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An abstract machine to control the execution of semi-independent concurrent computations

Virgil Eugene Wallentine
Iowa State University

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An abstract machine to control the execution of semi-independent concurrent computations

by

Virgil Eugene Wallentine

A Dissertation Submitted to the Graduate Faculty in Partial Fulfillment of The Requirements for the Degree of DOCTOR OF PHILOSOPHY Major: Computer Science

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CHAPTER I.

INTRODUCTION AND OVERVIEW

In recent years the emphasis placed on providing a computer utility which accommodates many concurrent users has given rise to many new problem areas. The basic premise of such a computer utility -- that the system provide users with:

1) an efficient environment for program development, debugging, and execution;
2) a wide range of problem solving facilities;
3) low-cost computing through the sharing of resources and information --

has given impetus to the development of computer systems which have the following common characteristics (4):

1) concurrency of (parallel) activities,
2) automatic resource allocation,
3) sharing - the simultaneous use of a resource by more than one process (computation),
4) multiplexing of resources (mutually exclusive access by a computation to a resource for an interval of time),
5) remote conversational access by users,
6) nondeterminacy (unpredictable ordering of events),
7) long-term storage.

There are many interesting questions to be answered in each of the above areas of which the following is but a sample.
1) Does such a system perform the functions expected of it?
2) Can the system be extended or contracted by the addition or re­moval of resources?
3) Is it possible for the system to "die"?
4) Can a system's demise be circumvented?
5) What control mechanisms are necessary to permit user intercom­munication?

Unfortunately, in many cases, the state of the art is not such that we can find definitive answers to these questions. A theoretical foundation has not yet been laid within which these questions can be posed in order to receive precise answers.

While the study of programming languages has benefitted greatly from various formal models of programming (see for example, Landin (15), McCarthy (19), and Lucas, Lauer, and Stigleitner (18)), no corresponding formal model has emerged to aid in the study of computer systems. Some models, such as Petri nets (21) and flow graph schemata (22) have been investigated with encouraging results, but such models have not yet been shown to realistically represent modern computer systems let alone provide insight for future systems.

This dissertation investigates the Vienna Definition Language (18) as a candidate for modeling computer systems. The Vienna Definition Language was originally developed for the formal definition of PL/I; however, it has proven to be generally applicable for language definition and has been used to formally define several programming languages. It is this author's belief that the Vienna definition method can also be applied
to the formal definition of computer systems.

The Vienna method is described by Wegner (23) and Lucas, Lauer, and Stigleitner (18). It consists of a language in which to describe operations on a data structure and an abstract machine which interprets all statements in the language. This machine is characterized by the set of states it may attain and a state transition function. The initial state of the machine is defined in terms of a given program, and the subsequent behavior of the machine is said to define the interpretation of the program. The abstract machine therefore attaches a meaning to programs and their constituent parts by defining the effect of their interpretation on the state of the machine.

In this dissertation we investigate the use of the Vienna method in formalizing one aspect of computer systems, namely the control of concurrent computations. The basic approach taken is to design an abstract machine whose structure is simple, yet sufficient to define the execution of parallel activities. The representation of the state of the machine coupled with the state transformation function provide a simple framework within which to study computer systems. Although more complex and more efficient machines can be designed, the necessary mechanisms for a concurrent control machine are presented in this simple model.

The flow of control during execution of a program emanates from the control structures available to the programmer. Thus, in Chapter II we investigate the control structures of current programming languages with particular emphasis on control structures useful for controlling concurrent computations. The semantics of the control structures are described
informally and examples of their use are presented.

In Chapter III we develop the architecture of the abstract machine. The information structure representing the state of the machine is defined in terms of the information accessible to all computations (common data through which the computations communicate) and that information which is local to each computation (data which affords a computation its independence). Transformation of the state of the machine is effected by the execution of an instruction in one of the currently active computations. The skeleton of a concurrent computations machine is completed with the specification of the machine's initial state, the structure of all final states, and the nondeterministic manner in which the computations and instructions within computations are chosen for execution.

In Chapter IV we abstract from the informal descriptions of Chapter II that set of primitive control mechanisms sufficient to implement the control structures presented in Chapter II. Control primitives are defined for the initiation of computations, the termination of computations, and the synchronization of computations.

In Chapter V the applicability of the machine and the set of task control primitives presented in Chapters III and IV is illustrated. Two simple programming languages which accommodate concurrent computations are given and are defined using the methods of Chapters III and IV.

In Chapter VI we discuss the strengths and weaknesses of the abstract machine as a framework in which to imbed the study of computer operating systems. Particular emphasis is placed on its conceptual simplicity.
CHAPTER II.

CONTROL FEATURES OF MULTITASKING PROGRAMMING LANGUAGES

The motivation for the specification of parallel computations in a program is not so much to make a particular program execute more efficiently as it is to relax the constraints on the order in which parts of the program are executed. A scheduling algorithm is therefore allotted more degrees of freedom in the ordering of executions and more freedom to effect greater efficiency in the allocation of resources. In this chapter we informally examine the control structures proposed in the literature which permit programmers to specify concurrent operations.

In order to make clear the terminology used, the following definitions are given. Control structures are the operations which specify the sequencing rules for programs or parts of programs. Computation will be used to indicate the sequence of instructions, specified by the programmer, the interpretation of which will produce the intended results. Concurrent computations are those which can be interpreted simultaneously and produce no unintended results. In some cases the order of interpretation of computations is dictated by inter-computation dependencies. The term semi-independent means that computations can be interpreted as independent entities except at certain points (where they are dependent on one another) of explicit communication.

The term task\(^1\) refers to the data structure - a set of cells containing data and its structural relationships - containing all the information

\(^1\)A synonym for task will be process.
necessary to perform the computation. It consists of the information accessible to a computation and the computation itself. A computation can be interrupted, blocked, and resumed at a later time as long as all the task information is saved. The control of concurrency of computations is affected by the synchronization of transformations.

Control structures of current multitasking programming languages are surveyed in order to identify the common properties among them. In order to gain some insight into the mechanisms necessary to implement control structures, we present in this chapter an informal description of the actions effected by the following control structures:

1) transfer statements,
2) procedure call,
3) block entry,
4) block/procedure exit,
5) task initiation,
6) task termination,
7) task synchronization.

The first four items on the above list are common to single task languages and are well understood. Since a task consists of a set of accessible cells and an instruction sequence representing a computation, the execution of a transfer statement affects a change of the instruction sequence to be executed and a change in the names and number of memory cells accessible to the task. Both items are associated with the label specified as the operand of the transfer statement.
Execution of a procedure call effects a change in both components of a task. The new instruction sequence is specified in the declaration of the procedure whose identifier is the operand of the call. The set of cells accessible to a procedure during execution is the union of the set specified at procedure declaration time and the set of cells associated with the actual parameters specified at the time of call. When a procedure body is activated by a call, some provision must be made to save the current components of the task. These components are to be restored upon procedure exit to enable the execution to be continued at the instruction following the procedure call.

The semantics of block entry only affect the cells accessible by the task. At entry, new cells (one for each identifier declared in the block) are created and the association between identifier and cell is established. If an identifier is declared which had a previous cell, the association is changed. Again the current task components must be saved for reinstatement at block exit.

Block and procedure exit instructions must, when executed, restore to the task those cells accessible prior to block entry and procedure call, respectively. No change is effected (other than the incrementation of the instruction pointer) to the instruction sequence on block exit. Procedure exit, however, must restore the instruction sequence which was effective at the time of procedure activation (i.e. procedure call).

The above four control structures have been formally defined by others (18) and there is general agreement on their usefulness. But there seems to be very little agreement on the control structures
necessary to specify the multitasking concept. Of the many control features cited in the literature, four types of synchronization control and three types of task creation and deletion are discussed. Most other control features of current programming languages are variations of the ones discussed.

One of the earliest proposed schemes for controlling concurrent computations was introduced by Conway (3) and later revitalized by Dennis and Van Horn (5). The basic control feature for parallel programming is specified by the statement

```
fork w;   
```

"w" is a label and "fork w" specifies a new task is to be initiated whose instructions are those at w and whose accessible memory cells are those accessible in the block in which w is declared. The termination of tasks is specified by

```
join t,w;   
```

where "t" is a counter under programmer control specifying the number of tasks to be terminated before a task - the last one to complete its computation - executes a transfer to "w".

Communication between concurrent computations is affected by access to a common data base. The ability to change such a data base requires some mechanism to assure a particular task has exclusive access to the common data. An instruction

```
lock d;   
```
effects such a lock on a data base where "d" is a variable lock indicator which is set to "on" if the data base is in use. The "lock d" sets "d" to "on" if not set. An instruction

unlock d;

sets the lock to "off".

A simple example of a program specifying concurrent computations is presented on the following page. This program specifies concurrent execution of the two computations

"a:=1", "c:=3"

and "b:=a+2"

followed by the single computation "d:=b+c". The intended value of "d" is 6 and the correct value is attained by synchronizing the computations by access to the common variable "a". It is set to "1" before the computation "b:=a+2" can proceed.

An informal description of its meaning follows. "t" is set to the value 2 in line 3 to indicate 2 computations are candidates to execute in parallel. A task is created in line 5 to set "a" to 1 and "c" to 3. The task executing line 5 continues to set "b" to "a+2" only after "a" has been set to "1" by the task assigned to execution of line 15. Note that the region of code which updates or checks the value of "a", which is the common data base, is protected by lock w. The concurrent
computations are

"b:=a+2"

and "c:=3".

Obviously "b" attains the correct value of 3 if and only if "a" is set to 1 first. One of the tasks will be assigned to the computation

"d:=b+c".

Only one task will be terminated since "t" has initial setting of 2.

1. begin integer t,a,b,c,d;
2. lock w; label wl, x, cycle;
3. t:= 2;
4. a:= 0;
5. fork w1;
6. cycle: lock w;
7. if a ≠ 1 then begin
8. unlock w;
9. goto cycle;
10. end;
11. b:= a + 2;
12. unlock w;
13. join t, x;
14. wl: lock w;
15. a:= 1;
16. unlock w;
17. c:= 3;
18. join t, x;
19. x: d:= b + c;
20. end
In another approach, Dijkstra (6) has proposed a set of control structures for specifying concurrent computations. His extensions to Algol include a sequence of n concurrent computations surrounded by the special statement bracket pair "parbegin" and "parend". This is interpreted as parallel execution of the constituent statements. He also presented two synchronization features which operate on special variables called "semaphores" which are initialized but not referenced by any operations other than the special task control operations "P(semaphore)" and "V(semaphore)". A program using these control structures and specifying the same concurrent computations as in the previous example is presented.

