Animal spirits and stock market dynamics

Dong-Jin Pyo

Iowa State University

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Animal spirits and stock market dynamics

by

Dong-Jin Pyo

A dissertation submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of
DOCTOR OF PHILOSOPHY

Major: Economics

Program of Study Committee:
Leigh Tesfatsion, Major Professor
Joydeep Bhattacharya
James R. Brown
Steve Kautz
Sergio H. Lence

Iowa State University
Ames, Iowa
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DEDICATION

To my parents: Chang-Ho Pyo and Sook-Yeo Park.
I would like to express my deepest gratitude to Dr. Tesfatsion for her guidance, patience and support throughout my entire graduate education. Her insight and passion for students are essential elements of this dissertation. I also would like to thank my committee members for their invaluable contributions to this work: Dr. Joydeep Bhattacharya, Dr. James R. Brown, Dr. Steve Kautz, and Dr. Sergio H. Lence.
CHAPTER 1. GENERAL INTRODUCTION

This dissertation consists of two independent studies—yet which are closely related—that mainly focus on interactions among traders, behavioral aspects in decision making process, and their impacts on stock market dynamics and other performance metrics of an economy.

From a methodological perspective, this dissertation—especially the second chapter—proposes a new way of modeling financial markets and economies; in its essence it strives to model an economy as a complex dynamic system in which heterogeneous agents—which are inherently subject to incomplete information and limited computational capabilities—are continuously interacting and updating their expectations on future uncertainties. This new modeling approach is expected to shed light on the new way of economic research on financial markets and macroeconomies.

The second core chapter of the dissertation studies the critical role of finance in the ‘innovation economy. It investigates how the financial sector and the fundamental values of firms are interrelated. Specifically, it introduces the possibility of a feedback loop between asset prices backed by animal spirits and fundamental values of a firm.

The key research question in this chapter is on whether a single animal-spirit shock can have persistent impacts on fundamental values of an economy. This type of question has been studied in the context of sunspot equilibria in the related literature. While previous studies show the existence of self-fulfilling equilibria arising from animal spirits that are defined as external coordination devices, this study centers its analysis around the possible interplay among asset prices, corporate external financing, corporate R&D innovation process, and corporate profitability rather than focusing on the existence of ad-hoc sunspot variables.

In this framework, the second chapter hypothesizes that animal-spirit shocks can have persistent impacts on a model economy: if financial sectors channel the funds provided by animal-spirit traders to innovation sectors, and innovation outcomes are endogenously determined, then expectations of the animal-spirit traders could possibly
become self-fulfilled. In this case, the commonly assumed conceptual distinction between fundamental asset prices and irrational asset price bubbles might be irrelevant in the times of increased interdependence between finance and firm innovation. In this sense, this chapter can be regarded as a computational manifestation-in a financial market context-of Keynes' animal spirit metaphor for aggregate fluctuations of an economy.

Specifically, the second chapter develops an agent-based computational model of a dynamic investment economy to examine the role of animal-spirit shocks in the determination of firm fundamental values. The economy is populated by traders with intertemporal utility objectives who engage in consumption, labor, and asset investment activities in an attempt to increase their utility over time, and by a corporate firm with an intertemporal profit objective that engages in R&D in an attempt to increase its profits over time. It is shown that a one-time animal-spirit shock, modeled as an abrupt purchase of additional IPO stock shares by one of the traders, can have persistent effects on the determination of firm fundamental values, measured as earnings per share, as well as on other critical system outcomes. Moreover, these effects can be amplified or contracted depending on the connectivity of this animal-spirit trader within a social network, and the extent to which traders desire to conform to the behaviors of other traders within this social network.

The contributions of the second chapter to the literature can be summarized as follows. First, it computationally demonstrates the existence of persistent impacts of animal-spirit shocks in the context of a fundamental feedback loop between real sectors and financial sectors. Second, it develops a flexible computational framework upon which interesting extensions can be made according to the specific needs of researchers. For example, it permits us to incorporate direct interactions among traders within an optimizing framework of a trader. In addition, it implements a model that can accommodate the actual modeling strategies and arrangements employed by financial institutions.

The third chapter develops a computational stock market model as a benchmark computational platform for the second chapter-in which each trader’s buying and selling decisions are endogenously determined by multiple factors: namely, firm profitability, past stock price movement, and imitation of other traders. Each trader can switch from being a buyer to a seller, and vice versa, depending on market conditions. Simulation findings demonstrate that the model can generate excess volatility, a fat-tail property, and the ARCH effect in stock returns. The results also suggest the importance of trader memory length for determining the stability of stock prices in response to dividend shocks.
The contributions of the third chapter can be summarized as follows. First, given heightened interests in behavioral aspects of trading strategies in stock markets, this chapter investigates the impacts of heuristic trading strategies on various stock market performance metrics. Second, it develops a simple but generically flexible computational stock market model upon which more complicated models could be developed.

The dissertation concludes with a chapter of general conclusions.
CHAPTER 2. THE IMPACTS OF ANIMAL-SPIRIT SHOCKS ON FIRM FUNDAMENTAL VALUES

2.1 Introduction to Chapter 2

This study develops an agent-based model of a dynamic investment economy in which firm fundamental values, measured as earnings per share, are endogenously determined within a feedback loop connecting IPO stock trades to R&D investment and hence to future firm profitability. The key finding is that a one-time animal spirit shock, introduced as an impulse shock at a single time point, can have substantial persistent effects on this feedback loop.\(^1\)

More precisely, the dynamic investment economy is populated by traders with intertemporal utility objectives who engage in consumption, labor, and asset investment activities in an attempt to increase their utility over time, and by a corporate firm with an intertemporal profit objective that engages in R&D and physical capital investment in an attempt to increase its profits over time. The traders in each trading period decide how to allocate their initial money holdings, wage earnings, and dividend payments among consumption and the purchase of two types of assets: namely, risky stock shares and risk-free bonds. The firm in each trading period decides whether or not to augment its current retained earnings by an additional IPO stock offering, and how to allocate its resulting internal funds between R&D and physical capital investment.

These trader and firm activities determine a fundamental feedback loop connecting IPO stock trades to R&D investment and hence to firm fundamental values, measured as ex-post earnings per share. A key finding of this study is that a one-time animal spirit shock, modeled as an abrupt purchase of additional IPO stock shares by one of

\(^1\)As a complement to the computational dynamic model, I also develop a analytical static model based on a rational expectation framework in Appendix A. The key result of the static model shows that there exists a unique symmetric rational expectation equilibria when agents have some degree of sentiment and strategic complementarity. It also suggests that an increase in the magnitude of the sentiment leads can induce a higher return of investment under a certain parametric condition.
the traders, can have persistent effects on the dynamic properties of this fundamental feedback loop.

In particular, this study demonstrates the possibility that an animal-spirit shock can turn out to be ex-post ‘rational’ when firm investments are dependent on external financing conditions. This finding suggests that, in a world where the investments of a firm depend partially on external finance, ex-ante non-rational initiatives in the financial sector can have persistent impacts on firm fundamental values and other economic performance measures. The conceptual distinction between an asset price bubble and an assets fundamental price thus becomes blurred, because an improvement in firm fundamentals can be facilitated by temporary asset price bubbles stemming from animal-spirit shocks.

Moreover, this study shows that the persistent effects of one-time animal-spirit shocks on the fundamental feedback loop can be amplified or contracted depending on the location and connectivity of the animal-spirit trader within a social network. The degree of this amplification or contraction depends on the extent to which traders desire to conform to the behaviors of other traders within this social network.

More precisely, the traders in the dynamic investment economy are structurally heterogeneous, in the sense that they have different locations and connectivity within a social network. A key finding is that a one-time animal-spirit shock induced by a trader has the most substantial persistent effects when traders display a moderate desire to conform to the actions of other traders in the social network. A low or high desire for conformity can attenuate these persistent effects. Moreover, assuming all traders have some desire for conformity, a higher connectivity for the animal-spirit trader results in greater persistent effects, all else equal.

This study builds on earlier works focusing on the potential role of animal-spirit shocks within macroeconomic systems (e.g., Keynes (1936), Akerlof and Shiller (2010)). The interesting feature of this study is that it investigates the possible role of animal-spirit shocks put in the middle of an open-ended feedback loop between financial sectors and real sectors. Once admitting the existence of the feedback loop, asset prices and fundamentals mutually affect each other. Thus it should be noted that the persistency of the impacts of external shocks to the system including animal-spirit shocks crucially depends on the feedback loop.

---

2This open-ended feedback loop between financial markets and real sectors is similarly suggested in Soros (2003) as the concept of reflexivity. See Bond et al. (2011) for the literature survey on the feedback between financial market and fundamentals.
As elaborated below in Section 3.2, the present study differs from this earlier work in its focus on the potentially persistent effects of one-time animal shocks within a dynamic investment economy that includes a detailed microfoundation modeling of trader and firm optimizing behaviors. Moreover, this study appears to be the first study to couple a study of animal spirit shocks with social network effects, thus permitting the investigation of social networks as potential augmenting or contracting influences on the propagation of animal-spirit shock effects.

The remainder of this study is structured as follows. Section 2.3 develops the dynamic model in which traders and a firm actively interact in a stock market. In Section 3.5, I present simulation results. Section 3.6 concludes with remarks.

### 2.2 Related Literature

To be sure, this study is not the first one that analyzes animal spirits as the key determinants of economic performance. Keynes (1936) postulates that macroeconomic fluctuations are largely due to spontaneous actions originating from optimism or pessimism. Despite of its familiarity to economists, the concrete definition of it in a theoretical context has not been established in the literature. As will see later, it varies by authors’ own interpretations and the specific settings of models. Keynes (1936) interprets animal spirits as entrepreneurs’ spontaneous actions with optimism, rather than inaction, which lack \textit{ex-ante} rational grounds—mathematical expectation—for taking such actions. In an effort to maintain the original flavor, an animal-spirit shock refers to a spontaneous action unsupported by logical reasoning or mathematical calculations in this study.

Akerlof and Shiller (2010)\textsuperscript{4} extend the notion of \textit{animal spirits} by linking it to the degree of an agent’s confidence regarding to unknown future events or psychological states of minds; they relate animal spirits to psychological states in which people do not optimize in using information available to them. They also argue that there exist asymmetric effects of confidence between economic downturns and upturns; the link

\textsuperscript{3}According to Keynes (1936), “There is the instability due to the characteristic of human nature that a large proportion of our positive activities depend on spontaneous optimism rather than on mathematical expectation.” (pp. 161)

\textsuperscript{4}The central role of animal spirits in aggregate fluctuations is also emphasized in Akerlof and Shiller (2010). Proposing the modern interpretations of the animal spirit, they relate it to the psychological aspects of economic decisions of human. In this study, the psychological component in an agent’s asset allocation problem is modeled into a formal preference structure that incorporates social interactions and heterogeneity among agents.
between changes in confidence and changes in income seems to be especially strong during economic downturns.

The role of animal spirits in a business cycle context has been analyzed in numerous studies. The first branch of studies are based on models with rational expectation equilibrium. Azariadis (1981) shows that an agent’s expectation itself can create aggregate fluctuations in business cycle and thus can become self-fulfilling. Benhabib and Farmer (1994) identify the property of increasing return to scale in production technology as a source for multiple optimal equilibrium paths to a unique steady state in a standard RBC model. The central message of it is that agents’ beliefs becomes crucial in aggregate fluctuations because of the indeterminacy of an equilibrium path.

Farmer and Guo (1994) investigate the impact of extraneous shocks to demand side on aggregate fluctuations. While they use sunspots and animal spirits interchangeably, the animal-spirit shocks are represented by random shocks to consumption. They show that even \textit{i.i.d.} sunspot shocks can generate persistent output dynamics. From the asset market perspective, Farmer (2011) introduces the beliefs in stock market participants as a key coordinating device for a unique equilibrium out of continuum of equilibrium arising from search and matching costs in labor markets.

Addressing difficulties in the separate identification of animal-spirit shocks in unexplained movements in aggregate consumption, Blanchard (1993) attributes the reduced confidence of general public to a key source for U.S. economy’s recession early in 1990s and its slow recovery. In contrast, Barsky and Sims (2012)\footnote{Barsky and Sims (2012) define the animal spirit as a noise component in the signal for aggregate productivity.} claim that the animal-spirit shock plays a minimal role in business cycle and its impacts are not significant.\footnote{In the literature, the animal spirits have been analyzed in the context of the information structure of an agent. We can attribute unexplained shocks either to noise or to foresight for future events. However, it should be noticed that under this framework agents are still ex-ante rational in the sense that their actions are derived in the context of a rational expectation equilibria. For an extensive survey on informational approach for this issue, see Ludvigson (2004).}

The second branch of related studies replaces a rational expectation assumption with various heuristic decision rules of agents. De Grauwe (2010) develops the reduced form of monetary macroeconomic model in which agents use heuristic forecasting rules.\footnote{He argues that justification for simple decision rule can be made in the sense that real world is too complex to understand.} In his model, animal spirits are modeled as a systemic upward bias or a downward bias in expectation of the output gap.\footnote{The bias in forecasting is subject to change in an adaptive way based on the fitness measure introduced by Brock and Hommes (1998).} He identifies the condition under which animal spirits
endogenously arise, and shows that strict inflation targeting by monetary authority can be a destabilizing factor in business cycle.

Assenza et al. (2009) investigate the impact of heterogeneous expectation on firm’s default probability, the subsequent default frequency, and the duration of the default. In their model, optimism or pessimism is represented in a lender’s expectation on a firm’s default probability, while the true default process is not common knowledge. Switching between various forecasting default probability are driven by reinforcement learning mechanism as in De Grauwe (2010). They show that even a small fraction of pessimistic traders can exert large effects on aggregate output and heterogeneous expectations can lead to the financial instability, which thus prolongs economic crises.

The majority of the asset pricing literature hinges on the presumption that an asset’s future fundamental values are exogenous; there is only one-way directional relationship from fundamentals to asset prices. This paper modifies this framework in a way that allows firm future cash flows to be influenced by the collective behavior of traders in an asset market. The realized firm fundamentals then feed into the formation of key decisions of traders. This setup may provide additional insights and new perspectives for related issues.\footnote{It is argued that, in an efficient market, temporary mispricing caused by any irrational motive is corrected and the asset price will converge to its fundamental value, leading to the ultimate demise of irrational traders (Friedman (1953). Under this setup, no definite conclusions emerge immediately to the claim raised by efficient market hypothesis advocates. For related studies, see De Long et al. (1990) and Hirshleifer et al. (2006).}

In the model, the mechanism through which the initial animal-spirit shock propagates its impact in the sequence of market activities includes a trader’s tendency to conform to the average investment behavior of his local neighborhood. I introduce this herd behavior to take into account the fact that the large part of investing in speculative assets involves social activities (Shiller (1984)). The presence of the impact of social interactions on investment behavior and the existence of herding behavior are frequently reported in numerous empirical studies (e.g., Shiller and Pound (1989), Hong et al. (2004)).

From a methodological perspective, this study makes a significant contribution to the asset pricing literature in the sense that it models the decision making procedure of an agent in a more realistic fashion; it permits an agent to make decisions with an optimizing principle as well as a forward-looking nature. At the same time, it rests on the assumption that an agent has limited knowledge regarding to his external environments and the decision rules of other agents. In this sense, this study is a natural extension,
which is tailored to quantify the impact of animal-spirit shocks in an asset market context, of Sinitskaya and Tesfatsion (2014)’s initiative that introduces the concept of constructive rationality into a formal DSGE model.\footnote{In the model, agents make decisions under their own limited information to achieve goals without any external coordination devices and rational expectations. Agents’ expectations are continuously updated corresponding to changes in external environments. Furthermore, their computational capabilities and information sets are not equivalent to those of a modeler. The constructive rational decision making procedure indeed corresponds to that of a real agent in a real world. Longerstaey (1996) provides a good example of the constructively rational decision making process of financial market practitioners from a risk management perspective.}

\section{Model}

\subsection{Overview: Market Process}

In this section, I describe a dynamic model in which a representative firm finances its capital investment and R&D investment through both the funds raised in a stock market and internal cash flow. There is a single non-perishable good that can be used for consumption, capital investment and R&D investment. The firm produces its output using labor and capital.

Figure 2.1 illustrates key market activities which occur during the period $[t, t+1)$.$\footnote{Note that internal times during the given period is expressed with a colon. For example, $t:1$ denotes a specific point of time when the first market activities take place.}$ At $t:1$, the firm and traders proceed to a single labor market for a forward labor contract (i.e., work now and get paid later).\footnote{In a following section, I assume, for simplicity, that a trader’s labor supply is not elastic w.r.t. a wage. They are simply assumed to provide a fixed labor supply. This assumption is made because our focus is not on an agent’s labor-leisure choice but rather on asset portfolio choices. This assumption can be easily relaxed by introducing a utility function which takes account into leisure and an additional decision domain for labor choice.} After the production takes place, the firm makes wage

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{flow_diagram.png}
\caption{Flow Diagram of the Model}
\end{figure}
payments, out of its output, to traders based on pre-specified labor contracts. Since all payment obligations of the firm are fulfilled, the firm’s profit is realized at this point.

