldots, x_m; q_1, \ldots, q_n; \theta_1, \ldots, \theta_m; \theta'_1, \ldots, \theta'_n; H_1, \ldots, H_n) \geq 0. \quad (8.6) $$

Expression 8.6 denotes the production as the m-th constraint. To make it stand separately from the other constraints, the production function is designated as the $f_e$ function of six groups of variables. The first group involves all the factors of production which are variable inputs in the situation under consideration; they are numbered 1 to r and des-
If the situation is any kind of short-run, there will be some processes or activities which are fixed in form. The second group of terms designates these by $x_1$ to $x_m$. Final outputs are designated by $q_1$ to $q_n$.

As discussed in Chapter VII, the large cooperative enterprise, like any business firm has some overhead activities which it feels essential but which are not directly assignable to any given output as a cost. These are designated by $z's$. Activities of an overhead type may well use variable inputs and indeed may compete for the use of fixed factors in the short-run. The large cooperative enterprise customarily provides some services for its members and (not always intentionally) for all the people of a rural community. These partake of the nature of a public good. The services in question are designated by $g's$. Finally, the cooperative large enterprise provides inputs of services to the homeplots of its members. These services are denoted by $h's$.

The other constraints. Individually or collectively, each of the foregoing variables may be subjected to constraints either directly or indirectly. Thus, the remaining $m' - 1$ constraining inequalities may be

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1. For more detailed information about the $y's$ and $x's$ see Expression 7.5.
2. See Equation 7.6.
3. See Equation 7.7.
4. See Expression 7.9.
5. See Expression 7.10.
written in vector notation as

\[ \mathbf{e}_1 (Y, X, Q, Z, \Theta, H) \geq 0 \]  
\[ \vdots \]  
\[ \mathbf{e}_{m-1} (Y, X, Q, Z, \Theta, H) \geq 0. \]

**Prices and the initial objective function**  
It is enough to note at this point that the \( P \) and \( V \) vectors used as prices of outputs and prices of inputs are given constants for the decision unit. These conditions can also be handled in the form of demand and supply functions. It would not make the problem theoretically more complicated but computationally and expression-wise it would.

The objective function, at least in short-run applications, is assumed to be that of profits

\[ \text{Max} \; \Pi = PQ - VY. \]

Expression 8.8 denotes that profits will be maximized.

**The maximum conditions**  
Expressions 8.6 and 8.7 constitute a general non-linear programming problem. If one assumes the set of feasible points defined by the constraint set is convex and the objective function is at least concave, then the maximum exists if conditions denoted by Expressions 8.9 through 8.11 hold

\[ \frac{\partial \Pi}{\partial q_1} + \sum_{i=1}^{m} \lambda_i \frac{\partial g_i}{\partial q_1} \leq 0 \]  
\[ \vdots \]  
\[ \frac{\partial \Pi}{\partial q_n} + \sum_{i=1}^{m} \lambda_i \frac{\partial g_i}{\partial q_n} \leq 0. \]
If $< \text{ holds in Expression 8.9 for the j-th commodity.}$
then $q_j = 0. \quad (8.10)$

If $g_i (Y, X, Q, Z, \Theta, H) > 0,$
then $\lambda_i = 0. \quad (8.11)$

The conditions stated in Expressions 8.9 through 8.11 are more restrictive than necessary to obtain a maximum point in non-linear programming. There are a number of less restrictive concavity conditions and correspondingly less restrictive convexity conditions for the constraint set which establishes the existence of, at least, a local maximum. If the conditions expressed above are met and the convexity and concavity conditions are also met, then the maximum is a unique or global maximum. The latter is not essential for the purpose at hand and, thus, the restrictive assumptions of convexity and concavity may be relaxed somewhat (50).\(^1\)

Expression 8.9 says essentially that the slope of the profits function plus the summation of the slopes of the constraints with respect to the independent variable times the Lagrange multiplier associated with each constraint must be less than or equal to zero. Maximum condition denoted by Expression 8.10 shows that if the inequality holds in Expression 8.9, then the output of the associated commodity must be zero in the profit-maximizing optimum solution. Expression 8.11 shows that

---

\(^1\)This topic has been well explored by a paper of Arrow and Enthoven (2) and by a paper of Arrow, Hurvicz and Uzawa (3).
if the available quantity of a constraining factor is not exhausted then its associated shadow price \((\lambda_j)\) must be zero.

One of the derivatives from the profit function can be written as

\[
\frac{\partial \pi}{\partial q_j} = p_j - \sum_{i=1}^r v_i \frac{\partial y_i}{\partial q_j}.
\]  

Equation 8.6 can be decomposed into many separate and possible related production functions. It was admissible in a compact model to write it as one equation at this point. Whenever it is helpful, and in some cases when it is logically necessary, one can easily decompose that function and make several functions out of it. Another essential point is here that it is most convenient to express the constraints as implicit functions.

In order to illustrate the purpose and applicability of this model, a set of simplifying assumptions is required to turn it into a linear model which can be treated with the techniques of linear programming.

The linear programming problem

It is instructive to go through the linear programming application in many respects. Of the many reasons why it is instructive, one is that at any optimal point all the relationships necessary for the linear model to be optimal would also hold for the non-linear model.

Requirements First, the total requirement vector for processes or activities must be obtained. Throughout Chapter VII, \(X\) was defined as a vector of direct requirements in terms of processes.\(^1\) Since the vector \(X\) denotes requirements of processes and since intermediate

\(^1\)See Expression 7.5.
processes are also used, a transformation is necessary to include both
direct and indirect processes

\[ \bar{X} = \bar{x}D \]  

(8.13)

where \( \bar{X} \) denotes the vector of total requirements of the \( m \) processes.
The matrix \( D \) is associated with intermediate processes and is \( m \) by \( m \)
in dimension. In Expression 8.13, \( xD_{ij} \) denotes the inputs of \( x_1, \ldots, x_m \)
used to produce a unit of \( x \). In the \( D \) matrix \( d_{ij} = 1 \) when \( i = j \). This
matrix containing 1's on the diagonal is the sum of the square matrix
of the intermediate product, \( \bar{D} \), and the identity matrix

\[ \bar{D} + I = \bar{D} \]  

(8.14)

**Constraints** The familiar set of constraints of a linear pro­
gramming problem is denoted by

\[ B - \bar{X} D A^t \geq 0 \]  

(8.15)

where \( B \) is the ordinary constraint vector of a linear programming
model. It is 1 by \( r \) in dimensions. The \( A \) matrix is the usual production
(transformation or input-output) coefficient matrix of linear programming
technique and this matrix is \( r \) by \( m \) in dimension. A single constraint
is designated by

\[ b_i - \bar{X} D A^t_{ij} \geq 0 \]  

(8.16)

and there are \( r \) constraints of this kind. Intrinsically, Expression
8.16 denotes a part of the decomposition of the production function
(Equation 8.6).

---

1Intermediate processes are defined here as processes used in
producing other processes.
The decomposition of the production function. From this point one could proceed in several ways. The shortest seems to be to state the following side relation

\[ \bar{x}_Q = Q \bar{y}, \quad (8.17) \]

where the final output vector, \( Q \), is 1 by \( n \) in dimension. The matrix denoted by \( \bar{y} \) is \( n \) by \( m \) in dimension and it establishes the relation between final outputs (\( Q \)) and corresponding processes (\( \bar{x}_Q \)). The latter matrix is also associated with the \( d \)'s of Equation 7.6. A typical element of Equation 8.17 is written as

\[ \bar{x}_i = Q \bar{y}_{ji}. \quad (8.18) \]

Equations 8.17 and 8.18 express a linear production relationship between the \( x \)'s and \( q \)'s. One could state it as an inequality and treat it as a constraint. The \( \lambda \)'s thus obtained would have no particular use; therefore this road is not chosen here. The approach followed is, in reality, a decomposition of the production function which was given in the non-linear model as Equation 8.6. A part of this decomposition is contained in Equation 8.17 with respect to the final output. Similar equations exist for over-head type activities (\( z \)'s), for cooperative public goods (\( \theta \)'s), and for processes of the large-scale collective enterprise which are required by members' homeplot farming (the large enterprise processes in the latter category will be denoted by symbol \( HP \) in this section). \(^1\)

The decomposition of the production function according to these

\(^1\)For further information about these three types of processes, see Expressions 7.7, 7.9 and 7.10.
three sets of processes is expressed as

$$\bar{X} = z \Delta,$$  \hspace{1cm} (8.19)

where $z$ is a vector with dimension 1 by $n'$. The matrix $\Delta$ is $n'$ by $m$ in dimension and its elements correspond to the $\theta$'s of Equation 7.7. An element of Equation 8.19 is given by

$$\bar{x}_i = z_{j_1}.$$

(8.20)

A similar equation holds for the $\theta$'s

$$\bar{X} = \theta \bar{E},$$

(8.21)

where the $\theta$ vector is 1 by $n''$ in dimension, thus matrix $\bar{E}$ is $n''$ by $m$; and an element is given by $\bar{x}_i = \theta_{j_1}$. This matrix connects the cooperative public goods ($\theta$) with the corresponding processes ($\bar{X} = \theta \bar{E}$).

The decomposed portion of the production matrix is expressed for large-scale activities devoted to all members' homeplot farms (as an aggregate the latter is denoted by $H$)

$$\bar{X}_H = HP \bar{H},$$

(8.22)

where the vector $HP$ is 1 by $M$, the matrix $\bar{H}$ is $M$ by $m$ in dimension ($M$ denotes the number of cooperative households as before). A typical elements of Equation 8.22 reads: $\bar{x}_i = HP_{j_1}$.

The relationships in Equations 8.19 through 8.22 could be stated independently as constraints but this alternative was not chosen here. Implicit values may be placed on the $z, \theta, HP$ vectors without directly utilizing Equations 8.19 through 8.22 in terms of constraints. The alternative selected is, in essence, the subtraction of the output levels (defined in the above equations) from Expression 8.15 (constraint
set) and the computation of implicit \( z, \theta, \) and HP values from the resulting solution.

**Variable costs** To define total variable cost, a linear relationship is assumed between the utilization of processes (\( x' \)'s) and the utilization of variable factors (\( y' \)'s)

\[
Y = XDT,
\]

where the matrix \( T \) is \( m \) by \( r \) in dimension and its elements correspond to the \( T_j \)'s in Expression 7.13 and in Equation 7.14.

Now total variable costs may be written as

\[
\text{total variable costs} = V QYDT,
\]

where \( V \) is the input price vector, \( Q \) is the final output vector, and the three matrices \((V, D, T)\) are defined by Expressions 8.17, 8.13 and 8.23 respectively. In Expression 8.23, \( \bar{X} \) has been replaced by the relation denoted by Equation 8.17 in order to derive total variable costs.

**The objective function** The objective is to maximize profits as given by

\[
\text{Max } \Pi = PQ - VY',
\]

where profit is expressed as the difference between the product of the final outputs and their prices, on one hand, and the product of variable factor inputs and their prices, on the other.

**The new constraints** A new set of constraints is derivable by subtracting the requirements of \( Z, \theta, \) and HP from the original set of constraints \((B)\)

\[
\bar{B} = B - \left[ ZA + \theta E + HP H \right] \left[ P \right]',
\]

where \( \bar{B} \) is the difference between the original constraints and the new ones.
This model gives the optimum values of the q's (final outputs) but the constraints are given in terms of \( \bar{X} \) (processes defined before). Now it is convenient to define

\[
\bar{A}' = A'D'\bar{Y}.
\]

(8.27)

Utilizing Equations 8.27 and 8.17, it is possible to replace Expression 8.15 by

\[
\bar{E} - QA' \geq 0.
\]

(8.15a)

Now the new constraints are stated directly in terms of the variables of the objective function.

The "fixed costs" of final outputs A method is necessary in order to calculate the variable costs encountered in the production of the predetermined levels of \( Z, \Theta, \) and \( HP \). These costs represent fixed costs with respect to the determination of the final output vector, \( Q \). This is a situation well known in business firms. Holdren (51, Chapter III) has used the term "discretionary fixed costs" to denote costs that are functionally variable in the usual short-run consideration but are not variable with respect to the decision at hand. In the recent context, discretionary fixed costs are expressed as

\[
V \left[ Z A + \Theta \bar{X} + HP \bar{H} \right] D' = \text{discretionary fixed costs.}
\]

(8.28)

This suggests that the implicit value that the cooperative firm is placing on activities \( Z, \Theta, \) and \( HP \) will be a function of the shadow price of the fixed constrained inputs plus the variable costs assignable. The marginal cost of \( Z, \Theta, \) or \( HP \) is simply the derivative of Expression 8.28 with respect to each variable.

Expression 8.15a establishes \( r \) constraints and their derivatives
with respect to the decision variable are

\[
\frac{\partial G}{\partial Q} = -\nabla D A'.
\]  

(8.29)

The evaluation of Z, Θ, and HP To evaluate the Z, the Θ, and the HP terms is a relatively simple procedure now. Each of these makes use of scarce fixed factors (b's) which have given a shadow price. Each of the Z, Θ, HP also utilizes variables inputs (y's) for which prices have already been calculated. Equations 8.19, 8.21, and 8.22 give the linear production function for Z, Θ, and HP in terms of the x's and in the b's. Equation 8.23 for Τ gives the use of y's with respect to the x's.

Now it is possible to compute the use of the fixed factors by Z, Θ, and HP as

\[
Z A' = B_Z.
\]  

(8.30)

\[
\Theta A' = B_{\Theta}.
\]  

(8.31)

\[
HP A' = B_{HP}.
\]  

(8.32)

The cost equations can also be set up for each of Z, Θ, and HP as

\[
V Z A' D_T + B_Z \lambda' = T C_Z.
\]  

(8.33)

\[
V \Theta A' D_T + B_{\Theta} \lambda' = T C_{\Theta}.
\]  

(8.34)

\[
V HP A' D_T + B_{HP} \lambda = T C_{HP}.
\]  

(8.35)

These equations show the total costs of variable and fixed factors utilized by Z, Θ, and HP respectively.

The maximum conditions The conditions for the maximum are given by the four relations

\[
p_j - V \nabla_j D_T - \lambda A'_{1j} \leq 0.
\]  

(8.36a)

\[
q_j (p_j - V \nabla_j D_T - \lambda A'_{1j}) = 0.
\]  

(8.36b)
\( \lambda_1 \left( \bar{b}_i - q \bar{A}_{ij} \right) = 0 \). \hspace{2cm} (8.36c)

\( \lambda_1 \geq 0 \). \hspace{2cm} (8.36d)

Relations denoted by Expression 8.36a essentially determine the value of the \( \lambda \)'s for the non-zero \( q \)'s. These \( \lambda \)'s form a vector which is 1 by \( r \) in dimension. Expression 8.36b states that for any output that is positive in the optimum solution, this relation must have a zero value. Expression 8.36c establishes the relation that the Lagrange multiplier, corresponding to any constraint which is not used up completely, will given zero value. Expression 8.36d requires that the \( \lambda \)'s be non-negative.

The optimum solution Once it is known which \( \lambda \)'s are non-zero and Expression 8.17 is incorporated in Expression 8.15a, the solution can be obtained for the decision variables, \( q \)'s.

The presentation of this model aims at two objectives. First, it may provide some initial steps in developing a systematic programming model adapted to the rather complex problems of cooperative optimization. Second, this optimization model of large cooperative enterprise will connect programming procedure with alternative objective functions in Chapter IX. Thus the next chapter gives double treatment to cooperative optimization tasks — concerning the large collective enterprise — by applying both the marginal and programming approaches.
CHAPTER IX. OPTIMIZATION IN THE LARGE COLLECTIVE ENTERPRISE

Several models with alternative objective functions may serve the optimization purposes of the large-scale collective portion of a farmers' cooperative.

