Effects of grate cooler dynamics on cement process simulation

Herbert Allen Johnson
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EFFECTS OF GRATE COOLER DYNAMICS ON CEMENT PROCESS SIMULATION

by

Herbert Allen Johnson

A Dissertation Submitted to the Graduate Faculty in Partial Fulfillment of The Requirements for the Degree of DOCTOR OF PHILOSOPHY

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1969
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\( h_4 \) heat transfer coefficient to gas from solids in kiln, BTU/ft\(^2\)-min-R°

\( h_5 \) heat transfer coefficient to wall from gas in kiln, BTU/ft\(^2\)-min-R°

\( h_7 \) heat transfer coefficient to solids from gas in cooler, BTU/ft\(^2\)-min-R°

\( h_8 \) heat transfer coefficient to surroundings from outside wall, BTU/ft\(^2\)-min-R°

\( H_{R \text{j}} \) heat of reaction based on component \( j \), BTU/lb \( j \)

\( K \) thermal conductivity of wall, BTU/ft-min-R°

\( L \) length, ft

\( M_i \) molecular weight of component \( i \), lb/lb-mole

\( m_i \) mass fraction of component \( i \) in the solid phase, lb \( i \)/lb clinkerable mass

\( n_i \) mass fraction of component \( i \) in the gas phase, lb \( i \)/lb \( N_2 \)

\( P \) pressure, lb/ft\(^2\)

\( R \) gas constant, lb-ft\(^3\)/ft\(^2\)-lb mole-R°

\( R_i \) rate of production by reaction of component \( i \) in solid phase, lb \( i \) produced/lb clinkerable mass-min
R_i\text{ rate of production by reaction of component i in gas phase, lb i produced/lb N}_2\text{-min}
t\text{ time, min}
Ta\text{ temperature of surrounding air, R}^\circ
Tg\text{ temperature of gas, R}^\circ
ti\text{ temperature of incoming gas in cooler, R}^\circ
tr\text{ reference temperature, R}^\circ
ts\text{ temperature of solids, R}^\circ
twg\text{ temperature of the wall in contact with the gas, R}^\circ
tws\text{ temperature of the wall in contact with the solids, R}^\circ
tu\text{ overall heat transfer coefficient, input air-solids in cooler, BTU/ft}^2\text{-min-R}^\circ
tv_i\text{ convection coefficient in heat transfer coefficient equation, BTU/ft}^2\text{-min-R}^\circ
vg\text{ velocity of gas, ft/min}
vi\text{ velocity of air through bed in cooler, ft/min}
v_s\text{ velocity of clinkerable mass, ft/min}
w\text{ width of cooler, ft}
x\text{ distance measured from solids feed end, ft}
z\text{ wall thickness, ft}
\beta\text{ flow rate of clinkerable solids, lb clinkerable solids/min}
\beta_t\text{ total solids flow rate, lb/min}
\epsilon_i\text{ emissivity}
\rho\text{ density of incoming gas in cooler, lb/ft}^3
\( \Psi \) flow rate of nitrogen through the kiln, \( \text{lb N}_2/\text{min} \)

\( \Psi_T \) total gas flow rate, \( \text{lb/min} \)

\( \sigma \) Stefan-Boltzmann constant, \( \text{BTU/ft}^2\text{-min-R}^4 \)

\( \Sigma_g \) sum of all components \( i \) in gas phase

\( \Sigma_s \) sum of all components \( i \) in solids phase
INTRODUCTION

Cement, a gray powder used as the binding agent in concrete, is manufactured from a mixture containing argillaceous materials such as clay and calcareous materials such as limestone. These materials are crushed and ground to a fine powder, correctly proportioned, and thoroughly intermixed to form a homogeneous mixture. This grinding and mixing operation may be done either with dry materials (the dry process) or with wet materials (the wet process). The mixture of raw materials is then fed into a rotary kiln where it is heated by a counter-current stream of hot gases.

The rotary kiln is a long, steel, cylindrical shell which is lined with a special refractory brick to withstand the severe effects of abrasion and high temperatures. The kiln, shown in Figure 1, is inclined so that materials fed in the upper end travel to the lower end where the fuel, pulverized coal or natural gas, is mixed with the incoming air and burned. In the upper section of the kiln chains are often attached to the walls so that they hang down in the gas stream and assist in the transfer of heat from the kiln gases to the raw materials.

The rotary kiln can be divided into several processing sections. The first thirty to forty-five percent of the kiln length acts as a drying and heating zone where water is
Figure 1. Rotary kiln and grate cooler
evaporated, and the temperature of the burden (the solid material passing through the kiln) is increased. The next thirty-five to forty-five percent of the kiln length serves as the calcining zone where the decomposition reactions of the carbonates occur. The final twenty to twenty-five percent of the kiln length is a clinkering and cooling zone. In this region the clinkering reactions occur due to the presence of the burning fuel in the gas stream. The resultant clinker is then cooled by the incoming fuel-air mixture.

The cement clinker leaving the kiln consists of hard granular particles ranging from 1/8 inch to several inches in diameter. This clinker is transported through a cooler (Figure 1) on a moving grate while a controlled stream of air passes through the bed of clinker. The heated air from the cooler is then used for burning the fuel in the kiln.

The cool clinker falls from the cooler grate onto a conveyor and is transported directly to the grinding mills. A small quantity of gypsum is added during grinding to control the setting of the cement. The finished product is then packaged for shipment.

Production of cement has grown steadily in the United States since the first plant was built in 1871. During this growing period, the demand for finished cement generally exceeded the available supply, the competition was insig-
significant, and the industry operated profitably. However in recent years the output of the cement industry has been much larger than the demand, and competition from other materials has increased. Also, customers are demanding that the finished cement have certain specified properties.

These problems have caused the cement industry to follow the example set by other industries who have been in similar situations. New manufacturing facilities have been built to improve production efficiency, and research investigations on the fundamental aspects of the process have been conducted.

The requirement of improved production efficiency was the basis for initiating this research project. Although the cement-making process is very old, the complexities of the system have limited the understanding of its operational characteristics. Since these characteristics are difficult to study when a production facility is in operation, an alternate method of investigation must be proposed.

Several research investigations have been conducted on the simulation of the rotary kiln through use of a set of mathematical equations. The results of this work were helpful in the understanding of the cement-making operation, but were limited because the complete process, namely the rotary kiln and the grate cooler, was not studied. This neglect of the grate cooler was a serious limitation of the simulation research that has been conducted on the process. The cooler
is known (by the operating personnel) to have a definite effect on the operation of the rotary kiln, but until now, no analysis of this effect has been presented.

The analysis of this problem was begun by derivation of a set of mathematical equations to describe the operation of the grate cooler. After the parameters of the system were defined, the response characteristics of this mathematical model were calculated. The rotary kiln was also simulated by a set of mathematical equations. After the system parameters and then the response characteristics of this model were determined, the two models were joined so that the effect of the grate cooler on the operation of the rotary kiln could be studied.
REVIEW OF PREVIOUS WORK

The historical development of the cement industry together with discussion of cement chemistry has been reviewed by Bogue (1) and Lea (26). These two books are excellent sources of general information on the cement process.

Process simulation

Grate cooler No published works on the simulation of the grate cooler were found. However, the importance of the grate cooler operation on the operation of the cement kiln has been noted (22).

Rotary kiln The operation of the rotary kiln has been investigated by developing equations that describe the two fundamental transfer processes, heat and mass transfer, that occur during production.

W. Gilbert (4-16) in a twelve-part series of articles entitled "Heat Transmission in Rotary Kilns" brought together the available scientific knowledge on heat and mass transfer and applied this knowledge to the manufacture of cement. A method to calculate the heat requirements in different sections of the kiln was developed and used to analyze three cement kilns. This method consisted of a summation of all heat requirements in a small section of the kiln. To obtain the overall heat requirements the heat effects from all the
small sections of the kiln were added together.

The most significant part of this work was done on radiation heat transfer. This work showed that radiation heat transfer in the kiln accounted for about eighty-eight percent of the heat transfer while convection and conduction accounted for the other twelve percent. An additional result was that "an increase in the kiln diameter adds to the percentage of heat transmitted by gas radiation and reduces the percentage transmitted by conduction".

The analysis of the temperature fluctuation in the kiln wall and the flow of material through the kiln was very inadequate. Gilbert did not explain how the graphs of the temperature fluctuations or the fluctuation depth into the wall were obtained. The pseudo-experimental analysis used to analyze the flow of material in the kiln was not realistic because the experimental apparatus did not even closely simulate the actual process.

Organization of this series of articles followed the sequence of development and did not convey the important basic ideas clearly. The discussion of too many simple calculation methods was included and this made the work tedious to follow. Also, one kiln heat balance using the final method of analysis would have been sufficient. Using three kilns to show how the method evolved did not add any significant value to the work.
Another series of articles on the heat balance in a rotary kiln was presented by W. T. Howe (18-21). His method of analysis is similar to the one used by Gilbert although Howe did not consider all the complex details covered by Gilbert. There are no significant research results in these articles.

Imber and Paschkis (23) have developed a theoretical analysis to describe heat transfer effects in a rotary kiln. The length of a rotary kiln required for certain processes was found to be dependent on the physical characteristics assumed for the burden. Unavailability of data for determination of heat transfer coefficients was noted, and collection of this data by companies operating kilns was recommended.

Two articles on the mathematical simulation of a rotary cement kiln were written by the same group of four persons. In the first article (27) the steady state simulation for a rotary kiln was discussed, and in the second (28) the unsteady state simulation was presented. For both cases the reactions in the kiln were considered to be evaporation of the water, decomposition of calcium carbonate, and formation of dicalcium silicate and tricalcium silicate.

A reaction mechanism where the order for all reactant concentrations was equal to one was assumed. They also assumed that no solids (dust) were transported in the gas
stream, that convective heat transfer, specific heat, and heat capacity coefficients were constant, that the velocity of the clinkerable material was constant throughout the length of the kiln, and that there was no heat accumulation in the walls of the kiln.

Radiation, convection, and conduction heat transfer coefficients were lumped together into one heat transfer coefficient. The numerical values for most of the constants required for solutions of the sets of equations were given in tabular form.

The steady state equations were solved on an analog computer. In both papers the authors presented a graph of the steady state temperature and composition profiles. These results were similar to the profiles in an actual kiln.

The unsteady state equations were solved on a digital computer but no discussion of the numerical methods was given. The variation over a time period of the gas temperature, the solids temperature, the exit temperatures of the solids and gases, the location of the calcining and clinkering zones, the drying and calcining rates, and the percentage of free lime were presented in graphical form. These graphs show the result of a twenty percent decrease in kiln rotational speed.

When the speed was decreased, the gas and solids temperatures throughout the kiln rose as the residence time of
the material in the kiln increased. The exit temperature of the gas rose quickly before decreasing to its steady state value while the solids temperature decreased and then began a slow rise to its steady state value. The decreases in rotational speed also caused the clinkering and calcining zones to move forward (toward the feed end) and the percentage of free lime to decrease.

The mathematical equations that were used for the simulation of the rotary kiln were derived from mass and energy balance over a differential increment of kiln length. However the equation derived for the gas phase energy balance is not correct. In the kiln there is a transfer of material from the solid phase to the gas phase due to chemical reactions. In these two articles on kiln simulation the energy required to raise this material to the gas phase temperature was considered to be related directly to the differential change in the solids mass flow rate over the differential increment. For the gas phase energy balance to be correct this energy change should also include a term that is directly proportional to the change in the differential solids mass flow rate over a differential increment of time.

A communication by R. E. Stillman on the simulation of

\[ \text{Stillman, R. San Jose, California. Cement kiln simulation using oxide chemistry. Private communication.} \]
a rotary cement kiln was an extension of the other two articles. The same assumptions and methods of derivation of the equations were used. There was, however, an extension of the reaction mechanism to include the decomposition of magnesium carbonate and the formation of tricalcium aluminate and tetracalcium aluminoferrite. The significant contributions of this work were the development and comparison of three methods of numerical solution of the mathematical model, a discussion on estimation of the parameters, and many graphical results.