Once the firm’s profit is realized, it distributes dividends out of its profit according to its dividend policy at a time point $t:4$. After receiving dividend payments in proportion to the number of shares currently held, traders liquidate risk-free assets, enjoying the return of $(1+r_f)$ out of a unit investment of cash. Then traders are assumed to consume a fraction of the current cash balance.

In the next point of time, the consumption of traders and stock exchanges occur at the same time. Notice that the firm also participates in the stock market, trying to issue new shares to finance its investments. In a generic context, the firm can either execute buyback its shares or dilute. However, I do not allow buybacks in the current study. Once exchanges are finalized in the stock market, and then traders finalize asset allocations by transferring remaining cash balances to the risk-free asset.

At the time of $t:7$, the firm divides the current cash balance, which is collected through the retained earnings and a new equity issue, into two possible uses: capital replenishment for the next period versus R&D which aims at upgrading its total factor productivity (TFP). Once all these activities are done, state variables for each trader and the firm are updated. Then the system enters into a new period.

### 2.3.2 Trader

For expositional simplicity, a trader’s portfolio problem is simplified to choosing between the risk-free asset and the stock issued by the representative firm. At the beginning, traders are endowed with initial amount of the stock and risk-free asset. Each trader is also endowed with a unit amount of time, which can be used for either labor or leisure. As briefly aforementioned, traders are assumed to provide an inelastic labor supply; no leisure is not included in a trader’s preference. Therefore, traders devote entire time endowments to labor. The sources of income include wages and dividends.

It should be noted that a trader can be characterized by either one of the two classifications: an animal-spirit trader versus an ordinary trader. The animal-sprit trader is the one who makes decisions without resorting to any sophisticated calculation. In subsequent sections, I denote an ordinary trader as a ‘trader’.

---

13I assume there is no limit to the supply of the risk-free asset. For a small open economy, the risk-free asset can be regarded as foreign assets with no risks.

14In subsequent sections, I denote an ordinary trader as a ‘trader’.

15The investment and consumption decisions of an animal-spirit trader are characterized by spontaneous buying or selling strategies and a fixed consumption rule.
Table 2.1: Summary of Variables: Trader

<table>
<thead>
<tr>
<th>Predetermined Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of outstanding shares $S_{it} \geq 0$</td>
</tr>
<tr>
<td>Number of risk-free assets $B_{it} \geq 0$</td>
</tr>
<tr>
<td>Real Money Balance $M_{it} \geq 0$</td>
</tr>
<tr>
<td>Endogenous Variables</td>
</tr>
<tr>
<td>Marginal propensity to consume $\kappa^*_{it;6} \in [0, 1]$</td>
</tr>
<tr>
<td>Portfolio weight on the firm’s stocks $x^*_{it;6} \in [0, 1]$</td>
</tr>
<tr>
<td>Exogenous Parameters</td>
</tr>
<tr>
<td>Conformity weight $\lambda_i \in [0, 1]$</td>
</tr>
<tr>
<td>Sensitivity to conformity factor $\eta \in [0, 1]$</td>
</tr>
<tr>
<td>Discount factor $\beta \in (0, 1]$</td>
</tr>
<tr>
<td>Animal-spirit shock size $s \in [0, 1]$</td>
</tr>
</tbody>
</table>

Contrast, the ordinary traders have specific goals; they try to maximize intertemporal utilities. In the following sections, I specify the configurations of each type of a trader in greater details. For a summary of variables associated with traders, refer to Table 2.1.

2.3.2.1 Trader Constraints

Unlike traditional macroeconomic models, the model developed in this study is based on sequence of various events as illustrated in Figure 2.1; this study does not allow simultaneous executions of actions that are invalid unless prerequisite actions are implemented. Therefore it is very important to specify constraints correctly in each time point in a single period. I first delineate the resource constraints faced by an individual trader along with the sequence of events during a given period.

At the time 0, traders are structurally identical except the number of links to other traders. Since this paper does not allow the endogenous selection of a neighborhood, the network structure is going to be exogenously set. In the beginning, a trader is endowed with initial amount of cash ($M_{i0}$)\(^{16}\), equity shares ($S_{i0}$) and the risk free asset ($B_{i0}$).

Each trader has a unit amount of time that can be used for labor supply and leisure. Since our focus is not on labor supply decisions of traders, I assume that each trader provides a constant labor supply ($l_{it} = 1$) for all $t$. Given the labor market clearing wage,\(^{16}\) Notice that a single good produced by the representative firm is used as a numéraire, which thus solely constitutes cash. All payments are made through the numéraire good.
the cash balance at $t:3$ is given

$$M_{it:3} = M_{it} + w_{it:3}$$  \hspace{1cm} (2.1)

where $w_{it:3}$ denotes a wage rate determined in time $t:1$. Once receiving payments from dividends ($d_{it:4}$) and the risk-free asset, the cash balance becomes

$$M_{it:5} = M_{it:3} + (1 + r_f)B_{it} + S_{it}d_{it:4}$$  \hspace{1cm} (2.2)

As briefly explained in the overview of the sequence of market activities, a trader’s consumption choice and asset allocation choice is made in the time $t:6$ at the same time. A trader consumes a fraction ($\kappa^*_{t:6}$), which is time-varying, of his current cash balance.\textsuperscript{17} So the consumption level at $t:6$ is determined as

$$C_{it:6} = \kappa^*_{t:6} M_{it:6}$$  \hspace{1cm} (2.3)

where $\kappa^*_{t:6} \in [0, 1]$ is a trader’s choice variable.

The mark-to-market wealth or potential money holding, which is a function of a stock price, of a trader is given by

$$W_{it:6}(P) = P S_{it} + (1 - \kappa^*_{t:6}) M_{it:6}$$  \hspace{1cm} (2.4)

where $P$ denotes a stock price and $S_{it}$ is a number of shares currently held. Given a trader puts $x^*_{it:6} \in [0, 1]$ fraction of his current wealth on stocks, it follows that

$$\tilde{S}_{it:7}(P) = x^*_{it:6} W_{it:6}(P) / P$$
$$\tilde{M}_{it:7}(P) = (1 - x^*_{it:6}) W_{it:6}(P)$$  \hspace{1cm} (2.5)

where $x^*_{it:6} \in [0, 1]$ is a trader’s choice variable, $\tilde{S}_{it:7}$ is a desired number of shares that a trader wishes to hold at the beginning of period $t:7$ and $\tilde{M}_{it:7}$ is a desired cash balance that a trader wishes to hold at the beginning of period $t:7$.

Because there is no further liquidity need for the trader, the cash balance at $t:7$ will be invested on the risk-free asset that yields a gross return of $(1 + r_f)$ for a unit cash investment:

$$B_{it:7} = M_{it:7}$$
$$M_{it:8} = 0$$  \hspace{1cm} (2.6)

\textsuperscript{17}A superscript ‘$*$’ is introduced to denote choice variables.
where $B_{it:t}$ denotes a number of the risk-free asset and $M_{it:t}$ is a realized real money balance.

### 2.3.2.2 Optimizing Trader Objective and Decision Procedure

The preference of an optimizing trader directly incorporates the social aspect of financial investment. There are two possible motives for why traders might consider others’ investment patterns important; the first one is related to informational motive and the second one involves psychological motive. Under incomplete knowledge about the firm’s future fundamental and uncertain returns from stock market investments, the moves made by other traders might outweigh private information (e.g., Bikhchandani et al. (1992)). On the other hand, people might feel uncomfortable when they behave in a significantly different manner within a peer group; the degree of emotional regret amplifies (reduces) when people fail while others succeed (fail) at the same time.

I assume that an individual trader derives utilities in part from conforming to the actions of others in a local neighborhood. This structure encapsulates a trader’s tendency to mimic the investment behavior of neighboring traders. Specifically, an ordinary trader’s instantaneous utility in a period $t$ is given by

$$u(x_{it:t}^*, \kappa_{it:t}^*, \lambda_i, \bar{x}_{it}) = (1 - \lambda_i)\ln C_{it:t}(\kappa_{it:t}^*) + \lambda_i v(x_{it:t}^*, \bar{x}_{it})$$

(2.7)

where $\bar{x}_{it}$ denotes the realized average portfolio weight on stocks of a trader $i$’s neighbors. Note that $v(\cdot)$ captures a local conformity factor and $\lambda_i \in [0, 1)$ is a trader’s subjective weight on it. The relative importance of the local conformity is determined by the variable $\lambda_i \in [0, 1)$.

One possible functional form that takes into account the tendency of local conformity is given as follows:

$$v(x_{it:t}^*, \bar{x}_{it}, \eta) = -\frac{\eta}{2} [x_{it:t}^* - \bar{x}_{it}]^2$$

(2.8)

---

---

---
where \( \eta \) captures the magnitude of psychological bitterness arising from the deviation. A trader’s intertemporal utility \( (U) \) is then defined as:

\[
U(\{x^{*}_{is:6}\}_{s=t}^{\infty}, \{\kappa^{*}_{is:6}\}_{s=t}^{\infty}; \lambda_i, \eta, \beta) \\
= u(\kappa^{*}_{it:6}, x^{*}_{it:6}) + \sum_{s=t+1}^{\infty} \beta^{s-t} u(\kappa^{*}_{is:6}, x^{*}_{is:6}) 
\]

(2.9)

Given budget constraints specified in Section 2.3.2.1, a trader’s problem mainly consists of choosing the sequence of \( \{x^{*}_{is:6}\}_{s=t}^{\infty} \) and \( \{\kappa^{*}_{is:6}\}_{s=t}^{\infty} \) to maximize the expected intertemporal utility:

\[
\begin{align*}
\text{maximize} & \quad E_t[U(\{x^{*}_{is:6}\}_{s=t}^{\infty}, \{\kappa^{*}_{is:6}\}_{s=t}^{\infty}; \lambda_i, \eta, \beta)] \\
\text{s.t.} & \quad (2.1) - (2.6) \\
& \quad 0 \leq x^{*}_{is:6} \leq 1, \forall s \geq t \\
& \quad 0 \leq \kappa^{*}_{is:6} \leq 1, \forall s \geq t
\end{align*}
\]

(2.10)

where \( \beta^{s-t} \) is a time-preference discount factor.

It should be acknowledged that traders in the model do not solve infinite-horizon intertemporal optimization problems under complete knowledge of the model as a modeller. Rather all traders in the model need to implement ‘locally constructive’ decision procedures without referring to the externally-imposed global coordination conditions (e.g., rational expectation equilibrium, market clearing conditions); traders do not know the true data generating process (DGP) of uncertainties and do not know the decision rules of other traders and the firm in advance. For example, traders do not know the DGPs of future wages, future stock prices and future dividends as well as the behavioral patterns of the traders in his local group.\(^{21}\)

The macro models based on the bounded rationality of agents evolve from relying on simple fixed behavioral rules to incorporating intertemporal optimizing principles and forward looking natures (e.g., Gaffeo et al. (2008), Sinitskaya and Tesfatsion (2014)). In the computational implementation of agents with intertemporal optimizing principles, assuming no presence of rational expectation requires us to implement dynamic programming in a practical sense.\(^{22}\)

\(^{21}\)It should be acknowledged that in this system the DGPs of future states are endogenously determined and traders consistently use the realized historical data to make good predictions on future events.

\(^{22}\)Dynamic programming with the rational expectation assumption severely suffers from the curses of dimensionality if the number of states increases. The approximate dynamic programming have emerged as responses to computational difficulties embedded in conventional dynamic programming.
In effort to find approximate solutions to various practical dynamic programming problems, the community of operational research has developed a stream of techniques under the name of approximate dynamic programming. A natural way to approximate an infinite horizon problem is to transform it into a finite horizon problem, which is usually called a rolling-horizon procedure (RHP). For the details of the RHP, refer to Appendix C.

Given a fixed horizon \( (h_i) \), a finite-horizon intertemporal utility is defined as:

\[
U\left(\{x_{i:t+h_i}^s\}_{s=t}^{t+h_i}, \{k_{i:s}^s\}_{s=t}^{t+h_i} ; \lambda_i, h_i, \eta, \beta \right) = u(k_{i:t+h_i}^s, x_{i:t+h_i}^s) + \sum_{s=t+1}^{t+h_i} \beta^{s-t} u(k_{i:s}^s, x_{i:s}^s)
\]  

(2.11)

A trader solves in each period the following dynamic stochastic programming:

\[
\begin{align*}
\text{maximize} & \quad \mathbb{E}_d[U(\{x_{i:s}^s\}_{s=t}^{t+h_i}, \{k_{i:s}^s\}_{s=t}^{t+h_i} ; \lambda_i, h_i, \eta, \beta)] \\
\text{s.t.} & \quad (2.1) - (2.6), \ t \leq s \leq t + h_i \\
& \quad 0 \leq x_{i:s}^s \leq 1, \ t \leq s \leq t + h_i \\
& \quad 0 \leq k_{i:s}^s \leq 1, \ t \leq s \leq t + h_i
\end{align*}
\]  

(2.12)

The critical step in RHP relies on how to approximate an expected value of future utilities. Given a set of agents’ belief about future uncertainties, a trader forms expectation by constructing Monte Carlo samples drawn from the current belief system. This is a central place in which the actual states of the world and agents’ belief systems interact. Note that the belief system of a trader is not stationary; it constantly changes according to the outcomes of markets. Then the belief system subsequently induces actions, leading to aggregate phenomena and so on. For the details of implementation of a trader’s belief update, refer to Appendix D.

### 2.3.2.3 Animal-Spirit Trader

An animal-spirit trader is different from an optimizing trader in terms of portfolio and consumption choices. I define an animal-spirit trader as an entity who makes these decisions without resorting to logical thinking or sophisticated calculations. While Keynes

\[23\] For an extensive coverage of various techniques in approximate dynamic programming, see Powell (2011). Approximate dynamic programming inherently involves various learning algorithms to implement it. The extensive sources associated with learning algorithms used in stochastic optimization can be found in Tesfatsion (2014).
(1936) focuses on animal spirits inherently embedded in the minds of entrepreneurs, I instead shift the focus of this study on the role of animal spirit traders as fund providers for firm investments. I consider this setup more relevant in the context of financial markets.

In this study, an animal-spirit shock is defined to be a purchase of stock shares not supported by logical reasoning or intertemporal optimization. An animal spirit shock is said to be optimistic if it exceeds what an optimizing trader would have selected under the same conditions.\textsuperscript{24}

The animal-spirits defined in the model should be distinguished from its counterparts in the current literature. In the literature analyzing animal spirits in the context of an agent information structure, agents are still rational decision makers operating under optimizing principles, responding to noisy signals. I intentionally give up this tradition to maintain Keynes (1936)’s original intention.

Specifically, the behavioral assumptions of an animal spirit trader are as follows:

- Use $s$ fraction of real money for purchasing stocks issued by the firm
- Use $(1 - s)$ fraction of real money for consumption

where $s \in [0, 1]$ denotes the size of an animal-spirit shock.\textsuperscript{25}

Note that an animal-spirit trader can be characterized by the following pair $(\tilde{T})$:

$$\tilde{T} = (s, v)$$

where $v \in \{1, \cdots, N\}$ is a location of the animal-spirit trader in an interacting network and $N$ denotes a total number of traders in the network.

2.3.3 Firm

In the model economy, there is a representative single firm that produces output and actively manages stock shares through the stock market. The firm finances its investment through internal cash flow and the repeated Seasoned Public Offerings (SPO) at the stock market.\textsuperscript{26} The proceeds from the retained earnings and a new equity issue

\textsuperscript{24}In the current study, the focus is on optimistic animal-spirit shocks.

\textsuperscript{25}This is so because the remaining cash after the consumption will be used for purchasing either stocks or risk-free assets. Also note that we denote the marginal consumption propensity of an animal-spirit traders as $\tilde{\kappa}_{it,t}$.

\textsuperscript{26}The SPO is different from Initial Public Offering (IPO). The SPO denotes a new equity issue by a publicly traded firm. The evidence that supports this assumption can be found in Brown et al. (2009), who show that internal cash flow and external equity injections are key sources for the investments of high-tech and young firms.
are used either physical capital accumulation or R&D investment. So the firm’s problem is summarized to choosing the size of a new equity issue and the investment composition between physical capital accumulation subject to diminishing marginal return and R&D investment that involves uncertainty. Table 2.2 summarizes variables associated with the firm.

In reality, a firm issues new shares through a primary market with the intermediation by investment banks. So there is distinct separation between a primary market and a secondary market. However, I do not make an explicit separation between these two markets and blend them into a single exchange market, creating an environment in which the firm has to compete with traders who wish to sell existing shares. Although this setup does not correspond to reality, there are two motivations for this setup.