1. begin integer a,b,c,d
2. semaphore sem; label cycle;
3. sem:= 1;
4. a:= 0;
5. parbegin
6. begin
7. cycle: P(sem);
8. if a $\neq$ 1 then begin
9. V(sem);
10. goto cycle;
11. end;
12. V(sem); b:= a + 2;
13. end
14. begin P(sem); a:= 1;
15. V(sem); c:= 3;
16. end
17. parend
18. d:= b + c;
19. end
The V-operation increases the value of its argument semaphore by 1 and it is considered an indivisible operation. The P-operation functions to decrease the value of its semaphore argument by 1 as soon as its resulting value will be non-negative. The completion of the P-operation - i.e. the decision that this is the appropriate moment to effectuate the decrease and the subsequent decrease itself - is also considered an indivisible operation.

Using the same task initiation and termination control features as Dijkstra, Hansen (9) has recently introduced more natural (in the author's opinion) synchronization features. Shared variables are referenced only within critical regions. The statement

\[
\text{region } v \text{ do } S
\]

indicates that statement "S" is executed whenever the current task owns the shared variable "v". The conditional critical region \( S \), indicated by

\[
\text{region } v \text{ when } B \text{ do } S
\]

is executed when the additional restriction, that condition \( B \) is interpreted as true, is met. Again, the same program is specified.

1. \text{begin } \text{var } a,b,c,d: \text{ integer;} \\
2. \text{var } v: \text{ shared lock;} \\
3. \text{a:= 0;} \\
4. \text{parbegin} \\
5. \text{\hspace{1em} region } v \text{ when } a \text{ do } b:= a + 2; \\
6. \text{\hspace{1em}begin region } v \text{ do} \\
7. \text{\hspace{2em}a:= 1;} \\
8. \text{\hspace{2em}c:= 3;} \\
9. \text{\hspace{1em}end;} \\
10. \text{parend;} \\
11. \text{d:= b + c;} \\
12. \text{end}
The tasking facilities of PL/I (12) and Fitzwater and Schweppe (7) specify the activation of procedures as tasks. The initiation of a task is a procedure call,

\[
\text{call proc (arg(1),...,arg(n)) task t, event e;}
\]

which specifies also an event variable. The state of the event variable is implicitly set upon completion of the assigned task. Task completion is implicit upon completion of the procedure. The synchronizing control is provided in the statements \text{signal (event)} and \text{wait (event)}. These statements set the event variable to "happened" and release the task, from a blocked list for the specified event, respectively. Again, the same concurrent computations are specified.

1. \text{begin integer a,b,c,d;}
2. \text{task t; event el, e2;}
3. \text{procedure pr(i); integer i;}
4. \text{begin a:= i;}
5. \text{signal el;}
6. \text{c:= 3;}
7. \text{end}
8. \text{begin}
9. \text{a:= 0}
10. \text{call pr(1) task t, event e2;}
11. \text{wait(el);}
12. \text{b:= a + 2}
13. \text{wait(e2);}
14. \text{d:= b + c;}
15. \text{end}
16. \text{end}
Many other control features have been proposed, but their inclusion would be of little additional value since most of them are combinations of the above. The interested reader may consult the task control structures of Oregano (2), SOL (13) (14), Algol 68 (17) or the control structures presented in a paper by Anderson (1) for further descriptions.
CHAPTER III.
THE CONTROL OF CONCURRENT COMPUTATIONS

This chapter presents the architecture of a class of abstract machines, called CONCOMs, whose structure is sufficient to control the execution of concurrent computations. Description of both the machine structure and the manner in which the structure is transformed by execution of instructions is presented using the Vienna Definition Language (18). An outline of the chapter is as follows:

1) The structure of the state of a CONCOM is presented.
2) The transformation of a state to the next state of a CONCOM is given.
3) And finally, the initial and all final states of a CONCOM are specified.

Complete definition of a CONCOM is accomplished by specifying the interpreter instructions whose execution effects the state transformation. This chapter thus presents the framework within which to define concurrent computation interpreters. The complete definition of such interpreters will be deferred until Chapter V.

The definition of a CONCOM machine will be given in terms of the objects from which the state of the machine is built, the structure of the objects, the manner of referencing the objects, an initial state, any final states and a state transformation function. In particular, we have the following definition of a CONCOM.
Definition 3.1

A concurrent computations (CONCOM) interpreter is a 6-tuple of sets and operations

\((EO, SEL, is-state, IS, FS, TF)\)

in which:

1) EO is a nonempty set of elementary objects;
2) SEL is a nonempty set of selectors;
3) is-state is a predicate defined over a set of objects 0 built from the sets EO and SEL;
4) IS is a special object, satisfying the predicate is-state, called the initial state;
5) FS is a set of objects, each satisfying is-state, called final states;
6) TF is a state transition (unary) operator whose argument satisfies is-state and which takes its values from the set of objects which satisfy is-state.

0 is the set of Vienna objects; IS and FS are elements of 0; and TF is the transformation function whose argument and values are states. EO and SEL are Vienna elementary objects and selectors. Specification of the other four components of a CONCOM follows.

The data structure which defines the state of the machine is structured in two levels. The global (immediate components) level contains all the information known only to the interpreting machine. That is, it is the common data through which all tasks communicate. The second level -- task local level -- contains information known only to a particular task.
No other task can reference this data. It is this data which is saved when a task must suspend execution and which is restored when a task resumes execution.

The global level of the state of the control machine specifies
1) the attributes of values known to the tasks in the system,
2) the memory cells in use and their current values,
3) an element specifying the currently executing task,
4) a list of tasks currently ready for execution, and
5) a set of lists of tasks which are blocked and waiting for the use of a resource.

The immediate components of the state of a CONCOM are as follows:
1) a unique name generator which supplies the index of a new cell (unique name) accessible to a task when the interpreter instruction un-name is invoked. Its unique selector is "s-n";
2) an attribute table which defines the type of information contained in each memory cell. Its unique selector is "s-at";
3) a denotation table which defines the relationship between cells and their values. Its unique selector is "s-den";
4) a task selector which identifies the unique name of the task whose instruction sequence has been selected for execution. Its unique selector is "s-task";
5) a concurrent computations component which is a collection of all those data structures called tasks which are currently available for execution. Its unique selector is "s-cc".
**Definition 3.2**

The state of a CONCOM satisfies the predicate is-state defined as:

\[
\text{is-state} = (\langle s-n: \text{is-integer} \rangle, \\
\langle s-at: \text{is-at} \rangle, \\
\langle s-den: \text{is-den} \rangle, \\
\langle s-task: \text{is-name} \rangle, \\
\langle s-cc: \text{is-cc} \rangle).
\]

Assuming a representative set of data types, the attribute table may contain designations for integer, boolean, label, and procedure variables. An additional data type is needed to specify the representation of a resource use lock variable. Each unique name of a cell selects from the attribute table the type of data contained in the cell.

**Definition 3.3**

The attribute table of a CONCOM has entries of the form \(\langle n: \text{is-type} \rangle\) where

\[
\text{is-at} = (\langle n: \text{is-type} \rangle | \text{is-n}(n)), \\
\text{is-type} = \text{INT} \lor \text{LOG} \lor \text{LABEL} \\
\lor \text{PROC} \lor \text{LOCK},
\]

\(n\) is the unique name of the cell, and \(\text{is-n}(n)\) tests the validity of \(n\) as a cell name of the form \(n^j\) (\(j\) is an integer).

The denotation table contains the values of the data types specified in the attribute table. The common data types such as integer and boolean have values as expected; but label, procedure, and lock variables have more interesting denotations. A label represents a new site of activity (new instruction sequence) and a new environment (new set of accessible cells). Furthermore, this new environment has an enclosing environment.
(due to the accommodation of block structure) which must be restored upon block exit. The label denotation must therefore designate all three components.

Procedure denotations must designate the new instruction sequence, its environment, and its formal parameters. The components of a procedure denotation have values that are in effect at the time the procedure is declared. The formal parameters and instruction sequence are specified in the procedure declaration. The environment in which the procedure is executed is in effect at declaration time and is copied into the environment component of the procedure denotation.

Since more detail on the use of resource use locks will be presented in Chapter IV, very little discussion follows. The basic hypothesis, though, is that each resource will have an associated lock which is set to "on" when the resource is in use and "off" when not in use. Therefore, when a task cannot seize a resource it must be blocked and placed on a list associated with the resource. The lock will therefore have a lock and an associated list of tasks waiting for the resource. The lock denotation must designate both components. The structure of these tasks will be discussed later; and the denotation table definition follows.

**Definition 3.4**

The *denotation table* of a CONCOM has entries of the form

\(<n:denotation>\)
where

\[
\text{is-den} = (\{\langle n:\text{is-integer} V \text{is-log} \\
\quad V \text{is-proc-den} V \text{is-label-den} \\
\quad V \text{is-lock-den}\} \mid \text{is-n}(n)),
\]

\[
\text{is-proc-den} = (\langle \text{s-attr:is-proc-attr}, \\
\quad \text{s-env:is-env}\rangle),
\]

\[
\text{is-proc-attr} = (\langle \text{s-parm-list:is-id-list}, \\
\quad \text{s-st:is-st}\rangle),
\]

\[
\text{is-label-den} = (\langle \text{s-st-list:is-st-list}, \\
\quad \text{s-env:is-env}, \\
\quad \text{s-d:is-dump}\rangle),
\]

\[
\text{is-lock-den} = (\langle \text{s-lock:is-log}, \\
\quad \text{s-ttab:is-ttab}\rangle),
\]

\[
\text{is-ttab} = (\{\langle n:\text{is-task} \mid \text{is-n}(n)\}),
\]

and \(n\) is a unique cell name.