The error in the gas phase energy balance that was discussed previously was not corrected in this article. An additional error involving the velocity terms for the gas phase and for the solid phase was introduced; however the author assumed that the velocities were not a function of time. The resulting equations were correct, but the derivation of these same equations is possible without adding this additional constriction.

Of the three methods of numerical solution of the kiln equations discussed, the first two methods were quite similar. In the first case a first order forward difference equation was used to approximate both of the derivative terms in the partial differential equations, while in the second case a similar difference equation was used in a set of ordinary differential equations.
The second set of equations, the lumped parameter model, was derived by considering the mass and energy balances over small finite sections of the kiln. The first set of equations, the distributed parameter model, was derived by considering the mass and energy balances over differential sections of the kiln. Thus the lumped parameter system is analogous to the distributed parameter system if the small finite sections are allowed to become differential sections, and if first order forward difference numerical equations are used to approximate all the derivatives.

The third method of numerical solution was based on the method of characteristics. The method of characteristics was used to determine the characteristic curves and characteristic equations of the distributed parameter system. The characteristic equations were then solved by a first order, forward difference, numerical scheme.

The calculation times for the three numerical methods were determined. The method of characteristics was about thirteen times faster than the other methods. This method was then used to determine all the response curves.

A significant portion of the work was devoted to discussion of parameter estimation. Reaction rate constants, heats of reaction, heat capacity, and heat transfer coefficients were estimated before the mathematical simulation of the rotary kiln was undertaken. These terms were all obtained
from other articles, and then some of them were changed so that the calculated results of the model would fit the experimental data.

Results of the steady state simulation include a simplified optimization method. For this calculation a performance index and profit index were defined, and then the maximum value of the profit index was determined for variations in clinker flow rate and clinker to fuel ratio. Also the effect of changes in the control parameters was presented. A base value was chosen at the optimum level, and then changes in the physical characteristics of the kiln with respect to changes from the base value were graphically displayed.

The significant conclusions that Stillman made using these results were than the coating thickness in the kiln has an optimum value, the kiln speed should be changed to maintain a constant solids area when the clinker flow rate is decreased but should be held constant when the clinker flow rate is increased, the kiln operates best when the greatest amount of heat from the cooler is recovered, and there is a specific feed composition for optimum operation.

The dynamic response of the gas temperature and solids temperature with respect to changes in kiln rotational speed, burden flow rate, fuel rate, and secondary air temperature were also presented. From these results Stillman concluded that
the gas and solids temperatures in the heating zone rather than the calcining zone would be better for sensing kiln dynamics because the temperature changes were greater, and that for the best operation the kiln speed and burden flow rate should generally be changed together to maintain the same holdup in the kiln.

Transfer processes

The development of a mathematical model to describe the cement process is dependent upon the description of many simultaneously occurring processes. Two of these processes are heat transfer in the kiln walls and mass flow through the kiln.

Heat transfer Weislehner (37, 38) has discussed the transfer of heat through the kiln wall. By assuming that the kiln radius was very large compared to the wall thickness the standard heat conduction equation was derived to describe the temperature profile in the kiln wall. This equation was solved by considering the wall to be a semi-infinite solid. An exponential solution together with a periodic boundary condition was used to determine the final result.

This analytical solution was used to demonstrate that there was a temperature fluctuation in the kiln wall during rotation, and that this effect penetrated the kiln wall to a
depth equal to about ten percent of the wall thickness. Further analysis of the solution showed that heat was transferred to the wall in the gas region and from the wall in the solids region.

Wachters and Kramers (36) have studied the heat transfer from the inside wall of a rotary kiln to the solids. All solid particles that are initially near the kiln wall were assumed to stay there. Using this assumption as a basis, the temperature profile equation for this small layer of material was derived. This result was used to show how the heat transfer coefficient could be determined.

The theoretical results were compared with some experimental observations. From these comparisons Wachters and Kramers concluded that one of the equations could be used for predicting the heat transfer coefficient between the material and the kiln wall.

**Mass transfer** Sullivan et al. (34) have experimentally studied mass flow through the kiln. Small kilns five and seven feet long and up to 19-3/4 inches in diameter, were used to study the effect of kiln slope, rate of feed, speed of rotation, length of kiln, diameter of kiln, angle of repose of the material, and size of the material on the time of passage of the material through the kiln. The effect of constrictions in different sections of the kiln was
also investigated. Sand was used to simulate the cement clinker.

The results of this investigation were presented graphically and were correlated with several empirical equations.

The significant deficiencies in this research work were that no chemical reaction occurred in the material moving through the kiln, and no dust effects were present.

Rutle (32) investigated material transport in a wet-process rotary kiln. Radioactive sodium, Na\(^{24}\), was used to indicate the path of the material in three full-sized kilns.

Rutle found that the material that was fed to the kiln at the same time had a different flow rate in the various sections of the kiln. This flow rate would increase gradually as the material moved through the chain section and on down the kiln. The flow rate reached a maximum value in the calcining section and then decreased to reach a minimum in the burning section. The flow rate then increased in the final section of the kiln. The flow rate of the material varied considerably from one kiln to another, but these variations could not be attributed to differences in kiln speed or diameter.

The changes in flow rate were assumed to be due to differences in nodule (material) grading. The finer the nodules were, the faster the flow rate would be. Rutle
surmised that part of the high flow rate in the calcining section could be attributed to the fluidizing effect of the carbon dioxide released in this zone.

The present design methods for rotating kilns were criticized and recommendations that the design be based upon the type of material fed to the kiln were made. Since different materials are assumed to produce nodules of different sizes, and since this nodule size is what determines the kiln performance, kiln designers were encouraged to consider this effect.

The research reported by Rutle is significant because it provides an understanding of mass flow in a rotary kiln. This was the first set of experimental data taken directly from an operating kiln. This research work would have been outstanding if the conditions under which the data were taken had been completely tabulated and if more situations had been investigated.

A theoretical description of the flow of material in a rotary kiln has been presented by Saeman (33). The experimental investigation of Sullivan, et al. (34) was analyzed, and then a diagram of a possible path of a kiln particle was constructed. Various geometric relationships were used to describe this path. Simplification of these relationships was made, and an equation similar to the one developed by Sullivan for the time of passage of material through the kiln
was developed.

Saeman assumed, like Sullivan, that a particle in the bed of material in a kiln remains stationary with respect to the kiln until it reaches the surface of the bed and then cascades down along the bed surface. However many particles in the bed tend to circulate around at one point in the kiln and do not cascade as often or as fast as Saeman assumed. Additional assumptions were that the flow rate of material was constant and that the physical characteristics of the particles did not change during transition through the bed.

Two other papers on the theoretical description of material transport in rotating cylinders have been written. In the first article (35) the authors, Vahl and Kingma, discuss material transport in horizontal cylinders. A method of derivation similar to the one used by Saeman (33) was used but because different mathematical simplifications were made, a different equation resulted. This equation and the laboratory data correlated very well.

In the second paper (25) Kramers and Croockewit used an analysis similar to that of Vahl and Kingma. An equation to predict material transport in inclined rotary kilns was developed which also differed from Saeman's equation because of the mathematical simplification that were made. Experimental data were presented show the applicability of the equation.
Longitudinal mixing of granular material flowing through a rotating cylinder has been studied by Rutgers. In his first paper (30) the particle movements, operating conditions, and longitudinal mixing in rotating cylinders were discussed, and articles pertinent to each subject were reviewed. The applicability of the general diffusion model to the continuous flow of granular solids was considered.

Rutgers, in his second article (31), discussed his experimental work. To study dispersion in a laboratory-scale, rotating cylinder long grain rice was used. Experimental variations in the control conditions of the rotating cylinders, cylinder design parameters, and properties of the solids were studied. From these results Rutgers determined longitudinal dispersion coefficients and concluded that a plug flow model with longitudinal diffusivity superimposed could be used to describe material transport in a rotating cylinder. The numerical value of this longitudinal diffusivity was found to be much lower than the values found in other types of fluid flow processes and reactors.
MATHEMATICAL MODEL DEVELOPMENT

For any process the development of a mathematical model in a microscopic or macroscopic sense can be initiated in one of two ways. The investigator can use his intuitive insights and academic understanding of a process to conceive a system of equations for the process under investigation. This method is dependent upon personal insight into the problem and the ability to recognize all significant factors in the process being studied.

The method can be quick, but as the problem to be analyzed becomes more complex, the possibility that some important facet will be overlooked increases. Hence this method, as a general guide for the mathematical analysis of a problem, is not recommended.

The alternative method is the application of a general set of equations to modelling problem through the use of appropriate limits and boundary values postulated for the system. This method, applicable both for the microscopic and macroscopic situations, requires a thorough understanding of the problem but does not permit overlooking important parts of the process because some simplifying assumption must be made for each part of the equation that is neglected.

The mathematical models that are discussed herein have been formulated by this latter method. In the simulation of the kiln model significant changes from models in previously
published works have resulted through the use of this method, while for the grate cooler the accuracy of the model was assured.

Mass and energy balances over a differential section of the process form the basis for the model of the moving grate cooler and for the model of the rotary cement kiln. The momentum balance for each process was neglected because its effect was assumed to be small when compared to the mass and energy changes.

Grate cooler simulation

The mathematical model of the grate cooler, the primary part of this work, was formulated on the basis of the following assumptions:

1) The cross-sectional area occupied by the gas is constant.
2) No solids are present in the gas stream.
3) The flow of gas through the grate is uniformly distributed, and the clinker depth on the grate does not alter this distribution.
4) The temperature of the gas under the grate is a constant, uniform value.
5) For the gas above the grate a temperature gradient exists only in the axial direction.
6) A particular point in the gas above the grate where the gas flow is zero can be characterized, and the "no flow point", shown in Figure 2, can be calculated by considering the ratio of the induced draft flow rate to the forced draft flow rate.

The most significant assumption in the list is the last one. The physical meaning of this assumption is that at some point in the gas phase above the grate, all the gas on the one side will eventually flow out as secondary air and all the gas on the other side will leave the system through the stack. This assumption also provides a basis for initiating the solution to the mass and energy balances for the gas phase.

The applicability of assumption four will be discussed later during the analysis of the grate cooler results. Assumption one is based on the relative importance of the change of clinker depth on the grate compared to the overall gas phase area. Assumptions two and five were made because the additional complexity of including these effects would yield only a small increase in accuracy.

The importance of the clinker bed depth with regard to the flow of air through the grate has been recognized by operating personnel. They were aware that increased depth on the grate causes the air flow to be restricted and finally limits the flow at a particular value. Assumption three was used because data to describe this phenomenon was not
Figure 2. Grate cooler differential increments
STACK GAS

INDUCED DRAFT

PRIMARY AIR & FUEL

KILN DRAFT

FORCED DRAFT

COOLER

CLINKER

FEED

GAS FLOW

NO FLOW POINT

GAS

SOLIDS

GAS

HEAT

MASS
available.

The mathematical model for the grate cooler was developed by considering a differential section of the cooler as shown in Figure 2. From this model and the model for the kiln to be given later the temperature profiles of each process could be determined. The temperature profiles are the most important characteristics of the process.