First, the determination of IPO price of new shares is multidimensional due to the complex nature of public offerings shaped by the structure of capital markets and a set of financial regulations etc. Modeling the determination of IPO price requires us to look further into the microstructure between banks and firms, which will definitely add more complications to the model. Second, combining these two separate markets reflects well the dependence of fund availability on the current market conditions.\footnote{For example, if the bearish sentiment dominates the stock market, it is less likely for the firm to raise additional funds from the stock market.}

\subsection*{2.3.3.1 Firm Constraints}

Before I specify the firm’s objective, I first describe the constraints faced by the firm in each point of time. At the time 0, the firm is endowed with initial capital ($K_0$), initial TFP ($A_0$), outstanding shares ($S_0$) and cash balance($M_{0f}$).

The firm produces its output using labor and capital. The production technology is represented by a standard Cobb-Douglas production function:

\begin{equation}
Y_{t:2} = A_t K_t^\alpha L_{t:1}^{1-\alpha}
\end{equation}

where $A_t$ represents total factor productivity (TFP). The cash balance of the firm at $t : 2$ becomes

\begin{equation}
M_{t:2} = Y_{t:2}
\end{equation}
Table 2.2: Summary of Variables: Firm

<table>
<thead>
<tr>
<th>Predetermined Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of outstanding shares</td>
</tr>
<tr>
<td>Capital stock</td>
</tr>
<tr>
<td>TFP</td>
</tr>
<tr>
<td>Real Money Balance</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Endogenous Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size of a new equity issue</td>
</tr>
<tr>
<td>Output</td>
</tr>
<tr>
<td>R&amp;D investment ratio</td>
</tr>
<tr>
<td>R&amp;D investment</td>
</tr>
<tr>
<td>Capital investment</td>
</tr>
<tr>
<td>Labor demand</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Exogenous Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retained-earings ratio</td>
</tr>
<tr>
<td>Capital income share</td>
</tr>
<tr>
<td>Discount factor</td>
</tr>
<tr>
<td>R&amp;D investment sensitivity</td>
</tr>
<tr>
<td>Upper bound on the size of new equity issuance</td>
</tr>
</tbody>
</table>

After paying wages to its workers, the firm’s cash balance changes as follows:

$$M_{t:3}^f = M_{t:2}^f - w_{t:1}L_{t:1}$$  \hspace{1cm} (2.16)

where $w_{t:1}$ is a market clearing wage in labor market and $L_{t:1}$ denotes total labor supply at the market clearing wage rate. The profit of the firm in period $t = [t : 1, t + 1)$ is defined as

$$\pi_{t:3} = Y_{t:2} - w_{t:1}L_{t:1}$$  \hspace{1cm} (2.17)

The firm has to decide what proportion of its profit is distributed as dividends to its shareholders. I assume that the firm retains a constant fraction of the current profit. Given the retained-earnings ratio ($\theta$), the total dividend payout is given

$$D_{t:4} = (1 - \theta)\pi_{t:3}$$  \hspace{1cm} (2.18)

Paying dividends also reduces the cash balance of the firm as follows:

$$M_{t:4}^f = M_{t:3}^f - D_{t:4}$$  \hspace{1cm} (2.19)
In addition to the internal cash, the firm can also raise funds by issuing new shares in the stock market. Given the size of new equity issue \((Z^*_t)\), an additional cash flow to the firm is given by the number of new shares multiplied by a closing stock price.\(^{28}\)

\[
M'_{t:6}(P) = M'_{t:4} + PZ^*_t
\]

where \(Z^*_t > 0\) is the firm’s choice variable and \(P\) denotes a market stock price.

Because there are no further liquidity needs for the firm, the firm’s final choice associated with the current cash balance is to divide it into physical capital investment and R&D investment. The first is going to directly increase its capital stock and the latter is to improve its TFP. From an accounting perspective, the capital stock is a tangible asset. In contrast, the TFP is an intangible asset for which an outside investor’s valuation is imprecise and is subject to errors. The current level of TFP of the firm is assumed not to be common knowledge to outside traders.

The planned R&D investment size \((\tilde{RD}_{t:6})\)\(^{29}\) and the planned physical capital investment size \((\tilde{I}_{t:6})\) determined by R&D ratio \((\rho_{t:6})\):

\[
\begin{align*}
\tilde{RD}_{t:6}(P) &= \rho^*_t M'_{t:6}(P) \\
\tilde{I}_{t:6}(P) &= (1 - \rho^*_t) M'_{t:6}(P)
\end{align*}
\]

The real money balance of the firm reduces to zero once both investments are made:

\[
M'_{t+1} = 0
\]

I assume the growth rate of TFP depends on the size of the firm’s actual R&D investment normalized by the firm size (i.e., R&D intensity):

\[
\ln A_{t+1} = \ln A_t + Pois(\tau RD_{t:7}/Y_{t:2})
\]

where \(Pois\) denotes a Poisson random variable with mean being \(RD_{t:7}/Y_{t:2}\), and \(\tau > 0\). Assuming no adjustment costs in capital investment, the next period capital is given as usual

\[
K_{t+1} = (1 - \delta)K_t + I_{t:7}
\]

where \(\delta > 0\) is a depreciation rate.

\(^{28}\)Note that the firm is not a sole seller of equity shares. It has to compete with other trade traders who wish to sell their existing shares. Therefore, there is no guarantee that the firm can fulfill the size of a new equity issue as planned.

\(^{29}\)A superscript \(\sim\) is introduced to denote a planned variable, not a realized variable.
2.3.3.2 Firm Objective and Decision Procedure

The firm’s intertemporal profit ($\Pi$) is given by:

$$\Pi({\{\rho^*_s\}_{s\geq t}}, \{Z^*_s\}_{s\geq t}; \theta, \beta_f, \tau, \delta, \alpha, \eta_{t;3}) =$$

$$\pi_{t;3} + \sum_{s=t+1}^{\infty} \beta^{s-t} \pi_s$$

(2.25)

The firm is assumed to maximize the expected intertemporal profit under constraints specified in 2.3.3.1:

$$\max_{\{\rho_{s;6}\}_{s \geq t}, \{Z^*_s\}_{s \geq t}} E_t[\Pi({\{\rho^*_s\}_{s\geq t}}, \{Z^*_s\}_{s\geq t}; \theta, \beta_f, \tau, \delta, \alpha)]$$

s.t.

$$- (2.15) - (2.24)$$

$$0 \leq Z^*_s \leq \bar{Z}, \forall s \geq t$$

$$0 \leq \rho^*_{s;6} \leq 1, \forall s \geq t$$

where $\bar{Z}$ is the upper bound for the size of a new equity issue.\(^{30}\) The firm’s problem can be summarized as choosing the sequences of $\{\rho_{s;6}\}_{s \geq t}, \{Z^*_s\}_{s \geq t}$ to maximize the expected sum of discounted profits.

As assumed for traders, the firm does not have complete knowledge about the model and is restricted to its own information: the firm does not know in advance the decision rules of traders and market clearing stock prices etc. Like traders, given a fixed horizon ($h_f$), the firm uses a rolling horizon procedure (RHP) to find approximate solutions to the following stochastic optimization problem as in Eq. (2.26).

$$\max_{\{\rho_{s;6}\}_{s \geq t}, \{Z^*_s\}_{s \geq t}} E_t[\Pi({\{\rho^*_s\}_{s\geq t}}, \{Z^*_s\}_{s\geq t}; h_f, \theta, \beta_f, \tau, \delta, \alpha, \eta_{t;3})]$$

s.t.

$$- (2.15) - (2.24)$$

$$0 \leq Z^*_s \leq \bar{Z}, \ t \leq s \leq t + h_f$$

$$0 \leq \rho^*_{s;6} \leq 1, \ t \leq s \leq t + h_f$$

(2.27)

Note that $h_f$ refers to the firm’s horizon and $\beta_f$ is a discounting factor. For details of the firm’s RHP and its belief update algorithm, refer to Appendix E and Appendix F.

---

\(^{30}\)This upper bound captures the existence of transaction costs involved in new equity issues. The endogenous determination of transaction costs of SPO can be modeled by explicitly introducing transac-

tion costs into the firm’s profit function. For the sake of simplicity, I abstract from this approach, which can be pursued in future research.
2.4 Simulations

2.4.1 Benchmark Case

Before I present the quantitative impacts of a single one-time animal-spirit shock, I first report the simulation results of the benchmark case. As suggested in Figure 2.2, there are no animal-spirit shocks for all periods and the degree of conformity is uniformly given as 0 for all traders in this case.

![Diagram](1)

![Diagram](2)

![Diagram](3)

![Diagram](4)

Figure 2.2: **Benchmark Case.** No animal-spirit shock and no conformity are assumed.

Figure 2.3 shows the simulated time series of key variables that might be of our interests. First of all, we can easily identify that the economy is growing over time as capital investment and R&D investment grow. However, the R&D investment exhibit more jagged fluctuations compared to the capital investment. Corresponding to a upward trend in the output, the aggregate consumption of the economy also grows.

The stock return series is characterized by periodic volatility clustering, which is consistent with what we observe in empirical stock return data. The similar volatility clustering phenomenon is also found in the size of public offerings—the number of new stock shares issued by the firm. Earnings-Per-Share (EPS), which I regard as a proxy measure for the firm fundamental value, is also growing over time.

2.4.2 Simulation Design

In this section, I simulate the model with varying degrees of key parameters, which might be of our key interests, so as to see whether there are persistent impacts of a single animal-spirit shock on firm fundamental values and other performance metrics. First of all, I specify the network structure of the model economy as in Figure 2.4, maintaining a small number of traders for simplicity. The parameter values maintained throughout all subsequent simulations are also presented in Table 2.3.
Table 2.3: Maintained Parameter Values

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of traders ($N$)</td>
<td>4</td>
</tr>
<tr>
<td>Risk-free rate ($r_f$)</td>
<td>0.03</td>
</tr>
<tr>
<td>Capital income ratio ($\alpha$)</td>
<td>0.3</td>
</tr>
<tr>
<td>Decay factor ($\psi$)</td>
<td>0.97</td>
</tr>
<tr>
<td>Upper bound for new issues ($\bar{Z}$)</td>
<td>10</td>
</tr>
<tr>
<td>Retained-earnings ratio ($\theta$)</td>
<td>0.5</td>
</tr>
<tr>
<td>Conformity weight ($\lambda$)</td>
<td>0.5</td>
</tr>
<tr>
<td>Trader’s forecasting horizon ($h$)</td>
<td>2</td>
</tr>
<tr>
<td>Firm’s forecasting horizon ($h_f$)</td>
<td>4</td>
</tr>
<tr>
<td>R&amp;D investment sensitivity ($\tau$)</td>
<td>1</td>
</tr>
<tr>
<td>Marginal consumption propensity of Animal-spirit traders</td>
<td>0.5</td>
</tr>
<tr>
<td>Monte Carlo sample number ($\bar{\omega}$)</td>
<td>10</td>
</tr>
<tr>
<td>Trader discount factor ($\beta$)</td>
<td>0.99</td>
</tr>
<tr>
<td>Firm discount factor ($\beta_f$)</td>
<td>0.99</td>
</tr>
</tbody>
</table>

In the following computational experiments, I introduce a single one-time animal-spirit trader into the pre-specified network, which is illustrated in Figure 2.5. In the figure, each node represent a trader, and an edge that connects two nodes implies two traders are linked. In each experiment I assume that conformity among traders are formulated, unless the conformity is equal to 0, after the arrival of a shock for comparison to the benchmark case.

Even though there are many important treatment factors that might be of our interests, I consider the following three parameters as key treatment factors properly discretized:

- Conformity ($\lambda$): $\Lambda = \{0, 0.1, 0.5, 0.9\}$
- Animal-spirit shock size ($s$): $S = \{0.25, 0.5, 0.75, 1\}$
- The location of an animal-spirit trader ($\upsilon$): $\Upsilon = \{1, 2, 3, 4\}$.

Note that there are 48 test cases are possible. This study considers a small subset of an entire parameter space $\Lambda \times S \times \Upsilon$. Each test case (TC) can be characterized by a triplet $(\lambda, s, \upsilon)$.

In the Sensitivity Design 1, I fix the location of an animal-spirit shock at 2. Given this shock location, I vary the degree of conformity ($\lambda$) and the size of the shock. I derive
Table 2.4: Sensitivity Design 1: $v = 2$

<table>
<thead>
<tr>
<th>$\lambda$ \ $s$</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>X</td>
<td>$TC_1$</td>
<td>$TC_5$</td>
<td>X</td>
</tr>
<tr>
<td>0.1</td>
<td>X</td>
<td>$TC_2$</td>
<td>$TC_6$</td>
<td>X</td>
</tr>
<tr>
<td>0.5</td>
<td>X</td>
<td>$TC_3$</td>
<td>$TC_7$</td>
<td>X</td>
</tr>
<tr>
<td>0.9</td>
<td>X</td>
<td>$TC_4$</td>
<td>$TC_8$</td>
<td>X</td>
</tr>
</tbody>
</table>

Note: $\upsilon$ denotes the location of an animal-spirit trader. A test case (TC) with a ‘X’ sign is not considered in this study.

Table 2.5: Sensitivity Design 2: $v = 3$

<table>
<thead>
<tr>
<th>$\lambda$ \ $s$</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>X</td>
<td>$TC_9$</td>
<td>$TC_{13}$</td>
<td>X</td>
</tr>
<tr>
<td>0.1</td>
<td>X</td>
<td>$TC_{10}$</td>
<td>$TC_{14}$</td>
<td>X</td>
</tr>
<tr>
<td>0.5</td>
<td>X</td>
<td>$TC_{11}$</td>
<td>$TC_{15}$</td>
<td>X</td>
</tr>
<tr>
<td>0.9</td>
<td>X</td>
<td>$TC_{12}$</td>
<td>$TC_{16}$</td>
<td>X</td>
</tr>
</tbody>
</table>

Note: $\upsilon$ denotes the location of an animal-spirit trader. A test case (TC) with a ‘X’ sign is not considered in this study.

Simulation results of key variables of in the form of Table 2.4, while the location of an animal-spirit shock is fixed at 3 in the Sensitivity Design 2.

2.4.3 Results

Now we come to the core of this study-simulating the economy populated with traders and the firm described. In this section, I report key results from aforementioned sensitivity designs. Readers should note that all numerical values denote the percentage gaps from the benchmark case.

This section starts with two particular test cases as to see whether there are persistent impacts of the optimistic animal-spirit shock on firm fundamental values and other metrics. Later I convey a general picture of how the magnitude of impacts of the shock vary by the degree of conformity, the shock size, and the shock location.

Figure 2.6 shows the impulse responses of key variables over 40 periods following
the single one-time optimistic animal-spirit shock from the Test Case 11 \((\lambda, s, \upsilon) = (0.5, 0.5, 3))\). First of all, at the time of arrival of the shock, the stock price dramatically increases due to a positive demand shock injected by the optimistic animal-spirit trader in the stock market. Although the impact of the shock on stock prices seems to eventually decay as the period goes further, it is quite persistent up to 40 periods.

This kind of pattern is also observed in the market capitalization; the impact of the optimistic animal-spirit shock on market capitalization seems to be very persistent given that the shock tends to slowly build up its impact over time after initial fluctuation and decay thereafter. Note that the gap in market capitalization is significant, reaching up to 10% at the maximum.

One notable finding is that both EPS and output maintain persistent gaps relatively over long periods. These facts clearly demonstrate that one-time optimism introduced by a specific trader in the asset market can indeed improve the fundamental of the model economy. The shock also slowly builds up its impacts on the aggregate consumption over the following period and gradually decays.

The impact of the optimistic animal-spirit shock on two types of the firm investment seemed to be somewhat mixed; the signs of gap in firm investments are not uniform provided that gaps fluctuate over periods. However, positive gaps dominate negative gaps during the majority of periods.

Figure 2.7 shows the gaps of key variables from the benchmark case for four different conformity values, suggesting how a trader’s conformity tendency play a role in determining the dynamics of an economy. We observe that there is a nonlinear relationship between the degree of conformity and the magnitude of the shock; the average gap tends to be greater as the degree of conformity gets higher up to 0.5. However, when the degree of conformity is very high, the size of the impact tends to decrease. For example, earnings-per-share gap is shown to be lowest and quickly disappears when the conformity is 0.9.

This can attributed to the less degree of liquidity caused by mimicking behavior of traders. As Pyo (2014) shows that liquidity dry-up in an asset market is closely associated with the extreme conformity in traders, the high level of conformity in a trader’s preference might have created less demand for stocks than cases where the conformity is moderate. This finding suggests that the magnitude of the impact of the animal-spirit shock on firm fundamental values crucially depends on the behavioral element of market participants.
In addition to the Test Case 11, I present the specific results of Test Case 3 \((\lambda, s, \upsilon) = (0.5, 0.5, 2)\) so as to demonstrate the importance of the animal-spirit trader’s topological position within a network. The Test Case 3 has a smaller animal spirit index than the Test Case 11, implying a less degree of optimism. Figure 2.8 shows the percentage gaps of key variables from the benchmark case over 40 periods.