Note that in Definition 3.4 the "s-parm-list" selector references the parameter list of a procedure declaration and that the values of the actual parameters will be added to the execution environment at procedure call time. Also note that the selector "s-env" references that set of accessible cells via the declared identifier. "s-attr" selects the attributes of a procedure — that is, the parameter list and statement list. The is-dump and is-task predicates are satisfied by task local components and are discussed later.

The "s-cc" selector retrieves those data structures called tasks. The tasks retrieved are those currently available for execution. That is, all tasks designated by a unique name in the concurrent computations component are on a ready list for execution. The next instruction to be executed can be selected from any one of the tasks on the ready list.
The ready list is really a task table since each task is selected by the unique name assigned to it at task initiation. Thus, assuming later definition of the predicate is-task, the concurrent computations component is defined.

**Definition 3.5**

The concurrent computations component of a CONCOM has entries of the form

\[ \langle n: \text{is-task} \rangle \]

where

\[ \text{is-cc} = \left\{ \langle n: \text{is-task} \rangle \ | \ \text{is-n}(n) \right\} \].

The tree structure representing the immediate components of the state S is in Figure 3.1. And the tree structures representing the procedure, label, and lock denotations are shown in Figures 3.2, 3.3, and 3.4.

*Figure 3.1. Immediate components of the state of a CONCOM*
Figure 3.2. Procedure denotation for cell whose unique name is $n_i$

Figure 3.3. Label denotation for cell whose unique name is $n_j$
Figure 3.4. Lock denotation for cell whose unique name is $n_k$ and which has $m$ tasks in its task table.

The task local information is that collection of data structures from which a task obtains its independent identity apart from all other tasks in the machine. Each task progresses in an independent manner except at explicit points of task intercommunication; and no assumptions of speed of other task execution are permitted. Therefore, a task will by necessity contain a sequence of instructions to be executed which represent the computation to be performed. It must also have access to that unique set of memory cells which contain values that are not accessible to other tasks and have access to memory cells common to other tasks. Furthermore, each task when executing has an enclosing environment of its
own which must be restored upon exit from the present environment. Hence the following definition of a task.

**Definition 3.6**

A task satisfies the predicate is-task, where

\[
\text{is-task} = (\langle \text{s-c:is-c} \rangle, \\
\langle \text{s-env:is-env} \rangle, \\
\langle \text{s-d:is-dump} \rangle).
\]

The environment of a task is specified by an environment table which defines the relationship between an identifier and the unique name of the cell containing its value. The selectors are identifiers and they retrieve the unique name of a cell. Every identifier referenced by the concurrent computation is a selector in the environment table of the executing task.

**Definition 3.7**

The environment component of a task has entries of the form

\[
\langle x: n_1 \rangle
\]

where \( x \) is an identifier and \( n_1 \) is a unique name. Therefore,

\[
\text{is-env} = \{ \langle \text{id:is-n} \mid \text{is-id(id)} \} \}.
\]

Many concurrent computations are specified in languages which have block structure and procedure declarations. And the execution of the instructions in a task may effect entry into and exit from blocks. The changing of the environment and sequence of instructions executed necessitate the saving of the environment component and the control component.
Therefore, a dump component is designated which contains both of these components plus a dump component which specified return information of the previously effective environment, control, and dump.

**Definition 3.8**

The dump component of a task satisfies the predicate is-dump.

\[
is\text{-dump} = (\langle s\text{-c:is-c}\rangle, \langle s\text{-env:is-env}\rangle, \langle s\text{-d:is-dump}\rangle).
\]

Each task is assigned a computation; and each computation is defined by a unique sequence of instructions. Therefore, the representation of a task must include a component containing a description of the program instructions to be executed. The control component is an object which is an element of the set $\mathbb{O}$ of composite objects which defines both the structure and sequence of instructions.

Instructions at a terminal vertex of the control tree (component) are available for execution; and the instruction at the terminal vertex chosen is deleted when executed. Execution proceeds to instructions at terminal vertices on the new control tree. When the only terminal vertex contains the null element $\Lambda$, then execution stops.

Even though use will be made of the macro format of instructions discussed by Wegner (23), the control tree contains the composite object representation of an instruction displayed in Figure 3.5. An instruction consists of three components, referred to as: the instruction name component selected by "s-in"; the argument list component "s-al"; and the
return information component "s-ri". Also associated with an instruction is a list of m possible successor nodes (further from the root of the tree) selected by "succ(1),\ldots,succ(m)" which are instructions.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{control_tree.png}
\caption{Structure of an instruction on the control tree}
\end{figure}

"s-al" selects the list of arguments of the instruction. These arguments are set to the values of value-returning instructions on successor nodes. Furthermore, a particular component of an argument may be selected. "s-ri" (return information) selects a two component structure which identifies the component selected for assignment by "s-comp"; its "s-addr" component indicates which arguments at which predecessor (closer to the root) nodes are to be assigned the value of this instruction. The following definition of the control component is recursive; but it is understood that the recursion is applied only a finite number of times.
Definition 3.9

The control component of a task satisfies the predicate is-c.

\[ \text{is-c} = \langle \text{s-in:is-in}, \langle \text{s-al:} (\{ \text{elem}(i): \text{is-obj} \land \neg \text{is-}A \land \text{is-integer}(i) \}) \rangle, \langle \text{s-ri:} (\langle \text{s-comp:is-sel}, \langle \text{s-addr:} (\{ \text{elem}(i): \text{is-arg-addr} \land \text{is-integer}(i) \}) \rangle, \{ \langle \text{r:is-c} \mid r \in \text{SUCC} \}) \rangle \text{V is-}A, \]

where

- is-\( A \) is satisfied by the null object \( A \);
- is-in is satisfied by an interpreter instruction;
- is-obj is satisfied by all elements of the set 0;
- is-sel \( \in \) SEL;
- is-integer is satisfied by the set of positive integers;
- SUCC = \{ succ(1), succ(2), --- \};
- is-arg-addr = (\langle \text{s-arg:is-integer}, \langle \text{s-pred:is-integer} \rangle \rangle).

Note that the "s-arg" selects an integer \( i \) indicating the \( i \)th element of an argument list is to be set to the value of this instruction. The "s-pred" selects an integer \( j \) indicating the \( i \)th element of the argument list of the \( j \)th predecessor node is to be set to the value of this instruction.

Keeping the structure of the machine state in mind, a transformation of a state in our sequentialized machine is effected by the execution of an instruction in any task available for execution. Furthermore in order to allow the interpreter to describe all possible implementations some arbitrariness in the selection of instructions, as in the Vienna interpreter, in tasks is a necessity. Two levels of nondeterminism in selecting the interpreter instruction to be executed are next presented.
In a system which allows concurrent execution, we assume the processor is assigned to the task whose execution at this point effects the most efficient use of the machine. The task scheduler picks such a task from the ready list of tasks. In a CONCOM that set of tasks is associated with the concurrent computations component of the state. Therefore, each time an instruction is executed the scheduler may be invoked to select a task to be executed. Hence the definition of the set of task names of tasks available for execution is given here.

**Definition 3.10**

The set

\[ TS(S) = \{ n \mid is-n(n) \& \neg is-A(s-c\cdot n\cdot s-cc(S)) \} \]

is called the set of task selectors of a given CONCOM.

The selection of an instruction for execution within a task is specified by a composite selector which selects the instruction at a terminal node of a task’s control tree. This set of control selectors is defined below.

**Definition 3.11**

The set

\[ CS(C) = \{ ISEL \mid ISEL \in SUCC^* \& ISEL\cdot s-c\cdot n\cdot s-cc \neq \Lambda \& \text{succ}(i)\cdot ISEL\cdot n\cdot s-cc = \Lambda, 1 \leq i \leq n \} \]

is called the set of control selectors of a given control tree C in a task n, where SUCC* is the set of all sequences of composite selectors that can be constructed from the set SUCC of Definition 3.9.
In a CONCOM each of the instructions at a terminal vertex of the control tree of each of the tasks identified by the set TS of Definition 3.10 in a state S may give rise to a different next state. The set of next states associated with a given state S will be denoted TF(S). The transformation function TF (of Definition 3.1) is effectively the next state function of the nondeterministic abstract machine called a CONCOM.

The instruction execution cycle of a CONCOM may be defined by specifying the function TF, which may then be defined in terms of the set of state transition functions associated with the terminal vertices of the control trees of all available tasks. Letting the state transition function be indicated by ST(S,ISEL,n), where the current state is S, the instruction to be executed is selected by ISEL, and the task containing the instruction to be executed is selected by n, then the transformation function can be defined formally.

**Definition 3.12**

The **transformation function** is defined as an element of the set of state transition functions

\[ \text{TF}(S) = \{ \text{ST}(S,\text{ISEL},n) \mid \text{ISEL} \in \text{CS}(s-c(n \cdot s-c(S))) \land n \in \text{TS}(S) \} \].

Since TF contains a state transition for every executable instruction in every available task, the execution of an instruction can be defined by specifying a selection rule for choosing an element of TF(S), together with a state transition function for individual instructions ST(S, ISEL, n). This selection rule is unspecified in the abstract machine since it corresponds to the algorithm for scheduling tasks. The state transition
function ST(S,ISEL,n) is defined in terms of the structure of the instruction -- illustrated in Figure 3.5 -- selected for execution.

Each terminal vertex of the control tree in task n has

1) an instruction component, \( \text{in} \), selected by "s-in*ISEL*s-c*n*s-cc(S)";

2) an argument list, \( \text{al} \), selected by "s-al*ISEL*s-c*n*s-cc(S)"; and

3) a return information component, \( \text{ri} \), selected by "s-ri*ISEL*s-c*n*s-cc(S)".

The argument list for an instruction with \( m_{in} \) arguments has \( m_{in} \) components selected by \( \text{elem}(1)\cdot\text{al}, \text{elem}(2)\cdot\text{al}, \ldots, \text{elem}(m_{in})\cdot\text{al} \).

The state transition function \( \text{ST}(S,ISEL,n) \) can be defined in terms of the following:

1) a function \( I_{in} \), which depends on the \( m_{in} \) arguments of the instruction;

2) the state \( S \) with the current instruction vertex deleted;

3) the selector \( ISEL \);

4) the selector \( n \) (task name); and

5) the return information component 
   "\( r_i = s-ri*ISEL*s-c*n*s-cc(S) \)".