The mass balances for the solids phase and the gas phase over a differential element of the cooler are,

\[ \frac{\partial \rho}{\partial x} + \frac{1}{c_p} \left( \frac{\partial \rho v_s}{\partial t} \right) = 0 \]  

(1)

\[ \frac{\partial \psi_T}{\partial x} + w v_i \rho_i = \frac{1}{c_p} \left( \frac{\partial \psi_T}{\partial t} \right) \]  

(2)

The energy balance for this system was also written over a differential element by simplifying the general energy balance and using a reference temperature. The equations for the solids phase and the gas phase were

\[ Ts \]

\[ - \frac{3}{3x} \left[ \beta \int_{T_s}^{T_g} C_p dT \right] - u w (T_s - T_i) - h_1 C_1 (T_s - T ws) \]  

(3)

\[ + h_I w (T_g - T_s) = \frac{3}{3t} \left[ \frac{\partial \psi_T}{\partial x} \right] \int_{T_x}^{T_s} C_p dT \]
Equations 3 and 4 were further modified by including the mass balances, and recognizing the similar functional relationship of all the gas heat capacity terms. From this modification the final equations used in the calculation were,

$$\frac{\partial T_g}{\partial x} + \frac{1}{V_g} \frac{\partial T_g}{\partial t} = \frac{1}{\beta C_P} \left\{ \frac{V_g}{T_g} \int T_g \, d\tau \right\}$$

$$+ U_w(T_s - T_i) + h_2 C_2(T_w - T_g) + h_7 w(T_s - T_g)$$

($5$)

$$- \frac{\partial T_s}{\partial x} + \frac{1}{V_s} \frac{\partial T_s}{\partial t} = \frac{1}{\beta C_P} \left\{ \frac{V_s}{T_s} \int T_s \, d\tau \right\}$$

$$+ U_w(T_s - T_i) + h_2 C_2(T_w - T_g) + h_7 w(T_s - T_g)$$

($6$)

**Rotary cement kiln simulation**

Although the kiln equations are more complex than those for the grate cooler, formulation of these equations was no more difficult. The following assumptions were used:

1) The cross-sectional area occupied by the gas is constant.
2) A particular set of reactions and rate equations characterize the chemical changes that occur in the solids.

3) No solids are present in the gas stream.

4) No radial or circular concentration and temperature gradients are present in the solid phase or in the gas phase.

5) The rate of burning of the fuel can be described by a position dependent rate equation.

The first assumption, as in the case of the cooler, was made because a change in solids area would be small when compared to the total area of the gas. The assumptions concerning reaction rate in both the solids and gas are required because insufficient experimental evidence is available to explain the processes in more detail. Although the amount of solids in the gas stream is large, experimental data on this phenomenon was not available. Hence assumption three was required. Assumption four was made because only the axial gradients are important to this analysis.

From these assumptions and the general mass and energy equations the solids mass and energy balance equations were derived for the differential section of the cement kiln shown in Figure 3. These equations were based, for the solids phase, on the total quantity of solid material, called
Figure 3. Rotary kiln differential increment
the clinkerable mass, that passes through the kiln and, for the gas phase, on the total flow of inert nitrogen gas. The components of the solids phase were all assumed to have the same velocity.

The solids mass balance for component $i$ was,

$$\frac{\partial}{\partial x} (\beta m_i) + R_i \frac{\beta}{V_s} = \frac{\partial}{\partial t} \left( \frac{\beta}{V_s} m_i \right)$$  \hspace{1cm} (7)

Since the clinkerable mass is the total mass that passes through the kiln, the mass balance used for this material was

$$- \frac{\partial \beta}{\partial x} = \frac{\partial}{\partial t} \left[ \frac{\beta}{V_s} \right]$$  \hspace{1cm} (8)

These two equations can be expanded and combined to produce an equation to determine the mass fraction of each component in the solid phase,

$$\frac{\partial m_i}{\partial x} + \frac{1}{V_s} \frac{\partial m_i}{\partial t} = \frac{R_i}{V_s} \hspace{1cm} i = 4, 5, ..., 16$$  \hspace{1cm} (9)

The total solids flow rate can be calculated from the relationship,

$$\beta_T = \beta (1 + m_4)$$  \hspace{1cm} (10)

For the gas the corresponding equations are,

$$\frac{\partial}{\partial x} (\psi n_i) + \frac{\psi}{V_g} R_i' + \frac{\beta}{V_s} R_i = \frac{\partial}{\partial t} \left( \frac{\psi n_i}{V_g} \right)$$  \hspace{1cm} (11)

$$\frac{\partial \psi}{\partial x} = \frac{\partial}{\partial t} \left( \frac{\psi}{V_g} \right)$$  \hspace{1cm} (12)
Expansion of terms and combination of these two equations yield the mass fraction equation for each component in the gas phase,

$$- \frac{\partial n_i}{\partial x} + \frac{1}{v_g} \frac{\partial n_i}{\partial t} = \frac{R_i'}{v_g} + \frac{\beta}{v_s} \psi R_i \quad i = 1, 2, \ldots, 5 \quad (13)$$

The sign on the term containing $R_i$ should be negative when $i$ is equal to four because $R_4$ was defined as a rate of depletion instead of rate of production. The equation for determination of the overall gas flow rate was,

$$\psi_T = \psi (1 + \sum_{i=1}^{5} n_i) \quad (14)$$

Energy balance equations for the kiln were derived, like those results of the cooler, from a general enthalpy balance based on a reference temperature. These equations for the solids phase and gas phase were respectively,

$$\frac{\partial}{\partial x} \left[ \beta \sum_{i=4}^{16} \left( m_i \int_{T_x}^{T_s} C_p dT \right) \right] - \beta \frac{\psi}{v_s} \left( \psi \sum_{j=4}^{5} \left( \sum_{i=1}^{5} \left( R_i \int_{T_x}^{T_s} C_p dT \right) \right) \right)$$

$$+ h_3 C_3 (T_{ws} - T_s) + h_4 C_4 (T_g - T_s) - \beta \frac{\psi}{v_s} \left[ \sum \Delta H_j \sum_{j=1}^{5} R_j \right]$$

$$= \frac{\partial}{\partial t} \left[ \frac{\beta}{v_s} \sum_{i=4}^{16} \left( m_i \int_{T_x}^{T_s} C_p dT \right) \right] \quad (15)$$
Again, by substituting the mass balance equations into Equations 15 and 16, the final solids energy equation was,

\[
\frac{3}{3x}\bigg[\sum_{i=1}^{5} n_{i} \int_{T_{g}}^{T_{s}} C_{p,i} dT \bigg] + \frac{3}{v_{s}} \bigg\{ \sum_{i=4}^{5} \int_{T_{g}}^{T_{s}} C_{p,i} dT \bigg\} 
- h_{3}c_{5}(T_{g}-T_{w}) - h_{4}c_{4}(T_{g}-T_{s}) - \frac{3}{v_{g}} \sum_{j} \Delta H_{R_{j}} R_{j}'
= \frac{3}{3t}\bigg[\frac{3}{v_{g}} \sum_{i=1}^{5} n_{i} \int_{T_{g}}^{T_{s}} C_{p,i} dT \bigg]\] (16)

and the final gas energy equation was,

\[
\frac{3}{3x} - \frac{1}{v_{g}} \frac{3}{3t} \bigg[\frac{5}{5} \bigg\{ \sum_{i=1}^{5} n_{i} C_{p,i} dT \bigg\} + \frac{h_{5}c_{5}}{v_{g}} (T_{g}-T_{w}) \bigg]\] (18)
PARAMETER ESTIMATION AND USE

In the preceding section the mathematical models for both the cooler and the kiln were derived. These models describe the basic transfer and transport mechanisms which take place. However, additional properties of the system must be determined before the models can actually be used to predict the process behavior. The material in this section covers the methods used for calculating wall temperature, heat transfer coefficients, and heat capacity for the cooler and kiln, and solids velocity and rate expressions for the proposed reactions for the kiln.

Grate cooler

Wall temperature During operation of the cooler the energy transfer through the wall was assumed to be a steady state process. The equations relating the temperature of the wall in contact with the solid and the temperature of the wall in contact with the gas to the ambient temperature were derived from the energy balance for the wall. These equations were,

\[ T_{ws} = \frac{T_{sh} (K + z_h) + K_h T_a}{h_0 K + h_1 (K + z_h)} \]  \hspace{1cm} (19)

\[ T_{wg} = \frac{T_{gh} (K + z_h) + K_h T_a}{h_0 K + h_1 (K + z_h)} \]  \hspace{1cm} (20)
Equations 19 and 20 were analyzed to see what significance each term would have on the final answer. Only the terms $h_1$ and $h_2$ were found to be important. The values of $h_8$, $T_a$, and $K$ have a very small effect on the calculated wall temperature when compared with coefficients $h_1$ and $h_2$.

**Heat transfer coefficients** The three heat transfer processes - solid to wall and grate, gas to wall, and gas to solid - are characterized by coefficients $h_1$, $h_2$, and $h_7$. These coefficients were computed from equations which included a constant term for the convective and conductive effects and a temperature dependent term for the radiative effect. These equations were,

$$h_1 = v_1 + \sigma \varepsilon_1 \left( \frac{T_{ws}^4 - T_s^4}{T_{ws} - T_s} \right)$$  \hspace{1cm} (21)

$$h_2 = v_2 + \sigma \varepsilon_2 \left( \frac{T_{wg}^4 - T_g^4}{T_{wg} - T_g} \right)$$  \hspace{1cm} (22)

$$h_7 = v_7 + \sigma \varepsilon_7 \left( \frac{T_s^4 - T_g^4}{T_s - T_g} \right)$$  \hspace{1cm} (23)

Values for the constant terms and the emissivities were estimated from values of these coefficients for similar, experimentally studied, processes.

**Overall transfer coefficient** The overall heat transfer coefficient, $U$, was used to calculate the heat transfer between the solid on the grate and the gas passing
through the solid, and was obtained from the results of an experimental analysis reported for a similar system (29). The results of that work, a correlation of heat transfer coefficient and bed depth, were incorporated into the model of the grate cooler by use of the following equation,

\[ U_{\text{new}} = \frac{U_{\text{old}} v_{\text{old}}}{v_{\text{new}}} \frac{v_{\text{new}}}{v_{\text{old}}} \]

(24)

By comparing the magnitude of each term in the cooler energy balances, Equations 5 and 6, the influence of \( U \) on the results was found to be at least six times greater than \( h_1, h_2, \) or \( h_7 \). Therefore the determination of \( U \) is critical to the accuracy of the grate cooler model. The effect of \( U \) on this model is shown graphically in Figure 4. These results, computed from the final grate cooler model, show that a 20% (1.44), 40% (1.68), and 60% (1.92) increase in the value of \( U \) over the base value of 1.20 causes a marked change in the temperature profile for the solids phase. The gas phase profile does not change significantly at the clinker discharge end of the cooler but does have an important effect at the opposite end of the cooler. A constant input clinker temperature of 2800\(^\circ\)R and a constant undergrate temperature of 520\(^\circ\)R were used to obtain these results.

The fact that \( U \) is the one significant parameter in the grate cooler model is both an advantage and a disadvantage in using this model. The disadvantage of the importance of a
Figure 4. Grate cooler temperature profiles for variations in the overall heat transfer coefficient

Curve A: $U = 1.20$
Curve B: $U = 1.44$
Curve C: $U = 1.68$
Curve D: $U = 1.92$
single parameter is that its value must be accurate or the results of the model will not provide an accurate simulation. The advantage of this condition is that all the research effort can be concentrated on methods of determining only one parameter.

**Heat capacity** The temperature dependent heat capacity equations used for the grate cooler were,

\[ C_{p\beta} = \lambda s_1 + \lambda s_2 \, T_s + \frac{\lambda s_3}{T_s^2} \]  \hspace{1cm} (25)

\[ C_{p\psi\tau} = \lambda g_1 + \lambda g_2 \, T_g + \lambda g_3 \, T_g^2 \]  \hspace{1cm} (26)

The values for \( \lambda s_1 \), \( \lambda s_2 \), and \( \lambda s_3 \) were estimated by adding a contribution for each component (24) based on its mass fraction in the clinker; while the values for \( \lambda g_1 \), \( \lambda g_2 \), and \( \lambda g_3 \) were obtained from tabulated data (17) for air.