The impluse responses of key variables in the Test Case 3 are similar to those found in the Test Case 11. The difference between two cases are clearly illustrated in Figure 2.9. Over the period following shock, Test Case 11 exhibits higher gaps on average for all key variables, which suggest the impact of the shock gets intensified when the shock is imposed on a trader who has a high centrality in a given network.

Recalling that the system-wide crisis back in 2007 is triggered by the collapse of a handful of investment banks which have relatively higher degrees of centrality in a complex wholesale funding network, this comparison clearly demonstrates the importance of the topological position of a market participant who faces an external shock for the determination of future trajectory of an economy.

Figure 2.10 summarizes key results of the sensitivity design, showing averages of single period percentage deviation from the benchmark case for key variables. From the Figure 2.10(a), we confirm that there is non-monotonicity in the effect of the degree of conformity on the firm fundamental value. The figure also suggests that, when the shock location has a higher degree, the magnitude of the impact tends to higher.

The impact on the fundamental volatility, which is measure by a percentage gap in EPS volatility, turns out to be greater when the conformity is low or moderate and the shock location is 3. This kind of pattern is also found in the market valuation of the firm as in 2.10(c).

Figure 2.10(f) suggests that the shock exerts the biggest impacts on the stock return volatility, which is measured as standard deviation of the realized stock returns during the periods following the shock, showing the average gap between 30 percent and 40 percent. The result also suggest that a high stock return volatility is associated with a low or a medium size of conformity and high degree of a shock location.

### 2.5 Concluding Remarks

Whether the animal-spirit sentiments of traders in financial markets can subsequently induce changes in the fundamentals of an economy is a fascinating topical issue for
economists. During the Great Recession (2007-2009) the collapse of several financial institutions led to contractions in many sectors of the world economy. Although the causes that triggered this systemic crisis are debatable, various researchers have reported that key market players with high network connectivity played a crucial role in spreading initial shocks throughout many regions of the world (Markose et al. (2012)).

In line with this observation, this study develops an agent-based computational model based on a social network of constructively rational traders who desire to conform to the actions of other traders in this network. A key simulation finding is that a single one-time animal-spirit shock in an asset market can have persistent impacts on firm fundamental values by influencing the firms stream of current and future real profits.

Simulation findings also show that the persistent effects of a one-time animal-spirit shock are amplified when the shock is induced by the animal-spirit actions of a trader with high connectivity in his social network, assuming traders in this network have moderate conformity levels. Too low or too high a conformity level can actually attenuate the effects of the animal-spirit shock.

However, this study does not claim that one-time animal spirit shocks will have persistent effects under any environmental setting. Additional simulations are needed to test systematically the effects of one-time animal-spirit shocks under alternative specifications for financial institutional arrangements, social network structures, utility and production functions, and key structural parameters such as the number of traders.

For example, one of the limitations of the dynamic investment economy developed in this study is that it excludes debt markets, which can be additional sources for firm external finance. It would be interesting to study the role of animal-spirit shocks when the market valuation of a firm in a secondary stock market is used as a key element in the risk assessment of a firms debt instruments (e.g., Merton (1973), Crosbie (2003)). In addition, permitting traders to choose both their social networks and their desired degrees of conformity with the behaviors of other traders in these networks, based on learning capabilities, would enrich the current study in many ways.

An extended modeling and analysis based on relationships among financial intermediaries (e.g., banks), equity markets, and the corporate financing decisions of firms will be undertaken in future research.
Figure 2.3: Simulated Paths of Key Variables in the Benchmark Case. The figure is based on 20 simulation runs.
Figure 2.4: **Network Structure in Computational Experiments**: The degree of a node is the number of edges connected to the node.

Figure 2.5: **Introducing a Single Animal-Spirit Shock**. A red node denotes an animal-spirit trader.
Figure 2.6: The Impacts of a Positive Animal-Spirit Shock: Test Case \(11(\lambda, s, \nu) = (0.5, 0.5, 3)\). The figures show percentage gaps from the benchmark case. The results are based on 20 simulation runs.
Figure 2.7: The Impact of a Positive Animal-Spirit Shock with a Varying Degree of Conformity: Test Case 11 \( [(\lambda, s, \upsilon) = (\lambda, 0.5, 3)] \). The figures show percentage gaps from the benchmark case and they are based on 20 runs of simulation.
Figure 2.8: The Impact of a Positive Animal-Spirit Shock: Test Case 3 $(\lambda, s, \nu) = (0.5, 0.5, 2)$. The figures show percentage gaps from the benchmark case and they are based on 20 runs of simulation.
Figure 2.9: **Comparison between Test Case 11 and Test Case 3.** The figures show percentage gaps from the benchmark case and they are based on 20 runs of simulation. The degree of conformity($\lambda$) is set as 0.5. The animal-spirit trader in Test Case 11 has a higher degree than the one in Test Case 3.
Figure 2.10: **Sensitivity Design Results**: Each value in a cell denotes an average of single period percentage gap from the benchmark case over 40 periods following a single one-time optimistic shock.
CHAPTER 3. A MULTI-FACTOR MODEL OF HETEROGENEOUS TRADERS IN A DYNAMIC STOCK MARKET

3.1 Introduction to Chapter 3

Over the last two decades behavioral finance has become an important component of academic finance. The hallmark of this progress is the conferment of the 2013 Nobel Prize in Economic Sciences to Robert Shiller along with Eugene Fama, who has a contrasting view on financial markets. Shiller has been in the front lines of behavioral finance (e.g., Shiller (2003)), while Fama has been a major proponent of the Efficient Market Hypothesis (EMH).

Witnessing the emergence of behavioral finance as alternative paradigm for explaining various phenomena in financial markets, this study builds a computational stock market that allows us to investigate various aspects of stock market when traders use simple behavioral trading rules, which are based on empirical observations on how people make financial investments. This study shows that traders with simple trading rules can replicate several stylized facts on stock returns such as excess volatility, ARCH effect, and fat-tail property.

The other contributing factor of the study is that it constructs a framework with a high degree of flexibility to design an experimental computational laboratory in which rational fundamental traders and behavioral traders compete, which permits two contrasting view of the Nobel laureates to be systematically explored. The extensions or modifications of the current framework for future studies could be easily tailored to specific environments of future studies. As one possible application of the framework, I carry out experiments with a particular dividend path, which suggests the impacts of exogenous dividend shock on stock market outcomes depend on memory length.

\footnote{For example, we can replace simple behavioral trading rules with more complex decision making rules that facilitate learning and intertemporal optimization.}
In the model, traders allocate their wealth between a risky stock and a risk-free asset based on heterogeneous information sets. The information set of each trader can include data on three factors: firm profitability, a past stock return movement, and the investment behavior of other traders with whom a trader interacts. The behavioral assumption on portfolio rebalancing used in this study is based on a mixture of network effect and momentum strategies.

The idea of network effects dates back to Keynes (1936)’s *beauty contest* metaphor for stock market investment. In line with Keynes’s insight, Shiller (2000) argues that social media plays a key role in magnifying fads, excitements at the early stage of spread of them. In addition to his argument, a social aspect of financial market investment is well documented in many empirical studies. Even though there is no explicit medium through which information flow across traders, one component of behavioral trading rules assumed in this study is based on an approximation of this beauty contest idea. Simulation results suggest that liquidity dry-up is closely related to mimicking behavior of traders.

This study not only constructs a flexible framework but also actually produces the simulated stock market performance metrics under different behavioral trading rules. I also investigate wealth dynamics across different types of traders. Key simulation findings are as follows. If all traders only take into account firm dividend as a key informational factor, and the dividend process is non-stationary, then long-memory dividend traders create a more volatile stock return process than short-memory dividend traders. For the stock market with traders who care about only a prior stock return, stock prices fluctuate in a cyclical pattern marked by a no-trade state at the peak of each cycle.

On the other hand, if the market is populated only with ‘beauty contest’ traders who try to mimic the average behavior of others, then the stock market collapses to a no-trade state after a few stock exchanges. Finally, when all traders place equal weight on the three factors, stock returns exhibit a more pronounced fat-tail property, with lower stock return volatility, relative to the case in which all traders only take into account firm profitability.

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2Shiller and Pound (1989)’s survey on professional institutional money managers shows that the interpersonal communications among investors is a crucial factor in choosing a particular stock. Hong et al. (2005) also report that the *word-of-mouth* effect exists in portfolio management of mutual funds in 15 big cities in the U.S. The literature also report that stock market participation of individual households is positively related to neighborhoods stock market participation (i.e., Hong et al. (2004), Brown et al. (2008)). Malloy et al. (2008) investigates the stock return implication of education networks among money managers and corporate board members.
In terms of trading performance, long memory dividend traders’ average wealth turns out to be highest compared to other types of traders. However, I do not find any significant differences in real wealth growth rates across different types of traders except for trend-following traders’ average wealth growth being most volatile.

The paper is structured as follows. Section 3.2 discusses, focusing on the contributions of this study, the closely related literature. Section 3.3 provides an overview of the model followed by a detailed description of the proposed model. Section 3.4 outlines the experimental design to be used for sensitivity testing. In Section 3.5, I present simulation results for six illustrative test cases. Section 3.6 presents concluding remarks.

3.2 Relationship to Existing Literature

Since the rational expectation revolution in the nineteen seventies, economists have made great progress in finding tractable ways to incorporate agent heterogeneity within their models. Heterogeneity is prevalent in all areas as well as economic domains. Economic theories or models that ignore evident heterogeneity are prone to lacking a proper microfoundation for explaining aggregate outcomes that are the results of coordination of interacting agents (Kirman (1992)).

The rediscovery of heterogeneity in human nature has led to propagation of heterogeneous agent-based computational modeling techniques in many sub-disciplines of economics. In particular, studies of financial markets have been in the lead by incorporating adaptive behavior of agents in the formation of expectations about future states.

While the current literature focuses on endogenous switching between a few trading types (i.e., chartists and fundamentalists), I rather view agents as standing somewhere

---

3 Kirman (1992) clearly elaborates on how the representative agent framework can be flawed in various contexts. A notable remark is on the question of the validity of policy implications drawn from representative agent models. Hommes (2011) also corroborates this argument by showing, in human-subject experiment, that homogeneous rational expectations do not occur in many market contexts.

4 For an overview of studies based on heterogeneous agents in diverse disciplines of ACE (Agent-based Computational Economics), see Tesfatsion and Judd (2006). Tesfatsion (2001) also provides a brief introduction to ACE. Arguing possible advantages of ACE methodologies, she explicitly defines the goal of ACE is to “demonstrate constructively how these global regularities might arise from the bottom up, through repeated local interactions of autonomous agents acting in their own perceived self-interests.”

5 For extensive coverage of intellectual endeavors made in financial ACE area, see Hommes (2006), Lebaron (2006) and Hommes and Wagener (2009). The prototype of agent-based artificial stock market can be found in Arthur et al. (1997). For studies on adaptive behavior based on genetic algorithms in asset market context, see Arifovic (1996), Chen and Yeh (2001), Kluger and McBride (2011), and Arthur et al. (1999). Endogenous switching between different forecasting rules are considered in numerous models, such as Brock and Hommes (1998), Lux and Marchesi (2000), and Chiarella and He (2002).
between extreme types. Rather than imposing endogenous selection among different forecasting rules I take a shortcut: demand or supply is a direct function of the factors described. This could be understood as there exists a sort of internal forecasting mechanism that converts the information set to a specific investment rule.\textsuperscript{6} Chiarella et al. (2009) take a similar view in their study of a double auction stock market. The distinctive feature of their model is that a trader forms an expectation about future stock returns based on multiple components, i.e., fundamentalist component, chartist component and noise component. What differentiates our model from Chiarella et al. (2009) is that I replace the fundamental price by the subjective perception of a trader on current firm profitability relative to the history of it.\textsuperscript{7} Subjective evaluation of each component does not go through expectation formation process. Rather, they are directly blended into portfolio choice.

One of key advantages of exploiting multiple factors in forming portfolio decision is that we have a flexible platform which allows us to explore over how heterogeneous responses to each of factors affect stock market dynamics. For example, it would be worthwhile to see how the system reacts to conflicting signals about asset valuation, i.e., low profitability coupled with past high stock return.

It should be noted that subjective comparison of current firm profitability relative to past profitability inherently involves selection of extent of past data usage (i.e., memory length). In this study the relative profitability of the firm is represented as normalized deviation of current dividend to moving average of dividend. Memory length in using past information and heterogeneous learning gain is shown to be crucial aspects of market dynamics in LeBaron (2001a), LeBaron (2001b) and LeBaron (2012). As we will see in the following sections, our model also generates quite different market dynamics under different schemes of memory length.

Another distinction between this study and previous stock market studies is that our model permits the endogenous determination of trading positions and no-trade states. A no-trade state is a situation where all traders are on the same side of trading direction for any values in the space of stock price. This feature has been rarely examined in a majority of computational stock market models since it is conventional to assume that there always is a fixed amount of stock shares that are ready to be supplied. This assumption is analogous to saying there is a continuous IPO market in each trading

\textsuperscript{6}This type of modeling approach is similarly implemented in Thurner et al. (2009).

\textsuperscript{7}I regard knowing the fundamental price of a stock share as being incompatible with the imperfect knowledge of the actual data generating process that determines firm’s fundamental value.
period. I circumvent this unrealistic convention by postulating that a trader’s demand or supply for stocks is dependent on his current state.

This study critically departs from the earlier literature in an additional way; the portfolio choice of a trader is directly influenced by the portfolio profile of linked traders. The mimetic behavior could be expressed as the following: *if my friends buy more stock, I would buy more.*

Analyses on indirect mimetic behavior of traders in the context of computational stock markets are found in Lux (1998) and Iori (2002). Iori (2002) develops a multi-agent stock market model under which trading decision depends on communication between traders and idiosyncratic shocks. She identifies that the imitating behavior and trading frictions are key elements of volatility clustering.

Compared to Iori (2002), Lux (1998) develops a model in which mimetic behavior is implemented in a less direct sense. In his model, conversion between optimistic chartist and pessimistic chartist is stochastically executed through a global variable, i.e., an opinion index. Taking a more drastic step, I model stock holdings of a trader as a function of stock holdings of agents within his interacting boundaries. The rationale for this type of assumption can be found in numerous works in the empirical finance literature that show the significance of social influence on financial investment behavior (e.g., Shiller and Pound (1989), Hong et al. (2004), Hong et al. (2005), Malloy et al. (2008), Brown et al. (2008)).

### 3.3 Model Description

#### 3.3.1 Overview

The model consists of a finite number \(N\) of traders repeatedly interacting in a dynamic stock market. There is a single risky asset (stock) and a single risk-free asset (bond). Traders are initially endowed with a mixture of risk-free bonds and stock shares. All traders in the model are wealth seekers in the sense that they keep rebalancing their asset portfolios, based on an observed information set, in the anticipation of wealth growth. Table 3.1 and Table 3.2 summarize the variables used throughout the model.

Figure 3.1 depicts a typical day in the life of a trader. A trader starts out his day by receiving dividend. Afterward, he goes out for interacting with his friends. From these mutual interactions, traders get informed of the current portfolio profiles of his friends. The profiles simply contain the stock holdings of his friends. Thereafter a trader
reads through his all financial accounts and newspapers to collect relevant information for a subsequent trading. The information set, $I_t$, includes the portfolio profiles of neighboring traders ($X_i^Z_t$), the current number of stock shares ($S_i$), the current number of bonds ($B_i$), the stock price of a previous trading period ($P_t$), the current wealth ($W_i$), the bond price ($Q_t$), dividend ($d_{t+1}$). 

Once a stock market opens up for trading, a trader $i$ computes his desired stock holding, which is measured as a percentage of his current wealth. I assume the trader’s desired stock holding as a function of three distinct factors in the information set previously collected: the portfolio profiles of neighborhood; firm’s profitability; and past stock return performance. A trader may have a different weight on each factor, depending on his nature, experiences and etc. Consequently, for each trader, the weights placed on three factors determine his trading type. These weights will constitute important treatment factors which will be systematically varied in the subsequent computational experiments.

Once a trader determines his desired stock holding, he forms his demand bid or supply offer for stock shares and submits this bid/offer to the stock market. Afterward, stock bids/offers are matched to achieve a market clearing solution consisting of a trading

---

8Note that all of the trader subsequent actions are conditional on this information set. The notation for an information set of a trader $i$ will be suppressed in the following sections for notational simplicity.
volume and a closing stock price. Subsequently, his new stock holding \((x_{it})\), which is defined as a ratio of cash value of shares to wealth, is realized. This step finalizes one day of a trader. In the model, all traders go through this daily routine. In the following subsections, I provide the detailed explanations for each component in the routine. Table 3.1 and Table 3.2 can be used for references in the following discussions.