Assume the instruction at the vertex \( ISEL*s-c*n*s-cc(S) \) has the following form:

\[
\begin{align*}
\text{inst}(x_1, x_2, \ldots, x_m) = & \quad p_1(x_1, x_2, \ldots, x_m, S) + a_1(x_1, x_2, \ldots, x_m, S) \\
& + p_2(x_1, x_2, \ldots, x_m, S) + a_2(x_1, x_2, \ldots, x_m, S) \\
& \quad \vdots \\
& + p_k(x_1, x_2, \ldots, x_m, S) + a_k(x_1, x_2, \ldots, x_m, S)
\end{align*}
\]
where \( p_1, p_2, \ldots, p_k \) are predicates and \( a_1, a_2, \ldots, a_k \) are the actions associated with the predicates. The state transition can now be defined formally.

**Definition 3.13**

The **state transition function** is defined by

\[
ST(S, ISEL, n) = I_n(elem(1) \cdot a_1, elem(2) \cdot a_1, \ldots, elem(m) \cdot a_1, \\
\delta(S; ISEL), ISEL, ri),
\]

where

\[
I_n(x_1, x_2, \ldots, x_n, S, ISEL, ri) = \\
p_1(x_1, x_2, \ldots, x_n, S) \Rightarrow a_1^*(x_1, x_2, \ldots, x_n, S, ISEL, ri) \\
p_2(x_1, x_2, \ldots, x_n, S) \Rightarrow a_2^*(x_1, x_2, \ldots, x_n, S, ISEL, ri) \\
\vdots \\
p_k(x_1, x_2, \ldots, x_n, S) \Rightarrow a_k^*(x_1, x_2, \ldots, x_n, S, ISEL, ri)
\]

and

\[
ISEL = "ISEL\cdot s-c\cdot n\cdot s-cc(S)"
\]

and

\[a_1^*\] is the state transition function associated with the action \( a_1 \) specified by the system designer (interpreter programmer),

and

\[\delta(S, ISEL)\] is the deletion operator of VDL which performs the function \( \mu(S; <isel:A>) \).

The state transitions \( a_1^* \) result from the execution of one of two kinds of instructions specified by \( a_1 \). If \( a_1 \) is a macro instruction the new state, \( S_{new} \), is obtained by replacing the "isel" component of the
state by $a_i$ as follows:

$$S_{\text{new}} = \mu(S; <\text{isel}; a_i>).$$

In the case $a_i$ is a value-returning instruction, its form is

$$\text{PASS: } e_0(x_1, x_2, \ldots, x_m, S)$$
$$s-sc_1: e_1(x_1, x_2, \ldots, x_m, S)$$
$$\vdots$$
$$s-sc_{\lambda}: e_{\lambda}(x_1, x_2, \ldots, x_m, S) .$$

The state transition $a_i$ involves evaluation of the expressions $e_i$, $i=0,1,\ldots,\lambda$. The value "val($e_0$)" is substituted in all addresses specified by $r_i$ and the values "val($e_i$)" , $i=1,\ldots,\lambda$, are substituted for each component $s-sc_i$. If $\text{new}_{\text{tct}}$ represents the control tree of the task being executed after the substitution of "val($e_0$)" in all return addresses, after modification of the specified state components the new state is defined as follows:

$$S_{\text{new}} = \mu(S; <s-c.n.s-cc(S):\text{new}_{\text{tct}}>,$$
$$<s-sc_1: \text{val}(e_1)>,$$
$$\vdots$$
$$<s-sc_{\lambda}: \text{val}(e_{\lambda})>).$$

The new control tree of the task currently being executed is the only component of the new state which is yet to be specified. Introduction of a function "pred"$^1$, which operates on composite selectors

---

$^1$This function is discussed by Wegner (23).
ISEL = s_1 * s_2 * --- * s_k and removes the rightmost selector, is necessary. Thus, pred(ISEL) = s_2 * s_3 * --- * s_k. The function pred^i has been defined as the i-fold composition of pred. Thus pred^i(ISEL) = s_{i+1} * s_{i+2} * --- * s_k.

If the components s-arg and s-pred of s-addr(ri) are respectively i and j, the control tree position to which val(c_e^i) is returned is defined by the following selector:

\[ s\text{-comp}(ri) \cdot \text{elem}(j) \cdot s\text{-al} \cdot \text{pred}^i(isel) \cdot s\text{-c} \cdot n \cdot s\text{-cc}(S) \]

The selection process is as follows:

1) "s\text{-c} \cdot n \cdot s\text{-cc}(S)" selects the control tree of the currently executing task n;
2) "\text{pred}^i(isel) \cdot s\text{-c} \cdot n \cdot s\text{-cc}(S)" selects predecessor node i of the current instruction being executed;
3) "\text{elem}(j) \cdot s\text{-al} \cdot \text{pred}^i(isel) \cdot s\text{-c} \cdot n \cdot s\text{-cc}(S)" selects the jth element of the argument list of predecessor i;
4) "s\text{-comp}(ri) \cdot \text{elem}(j) \cdot s\text{-al} \cdot \text{pred}^i(isel) \cdot s\text{-c} \cdot n \cdot s\text{-cc}(S)" selects the component of the element of the argument list to which the value is to be assigned.

**Definition 3.14**

The next state \( S_{\text{next}} \) of a CONCOM attained after execution of an instruction in state \( S \) is defined by the VDL statement...
where two cases exist.

1) if \( a_1 \) is a macro instruction then \( \ell = 0 \) and

\[
\text{new}_{\text{tct}} = \mu(\delta(s-c\cdot n\cdot s-cc(S);isel);a_1)
\]

2) if \( a_1 \) is a value-returning instruction then \( \ell = \text{number of expressions in the state transition function } a_1^* \) and

\[
\text{new}_{\text{tct}} = \mu(\delta(s-c\cdot n\cdot s-cc(S);isel);
\{<\text{comp}(i,j);\text{val}(e_i)> \mid s-\text{arg}\cdot s-\text{comp}(ri) = j
\& s-\text{pred}\cdot s-\text{comp}(ri) = i\},
\]

where

\( \text{comp}(i,j) \) is the selector

\( s-\text{comp}(ri)\cdot\text{elem}(j)\cdot s-\text{al}\cdot\text{pred}^i(isel) \)

\( s-c\cdot n\cdot s-cc(S) \).

The specification of a CONCOM is not complete at this point. The IS and FS components of the 6-tuple of Definition 3.1 have yet to be specified. IS is the initial state of a CONCOM. It specifies the contents of both global and local levels of the CONCOM before execution of any instructions. The set of final states, FS, contains the specification of the form of states \( S_1, S_2, \ldots, S_m \) which have no succeeding state. That is,

\[
\text{TF}(S_i) = \Lambda, \text{ for } i = 1, \ldots, m.
\]
The IS specifies one active task $n_1$ to execute a program. It initially has no accessible cells and no enclosing site of activity. Therefore, the denotation and attribute components of the state are null; and the environment and dump components of the task $n_1$ are null. But since a task $n_1$ is active, the current task component selects $n_1$; and the unique name generator component has value "2" so that no additional cell will have the unique name $n_1$.

**Definition 3.15**

The initial state of a CONCOM satisfies the predicate IS.

$$ IS = \mu_0(<s\text{-}task:n_1>, $$

$$ <s\text{-}n:2>, $$

$$ <s\text{-}cc:\mu_0(<s\text{-}c:\text{int-prog}(t)>)>), $$

where

\text{int-prog}(t) is an interpreter macro instruction whose expansion emanates the information structure transformations necessary to evaluate the program $t$ to be executed.

The set of final states all have one common attribute. The control trees of all currently active tasks are null. Therefore, the scheduling algorithm will find no elements in the set of next states, because no state transitions will be specified.

**Definition 3.16**

The set of final states of a CONCOM is a collection of objects which satisfy the predicate is-final-state.
FS = \{S \mid \text{is-final-state}(S)\},
where
\text{is-final-state}(S) =
\langle \text{s-den}\text{:is-den}, \text{s-at}\text{:is-at},
\langle \text{s-n}\text{:is-integer}, \text{s-task}\text{:is-n},
\langle \text{s-cc}\text{:\{n\text{:is-task} \mid s-c\cdot n\cdot s-cc}(S) = \lambda \rangle \rangle \rangle \).
CHAPTER IV.
CONCURRENT COMPUTATION PRIMITIVES

The control of concurrent computations emanates from the control structures available to the programmer. Of the control structures presented in Chapter II, techniques for implementing (and describing) transfer, procedure call, block entry, block exit and procedure exit are well understood and discussed elsewhere (18). It is the methods of implementing (and describing) the control of parallel activities (in the abstract machine) which are the concern of this and the succeeding chapter.

The implementation of control structures involves the establishment of an appropriate environment in which the instructions that perform the sequencing operations are to be executed. In the implementation of block structured languages, for example, a new environment must be established whenever a block is entered. In most block structured languages this establishment of a new environment can be described (and implemented) with the aid of a pushdown stack. The establishment of a new environment amounts to allocating (i.e. pushing) storage on top of the stack (together with the establishment of the proper linkage to items already in the stack).

In programming languages which accommodate parallel activities and in which environments may be shared among these parallel activities, great care must be taken when instructions interact with a common environment. In particular, in order that information not be incorrectly modified, certain precautions must be taken to insure that only one instruction modify a sharable environment at one time. Part of the guarantee that
this constraint is followed is obtained by assuming that certain instructions which control concurrent computations are uninterruptable. Such instructions will hereafter be referred to as primitive instructions. The main thrust of the current chapter will be a presentation of primitive instructions which initiate, terminate, and synchronize parallel activities (i.e. tasks).

Initiation of a parallel activity involves the creation of a special data structure called a task, followed by its placement on a special list called the ready list. In a CONCOM, the creation of a task is accomplished by creating a tree consisting of an environment component, a dump component, and a control component. This tree is assigned a unique name and attached as a subtree on the concurrent computations component of the state of the CONCOM. Placement on this component, selected by "s-cc", amounts to adding this task on the ready list. (Note that the initiation of a parallel activity results in the creation of a unique task with its own unique name. If the newly created task is to be associated with an identifier, this association must, of course, be made in some appropriate environment.)