Optimization in the Pure Enterprise Model

The pure enterprise version of the large-scale portion of a farmers' cooperative is conceived of as the usual commercial farm in the United States or as a typical state-owned farm (firm) in Hungary. This model of large-scale collective farming has the objective function of maximizing net profits which is the difference between gross revenue and production costs. Given prices, resource availabilities and technological possibilities, the profit-maximizing conditions will determine the optimal levels of outputs and inputs as well as the optimal size of farm operation in this model.¹

An application of marginal analysis

The objective function of a profit-maximizing cooperative enterprise by means of the square root production function expressed by Equation 8.5 is formulated as

$$\max_{\bar{M}} = P_q (a - b\bar{M} + c\bar{M}^5) - V_M \bar{M}, \quad (9.2)$$

where $V_M$ stands for the average price of a unit of $M$ defined as $M_o$ in

¹To optimize under the conditions of different farm structures, Heady (148) applies a production function exhibiting the ranges of both increasing and decreasing returns to scale in mathematical form

$$Y = aR + bR^2 - cR^3, \quad (9.1)$$

where $R$ is the ancestor of $M$ which this paper operates with.
Expression 8.1.

Maximum profit and optimal magnitude of \( M \) (as a farm size indicator) can be determined from the derivative of Equation 9.2 taken with respect to the independent variable

\[
\frac{\partial \pi}{\partial M} = -bP + 0.5cPqM^{-5} - V_M.
\]  

(9.3)

To calculate the profit maximizing level of input, Equation 9.3 is equal to zero and the new expression is solved for \( M \)

\[
-bP + 0.5cPqM^{-5} - V_M = 0
\]  

(9.4a)

\[
0.5cPqM^{-5} = bP + V_M
\]  

(9.4b)

\[
M^{-5} = \frac{bP}{0.5cPq} + \frac{V_M}{0.5cPq}
\]  

(9.4c)

\[
M^{-5} = 2bc^{-1} + 2c^{-1} V_M P_q^{-1}
\]  

(9.4d)

\[
M = (2bc^{-1})^{-2} + (2c^{-1} V_M P_q^{-1})^{-2}
\]  

(9.4e)

\[
M = 0.25b^{-2}c^2 + 0.25c^2 V_M P_q^{-2}
\]  

(9.5)

Equation 9.5 gives the optimal farm size in terms of the profit-maximizing input magnitude for a large cooperative enterprise having a square root production function under the specified conditions.

With the help of Equation 9.5 it is possible to determine separately the nature of the relationships between \( M \), on the one hand, and \( b, c, V_M, P_q \), on the other hand. These relationships are characterized by
Expressions 9.8 and 9.9 reflect the logic of economics according to which firm size increases (decreases) when output price increases (decreases) and firms become smaller (larger) if their inputs are more expensive (cheaper), assuming unchanged conditions otherwise. In the given context, b behaves like $V_M$ and c acts like $P_q$ with respect to optimal farm size.

By extending the mathematical formulation, Equation 8.5 may develop into a square root function incorporating two inputs ($X$ and $Z$) and their positive interaction

$$q = a + eXZ + cx^5 + dZ^5.$$  \hspace{1cm} (9.10)

After partially differentiating the new profit function, one can derive for maximum profit and optimal farm size the conventional first-order conditions (marginal profit is zero; the marginal value product of each resource is equal to its price; the marginal rate of technical substitution between two inputs equals the inverse ratio of their prices).

The optimal state of this sort may exist when prices are constant parameters for the farm decision unit and no input is of limited availa-
bility. In addition to these, decreasing returns to scale (diminishing marginal products of each input) must be in existence which have been ensured by the chosen (square root) type of the production function.

If capital availability is limited, then marginal conditions will still prevail; however, farm size has become smaller because a lesser quantity of each input is used.

A numerical example of linear programming

This section contains a short numerical example which shows the application of the programming technique and also demonstrates the efficacy of the linear programming model developed in Chapter VIII.

The initial set of data According to Equations 8.13 and 8.14 the $D_j$ matrix and the $D$ matrix assume general forms

$$D = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}$$

and

$$D = \begin{bmatrix} 1 & d_{12} \\ d_{21} & 1 \end{bmatrix}$$

In order to have a brief example it is assumed that $d_{12} = d_{21} = 0$. By this assumption matrix $D$ has become the identity matrix and the intermediate processes have been eliminated from the example.

The chosen form of the usual $A'$ matrix indicates that there are two constraining factors ($b$'s) and two processes ($x$'s) in this simplified example.
\[ A' = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}. \]  

(9.13)

The two constraining factors are given numerical values as

\[ b_1 = 100 \]
\[ b_2 = 250. \]  

(9.14)

According to the data in Equation 9.14, an upper limit was placed on each of the two processes utilized

\[ x_1 = 500 \]  
\[ x_2 = 250. \]  

(9.15)

The form of matrix \( \nabla \) defined by Equations 8.17 and 8.18 indicates that both processes are utilized in producing both products (q's)

\[ \nabla = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}. \]  

(9.16)

Matrix \( A' \) defined by Equation 8.27 relates the b's and the q's and takes the form

\[ A' = \begin{bmatrix} 5 & 1 \\ 3 & 2 \end{bmatrix}. \]  

(9.17)

The \( T \) matrix defined in connection with Expressions 7.13 and 8.23 relates the x's and the y's (the variable factors) in form

\[ T = \begin{bmatrix} 1 & 3 \\ 20 & 4 \end{bmatrix}. \]  

(9.18)

The dimensions of this matrix show that there are two y's (\( y_1 = \) non-labor, \( y_2 = \) labor) in this example and both processes utilize both variable factors.

The price vector of the variable factors is assumed to be
\[ V = \begin{bmatrix} 1 & 2 \end{bmatrix} \cdot \tag{9.19} \]

The price of the two final outputs are
\[ p_1 = 56 \tag{9.20} \]
\[ p_2 = 138 \cdot \]

The value weight of a unit of outputs. On the basis of the above information, Expression 8.36 can be given numerical values for the first output \( q_1 \) as
\[ 56 - \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 20 & 4 \end{bmatrix} - \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} \begin{bmatrix} 0.5 \\ 1.3 \end{bmatrix} \leq 0 \ . \tag{9.21} \]

The calculation gives 49 as variable costs; therefore, the familiar \( c \) value is 56 - 49 = 7 for \( q_1 \). This is the net price or the value weight or profits associated with one unit of output. The \( q_1 \) value must be equal to the product of the last two matrices of Expression 9.21, i.e.
\[ 0.5 \lambda_1 + 1.3 \lambda_2 = 7 \cdot \tag{9.22} \]

For the second output \( q_2 \) Expression 8.36 takes the numerical form
\[ 138 - \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 20 & 4 \end{bmatrix} - \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} \begin{bmatrix} 1 \\ 2.2 \end{bmatrix} \leq 0 \ . \tag{9.23} \]

These data give 126 as variable costs and \( c_2 = 12 \). The twin expression of Equation 9.22 for \( q_2 \) is written
\[ \lambda_1 + 2.2 \lambda_2 = 12 \cdot \tag{9.24} \]

The shadow prices. The simultaneous solution of Equations 9.22 and 9.24 gives the numerical values
\[ \lambda_1 = 1 \tag{9.25} \]
\[ \lambda_2 = 5 \cdot \]
These two numerical values are the so-called shadow prices for the two constraining resources; the total value is $\lambda_1 b_1 = 100$ for the first resource and $\lambda_2 b_2 = 1,250$ for the second (considering the endowment expressed by Equation 9.14). The positive values of $\lambda_1$ and $\lambda_2$ indicate that both the constraining resources are used up completely by the final (optimal) plan.

**Optimum output levels and maximum profits**

Given all the information obtained, the optimum level of outputs can be computed by utilizing Expressions 8.17 and 8.15a

\[
\begin{align*}
q_1 &= 150 \\
q_2 &= 25.
\end{align*}
\]  

(9.26)

Together with the $c_1$ and $c_2$ values, Equation 9.26 gives the sum of the maximum net profits ($\pi^0$)

\[
\pi^0 = 7(150) + 12(25) = 1,350.
\]  

(9.27)

The numerical value of the optimum solution can be checked by

\[
\lambda_1 b_1 + \lambda_2 b_2 = 100 + 1,250 = 1,350.
\]  

(9.28)

**Quantity and cost of variable factors used**

Knowing the optimum $q_1$ and $q_2$ values and utilizing Equations 9.16 and 9.18, the optimal level of the variable factors can be calculated from Expression 9.26

\[
\begin{bmatrix} 150 & 25 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 20 \\ 4 \end{bmatrix}.
\]  

(9.29)

This expression results in numerical values

\[
\begin{align*}
y_1^0 &= 5,500 \\
y_2^0 &= 2,500.
\end{align*}
\]  

(9.30)

The total value of the utilized labor input ($y_2$) is computed from
Equations 9.19 and 9.30 in terms of the total wage bill
\[ v_2 y_2^0 = 2 (2,500) = 5,000. \]

The amount of the cooperative surplus Utilizing the solution values in Equations 9.27 and 9.31, the maximum amount of cooperative surplus (gross income) obtained by this pure enterprise model is given by
\[ s^0 = \Pi^0 + v_2 y_2^0 = 1,350 + 5,000 = 6,350. \]

Cooperative surplus per unit of labor input is computed from Equations 9.32 and 9.30 as
\[ \frac{s^0}{L} = \frac{6,350}{2,500} = 2.54. \]

This data will later serve the purpose of a very instructive comparison.

Optimization and the Cooperative Model of the Family-Farm Type

It is assumed by a large group of Marxian and non-Marxian economists that the large cooperative enterprise has been organized to give maximum benefits to the cooperative members who seek maximum returns on their labor input. The model expressing this idea be termed as the family-farm type model for a large cooperative enterprise. The objective function of this model is associated with the synonymous concepts of gross income, net value product, and cooperative surplus defined in Chapter VII. Exclusively the first term is used in the socialist countries; the last one will be applied in this section.

Results from marginal analysis

The amount of cooperative surplus per cooperative member is given by
\[ S = \left( P_q - rV_x^M \right) M^{-1}, \]
where $S$ denotes the total amount of the cooperative surplus (defined by Equation 7.32) divided by the number of cooperative members; $M^{-1}$ indicates that the terms in parenthesis are divided by the number of cooperative members (the other symbols are explained in connection with Expression 8.2). The mathematical form of Equation 9.34 hints that this model does not consider direct wages for the cooperative members. It also shows, explicitly, that cooperative surplus has clearly a residual nature which is characteristic for the income of family farms.

On the basis of a square root production relationship, the objective function of this model is set up as

$$\text{Max } S = \left[ \frac{P}{M} (a - bM + cM^2) - rvx \right] M^{-1}.$$  \hspace{1cm} (9.35)

Expanding the terms and dividing by $M$ in Equation 9.35 and taking derivative will give the results

$$\text{Max } S = \frac{aP}{qM} M^{-1} - bP + cP q M^{-5} - rvx$$ \hspace{1cm} (9.36)

$$\frac{\partial S}{\partial M} = -\frac{aP}{qM^2} M^{-2} - \frac{5cP}{q} M^{-1.5}.$$ \hspace{1cm} (9.37)

Setting Equation 9.37 equal to zero, the optimal value of $M$ may be calculated

$$-\frac{aP}{q} M^{-2} - \frac{5cP}{q} M^{-1.5} = 0$$ \hspace{1cm} (9.38)

$$aP M^{-2} = \frac{5cP}{q} M^{-1.5}$$ \hspace{1cm} (9.38a)

$$M^{-5} = \frac{5a^{-1}}{c}$$

$$M = (\frac{5a^{-1}c}{5})^{-2}$$ \hspace{1cm} (9.38b)

$$M = 4a^2 c^2.$$ \hspace{1cm} (9.39)
Equation 9.39 shows that the optimum size is insensitive to prices of output and nonlabor inputs in the pure family-farm model of the cooperative enterprise. (The terms of $P_q$ and $V_x$ have not shown up in Equation 9.39.) This finding is due to the nature of this cooperative model and it may not be attributed to the type of the production function applied. The optimal solution has exactly the same nature as Heady's (44) solution based on a different production function (Equation 9.1) in the framework of the given cooperative model.

In his own practical experience covering more than two decades of cooperating farming, the author has never met a single cooperative member or cooperative manager whose ideas about production scale and farm size are in harmony with the notion formulated by Equation 9.39. However, he admits that some public administrators and policy makers have shown attitudes and behavior which are fairly consistent with the essence of this equation. At the same time, the economic literature is quite "rich" in authors who are in favor of a cooperative model of the family-farm type and who consider the maximization of the per capita cooperative surplus as the only perceivable objective function in a cooperative model. First in the technical literature, Heady has shown the link between the nature of the cooperative model, the type of the cooperative objective function expressed in Equation 9.34, on one the hand, and the size-insensitivity to prices formulated by Equation 9.39, on the other hand. Empirical studies may demonstrate the correspondence between cooperative theories associated with Equation 9.34 and practical policies implicitly based on Equation 9.39.
A new interesting feature of the given model is explored by means of other production functions. Using the homogeneous version of the square root Equation 8.5, the objective function becomes (instead of Equation 9.35)

\[ \text{Max } S = \left[ P_q \left( -bM + cM^{1.5} \right) - rV_x \right] M^{-1} \]  

(9.40)

Following the steps expressed by Equations 9.36 through 9.39 will give the results

\[ \text{Max } S = -bpq + cpq M^{1.5} - rV_x \]  

(9.41)

\[ \frac{dS}{dM} = -0.5 cpq M^{1.5} = 0 \]

(9.42)

\[ M = 0 \]  

(9.43)

Expression 9.43 suggests that per capita cooperative surplus is "theoretically" maximum at \( M = 0 \); it means that for the objective function of the family-farm-type cooperative model, the value of \( S \) declines definitely for all positive values of \( M \). In this model the cooperative enterprise should be as small as possible and if \( M \) denotes labor-supplying members, the "collective-cooperative" enterprise would be a "one-person" operation. If the cooperative model of the family-farm type is the case, a production function of constant scale returns (as Equation 8.1) will also give a solution expressed in Equation 9.43 (44, p. 24). In contrast, it may be recalled that the pure enterprise model has no limit to the optimal cooperative size.

Applying the above homogeneous square root equation again, an optimal farm size is calculated for the pure enterprise model.
\[
\text{Max } \Pi = P \left( -bM + cM^5 \right) - V_M M \\
\text{Max } \Pi = -bP M + cP M^5 - V_M M \\
\frac{\partial \Pi}{\partial M} = -bP q + 0.5cP M^{-5} - V_M = 0 \\
M = 0.25b^2c^2 + 0.25c^2p^2 v^2.
\]

Comparison of the two solutions (Equations 9.43 and 9.46) detects that the pure enterprise model has an optimal farm size larger than the family-type model of the large cooperative enterprise unit. The incomparability of Equations 9.45 and 9.39 (originating from the selected particular functional form) necessitated a simplified comparison in which one of the compared "optimal cooperative" sizes was zero. Heady (43, 44) avoided trivial procedure of this sort and derived comparable nonzero values for M in his models by applying a quadratic production function \((q = -k + aM - bM^2 \text{ in my notation})\).

To summarize, the optimal size for the family-farm-type cooperative model is not influenced by prices of output and nonlabor inputs. It is only a function of the parameters of the production function. (The value of \(P_q\) and \(V_x\) does not enter in Equation 9.39.) As the objective function of this model designates in Equation 9.35, the magnitude of \(P_q\) and \(V_x\) does influence the sum of returns (cooperative surplus) per cooperative member but either of these prices does not influence the optimal size or the optimum number of cooperative members (in Equation 9.39). If \(P_q\) is lowered and/or \(V_x\) is raised, cooperative members will of course be given a lower income but the optimal size of their cooper-
ative operation is unaffected by any price change.