**Equipment capacity and dimensions** Data on the capacity and dimensions of the grate cooler were gathered at a cement-making plant. These data together with the data for the equations that have just been discussed are tabulated in Appendix I.

**Rotary kiln**

**Wall temperature** The wall temperatures for the kiln were calculated from equations similar to Equations 19 and 20.

for the cooler. To derive these equations the curved wall of the kiln was assumed to be a flat slab. Since the kiln was rotating, the wall temperature used in the kiln simulation was calculated from a weighted average. This average was found by multiplying the fraction of wall surface in contact with a particular phase times the temperature of the wall in contact with that phase, and adding these results for the gas and solids phases together.

**Heat transfer coefficients**

The heat transfer relationships used for the kiln are of the same form as those used for the cooler.

\[
H_3 = V_3 + \sigma \varepsilon_3 \left( \frac{T_{WS}^4 - T_S^4}{T_{WS} - T_S} \right) \\
H_4 = V_4 + \sigma \varepsilon_4 \left( \frac{T_g^4 - T_S^4}{T_g + T_S} \right) \\
H_5 = V_5 + \sigma \varepsilon_5 \left( \frac{T_g^4 - T_{Wg}^4}{T_g + T_{Wg}} \right)
\]  

(27)  

(28)  

(29)

Coefficients used in these equations vary with position in the kiln because of the chemical reactions that occur in the different sections. Values for these coefficients were estimated from the values of these coefficients for similar processes that have been experimentally studied.

**Heat capacity**

The heat capacity equations used for the kiln simulation were similar to the grate cooler equations, Equations 25 and 26. The equations for the kiln
Values for the coefficients in these equations were obtained from Kelley (24) for the solids components and from Hougen, Watson, and Ragatz (17) for the gas components. The coefficients for \((\text{CaO})_4(\text{AL}_2\text{O}_3)(\text{Fe}_2\text{O}_3)\) were not available so a contribution for \(\text{CaO}, \text{AL}_2\text{O}_3, \) and \(\text{Fe}_2\text{O}_3\) based on the mass fraction of each component in \((\text{CaO})_4(\text{AL}_2\text{O}_3)(\text{Fe}_2\text{O}_3)\) were used to estimate them.

**Solids velocity** The shape of the solids velocity profile was determined from the results of a radiotracer study of solids distribution reported by Rutle (32) for a wet process kiln. The boundary conditions chosen were,

1. \(\frac{dv_s}{dy} = 0 \quad y = y_1\)
2. \(\frac{dv_s}{dy} = 0 \quad y = y_2\)
3. \(v_s = v_{s1} \quad y = y_1\)
4. \(v_s = v_{s2} \quad y = y_2\)
5. \(v_s = v_0 \quad y = 0\)
6. \(\int_0^1 \frac{L}{v_s} dy = R_T\)
where $y$ is the dimensionless length $x/L$, and $R_T$ is the total residence time, in minutes, of the material in the kiln.

In evaluating the above conditions the value for $v_0$ was calculated using a feed density of 75 lb/ft$^3$, a feed rate of 1400 lb/min, and a 10% loading of the kiln. This value was $1.64841$. Numerical values used for the other conditions were,

$$y_1 = 0.2 \quad v_{s_1} = 1.50$$

$$y_2 = 0.7 \quad v_{s_2} = 3.15$$

$$R_T = 195 \quad L = 400$$

These numbers were determined from Rutle's data (32).

The kiln velocity data were fitted to a curve having the form,

$$v_s = \frac{v_0}{B_0 + B_1 y + B_2 y^2 + B_3 y^3 + B_4 y^4 + B_5 y^5} \quad (32)$$

This equation does not depend on temperature and therefore would not be adequate where wide variations in process conditions, such as startup, are to be simulated.

The boundary conditions and Equation 32 were used to formulate the matrix equation;
This equation was then used to determine $B_1$, $B_2$, $B_3$, $B_4$, and $B_5$.

The solids velocity profile used for the general steady state kiln result of Figure 14 is shown in Figure 5.

**Solids reaction scheme**  
To apply the mass and energy balances to the simulation of the rotary cement kiln the chemical reactions occurring in the process must be known. The reaction scheme for the process is very complex, however, and is not well understood. But according to Lea (26), the following set of reactions are adequate to describe the process.
Figure 5. Velocity profile for steady state rotary kiln

Initial velocity = 1.659249 ft/min
A. \( \text{CH}_4 + 2\text{O}_2 \rightarrow \text{CO}_2 + 2\text{H}_2\text{O} \)

B. \( \text{H}_2\text{O} \) [solids] \( \rightarrow \) \( \text{H}_2\text{O} \) [gas] \( (520-950^\circ\text{R}) \)

C. \( \text{MgCO}_3 \rightarrow \text{MgO} + \text{CO}_2 \) \( (1450-1650^\circ\text{R}) \)

D. \( \text{CaCO}_3 \rightarrow \text{CaO} + \text{CO}_2 \) \( (1580-2400^\circ\text{R}) \)

E. \( 2\text{CaO} + \text{SiO}_2 \rightarrow (\text{CaO})_2 \text{SiO}_2 \) \( (1950-2700^\circ\text{R}) \)

F. \( \text{CaO} + (\text{CaO})_2 \text{SiO}_2 \rightarrow (\text{CaO})_3 \text{SiO}_2 \) \( (2600-3100^\circ\text{R}) \)

G. \( 4\text{CaO} + \text{AL}_2\text{O}_3 + \text{Fe}_2\text{O}_3 \rightarrow (\text{CaO})_4(\text{Al}_2\text{O}_3)(\text{Fe}_2\text{O}_3) \) \( (2350-2700^\circ\text{R}) \)

H. \( 3\text{CaO} + \text{AL}_2\text{O}_3 \rightarrow (\text{CaO})_3 \text{AL}_2\text{O}_3 \) \( (2350-2800^\circ\text{R}) \)

The temperature ranges given with the reactions for the solids phase are the intervals in which each reaction is assumed to occur.

**Component notation**

To simplify the notation for the rate equations the following number code was used to designate the various chemical constituents.

**Gas phase**

1. \( \text{N}_2 \)
2. \( \text{CH}_4 \)
3. \( \text{O}_2 \)
4. \( \text{H}_2\text{O} \)
5. \( \text{CO}_2 \)
Solid phase

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The quantities given in the brackets are the shorthand notations commonly used to designate the given compounds.

Reaction rate equations The reaction rate expressions used for the solids phase were,

\[
R_4 = -k_B m_4 \\ (34)
\]

\[
R_5 = -R_6 \frac{m_5}{M_6} - R_7 \frac{m_5}{M_7} \\ (35)
\]

\[
R_6 = -k_C m_6 \\ (36)
\]

\[
R_7 = -k_D m_7 \\ (37)
\]

\[
R_8 = -R_7 \frac{m_8}{M_7} - R_{13} \frac{2m_8}{M_{13}} - R_{14} \frac{m_8}{M_{15}} - R_{15} \frac{3m_8}{M_{15}} - R_{16} \frac{4m_8}{M_{16}} \\ (38)
\]
These expressions were presented in a previous study. They provide a good representation of the reaction scheme but probably do not reflect the actual chemical mechanism that occurs.

**Kinetic coefficient equation** The values of $k_i$ used in these rate equations were computed from a temperature dependent relationship.

$$k_i = \frac{C_k i}{1 + AK_i e^{-\left(\frac{E}{k_B}T_s\right)}}$$

---

Values of $AK_i$, $CK_i$, and $EK_i$ along with Equation 47 have been proposed to explain the kiln reaction system. For this investigation some of these values were changed so that the reactions would occur only in the temperature ranges listed previously.

**Burning rate** The burning rate for the fuel (natural gas assumed to be all $CH_4$) was calculated from the equation:

$$R' = \frac{(CK^g)(AK^g)(L-x)(v_g)}{1 + (AK^g)^2(L-x)^2}$$

(48)

The other rates for the gas phase were,

$$R'_1 = 0$$

(49)

$$R'_2 = -\frac{R'_1}{M_3}$$

(50)

$$R'_3 = -\frac{R'_1}{M_4}$$

(51)

$$R'_4 = -\frac{R'_1}{M_5}$$

(52)

The burning rate, computed from Equation 48 for the general steady state kiln result of Figure 14, is presented graphically in Figure 6. The form of this burning rate equation was assumed after the experimental results of Fristrom and Westenberg (3) were analyzed. The constants $AK^g$ and $CK^g$ were varied until the gas temperature profile was computed correctly.
Figure 6. Burning rate for steady state rotary kiln
Heat of reaction The heat of reaction data for the chemical reactions occurring in the kiln were obtained from Hougen, Watson and Ragatz (17). These data were based on a reference temperature of 528°C and had a negative magnitude for exothermic reactions.

Equipment capacity and dimensions The capacity and dimensions of the rotary kiln were gathered\(^1\) at a cement-making plant. These data and the data for the equations that have just been discussed are tabulated in Appendix II.

Grate cooler

Analytical method The solids mass and energy balances for the grate cooler, Equations 1 and 5, have the form:

$$\frac{\partial \theta}{\partial x} + \frac{1}{V_s} \frac{\partial \theta}{\partial t} = F$$

(53)

where the vector $\theta(x,t)$ has as components the solids flow rate, $\dot{S}$, and the solids temperature, $T_s$. This form was determined because the velocity of the solids were not time dependent, and so the $\frac{\partial V_s}{\partial t}$ in Equation 1 was zero.

The gas mass and energy balances for the grate cooler, Equations 2, and 6, have the form:

$$-\frac{\partial \phi}{\partial x} + \frac{1}{V_g} \frac{\partial \phi}{\partial t} = G$$

(54)

where the vector $\phi(x,t)$ has as components the gas flow rate, $\dot{G}$, and the gas temperature, $T_g$. To develop this form from Equation 2 the effect of the term $\frac{\partial V_g}{\partial t}$ in that equation was assumed to be negligible.

Equations 53 and 54 were solved by the method of characteristics (2). This method was applied by observing that the left hand side of Equation 53 [Equation 54] represented the derivatives of $\theta[\phi]$ in the direction, $1 : \frac{1}{V_s}$ [ $-1 : \frac{1}{V_g}$]. Now the curves in the x-t plane that have
tangents at each point in the plane have the directions
given above. This family of curves, defined by,
\[ \frac{dt}{dx} = \frac{1}{v_s} \]  (55)
for Equation 53 and,
\[ \frac{dt}{dx} = -\frac{1}{v_g} \]  (56)
for Equation 54 have the property that along then \( \theta[\phi] \)
satisfies the ordinary differential equations;
\[ \frac{d\theta}{dx} = F \]  (57)
\[ \frac{d\phi}{dx} = -G \]  (58)

The families of curves defined by Equations 55 and 56 are
known as characteristic curves.