Since the interruption of a task's creation and placement on the ready list might result in incorrect data structures, these actions must be made uninterruptable. Thus, a special primitive instruction "init-task" is defined to perform these functions. The execution of the instruction init-task(arg1, arg2, arg3, arg4) generates a new branch on the "s-cc" component of the machine state S. The arguments for the init-task
are defined as follows:

arg1 - the unique name of a task which is created for each task initiation;

arg2 - the data structure consisting of the cells accessible to the newly created task which are created or copied from an existing environment;

arg3 - the control tree (which is created in the form specified in Definition 3.9) containing the interpreter instructions which are to implement the specified computation;

arg4 - the dump component which is created (in the form specified by Definition 3.8) or copied from a denotation.

**Definition 4.1**

The primitive function which initiates a new task in a CONCOM is defined by the VDL instruction *init-task*.

\[
\text{init-task}(n, \text{CON}, \text{ENV}, \text{DUMP}) = \\
\text{s-cc} \cdot \mu(s-cc(S); <n, \mu_0( <s-env:ENV>, \\
\quad <s-c:CON>, \\
\quad <s-d:DUMP>)).
\]

where

- \( S \) is current machine state,
- \( n \) is the unique name of the task being created,
- \( \text{CON} \) is the control tree of the task being created,
- \( \text{ENV} \) is the environment in which the task is to be executed,
- \( \text{DUMP} \) is the stack of task local composite objects \( \mu_0(<s-c:is-c>, <s-d:is-d>, <s-env:is-env>) \) which defines the enclosing site of activity.
The action of terminating a task in a CONCOM is accomplished by deleting from the "s-cc" component of the state that tree identified by the task's unique name. In VDL this is accomplished by creating a new "s-cc" component which contains all tasks but the task to be terminated. Then when a task is selected for execution, from the set TF(S), the control tree of this task is null and not available for execution.

A word of caution is in order regarding the termination of tasks. If the task to be terminated is the only active task then the interpretation of the entire program will be stopped prematurely by termination of the task. Normally, the intent in such a case is that this (solely active) task be assigned to continue the ensuing (parent) computation. To accommodate this situation a counter is maintained which keeps track of the number of currently active tasks and is used in task termination.

Termination is accomplished by the execution of the CONCOM primitive \texttt{terminate} (arg), where arg is the count of the number of active tasks. It performs a decrement of the count; and if it then has value 1, this task is assigned the ensuing computation by default. If not 1, then this task is deleted from the ready list. Note the necessity for uninterruptible execution between decrementing the counter and removable from the list. \texttt{terminate} can now be defined formally.

\textbf{Definition 4.2}

The primitive function which terminates m-1 tasks in a CONCOM is defined by the VDL instruction \texttt{terminate}. 

terminate(ctr) =
\[(\text{ctr} \cdot \text{s-den}(S) \leq 1) \rightarrow\]
\[\text{s-den: } \mu(\text{s-den}(S);<\text{ctr}\cdot\text{s-den}(S)-1>)\]
\[T \rightarrow\]
\[\text{s-cc: } \mu(A;\{<n:n\cdot\text{s-cc}(S)> | n \neq \text{s-task}(S)\})\]
\[\text{s-den: } \mu(\text{s-den}(S);<\text{ctr}\cdot\text{s-den}(S)-1>)\),

where
"ctr" is the denotation of the count of the number of currently assigned tasks (initial value is m),
and "s-task" is the CONCOM state component which designates the task executing this primitive.

Communication between tasks in a CONCOM is provided by access to common variables. Tasks n₁ and n₂ can access common variables v₁, v₂, ..., vₘ if the "s-env" component of both n₁ and n₂ has selectors v₁, v₂, ..., vₘ and the unique names (cells) assigned to v₁, v₂, ..., vₘ are the same in both n₁ and n₂.

Since tasks can have access to common cells, each time a task wishes to send a message it merely sets the value of a common cell to a predetermined value. The receiving task needs only to check the value of the cell. If it has attained the agreed upon value, the message has been set and execution proceeds. If the cell does not contain the predetermined value, the receiving task must loop on the check until the value is attained.

But looping on the test, called a "busy" test, is inefficient; and no guarantee can be given that access to the common cells is mutually exclusive. That is, the value may be changed by some other task between the time it is tested and the time it may be set again, possibly to another predetermined value to indicate the receipt of the message.
Therefore, in order to provide mutually exclusive access to a cell, it is treated just like any other resource. Each resource has an associated use lock. All tasks which have access to the common cell must agree to synchronize their use of the common cell according to the following rules.

1) Each task must set the lock before referencing the cell. If the lock is already set, the task must block itself until the lock is reset.

2) Each task must not leave the lock set indefinitely. The task will reset the lock as soon as its exclusive access restriction is relaxed.

This mechanism provides for exclusive access to cells between the setting and resetting of the lock.

Note that the setting of a lock is really a request for a resource, and it must perform two functions. It must set the lock if it isn't presently set. If it is set, the current task must be blocked and remembered so that it may be resumed when the lock is cleared. Using the structure of a lock from Definition 3.4, the block of a task is indicated by placing it in the table of tasks ("s-ttpat" component of a lock denotation) associated with this lock. It must then be deleted from the ready list, since it cannot be executed. Since interruption between the steps of testing of a lock, setting it, and moving a task to a blocked list may cause incorrect synchronization, the requesting of a resource is a primitive action and is accomplished by the primitive, request (lock), specified in Definition 4.3.
**Definition 4.3**

The primitive function necessary for a task to request common resources is defined by the VDL instruction `request (lock)`.

\[
\text{request (lock)} = \\
(s\text{-lock} \cdot \text{lock} \cdot s\text{-den}(S)) \rightarrow \\
\text{s-den: } \mu(s\text{-den}(S); \\
\langle s\text{-ttab} \cdot \text{lock} \cdot s\text{-task}(S) \cdot s\text{-cc}(S) \rangle) \\
\text{s-cc: } \mu(\lambda; \\
\{<n; n \cdot s\text{-cc}(S) > \\
| n \neq s\text{-task}(S) \& \text{is-task}(n \cdot s\text{-cc}(S))) \} \\
T \rightarrow \\
\text{s-den: } \mu(s\text{-den}(S); \langle s\text{-lock} \cdot \text{lock} : \text{on} \rangle),
\]

where

"lock" is the unique name associated with the pertinent resource use lock.

If it is again observed that the setting of a lock is a request for a resource, then upon the release of a resource (resetting a lock) all tasks currently waiting for the use of the resource become candidates to seize the resource. It is natural then that a scheduler be associated with each category of resources (and thus their associated locks). Each scheduler then may pick from the set of blocked tasks, \( BT(\text{lock}) \),

\[
BT(\text{lock}) = \{ n \mid n \cdot s\text{-ttab} \cdot \text{lock} \cdot s\text{-den}(S) \neq \lambda \\
\& \text{is-task}(n \cdot s\text{-ttab} \cdot \text{lock} \cdot s\text{-den}(S)) \}.
\]

Each task whose name \( n \) is an element of the set, \( n \in BT(\text{lock}) \), is a candidate to obtain the resource.
Just as the requesting of a resource is a primitive action so is the releasing of a resource. The `release (lock)` instruction of Definition 4.4 specifies the action of the interpreter in resetting a lock or scheduling for execution one of the tasks on the blocked list for the resource use lock "lock". The function `sel(set)` of Definition 4.4 is left unspecified in this definition as it is the selection function on the set "set" which corresponds to the scheduling algorithm for the pertinent lock. Again, note that incorrect execution may occur if `release` is interrupted between the testing of the lock and movement of the task from the blocked to the ready list.

**Definition 4.4**

The primitive function necessary to allocate resources to competing tasks is described by the VDL instruction `release (lock)`.

```vdl
release (lock) =
    is-λ(s-ttab·lock·s-den(S))
    → s-den: μ(s-den(S);<s-lock·lock:"off">)
    T →
    s-cc: μ(s-cc(S);
          {<n:sel(BT(lock))·s-ttab·lock·s-den(S)>
           | is-task(n·s-ttab·lock·s-den(S))
           & n = sel(BT(lock))}
    s-den: μ(s-den(S);
          {<n·lock:∧>
           | is-task(n·s-ttab·lock·s-den(S))
           & n = sel(BT(lock))}
```
where

"sel" is a VDL primitive function which selects
a unique task name, from the set BT (lock),
whose task is to be scheduled for execution;

\[
BT(\text{lock}) = \{ n \mid n \cdot s - \text{ttab} \cdot \text{lock} \cdot s - \text{den}(S) \neq \emptyset \\
\quad \& \ \text{is-task}(n \cdot s - \text{ttab} \cdot \text{lock} \cdot s - \text{den}(S)) \};
\]

and "lock" is the denotation of the resource use lock
of the requested resource.
CHAPTER V.
APPLICATION OF THE ABSTRACT MACHINE

Chapter III presented the architecture of a class of abstract machines called CONCOMs. The design specifications were the following.

1) The machine must be able to direct the control of concurrent computations whose execution must synchronize at explicit points in time.

2) The structure must be sufficiently simple so that the underlying concepts of concurrent control are easily understandable.

In Chapter IV we presented a set of CONCOM primitive instructions which are sufficient for the control of concurrent computations. It is the purpose of this chapter to illustrate the applicability of the class of CONCOMs and the proposed primitives to the problem of concurrent control. Implementations of two block structured, multitasking programming languages, STAL and SIMPAL, are presented. That is, machines (CONCOMs) are specified which interpret the control structures of multitasking languages. The task control structures of STAL and SIMPAL differ significantly and these differences are reflected in their respective implementation models. The primitive task control mechanisms of Chapter IV are shown to be sufficient to implement both models. Note that the interpreter instructions which are common to single and multiple task languages are very similar to those presented in EPL (23). They are presented here only for completeness.