The optimal size or dimension for the pure enterprise model is a function of output and input prices. (The value of both $P_q$ and $V_M$ or $V_x$ is involved in Equations 9.5 and 9.46.) In this model the optimal volume of production will decrease as $P_q$ decreases or $V_M$ increases, and farm size will increase as a result of higher $P_q$ and lower $V_M$. ($V_M$ stands for the prices of all the aggregate inputs here.)

In a given period, the optimum-scale cooperative of the pure enterprise model would be larger (its $M$ will have a larger value) than the corresponding cooperative of the family-type model; the actual difference in size is influenced by the price of output and nonlabor inputs as well as by wages ($P_q$, $V_x$, and $V_m$).

Conceiving these two cooperative models as twins, "the enterprise twin" in this paper corresponds to Domar's (16) "capitalist twin" and to Heady's (43,44) commercial or state farm model. The "family-farm resembling" twin is born in the same family as Domar's producer cooperative and Heady's pure cooperative.¹

The solution values of linear programming

For the cooperative model of the family-farm type, linear programming procedure will be applied on the basis of the simple, contrived, initial data which exemplified the pure enterprise model of

¹It is also very important to distinguish between the cooperative model of family-farm type and the original family farm model since the family farm almost always operates as an enterprise unit.
cooperative farming. Expressing the essence of the family-farm-type model, this example will not put a price on the labor factor. It will change vector V in Equation 9.19 into a scalar. This alteration will not influence Expressions 9.13, 9.18, and 9.20 but it will result in completely different solution values.

The value weights In the recent programming model, Expression 9.47 will take the place of the first part of Expression 9.21 of the pure enterprise model (the second part is unchanged and will not be repeated)

\[ 56 - 1 \begin{bmatrix} 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 20 \end{bmatrix} \]  
(9.47)

This will give 23 as variable costs for \( q_1 \) and \( c_1 = 33 \). Thus Equation 9.22 is changed to

\[ 5\lambda_1 + 1.3\lambda_2 = 33 \]  
(9.48)

Expression 9.23 will be altered partially as

\[ 138 - 1 \begin{bmatrix} 2 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 20 \end{bmatrix} \]  
(9.49)

which provides a variable cost of 84 for \( q_2 \) and \( c_2 = 54 \). Thus Equation 9.40 is replaced by

\[ \lambda_1 + 2.2\lambda_2 = 54 \]  
(9.50)

The shadow prices The simultaneous solution of Equations 9.48 and 9.50 shows that the first constraining fixed factor \( (b_1) \) has not been exhausted by the optimal program, therefore, \( \lambda_1 = 0 \). The actual value of \( \lambda_2 \) will be computed later after the optimal \( q \)'s and the maximum surplus value are calculated. Now just the data is given (which
will be calculated by Equation 9.57)

\[ \lambda_1 = 0 \quad (9.51) \]
\[ \lambda_2 = 25.38. \]

**Optimum output levels** In this numerical example the second output will not be brought into the optimum solution by the programming procedure. The profit (surplus) maximizing output levels are

\[ q_1 = 192 \frac{4}{13} \quad (9.52) \]
\[ q_2 = 0. \]

**Quantity and costs of variable factors** Instead of Equation 9.15, this model will operate with \( x \) values obtainable from Equations 9.16 and 9.52

\[ x_1 = 3 (192 \frac{4}{13}) = 576 \frac{12}{13} \quad (9.53) \]
\[ x_2 = 192 \frac{4}{13}. \]

The optimal levels of variable factors used are computed as

\[ y_1^o = 576 \frac{12}{13} + 20 (192 \frac{4}{13}) = 4,423 \frac{1}{13} \quad (9.54) \]
\[ y_2^o = 3 (576 \frac{12}{13}) + 4 (192 \frac{4}{13}) = 2,900. \]

The total wage bill which is an implicit normative category in this model can also be calculated

\[ v_2 y_2^o = 2 (2,900) = 5,800. \quad (9.55) \]
Maximum surplus  Equations 9.53 and the $c_1$, $c_2$ values give maximum gross income (cooperative surplus) attainable in this model

$$S^0 = 33 \left( 192 \frac{4}{13} \right) = 6946.66$$  \hspace{1cm} (9.56)

Now — referring to Equation 9.14 — the calculation of the shadow price of the exhausted resources ($b_2$) can be illustrated

$$\lambda_2 = \frac{6946.66}{250} = 25.38$$  \hspace{1cm} (9.57)

Cooperative surplus per unit of labor input is given by

$$\frac{S^0}{L} = \frac{6946.66}{2900} = 2.18$$  \hspace{1cm} (9.58)

Maximum profits  Equations 9.55 and 9.56 give the maximum profit ($\Pi^0$) attainable in this cooperative model

$$\Pi^0 = S^0 - v^0 = 6946.66 - 5800 = 548.66$$  \hspace{1cm} (9.59)

A Comparison of Two Cooperative Models

Values in the optimum plan

The results achieved for the two cooperative models by the linear programming technique may be compared in several aspects although a simplified example may not serve as a final argument in any discussion. Without far-reaching direct conclusions, the major dissimilarities are listed and commented on below. For the sake of brevity, the pure enterprise model of the cooperative farm is called Model I, and the cooperative model of the family-farm type is named Model II.

The major characteristics and differences in the optimum plan of
the two cooperative models are the following:

1. The \( c \) values are, of course, larger in Model II since its variable costs do not include wages.

2. Both the limited fixed factors are exhausted by the optimal plan in Model I; only one of the two gets this solution in Model II.

3. Both outputs are produced in the optimum product mix of Model I; only one final output is included in the optimal plan of Model II. (Here a warning is especially due against any generalization of this example.)

4. Each model utilizes one of the variable factors in a larger quantity and uses the other variable factor in a smaller quantity.

5. The total amount of profits obtainable is much higher (more than twice) in Model I.

6. The total amount of cooperative surplus (gross income) is almost the same in both models; it is 6,350 in Model I and 6,346.66 in Model II.

7. Finally, the cooperative surplus (gross income) per unit labor input (member) is lower in Model II which claims the declared objective of maximizing this value.

The numerical examples offer understandable explanations for the last major (and perhaps unexpected) difference by indicating that resources may remain unutilized and the output mix may be suboptimal; therefore, much lower profits can be achieved and only a smaller per-capita gross income can be "earned" in Model II. In other words, Model II does not seem to be organized for the maximum benefit of its members. In addition, this model may not provide maximum profits for developing the large collective enterprise and may not produce an optimum output with efficiently utilizing the limited resources.
Some considerations based on programming approach (Model I)

It has not been customary to utilize the profit-maximizing pure enterprise model in cooperative optimization. In spite of this fact, economists should recognize the evident advantages of this model, especially if it is formulated in the framework of a linear programming model. In this case the linear programming procedure discussed in Chapter VIII is completely applicable and the objective function is exactly the same as defined in Equation 8.25, namely:

\[ \text{Max } = PQ - VY^* \]

Linear programming seems to be the most powerful technique for a large-scale multiproduct cooperative enterprise for calculating the profit-maximizing quantities and mix of outputs to produce, and for computing the optimizing quantities and mix of inputs to apply and purchase.

The optimal solution obtained by linear programming provides the shadow prices of the scarce resources. These prices are of great importance in the efficient allocation of inputs among different branches of the large collective enterprise.

To achieve efficient intrafarm allocation of resources, a large cooperative enterprise should consider as cost each scarce input used. By no means can labor be an exception, although direct wage systems are not applied in many cooperatives and the "civil rights" of formal wages paid to cooperative members are vehemently refuted by quite a few economists. This point will be taken up and discussed further in connection with Model II and other cooperative models.
Programming approach and its lessons (Model II)

The cooperative model of the family-farm type suggests that the value of output minus nonlabor costs divided by the number of cooperative members should be maximized on the part of the large collective enterprise. In a linear programming framework, this objective would have the same functional form as Equation 8.25, but the content of the cost-term \((VY^e)\) would be different. The input price vector, \(V\), and the variable input vector, \(Y\), are simply interpreted as nonwage and nonlabor items.

In the standard linear programming model, the maximization of the total cooperative surplus is equivalent to the maximization of this surplus per cooperative member since here the labor supplied is a limited resource rather than a decision variable. Heady and Agrawal (45) have developed the technique of linear fractional functional programming in order to optimize an objective function which is a ratio of two linear functions of decision variables, subject to linear constraints.

The cooperative model of the family-farm type presumes that large collective enterprises do not consider their members' labor input as production cost. The category of cooperative surplus (or gross income) lumps basically different income shares into the same inseparable basket. It does not distinguish between profit returns and implicit wage returns which are of rather different nature from a functional point of view. Profit is, in reality, a return to the collective of the cooperative members as an economic organization embodied in their large common enterprise. The wage return is logically return on their labor inputs to the
individual members-laborers as such.

A cooperative member may allocate his labor among different employment opportunities, including his work in the large collective enterprise. But the large cooperative enterprise has no other economic alternatives than to utilize its members' labor, its capital, land, managerial resources and to pursue profits for the benefit of its members in accordance with social goals and priorities.

Maximum cooperative surplus as an economic objective seems to be the consequence of a functional falsification without any ideological connotation. If profits and wages are placed into the same boat, with no distinction between them, the economic reasons for cooperation may think into the "dim ocean" of cooperative surplus.

Agricultural economists in my country, Hungary, should follow the practical path of cooperative managers in recognizing that the proper "cooperative" wage rate for calculations is the well known alternative (or opportunity) cost in most cases. It is what the cooperative member and employee could earn in comparable employment outside the large collective enterprise.

Optimum Intensity in Farming

Increasing yield from a given (or even diminishing) area of arable land may be the necessary aspect of farming as is the unquestionable case in Hungary. Under these circumstances the optimum utilization of available land plays an important role.
The applicability of marginal analysis

The amount of any single input or that of an aggregate input category (defined as \( M_o \) in Expression 8.1) which maximizes per hectare output can be determined by the standard optimization technique after an appropriate production function has been selected. Now one may assume that Equation 8.5 is characteristic for the input-output relationship of a technical unit (1 hectare land) of agricultural production and may also assume \( M_o \) in the place of \( M \).

The derivation of the output-maximising condition (\( MPP = 0 \)) will give the optimum amount of input per unit of land area

\[
q = a - bM_o + cM_o^5
\]  
(9.60)

\[
\frac{\partial q}{\partial M_o} = -b + 5cM_o^4 = 0
\]  
(9.61)

\[
M_o = \frac{a - b}{5c}.
\]  
(9.62)

Equation 9.62 shows the level of aggregate input (or a single input as fertilizer) which gives rise to maximum output per hectare if Equation 9.60 is the relevant production function for this technical unit.

The optimum level of intensity can be computed by the usual optimization technique. This optimal intensity may be conceived of as the amount of an input (for example, fertilizer denoted by \( Z \)) per hectare which maximizes return from expenditures spent on this input. The farm unit is assumed to have limited capital in this case.

Applying Heady's (43) recent example in a generalized form, first
"gross returns from fertilization" are calculated by the help of a quadratic response function in order to find the optimal intensity of fertilizer use. Then this return from fertilization is divided by the total cost of fertilizer use. Thus the resulting rate of gross return function, GR*, for fertilizer expenditure is derived as

\[
GR^* = \frac{P_q(aZ - bZ^2)}{F + V_Z Z}
\]  

where Z stands for fertilizer used on one hectare of land, \( V_Z \) denotes the price of fertilizer and F is the fixed cost of fertilization per land unit.

Taking the derivative of \( GR^* \) with respect to \( Z \), equating it with zero and solving for the value of \( Z \) will give the optimal intensity expressed by the per-hectare amount of fertilizer which maximizes gross return from expenditures spent on buying and applying fertilizer.

An important physical aspect of optimal intensity may be shown by computing the substitution rate between fertilizer and land. In this two-resource case, the production function is assumed to be given by

\[
\frac{q}{D} = \frac{aZ}{D} - b \left( \frac{Z}{D} \right)^2
\]  

where \( D \) denotes the units of land, \( q, Z, a, b \) are output, fertilizer, and constants respectively (as before). Multiplying and dividing by \( D \) will result in

\[
q = aZ - bZ^2D^{-1}
\]

Both Equations 9.64 and 9.64a express output as a function of fertilizer
and land. From Equation 9.26a one can easily calculate the marginal physical products of the two inputs and their ratio will represent the marginal rate of technical substitution between fertilizer and land.

\[ \text{RTS}_Z \text{ for } D = \frac{\text{MPP}_Z}{\text{MPP}_D} = \frac{a - 2bD^{-1}}{b^2D^2} = \frac{aD^2}{b^2} - \frac{2D}{b^2} \]  

(9.65)

Equation 9.65 shows that fertilizer substitutes for land at a diminishing rate.

To bring economic indicators into the picture, one should set RTS equal to the corresponding price ratio. Since the market price for farm land is not given in the Eastern European countries, it must be adopted either from the dual solution of a linear programming model or from the statistics of land prices paid for alternative uses (for example, home and factory sites).

Another variant of optimal intensity can be computed from the regular profit maximizing condition (applied on per hectare basis) which equates the marginal value product of an input with its price. This principle is applicable to each kind of input. In the cooperative model of the family-farm type, however, the optimal intensity of labor input may be derived either with the help of a normative wage or by finding the state where the marginal product of labor is zero (if available labor were unlimited and completely "free" as well as if "ceteris paribus" conditions prevailed).
Programming aspects

In a programming framework, the optimum intensity model depicts a situation where only land is considered as a scarce factor and the objective function of the large cooperative enterprise is to maximize the output per land unit. This amounts to the same programming problem as maximizing total output, with the only special feature being that labor is considered in unlimited supply.

In today's Hungary it is hard if not impossible to conceive of a cooperative situation in which optimum intensity would be a rational decision rule. A cooperative of this type would have to be an almost Robinson-Crusoe-type economy with a labor supply unlimitedly abundant relative to land. This cooperative would have a surplus of labor whose societal marginal product approaches zero, and it would purchase no external inputs for farming. Under these rather unrealistic conditions, the profit function would assume the form of maximising the rent on the scarce land.

A variant of this intensity model is the case where there are no inputs from nonfarm sectors to agriculture and land is not scarce but labor is scarce. Once again one maximizes rent on land associated with labor up to the point where the marginal product of land approaches zero.

If the objective function of the intensity model is a ratio of two linear functions of decision variables (in a form resembling Equation 9.63), then linear fractional functional programming (45) may come into the picture again.

Aspects of the intensity model cast light on the relative rarity
of modern cases where output maximization is a relevant rule. This rule be analyzed under cooperative conditions in the next section.

Maximum Output Model

Results from marginal analysis

If a cooperative enterprise wishes to maximize production subject to cost constraints, it may treat this optimization problem with the usual technique of marginal analysis. Assuming linear relationships, one output and two inputs, the relevant functions are given by

\[ q = f(M, K) \]  \hspace{1cm} (9.66)

\[ C = V_M M + V_K K + F \]  \hspace{1cm} (9.67)

where \( M \) and \( K \) are labor and capital inputs with unit price \( V_M \) and \( V_K \) respectively, and \( F \) represents fixed costs. The objective function (\( \Phi \)) will maximize output subject to limited capital (\( \bar{C} \)) which is entirely used up according to Equation 9.67

\[ \text{Max } \Phi = f(M, K) + (\bar{C} - V_M M - V_K K - F) \]  \hspace{1cm} (9.68)

Partial differentiation results in

\[ \frac{\partial \Phi}{\partial M} = \frac{\partial f}{\partial M} - \lambda V_M \]  \hspace{1cm} (9.69)

\[ \frac{\partial \Phi}{\partial K} = \frac{\partial f}{\partial K} - \lambda V_K \]  \hspace{1cm} (9.70)

\[ \frac{\partial \Phi}{\partial \lambda} = \bar{C} - P_M V_M - P_K V_K - F \]  \hspace{1cm} (9.71)

Dividing Equation 9.70 by Equation 9.69 will give the first-order
conditions for maximum output which may be written in two alternative forms

\[
\frac{\text{MPP}_M}{\text{MPP}_K} = \frac{\text{RTS}_M}{\text{RTS}_K} \quad \text{for } K = \frac{V_M}{V_K}, \tag{9.72a}
\]

\[
\frac{\text{MPP}_M}{V_M} = \frac{\text{MPP}_K}{V_K} = \lambda. \tag{9.72b}
\]

In Equation 9.72b the Lagrangean multiplier, \( \lambda \), is interpreted as the total derivative of output with respect to costs (the reciprocal of the marginal costs).