The set of ordinary differential equations for the
solids phase that was solved along the characteristic
curves, Equation 55, was,
\[ \frac{d\delta}{dx} = 0 \]  (59)
\[ \frac{dT_s}{dx} = \left[ \frac{1}{\beta C_p} \right] [U_w(T_i-T_s) + h_{1c}(T_w-T_s) + h_{1w}(T_g-T_s)] \]  (60)

while the corresponding set of equations for the gas phase
that was solved along the characteristic curves, Equation 56,
was,

\[ \frac{dv_T}{dx} = -w v_i \rho_i \]  \hspace{1cm} (61)

\[ \frac{dT_g}{dx} = \left[ \frac{1}{v_T^2 C_p v_T} \right] [w v_i \rho_i \int_{T_g}^{T_i} C_p v_T dT] \]

\[ + U w (T_s - T_i) + h_2 C_2 (T_w - T_g) \]

\[ + h_w (T_s - T_g) \]  \hspace{1cm} (62)

The gas velocity in the cooler was significantly greater than the velocity of the solid. Therefore the equation for the gas phase characteristic curves, Equation 56, after it was compared to the equation for the solids phase characteristic curves, Equation 55, was changed to the equation,

\[ dt = 0 \]  \hspace{1cm} (63)

**Numerical method** To solve the ordinary differential equations which have the form,

\[ \frac{d\phi}{dx} = G(x, \phi) \]  \hspace{1cm} (64)

a difference scheme based on Euler's equation,

\[ \phi_{n+1} - \phi_n = [x_{n+1} - x_n] G(x_n, \phi_n) \]  \hspace{1cm} (65)

was used.
The difference equations for the solids phase, from Equations 55, 59, and 60, were,

\[ t_{x+1,t+1} - t_{x,t} = (x_{x+1,t+1} - x_{x,t}) \left( \frac{1}{v_{s,x,t}} \right) \]  

(66)

\[ \beta_{x+1,t+1} - \beta_{x,t} = 0 \]  

(67)

\[ T_{s,x+1,t+1} - T_{s,x,t} = (x_{x+1,t+1} - x_{x,t}) \left( \frac{1}{\beta_{x,t}C_{p,x,t} \beta_{x,t}} \right) [U_{x,t}, t^w] \]

(68)

The following equation was used to define \( \Delta t \):

\[ \Delta t = t_{x+1,t+1} - t_{x,t} \]  

(69)

This equation was substituted into Equations 66 and 68 to obtain,

\[ x_{x+1,t+1} = x_{x,t} + (\Delta t) v_{s,x,t} \]  

(70)

\[ T_{s,x+1,t+1} = T_{s,x,t} + (\Delta t) \left[ \frac{x_{x,t}}{\beta_{x,t}C_{p,x,t} \beta_{x,t}} \right] [U_{x,t}, t^w] \]

(71)

\[ (T_{i} - T_{s,x,t}) + h_{c1}(T_{ws,x,t} - T_{s,x,t}) + h_{w}(T_{q,x,t} - T_{s,x,t}) \]

The difference equations for the gas phase, from Equations 63, 61, and 62, were,

\[ t_{x+1,t+1} - t_{x,t} = 0 \]  

(72)
The interval, $\Delta x$, was defined as,

$$\Delta x = x_{x+1,t+1} - x_{x,t+1}$$

(75)

and was then substituted into Equations 73 and 74 to give the following results:

$$\psi_{T_{x+1,t+1}} = \psi_{T_{x,t+1}} - (\Delta x) [wv_i \rho_i]$$

(76)

$$T_{g_{x+1,t+1}} = T_{g_{x,t+1}} - (\Delta x) \left[ \frac{1}{C_{P_{T_{x,t+1}}}} \right] [wv_i \rho_i]$$

(77)
To compute results from the grate cooler difference equations, Equations 67, 70, 71, 76, and 77, the initial points on the characteristic curves had to be determined. The locus of these points was called the initial curve.

The solids characteristic curves have the initial curves:

\[ \theta(0,t) = \theta_1 \]  
\[ \theta(x,0) = \theta_2 \]  

(78a)  
(78b)

where the components of the vectors, \( \theta_1 \) and \( \theta_2 \), represent the flow rate and temperature of the solids. Values for the points along the initial curves, Equation 78a, which represent the conditions of the clinker leaving the kiln, were gathered at a cement plant\(^1\). These values are listed in Table 4. The values for the points along the curves, Equation 78b, were estimated for the first computations. These estimates, the other initial curves, and the grate cooler difference equations were used to calculate a steady state result. For further calculations this result was used to provide the numerical values along this initial curve.

The gas characteristic curves have initial curves that pass through the "no flow points". These curves were,

\[ \phi(N_f, t) = \phi_{NF} \]  \hfill (78c)

where \( N_f \) designates the position of the "no flow point", and where the vector, \( \phi_{NF} \), has components which represent the flow rate and temperature of the gas at the "no flow point" was obviously zero. However, the temperature of the gas at this point was not explicitly defined. Therefore this temperature was calculated by assuming that a small strip of width \( \Delta x \) existed at the "no flow point". The following energy balance was derived for this strip by assuming that the temperature of the gas in the strip was dependent only on the energy brought in by the incoming air and the energy transferred to this gas from the clinker as it passed through the grate:

\[ T_i h w p_i C_p \Delta x + U w (T_s - T_i) \left( \frac{h}{V_i} \right) \Delta x = T_g C_p h w p_i \Delta x \]  \hfill (79a)

Energy that was transferred to the wall or out of the strip was neglected. The resulting equation for the gas temperature at the "no flow point" in the cooler was,

\[ T_g = \frac{T_i C_p \rho_i v_i}{U(T_s - T_i) \rho_i v_i} \]  \hfill (79b)

For the first increment on either side of the "no flow point" the difference equation for the gas temperature, Equation 77, could not be used. Equation 6, which was used
to obtain Equation 77, was derived by substitution of the gas mass balance, Equation 2, into the gas energy balance, Equation 4. In the ensuing mathematical manipulations both sides of the equation were divided by the gas flow rate term, \( \psi_T \). Therefore Equation 6 was valid only at points where the gas flow rate was not zero and so the corresponding difference equation, Equation 77, could not be used for the gas temperature calculations in the first increment.

The gas energy balance, Equation 4, was solved in the same manner as Equation 6. The resulting equation was,

\[
\frac{d}{dx}(\psi_T \int_{\text{Tr}}^{T_g} C_p \psi_T \,dT) + \left[ \int_{\text{Tr}}^{T_i} C_p \,dT + Uw (T_s - T_i) \right] \frac{h_2 C_2 (T_g - Twg) - h_2 \psi (T_g - T_s)}{h_2 C_2 (T_g - Twg) - h_2 \psi (T_g - T_s)} = 0
\]

This equation had the following difference form;

\[
\begin{align*}
\left[ \psi_T \int_{\text{Tr}}^{T_g} C_p \psi_T \,dT \right]_{x+1, t+1} - \left[ \psi_T \int_{\text{Tr}}^{T_g} C_p \psi_T \,dT \right]_{x, t+1} &= -h_2 C_2 (T_g x, t+1 - T_g x, t+1) \\
\left[ w_i \rho_i \int_{\text{Tr}}^{T_g} C_p \,dT + Ux, t+1 w(T_s x, t+1 - T_i) - h_2 C_2 (T_g x, t+1 - Twg x, t+1) \right] &- h_2 \psi (T_g x, t+1 - T_s x, t+1) \\
\end{align*}
\]

The first term on the right in Equation 81 was dropped because \( \psi T_{x, t+1} \) was equal to zero at the "no flow point".
Hence the equation that was used to calculate the gas temperature at the end of the first increment on either side of this point was,

\[ \frac{\Psi}{T} \int_{T_x}^{T_i} C_p \frac{dT}{T} \cdot x+1, t+1 = -(\Delta x) \int_{T_x}^{T_i} C_p \frac{dT}{T} + \]

\[ U_x, t+1(w)T_s x, t+1 - T_i = h_2 C_2(T_g x, t+1 - T_w g x, t+1) \]

\[ - h_7 w(T_g x, t+1 - T_s x, t+1) \]  \hspace{1cm} (82)

Since the gas temperature is required for determination of the heat capacity in Equation 82, an iteration scheme averaged over the gas temperature values at the beginning and end of the increment was used to calculate the gas temperature at the end of the increment.

In addition to the numerical values for the initial curves, values for \( \Delta t \) and \( \Delta x \) were required for solving the grate cooler equations. Decreasing values of these two terms were used to compute solutions of these difference equations. When this decrease did not change these solutions significantly, values of \( \Delta x \) and \( \Delta t \) used in the last calculations were selected for later work. For the grate cooler equations these numbers were 1.0 foot and 0.125 minute.

The computational process was started by using Equation 70 to calculate the position of the solids characteristic curves on the next time line. Then Equations 67 and 71 were
used to calculate values for the solids flow rate and temperature at these points. When these calculations were completed, the solids temperatures at these new points were interpolated linearly to points that were spaced at an interval of $\Delta x$.

The "no flow point" on this new time line was calculated next. By using a ratio of the forced draft air flow rate minus the induced draft air flow rate to the forced draft flow rate, the location of this point on the gas characteristic curve was determined. Equation 79b was used to calculate the value of the temperature at this point, and Equation 82 was used along with Equation 76 to calculate the flow rate and temperature for the first point on both sides of the "no flow point".

Equations 76 and 77 were then used to compute the flow rate and temperature of the gas for all the remaining points on the gas characteristic curve. The gas temperatures calculated for these points were interpolated linearly to the points calculated by Equation 70, and then the computational process was repeated for the next time line.

Regularly spaced intervals were used on the gas characteristic curves rather than the intervals determined by Equation 70 so that the "no flow point" could be found easily and so that the gas temperature at this point could be calculated easily. The average values for the solids temperature were
used in Equation 77 to minimize the effect of the interpolation process.

The computations for the grate cooler difference equations were made on an IBM 360 System Model 65 computer. The solution was calculated about 54 times faster than real time.

Rotary kiln

Analytical method  The method of characteristics was also used to solve the rotary kiln mass and energy balances, Equations 8, 9, 12, 13, 17, and 18. For the solids phase in the kiln the vector \( \theta(x,t) \) had as components the flow rate, mass fractions, and temperature. The vector \( \phi(x,t) \) had similar components for the gas phase.

The equation for the characteristic curves for the solids phase was,

\[ \frac{dt}{dx} = \frac{1}{v_s} \]  \hspace{1cm} (83)

and the equation for the characteristic curves for the gas phase was,

\[ \frac{dt}{dx} = -\frac{1}{v_g} \]  \hspace{1cm} (84)

The set of ordinary differential equations that was solved along the characteristic curves for the solids phase, Equation 83, was,

\[ \frac{d\theta}{dx} = 0 \]  \hspace{1cm} (85)
\[
\frac{\text{dm}_i}{\text{dx}} = \frac{R_i}{v_s}, \quad i = 4, 5, \ldots, 16
\]  

\[
\frac{dT_s}{dx} = \left[\frac{1}{16}\right] \left[-\frac{h_3 C_3}{\beta} (T_{ws} - T_s) + \frac{h_4 C_4}{\beta} (T_g - T_s) \right] 
\begin{aligned}
& \sum_{i=4}^{16} S_i (m_i C_{p_i}) \\
& - \frac{1}{v_s} \left[ \sum_{i=4}^{5} \frac{T_s}{T_r} \right] 
\end{aligned}
\]  

while the set of ordinary differential equations, that was solved along the gas characteristic curves, Equation 84, was,

\[
\frac{d\psi}{dx} = 0
\]  

\[
- \frac{dn_i}{dx} = \frac{R_i'}{v_g} + \frac{\beta R_i}{v_s \psi}, \quad i = 1, 2, \ldots, 5
\]  

\[
\frac{dT_g}{dx} = \left[\frac{1}{5}\right] \left[ \sum_{i=1}^{5} \left( \frac{R_i}{v_g} + \frac{\beta R_i}{v_s \psi} \right) \right] \frac{T_g}{T_r} 
\begin{aligned}
& \sum_{i=1}^{5} q_i (n_i C_{p_i}) \\
& - \frac{\beta}{v_s} \left[ \sum_{i=4}^{5} \frac{T_s}{T_r} \right] 
\end{aligned}
\]  

The velocity of the gas in the kiln was significantly greater than the velocity of the solid. By applying the same
analysis that was used on the grate cooler system, the equation for the gas phase characteristic curves Equation 84, was changed to the equation,
\[ \frac{dt}{d} = 0 \] (91)