The first application of the abstract machine is to the interpretation of programs written in the simple tasking language STAL. Informally,
a STAL program consists of a single block containing a declaration part and a statement part. The declarations in a block may include variables having integer or logical values and declarations of procedures, labels or locks. Statements may be assignment statements, conditional statements, procedure calls, goto statements, blocks, and task control statements. The concurrent control statements of STAL are the fork, join, lock and unlock statements discussed in Chapter II. STAL does not have arrays and restricts actual parameters to identifiers. All procedure call parameters are passed by reference as in Fortran or PL/I. The principal productions of the abstract syntax of STAL are given in Table 5.1.

The specification of a CONCOM to interpret STAL programs, a machine which will be referred to as a STAL machine, assigns meaning to the programs. The interpreter instructions are described in VDL interspersed with an English description. The reader will note that a complete definition of the STAL machine is also illustrated in Appendix I, without English description, for more compact reference.

The execution of all STAL programs starts in the initial state, IS, of a CONCOM which has but one task active and only one terminal instruction vertex. The instruction selected for execution is int-prog(t),

\[(ST1) \quad \text{int-prog}(t) = \text{int-block}(t),\]

where \(t\) is a structure satisfying the predicate "is-prog" as defined in Table 5.1. "int-prog" is a macro-instruction which causes itself to be replaced by the instruction \(\text{int-block}(t)\), since a program is simply a block. It is defined by (ST1). Thus the execution of the instruction
<table>
<thead>
<tr>
<th>Rule</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A1)</td>
<td><code>is-program</code> = <code>is-block</code></td>
</tr>
<tr>
<td>(A2)</td>
<td><code>is-block</code> = <code>(&lt;s-dec-pt:is-dec-pt&gt;,&lt;s-st-list:is-st-list&gt;)</code></td>
</tr>
<tr>
<td>(A3)</td>
<td><code>is-dec-pt</code> = `([<a href="">id:is-attr</a></td>
</tr>
<tr>
<td>(A4)</td>
<td><code>is-attr</code> = <code>is-var-attr V is-proc-attr V is-lock-attr</code></td>
</tr>
<tr>
<td>(A5)</td>
<td><code>is-var-attr</code> = <code>{{INT, LOG}}</code></td>
</tr>
<tr>
<td>(A6)</td>
<td><code>is-proc-attr</code> = <code>(&lt;s-parm-list:is-id-list&gt;,&lt;s-st:is-st&gt;)</code></td>
</tr>
<tr>
<td>(A7)</td>
<td><code>is-lock-attr</code> = <code>LOCK</code></td>
</tr>
<tr>
<td>(A8)</td>
<td><code>is-label-attr</code> = <code>LABEL</code></td>
</tr>
<tr>
<td>(A9)</td>
<td><code>is-st</code> = <code>is-lab-st V is-unlab-st</code></td>
</tr>
<tr>
<td>(A10)</td>
<td><code>is-lab-st</code> = <code>(&lt;s-lab:is-id&gt;,&lt;s-st:is-unlab-st&gt;)</code></td>
</tr>
<tr>
<td>(A11)</td>
<td><code>is-unlab-st</code> = <code>is-assign-st V is-cond-st V is-proc-call</code></td>
</tr>
<tr>
<td>(A12)</td>
<td><code>is-assign-st</code> = <code>(&lt;s-lpart:is-var&gt;,&lt;s-rpart:is-expr&gt;)</code></td>
</tr>
<tr>
<td>(A13)</td>
<td><code>is-expr</code> = <code>is-const V is-var V is-bin</code></td>
</tr>
<tr>
<td>(A14)</td>
<td><code>is-const</code> = <code>is-log V is-integer</code></td>
</tr>
<tr>
<td>(A15)</td>
<td><code>is-var</code> = <code>is-id</code></td>
</tr>
<tr>
<td>(A16)</td>
<td><code>is-bin</code> = <code>(&lt;s-opl:is-expr&gt;,&lt;s-op?is-fixpr&gt;,&lt;s-op:is-op&gt;)</code></td>
</tr>
<tr>
<td>(A17)</td>
<td><code>is-op</code> = <code>{+, &amp;}</code></td>
</tr>
<tr>
<td>(A18)</td>
<td><code>is-cond-st</code> = <code>(&lt;s-expr:is-expr&gt;,&lt;s-then-cl:is-st&gt;,&lt;s-else-cl:is-st&gt;)</code></td>
</tr>
<tr>
<td>(A19)</td>
<td><code>is-proc-call</code> = <code>(&lt;s-id:is-id&gt;,&lt;s-arg-list:is-id-list&gt;)</code></td>
</tr>
<tr>
<td>(A20)</td>
<td><code>is-goto-st</code> = <code>is-id</code></td>
</tr>
<tr>
<td>(A21)</td>
<td><code>is-fork-st</code> = <code>is-id</code></td>
</tr>
<tr>
<td>(A22)</td>
<td><code>is-join-st</code> = <code>(&lt;s-ctr:is-var&gt;,&lt;s-lab:is-id&gt;)</code></td>
</tr>
<tr>
<td>(A23)</td>
<td><code>is-lock-st</code> = <code>is-id</code></td>
</tr>
<tr>
<td>(A24)</td>
<td><code>is-unlock-st</code> = <code>is-id</code></td>
</tr>
</tbody>
</table>
\texttt{int-prog}(t) always yields a new single vertex control tree containing \texttt{int-block}(t).

Execution of \texttt{int-block}(t) -- as defined in (ST2) -- involves four actions:

1) saving the task status on the pushdown component "s-d" of the executing task;

2) creation of a new four-vertex (one terminal vertex) control tree -- for the executing task -- that will update this task's environment table and the global denotation and attribute components of the state;

3) execution of the statement list of this task in its new environment;

4) exiting the block and restoring the saved environment.

The \texttt{int-block}(t) is a value-returning instruction which returns a null value and modifies the dump and control component of the executing task.

Before proceeding with the formal definition, some notational conveniences are described. The three components of a task are used often and their composite selectors will have the designated shorthand that follows:

"CON" will stand for the control tree of the currently executing task and its composite selector is "s-c\cdot s\cdot task(S)\cdot s-cc";

"ENV" will stand for the environment table of the currently executing task and its composite selector is "s-env\cdot s\cdot task(S)\cdot s-cc"; and "DUMP" will stand for the dump component of the currently executing task and its composite selector is "s-d\cdot s\cdot task(S)\cdot s-cc".
(ST2) \[\text{int-block}(t) =\]
\[
\text{DUMP: } \mu_0(\langle s\text{-}env: \text{ENV}(S)\rangle, \langle s\text{-}c: \text{CON}(S)\rangle, \langle s\text{-}d: \text{DUMP}(S)\rangle) \\
\text{CON: exit;} \\
\text{int-st-list}(s\text{-}st\text{-}list(t)); \\
\text{int-dec-pt}(s\text{-}dec\text{-}pt(t)); \\
\text{update-env}(s\text{-}dec\text{-}pt(t))
\]

After the execution of an int-block\((t)\), the control tree of the currently executing task will have the form shown in Figure 5.1. Since execution is from the terminal vertex, the environment of this task will first be updated by the variables declared in the current block. The \text{int-dec-pt} instruction of (ST7), when executed updates the denotation and attribute tables of the state. In the new environment, the statement list of the new block is then executed by the interpreter instruction \text{int-st-list} of (ST10). The \text{exit} instruction of (ST20) will cause the reinstatement of the control, environment, and dump components (of this task) which were effective at the time of block entry.

\[
\text{CON}(S) \\
\text{exit} \\
\text{int-st-list}(s\text{-}st\text{-}list(t)) \\
\text{int-dec-pt}(s\text{-}dec\text{-}pt(t)) \\
\text{update-env}(s\text{-}dec\text{-}pt(t))
\]

Figure 5.1. Control tree after block entry
Updating the environment by the \texttt{update-env} instruction,

\begin{equation}
\text{update-env}(t) = \null; \\
\quad (\text{update-id}(id,n); \\
\quad \quad n: \text{un-name} \mid id(t) \neq A),
\end{equation}

is the process of adding an entry for each variable declared in this block by the \texttt{update-id} instruction defined in \textit{(ST5)}. Execution of an \texttt{update-env}(t) for an argument, \( t \), which satisfies the predicate "is-dec-pt", generates a 2m-vertex tree whose predecessor node has a \texttt{null} instruction. The 2m-vertices consist of \( m \) \texttt{update-id}(id)\texttt{instructions} for the \( m \) variables declared in this block and \( m \) \texttt{un-name} instructions (defined in \textit{(ST4)}) to generate unique names for each of the declarations. The form of the control tree is given in Figure 5.2 for the declaration of the \( m \) identifiers \( id_1, id_2, \ldots, id_m \).

\begin{figure}[h]
\centering
\begin{tikzpicture}
\node (null) at (0,0) {null};
\node (id1) [below left of=null] {\texttt{update-id} (id_1, v_1)};
\node (id2) [below right of=id1] {\texttt{update-id} (id_2, v_2)};
\node (id3) [below right of=id2] {\ldots};
\node (idm) [below right of=id3] {\texttt{update-id} (id_m, v_m)};
\node (vn1) [below of=id1] {v_1: \texttt{un-name}};
\node (vn2) [below of=id2] {v_2: \texttt{un-name}};
\node (vnm) [below of=idm] {v_m: \texttt{un-name}};
\draw [->] (null) -- (id1);
\draw [->] (id1) -- (vn1);
\draw [->] (id2) -- (vn2);
\draw [->] (id3) -- (null);
\draw [->] (idm) -- (vnm);
\end{tikzpicture}
\caption{Control tree after execution of \texttt{update-env}(t), where the block \( t \) has \( m \) declarations}
\end{figure}
is a value-returning instruction which simply increments the unique name generator by 1 and returns the unique name whose subscript is the previous value of the component "s-d(S)" as defined in (ST4). Execution of instructions at vertices $v_1, v_2, \ldots, v_m$ in Figure 5.2 generate $m$ unique names which are returned to the parameter positions $v_1, v_2, \ldots, v_m$ of the update-id instruction.

Execution of the instruction update-id(id,n),

\[
\text{(ST5) } \quad \operatorname{update-id}(id,n) = \\
\text{ENV: } \mu(\text{ENV}(S);<id:n>),
\]

simply updates the environment of the currently executing task by the selector-object pair $<id:n>$, where $id$ satisfies the predicate "is-id" and $n$ is the unique name returned by un-name. When all of the $m$ update-id instructions have been executed, the control tree has only one terminal vertex and its instruction component is null. Execution of a null performs a no-operation function.