Output maximization procedure is altered in the cooperative model of the family-farm-type to such extent that labor costs are not considered. Since a relevant production function is inconceivable without labor input, some calculative price (alternative wage) should be used for labor; otherwise no solution could be derived in this cooperative model containing two inputs. If more than two input categories are involved but labor is given no price, then the conclusion still remains the same. The next model to be developed will determine a maximum (potential) return on the unit labor input of the cooperative members as some kind of cooperative "wage" in terms of solution values.

Considerations from a programming viewpoint

In a programming approach, the output-maximizing model is considered a variant of the intensity model in the sense that both labor and land have a fixed quantity and for some reason or other the decision-maker does not want to put prices on these inputs. In general it
would of course lead to misallocation of scarce resources and to inefficient resource use if some inputs were priced and others were not. This point has been discussed in connection with the cooperative model of the family-farm type.

If there existed a "free" and practically unlimited capital supply from outside to the farm decision units but labor and land were fixed in quantity, it would seem obvious that one should simply maximize output relative to the fixed factors. This would probably not result in resource misallocation in the event of a single output. However, even in the unrealistic case of a super-specialized farm, if the only product is produced by several processes (or technologies) resources could easily be misallocated by not considering land or labor costs.

When labor and land are both scarce and more than one output is produced, then a manager (even that of a freely and abundantly equipped state farm) must charge both labor and land — at least implicitly for intrafarm use — in order to choose the correct product mix which maximizes the value of output and in order to select the mix of the most efficient production processes (technologies) for minimizing the cost of whatever output mix that is optimal.

In general, when there are at least two factors that can be substituted for each other in many different uses, and there are various ways of producing the same product, or there exists the possibility of producing different products with different processes (technologies), then an allocation problem is always in existence. If an allocation problem arises, then optimal allocation can be achieved only by utilization of
prices of some sort; thus at least shadow — or intrafarm — prices are required. Regardless of how many factors are involved, for example if the government is willing to grant all the necessary capital goods to a state farm with no charge (as was the case for quite a long time in Hungary), this farm could not optimize in any meaningful economic sense without considering these capital factors as costs.

The farm situation depicted before is optimally adaptable to treatment by mathematical programming. The more inputs and the more outputs involved, the more essential and the more powerful the programming technique is in every respects.

"Efficiency" Models of a Cooperative Firm

In the socialist countries cooperative farms are very often given the politico-economic slogan of efficiency in a specific form: "to obtain the greatest possible output at least cost." According to the author's judgement, this tautology may essentially express the minimum cost principle termed as "lex minimi" in economics. At the same time Shubik (74) and others should be praised for saying that the cited goal or inspiration is nonsense as no operative meanings can be attributed to it. While "the maximum output at minimum cost" is operationally undefined, it is completely meaningful to ask for the greatest output for a fixed (given) cost, and it is similarly reasonable to seek for the least cost of a fixed output.
Cost minimization and maximum unit price for cooperative labor

Under general circumstances, the minimum cost of a given output is determined as subject to the production function. Operating with Equations 9.66 and 9.67, the cost minimizing objective function is expressed by

$$\text{Min } \psi = V_h M + V_r K + F + \lambda \left[ q - f(M,K) \right] , \quad (9.73)$$

where $q$ denotes that the quantity of output is predetermined. Successive partial differentiation and division will result in the same first-order conditions for minimum cost as have been previously computed for maximum output (Equations 9.72a and 9.72b). The same problems have also arisen with respect to the price of labor input in the cooperative model of the family-farm type as discussed before. In the one output, two input case some sort of labor price is also required in order to achieve the standard geometrical (tangency) solution because the price line cannot be graphed without it.

In addition to the alternative cost and the programming-provided shadow price, a third kind of labor price (wage) may also be derived in the cooperative model of the family-farm type. The latter is based on the concept of cooperative surplus and is analogous to a certain product price elaborated by Helmberger and Hoos (48) for members of marketing cooperatives.

In a large-scale cooperative enterprise (assumed for the time being as a family-farm type version of the cooperative models) various other inputs are associated with $M$ (members' total labor input), to
produce a final good, \( q \). The production function is given by:

\[ q = q (X_1, \ldots, X_i, \ldots, X_N, H[H]), \tag{9.74} \]

where the \( X \)'s represent nonlabor inputs and \( H \) stands for all the home-plot farms belonging to the given large cooperative enterprise.

The following assumptions will serve in the model to be developed:

1. the production function is a regular single-valued function which specifies all the technologically efficient methods of production;

2. the marginal physical products are nonnegative in the relevant range;

3. the isoquants are smooth and convex to the origin;

4. all \( X \)'s are bought and \( q \) is sold in a perfectly competitive market and/or their prices are constant parameters for the cooperative;

5. the number of cooperative members is fixed and the large collective enterprise is able to utilize all the labor that its members supply;

6. all cooperative members receive the same return per unit of \( M \) (and this return is denoted by \( V_M \)).

In addition to the above assumptions, it is also assumed that the objective of the cooperative organization is to maximize the return (unit price) of \( M \), for any amount of \( M \) which the individual members wish to supply, but subject to the restraint that all nonlabor costs, including fixed costs, \( F \), are met.

If the large cooperative firm acts according to the pure enterprise model, its profit equation would be shown in the given context by

\[ \Pi = P_q q - \sum_{i=1}^{N} V_i X_i - V_M M - F. \tag{9.75} \]
Recalling that the large cooperative firm views $M$ as a parameter beyond its control, Equation 9.75 may be rewritten in the form resembling the concept of the cooperative surplus ($S^*$)

$$S^* = P_q q - \sum_{i=1}^{N} V_i x_i - P$$

(9.76)

An alternative form of this cooperative surplus is given by

$$S^* = V M$$

(9.77)

where $M$ is a parameter for the large cooperative as $P_q$ and $V_i$ but $V_M$ is not so. Clearly, a maximum $S^*$ will determine a maximum $V_M$. The maximization of $S^*$ implies the fulfillment of a cost minimization condition and an optimum output requirement. On the track of Helmberger and Hoos (48) it is possible to show that these conditions are exactly analogous to those which must be satisfied by a profit maximizing competitive firm.

Although the amount of members' labor input is fixed, various alternative levels of $q$ may be feasible considering different technologies, timing, etc. To achieve a maximum $S^*$, any level of $q$ produced must be produced at a minimum variable cost (excluding labor cost) when the sum of all nonlabor variable costs, $C^*$, is defined by

$$C^* = \sum_{i=1}^{N} V_i x_i$$

(9.78)

With Equation 9.78, the cost minimizing Lagrangean term is written as

$$\text{Min } C^* = \sum_{i=1}^{N} V_i x_i - \lambda \left[ \alpha(x_1, \ldots, x_i, \ldots, x_N | M, H) - q \right]$$

(9.79)
In order to have \( C^* \) as a minimum, two conditions must be satisfied

\[
\frac{\partial C^*}{\partial x_1} = 0 \quad (9.80a)
\]

and

\[
V_1 = \lambda \frac{\partial q}{\partial x_1}, \quad (9.80b)
\]

where \( i = 1, \ldots, N \). Costs minimization for any given amount of \( q \) implies that the cost-productivity ratio (the ratio of input price to the marginal physical product) must be equal for all nonlabor productive services.

In the recent "efficiency" model each \( q \) will be associated with a corresponding minimum variable nonlabor cost of production according to a specific cost function

\[
C^* = C^*(q), \quad (9.81)
\]

where \( C^* \) denotes the variables costs of nonlabor nature again. Equation 9.76 may be rewritten by substituting Equation 9.81

\[
S^* = P_q q - C^*(q) - F. \quad (9.82)
\]

If \( S^* \) is to be a maximum, the analogous conditions to Equations 9.80a and 9.80b must hold in form

\[
\frac{dS^*}{dq} = 0 \quad (9.83)
\]

and

\[
P_q = \frac{dC^*}{dq}. \quad (9.84)
\]

Thus a maximum \( S^* \) implies that output price equals marginal cost. (The marginal cost curve is assumed to have a positive slope at its inter-
section with the horizontal price line.)

Still remaining on the track of Helberger and Hoos (48) let \( q^0 \) be the output connected with the maximum \( S^* \), and let \( AC^0 \) denote the corresponding average total nonlabor cost of production. At the maximum, therefore, \( S^* = (P_q - AC^0) q^0 \) and since \( S^* = V_M^* \) (defined in Equation 9.77) thus

\[
V_M^* = \frac{(P_q - AC^0) q^0}{M}. \tag{9.85}
\]

For any given level of \( M \), the large cooperative firm will select the level of \( q \) which maximizes \( V_M^* \). In this vein, Equation 9.47 expresses a unique functional relationship between the maximum return (unit price) of \( M \) (denoted by \( V_M^* \)) and the level of \( M \). In another form it may be written as

\[
V_M^* = g(M). \tag{9.86}
\]

Under the specified conditions of this "efficiency" model of the cooperative, \( V_M^* \) shows the maximum remuneration (termed as gross income or dividend by most Marxian authors) the cooperative firm can return for its members' unit labor input after covering fixed and nonlabor variable costs in the case of various quantities of total labor which the cooperative members might be willing to supply. Solution values for \( V_M^* \) may be called maximum (potential) cooperative "wage" or gross-unit-income payable to members at various levels of their labor supply.

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1 Helberger and Hoos (48) call this relationship the short-run net returns function.
Programming and cost minimization

Cost minimization included in the "efficiency* model seems to be a frequently encountered decision rule. A careful examination of economic programming models shows that the cost-minimizing objective function as such is simply incomplete. It leaves unanswered the question of the optimal output level and that of the optimum output mix of the enterprise. If the target mix and levels of outputs are previously specified, then the cost-minimizing model is automatically reduced to a special case of the profit maximizing model.

Costs are minimized for every output mix that is produced in the framework of a programming model of profit maximization. However, it should be emphasized that an arbitrary pre-fixing of the enterprise production target will generally lead to inefficiencies from a macro-economic viewpoint. In other words, a given farm might produce what it produces at the least cost but this would not be the least-cost output for the farm industry as a whole nor would it be a social-welfare-maximizing output by any other means than by chance.

The programming language expresses that in Equation 8.25, \( VY' \) is minimized with a given \( PQ \) by a cost-minimizing model. Since the vector \( P \) and the vector \( Q \) are both given (the latter as a target), then the problem under consideration is reduced to the case of maximizing \(-VY'\). The first term (\( PQ \)) would simply drop out of Equation 8.25.

\[\text{Equation 8.25}\]

\[VY' = \text{minimized with a given PQ by a cost-minimizing model.}\]

\[\text{Since the vector P and the vector Q are both given (the latter as a target), then the problem under consideration is reduced to the case of maximizing \(-VY'\). The first term (PQ) would simply drop out of Equation 8.25.}\]

\[1\text{Kaldor and Saupe (56) conducted a deep analysis of the efficiency of the farm industry in the North Central Region of the United States. The authors paid special attention to the lack of fulfillment of the minimum cost requirement.}\]
Whatever output a farm might obtain to maximize its profits, it must also minimize its production costs. In the "efficiency" model, the cost term is minimized regardless the value of the output, and if an arbitrary PQ is determined, then the profit maximization problem is solved by cost minimization.

The "efficiency" models evidently depicted short-run, one-period situations in which accumulation (investments to increase productive capacity) are not considered directly. The next section will turn attention to the long-run multiperiod cases.

Problem of Cooperative Growth Models

A concept of cooperative growth

Large-scale cooperative enterprises reflect the heavily-weighted objective of steady economic and financial growth which is an integral part of the agricultural development in socialist countries. For a large cooperative firm, economic growth may be defined very broadly as the changes in its financial position that permit an increase in its productive capacity to satisfy the needs and desire of its members. As a rule, aggregate output may be used to denote and quantify economic growth both in general and in a cooperative sense. With respect to the cooperative conditions, three further qualifications might be helpful:

1. The rate of growth of aggregate output should not be lower than the rate of increase in cooperative membership (if the latter prevails), thus a higher per capita output must be ensured as an index of economic growth.

2. The increase in aggregate output should not be associated with
a substantial worsening of the financial position (indebtedness).

3. The growth of aggregate output should not be linked to a significantly increasing inequality of the cooperative income distribution.

As expressed in the basic definition and emphasized in the second qualification, the economic concept of cooperative growth involves an improving (or at least non-worsening) financial position. Thus finding the present value of future income streams is crucial from the viewpoint of economic growth. This way one arrives at the problem of discounting which lies in finding the present (capital) value of given sums (revenue magnitudes) that are available in successive years of a chosen period.

The maximization of capital value as an objective

In the economic literature (43, 54, 84) it has been suggested that the proper objective function for a firm is that of maximizing the firm's net worth (capital value) over time.

Objective functions discussed up to this point are relevant for short-run decisions and they do not consider capital investment. In the cooperative case, the acquisition of new members is an investment in human capital and this is similar to an increase in the technical skills of members.¹ Both would have direct pay-offs to the cooperatives.

The maximization of capital value would be a perfectly sufficient guide to investment decisions if the only objective of the cooperative were that of making profits in the long-run. However, in the cooperative

¹In his lectures at ISU, Kaldor discusses various social aspects of investment in human capital, including the costs and benefits of human capital formation.
case particularly there are many other objectives to be considered. Members' aspirations and life styles as well as their social security and opportunity for their children are all relevant factors. The likelihood that the cooperative farm will remain a viable organization implies that its major objective may at times not coincide with long-run profit maximization. The latter is just another term for maximizing present capital value.

In some aspects, a large cooperative firm is certainly more than a typical business enterprise because it furnishes personal income and services to its members and it considers members' personal goals and aspirations in a more direct and more encompassing fashion than a private capitalistic firm mostly does concerning workers and employees.

Since growth of the large cooperative firm or investment in this enterprise directly competes with members' personal income and consequently with their individual personal savings, there is introduced another problem in any long-run consideration. The business firm portion of the cooperative farm can only grow from withheld profits not paid out to the cooperative members; therefore, it must clearly offer an expected return not only in profits but also in amenities. These expected returns should be at least equal to the expected money returns on members' individual private savings.

A growth model constructed to reflect all the relevant foregoing considerations would have the same mathematical form as that of the ordinary profit-maximizing enterprise model but the discount factor would possess crucial importance and might well be different from its
usual value connected closely with the market rate of interest.

*Present value* in cooperative optimization

Assuming \( T \) periods of time (\( t = 1, \ldots, T \)) and \( R_t \) as prospective revenue at the end of the \( t \)-th year, the total present (capital) value of \( R_t \) is the sum

\[
PV = \sum_{t=1}^{T} R_t (1 + r)^{-t}, \tag{9.87}
\]

where \( PV \) denotes present value and \( r \) is the rate of interest per annum.

If three years are only considered (in a simple example), then the present capital value of a recent farm investment is given by

\[
PV = R_1 (1 + r)^{-1} + R_2 (1 + r)^{-2} + R_3 (1 + r)^{-3} + K^*, \tag{9.88}
\]

where \( K^* \) denotes the capital value of the given investment (for example, a purchased farm machine) at the end of the time period considered. If the investment is not utilized according to its original designation, then \( K^* \) will stand for the familiar "scrap value" of the capital good in question.