### Table 3.1:
Summary of Endogenous Variables

<table>
<thead>
<tr>
<th>List Endogenous Variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of bonds</td>
<td>(B_{it} \in \mathbb{R}_+)</td>
</tr>
<tr>
<td>Stock price</td>
<td>(P_t)</td>
</tr>
<tr>
<td>Stock return</td>
<td>(r_t = \frac{P_t - P_{t-1}}{P_{t-1}})</td>
</tr>
<tr>
<td>Number of stock shares</td>
<td>(S_{it} \in \mathbb{R}_+)</td>
</tr>
<tr>
<td>Realized portfolio weight on stock</td>
<td>(x_{it} = \frac{P_t S_{it}}{P_t S_{it} + Q_t B_{it}} \in [0, 1])</td>
</tr>
<tr>
<td>Temporary desired portfolio weight on stock</td>
<td>(\hat{x}_{it} \in [0, 1])</td>
</tr>
<tr>
<td>Final desired portfolio weight on stock</td>
<td>(x^*_{it} \in [0, 1])</td>
</tr>
<tr>
<td>Portfolio weights of neighborhood</td>
<td>(X^Z_i = {x_{jt}}_{j \in Z_i})</td>
</tr>
<tr>
<td>Wealth</td>
<td>(W_{it})</td>
</tr>
</tbody>
</table>

### 3.3.2 Dividend Payout

As described above, a trader starts out his day by receiving dividends for stock shares currently held. One notable assumption on dividend payments is that they are automatically converted to the risk-free bonds before submitting a bid or an offer for stock shares. This assumption is pivotal for the overall market dynamics because the dividend payments function as persistent disturbances in the current portfolio position of a trader, which may lead him to adjust his portfolio in a continuous fashion.

I assume the logarithm of the dividend \((d_t)\) follows a random walk with a drift \((\bar{d})\):

\[
\log(d_t) = \log(d_{t-1}) + \bar{d} + \sigma \epsilon_t
\]

where \(\sigma\) affects a volatility of the dividend process, and \(\epsilon_t\) is a Gaussian white noise term. Once the current dividend is paid out to the trader, it is recorded in the information set \(I_{it}\) of trader \(i\) for subsequent use in portfolio rebalancing.
Table 3.2:  
Summary of Exogenous Variables

<table>
<thead>
<tr>
<th>Exogenous Variables</th>
<th>admissibility conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weights on neighborhood</td>
<td>( a_{ij} = {a_{ij}} \in \mathbb{Z}<em>i \text{ s.t. } \sum</em>{j=1}^{N_i} a_{ij} = 1 )</td>
</tr>
<tr>
<td>Dividend</td>
<td>( d_t )</td>
</tr>
<tr>
<td>Moving average of dividend</td>
<td>( d_t^h_i = (\sum_{k=t-h_i}^{t} d_k) / h_i )</td>
</tr>
<tr>
<td>Memory length</td>
<td>( h_i &lt; \infty )</td>
</tr>
<tr>
<td>Total number of traders in the market</td>
<td>( N \in \mathbb{R}_+ )</td>
</tr>
<tr>
<td>Number of traders in neighborhood</td>
<td>( N_i =</td>
</tr>
<tr>
<td>Price of bonds</td>
<td>( Q_t = \frac{1}{r_f} )</td>
</tr>
<tr>
<td>Risk-free rate</td>
<td>( r_f \in [0, 1] )</td>
</tr>
<tr>
<td>Tolerance level</td>
<td>( Tol_i \in [0, 1] )</td>
</tr>
<tr>
<td>Set of interacting traders</td>
<td>( Z_i )</td>
</tr>
<tr>
<td>Weight on network factor</td>
<td>( \alpha_i \in [0, 1] )</td>
</tr>
<tr>
<td>Weight on dividend factor</td>
<td>( \beta_i \in [0, 1] )</td>
</tr>
<tr>
<td>Aggression parameter of dividend factor</td>
<td>( \gamma_i^f \in \mathbb{R}_+ )</td>
</tr>
<tr>
<td>Aggression parameter of technical factor</td>
<td>( \gamma_i^g \in \mathbb{R}_+ )</td>
</tr>
<tr>
<td>Stochastic shock on the dividend</td>
<td>( \epsilon_t \sim N(0, 1) )</td>
</tr>
</tbody>
</table>

3.3.3 Social Interaction

As mentioned in the introduction, the salient feature of the model in this study is the existence of mutual interactions among traders that affect stock holdings in every trading period. The network structure, which defines a channel through which the interactions among traders occur, is one of the crucial features of the model. In this study, I incorporate a Small-World Network (henceforth, SWN) as the network structure that I impose. The SWN is an extension of locally connected networks with a small number of traders having distant links to other traders in different local networks. It has been emerging as a good description of a realistic social network structure, and has been widely applied in different contexts.\(^9\)

Figure 3.2 illustrates how the social network among traders is structured in the model. Each node represents an individual trader. An edge which connects two nodes implies that two traders are linked. If traders are linked, they both affect each other in portfolio rebalancing.

Figure 3.2: The Network Structure of Traders. This figure illustrates how 100 traders are linked. Note that each node represents a single trader.

Let $T$ denote the set of all traders in the market. Formally, the set $Z_i \in T$ is defined as follows.

Definition 1.

$$Z_i = \{ j \mid q_{ij} = 1, \forall j \in T \}$$

(3.2)

where $q_{ij} = 1$ if $i$ and $j$ are linked. Otherwise, $q_{ij} = 0$.

The network structure assumed here is an undirected graph in which the direction of a link is not of importance. Formally, we can express the network structure as a symmetric matrix (e.g., 
\[
\begin{pmatrix}
1 & 1 & 0 \\
1 & 1 & 1 \\
0 & 1 & 1
\end{pmatrix}
\])

I assume trader $i$ regards himself as an element of $Z_i$. Therefore all diagonal elements are 1 in a matrix representation of the network.

The interaction yields the portfolio profiles of neighborhood traders ($X_{it}^{Z_i}$), which is simply a vector of stock holdings of traders in interactions. The portfolio profiles for the traders in $Z_i$ are assumed to be included in the information set for trader $i$.

3.3.4 Desired Stock Holding

In rational expectation asset pricing models, the optimal portfolio weight can be easily computed using only the expected return and volatility of an asset.\textsuperscript{10} Unlike this conventional approach, I rather exploit behavioral assumptions that enable us to embrace

\textsuperscript{10}In other words, the coordination among agents is made only through global variables.
a higher degree of heterogeneity. The factors included in the model need not be restricted to the specific factors used in the current study.

In this stock market model, the key process in the daily routine of a trader is how a trader calculates his provisional\textsuperscript{11} desired stock holding as a percentage of his wealth, at the beginning of the period. As will be clarified below, I assume that the provisional desired stock holding $(\hat{x}_{i,t+1})$ of a trader $i$ is a function of three factors: firm profitability; a past stock return; and the investment behavior of other traders. These factors are called as ‘dividend factor’, ‘technical factor’, and ‘network factor’, respectively.

The dividend factor enters into a trader’s consideration as a signal for a firm’s future profitability. A trader takes into account of the relative profitability of the firm, which is expressed as a deviation of the current dividend from a moving average of past dividends. This factor influences an agent’s decision as follows: a positive deviation of from the moving average signals traders to increase their stock holding, and vice versa.

The technical factor is the stock return $(r_t)$\textsuperscript{12} realized in the previous trading period. Specifically, I assume the positive stock return in the previous trading period causes the trader to rebalance his portfolio in favor of stocks over bonds. This type of behavioral pattern has been ascribed to momentum traders in the computational literature or to leveraged financial institutions with risk regulations.\textsuperscript{13}

The network factor, which is a byproduct of interactions, gets traders infused with a beauty contest environment that leads them to mimic the behavior of other traders in their neighborhood sets. For example, if one trader finds that weighted average of stock holdings in his networking boundary has increased, then (all else equal) he will rebalance his portfolio in favor of more stocks.

Summing up, the provisional desired stock holding $(\hat{x}_{i,t+1})$ for period $t + 1$ is specifically determined as follows:

$$\hat{x}_{i,t+1} = [a_i x_t z_t]^\alpha_i [f(d_t - \bar{d}_h i)]^\beta_i [g(r_t)]^{1-\alpha_i-\beta_i}$$ (3.3)

where $a_i$ is a vector of weights on the portfolio profiles of neighboring traders, $\bar{d}_h i$ is an $h_i$ periods moving average of dividend, $\alpha_i$ is a weight on the network factor and $\beta_i$ is a weight.

\textsuperscript{11}As shown in the figure, the final desired stock holding will be determined in a subsequent step.
\textsuperscript{12}As in the financial literature, the stock return is defined to exclude dividend payments. This is usually done since dividend payments are irregular. For modeling purposes, this is to capture a pure price movement impact on the demand of a trader.
\textsuperscript{13}A price increase in a risky asset leaves additional room for capital buffers, which leads to more purchases of a risky asset. For more details, see Shin (2010).
on the dividend factor. The memory length, $h_i$, of a trader $i$ determines the extent of past dividend data usage in forming the dividend moving average. This parameter is one of the key variables of the model. For example, a higher value of memory length implies the use of longer time series of dividend in computing the dividend moving average. As will be seen, the dynamics of stock market depend strongly on the choice of $h_i$. A high degree of trader heterogeneity can be implemented by varying the weights ($\alpha_i, \beta_i$) assigned to the network factor and dividend factor for each trader $i$. Depending on the values of these weights, a trader can be categorized as one of the following four trading types:

**Definition 2.** A trader $i$ is a ‘dividend trader’ iff $\alpha_i = 0, \beta_i = 1$.

**Definition 3.** A trader $i$ is a ‘technical trader’ iff $\alpha_i = 0, \beta_i = 0$.

**Definition 4.** A trader $i$ is a ‘network trader’ iff $\alpha_i = 1, \beta_i = 0$.

**Definition 5.** A trader $i$ is a ‘hybrid trader’ iff $\alpha_i \in (0, 1), \beta_i \in (0, 1)$.

The functional forms of $f$ and $g$ in (3.3) are given by (3.4) and (3.5):

$$f(z) = \frac{1}{1 + exp(-\gamma_i^f z)} \quad (3.4)$$

$$g(z) = \frac{1}{1 + exp(-\gamma_i^g z)} \quad (3.5)$$

where $\gamma_i^f$ is an aggression parameter for the dividend factor and $\gamma_i^g$ is an aggression parameter for the stock return factor. In other words, the functions $f, g$ are response functions which determine how aggressively a trader reacts to innovations in the dividend factor and the stock return factor. These response functions map the real line onto the open unit interval (i.e, $f, g : R \rightarrow (0, 1)$) in a monotonically increasing manner. Figure 3.3 illustrates how the curvatures of $f$ and $g$ change with changes in the aggression parameters ($\gamma_i^f, \gamma_i^g$).

### 3.3.5 Systemic Inertia

After computing the provisional desired stock holding ($\hat{x}_{i,t+1}$) for period $t + 1$, a trader $i$’s next task in period $t$ is to determine the final desired stock holding ($x_{i,t+1}^*$). I assume that the desired stock holding changes only if the provisional desired stock holding ($\hat{x}_{i,t+1}$) deviates significantly from the current stock holding ($x_{it}$).
Specifically, it is modeled by introducing the systemic inertia into the portfolio rebalancing: a tolerance level \((Tol_i)\) of a trader \(i\) acts as a proxy for this inertia, which dampens the possibility of frequent trading.\(^{14}\) By this construction, we infuse the model with additional source of heterogeneity.\(^{15}\)

The final desired stock holding \((x_{i,t+1}^*)\), which will be the basis for a bid or an offer for stock shares, is determined as follows:

\[
x_{i,t+1}^* = \begin{cases} 
  x_{i,t}, & \text{if } |\hat{x}_{i,t+1} - x_{i,t}| \leq Tol_i \\
  \hat{x}_{i,t+1}, & \text{otherwise}
\end{cases}
\]

(3.6)

Note that (3.6) prevents frantic trading behavior in the sense that it dampens the frequency of desire to trader further.

\(^{14}\)Although trading frictions are not explicitly modeled in this study, the introduction of a tolerance level implicitly brings a similar effect of having trading frictions prevalent in the market.

\(^{15}\)I checked that the presence of heterogeneity in the threshold level is a key source of market liquidity. Even when the only structural differences among traders are their tolerance levels, I observed that exchanges among traders occur.
3.3.6 Endogenous Switching between Buying and Selling

Given a desired stock holding \( x_{i,t+1}^* \) for period \( t + 1 \), a trader \( i \)'s next task is to translate this desired stock holding into the number of shares using the prevailing stock price:

\[
S_{it}^*(P_{t+1}) = \frac{W_{it}x_{i,t+1}^*}{P_{t+1}} \tag{3.7}
\]

Subsequently, given \( S_{i,t+1}^* \), a trader forms his demand or supply of stocks according to the following rule:

\[
\Delta S_{it}^*(P_{t+1}) = \frac{W_{it}x_{i,t+1}^*}{P_{t+1}} - S_{it} \tag{3.8}
\]

Let \( \phi_{i,t+1} \) be an index for trading direction at the beginning of trading period \( t + 1 \). Let \( \phi_{i,t+1} = 1 \) if a trader wishes to buy, and let \( \phi_{i,t+1} = -1 \) if trader \( i \) wishes to sell. Otherwise, let \( \phi_{i,t+1} = 0 \). Then it follows that

\[
\phi_{i,t+1} = \begin{cases} 
1 & \text{if } \Delta S_{it}^*(P_{t+1}) > 0 \\
-1 & \text{if } \Delta S_{it}^*(P_{t+1}) < 0 \\
0 & \text{if } \Delta S_{it}^*(P_{t+1}) = 0
\end{cases} \tag{3.9}
\]

Eq. (3.9) implies that a trader has a unique switching price \( (P_{i,t+1}^s) \) at which his trading direction changes. We can easily observe that the heterogeneity in the switching price is the definitive source of exchanges for stock shares. For instance, if all traders collapse to the same switching price, the no-trade state emerges.

Figure 3.4 shows how the demand or the supply for stock changes with variations in the stock price. The left side of the red vertical line denotes the selling domain, while the right side of this line denotes the buying domain. It clearly demonstrates that the trading direction of a trader is endogenously determined by the prevailing stock price.

3.3.7 Market Price Determination

In this study it is assumed that there is a Walrasian auctioneer who adjusts the stock price in order to clear the market using a tâtonnement process. When the auctioneer announces a stock price, traders make bids or offers in accordance with the announced price. The auctioneer then adjusts the stock price until the stock market clears. However,
Figure 3.4: **Bid/Offer Curve of a Trader.** The depicted curve is the graph of $\Delta S^*_i(P)$ as a function of stock price $P$. Trader $i$ switches his trading position at the point where the curve and the vertical line intersect.

I restrict an increment in the tâtonnement process to be unity, which implies that the market price cannot be infinitely fine-tuned to perfectly clear the market. Therefore, in this setting there is no guarantee that the market clearing stock price exists. If it does not exist, the auctioneer closes the trading period with the unique price that minimizes excess demand or excess supply as follows:

$$P^*_{t+1} := \arg \min \sum_i \Delta S^*_i(P_{t+1})$$  \hspace{1cm} (3.10)

The rationale for this restriction is to achieve a reconciliation between the ideal Walrasian equilibrium world and the real stock market that is frequently characterized by uncleared bids and offers in the order book. If there are uncleared bids or offers, a rationing is executed in a random fashion. For an excess demand (supply) case, the rationing of stock shares is put into effect only for buyers (sellers). Given this rationing scheme, a trader may end up being only partially successful in achieving the desired portfolio rebalancing.
3.4 Experimental Design

3.4.1 Specification of Treatment Parameter Values

The model is quite flexible in terms of allowing investigation of market dynamics under various settings. To demonstrate it as a flexible platform, I consider six simple cases as clear illustrations of the proposed model. One treatment factor varied across these cases is memory length \( h_i \) upon which the dividend moving average is computed: short-memory length (i.e., ten trading periods) versus long-memory length (i.e., one-hundred trading periods). Two additional treatment factors varied across these cases are the weight \( \alpha_i \) on the network factor and the weight \( \beta_i \) on the dividend factor in the determination of the provisional desired stock holdings \( \hat{x}_{i,t+1} \) for each trader \( i \); see (3.3).

Table 3.3 lists the six cases studied in our simulation experiments. Since exploring all possible pairs of values for \( \alpha \) and \( \beta \) is prohibitive, I restrict our analysis to three pure types of traders who consider only one factor in the determination of their provisional desired stock holdings \( (\alpha_i = 0 \text{ or } \beta_i = 0) \) and one hybrid trader type who places equal weight on all three factors in the determination of his provisional desired stock holdings \( (\alpha_i = \beta_i = 1/3) \).