**Numerical method**

To solve these ordinary differential equations Euler's difference equation, Equation 65, was applied to the solids phase equations, Equations 83, 85, 86, and 87. The difference equations for this phase were,

\[ \frac{x_{x+1,t+1} - x_{x,t}}{x_{x,t}} = (x_{x+1,t+1} - x_{x,t}) \frac{1}{v_{s,x,t}} \] (92)

\[ \frac{\beta_{x+1,t+1} - \beta_{x,t}}{x_{x,t}} = 0 \] (93)

\[ \frac{m_{x+1,t+1} - m_{x,t}}{x_{x,t}} = (x_{x+1,t+1} - x_{x,t}) \frac{R_i}{v_{s,x,t}} \] (94)

\[ \frac{T_{s,x+1,t+1} - T_{s,x,t}}{x_{x,t}} = (x_{x+1,t+1} - x_{x,t}) \frac{F_{x,t}}{v_{s,x,t}} \] (95)

where the term \( F_{x,t} \) was defined by the equation,

\[ F_{x,t} = \frac{1}{16} \sum_{i=1}^{4} \frac{h_i C_i}{T_{w,s,x,t} - T_{s,x,t}} \]

\[ + \frac{h_4 C_4}{B_{x,t}} (T_{q,x,t} - T_{s,x,t}) - \frac{1}{v_{s,x,t}} \] \( \sum_{i=4}^{5} \) \( R_i \) \( C_{p,i} \) \( T_{s,x,t} \))

\[ + \frac{16}{5} \sum_{i=4}^{5} \frac{T_{s,x,t}}{T_{x,t}} \] \( C_{p,i} \) \( T_{s,x,t} \))

\[ + \sum_{i=4}^{5} R_i \] \( C_{p,i} \) \( T_{s,x,t} \))

\[ + \sum_{j=4}^{5} \sum_{i=4}^{5} \]
Equation 69, the defining equation for \( \Delta t \), was used in Equations 92, 94, and 95 to produce the following equations:

\[
x_{x+1,t+1} = x_{x,t} + (\Delta t) v_{s_{x,t}}
\]  
(97)

\[
m_{i_{x+1,t+1}} = m_{i_{x,t}} + (\Delta t) [R_{i_{x,t}}]
\]  
(98)

\[
T_{s_{x+1,t+1}} = T_{s_{x,t}} + (\Delta t) [v_{s_{x,t}} F_{x,t}]
\]  
(99)

The difference equations for the gas phase, from Equations 91, 88, 89, and 90 were,

\[
t_{x+1,t+1} - t_{x,t+1} = 0
\]  
(100)

\[
\psi_{x+1,t+1} = \psi_{x,t+1} = 0
\]  
(101)

\[
n_{i_{x+1,t+1}} - n_{i_{x,t+1}} = -(x_{x+1,t+1} - x_{x,t+1}) \left[ \frac{R_{i_{x,t+1}}}{v_{s_{x,t+1}}} \right]
\]

\[
+ \frac{\beta_{x,t+1} R_{i_{x,t+1}}}{v_{s_{x,t+1}} \psi_{x,t+1}}
\]  
(102)

\[
T_{g_{x+1,t+1}} - T_{g_{x,t+1}} = (x_{x+1,t+1} - x_{x,t+1}) [G_{x,t+1}]
\]  
(103)

where the term \( G_{x,t+1} \) was defined by the equation,
Equations 93, 97, 98, 99, 101, 102, and 103 and the initial curves for this system were used to compute the rotary kiln results.

The initial curves for the solids characteristic curves were,

\[ \theta(0,t) = \theta_3 \]  \hspace{1cm} (105a)

\[ \theta(x,0) = \theta_4 \]  \hspace{1cm} (105b)

where the components of the vectors \( \theta_3 \) and \( \theta_4 \) were the flow rate, mass fractions, and temperature of the solids. Values for the points along the initial curves, Equation 105a, which represent the conditions of the kiln feed, were
gathered at a cement plant. These values are listed in Table 14. The values for the points along the other curves, Equation 105b, were estimated for the first computation, and then the difference equations and other initial curves were used to calculate a steady state result. For all further computations this result was used to provide the values for the points along these initial curves.

The initial curves for the gas characteristic curves were,

$$\phi(L,t) = \phi_5$$  \hspace{1cm} (105c)

where L is the length of the kiln. The components of the vector $\phi_5$ were the flow rate, mass fractions, and temperature for the primary air, secondary air, and fuel. The values for the points along these curves were also gathered at a cement plant. These values are given in Table 14.

The value of $\Delta t$ for the rotary kiln difference equations was selected by comparing solutions computed with different values of $\Delta t$. The value that produced a stable, convergent, solution was used for all the remaining computations. For the rotary kiln equations this value was 2.0 minutes.

The computational process for the kiln equations was

---

initiated by using Equation 97 to calculate the position of the solids characteristic curves on the next time line. Then Equations 93, 98, and 99 were used to calculate values for the solids flow rate, mass fractions, and temperature at these points.

The calculations along the gas characteristic curve, Equation 100, were made for the flow rate, mass fractions, and temperature of the gas by Equations 101, 102, and 103. These calculations began at the initial points on the gas characteristic curve and were performed for the points that were determined by Equation 97. When values for all the points on this characteristic curve were determined, the sequence of calculations was repeated for the next time line.

The computations for the rotary kiln difference equations were also made on an IBM 360 System Model 65 computer. The solution was computed about 116 times faster than real time.
RESULTS OF PROCESS SIMULATION

Grate cooler

The selection of the overall heat transfer coefficient, U, has been shown in Figure 4 to be critical in the grate cooler simulations. This coefficient was also found to be much more significant to these equations than any of the other heat transfer coefficients.

Undergrate temperature profile  A constant undergrate temperature for the air was assumed when the grate cooler equations were developed. To check the applicability of this assumption two results were calculated. These results are shown in Figure 7. Curve A is the general steady state cooler result using an undergrate temperature of 520°R. Curve C is the steady state result of the same model except that the undergrate temperature was assumed to have a profile starting at 520°R at the air inlet (x = 0) and increasing linearly to 572°R at the end of the cooler (x = L). These curves confirm the assumption that a constant undergrate temperature is adequate to describe the profile. The use of a linear profile did not alter the solutions significantly.

Process variables  The dynamic response of the grate cooler model was studied by introducing changes in both uncontrollable and controllable process variables. There
Figure 7. Temperature profiles for variation in the undergrate temperature for the grate cooler

Curve A: Steady state result using a constant undergrate temperature of 520°F

Curve C: Steady state result using a linear undergrate temperature profile

\[ T_i = 520 + 52(x/L) \]
are three uncontrollable inputs to the grate cooler (the undergrate temperature, the clinker temperature, and the clinker flow rate) and three controllable inputs (the grate speed, the induced draft, and the forced draft).

In Figure 8 the effect of a change in the undergrate temperature is shown. The initial steady state profiles in curve A, were calculated for an undergrate temperature of 520°R, while the two sets of response curves, labeled B and C, were determined for changes in the undergrate of 494°R (5% decrease) and 468°R (10% decrease) respectively. These changes produce rather significant changes in the secondary air temperature, the stack air temperature, and the clinker outlet temperature. However the temperature change from 520°R to 468°R represents a difference of 52°. Such a large temperature change would probably not be expected to occur over a short interval, such as a few hours. Hence the undergrate temperature changes would be most significant over a seasonal period, say summer to fall to winter.

The response of the grate cooler to a change in clinker temperature is given in Figure 9. The effects of variations in clinker temperature are probably the most important changes to study since this temperature is always varying in the manufacturing process. For a change in the clinker temperature from 2800°R (Curve A) to 2660°R (Curve B) or 2520°R (Curve C) the effect on the secondary air temperature,
Figure 8. Response characteristics and steady state temperature profiles for variation in undergrate temperature for the grate cooler

Curve A: 520°R (initial value)
Curve B: 494°R (5% decrease)
Curve C: 468°R (10% decrease)
Figure 9. Response characteristics and steady state temperature profiles for variation in clinker temperature for the grate cooler

Curve A: 2800°R (initial value)
Curve B: 2660°R (5% decrease)
Curve C: 2520°R (10% decrease)
stack air temperature, and clinker outlet temperature is very large. Hence a change in the temperature of the clinker leaving the kiln can have a significant effect on how the cooling process occurs. Unfortunately, as for most characteristics of the cement process, the effect of variations in the cooling process has not been studied experimentally so adequate criteria for cooling have not been established.

Comparison of the secondary air temperature curve with the stack air temperature curve shows the result of the "no flow point" assumption. After that portion of the solid material where the temperature change occurs has passed the "no flow point", the secondary air temperature ceases to change while the stack air temperature just starts changing. The abrupt change in the clinker outlet temperature reflects the residence time requirement for the solid in the cooler.

Variation in the clinker flow rate going to the cooler is caused by changing the flow rate of the feed to the kiln or by changing the rotational speed of the kiln. In Figure 10 the effect of a change in clinker flow rate on the grate cooler model is shown. This change from 1400 lbs/min to 1330 lbs/min (Curve B) and then to 1260 lbs/min (Curve C) causes the grate cooler to respond in a manner similar to that for a change in the clinker temperature. The overall
Figure 10. Response characteristics for variation in the clinker flow rate for the grate cooler

Steady state: 1400 lbs/min (initial value)
Curve B: 1330 lbs/min (5% decrease)
Curve C: 1260 lbs/min (10% decrease)
changes in secondary air temperature and stack air temperature are not greatly different, but the overall change in the clinker outlet temperature is significantly smaller.

The decrease in the flow rate, when compared to the decrease in clinker temperature (Figure 9), does not change the operation of the cooler significantly but can cause a large change in the secondary air temperature. Hence a clinker flow rate change can be considered to produce a more significant variation in the kiln operation (due to the secondary air temperature change) than in the cooler operation (due to the small variation in the outlet clinker temperature).

A rather typical control change for the cement process has been to decrease the rotational speed of the kiln so that the cross-sectional area of solid remains constant when a feed rate variation occurs. Consideration of the response characteristics given in Figures 9 and 10 can provide an indication of how this control method would affect the operation of the cooler. When the rotational speed decreases, for example, the flow rate of clinker to the cooler also decreases. However, the temperature of the clinker increases because of its longer residence time in the kiln. This increase would produce an effect that was opposite to the one shown in Figure 9. When this increase in clinker temperature and a decrease in clinker flow rate (Figure 10)
occur at the same time, the secondary air temperature and stack air temperature responses due to the increased clinker temperature tend to cancel the secondary air temperature and stack air temperature responses due to the decreased clinker flow rate. The clinker outlet temperature is still influenced by the clinker temperature from the kiln since the flow rate of clinker does not affect the clinker outlet temperature significantly. Therefore this cancelling effect of the cooler responses decreases the influence of the cooler on the kiln process, and the use of this type of control is particularly effective.

The next three changes are due to variations in the mechanical operation of the cooler and are controllable. In Figure 11 the response curves for grate speeds of 8.55 ft/min (Curve B) and 8.1 ft/min (Curve C) are given. The curves for secondary air temperature and stack air temperature again show the effect of the "no flow point" assumption.

The most significant change occurs in the clinker outlet temperature. The results displayed in Figure 11 show exactly what effect a change in the residence time (the basic characteristic of the cooler that the grate speed controls) will have on the cooling operation. Although the grate speed can be controlled easily, the response of the grate cooler indicates that changes in this parameter must be judiciously
Figure 11. Response characteristics for variation in the grate speed for the grate cooler

Steady state: 9.0 ft/min (initial value)
Curve B: 8.55 ft/min (5% decrease)
Curve C: 8.1 ft/min (10% decrease)
chosen so that the requirements for cooling of the clinker are not altered radically.

The last two variations studied concerned the air stream that passes through the grate and cools the clinker. In the first variation, shown in Figure 12, the induced draft flow rate was changed from a steady state value of 4370 lbs/min to 4180 lbs/min (Curve B) and 3895 lbs/min (Curve C). These rates were as close to a 5% and 10% decrease as could be selected since the cooler calculations were made only at finite points and since the "no flow point", which was calculated from the induced and forced draft air flow rates, had to be one of these points. These changes produced instantaneous decreases of moderate size in the secondary air temperature and stack air temperature, while the effect of these changes on the clinker outlet temperature was negligible. The temperature profiles for the grate cooler show how the "no flow point" (the dip in the gas profile) moves when the induced draft flow rate is changed and the forced draft flow rate is kept constant at 6650 lbs/min.