To consider multiperiod cases and growth aspects, a large cooperative farm can formulate a maximizing objective function from Equation 9.49, and after specifying the relevant restraints, it may call for optimization techniques.

Similarly to Shubik's (74) management-run corporation, a large cooperative firm may pursue its objective by attempting to maximize the value of the discounted income stream paid to its members over some time period subject to the constraint that the risk of indebtedness
(insolvency, bankruptcy) is kept below a specified level. Now the optimization is set up as

\[
\text{Maximize } \sum_{t=0}^{T} p^t v_t \tag{9.89}
\]

subject to

\[
\prod_{t=1}^{T} \left[ 1 - p_t \left( E_t \leq U \right) \right] \geq k,
\]

where

- \( \rho \) = discount factor,
- \( v_t \) = members' income in period \( t \).

The restraint in Expression 9.89 indicates that the probability of survival (avoiding insolvency) for \( T \) periods will be at least \( k \).

Insolvency condition is specified as

\[ E_t \leq U, \tag{9.90} \]

where \( E_t \) denotes the value of the collective funds (called Indivisible Funds of the large cooperative enterprise in the Eastern European countries) and \( U \) stands for the sum of current debt and liabilities of the cooperative firm. Expression \( p_t (E_t \leq U) \) is, of course, the probability that the cooperative firm will be bankrupted in the \( t \)-th period.

To evaluate the yearly income stream of cooperative members a discount factor, \( \rho \), is applied in Expression 9.89; this factor has the usual form \( \rho = (1 + r)^{-1} \) where \( r \) is the rate of interest.

---

1 The upper-case Greek pi denotes that each periods (\( t = 1, \ldots, T \)) is considered separately.
While its relevant magnitude is far from being determined or agreed upon, some economic aspects of \( p \) may definitely be recognised under socialist conditions. Everywhere the discount rate takes into account time dimensions and alternative use of investment funds. Both are especially important in Hungary where capital is extremely scarce for most cooperative farms. When a cooperative firm ties up capital in any given investment, it is prevented from using this money for alternative purposes.

In particular, it could put this money in a bank savings account and could have a return on this investment, a year hence, of 3.5 per cent or 5 per cent. Thus a forint invested this way today would be worth 1.035 or 1.05 forints a year hence, or alternatively, a forint return a year hence may be equated to \( p = (1.035)^{-1} \) or \( p = (1.05)^{-1} \) forints today. The concept of \( p \) known as the discount factor has nothing which is methodically special under socialist conditions. In a large-scale enterprise it is also clear that \( p \) should not be neglected for long time horizons which are an especially characteristic feature of cooperative farm organization.

All types of cooperative models presented in this chapter may prevail side by side at a given time when large cooperative firms aim at different objectives and have various sets of restraining factors since their political, social make-up, and economic aspirations are far from being unified. It is also highly perceivable that a given cooperative may change objective functions when its inner organizational structure and its economic power is strengthening in harmony with general agricultural development and national economic growth. Considering the
large-scale portion of farmers' cooperatives only, an observer can recognize that elements of the pure enterprise model are gaining place and those of the family-farm-type model are losing weight while those of other models are present mostly as constraints in Hungary's cooperatives today.
CHAPTER X. ECONOMIC WELFARE AND OBJECTIVES
OF COOPERATIVE HOUSEHOLDS

This chapter is addressed to the economic welfare of the cooperative household (family) considered as a certain type of tightly-knit interacting organization. The first step in the analysis will be to set up an objective function containing the decision variables open to the cooperative household. Second, the pay-offs to the household for its various employment choices will be defined. Third, the constraints impinging on the labor supply alternatives of the household are developed. Finally, the economic welfare (objective) function of the cooperative household will be maximized subject to the relevant constraints.

It is to be emphasized that an economic welfare function is going to be developed and not the much wider, so-called social welfare function of the cooperative households.

Decision Variables in the Objective Function

The level of the economic welfare of a cooperative household is determined by two groups of variables. The first group, which this chapter focuses on, consists of decision variables associated with employment alternatives. The second group discussed in much shorter form is made up of variables determined by the cooperative or by the state. This latter group contains variables which are not decision variables from the viewpoint of a cooperative household unit; but these variables determine the alternatives open to the household, and are decision variables for the cooperative decision unit and/or the state.
The variables that are functionally related to the welfare objectives are grouped as

\[ \omega_i = \omega_i^1 \begin{pmatrix} l_{1j1} & l_{1j1}^i & l_{1j1}^* & l_{1j1} \end{pmatrix}, \]  \hspace{1cm} (10.1) \]

where

\[ \omega_i = \text{the economic welfare of the } i\text{-th cooperative household.} \]

In Equation 10.1 the first decision variable, \( l_{1j1} \), denotes the hours of labor of particular (the \( j \)-th) kind which is supplied to the large cooperative enterprise by person \( i \) in the \( i \)-th household who is a member of the cooperative.

\( \mathcal{L}_{ei} \) denotes the matrix of all kinds of labor supplied to the large cooperative firm by all members of the \( i \)-th household. The dimensions of this matrix are \( s \) by \( r \), if \( s \) family members provide \( r \) kinds of labor input. In Equation 10.1 the second term, \( \mathcal{L}_{ei}^i \), has the same meaning as the \( \mathcal{L}_{ei} \) matrix with the cooperative member excluded. In other words, the first row of the latter matrix is deleted, therefore, \( \mathcal{L}_{ei}^i \) has dimensions of \((s-1)\) by \( r \). If in a household there is more than one cooperative member, then \( \mathcal{L}_{ei} \) will change its dimensions accordingly.

The third term of Equation 10.1, \( \mathcal{L}_{1i}^* \) stands for the matrix of non-cooperative employment of all household (family) members; its dimensions are \( s \) by \( r^* \) where \( r^* \) kind of labor are supplied to non-cooperative employers. Finally, \( \mathcal{L}_{1i} \) denotes the labor performed by the household members on their own homeplot farm; this matrix is of \( s \) by \( r^m \) in its dimensions.
Evaluation of Alternative Employment Opportunities

The four decision variables involved in Equation 10.1 symbolize four categories of labor supply. Each of these labor (employment) categories provides the household a specific return. These returns are evaluated by Equations 10.2 through 10.5. The first equation gives the evaluation of the cooperative employment

\[ Z^{(1)} = (v_{eij} + r_{ei} + F_e) \]  

(10.2)

where

\[ Z^{(1)} = \text{the value to the household of a cooperative member's employment in the collective enterprise.} \]

In Equation 10.2, \( v_{eij} \) expresses the wage return to the cooperative member for a unit of work in the large cooperative enterprise. The next term, \( r_{ei} \), stands for the profit-share going to the cooperative household from the large enterprise. This share is proportional to the number of hours worked by a cooperative member. The last term, \( F_e \) (its first letter may be thought of as referring to fringe benefits), denotes non-wage benefits accruing to the laborer which vary with the hours worked in the large enterprise.  

\footnote{It should be noted that the relationship is simplified between the \( \eta_{ei}, F_e \) (and \( \eta^*, F^* \)), on the one hand, and the \( Z \)'s, on the other. It is assumed here that the relationship is linear. In many cases, however, this relationship is far from linear; in particular the \( \eta \)'s may have threshold values and the \( F \)'s may form a stepped-function with respect to the number of hours worked. In other words, profit-share and some fringe benefits are usually available only to a person whom one might call a regular full-time employee. This has the prime objectives of attaching laborers more securely to a particular employment.}
A typical cooperative household involves persons who are not cooperative members but supply labor to the large cooperative enterprise. Equation 10.3 evaluates the return on the labor of these family members

\[ Z^{(2)} = (V_{e1} + F_e), \]  

(10.3)

where

\[ Z^{(2)} = \text{the value to the household of a unit of employment of other family members in the large cooperative enterprise.} \]

In Equation 10.3, \( V_{e1} \) denotes the vector of net wages paid to the i-th household for employment of its member by the large collective enterprise; this vector has \( r \) elements if the household members perform \( r \) kinds of job \( (j = 1, \ldots, r \) in the case of this household which has a cooperative member employed in the \( j \)-th job of the large collective enterprise). \( F_e \) has already been defined.

Some members of a cooperative household are generally working as non-cooperative employees. The return on their labor services is evaluated by

\[ Z^{(3)} = (V^* + \Pi^* + F^*), \]  

(10.4)

where

\[ Z^{(3)} = \text{the value to the household of a unit of employment of all family members working in non-cooperative organizations.} \]

In Equation 10.4 \( V^* \), \( \Pi^* \), and \( F^* \) are of the same basic economic content as defined by these symbols before, and they are related to non-cooperative employment again. The vector, \( V^* \) has \( r' \) elements since this is the corresponding number of kinds of labor assumed.
The work performed on its own homeplot farm of the i-th cooperative household can be evaluated by

$$z^{(i)} = (Pf_i + Q), \quad (10.5)$$

where

$$z^{(i)} = \text{the value to the household of a unit of labor on its own homeplot farm.}$$

In Equation 10.5, $P$ is an n-dimensional vector of prices of outputs which are produced on the homeplot farm. Symbol $f_i$ stands for the matrix of marginal physical products of various labor inputs that are supplied by the household to its own homeplot farm. This matrix is $r^n$ by n in dimensions since there are assumed to be $r^n$ kinds of works and n varieties of outputs on the homeplot farm. Matrix $Q$ denotes the marginal recreational value associated with each family member's work performed on the household farm. The $Q$ matrix is $s$ by $r^n$ in its dimensions.

The Set of Relevant Constraints

As discussed before, each cooperative member is constrained to supply a certain minimum amount of labor hours to the large collective enterprise. This constraint is expressed by

$$l_{ij1} - \overline{r}_i \geq 0 , \quad (10.6)$$

where

$$\overline{r}_i = \text{the minimum labor quantity to be supplied per annum by a cooperative member to the large collective enterprise.}$$

The first term in Equation 10.6 spells out that person 1 belonging to the i-th household, the cooperative member, is assigned to job j in the large collective firm.
Leisure or recreation time will not enter the objective function directly. However, it is worthwhile to consider the fact that if \( r_{ik}^0 \) is the number of recreation hours for the \( k \)-th members of the \( i \)-th household and \( \sigma_{ik} \) is the vector of hours available for this family member to be divided between work and leisure, then the equation for leisure (recreation) of the \( k \)-th member of the \( i \)-th household reads as

\[
r_{ik}^0 = \sigma_{ik} - \left[ L_{eik} 1 + L_{ik}^0 1 + L_{ik} 1 + L_{hk} 1 \right] .
\]  

(10.7)

The first three terms in brackets are defined in connection with Equation 10.1. The fourth term, \( L_{hk} \), denotes the obligatory household duties for the \( k \)-th member of this household. (These duties are especially worth considering in the case of the housewives of cooperative peasant families.) In each bracketed term, the 1's denote that only one family member is taken care of by Equation 10.7.

Leisure (recreation) must, of course, be a nonnegative quantity and, therefore, the leisure constraint for the \( k \)-th family member takes the form

\[
\sigma_{ik} - \left[ L_{eik} 1 + L_{ik}^0 1 + L_{ik} 1 + L_{hk} 1 \right] \geq 0 .
\]  

(10.8)

There are \( s \) expressions of this type — one for each member of the cooperative household.

Maximum Conditions and Relevancy

On the basis of evaluations (Z's) and constraints discussed, one can derive the maximum conditions for the objective function expressed by Equation 10.1. A typical member of this maximum condition set is given by
The first term of Equation 10.9 is the evaluation of a cooperative member's employment in the collective enterprise times the derivative of the objective (welfare) function with respect to the work performed in this employment. The second term, \( \lambda_{s+1} \), is equal to zero if the given cooperative member works more than the minimum requirement (i.e., if the inequality holds in Expression 10.6). In this case Expression 10.9 becomes exactly like the maximum conditions for other family members working in the large collective enterprise. This condition for the \( s \)-th family member (helper) is expressed as

\[
\frac{\partial \omega_1}{\partial x} \left( \frac{1}{2} \right) = \lambda_s \leq 0. \tag{10.10}
\]

If the equality holds in Expression 10.6 and \( \lambda_{s+1} \) is positive in Expression 10.9, then the following relation appears

\[
z_{11} \frac{\partial \omega_1}{\partial x} + \lambda_{s+1} = z_{11} \frac{\partial \omega_1}{\partial x} = z_{11} \frac{\partial \omega_1}{\partial x} \tag{10.11}
\]

Under the conditions of this model the cooperative member would supply only the required minimum of labor to the large collective enterprise if his cooperative implicit wage plus his "profit-share" (dividend) were lower than the implicit wages of other employment.

The case where \( \lambda_{j1} = \frac{1}{2} > 0 \) and, therefore \( \lambda_{s+1} = 0 \) can occur if and only if the implicit wage rates are equal in the different occu-

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1 Similar conditions are derivable by using \( z^{(3)} \) and \( z^{(4)} \).
Equation 10.11a suggests that a cooperative member of "homo oeconomicus" type may take all possible combinations of occupations or may work just in his cooperative employment. Equation 10.11 states that the collective enterprise provides an inferior form of employment. Undoubtedly neither is the case in the real world and this suggests courses of action make the model more realistic.

The model elaborated so far assumed a linear monetary equivalent for the $F$ and $Q$ variables. In this model, pleasure or pain of working in various employments could be given a monetary evaluation and the individual's "utility" for varying work conditions of different employments would not change with the time length (number of hours) of work. This feature of the model suggests that the individual's evaluation of the different employment opportunities (each $Z$ above) is a vector rather than a single dimensional scalar.

It should also be recognized that an individual is not free to work as many hours as he would like in most forms of conventional employment. Whenever one has an hourly-wage-type of employment there is usually both a minimum and a maximum number of work hours specified. The former is evident in cooperative employment as stated in Expression 10.6. In most other employments, however, the upper limit is more frequently encountered. Thus including a maximum constraint on work
hours will also make the model relevant to a large set of situations.

Another shortcoming of the above model is the fact that the amount of leisure (defined as \( r^0 \) is Equation 10.7) would intrinsically have zero value. On the road to a more realistic model, in order to incorporate the work-leisure choice it is also necessary to alter the objective function.\(^1\)

The More Realistic Model

**Objective function**

On the basis of previous reasoning Equation 10.1 is to be replaced as the objective function in a more realistic model by

\[
\omega_i = \omega_i(l_{ij1}, l_{ei}, l_{i1}, l_{d1}, R) ,
\]

(10.1a)

where

\( R \) = the vector of leisure time for all the \( s \) individuals belonging to the \( i \)-th cooperative household.

**Evaluation**

The evaluation of a labor hour for the household becomes a vector for each kind of employment and is denoted by Equations 10.2a through 10.5a.

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\(^1\) In this model there is no necessity to assume that only leisure is associated with pleasure (utility) and work is always linked to pain (disutility). All it is necessary to specify is that leisure has some positive value and any pleasure associated with it diminishes after some number of hours spent on leisure (recreation) and likewise for work.
The evaluation of the $F$ and $O$ type variables on the part of the household is a function of the pleasure (utility) of these things to the individual family members being exposed to them. This approach does not rely upon any particular concept of household organization and thus implies no assumptions about different members' evaluation of working places in the objective (welfare) function of their household. Vector, $\epsilon$, is defined

$$\epsilon_1 = \frac{\partial \omega_1}{\partial \omega_1} .$$

In Equation 10.12 $\epsilon_1$ expresses the marginal change in the household welfare that is induced by the marginal change in the welfare of a (number 1) member of this household; and $E$ is the $s$ component vector corresponding to $\epsilon_1$.