<table>
<thead>
<tr>
<th>Case Number</th>
<th>Type Description</th>
<th>( \alpha_i )</th>
<th>( \beta_i )</th>
<th>( h_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>long-memory dividend trader</td>
<td>0</td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>short-memory dividend trader</td>
<td>0</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>Technical trader</td>
<td>0</td>
<td>0</td>
<td>n.a.</td>
</tr>
<tr>
<td>4</td>
<td>Network trader</td>
<td>1</td>
<td>0</td>
<td>n.a.</td>
</tr>
<tr>
<td>5</td>
<td>long-memory hybrid trader</td>
<td>1/3</td>
<td>1/3</td>
<td>100</td>
</tr>
<tr>
<td>6</td>
<td>short-memory hybrid trader</td>
<td>1/3</td>
<td>1/3</td>
<td>10</td>
</tr>
</tbody>
</table>

Notes: The network and technical trader cases do not involve a data truncation issue. Hence, a varying degree of memory length \( (h_i) \) is not considered for these cases.

3.4.2 Specification for Maintained Parameter Values

In this section, I provide the specific values of exogenous variables for which each case in Table 3.3 is implemented. Table 3.4 lists specific values of exogenous variables.

\[16\] For the definition of \( h_i \), see Section 3.3.4.
The dividend process (3.1) is calibrated to Shiller’s monthly real dividend data\textsuperscript{17}, yielding $d = 0.0014$, $\sigma = 0.0072$. The number of traders ($N$) in this stock market model is an important dimension we have to consider. Given that a very small number of traders would give a low chance of having the diversity in the market, I set the total number of traders to be 100. I find that this is a reasonable number that allows the model to have enough diversity in terms of distributions of wealth, the number of stock shares, and the number of risk-free bonds. At the initialization step, the initial endowment $S_{i0}$ and $B_{i0}$ of stocks and bonds for each trader $i$ are drawn from uniform distributions, i.e., $S_{i0} \sim \text{Uniform}(S_L, S_U)$, $B_{i0} \sim \text{Uniform}(B_L, B_U)$.

For simplicity, the return on the risk-free bond ($r_f$) is exogenously given as 0 %. I assume there is no upper bound for the total supply of risk-free bonds. Also, I assume the weights ($\alpha_i$, $\beta_i$) on factors are all equal across hybrid traders. A tolerance level ($Tol_i$) is randomly drawn for each trader from a uniform distribution bounded between $Tol_L$ and $Tol_U$. I set memory length to be 10 trading periods for the short-memory case and 100 trading periods for the long-memory case.\textsuperscript{18} For the network factor, I assume, for simplicity, that a trader weighs equally the portfolio profiles of his neighborhood traders, which means each neighborhood trader’s stock holding gets a weight of $1/N_i$. The ranges of tested values for the three trader attributes ($\alpha_i, \beta_i$) selected as treatments factors are given in Table 3.3.

3.5 Simulation Results

In this section, I present the simulated stock market dynamics in which only a single type of traders exists.\textsuperscript{19} The stock market dynamics along with the statistical properties of stock returns will be presented for each case, and comparisons between the cases will be made. Even though the stock market with the single type of traders seems to be unrealistic, these experiments would provide a general picture of how the stock market evolves, and would serve as benchmarks on which extensions could be developed for future studies.

\textsuperscript{17}http://www.econ.yale.edu/ shiller/data.htm. Data from 1950:1-2012:12 are used.
\textsuperscript{18}A heterogeneous memory length is a very critical aspect in the asset market dynamics. For simplicity, this study does not consider heterogeneous memory length or evolutionary learning algorithms. This topic would deserve a separate future study. For interested readers, refer to LeBaron (2001a), LeBaron (2001b), LeBaron (2012), and Mitra (2005).
\textsuperscript{19}Note that all traders are characterized by the same specification of weights on factors for each case. Refer to Table 3.3.
Table 3.4: Maintained Parameter Values

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of traders (N)</td>
<td>100</td>
</tr>
<tr>
<td>Risk-free rate (r_f)</td>
<td>0</td>
</tr>
<tr>
<td>Aggression parameter for dividend factor (\gamma_f^i)</td>
<td>5</td>
</tr>
<tr>
<td>Aggression parameter for technical factor (\gamma_g^i)</td>
<td>5</td>
</tr>
<tr>
<td>Initial stock price (P_0)</td>
<td>166</td>
</tr>
<tr>
<td>Initial dividend (d_0)</td>
<td>10</td>
</tr>
<tr>
<td>Long-memory length (h_i^{long})</td>
<td>100</td>
</tr>
<tr>
<td>Short-memory length (h_i^{short})</td>
<td>10</td>
</tr>
<tr>
<td>Drift in the dividend process (\bar{d})</td>
<td>0.0014</td>
</tr>
<tr>
<td>Volatility of the dividend process (\sigma)</td>
<td>0.0072</td>
</tr>
<tr>
<td>Lower bound of initial stock number (S_L)</td>
<td>0</td>
</tr>
<tr>
<td>Upper bound of initial stock number (S_U)</td>
<td>100</td>
</tr>
<tr>
<td>Lower bound of initial bond number (B_L)</td>
<td>0</td>
</tr>
<tr>
<td>Upper bound of initial bond number (B_U)</td>
<td>100</td>
</tr>
<tr>
<td>Lower bound of tolerance level (Tol_L)</td>
<td>0</td>
</tr>
<tr>
<td>Upper bound of tolerance level (Tol_U)</td>
<td>0.1</td>
</tr>
<tr>
<td>Neighborhood weights (a_i)</td>
<td>(\frac{1}{N_i}, \frac{1}{N_i}, \ldots, \frac{1}{N_i}, \frac{1}{N_i})</td>
</tr>
</tbody>
</table>

Notes: The neighborhood weights vary by trader since a trader’s number of links to other traders is heterogeneous. A trader \(i\)’s initial numbers of stocks and bonds will be maintained throughout all test cases. For specific values, readers can obtain the file at the author’s website: https://sites.google.com/site/djpyo0425/research.

3.5.1 Case 1 and Case 2: Dividend Trader

For the dividend trader cases, I divide them into two sub-cases depending on memory length: long-memory versus short-memory. The simulated times series for key endogenous variables for the dividend trader cases are shown in Figure 3.5 and Figure 3.6. In those figures, the top panels show the simulated series of stock prices and the middle panels show the simulated stock returns series. Finally, the bottom panels exhibit the series of trading volumes.

In both cases, the stock prices tend to trace out an upward trend in the dividend process, while the stock market with long-memory dividend traders generates more volatile stock price fluctuations. At the first glance, this seems to stand in sharp contrast with the intuition in the sense that it implies more use of the historical data creates higher stock return volatilities.
However, this is a natural consequence of stochastic process of dividend. As specified previously, the dividend process follows a random walk with a positive drift. Given that the dividend moving average based on the long-memory length moves slowly than the short-memory length in response to new realization of the dividend in the current period, it is highly likely that the currently realized dividend differs much greater from the moving average in the long-memory case.

This greater discrepancy creates more rooms for the portfolio rebalancing. In other words, given that dividend follows a non-stationary process, the dividend moving average based on the long-memory scheme is prone to being irrelevant for evaluating the current profitability of the firm. Therefore, comparing the current dividend level to the long-memory moving average solicits more reactions from traders. This might cause more jagged fluctuations in stock prices.

The difference in stock return volatilities can be also verified by the stock return distributions and box plots in Figure 3.7 and Figure 3.8. The simulated moments of stock returns for the dividend trader cases are presented in Table 3.5.

The stock return distributions are characterized by being leptokurtic given that the excessive kurtosis is positive for two dividend trader cases, suggesting the existence of fat-tail in the stock return distributions. The table also shows that the long-memory case exhibits a greater dispersion in the stock return distribution than the short-memory case.

On the other hand, the extreme values of stock returns are more observed in the stock market with short-memory dividend traders. This finding is further verified by observing box plots of simulated stock returns in Figure 3.7 and Figure 3.8. Comparing the simulated moments to those of the dividend process, both the long-memory dividend trader case and short-memory dividend trader case exhibit the excessive volatility and fat tail properties observed in actual stock return data, while the first moments are similar to the drift in the dividend process. These results, indeed, are in line with Shiller (1981)’s empirical observations.

To check the existence of conditional heteroscedasticity in stock return volatility, I carry out ARCH effect tests proposed by Engle (1982). I reject the null hypothesis that there are no ARCH effects in stock return volatilities. Table 3.6 presents LM test statistics for various lags, showing F-statistics of these tests are highly significant.

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20Note that last 3000 observations out 5000 observations are used in plotting histogram of stock returns and box plot throughout all results.
3.5.2 Case 3: Technical Traders

As a next pure type trader, I report the simulation results for runs of the stock market populated only with technical traders. In this run, all traders are heterogeneous in terms of their initial endowments and levels.\textsuperscript{21}

Technical traders anchor on the stock return in rebalancing their portfolios. In terms of the memory length, technical traders have super-short-memory lengths in the sense that the stock returns in other periods do not matter, except for the previous trading period. By construction, technical traders inject positive feedback into stock prices for they demand more stock shares after a large stock price increase, and conversely.

Figure 3.9 shows the simulated series of key variables of the stock market with technical traders. The top panel shows the simulated stock price series, which exhibits a quite stable cyclical pattern with an upward trend. The upward trend seems to reflect the wealth effect generated by the periodic dividend payments.\textsuperscript{22} One notable finding in

\textsuperscript{21}As in the dividend trader case, I observed that the heterogeneity in the tolerance level solely can generate stock exchanges among traders. And it should be noted that initial conditions are identical across different cases except trading styles.

\textsuperscript{22}In a run with the dividend being zero during all periods, I found that the upward trend vanishes.
Investigating the volume of trading clearly shows that cyclical ups and downs in stock prices accompany with the same pattern of trading volume. The trading volume and the stock price fluctuations are highly correlated in this case. As shown in Figure 3.10, the stock market is marked by the frequent dominance of one type of market forces.

At this point, we have to ask what actually triggers the collapse in stock prices, and make exchanges resume after no-trade states. It is intuitively unclear about this cyclical pattern. However, the detailed investigation of simulated data provides us with the clue about this cyclical pattern. The peak in stock prices is always followed by the no-trade states. In principle, the no-trade state leads to the indeterminacy of a stock price in that period. As construction, I assume that traders evaluate, during no-trade states, their

---

23 The no-trade states are marked by discontinuous portion in the figure.
Table 3.5:
Stock Return Statistical Properties: Dividend Traders

<table>
<thead>
<tr>
<th></th>
<th>1st moment</th>
<th>2nd moment</th>
<th>Skewness</th>
<th>Excessive Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long-memory</td>
<td>0.0015</td>
<td>0.0127</td>
<td>0.5885</td>
<td>5.7744</td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.0006)</td>
<td>(0.1863)</td>
<td>(0.779)</td>
</tr>
<tr>
<td>Short-memory</td>
<td>0.0015</td>
<td>0.0099</td>
<td>1.4434</td>
<td>8.5657</td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.0003)</td>
<td>(0.2097)</td>
<td>(1.4881)</td>
</tr>
</tbody>
</table>

Note: Values in parentheses denote standard deviation from 1000 runs under the calibrated dividend process. Excessive kurtosis is defined as the fourth moment of a distribution less the fourth moment of a standard normal distribution.

Table 3.6:
ARCH Effect Tests for Stock Returns: Dividend Traders

<table>
<thead>
<tr>
<th></th>
<th>$l = 1$</th>
<th>$l = 2$</th>
<th>$l = 3$</th>
<th>$l = 4$</th>
<th>$l = 5$</th>
<th>$l = 6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long-memory</td>
<td>606.3</td>
<td>310</td>
<td>214.6</td>
<td>163.1</td>
<td>130.6</td>
<td>108.8</td>
</tr>
<tr>
<td></td>
<td>(2.2e-16)</td>
<td>(2.2e-16)</td>
<td>(2.2e-16)</td>
<td>(2.2e-16)</td>
<td>(2.2e-16)</td>
<td>(2.2e-16)</td>
</tr>
<tr>
<td>Short-memory</td>
<td>79.8</td>
<td>40.91</td>
<td>27.3</td>
<td>20.75</td>
<td>16.72</td>
<td>14.1</td>
</tr>
<tr>
<td></td>
<td>(2.2e-16)</td>
<td>(2.2e-16)</td>
<td>(2.2e-16)</td>
<td>(2.535e-16)</td>
<td>(6.753e-16)</td>
<td></td>
</tr>
</tbody>
</table>

Note: Estimates are F-statistics of LM tests proposed by Engle (1982). Values in parentheses denote $p$-values of F-statistics of LM tests. Notice that $l$ denotes a number of lags in each test.

wealth based the stock price in the period followed by the no-trade period.\(^{24}\) In addition to that, I further assume that technical traders perceive stock returns in the no-trade state periods as zero. This leads to the greater difference between the current portfolio and the desired portfolio in the periods following no-trade periods.\(^{25}\)

### 3.5.3 Case 4: Network Traders

The other intriguing component of the model is that it captures the mimetic behavior of a trader by incorporating the neighborhood portfolio profile into the portfolio choice of a trader. In this section, I report simulation findings from runs of the stock market

\(^{24}\)This pricing rule is arbitrary and the market dynamics will definitely depend on the specific pricing rule in no-trade states. The simple pricing rule adopted in this study actually prevents a complete explosion or a bust in stock prices during relatively short periods in case of the stock market with technical traders

\(^{25}\)The no-trade state poses delicate issues which have been rarely dealt within the earlier computational stock market models. The existing asset pricing models systemically excludes the occurrence of no-trade states since it is assumed that there is always a fixed number of stock shares supplied.
Figure 3.9: **Stock Market Dynamics: Technical Trader (Case 3)**. The horizontal x-axis denotes trading periods. The disconnected portions of plots denote the periods in which no trades occur.

model with network traders linked under SWN.

The top panel of Figure 3.11 shows the simulated stock prices during the first 100 trading periods. After a few of adjustment periods, the stock prices stay constant as no-trade states continue. The simulated trading volume series is presented in the bottom panel of Figure 3.11. It suggests that the difference between a trader’s portfolio weight and his neighborhoods’ portfolio weights quickly disappear by the initial rounds of stock exchanges. This can be verified in Figure 3.12, which shows the time paths of $x_{it}$ for three traders in the run. Trader 1 and Trader 2 are directly linked, while Trader 3 has no direct links with two other traders. Traders 1 and Trader 2 end up with the similar level of portfolio weights after one trading period. On the other hand, Trader 3 remains still below that level. A downward trend in $x_{it}$ for all trader is due to the subsequent
no-trade states and the continuous dividend payments in the form of risk-free bonds.

It is interesting to observe that trades do not resume even after dividend payment. This may happen because, once a pure-network trader conforms his portfolio weight to those of his neighboring traders, he will not engage in further trades unless the dividend payments disturb his wealth in a way that makes $x_{it}$ deviate significantly from the average stock share holdings of his neighborhood traders. Since traders must be outside their tolerance levels before they will change their current stock holdings, the small perturbations caused by dividend payments generally do not result in further trades.

Even though the stock market populated with network traders produces simple results, I expect the role of network traders would not be negligible for the stock market in which network traders interact with other types of traders. Mimicking other traders’ investment behavior might give rise to complex market dynamics.

Figure 3.10: Trading Profile: Technical Trader (Case 3). The horizontal x-axis denotes trading periods.
Case 5 and Case 6: Hybrid Trader Cases

In this section, I report results from the cases in which traders consider all three factors. As in the dividend trader cases, we also have two experiment environments depending on memory length: long-memory length versus short-memory length. The simulated run of the stock market with the long-memory hybrid traders is presented in Figure 3.13, and that of stock market with short-memory hybrid traders is shown in Figure 3.14. In those figures, the top panels show simulated stock prices and the middle panels show simulated stock returns. The bottom panels show the simulated volume of trade. Table 3.7 summarizes the moments of the simulated stock returns. Unlike the dividend trader case, there is not a substantial difference in the second moments between the two cases, while the fat tail property is more pronounced in the long-memory case.

Comparing these hybrid trader cases to dividend trader cases, the stock market with hybrid traders seems to generate a less volatile stock return process, while extreme values are more frequently observed in a long-memory hybrid trader case. The distributions of
simulated stock returns and the box plots in Figure 3.15 and Figure 3.16 suggest that positive extreme values are more frequent than negative extreme values. The hybrid trader cases yield an asymmetric distribution of stock returns with skewness towards positive values.

The simulated moments of stock returns, shown in Table 3.7, conform to the first moment and the second moment of the dividend process, while the third moment and the fourth moment are not consistent with a normal distribution.