In Figure 13 the results of changing both the forced draft and induced draft flow rates are shown. These combination changes cause the secondary air temperature to rise significantly; however, there was virtually no change in the clinker outlet temperature.
Figure 12. Response characteristics and steady state temperature profiles for variation in the induced draft air flow rate from the grate cooler

Curve A: 4370 lbs/min (initial value)
Curve B: 4180 lbs/min
Curve C: 3895 lbs/min
Figure 13. Response characteristics and steady state temperature profiles for variation in the induced draft air flow rate and forced draft air flow rate for the grate cooler

<table>
<thead>
<tr>
<th></th>
<th>Forced draft</th>
<th>Induced draft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curve A:</td>
<td>6650 lbs/min (initial value)</td>
<td>4370 lbs/min</td>
</tr>
<tr>
<td>Curve B:</td>
<td>6317 lbs/min (5% decrease)</td>
<td>4560 lbs/min</td>
</tr>
<tr>
<td>Curve C:</td>
<td>5985 lbs/min (10% decrease)</td>
<td>4845 lbs/min</td>
</tr>
</tbody>
</table>
From this analysis of changes in the air flow rates, two results are evident. First, the air flow rates cannot effectively control the cooling operation, and second, since variations in the flow rates do not upset the cooling operation, these variations can be used to control the secondary air temperature.

Rotary kiln

Temperature profile  The steady state temperature profiles for the solids phase and for the gas phase are shown in Figure 14. The rapid rise in the gas temperature at the kiln outlet end is due to the heating effect of the flame, while the rapid decline at the feed end is due to the increased heat transfer effect of the chain system.

Solids reaction  In Figure 15 the steady state composition profiles for the solids phase in the rotary kiln are graphically displayed. These composition profiles are not intended to be an exact representation of the reaction effects in a cement kiln, because no method is presently available to collect the data necessary to establish these effects. However, the results shown here do have input and output values that are similar to operating installations and do represent a good approximation to the actual internal operation.
Figure 14. Steady state temperature profiles for the rotary kiln
Figure 15. Steady state composition profiles for the rotary kiln

Curve A: CaCO$_3$ as CaO
Curve B: SiO$_2$
Curve C: H$_2$O
Curve D: MgCO$_3$ as MgO
Curve E: C$_2$S
Curve F: C$_3$S
Curve G: CaO
Curve H: C$_4$AF
Curve I: C$_3$A
RESULTS OF COOLER-KILN SIMULATION

The results of the simulation of the grate cooler and rotary kiln are useful in establishing the response characteristic of the individual units; however, to study the complete cement process the combination must be analyzed together.

Although the effects of several parameters such as feed composition, fuel rate, and air rate could be investigated, the effect of a variation in the feed rate was chosen in this work because it produces the most significant variation in actual process situations.

In Figure 16 a cooler-kiln response to a 10% decrease in feed rate with a corresponding decrease in rotational speed (so that the cross-sectional area remains constant) is shown together with the response for the kiln simulation for the same changes. For the kiln the secondary air temperature was kept constant at the steady state value since no grate cooler effect was to be included.

These results (Figure 16) on the effect of the grate cooler on the response of the system for the stated variations are extremely interesting. According to the three criteria of operating performance (temperature of solids leaving the kiln, temperature of gas leaving the kiln, and the free lime concentration of the clinker) presented in Figure 16 the grate cooler has a negligible effect on the operation of the rotary kiln. Operating personnel in the cement industry stated that
Figure 16. Response characteristics of the rotary kiln for a 10% decrease in feed rate and a decrease in rotational speed to keep a constant cross-sectional area

Curve A: rotary kiln with grate cooler
Curve B: rotary kiln without grate cooler
the grate cooler dynamics have a definite effect on the response of the rotary kiln aside from the obvious effect that the grate cooler is necessary for recovery of heat from the clinker. However the results in Figure 16 appears to refute this statement.

There are several possible explanations for these contradictory views. Although these operators have stated that the grate cooler dynamics have a significant effect on the kiln, few actual measurements have been taken to substantiate the information or to indicate the magnitude of the effect. Even though experimental evidence is not available to decisively prove their views, the experience of these operators cannot be easily disregarded.

In the description of the burning process in the kiln (Figure 6 and equation 43) no effect of the temperature of the primary and secondary air on the burning characteristics of the flame is included. If the secondary air temperature changes because of the dynamics of the grate cooler (Figure 16), the present representation of the burning rate would not reflect the changes that this variation would produce. Therefore a significant influence on the operation of the rotary kiln would have been left undetected.

Another explanation for the contradiction between the results shown in Figure 16 and the information gathered from operating personnel is that Figure 16 is correct but that
the basis of the explanation of the results was centered on the wrong part of the process. Rather than using the rotary kiln mathematical model as a basis for analyzing this contradiction, the mathematical model of the grate cooler should be used for the basis of the explanation. The effect of a decrease in clinker rate to the grate cooler combined with an increase in temperature of the clinker has already been discussed. The result was a cancelling of the two effects so that only a small change in the operation of the cooler was expected. This same effect occurs in this present situation, and hence the addition of the grate cooler model to the system would not significantly alter these results.

Throughout this discussion of the cement process the common control technique of keeping a constant cross-sectional area in the kiln by variation in the rotational speed has been noted. In Figure 17 an initial analysis of this method is given. The response of the cooler-kiln system for a 10% decrease in feed rate with and without a rotational speed change are shown.

The most significant result in this figure is the response curve for the temperature of the gas leaving the kiln when the rotational speed does not change. This temperature increases and then begins to level out at a new steady state value. However, after approximately 166 minutes the tempera-
Figure 17. Response characteristics of the rotary kiln with grate cooler for a 10% decrease in feed rate

Curve A: rotational speed decrease to keep a constant cross-sectional area

Curve B: no rotational speed change
ture begins to increase at a rapid linear rate. This 166 minute point corresponds to the position in the kiln where the solids, having a different cross-sectional area because of a change in feed rate, enter the clinkering and burning zone.

Because an approach to steady state is indicated by the temperature of the gas leaving the kiln when no rotational speed change is made and because this same result does not occur as quickly when the rotational speed is changed immediately, the necessity of making this control change immediately after a flow rate change is made is questionable. These results indicate that there may be a better time to make the adjustment.
CONCLUSIONS

A thorough examination of all the results of this study leads to the general conclusion that the development and analysis of the mathematical model for the grate cooler and simulation of the grate cooler-rotary kiln operation, along with modifications in existing mathematical models for the rotary kiln, were the significant results of this work. The specific conclusions garnered from this research work are,

1) The overall heat transfer coefficient, U, for transfer of heat between the incoming air and the clinker is the most significant experimentally determined parameter in the grate cooler model.

2) The undergrate air temperature in the grate cooler does produce significant changes in the cooling process; however, the magnitude of the temperature variation required for these changes limits the effect to seasonal periods.

3) The temperature of the clinker leaving the kiln has a significant effect on the operation of the grate cooler.

4) A change in the clinker flow rate can be expected to produce a more important effect on the kiln operation than on the cooler operation.
5) A simultaneous increase in clinker temperature and decrease in clinker flow rate (which occurs when the rotational speed of the kiln is changed to keep a constant cross-sectional area) tend to cancel so that a smaller effect on the grate cooler operation occurs than the changes would individually produce.

6) The grate speed can be used to regulate the operation of the grate cooler, but the sequence of control changes must be carefully selected so that the cooling operation is not adversely affected.

7) The forced draft air flow rate and induced draft air flow rate cannot be effectively used to control the cooling operation of the grate cooler but can be used to control the secondary air temperature.

8) The grate cooler dynamics did not significantly alter the operation of the rotary kiln when a 10% decrease in feed rate was made.

9) Additional experimental verification of operator intuition and additional research on the burning process in the rotary kiln will be required before the effect of grate cooler dynamics on the rotary kiln operation can be conclusively demonstrated.
10) Investigation of new control methods using a grate cooler-rotary kiln mathematical simulation could significantly alter accepted control practices.
RECOMMENDATIONS

Continuation of research on the cement process is the basic recommendation derived from this investigation. The need for further research, particularly experimental research, is made evident by the development of the mathematical model to describe the process. To initiate this development an understanding of the process is required so that the mathematical equations can be formulated. To solve the mathematical equations the physical parameters of the process are required. To demonstrate the applicability of the model to actual process situations, response of the process to upsets must be recorded. Hence experimental evidence of process performance is definitely needed.

In situations where experimental evidence cannot possibly be gathered the mathematical model can be studied to provide information on the process. However this method requires large amounts of computer time to perform the necessary search sequences. Operator experience can also be useful. Detailed discussions with operating personnel from many plants need to be gathered and systematically analyzed so that similarities can be noted, and operator response to kiln upsets evaluated.

Specific areas for experimental, computational, and operational investigation that are recommended are the burning
process in the kiln, the temperature profiles in the kiln and cooler, the overall heat transfer coefficient in the cooler, the solids velocity in the kiln as a function of the solids temperature and composition, and the heat transfer coefficients in the kiln and cooler.
ACKNOWLEDGMENTS

The author wishes to express his gratitude to Dr. L. E. Burkhart for his guidance and counsel given on this project. Opportunity to study and analyze any portion of this project was always present, and investigations into unknown areas were encouraged.

Plant data used in this work was provided by Northwestern States Portland Cement Company. Additional information on the operation of the cement process was provided by Paul Gilbert and Ralph Draves of this company.

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LITERATURE CITED


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APPENDIX I

Cooler Parameters

The parameters that were used to calculate the basic cooler results are tabulated in this section. Variations of particular parameters to demonstrate the response of the cement process are given in the discussion of the results.

These data are grouped according to function and are equated to the representative term from the list of nomenclature.

Table 1. Physical parameters for cooler

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>C_1</td>
<td>6.0 ft</td>
</tr>
<tr>
<td>C_2</td>
<td>14.75 ft</td>
</tr>
<tr>
<td>h</td>
<td>4.5 ft</td>
</tr>
<tr>
<td>L</td>
<td>70.0 ft</td>
</tr>
<tr>
<td>W</td>
<td>6.0 ft</td>
</tr>
<tr>
<td>z</td>
<td>0.6 ft</td>
</tr>
</tbody>
</table>

Table 2. Energy transfer parameters for cooler

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A_g_1</td>
<td>0.220207</td>
</tr>
<tr>
<td>A_g_2</td>
<td>0.337548 x 10^{-4}</td>
</tr>
<tr>
<td>A_g_3</td>
<td>-0.28267 x 10^{-8}</td>
</tr>
<tr>
<td>A_s_1</td>
<td>0.2135</td>
</tr>
<tr>
<td>A_s_2</td>
<td>0.2177 x 10^{-4}</td>
</tr>
<tr>
<td>A_s_3</td>
<td>-0.10781 x 10^{-8}</td>
</tr>
<tr>
<td>h_g</td>
<td>0.42 BTUs/ft^{2}-min- R^0</td>
</tr>
<tr>
<td>V_1</td>
<td>0.45</td>
</tr>
<tr>
<td>V_2</td>
<td>0.40</td>
</tr>
<tr>
<td>V_7</td>
<td>0.10</td>
</tr>
<tr>
<td>ε_1</td>
<td>0.20</td>
</tr>
<tr>
<td>ε_2</td>
<td>0.20</td>
</tr>
<tr>
<td>ε_7</td>
<td>0.10</td>
</tr>
</tbody>
</table>
Table 2 (Continued)

K = 0.0125 BTUs/ft-min-R°
Ta = 520 R°
U = 1.2 BTUs/ft²-min- R°

Table 3. Miscellaneous parameters for cooler

P = 2116.8 lbs/ft²
R = 1545.0 lbs ft³/ft²-lb mole- R°
Δt = 0.125 min
Δx = 1.0 ft
σ = 0.28567 x 10⁻¹⁰ BTUs/ft²-min- R°⁴
Tolerance for temperature iteration at "no flow point" = 0.5 R°

Table 4. Initial conditions for steady state cooler results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clinker flow rate</td>
<td>1400 lbs/min</td>
</tr>
<tr>
<td>Clinker temperature</td>
<td>2800 °R</td>
</tr>
<tr>
<td>Grate speed</td>
<td>9 ft/min</td>
</tr>
<tr>
<td>Undergrate temperature</td>
<td>520 °R</td>
</tr>
<tr>
<td>Forced draft flow rate</td>
<td>6650 lbs/min</td>
</tr>
<tr>
<td>Induced draft flow rate</td>
<td>4370 lbs/min</td>
</tr>
</tbody>
</table>
APPENDIX II

Kiln Parameters

In this section the parameters that were used to calculate the kiln results are tabulated.