After deletion (execution) of update-env from the control tree, the int-dec-pt(t) instruction,
(ST6) \[ \text{int-dec-pt}(t) = \]
\[ \text{null; } \]
\[ \{ \text{int-dec}(\text{id}, \text{id}(t), s-st-list(t)) \mid \text{id}(t) \neq A \]
\[ \& \text{is-id}(\text{id}) \}, \]

becomes eligible for execution. Its argument, t, satisfies the predicate "is-dec-pt" and its function is to update the denotation and attribute tables, with the assumption that the environment component of the task has already been updated. Of the types of declarations considered, integer and logical do not require denotation updating; but lock, label, and procedure denotations must be initialized at block entry. All types require attribute table updating.

The \text{int-dec-pt}(t) instruction creates a control subtree with a null instruction at the root and a branch for each variable declared in this block whose vertex consists of an instruction \text{int-dec} for updating the attribute and denotation table entries as shown in (ST6). The first parameter of \text{int-dec}(\text{id}, \text{type}, \text{st}) specifies the variable name, the second parameter specifies the attributes of the declared variable, and the third specifies the statement list of the block. Note that the second argument, "\text{id}(t)"/, selects the attributes of declaration, which are defined by (A3) of Table 5.1.

The instruction \text{int-dec}(\text{id}, \text{attr}, \text{st})

(ST7) \[ \text{int-dec}(\text{id}, \text{attr}, \text{st}) = \]
\[ \text{is-var-atrr}(\text{attr}) \rightarrow \text{s-at: } \mu(s-at(S);<\text{id}\cdot\text{ENV}(S):\text{attr}>) \]
is-proc-attr(attr) → s-at: \( \mu(s-at(S);<id\cdot ENV(S):PROC>) \)
s-den: \( \mu(s-den(S);
<id\cdot ENV(S):v_0(<s-attr:attr>,
<s-env:ENV(S)>)>) \)

is-lock-attr(attr) → s-at: \( \mu(s-at(S);<id\cdot ENV(S):LOCK>) \)
s-den: \( \mu(s-den(S);<id\cdot ENV(S):"off">) \)

is-label-attr(attr) → update-lab(v,id,st);
v:find(id,st,1),

effects the updating of the attribute table for all the new unique names created for the declarations in this block. As described in (ST7) it sets denotations for lock, procedure, and label variables. But no value is set for integer and logical variables since no static initialization of variables is permitted. If the attribute parameter satisfies one of the predicates "is-proc-attr", "is-lock-attr", or "is-label-attr", then denotations which satisfy the predicates "is-proc-den", "is-lock-den", and "is-label-den", respectively, must be generated.

The environment of a procedure denotation is the current environment (after the update has taken place). Therefore, variables declared in the current block are known to the procedure. The only component of a lock denotation which is set at declaration time is its logical value. It is set to "off" since it is associated with a resource that is yet to be requested.

The label denotation has dump, environment, and statement list components which must be set at declaration time. The environment and dump components are set from the current task's environment and dump components.
But the statement list must be that list of statements starting at the specified label and all the statements following in the statement list of this block. Therefore, an instruction \( \text{find}(id, st, i) \),

\[
\text{find}(id, st, i) = \\
(\text{length}(st) = i \lor \text{s-lab} \cdot \text{elem}(i) \cdot \text{st} = \text{id}) \\
\rightarrow \text{PASS: } i \\
T \rightarrow \text{find}(id, st, i+1),
\]

is executed iteratively, starting at the first statement of this block, to search for the index of the statement whose label is "id". The value returned is the index of the first such label or the index of the last statement in case the label does not exist in this block.

The instruction \( \text{update-lab}(n, id, st) \),

\[
\text{update-lab}(n, id, st) = \\
\text{s-at: } \mu(s \cdot \text{at}(S); \langle id \cdot \text{ENV}(S) : \text{LABEL} \rangle) \\
\text{s-den: } \mu(s \cdot \text{den}(S); \nu_0 \langle id \cdot \text{ENV}(S) : \\
\nu_0(\langle \text{s-env} : \text{ENV}(S) , \langle \text{s-d} : \text{DUMP}(S) , \\
\text{s-st-list: } \\
\nu(\lambda : \langle \text{elem}(i) : \text{elem}(j) \cdot \text{st} \\
| i < j < \text{length}(st) \& j > n \rangle) \rangle)),
\]

sets the attribute table entry for the unique name associated with id, sets the environment and dump components from the currently active ones, and sets the statement list component to be the statement identified by "id" and all following statements in this block.

After the execution of all the \text{int-dec}, \text{find}, and \text{update-lab} instructions have been executed, the control tree contains an \text{int-st-list}(t)
instruction at its terminal vertex. The argument \( t \) satisfies the predicate "is-st-list"; and execution of the statements in this block in the order in which they fall in the list is specified by the definition of \( \text{int-st-list} \),

\[
\text{(ST10)} \quad \text{int-st-list}(t) =
\begin{align*}
\text{is-\Lambda}(t) & \Rightarrow \text{PASS: } \Lambda \\
T & \Rightarrow \text{int-st-list}(\text{tail}(t)); \\
\text{int-st}(\text{head}(t)).
\end{align*}
\]

This instruction (ST10) causes execution of the first statement of the list followed by the recursive execution of \( \text{int-st-list} \) for the remaining statements.

Enough basis has been laid to now consider the execution of instructions (specified by the programmer) in the environment created so far. The types of instructions possible are given in Table 5.1 in (All). In defining \( \text{int-st} \),

\[
\text{(ST11)} \quad \text{int-st}(t) =
\begin{align*}
\text{is-assign-st}(t) & \Rightarrow \text{int-assign-st}(t) \\
\text{is-cond-st}(t) & \Rightarrow \text{int-cond-st}(t) \\
\text{is-proc-call}(t) & \text{ \& } (\text{at}_t = \text{PROC}) \\
& \Rightarrow \text{int-proc-call}(t) \\
\text{is-goto-st}(t) & \text{ \& } (\text{lat}_t = \text{LABEL}) \\
& \Rightarrow \text{int-goto-st}(t) \\
\text{is-fork-st}(t) & \text{ \& } (\text{lat}_t = \text{LABEL}) \\
& \Rightarrow \text{int-fork-st}(t)
\end{align*}
\]
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\[
is\text{-join-st}(t) \& (lat^t = \text{LABEL}) \\
\& (s-\text{ctr}(t)(\text{ENV}(S)) \cdot s-at(S) = \text{INT}) \\
\rightarrow \text{int-join-st}(t)
\]

\[
is\text{-lock-st}(t) \& (at^t = \text{LOCK}) \\
\rightarrow \text{int-lock-st}(t)
\]

\[
is\text{-unlock-st}(t) \& (at^t = \text{LOCK}) \\
\rightarrow \text{int-unlock-st}(t)
\]

\[
is\text{-block}(t) \rightarrow \text{int-block}(t),
\]

assume the abbreviation "at^t" for the composite selector 
"s-id(t)(\text{ENV}(S)) \cdot s-at(S)" -- which selects the attributes of the procedure identifier and lock identifier "id" in a procedure call and lock or unlock statement t, respectively. Furthermore, assume the abbreviation "lat^t" denotes the selection of the label component of a fork, join or goto statement t which has the composite selector "s-lab(t) \cdot \text{ENV}(S) \cdot s-at(S)". Note that a procedure call, a lock and an unlock statement are valid only if the identifier specified as the procedure identifier and lock identifier, respectively, (selected by "at^t") have the PROC and LOCK declaration attributes. In the same vein -- error checking -- the label identifier of a fork, join and goto statement must have a LABEL attribute from its declaration or these statements are not legitimate. Verification that the counter specified in a join statement is an integer is accomplished before interpretation of the join.

The execution of an assignment statement involves the evaluation of the expression, which constitutes its right part, and assignment of its value to the denotation of the unique name associated with the left part, a variable, of the statement. Execution of a conditional statement
proceeds with evaluation of the logical expression followed by the execution of the then-clause if the value is true and execution of the else-clause if false. The procedure call is effected by saving the current environment, dump, and control of the currently executing task and installing the environment and control of the specified procedure. Execution of a transfer instruction must install the environment, dump, and control of the label specified in the statement. The denotation of the label contains this information. Execution of a block involves recursive execution of the int-block instruction of (ST2).

The execution of the multitasking control feature fork of STAL involves creating a new task -- with its own environment, dump, and control components -- and making this task ready for execution. Execution of a join instruction must delete the currently executing task from the system if it is not the last task to complete its computation in this concurrent computation specification. Deletion from the system involves destroying all aspects (components) of the task. Task synchronization is accommodated via execution of the lock instruction -- which moves the task requesting an unavailable resource to a blocked list for the associated resource use lock -- and the execution of the unlock instruction -- which controls allocation of the resource of the specified resource use lock.

Assuming the abbreviation "n_t" for the selector "s-lpar(t)•ENV(S)", which selects the denotation of the unique name of the identifier of the variable in the left part of an assignment statement, we define the
int-assign-st instruction,

(ST12) \[
\text{int-assign-st}(t) = \\
is-\text{var-attr}(n^t(s-\text{at}(S))) \\
\rightarrow \text{assign}(n^t,v); \\
v:\text{int-expr}(s-\text{rpart}(t)) \\
T \rightarrow \text{error}.
\]

It expands into the evaluation of the right part expression by the \text{int-expr} of (ST14) followed by the assignment to \(n^t\) by \text{assign} of (ST13). Note that a valid assignment statement exists only if the attribute of \(n^t\) is integer or logical.

Execution of the \text{assign} statement,

(ST13) \[
\text{assign}(n,v) = \\
s-\text{den}: \mu(s-\text{den}(S); \\
\langle n: \text{convert}(v,n(s-\text{at}(S)))\rangle),
\]

where "convert\((n,v)\)" is a VDL primitive function which converts the value \(v\) to the representation specified in the attribute table for the unique name \(n\),

updates the denotation of \(n^t\) by the value returned from the \text{int-expr} instruction.