As done in dividend trader cases, I also conduct ARCH effect tests for stock returns generated in hybrid trader cases. I reject the null hypothesis that there are no ARCH effects in stock return volatilities. Table 3.8 shows that F-statistics of these tests are highly significant for all lags considered.

3.5.4 Post-Earnings Shock Dynamics

In this section, I investigate how earnings shocks leading to periods when no dividends are paid affects the stock price dynamics. In these experiments, only difference is made on the dividend process: I construct an experimental environment in which the firm goes through a recession, which forces it not to make dividend payments to shareholders.
Figure 3.13: **Stock Market Dynamics: Long-Memory Hybrid Trader (Case 5).** The horizontal x-axis denotes trading periods.

Figure 3.14: **Stock Market Dynamics: Short-Memory Hybrid Trader (Case 6).** The horizontal x-axis denotes trading periods.

for four trading periods. Specifically, negative earnings shocks begin at time $t=100$ and continue until $t=103$, referred to below as the recessionary phase. The dividend process then reverts to its normal path, referred to below as the recovery phase.

Figure 3.17 compares stock price fluctuations between the long-memory dividend trader case and the long-memory hybrid trader case. It clearly shows that, for the long-memory hybrid trader case, the stock price falls to a lesser extent than the long-memory dividend trader case. The interesting finding is that stock prices overshoot to a greater extent during the recovery phase from the recession. This implies the long-memory

<table>
<thead>
<tr>
<th></th>
<th>1st moment</th>
<th>2nd moment</th>
<th>Skewness</th>
<th>Excessive Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long-memory</td>
<td>0.0014</td>
<td>0.0062</td>
<td>4.9082</td>
<td>31.5835</td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.0004)</td>
<td>(0.47)01</td>
<td>(6.3364)</td>
</tr>
<tr>
<td>Short-memory</td>
<td>0.0017</td>
<td>0.0064</td>
<td>4.7670</td>
<td>28.8932</td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.0004)</td>
<td>(0.3735)</td>
<td>(5.3781)</td>
</tr>
</tbody>
</table>

Note: Values in parentheses denote standard deviation from 1000 runs under calibrated dividend process. Excessive kurtosis is defined as the fourth moment of a distribution less the fourth moment of a standard normal distribution.
hybrid trader case generates an asymmetry in stock prices between the recessionary phase and the expansionary phase.

The stock price asymmetry in response to shocks can be explained by interactions between the *washing-out* effect and the positive-feedback effect, both of which are caused by the technical factor and the network factor in $\hat{x}_{it}$. It seems that the washing-out effect dominates the stock price dynamics at the beginning of the recessionary phase, while the positive-feedback effect dominates the stock price dynamics during the recovery phase. The sources of these two effects are the same, but the timings of occurrence differ. At the beginning of the recessionary phase, traders have a strong desire to sell stock shares because their current zero dividend deviates significantly from the dividend moving
Table 3.8: ARCH Effect Tests for Stock Returns: Hybrid Traders

<table>
<thead>
<tr>
<th></th>
<th>$l = 1$</th>
<th>$l = 2$</th>
<th>$l = 3$</th>
<th>$l = 4$</th>
<th>$l = 5$</th>
<th>$l = 6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long-memory</td>
<td>3,991</td>
<td>3,224</td>
<td>2,358</td>
<td>1,785</td>
<td>1,427</td>
<td>1,189</td>
</tr>
<tr>
<td></td>
<td>(2.2e-16)</td>
<td>(2.2e-16)</td>
<td>(2.2e-16)</td>
<td>(2.2e-16)</td>
<td>(2.2e-16)</td>
<td>(2.2e-16)</td>
</tr>
<tr>
<td>Short-memory</td>
<td>2,575</td>
<td>1,943</td>
<td>1,434</td>
<td>1,102</td>
<td>883.3</td>
<td>736.1</td>
</tr>
<tr>
<td></td>
<td>(2.2e-16)</td>
<td>(2.2e-16)</td>
<td>(2.2e-16)</td>
<td>(2.2e-16)</td>
<td>(2.2e-16)</td>
<td>(2.2e-16)</td>
</tr>
</tbody>
</table>

Note: Estimates are F-statistics of LM tests proposed by Engle (1982). Values in parentheses denote $p$-values of F-statistics of LM tests. Notice that $l$ denotes a number of lags in each test.

average. On the contrary, at the recovery phase other factors amplify the urge to buy more shares, which eventually leads to overshooting in stock prices.

The short-memory case is presented in Figure 3.18. As opposed to the long-memory case, the overshooting in stock prices is more pronounced for the short-memory dividend trader case. This implies that the dramatic innovation in earnings in the recovery phase gets more amplified in the short-memory dividend trader case. This is because earnings performance in the recession periods dominates in the moving average based on short-memory compared to those in relatively distant periods.

Figure 3.17: Recovery Phase Dynamics: Long-Memory Cases

Figure 3.18: Recovery Phase Dynamics: Short-Memory Cases
3.5.5 Wealth Dynamics

Figure 3.19: **Average Wealth by Trading Type** ($AvgM_t$). Figures are based on 100 simulation runs. The benchmark denotes the average wealth in which stock shares are equally distributed to all traders and no trades occur. Note that real wealth levels are normalized by a constant number for easier illustrations.

In this section, I consider the real wealth\(^{26}\) dynamics across different trading types. In this experiment, I populate the stock market with multiple trading types. Note that traders are structurally same except trading type and tolerance level parameter ($Tol_i$). Let $AvgM_t^r$ denote the cross-sectional average real wealth of traders of a specific trading type in period $t$ for the $r$th run of the simulation.\(^{27}\) Figure 3.19 shows, depending

\(^{26}\)In this simulation, the real wealth refers to individual trader’s cash balance after each trading period.

\(^{27}\) Note that $AvgM_t^r = \sum_{i=1}^{100} M_{it}^r / 100$ where $M_{it}^r$ is individual trader $i$’s real wealth in period $t$ for the $r$th run.
the trading type, an average real wealth of traders ($AvgM_t$), which is averaged across multiple runs in each period.\footnote{The average real wealth of traders is calculated as $AvgM_t = \sum_{r}^{N_r} AvgM_t^r / N_r$, where $N_r$ is the number of simulation runs.}

According to Figure 3.19, the long memory dividend traders’ average wealth is highest among 6 types of traders, while network traders most underperform. Table 3.9 presents the average wealth growth rate of each trading type. In terms of wealth growth, I don’t observe any discernible differences among different trading types except that technical traders’ wealth growth exhibits the highest volatility.

There is a caveat to making inferences from the results in Figure 3.19: it is inappropriate to conclude that one trading type is superior to other types by simply observing these results. To check the supremacy of one strategy to others, we have to introduce an evolutionary market environment in which the composition of traders is dynamically changing according to some performance measures.

In other words, it is closely related to the question of whether switching from one trading type to another trading type yields a higher wealth growth, while all other traders are also simultaneously contemplating possible moves. In this environment, a stock market is inherently dynamic. Thus, the superiority of one trading type should be investigated in a context that permits a dynamically changing composition of traders.\footnote{In this context, LeBaron (2001a) delivers a counterargument against Friedman’s natural selection hypothesis by reminding us that the population of the market itself is dynamically changing. He raises the question of ‘who is rational’ in ‘what sense’: “In Friedman’s world, rational traders have started off rational world. So the small infusion of irrational traders doesn’t alter the whole picture of market. But if we start the market off in other way such as market dominated by short-memory traders, the story would be totally reversed.”}

My future research will focus on the possibility of emergence of a stable composition of trading types, including the possible dominance of one trading type.

### 3.6 Concluding Remarks

This study develops a simple yet flexible stock market model permitting the comparative study of different types of stock trading behaviors in relation to market performance. Certain types of behaviors have been shown to result in stock return outcomes matching the stylized facts of actual stock markets.

Depending on the choice of information set\footnote{The full information set consists of three distinct elements: dividend as a measure for firm profitability, past stock return movements, and imitation of neighborhood traders, all of which are shown to be important in many empirical studies. See Section 3.3.4 for references for the studies which shows the significance of these factors.} from which traders anchor for portfolio
Table 3.9: 
Growth Rate of Average Wealth

<table>
<thead>
<tr>
<th>Traders</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long-memory Dividend Trader</td>
<td>0.15 %</td>
<td>0.07 %</td>
</tr>
<tr>
<td>Short-memory Dividend Trader</td>
<td>0.14 %</td>
<td>0.07 %</td>
</tr>
<tr>
<td>Technical Trader</td>
<td>0.13 %</td>
<td>0.12 %</td>
</tr>
<tr>
<td>Network Trader</td>
<td>0.13 %</td>
<td>0.09 %</td>
</tr>
<tr>
<td>Long-memory Hybrid Trader</td>
<td>0.14 %</td>
<td>0.07 %</td>
</tr>
<tr>
<td>Short-memory Hybrid Trader</td>
<td>0.13 %</td>
<td>0.07 %</td>
</tr>
</tbody>
</table>

Note: These estimates are based on 100 simulation runs. In each run, the growth rate of average wealth is based on \( \text{Avg} M^r_t \), and the initial 2000 observations are discarded to eliminate the effects of initial conditions; i.e., \( \text{Mean} = \frac{(\sum_{r=1}^{N_r} \sum_{t=2001}^{T} \Delta \text{Avg} M^r_t)}{(N_r + 3000)} \) where \( N_r = 100 \) and \( T = 5000 \).

rebalancing, traders are modeled into three different trading types: dividend trader, technical trader, and network trader.\(^{31}\) Furthermore, the endogenous trading decision of buying and selling, coupled with a stock rationing scheme in case of the nonexistence of a market clearing price, is another distinguishing feature of the model.

This study shows that stock market performance metrics are quite sensitive to the trading types of traders and memory length. Specifically, if all traders only consider the firm dividend, and the dividend process is non-stationary, then long-memory traders make the stock return process more volatile than short-memory traders. If all traders only consider a past stock return, stock prices exhibit a cyclical pattern. On the other hand, if all traders simply mimic the choices of their neighborhood traders, the stock market converges to a no-trade state after short periods of stock exchanges. Finally, the fat-tailed property in the distribution of stock returns is more pronounced when traders place equal weight on the three factors than when traders place weight only on firm profitability.

The model is subject to several limitations. First of all, the feature of no learning capabilities is unrealistic in the sense that real-world agents make constant adaptations to the ever-changing environment. For example, when the market consists only of pure technical traders, who only consider a past stock return movement, there is a possibility to exploit the resulting clear pattern in stock prices. One possible way to overcome this limitation is to introduce learning algorithms for the formation of weights on the three factors.

\(^{31}\)See Section 3.3.4 for the exact definitions of trading types.
factors. Specifically, instead of assuming fixed weights on the factors determining trading behavior, traders could learn how to set these weight by some performance measures. This type of extension opens the door to capturing both heterogeneity and the adaptive behavior of traders.

The other limitation of the model is the fact that, unlike traditional risk-based asset pricing models, the model does not take into account the risk preferences of traders. To introduce a volatility measure as one of factors would be an effective way of overcoming this limitation. Additionally, a different measure of firm profitability, such as dividend yield, could be used, provided the stylized fact that dividend yield predicts future asset returns for several asset classes (Cochrane (1993)).

Various interesting extensions of this work can be undertaken. First, this stock market model can be appropriately embedded into a macroeconomic model in which the dividend of a firm is endogenously determined by the firm’s innovation endeavor. Second, the stock market model can be generalized to permit different types of traders to compete for survival. It would be interesting to see whether particular types of trading eventually dominate the market in this evolutionary setting. Third, allowing traders to choose their own trade networks endogenously is another intriguing application. This extension would make it possible to explore how the stock market dynamics and the network properties are inter-related and coevolve. Finally, the study on stock market performance from competition between rational *fundamental* traders and behavioral traders described in the model would be an additional contributing factor to the EMH debate.
CHAPTER 4. GENERAL CONCLUSIONS

This dissertation develops agent-based computational models so as to quantify the impacts of animal-spirit shocks on fundamental values and the impacts of heuristic trading strategies of traders on stock market dynamics.

In the history of economic thoughts, the importance of psychological aspects in a macroeconomic context is first introduced by Keynes (1936) who argues:

There is the instability due to the characteristic of human nature that a large proportion of our positive activities depend on spontaneous optimism rather than on mathematical expectation.

Interestingly this old but radical proposition is revived by Akerlof and Shiller (2010) who embrace Keynes’s idea in a broader context:

We must pay attention to the thought patterns that animate peoples ideas and feelings, their animal-spirits. We will never understand economic events unless we confront the fact that their causes are largely mental in nature.

Following the tradition that places animal spirits as central sources for economic dynamics of modern economies, the second chapter of this dissertation analyzes the impact of animal-spirit shocks in the context of interaction between financial sectors and real sectors. The key findings suggest that a temporary animal-spirit shock can have persistence impacts on fundamental values of an economy. It also shows that the magnitude of the impacts of the animal-spirit shock crucially depends on the location of the shock and the degree of a behavioral aspect of decision making-conformity. Additional interesting result is that there is a nonlinear effect in the degree of conformity; the magnitude of impacts of the shock is not monotonic in the degree of conformity.

The key findings in the second chapter yield an important implication; in a highly interconnected society in which the flow of physical goods and information are mediated by direct interactions among various entities, multiple idiosyncratic shocks may not be
canceled out each other so that ultimate impacts of multiple shocks would be larger than what we might expect from a representative agent framework. This suggests that the correct configuration of a trading network in which agents are interacting is an essential element in assessing final impacts of any external shocks. Provided that the second chapter does not consider the presence of multiple animal-spirit shocks, future research should include additional simulations in which multiple shocks are imposed on a relatively large network of traders.

Another notable feature of the model in the second chapter of this dissertation is that it infuses the model not only with animal-spirit traders but also with traders whose decisions are based on an optimizing principle within the framework of constructive rationality. I expect that this modeling strategy bridges the gap between mainstream economic models and agent-based computational models, which ultimately moves forward to the reconciliation of two different modeling approaches.

It should be acknowledged that this dissertation is not the final completion of a long journey for the key topics covered in this dissertation; it rather sets out a new computational framework and, at the same time, raises an important question in the sense that it provides computational test beds for the quantification of the impacts of animal-spirit shocks. Thus empirical verification of the theoretical results derived in the second chapter is highly desirable for empirical validations of the theoretical model. For example, a rigorous econometric modeling on empirical causal relationship between the measure of animal spirits and fundamental values is expected to strengthen the significance of this dissertation.

The third chapter develops a flexible agent-based computational stock market model, which provides a benchmark computational platform for the second chapter. In the model, a traders buying or selling decision of stock is endogenously determined by three key factors; firm profitability, past stock price movement and other traders’ investment behaviors. Key simulation findings show that the higher volatility of stock returns is related with a memory length with which a trader processes information. Furthermore, they also suggest that no-trading state is closely associated with the system-wide extreme conformity in investment on risky assets.

The promising future research direction that might be of our interests will involve the endogenous formation of a social network. In line with recognition of the fact that people are constantly moving and their decisions on with who to interact are, in part, outcomes of deliberate contemplations, permitting an evolving network featured with
learning capabilities of agents will definitely enrich the models in a significant way. I expect that the new methodology and the framework that I propose in this dissertation will pave the way for this new adventure.
APPENDIX A. RATIONAL EXPECTATION MODEL OF ANIMAL SPIRITS

In this section I develop a simple static model to investigate—in a rational expectation setting—the impact of sentiments in an agent’s expectation on the fundamental of a project. The outcome of a project is endogenously determined by the joint actions of all agents in the economy. As will see later, there is a unique rational expectation equilibrium under the presence of systemic optimism or pessimism.

There is a unit mass of investors. An investor $i$ is endowed with a unit of cash that can be used for investment in the project. An investor $i$’s action space is $S_i = \{\text{Invest, NotInvest}\}$. Let $x_i(l)$ denote an investor $i$’s belief on a payoff from the project. I assume that $x_i(l)$ is sum of the fundamental value of the project ($\theta(l)$) and an idiosyncratic noise. This noise captures a non-fundamental belief, which I interpret as animal spirits, on the profitability of the project. The belief on the payoff structure is given as

$$x_i(l) = \begin{cases} 
\theta(l) + \epsilon_i, & \text{if Invest} \\
0, & \text{if NotInvest}
\end{cases} \tag{A.1}$$

where $l \in L = [0, 1]$ is a proportion of agents who invest in the project and $\epsilon_i \sim N(0, \sigma^2)$. Note that $\epsilon_i$ corresponds to an animal-spirit component in an investor $i$’s belief; a positive (negative) $\epsilon_i$ implies optimism (pessimism). I assume, for simplicity, the project technology is linear in a participation rate ($l$) such that $\theta(l) = \alpha l$ where $\alpha \in (1, 2)$. This assumption implies that as more investors participate in the project, the Invest action yields a higher payoff.