The denominator of the fraction in Equation 10.12 denotes the marginal change in the welfare of individual 1 as a result of a marginal change in his working conditions. This is expressed in the last term of Equations 10.2a where
The principle of diminishing marginal pleasure (utility) appears for person 1 as

\[ \frac{\partial^2 w_1}{\partial F_{e1} \partial l_{1j1}} < 0. \]  

(10.14)

The matrix \( A^{(2)} \) stands for the marginal welfare changes of all family members (except person 1) and it is \((s - 1)\) by \(r\) in dimensions. A typical element of this matrix for the \(k\)-th individual who performs a job of \(j\)-th type in the large cooperative enterprise is given by

\[ a_{kj}^{(2)} = \frac{\partial w_k}{\partial F_{ej}}. \]  

(10.15)

The \( A^{(3)} \) and \( A^{(4)} \) matrices are interpreted similarly. A like procedure is applicable for leisure. In Equation 10.16 the vector \( d \) denotes the evaluation of a person of his own leisure. In Equation 10.17 vector \( D \) denotes the household's evaluation for the vector \( d \); in symbols

\[ d_k = \frac{\partial w_k}{\partial x_k}. \]  

(10.16)

and

\[ D_{1k} = \frac{\partial w_k}{\partial d_k}. \]  

(10.17)
Constraints

The more realistic model contains leisure constraints

\[ \sigma_1 - (I L_{e11} + I L_{11}^* + I L_{11} + I L_{H1} + r_1^o) \geq 0 \]  
\[ \sigma_k - (I L_{e1k} + I L_{1k}^* + I L_{1k} + I L_{Hk} + r_k^o) \geq 0 \]  
\[ \sigma_s - (I L_{e1s} + I L_{1s}^* + I L_{1s} + I L_{Hs} + r_s^o) \geq 0 \],

where \( I \) stands for the unit vector.

A set of maximum constraints is also required for various forms of employment. This set does not include the work in homeplot farming since there is no institutional upper boundary on the number of hours of homeplot (or household) work. There is a "choice" upper limit for homeplot work introduced by Equation 10.8a.

In every non-homeplot form of employment, it is highly possible that there is an upper boundary on the number of hours which can be worked per annum. This is particularly important where a cooperative member is assigned to work in the plant cultivation section of the large collective enterprise (as discussed in Chapter VI and mapped in Figure 6.2). A member of a plant cultivation brigade is usually "unemployed" during a large part of the year. At the same time, another cooperative member working in intensive livestock operations or in a machine repair shop or in the cooperative administration is regularly employed throughout the year, usually working the official 48 hours per week.

The maximum constraints on work hours are expressed in this model by
Expression 10.18 stands for the upper boundary on the work which is supplied by a cooperative member of the $i$-th household to the large collective enterprise. Expression 10.19 denotes the maximum constraints on the work of a person who belongs to the $i$-th household and as a non-member supplies labor to the large enterprise. This case covers $s-1$ persons and $r$ kinds of work. Expression 10.20 defines the maximum labor constraints on non-cooperative employment and refers to $s$ persons and $r^*$ jobs in its general form. All the three maximum constraints are intrinsically formulated as per annum quantities.

**Optimization and Household Equilibria**

In order to maximize Expression 10.1a subject to all the relevant constraints formulated by Expressions 10.6, 10.8a and 10.18, 10.19, 10.20, the following condition are to be satisfied

\[ \alpha x_1^{(1)} + \xi_1 s_1^{(1)} + \Delta - \lambda_1 - \mu_{1j1}(e) \leq 0, \]  
\[ \alpha x_j^{(2)} + \xi_k a_k^{(2)} - \lambda_k - \mu_{ijk}(e) \leq 0, \]  
\[ \alpha x_j^{(3)} + \xi_k s_k^{(3)} - \lambda_k - \mu_{ijk}(*) \leq 0, \]
Each of the five Expressions 10.21 through 10.25 is a sample expression for each of the independent variables. In this set each of the sample expressions, save the first one, is identical to the others in its structure. Expression 10.21 is exceptional since there is only one variable in that set, being the cooperative member's employment in the large collective enterprise. The $\lambda$'s are identified with the general time constraint placed by the elementary recreation needs on each individual. The $\Delta$ is the Lagrangean multiplier associated with the minimum labor supply required from the cooperative member in question. The $\mu$'s are identified with the maximum constraint on work hours where that is relevant.

The optimization model set up this way seems to be satisfactory to predict relevant rational behavior on the part of the cooperative household.

Case 1

Among typical cases, one arises when:

- a cooperative member works more than his minimum labor requirement in the large collective enterprise;

- either there is no institutional upper limit on his work hours or his total work does not amount to that many hours in the large collective enterprise;

- he does not supply labor outside his cooperative organization but he works on his homeplot farm where is no upper limit on the work hours;
— this cooperative member naturally has a positive amount of leisure time.

In such a situation described (and called Case 1), the equilibrium relationships are given by

\[ a_{x1}^{(1)} + \xi_{1} a_{11}^{(1)} - \lambda_{1} = 0 \]  

\[ a_{xj}^{(4)} + \xi_{1} a_{j1}^{(4)} - \lambda_{1} = 0 \]

\[ d_1 D_{11} = -\lambda_{1} = 0. \]

The three equations in Expression 10.26 may also be written as

\[ a_{x1}^{(1)} + \xi_{1} a_{11}^{(1)} = a_{xj}^{(4)} + \xi_{1} a_{j1}^{(4)} = d_1 D_{11}. \]  

(10.26a)

In Case 1, the individual member of a cooperative, works both in the large collective enterprise and on his own homeplot farm, and he also enjoys leisure time. The values of the monetary return on his labor are \( a_{x1} \) and \( a_{xj} \); the values of non-monetary returns are the two terms involving \( \xi_{1} \) and the term involving \( D_{11} \). The equilibrium situation is simply characterized by saying that the value to the household of different employment must be equated at the margin. This result reads directly from Equation 10.26a.

Case 2

A new situation arises where

— the cooperative member encounters the maximum constraint on his work performed in the large collective enterprise (for example, he is assigned to work in plant cultivation there);
— he works on his homeplot farm, too;
— while there is, of course, no maximum constraint on the positive magnitude of his leisure time.

The single significant difference here (in comparison with Case 1) is that a new negative term appears in the first equality of the equilibrium conditions

\[
\alpha x_1^{(1)} + \xi_1 a_{11}^{(1)} - \lambda_1 - \mu_{11}^{(e)} = 0 \tag{10.27}
\]

\[
\alpha x_j^{(4)} + \xi_1 a_{11}^{(4)} - \lambda_1 = 0
\]

\[
d_1 D_{11} - \lambda_1 = 0.
\]

Equations 10.27 can also be expressed as

\[
\alpha x_1^{(1)} + \xi_1 a_{11}^{(1)} - \mu_{11}^{(e)} = \alpha x_j^{(4)} + \xi_1 a_{11}^{(4)} = d_1 D_{11} \tag{10.27a}
\]

or

\[
\alpha x_1^{(1)} + \xi_1 a_{11}^{(1)} \geq (\alpha x_j^{(4)} + \xi_1 a_{11}^{(4)}) = d_1 D_{11} \tag{10.27b}
\]

The last term on the left-hand side of the first line of Equations 10.27 and also in Expression 10.27a is a negative term in evaluating collective (large-scale) cooperative employment. This term signifies that the marginal value of collective cooperative employment to the given household is greater the marginal value of working in other employment including the homeplot farming. If the cooperative member were given the opportunity, he would simply work more hours in the large-scale collective enterprise.
Case 3

A third case occurs when the cooperative member works only the minimum numbers of hours required from him by the General Assembly and has other employment. In other words, this is the case when $\Delta$ is non-zero in the corresponding conditions based on Expression 10.21. The new situation changes the top line in Expression 10.26 or 10.27 into

$$\alpha x_1^{(1)} + \epsilon_1 a_{11}^{(1)} + \Delta - \lambda_1 = 0. \quad (10.28)$$

The remaining two lines would be same as in Expression 10.26 or 10.27, and therefore, are not repeated here. But it is essential to give the relevant new relationships which replace Expressions 10.26a and 10.27b

$$\alpha x_1^{(1)} + \epsilon_1 a_{11}^{(1)} \leq (x_j^{(4)} + \epsilon_i a_{i1}^{(4)}) = d_i D_{i1}. \quad (10.28a)$$

Expression 10.28 looks very much like 10.27 except that the first constraint appearing is positive. In Case 3 the minimum labor requirement exists in terms of an effective constraint since Expression 10.28a indicates that the value of the last unit of work performed in the collective cooperative employment is less than the marginal value of other employment for the cooperative household.

Cases for family members

A simple alteration of the above three cases can consider all the possible cases for family members who are not a member of the cooperative. Cases 1 and 2 are essentially identical with those for other
family members. The crucial point is that the same individual has multiple employment from two essentially different sources. In one of these employment opportunities the individuals are constrained by the nature of the job (for example, plant cultivation) or by the law to work only certain maximum hours. But some individuals desire (or need) to work more and, thus, take a second job.

Additionally, if an individual obtains diminishing marginal welfare from his work in one employment, he may work in another job selected on the basis of his own welfare evaluation.

Recreation value of homeplot farming

Particularly in the case of the homeplot farm, the monetary return on labor could be zero and people would still work there for the sheer recreational value, for leisure. This behavior is probably observed only among those who have other employment, sufficient non-labor income, or who are not self-supporting.

Under the above conditions it is not unrealistic to conceive the existence of a homeplot "farm" simply because people attach a sufficiently large recreation value to their homeplot activities although these give no explicit monetary reward.

The Economic Welfare Function of Cooperative Households

In a general sense, the welfare of a household (or of an individual) is its (his) perceived well-being. For this very general concept, considering the decision unit and its total environment in complete interaction, there has been reserved the term "social welfare function." Furthermore, Equations 10.1 and 10.1a listed only a part of economic
variables affecting the total (social) welfare of cooperative household. Even a more general function of economic welfare would directly incorporate the quantities consumed. This chapter has not considered these.

There are, however, a great number of relevant non-decision variables (from the viewpoint of a household) which affects its well-being and/or influences the optimal choice among its decision variables. These are the variables which do not vary with the independent variables of Equations 10.1 and 10.1a. Objective functions expressed in those two equations can be conceived as being embedded in a more general (larger) function that may be called the economic welfare function of a cooperative household. The rest of this function consists of non-decision variables for the household and is expressed by

\[ \omega'_1 (W_{t1}, W_{t1}, W_{t1}, F_{t1}, F_{t1}, W_{t1}, F_{t1}, F_{t1}, I_e, \Omega_e, \Omega_s) \]  

(10.29)

The first three variables have been encountered in Chapter VII (in Expression 7.3). Their meaning remains the same in Expression 10.29. Welfare payments and social services evidently affect the well-being of the benefited household. Symbol \( F_e \) stands for the amenities accompanying the collective cooperative employment which amenities do not vary with a person's work hours. This variable could include the pleasant atmosphere of the so-called cooperative democracy where a cooperative member has an explicit role in determining resource allocation, income distribution and in determining his work and home environment. Working conditions in their usual sense are also reflected
by this variable. The term $F^*$ is similarly defined for non-cooperative employment. In the latter case, of course, one would not have certain advantages of cooperative membership especially with respect to participation in managerial and entrepreneurial decisions.

Variable $\psi_0$ symbolizes the public goods provided by the cooperative of the $i$-th household. Some of them are available only to cooperative members or to cooperative households but many of them are provided to the whole community in which the cooperative is located, such as parks, play-grounds, cultural and sport facilities, or even flood control and quite a few other things of this nature which are really cooperatively created public goods. The variable $\psi_0$ is a like variable for the public goods provided by the government, representing the interests of the whole society.

The variable $H_{PL}$ denotes the provision of the homeplot land and also the cooperative services connected with household farming. It is essentially a vector describing the quantity and quality of the land as well as the productive, marketing and managerial services provided by the large collective enterprise. As discussed in Chapter VI and VII homeplot farming is a rather unique economic phenomenon and it is of great importance for people preferring agricultural cooperatives.

The symbol $I_0$ denotes the implicit share of the given household in the large-scale collective investments. Implicitly it would be a vector of anticipated investments over some time horizon and is extremely important in determining the future well-being of cooperative households. Their future income streams directly depend on the quantity
and nature of collective investments originating from cooperative
profits via collective decisions.

The term $\Omega_c$ represents the interests and well-being of all the
other households in the cooperative which the $i$-th household belongs
to. $\Omega_S$ is a similar variable for the welfare of all people in the
society. These two variables denote that if the welfare of other house-
holds would improve with no deterioration in the welfare of the $i$-th
household, then the latter household would consider its well-being
enhanced. In other words, the first-order partial derivative of $\Omega_c$
and $\Omega_S$ is positive with respect to $\omega_i$

$$\frac{\partial \omega_i}{\partial \Omega_c} > 0; \quad (10.30)$$

and

$$\frac{\partial \omega_i}{\partial \Omega_S} > 0. \quad (10.31)$$

Finally, the complete economic welfare (objective) function can
be set up for a cooperative household by combining Expressions 10.1a
and 10.29

$$\omega_i^0 = f (l_{i1}, L_c^0, L_1^*, R_1 \mid W_e, W_w^r, P_r, F_r, F^*,$$

$$\psi_c, \psi_S, HP_e, I_o, \Omega_c, \Omega_S), \quad (10.32)$$

where

$$\omega_i^0 = \text{the complete economic welfare function of the } i\text{-th coo}$$

$\text{op}$$

$\text{pe}$$

$\text{e}$$

$\text{r}$$

$\text{ative household.}$
The separating bar stands between the decision variables (first group) and the non-decision variables (second group). The latter are beyond the decision-making scope of a cooperative household unit and they do not vary directly with the labor supply of the given household.

**Labor Supply in the Short-run and in the Long-run**

The derivation of labor supply curves is fairly simple in the framework of the model presented. The labor supply of a cooperative household responds parametrically to the household evaluation terms. Any labor demanding organization, whether it is the large collective enterprise or a non-cooperative employer, can vary the wage rate and/or it can change the amenities associated with the employment. A cooperative farm can offer, for example, day care in summer for children and effect the last but one term of Expressions 10.8a (the obligatory household duties) in order to free time for housewives to be employed in peak work seasons. Since some household duties are indispensable and form a parameter in essence, they cannot be considered as an independent variable.

By wage and nonwage variations, the large collective enterprise and any other firm can establish a labor supply function.\(^1\)

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\(^1\) Varying Holdren’s (50, 51) term one may speak of wage and nonwage-offer variations. He defines styling, product quality, sales promotion, and advertising as nonprice-offer variations since these may also be varied by a seller in addition to varying the price.

Holdren (51) essentially postulates roughly circular areas of sales coverage for certain retail stores. The store attracts its customers by varying the price and nonprice aspects of its offer. As the store reaches out farther, it enlarges its circle. Its territorial increment is multiplied by population density and other demographic factors determining the demand function for the store.
Some variables listed in Expression 10.29 affect the labor supply by influencing household member's choice between labor and leisure. Both cooperative and social public goods may provide particular services such as better transportation which makes work easier as in the case of diminishing household duties discussed before.

A particular set of variables of Expression 10.29 would also operate to make households prefer cooperative membership and large-scale collective employment. These variables are $W_e$, $F^*$, $V_R$, $HP_e$, $I_e$ and $Q_e$. An increase in any of the latter variables would enhance the value of cooperative membership and would increase the labor supply to the large collective operation on the part of individual households. Generally in this group there are only two variables which can be manipulated by a non-cooperative employer; these are $W^*$ and $F^*$. It would appear that the cooperative organization may have "more strings to its bow" in terms of attracting members and employees than other employers.