Table 5. Heat capacity coefficients for gases

<table>
<thead>
<tr>
<th>Component i</th>
<th>$A_{1i}$</th>
<th>$A_{2i}$</th>
<th>$A_{3i}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2304</td>
<td>0.2753 $\times 10^{-4}$</td>
<td>-0.7602 $\times 10^{-9}$</td>
</tr>
<tr>
<td>2</td>
<td>0.1998</td>
<td>0.6376 $\times 10^{-3}$</td>
<td>-0.8622 $\times 10^{-7}$</td>
</tr>
<tr>
<td>3</td>
<td>0.1912</td>
<td>0.5498 $\times 10^{-4}$</td>
<td>-0.9694 $\times 10^{-8}$</td>
</tr>
<tr>
<td>4</td>
<td>0.3960</td>
<td>0.8139 $\times 10^{-4}$</td>
<td>-0.7863 $\times 10^{-9}$</td>
</tr>
<tr>
<td>5</td>
<td>0.1440</td>
<td>0.1280 $\times 10^{-3}$</td>
<td>-0.2395 $\times 10^{-7}$</td>
</tr>
</tbody>
</table>

Table 6. Heat capacity coefficients for solids

<table>
<thead>
<tr>
<th>Component i</th>
<th>$A_{1i}$</th>
<th>$A_{2i}$</th>
<th>$A_{3i}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1.006</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>6</td>
<td>0.2208</td>
<td>0.9091 $\times 10^{-4}$</td>
<td>-0.1598 $\times 10^{5}$</td>
</tr>
<tr>
<td>7</td>
<td>0.2496</td>
<td>0.2908 $\times 10^{-4}$</td>
<td>-0.2007 $\times 10^{5}$</td>
</tr>
<tr>
<td>8</td>
<td>0.2081</td>
<td>0.1070 $\times 10^{-4}$</td>
<td>-0.9013 $\times 10^{4}$</td>
</tr>
<tr>
<td>9</td>
<td>0.2399</td>
<td>0.1795 $\times 10^{-4}$</td>
<td>0.0</td>
</tr>
<tr>
<td>10</td>
<td>0.2525</td>
<td>0.2398 $\times 10^{-4}$</td>
<td>0.3214 $\times 10^{3}$</td>
</tr>
<tr>
<td>11</td>
<td>0.2697</td>
<td>0.1537 $\times 10^{-4}$</td>
<td>-0.2663 $\times 10^{5}$</td>
</tr>
<tr>
<td>12</td>
<td>0.1471</td>
<td>0.6470 $\times 10^{-4}$</td>
<td>-0.7202 $\times 10^{4}$</td>
</tr>
<tr>
<td>13</td>
<td>0.1867</td>
<td>0.3555 $\times 10^{-4}$</td>
<td>0.0</td>
</tr>
</tbody>
</table>
Table 6 (Continued)

<table>
<thead>
<tr>
<th>Component i</th>
<th>$A_{1i}$</th>
<th>$A_{2i}$</th>
<th>$A_{3i}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>0.2184</td>
<td>0.2098 x 10^{-4}</td>
<td>-0.1440 x 10^5</td>
</tr>
<tr>
<td>15</td>
<td>0.2305</td>
<td>0.9418 x 10^{-5}</td>
<td>-0.1440 x 10^5</td>
</tr>
<tr>
<td>16</td>
<td>0.2009</td>
<td>0.2943 x 10^{-4}</td>
<td>-0.1211 x 10^5</td>
</tr>
</tbody>
</table>

Table 7. Heat transfer convection coefficients and emissivities

Defining equations:

\[ V_3 = HC_1 + HC_2(x/L) \]
\[ V_4 = HC_3/(1.0 + HC_4(x/L)) \]
\[ V_5 = HC_5 + HC_6(x/L) \]
\[ \epsilon_3 = HC_7 + HC_8(x/L) \]
\[ \epsilon_4 = HC_9 + HC_{10}(x/L) \]
\[ \epsilon_5 = HC_{11} + HC_{12}(x/L) \]

$HC_1 = 0.80$  \hspace{1cm} $HC_7 = 0.85$
$HC_2 = -0.72$ \hspace{1cm} $HC_8 = -0.75$
$HC_3 = 6.0$  \hspace{1cm} $HC_9 = 0.30$
$HC_4 = 75.0$ \hspace{1cm} $HC_{10} = -0.20$
$HC_5 = 1.0$  \hspace{1cm} $HC_{11} = 0.70$
$HC_6 = -0.8$ \hspace{1cm} $HC_{12} = -0.40$
### Table 8. Heats of reaction

<table>
<thead>
<tr>
<th>Base component j</th>
<th>( H_{R_j} ) (BTUs/lb.) @ 528 °R</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-21,506.0</td>
</tr>
<tr>
<td>4</td>
<td>988.0</td>
</tr>
<tr>
<td>6</td>
<td>581.0</td>
</tr>
<tr>
<td>7</td>
<td>765.0</td>
</tr>
<tr>
<td>9</td>
<td>-311.4</td>
</tr>
<tr>
<td>13</td>
<td>2.707</td>
</tr>
<tr>
<td>16</td>
<td>-45.0</td>
</tr>
<tr>
<td>15</td>
<td>-16.0</td>
</tr>
</tbody>
</table>

### Table 9. Kinetic coefficient equation constants

<table>
<thead>
<tr>
<th>Reaction i</th>
<th>( A_{K_i} )</th>
<th>( C_{K_i} )</th>
<th>( E_{K_i} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.2</td>
<td>-0.005</td>
<td>0.0</td>
</tr>
<tr>
<td>B</td>
<td>0.4050 \times 10^5</td>
<td>0.44</td>
<td>0.01</td>
</tr>
<tr>
<td>C</td>
<td>0.3300 \times 10^{17}</td>
<td>2.57</td>
<td>0.0216</td>
</tr>
<tr>
<td>D</td>
<td>0.1260 \times 10^{10}</td>
<td>3.70</td>
<td>0.0745</td>
</tr>
<tr>
<td>E</td>
<td>0.2040 \times 10^{10}</td>
<td>6.67</td>
<td>0.0931</td>
</tr>
<tr>
<td>F</td>
<td>0.2700 \times 10^{13}</td>
<td>0.90</td>
<td>0.0099</td>
</tr>
<tr>
<td>G</td>
<td>0.1547 \times 10^{20}</td>
<td>146.17</td>
<td>0.01714</td>
</tr>
<tr>
<td>H</td>
<td>0.9490 \times 10^{16}</td>
<td>8.0</td>
<td>0.01285</td>
</tr>
</tbody>
</table>
Table 10. Molecular weight of components

<table>
<thead>
<tr>
<th>Component i</th>
<th>Molecular weight</th>
<th>Component i</th>
<th>Molecular weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>28.02</td>
<td>9</td>
<td>60.06</td>
</tr>
<tr>
<td>2</td>
<td>16.04</td>
<td>10</td>
<td>40.32</td>
</tr>
<tr>
<td>3</td>
<td>32.00</td>
<td>11</td>
<td>101.94</td>
</tr>
<tr>
<td>4</td>
<td>18.02</td>
<td>12</td>
<td>159.70</td>
</tr>
<tr>
<td>5</td>
<td>44.01</td>
<td>13</td>
<td>172.22</td>
</tr>
<tr>
<td>6</td>
<td>84.33</td>
<td>14</td>
<td>228.30</td>
</tr>
<tr>
<td>7</td>
<td>100.09</td>
<td>15</td>
<td>270.18</td>
</tr>
<tr>
<td>8</td>
<td>56.08</td>
<td>16</td>
<td>485.96</td>
</tr>
</tbody>
</table>

Table 11. Velocity equation constants

\[ B_0 = 1.0 \quad B_3 = -41.55159 \]
\[ B_1 = 0.1423262 \quad B_4 = 58.56255 \]
\[ B_2 = 7.933028 \quad B_5 = -25.41901 \]

Initial velocity = 1.659249 ft/min (Kiln 9.95% filled)
Table 12. Physical parameters for rotary kiln

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_3$</td>
<td>9.74 ft</td>
</tr>
<tr>
<td>$C_4$</td>
<td>8.70 ft</td>
</tr>
<tr>
<td>$C_5$</td>
<td>27.95 ft</td>
</tr>
<tr>
<td>$L$</td>
<td>400 ft</td>
</tr>
<tr>
<td>$Z$</td>
<td>0.60 ft</td>
</tr>
</tbody>
</table>

Percentage of area occupied in kiln initially = 9.95%

Bulk density of kiln feed = 75 lbs/ft$^3$

Inside diameter of kiln = 12.0 ft

Slope of kiln = 0.036458 ft/ft

Table 13. Miscellaneous parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta t$</td>
<td>2.0 min</td>
</tr>
<tr>
<td>$K$</td>
<td>0.0125 BTUs/ft-min-°R</td>
</tr>
<tr>
<td>$T_a$</td>
<td>520. °R</td>
</tr>
<tr>
<td>$h_g$</td>
<td>0.42 BTUs/ft$^2$-min-°R</td>
</tr>
</tbody>
</table>

Tolerance for cutoff of chemical reactions 0.00001 lbs i/lb
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feed rate</td>
<td>1400 lbs clinkerable solids/min</td>
</tr>
<tr>
<td>Feed temperature</td>
<td>520 °R</td>
</tr>
<tr>
<td>Feed composition</td>
<td></td>
</tr>
<tr>
<td>( m_4 )</td>
<td>0.1833</td>
</tr>
<tr>
<td>( m_6 )</td>
<td>0.0790</td>
</tr>
<tr>
<td>( m_7 )</td>
<td>1.1744</td>
</tr>
<tr>
<td>( m_9 )</td>
<td>0.2080</td>
</tr>
<tr>
<td>( m_{10} )</td>
<td>0.0610</td>
</tr>
<tr>
<td>( m_{11} )</td>
<td>0.0360</td>
</tr>
<tr>
<td>Fuel rate ((CH_4))</td>
<td>204.8281 lbs/min</td>
</tr>
<tr>
<td>Fuel temperature</td>
<td>860 °R</td>
</tr>
<tr>
<td>Primary air rate</td>
<td>1660.7737 lbs/min</td>
</tr>
<tr>
<td>Primary air temperature</td>
<td>860 °R</td>
</tr>
<tr>
<td>Grate speed</td>
<td>9 ft/min</td>
</tr>
<tr>
<td>Undergrate temperature</td>
<td>520 °R</td>
</tr>
<tr>
<td>Forced draft flow rate</td>
<td>6650 lbs/min</td>
</tr>
<tr>
<td>Induced draft flow rate</td>
<td>4150 lbs/min</td>
</tr>
</tbody>
</table>