Given the expectation on the participation rate ($l^*$) in the project, the belief on net payoff from Invest is given as $x_i(l^*) - 1$. The net payoff from NotInvest action is simply 0. Therefore, the best response of an investor is given as:

$$s_i(l^*) = \begin{cases} 
\text{Invest,} & \text{if } x_i(l^*) > 1 \\
\text{NotInvest,} & \text{if } x_i(l^*) \leq 1
\end{cases} \tag{A.2}$$
The remaining task is to specify the equilibrium expectation on the participation rate \( \overline{l}^* \). Let \( \overline{l} \) denote an investor \( i \)'s expectation on the participation rate. From the perspective of an individual investor, knowing the strategy of others in Eq. (A.2), \( \overline{l} \) is given

\[
\overline{l} = \text{Prob}(x_j(l^*) > 1) = 1 - \text{Prob}(x_j(l^*) \leq 1)
\]

(A.3)

Because of the normality assumption on \( \epsilon_i \), Eq.(A.3) is simplified to

\[
\overline{l} = 1 - \Phi((1 - \theta(l^*)) / \sigma)
\]

(A.4)

where \( \Phi \) is the cdf of a standard normal distribution. Then we can easily prove that there exits a unique rational expectation equilibria:

**Proposition 1.** Assuming a symmetric equilibria \( \overline{l} = l^* \), the equilibrium expectation on the participation rate \( l^*(\sigma, \alpha) \) is the fixed point of a mapping \( f(l) = 1 - \Phi((1 - \theta(l)) / \sigma) \) and it is unique.

**Proof:** Given \( \theta'(l) > 0 \), \( f(l) \) is monotonically increasing in \( l \in L = [0, 1] \). And notice that \( f(l) \) is a contraction mapping on \( L \). By the Banach Fixed Point Theorem, there is a unique fixed point \( l^*(\sigma, \alpha) \in L \) such that \( l^* = f(l^*) \)

For the subsequent analysis I drop a subscript \( i \) since we’re considering a symmetric equilibria. Although we have a unique equilibria when agents have some degree of optimism or pessimism, in the following limiting case, multiple equilibria arise as expected.

**Lemma 1.** For a infinitely small standard deviation in animal spirits (ie., \( \sigma \rightarrow 0 \)), a multiple equilibria arises; either \( \lim_{\sigma \rightarrow 0} l^*(\sigma, \alpha) = 0 \) or \( \lim_{\sigma \rightarrow 0} l^*(\sigma, \alpha) = 1 \).

The ex-post participation rate \( \overline{l} \) is given as

\[
\overline{l} = \text{Prob}(x(l^*(\sigma, \alpha)) > 1) \\
= 1 - \text{Prob}(x(l^*(\sigma, \alpha)) \leq 1) \\
= 1 - \text{Prob}(\theta(l^*(\sigma, \alpha)) + \epsilon \leq 1) \\
= 1 - \Phi((1 - \alpha l^*(\sigma, \alpha)) / \sigma)
\]

(A.5)

By Eq. (A.5) and Lemma 1, it follows that

**Lemma 2.** For a infinitely small standard deviation in animal spirits, the expectation of the participation rate becomes self-fulfilled; \( \overline{l}(\lim_{\sigma \rightarrow 0} l^*(\sigma, \alpha) = 0) = 0 \) and \( \overline{l}(\lim_{\sigma \rightarrow 0} l^*(\sigma, \alpha) = 1) = 1 \).
The ex-post fundamental is given as

$$\theta(\bar{l}) = \alpha \bar{l}$$  \hspace{1cm} (A.6)

Then it follows that

**Proposition 2.** If $1 - \alpha l^*(\alpha, \sigma) > 0$, then the increasing standard deviation of animal spirits is fundamental-enhancing, which thus implying welfare-enhancing.

**Proof:** Note that $\partial \bar{l}/\partial \sigma = \alpha \Phi'(\cdot) (\partial l^*(\sigma, \alpha)/\partial \sigma) + \Phi'(\cdot) (1 - \alpha l^*(\sigma, \alpha))/\sigma^2 = \Phi'(\cdot) (\alpha (\partial l^*(\sigma, \alpha))/\partial \sigma) + (1 - \alpha l^*(\sigma, \alpha))/\sigma^2$ where $\Phi'(\cdot) = \Phi'\left(\frac{1 - \alpha l^*(\sigma, \alpha)}{\sigma}\right)$. Given that $l^*(\sigma, \alpha)$ is a solution to (A.4), by implicit function theorem $\partial l^*(\sigma, \alpha)/\partial \sigma = \frac{\Phi'(\cdot)}{1 - \alpha l^*(\sigma, \alpha)} \frac{1 - \alpha l^*(\sigma, \alpha)}{\sigma^2}$ where $\Phi'(\cdot) = \Phi'\left(\frac{1 - \alpha l^*(\sigma, \alpha)}{\sigma}\right) > 0$. Notice that $1 - \alpha \Phi'(\cdot) > 0$ if $1 - \alpha l^*(\alpha, \sigma) > 0$. Therefore, if $1 - \alpha l^*(\alpha, \sigma) > 0$, then $\partial \bar{l}/\partial \sigma > 0$, which subsequently implies $\partial \theta(\bar{l})/\partial \sigma > 0$. 

Figure B.1: Model Architecture. This figure illustrates key classes in the dynamic model in the UML (Unified Modeling Language) representation.
APPENDIX C. ROLLING HORIZON PROCEDURE: OPTIMIZING TRADER

The rolling horizon procedure (RHP) in stochastic optimization problems circumvents the curse of dimensionality arising from traditional infinite-horizon dynamic programming by reformulating an original problem into a new one with a relatively short horizon \((h)\). For expositional simplicity, I drop a subscript \(i\), which is an index for an agent, in following discussions.

The basic idea of the RHP is that in the current period \(t\) a trader can choose his vector-valued action \(a_t = \{\kappa_{t,6}, x_{t,6}\}\), after which he simulates the future states up to the pre-specified horizon. The trader repeats this process by optimizing over the interval from \(t+1\) to \(t+h+1\) with the continuously updated belief structure for future uncertainties.

The critical step in the RHP involves an approximate calculation of the expected value of stream of future utilities. Although there is no guarantee that each agent has a correct belief system regarding to true data generating processes for future unknown variables, he is able to calculate his own approximate value of the expected utilities by simulating many possible scenarios (i.e., Monte Carlo sampling).

The Monte Carlo sampling strategy in the RHP is equivalent to the basic philosophy of modern stress testing techniques adopted by financial market practitioners and policy makers.\(^1\) From the perspective of an individual trader in the model, random variables, of which trader should might consider important, includes future dividends, future stock prices, future wages and a peer group’s investment pattern. In its principle, trader can obtain sample paths of these random variables up to the horizon as many as possible once he specifies his subjective probabilistic distributions of them. The subjective probabilistic distribution and its evolution are indeed the core components of belief structures of traders. See Appendix D to see how a trader’s belief system is updated through time.

\(^1\)This “What-If” thinking indeed constitutes a core of many stress tests. Variations in different stress testing schemes are due to specific modeling assumptions on the dependence structure of future uncertainties and endogenous responses of agents within a system.
Let’s suppose that current time period is $t$. Let $\omega$ denote an index for a sample realization of all random variables $(P_s, d_s, w_s, \bar{x}_s)$ for $s$ subject to $t+1 \leq s \leq t+h$. Traders generate a finite sample $\Omega$ of possible future outcomes by Monte Carlo sampling. Then it follows that $\Omega = \{(P_s(\omega), d_s(\omega), w_s(\omega), \bar{x}_s(\omega))^{t+1}_{s=t+1} : \omega = 1, 2, \cdots, \bar{\omega}\}$. Consequently, traders create additional decision variables dependent on each scenario and each time, $(\kappa_{s,6}(\omega), x_{s,6}(\omega))^{t+1}_{s=t+1}, \forall \omega \in \Omega$.

The approximated intertemporal utility ($\tilde{U}$) in this reduced optimization problem is defined as:

$$\tilde{U}(x_{t,6}, \kappa_{t,6}, \{x_{s,6}(\omega)\}^{t+1}_{s=t+1}, \{\kappa_{s,6}(\omega)\}^{t+1}_{s=t+1}; \Omega, h, \lambda, \eta, \beta)$$

$$= u(\kappa_{t,6}, x_{t,6}) + \sum_{\omega \in \Omega} p(\omega) \sum_{s=t+1}^{t+h} \beta^{s-t} u(\kappa_{s,6}(\omega), x_{s,6}(\omega))$$

(C.1)

where $p(\omega)$ denotes a probability that a sample path $\omega$ happens and $u(\cdot)$ is a generic utility function. If we draw a reasonable number of Monte Carlo sample paths, we can set $p(\omega) = |\Omega|$.

Note that having Monte Carlo samples of random variables transforms the initial stochastic problem into a new multivariate deterministic optimization problem as follows:

$$\text{maximize}_{\kappa_{t,6}, x_{t,6}, (\kappa_{s,6}(\omega), x_{s,6}(\omega))^{t+1}_{s=t+1}} \tilde{U}(x_{t,6}, \kappa_{t,6}, \{x_{s,6}(\omega)\}^{t+1}_{s=t+1}, \{\kappa_{s,6}(\omega)\}^{t+1}_{s=t+1}; \Omega, h, \lambda, \eta, \beta)$$

s.t.

$$0 \leq x_{t,6} \leq 1$$

$$0 < \kappa_{t,6} \leq 1$$

$$0 \leq x_{s,6}(\omega) \leq 1, \ t < s \leq t+h, \forall \omega \in \Omega$$

$$0 < \kappa_{s,6}(\omega) \leq 1, \ t < s \leq t+h, \forall \omega \in \Omega$$

(C.2)

where $p(\omega)$ denotes a probability that a sample path $\omega$ happens and $u(\cdot)$ is a generic utility function. If we draw a reasonable number of Monte Carlo sample paths, we can set $p(\omega) = |\Omega|$.

Note that constructing a single Monte Carlo scenario for future dividends, stock prices and wages is a multi-staged process. Rather than constructing a Monte Carlo sample of variables directly, we first draw the rates of change of random variables, then we impute level-variables in accordance with the current states and the scenario drawn.
In the RHP (Rolling Horizon Procedure), modeling a trader’s belief regarding to unknown future variables is a key component in establishing an approximate value of expectation.\textsuperscript{1} The belief structure of a trader on the vector of random variables (i.e., stock return \((r_1)\), dividend growth \((r_2)\) and wage growth \((r_3)\)) is assumed to be represented by a conditionally joint normal distribution. The belief, which is represented by \((\mu_\beta, \sigma_\beta)\), on the average portfolio weight of neighboring traders is represented by a beta distribution, \(Beta(\mu_\beta, \sigma_\beta)\).

Let \(\xi_t = (\mu_t, \Sigma_t)\) denote the trader’s belief regarding to the expected values and covariances of a vector \(r = \{r_1, r_2, r_3\}\). As widely applied in financial market practitioners, the belief structure will be updated based on the historical data of variables concerned. Specifically, traders update their beliefs under VCV (Variance-Covariance) scheme, which is developed by RiskMetrics\textsuperscript{TM}.\textsuperscript{2} Under the VCV scheme, traders update a mean vector and a covariance matrix by taking a geometric average of past realizations of random variables.\textsuperscript{3}

Let \(\sigma_{ij}^t\) denote the estimate of covariance between a random variable \(i\) and a random variable \(j\) for \(i, j = 1, 2, 3\) in a period \(t\). Then the covariance matrix is updated in the following recursive way.

\[
\sigma_{ij}^{t+1} = \psi \sigma_{ij}^t + (1 - \psi)(r_{it} - \mu_{it})(r_{jt} - \mu_{jt})
\]  

(D.1)

where \(\psi \in [0, 1]\) is a decay factor and \(r_{it}\) is a realized value of random variable \(i\) in a period \(t\). If decay factor is equal to 0, this scheme is equivalent to a simple adaptive expectation.

\textsuperscript{1}For expositional simplicity, we drop a subscript \(i\) in the subsequent discussions.

\textsuperscript{2}This procedure constitutes a core elements of risk managements of many financial institutions in calculating VaR.

\textsuperscript{3}Longerstaey (1996) shows that there are no significant differences between the geometric average approach and GARCH-type modeling on stock return volatilities.
In a similar fashion, the mean vector is updated as follows:

$$\mu_{t+1} = \psi \mu_t + (1 - \psi) r_t$$

where $r_t$ is a vector of realized values of a stock return, a dividend growth and a wage growth in a period $t$.

The belief structure on the average portfolio choice of neighborhood $(\mu_\beta, \sigma_\beta)$ is also updated in a similar fashion:

$$\sigma_{\beta,t+1} = \psi \sigma_{\beta,t} + (1 - \psi) (\bar{x}_{it} - \mu_{\beta,t})^2$$

$$\mu_{\beta,t+1} = \psi \mu_{\beta,t} + (1 - \psi) \bar{x}_{it}$$

where $\bar{x}_{it}$ is the realized average portfolio choice of a trader $i$’s neighboring group.
APPENDIX E. ROLLING HORIZON PROCEDURE: FIRM

The rolling horizon procedure for the firm is quite similar to that of traders except the composition of random variables that might be of interests to the firm in computing the approximate value of the expectation. From the perspective of the firm, future stock prices and future TFPs are key uncertainties involved in the firm’s decision making procedure.

Let $\omega$ denote an index for a sample realization of future stock prices $\{P_s\}_{s=t+1}^{t+hf}$ for $s$ subject to $t < s \leq t + h$. The firm generates a finite sample $\Omega$ of possible future outcomes by Monte Carlo sampling. Then it follows that $\Omega = \{(P_s(\omega))_{s=t+1}^{t+hf} : \omega = 1, 2, \cdots, \bar{\omega}\}$. The firm creates additional decision variables dependent on each scenario and time, $(\rho_{s,6}(\omega), Z_{s,6}(\omega))_{s=t+1}^{t+hf}$, $\forall \omega \in \Omega$.

Given the sample, the firm’s approximated intertemporal profit is defined as:

$$
\Pi(\rho_{t,6}, Z_{t,6}, \{\rho_{s,6}(\omega)\}_{s=t+1}^{t+hf}, \{Z_{s,6}(\omega)\}_{s=t+1}^{t+hf}; \Omega, \theta, \beta_f, \tau, \delta, \alpha, \pi_{t,3}) = \pi_{t,3} + \sum_{\omega \in \Omega} p(\omega) \left\{ \sum_{s=t+1}^{t+hf} \beta_s^{s-t} \pi_{s,3}(\omega) \right\}
$$

(E.1)

where $p(\omega)$ denotes the probability that a scenario $\omega$ takes place. For a reasonable number of Monte Carlo sample paths, we set $p(\omega) = |\Omega|$. Note that having Monte Carlo samples of random variables transforms a stochastic problem into a deterministic optimization problem as follows:

$$
\begin{align*}
\text{maximize} & \quad \Pi(\rho_{t,6}, Z_{t,6}, \{\rho_{s,6}(\omega)\}_{s=t+1}^{t+hf}, \{Z_{s,6}(\omega)\}_{s=t+1}^{t+hf}; \Omega, \theta, \beta_f, \tau, \delta, \alpha, \pi_{t,3}) \\
\text{s.t.} & \quad (2.15) - (2.24), t \leq s \leq t + hf, \forall \omega \in \Omega \\
& \quad 0 \leq Z_{t,6} \leq \bar{Z} \\
& \quad 0 \leq \rho_{t,6} < 1 \\
& \quad 0 \leq Z_{s,6}(\omega) \leq \bar{Z}, t < s \leq t + hf, \forall \omega \in \Omega \\
& \quad 0 \leq \rho_{s,6}(\omega) < 1, t < s \leq t + hf, \forall \omega \in \Omega \\
\end{align*}
$$

(E.2)
APPENDIX F. BELIEF UPDATE PROCESS: FIRM

The belief update procedure of the firm is identical to that of a trader. The belief of the firm on the stock return \( r \) is also assumed to be represented by a conditionally normal distribution.

Let \( \xi_t = (\mu_t, \sigma^2_t) \) denote the firm’s time \( t \) belief regarding to the stock return. The firm updates its belief by taking a geometric average of the historical realizations of stock returns. Let \( \sigma^2_t \) denote the estimate of the volatility of its stock prices. Then the firm updates the estimate of the volatility in the following recursive way.

\[
\sigma^2_{t+1} = \psi \sigma^2_t + (1 - \psi)(r_t - \mu_t)^2 \tag{F.1}
\]

where \( \psi \in [0, 1] \) is a decay factor and \( r_t \) is a realized stock return in a period \( t \).

In a similar fashion, the firm updates the mean of conditional normal distribution of stock returns as follows:

\[
\mu_{t+1} = \psi \mu_t + (1 - \psi)r_t \tag{F.2}
\]

where \( r_t \) is a realized values of a stock return in a period \( t \).
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