Almost uniquely the large collective enterprise does affect its long-run labor supply by providing education grants on the account of variable $W_e$ to its members' children and practically supports every member to get vocational training. One might even suggest that a cooperative organization can also influence the long-run labor supply by better and cheaper housing facilities which evidently affect the birth rate.

It should be emphasized that the work-leisure choice is affected by many of the variables of Expression 10.29 in a more or less indirect
but extremely important fashion. As education is enhanced by expend-
itures on public goods, people-to-people interaction may be improved
in many other aspects. As more and more of the public services directly
connected with education and recreation are provided, the quality of
leisure is also increasing. People should become more secure in their
economic future as expenditure on the welfare funds and on collective
investments are enhanced. It is difficult to overemphasize the impor-
tance of the non-decision variables of the complete economic welfare
function (described by Equation 10.32) in things and feelings that
make for a really good life.
CHAPTER XI. COOPERATIVE EQUILIBRIUM AND ECONOMIC POLICY

It would be possible to set up a unified economic welfare function for the cooperative farm as an organizational unit. The large collective enterprise would be depicted as allocating its profits among investments, members' shares (dividends), cooperative welfare funds, cooperative public goods, and provisions to homeplot farming. At the same time each cooperative member's objective (welfare) function would be amalgamated with each other member's objective (utility) function by means of voting and bargaining processes. The unified economic welfare function, to be maximized, would imply some configuration of the above named variables. It is not the purpose of this study to inquire deeply into the nature of a unified cooperative welfare function but rather to derive an objective function for both portions of the cooperative farm that might have long-run applicability.

In this chapter it will be assumed that the amount of investment gets determined for any point of time and that homeplot farm inputs other than members' own labor are also determined for each time. Given investment in both the large collective and the small homeplot portions, one can be sure that maximization of the present value of the total cooperative farm organization would maximize its social surplus value produced (defined alternatively by Equations 7.29 and 7.30). In order to avoid the problem of discounting all future costs and returns, it

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1 In many cases the provision of homeplot lands and services contributing to homeplot farming competes for future cooperative profits rather than for current profits.
is conceivable that a one-period analysis is undertaken with the period sufficiently long to cover the entire future time horizon, or a typical period is selected some time in the future. Thus, the problem at hand can be reduced to one of maximizing one-period profits where all inputs are variable.

An extremely important two-fold consideration must be built into the unified long-run cooperative model. For a variety of reasons, members' labor should be priced at the alternative wage rate. If members' labor is not priced at all and is not allocated according to an optimal farm plan, then the result will be inefficient resource use, suboptimal output mix and insufficient rentability, as pointed out in Chapter IX. Furthermore, an incompetent cooperative management could pay the members less than their alternative wages and thus could hide the fact that the collective enterprise was failing to make a viable profit. In order to avoid this, cooperative labor input will be calculated at its alternative wage in the model.

For exactly the same set of reasons, the foregoing argument must be applied to cooperative capital input. In order that the cooperative management be accountable to the membership as well as to society as a whole, and in order to preclude inefficient resource allocation, both cooperative labor and cooperative capital must be computed at what they would return to society in other economic pursuits outside cooperative farming.
Maximization of Social Surplus Produced by a Cooperative

At every point in time the cooperative firm — as an organizational unit — has a finite amount of land, labor and capital to allocate. Additionally, it purchases inputs from other producers and/or may hire workers and employees. Under these conditions, the objective function becomes

$$\text{Max} \quad \Pi = PQ - LW^* - Kr^* - VY$$

where $W^*$ is the opportunity wage rate, $r^*$ denotes the alternative rate of returns on capital, and $PQ, VY$ are essentially the same as before. (The former stands for gross return; the latter denotes expenditures on purchased variable inputs in this case.)

The quantity of outputs is subject to the side relation representing the production function

$$Q = Q (A, L, K, Y)$$

where $A$ denotes land and $L$ symbolizes labor input. A typical equation referring to a single output is given by

$$q_j = q_j (A, L, K, Y)$$

The three restraints are given by Expressions 11.4, 11.5, and 11.6 as

$$A^0 - A \geq 0$$  \hspace{1cm} (11.4)

$$L^0 - L \geq 0$$  \hspace{1cm} (11.5)

$$K^0 - K \geq 0$$  \hspace{1cm} (11.6)

The maximum for this system occurs when the following five conditions are met
\[
p_j \frac{\partial q_j}{\partial \lambda} = \lambda_1 \leq 0 ; \\
p_j \frac{\partial q_j}{\partial \mu} = \nu - \lambda_2 \leq 0 ; \\
p_j \frac{\partial q_j}{\partial \lambda^*} = r^* - \lambda_3 \leq 0 ; \\
p_j \frac{\partial q_j}{\partial \gamma} = \gamma \leq 0 ; \\
q_j \geq 0 .
\]

In the above set of expressions \( \lambda_1 \) is associated with land, \( \lambda_2 \) with labor and \( \lambda_3 \) with capital. These \( \lambda \)'s may be interpreted as the shadow price (rent) of land, labor, and capital respectively. A greater-than-zero value for any \( \lambda \) denotes that the corresponding factor is exhausted by the optimal plan.

If any output (\( q_j \)) is produced (i.e. the inequality holds in Expression 11.11), then the corresponding terms equal zero in Expressions 11.7 - 11.10 (i.e. the equality holds there). If output \( j \) is considered to be produced and Expressions 11.7 through 11.10 are rewritten in equality form, the following results will be obtained

\[
p_j \frac{\partial q_j}{\partial \lambda} = \lambda_1 ; \\
p_j \frac{\partial q_j}{\partial \mu} = \nu^* + \lambda_2 ;
\]
The interpretation of Equations 11.12 and 11.15 is straightforward. The former states that the marginal value product of land must be equal to the shadow price (rent) of the land in equilibrium. The latter expresses the conventional profit-maximization conditions that the marginal value product of any purchased input equals its price in an optimum situation; it says that a net-revenue-maximizing firm utilizes purchased inputs up to the point where the values of their marginal products are equated to their prices.

Equation 11.13 says that the marginal product of cooperative labor equals the sum of the alternative wage rate and its shadow price ($\lambda_2$). Since only a limited quantity of labor is available to the cooperative farm, the implicit wage must theoretically include some alternative factor ($\lambda_2$) in addition to the opportunity wage rate ($v^*$). This alternative factor ($\lambda_2$) may of course be zero if the quantity of cooperative labor is not exhausted.

A large positive $\lambda_2$ indicates that there is too little labor available in the given cooperative for efficient utilization. In other words, net national product and/or total net social surplus could be increased if labor were transferred from other economic sectors into this cooperative. If $\lambda_2$ is just "barely" positive, it means that the cooperative labor force is fully utilized and that cooperative members
are earning just their alternative wage rate.

Similar considerations exist for Equation 11.14. A positive $\lambda_3$ means that the rate of return on cooperative capital exceeds the rate of capital return obtainable elsewhere in the economy, therefore, the cooperative members in question are earning profits greater than the return to capital in other pursuits. To maximize net national product and/or total social surplus, one should transfer capital into such a cooperative organization.

The Implications of Homeplot Farming

Homeplot farming has not been incorporated into the developing model so far. It would have been included if the household welfare function of Chapter X explicitly contained the assumption that the non-monetary, recreational value of homeplot works might exceed the monetary value of fringe benefits (F value) associated with any other employment. Furthermore, for social policy reasons cooperative members should not be precluded from working as many hours as they would like on their own homeplots. If this special proviso were missing, the model builder could say: the social net surplus is maximized on the cooperative farm (including all the homeplots) when the conditions of Expressions 11.7 through 11.11 are met, with the additional proviso that Expression 11.7 would be identical for the large collective enterprise and for the homeplot "sector" (as well as among homeplot farms). In both portions of the cooperative farm organization, labor and capital would also be bound by Expression 11.8 and 11.9 equally, and in addition Expressions 11.10 and 11.11 would now be binding on homeplot farming as well.
With the adopted proviso that cooperative members may work as much as they like on their own homeplots, the model under consideration must accept somewhat different conditions.

**Equilibrium without recreational value for homeplot activities**

In the first case, one may assume that the strongest motivation of a homeplot holder is to produce farm products for his own use and for sale in such a fashion as to maximize returns on his inputs. If there is no recreational value attached to homeplot farming, then Equations 11.12 through 11.14 will be applied in the i-th homeplot farm in somewhat modified form as

\[
p_j \frac{\partial q_i}{\partial A_i} = \lambda_1. \quad (11.12a)
\]

\[
p_j \frac{\partial q_i}{\partial L_i} = v^* + \lambda_2. \quad (11.13a)
\]

\[
p_j \frac{\partial q_i}{\partial K_i} = r^* + \lambda_3. \quad (11.14a)
\]

Equation 11.12a states the equilibrium condition that a land unit assigned to the i-th member's homeplot should earn at least as high a rent as it does in the large collective enterprise. The case is exactly the same for large-scale capital services utilized by the homeplot farms (Equation 11.14a); likewise for labor supplied to homeplot farms including the cooperative members' own labor (Equation 11.13a).

To state the above arguments differently and explicitly, the cooperative farm organization would have to be irrational (inefficient...
in the Paretian sense) if there were no recreational or other non-monetary value attached to homeplot activities and the two sets of conditions stated by Expressions 11.7 through 11.11 and by Equations 11.12a through 11.14a were not met.

**Optimum with recreational (non-monetary) value**

The optimum or efficiency conditions are vastly different if the cooperative members want to maximize their individual welfare (utility) on their homeplot by producing for their use and/or for market; at the same time they also desire getting recreational value or other non-monetary satisfaction from their homeplot farm activities. In this case the equilibrium conditions are

\[
p_j \frac{\partial q_j}{\partial l_i} \geq \lambda_1, \quad (11.12b)
\]

\[
p_j \frac{\partial q_j}{\partial k_i} \geq r^* + \lambda_3, \quad (11.13b)
\]

\[
p_j \frac{\partial q_j}{\partial l_1} \geq v^* + \lambda_2, \quad (11.16)
\]

It is worthwhile to consider Expression 11.16 first because the preceding three are explained more easily after this last one has been discussed. In Expression 11.16 the implicit money wage for homeplot work can be less than the implicit wage rate obtained in the large
collective enterprise. The reason for this is the fact that cooperative members' welfare evaluation (utility) attached to homeplot work — per hour — is greater than their satisfaction (utility) associated with any other work in the large collective enterprise.

If the situation just now depicted prevailed and inputs on homeplot farms were calculated as they are in the large collective firm, then more labor would be combined with other inputs on homeplots than in the collective enterprise. In such a case, the returns to land, to capital, and to labor other than cooperative members' own labor, are implicitly larger in the homeplot farm portion of the cooperative farm than in the large collective portion. (The latter conditions are given by Expressions 11.12b, 11.13b and 11.14b.)

Homeplots for recreation

A third case depicted in Chapter VI is also conceivable — at least in the remote future — where cooperative members want homeplots exclusively for recreation or merely because it makes them feel good. In the framework of this model, the cooperative could with all due logic vote to assign homeplots and even provide free or low-priced services (covered from the collective profits, of course) to homeplot activities regardless of whether any market-valued output, either for self-provision or for sale, were produced on members' homeplots.

Possible Conflicts between Cooperative Management and Members

Practical evidence as well as theoretical considerations suggest that there may be significant conflicts between cooperative members and cooperative managers (both with or without membership) with respect to
the allocation of capital.

The salary and social prestige of cooperative managers, and their own feelings of self-worthiness, are highly correlated with running a fast growing, profitable enterprise. Sometimes capital rationing might be in the direct, short-run interest of cooperative members in order to force a greater rate of return; but it can never be in the interest of the management (especially, of the salaried managers) because that would mean operating a smaller and/or less rapidly extending enterprise.

Another inherent conflict expresses itself repeatedly in connection with homeplot farming. Management would always be inclined to oppose allocating any resource (land, feed, seed, etc.) and service to homeplot farming since these curtail the productive capacity available for the large collective operations at any given time. On the other hand, it seems to be in the direct interest of the management that cooperative members as individuals be as dependent on the large collective enterprise as possible. In this respect management may count on members' homeplot farming.

There also exists another interconnectedness between the two portions of the cooperative farm and most managers usually apply a larger weight there. Most periods of peak activity (especially autumn harvesting and cultivation of wine-grape plantations) are the same on the homeplots as in the large collective enterprise. This phenomenon motivates the cooperative managers to restrict and reduce the scope of homeplot farming. It is not self-evident at all how managers'
interest and success can be enhanced by the existence and coordinated development of properly-sized homeplot farms. Thus there may always be present an essential conflict of interests and this is one of the problems that must be settled by voting and bargaining processes (which finally lead to a conceptual unified economic welfare function of the cooperative farm as a whole).

Cooperative members are constrained by the fact that better management responds directly to its own better opportunities. In order to obtain and retain the best possible management, the cooperative members would be voluntarily restrained in their desire to expand homeplot farming and to increase their individual shares from profits — such as dividend, cooperative welfare benefits, and cooperative public goods — at the expense of the collective investments.

All these inherent conflicts of interests may well be moderated, although not completely eliminated, if each cooperative manager obtained cooperative membership.

Conflicting Social and Cooperative (Group) Interests

Several sources of conflicts emerge between the cooperative members as a (more or less) coherent group and society as a whole.

Personal income and capital supply

Given the number of cooperative members, it is the immediate interest of the cooperative membership to get as high a return for labor input as possible. This might appear to be the members' interest concerning cooperative capital, but it would be a misinterpretation. Cooperative members are not really interested in the rate of return per
unit capital. The cooperative members as a group are interested in their total earnings which would be, of course, consistent with maximizing their implicit wage. But to maximize their implicit wage, the cooperative members would utilize successive capital inputs until it resulted in zero marginal productivity, if capital were unlimited and costless. Thus there exists a balancing factor in such a sense that a cooperative group wants to utilize exactly that quantity of capital which maximizes total earnings out of the social net surplus produced. However, members' earnings are limited with respect to their profit-share constituent directly by capital needs since it is from the profit that cooperative capital must come.

In any case, the cooperative members would naturally like to get as high an implicit wage plus profit-share as is commensurate with maintaining such a desired status over a long period of time. The correct answer might be the quantity of capital investment that equates the marginal value product of cooperative capital with the social rate of return on capital ($r^*$). It would be a sheer accident if, in the absence of borrowing ability on a skillfully planned capital market, there would be utilized exactly the quantities of capital which equate their internal rates of return with the social rate of return.

Prices and income distribution

Clearly it is also in the immediate interest of the cooperative groups to have the price of what they sell as high as possible and the price of what they buy as low as possible. At the same time, the central authority must exercise a proper control over these (and other) group
motivations in the social interests of efficiency and equity.\footnote{The problems of efficiency and equity are discussed from various aspects of theory and policy by Heady and Kaldor in their lectures at ISU.}

Too high prices for farm product — regardless of income distribution objectives — would allocate excessive capital and labor to agriculture. If farm prices are too low, then the result is just the opposite. Clearly, neither is in the long-run interest of the whole society because net national product and national income are reduced by both policies.

In the interest of the whole society, cooperatives must be prevented from becoming people's (i.e., members') monopolies in their economic behavior. Considering both possible deviations, the economists should advise the policy makers that the rate of return on cooperative capital and the "cooperative wages" of labor be kept at the alternative (comparable non-cooperative) level of both of these. This could be done in a fashion which does not negate cooperative (group) initiative and individual ingenuity or anything of that kind.

If a cooperative is more efficient (for example, because of applying cost-saving technologies) than other farms utilizing approximately the same natural and economic resources, it should be permitted to retain its extra income. This is not against the social interest at all, especially where more and cheaper food is really needed as in Hungary. But if more fertile land, advantageous location and other non-controllable resources result in apparently higher returns to the cooperative farm,
the excess should be taxed away. On the other hand, cooperatives operating under extremely unfavorable natural conditions (on pure sandy soils, in mountain areas, etc.) deserve special attention and definite economic support on the part of society. In other words, natural "gifts" and natural "handicaps" should be equalized in favor of an equitable income distribution. At the same time, cooperative "grit" must be rewarded properly.

Relevant efficiency and equity considerations lead directly to the current problems of price policy and central planning which are generally embedded in economic policy and particularly in the new, reformed economic mechanism of Hungary.

The Reform of Economic Mechanism

The system of new mechanism introduced to the Hungarian economy on January 1, 1968 meant essential modification of the system of central planning and pricing. This economic reform was, at the same time, a social and political reform. During its preparation, the attention of various political power factors and individuals was focused mainly on the efficiency and welfare aspects connected with the reform. On the other hand, the reform profoundly affected the structure of preferences, both personal and institutional, which was based on earlier economic conditions.

1 In Hungary this group of cooperatives is given tax allowance, investment credits in various forms, and in addition, if necessary, temporary subsidies to provide a guaranteed level of annual personal income.
Learning from experience in economic planning

In the main, the national economic plan is an economic policy conception relating to a certain time period, expressed by complex programs of actions. It is only possible to act efficiently and with foresight on the basis of coordinated plans and programs.

Learning from experience in economic planning has led the Hungarian economic theorists to the following conclusions\(^1\) (6, 32):

1. At the time of the so-called personal cult (prior to 1953) the ruling theory was that the most important priorities should be determined by one or a few persons. As a consequence, analysis of the main economic policy interrelations, the setting of alternative variants, and research on the interdependent decision-making factors were disregarded. Hence application of modern planning methods was out of the question.

2. This arbitrary way of making decisions led to incorrect economic concepts which disregarded the proper mechanism of modern economic processes.

3. The speedy setting up of the planning apparatus brought about the excessive concentration of government administrative activities and, consequently, an immense planning bureaucracy emerged.

4. The planning bureaucracy became engrossed in the detailed elaboration of minute and formal decisions having no primary importance.

\(^1\)Among the Hungarian economists specialized in economic policy and socialist planning Nyers (18), Bognár (6, 7, 8), Friss (32, 33), Vajda (81) and Kornai (59) are best known abroad.
5. Economic difficulties were not attributed to wrong economic policy concepts but to the assumption that the planning was not sufficiently detailed.

During the last decade the theoretical concepts of planning have changed to a very large extent. The main essence of the present concept is that political leadership enlists the nation's eminent experts in working out the economic plans in order to improve the system of economic administration, and invites them and the planning institutions to submit proposals (variants), while reserving the right to choose among alternative variants. In this way there have been created the situations discussed by Kaldor (55) as "selection of efficient programs."

The present concept of socialist planning focuses attention on efficiency and welfare (especially on the real income and the living standard) aspects of the economy. The questions of interconnection between economic sectors and branches are also emphasized. The immediate goal is to prepare plan variations which facilitate central economic decisions only concerning those problems which are relevant at the national level. The recent theory of central planning aims at the correct solution of nation-wide problems. Simultaneously, instead of an "overgeneralized" improvement it aims at finding a clear-cut optimum solution for the chosen task.

Of the recently applied mathematical methods utilizing experiences and concepts from abroad as well as theoretical works of Hungarian

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1 The problems of food economy and agricultural policy are analyzed mainly by Erdei (21), Dimény (15), Kazareczki (57), Csizmadia (14), Marton (65), Csendes (10), Vági (80) and László (63).
economists, the following should be listed:

(a) Various planning models, including operation-research models, which can help in finding optimum economic-policy solutions and measures;

(b) economic models with well-defined parameters, mainly to improve forecasts;

(c) comprehensive programming models concerning single industrial branches or complex firms (farms);

(d) input-output models which help the planners co-ordinate the production of different branches and thus ensure equilibrium;

(e) a simple growth model of the Harrod-Domar types for the determination of the rhythm of growth is applied according to which the growth rate depends upon the investment quota (ratio of investments in the national income) and the capital-output coefficient (investments needed for the unit-increase of national income);

(f) production functions determining the growth rates of branches by representing the impact of investments of a given branch on the output of other branches.

It goes without saying that the different classical equilibria of the national economy (expressed, for example, as the balance of budgetary revenues and public expenditures) will play a considerable role in future planning. It is even more true concerning the equilibrium between aggregate demand and aggregate supply (which are often named purchasing power and commodity fund in the technical terms of planning). The elaboration of material balances (covering 360 commodities) and detailed plans of technical supply is, of course, continued.

The economic role of a socialist government is partially based on its strong income redistribution effect. Social welfare, public education, the development of culture and of the infra-structure, rests

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1Among the Hungarian authors of mathematical economics, Kornai (59), Ganczer (34), Sebestyen (72, 73) and Csáki (9) may be mentioned here.
in this redistribution, as does the sophisticated network of the research and developmental programs including regional and urbanization plans. It is the reservoirs of income redistribution from which individuals and various social strata want a share, with or without justification.

**Price system and socialist price policy**

The Hungarian reform of the economic mechanism attempted to gradually set up a price system in which most prices are determined by the market mechanism. This is a consequence of the twin principle which served as a foundation for the reform, namely, that

- there can be achieved a more purposeful economic development with better utilizing national economic planning by the state;
- there can be obtained a more efficient economic organization by utilizing the market mechanism in the framework of a centrally planned economy.

Accordingly, the economic reform created a connection between the national economic plans and the market in a novel manner. The former policy of narrowing down the assertion of market phenomena gave to the deliberate utilization of the market as a regulator. Now the position was taken that the progress of the economy should be promoted not mainly by administrative price constructions but with the aid of price mechanisms adequately indicating the economic processes taking place in the market (12).

The extent to which the price mechanism asserts itself depends on the structure of the national economy. It is asserted most completely
in the "pure" market model and is most limited in the "pure" planned model. The latter in its extreme form goes so far as to eliminate the market, and here prices provide only a basis for accounting. One should be distinguish clearly between market price, which regulates market processes, and accounting prices which are completely independent of market relations. One may also define three separate forms of the latter:

1. The constant price and plan price applied in statistics and planning with the intention of making the production outputs of various periods comparable by eliminating the effects of price changes.

2. Fictitious price, which embraces the various forms of computed price systems based on inputs such as the value-proportionate price system or the shadow price system based on optimum calculations.

3. Internal accounting prices which are intended to promote cooperation within the firms.

In the planned national economy of Hungary the market price appears also in three different forms. A narrowing scope of government-fixed prices is established for basic raw materials, staple foods, and some important consumer goods. A maximum price or a directive price is set on a considerable portion of the other products. The prices of the third group are free; that is they are made a matter of agreement between buyers and sellers. Fixed and maximum prices are primarily applied to finished products, while within the framework of direct cooperation (contract) between firms and on the market of agricultural products, a wider scope is given to the third category of prices negotiated by buyers and sellers.
In a socialist economy, price is considered as a means of economic calculations as well as a means of regulating demand and income. As Csikos-Nagy (13) emphasized, these functions are closely connected with the triple task of national economic policy:

1. to ensure an optimal rate of economic growth;
2. to maintain conditions of economic equilibrium and stability;
3. to make sure that there is a regular rise in the standard of living together with an equitable income distribution.

Kornai (60) properly emphasized that the price system connects individuals with each other, acts on them, influences their decisions; thus its effects cannot be analyzed without models of human behavior. In this vein the models of some Hungarian economists as well as those of their predecessors — von Neumann, Leontief, Khantorovich and others — have a considerable role not in formulating the price theory, but in analyzing the real processes of the economy.

The reforms of the economic mechanism in Hungary and in other socialist countries are interpreted by many economists (at home and abroad) as a transition from the model based on central mandatory instructions to a decentralized market model. In the author's opinion the recent economic changes taking place in these countries can be better defined as a transition from a socialist-economic model with barter-type features to one with clear-cut market-economy characteristics.

In Hungary the economic reform was followed by the reform of the national science policy. The next step is going to be the reform of
the state and local government administrative structure. In the preparation of this reform the starting-points are to give correct and operational answers to the following questions: How far should the power of government extend? What are the methods and instruments power may or must use, and how can this activity be co-ordinated with the interest of the individual citizens and various social strata and with public opinion? The reform of the administrative structure indicates that the present government lays equal stress on initiative and on equilibrium in the political life of Hungary.
CHAPTER XII. SUMMARY AND CONCLUSIONS

Summary and Conclusions of this Study

The core objective of this study has been to analyze the historical development and socio-economic determinants of cooperative farming, to derive optimum solutions that may characterize the cooperative form of farm organization by reflecting its economic ends and capacities. This study is concerned with the objectives and interests of the large-scale collective enterprise, with those of the individual cooperative members (households), and also with the objectives and constraints of the cooperative organization (decision) unit as a whole. Current economic problems of the author's country, Hungary, have directly motivated this study.

Chapter I serves as introduction, providing the background and major objectives of this recent study.

Part One, including four chapters, sets the stage by describing the transformation and recent organizational structure of Hungarian agriculture.

Chapter II discusses the agrarian reform of 1945 and concludes that together with the enormous latifundia system, most of the quantifiable problematic gaps disappeared but "the ill effects of minifundia" started to show up shortly afterwards.

Chapter III analyzes the socialist transformation of Hungarian agriculture which took place in the period of 1948-1961, largely during the last two years, and characterizes the portion and role of state, cooperative and private sectors in the economy as well as the changes
in the level of economic development.

Chapter IV focuses on the scale-return implications to the land reform and on the scale-size aspects of the socialist transformation. This chapter shows the radical changes in farm size and in the number of farm decision (management) units and concludes that the recent farm organization is best characterized by the 208 state farms having 4,623 hectare cultivated land and 756 workers as an average, and by the 2,840 farmers' cooperatives possessing 1,672 hectare land and 379 members as an average.

Chapter V sets up objectives and constraints — in a simplified programming framework — which might implicitly have existed at the time of the land reform and during the socialist transformation. This chapter may also serve as a mathematical summary of the whole Part One of this study.

Part Two consists of two chapters and scrutinizes the major socio-economic characteristics of farmers' cooperatives and also develops various cooperative models.

Chapter VI elaborates a comprehensive survey on the dual character of the farmers' cooperative as a social organization, on the one hand, and as an economic organization, primarily a business firm (farm enterprise), on the other; cooperative members being both partners (co-owners) and laborers (employees) simultaneously. This chapter also argues that the cooperative farm includes two constituent parts: the large-scale collective firm and the small-scale homeplot farms of cooperative members (the latter being owned and largely individually operated). This
chapter draws the conclusion that the large collective enterprise is to be considered as a special type of business firm associated with collective decision-making at the top level and characterized by the principle of the so-called cooperative democracy (self-management) from the viewpoint of state control.

In Chapter VII a general behavioral model of cooperative farming has been formulated, then — as the central construction — a linear programming model of the farmers' cooperative has been set up and is followed by a simple cooperative model placing emphasis on investment and technology. At the end of this chapter a separate section provides a relatively large set of identities that quantify the economic categories prevailing in socialist cooperative farming. This chapter concludes that mathematical programming models may serve as a basis for further research orientation in the economic theory of cooperative farming.

Part Three, incorporating four chapters, discusses cooperative optimization at three levels: the large collective enterprise; the individual cooperative members (households); the cooperative organization as a whole.

Chapter VIII is concerned with the general theoretical and methodological aspects of cooperative optimization. Following a short review of technical literature, this chapter discusses alternative cooperative objectives and production functions. It sets up a non-linear programming model together with a linear programming example (in algebraic form). In this chapter the major conclusions state that systematic
programming approaches can be applied to the rather complex problems of cooperative optimization and mathematical programming may serve as a powerful tool of various optimization procedures, especially with respect to the large-scale collective enterprise.

Chapter IX deals with large-scale collective firm optimization in the framework of six cooperative models: 1. the pure enterprise model which maximizes net profit (similar to socialist state farms or private commercial farms); 2. the cooperative model of the family-farm type which maximizes the returns on members' labor input as a residual income (the difference between gross revenue and total non-labor costs); 3. the intensity model with the objective of maximizing output per land unit; 4. the maximum output model for total value-product maximization with given resource endowment; 5. the "efficiency" model with cost-minimization or "cooperative-wage" maximization objectives; 6. the growth model aiming at cooperative capital value maximization in multiple time periods. This chapter provides algebraic examples for cooperative optimization with different objective functions. Specifically it gives double optimization treatment — by applying both the marginal and programming approach in order to derive numerically comparable solutions — to the first and the second cooperative models.

Chapter X views the economic objectives of the cooperative household. It sets up an objective function containing the decision variables open to the cooperative household and discusses the pay-offs of various employment choices, in addition to the constraints which are impinging on the labor supply alternatives. In this chapter the economic welfare
(objective) function of the cooperative household is maximized, subject to relevant constraints, and, finally, some properties of short-run and long-run labor supply functions are briefly discussed.

Chapter XI is concerned with cooperative equilibrium and economic policy. It formulates an objective function with long-run applicability for both the collective and homeplot portions of the cooperative farm. In this chapter the social surplus (total profit) produced by a cooperative is maximized, and homeplot farm activities are considered in three variations. First, the latter are given only market evaluation (monetary value); next, both market and recreational (non-monetary) values are applied; and third, homeplot activities are treated exclusively as recreation and leisure. After listing conflicting interests between the cooperative management and cooperative members, as well as between cooperative groups and the whole society, this chapter analyzes recent economic-policy problems of Hungary in light of the reform of the economic mechanism (introduced in 1968) which essentially modified the previous system of central planning and pricing. This section concludes that the recent economic changes taking place in the Eastern European countries may be best defined as a transition from a socialist-economic model with barter-type features to one with clear-cut market-economy characteristics.

Chapter XII provides a summary of the study and concludes that its objectives have been achieved.
Concluding Remarks

The history of science shows that most relevant theories like major discoveries were developed by several people almost simultaneously. It is an open question whether Newton or Leibnitz discovered differential and integral calculus, but it is positively known that Pascal and Fermat were also pretty close to this discovery. Why? Because productive relations and particularly social conditions of the time indicated the need for a new physics. On the other hand, the new physics associated with Galileo, which put movement at the center of physics, demanded a new mathematics to measure movement. And this need, this "demand," set several men on the road to finding a new mathematics. As we know, Newton or Leibnitz found it, but very likely many others also looked for it at the same time.

In recent years, the Eastern European countries introduced numerous major and minor economic reforms. These reforms expressed the efforts of experts and scholars engaged in the field of the social sciences. Scientific criticism and new theoretical approaches played an important part in disclosing the changes which became necessary. Advances in the understanding of economic theory provided a set of answers to questions that had arisen from social needs. At present, it is impossible to show who made the first steps toward the discovery of the "new" economic principles. (In the so-called Western World, Lieberman's article received the loudest echo.)

Properly understanding and developing the economic theory of cooperative farming can provide particular answers to relevant questions
reflecting social needs. From Iowa to Siberia all sorts of people, among them devoted economists who live in Samuelson's (77, p. XIII) "golden age for analytical economics" want to satisfy this need. The cooperative form of farm organization provides special tasks for agricultural economists in our days when as Heady (43) formulates:

"Farming has catapulted from an occupation based on family-transmitted art and historic experience to a highly advanced scientific activity resting on continuously changing capital technologies and extending to the future. Its scientific complexity now approaches or surpasses the engineering characteristics of many industrial activities."

Pascal said that after having finished his work he generally found out what he should have begun with. Perhaps many economists of my country are in a similar situation and are rewriting their "papers," sparing neither trouble nor pain. The major requirement for doing this is given by Shakespeare: "the readiness is all." If this study stimulates fresh ideas and further research, and inspires readiness for new voyages of discovery, it will have entirely achieved its innermost purpose.
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