Execution of \text{int-expr}(t)

(ST14) \[
\text{int-expr}(t) = \\
is-\text{bin}(t) \rightarrow \text{int-bin-op}(s-\text{op}(t),a,b); \\
a:\text{int-expr}(s-\text{op1}(t)), \\
b:\text{int-expr}(s-\text{op2}(t))
\]
is-var(t) &
is-var-attr(n_t·s-at(S))
→ PASS: n_t·s-at(S))
is-const(t) → PASS: val(t)
T → error,

evaluates binary operations, variables and constants whose structures are
given by (A14), (A15), and (A16) of Table 5.1. The evaluation of a
binary operation involves the macro expansion which evaluates both operands,
which are expressions, and then the application of the operator to these
values. Constant evaluation involves only returning the value of the
constant; and variable evaluation involves the return of the value of the
variable, in the current environment, from the denotation table. Note
the "n_t" is now an abbreviation for the unique name of the variable in
the current environment and is the selector "t·ENV(S)".

The application of an operator to its operands is specified by

\[
\text{int-bin-op},
\]

(ST15) \hspace{1cm} \text{int-bin-op}(op, op1, op2) =

\hspace{1cm} op = '+' → PASS: op1 + op2

\hspace{1cm} op = '&' → PASS: op1 & op2.

Interpretation of the conditional statement t is specified by

\[
\text{int-cond-st}(t),
\]

(ST16) \hspace{1cm} \text{int-cond-st}(t) =

\hspace{1cm} \text{branch}(v, s-then-cl(t), s-else-cl(t));

\hspace{1cm} v::\text{int-expr}(s-expr(t)).
It is defined using an instruction \texttt{branch}(v,a,b),

\begin{align*}
\text{(ST17)} \quad \text{branch}(v,\text{st1},\text{st2}) &= \\
&= \text{convert}(v,\text{LOG}) \rightarrow \text{int-st}(\text{st1}) \\
&\quad T \rightarrow \text{int-st}(\text{st2})
\end{align*}

which selects which statement, \(a\) or \(b\), is to be executed depending upon the value \(v\) to be true or false, respectively.

Execution of a \texttt{goto} statement must install in the environment and dump components of the currently executing task, the environment and dump components of the block in which the label is declared. The denotation of a label (in the form given in Definition 3.4) has been set by interpreter instruction (ST9); and their assignments to the current components is necessary. Use of the abbreviations "lenv" and "ldump" for the selection of the label's environment, "s-env\_lab\_s-den(S)", and for the selection of the label's dump component, "s-d\_lab\_s-den(S)", respectively, allow for the definition of \texttt{int-cond-st}(t) in (ST18). Note that "lab" is an abbreviation for "s-lab(t)\_ENV(S)" which selects the unique name associated with the label of the goto statement.

The specification of the control component for execution at the specified label is accomplished by placing an \texttt{int-st-list}(t) on the current control tree, where \(t\) is the list of statements to be executed in the block in which the label is defined. This list of statements is specified in the "s-st-list" component of the label's denotation. After execution of the block in which the label is declared, restoration of the control, environment, and dump will be accomplished by execution of the
exit statement defined in (ST19). Note that the control component must be of the form specified in Definition 3.9 and is constructed in such a manner in the \textit{int-goto-st},

\begin{align*}
\text{(ST18) } \quad \text{int-goto-st}(t) = \\
\quad \text{ENV: } \text{lenv} \\
\quad \text{DUMP: } \text{ldump} \\
\quad \text{CON: } \mu_0(<s\text{-in:exit}, \\
\quad \quad <\text{succ}(1): \\
\quad \quad \quad \mu_0(<s\text{-in:ist-list}> \\
\quad \quad \quad \quad <s\text{-al:}\mu_0(<\text{elem}(1):s\text{-st-list}\cdot\text{lab}>)>)>).}
\end{align*}

The structure of the task $n_i$ after execution of a goto statement is illustrated in Figure 5.3.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{task_structure.png}
\caption{Task structure after execution of a transfer statement in task $n_i$}
\end{figure}
Interpretation of a procedure call involves pushing down one environment, installing a second environment, and updating the new environment with the formal/actual parameter correspondences. The parameter t of the instruction \texttt{int-proc-call(t)},

\begin{equation}
\texttt{int-proc-call(t)} =
\begin{cases}
(\text{length(arglist) = length(plist)}) \\
\text{DUMP: } \mu_0(<s\text{-env:ENV(S)>}, \\
\quad <s\text{-c:CON(S)>}, \\
\quad <s\text{-d:DUMP(S)>}) \\
\text{ENV: } \mu(\text{env;} \\
\quad \{<\text{elem}(i)(plist) \\
\quad :\text{elem}(i)(arglist)(\text{ENV(S)}) > \\
\quad | \ 1 \leq i \leq \text{length(plist)})\}) \\
\text{CON: } \texttt{exit;} \\
\quad \texttt{int-st(st)}
\end{cases}
\end{equation}

satisfies the predicate "is-proc-call" and has the syntax of (A17) of Table 5.1. The procedure denotation selected by "s-id", as defined in Definition 3.4, is quite complex; and introduction of the following abbreviations simplifies the definition of \texttt{int-proc-call}.

1) Let "n = s-id(t)•ENV(S)" denote the unique name associated with the procedure identifier of the calling procedure.

2) Let "den = n•s-den(S)" denote the selection of the procedure denotation that has the syntactic form of Definition 3.4.

3) Let "plist = s-parm-list(t)•s-attr(den)" select the list of formal parameters of the procedure denotation.
4) Let "env = s-env(den)" select the environment of the procedure denotation.

5) Let "arglist = s-arg-list(t)" select the actual parameter list of the procedure call statement.

6) Let "st = s-st·s-attr(den)" select the statement of the procedure denotation.

Execution of the \texttt{int-proc-call} proceeds only if the length of the actual and formal parameter lists are equal. Then the current task's components are placed in the dump; the environment of the procedure statement is installed; it is updated by the actual parameters; the control component is set to interpret the procedure statement "st"; and return from the procedure is accomplished by placing the \texttt{exit} statement on the control tree. Note that the updating of the environment by the actual parameters is accomplished by passing the unique name \(n\) of the actual parameter. In VDL this involves selection of the selector-object pair <\texttt{id}:n>, where "id" is the identifier of the actual parameter from the current environment "ENV(S)" and attaching it to the new environment "env". The control tree of the currently executing task \(n_1\) after a procedure call is illustrated in Figure 5.4.

```
s-c\cdot n_1\cdot s-cc(S)
  \begin{array}{ll}
    \text{exit} \\
    \text{int-st(st)}
  \end{array}
```

Figure 5.4. Control tree after the execution of a procedure call in task \(n_1\)
Both block entry and procedure call control structures — as defined in (ST2) and (ST19), respectively — save the control, environment, and dump components of the current task. The `exit` instruction,

(ST20) \[
\text{exit} = \\
\text{ENV: } s\text{-env}(\text{DUMP(S)}) \\
\text{CON: } s\text{-c}(\text{DUMP(S)}) \\
\text{DUMP: } s\text{-d}(\text{DUMP(S)}),
\]

restores these components. Each component is restored from the current dump component.

In the following discussion of STAL instructions to initiate, terminate and synchronize computations, the STAL program \( P \),

1. \( P: \) \text{begin} \text{ label} \text{ wl,w2,x;}
2. \text{integer} \text{ t; lock} \text{ w;}
3. \text{t:=} 3;
4. \text{fork} \text{ wl;}
5. \text{fork} \text{ w2;}
6. \text{stl;}
7. \text{lock} \text{ w;}
8. \text{PC;}
9. \text{unlock} \text{ w;}
10. \text{st2;}
11. \text{join} \text{ t,x;}
12. \text{wl: st3;}
13. \text{lock} \text{ w;}
14. \text{PC;}
15. \text{unlock} \text{ w;}
16. \text{st4;}
17. \text{join} \text{ t,x;}

will be used to illustrate such actions. In program P "sti" stands for STAL statement i and "PC" stands for program code consisting of STAL statements. The program specifies 3 concurrent computations followed by 1 computation indicated by "REST". Access to the program code PC is mutually exclusive.

In Chapter II we said that execution of the fork statement involves the creation of a new task to be assigned to the computation specified at the label "ψ". And that execution of the current task is to continue at the following statement. Therefore, the expansion of int-fork-st,

\[
\text{(ST21)} \quad \text{int-fork-st}(t) = \\
\quad \text{init-task}(v, \mu_0(\langle s-\text{in:exit}, \langle \text{succ}(i):\mu_0(\langle s-\text{in:int-st-list}, \langle s-\text{al:st} \rangle) \rangle), \\
\quad \text{lenv,ldump});} \\
\quad v: \text{un-name},
\]

into the task initiation primitive init-task of Definition 4.1 and the name function un-name effects the creation of a task with a unique name. Following execution of un-name and init-task, both the new task and current task are available for execution; and one of the terminal vertices of the current task's control tree specifies execution of the next statement. An illustration of the concurrent computations component of the
state is given in Figures 5.5 and 5.6 before and after initiation of a task, respectively. In Figure 5.5 interpretation of \textit{int-st(fork wl)} at line 4 of program P has already been executed.

The environment and dump components of the new task are the environment and dump components of the denotation of the specified label "\texttt{x}". The abbreviations "lenv" and "ldump" are used – as in (ST18) – to select the respective components which are passed as arguments "arg3" and "arg4" to \texttt{init-task} (arg1, arg2, arg3, arg4). "arg1" is the unique task name returned from \texttt{un-name}. "arg2" has the structure of a control tree of Definition 3.9 as illustrated in Figure 5.6. Again the abbreviation "\texttt{st}" is used as selector of the statement component of the label's denotation.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{control_tree.png}
\caption{Control tree of task $n_1$ before task initiation}
\end{figure}
Figure 5.6. Concurrent computations component of state S after initiation of task $n_j$ in task $n_i$

Execution of the $\text{int-join-st}(t)$,

$$(ST22) \quad \text{int-join-st}(t) = \text{int-goto-st}(s\text{-lab}(t));$$

$$\quad \text{terminate}(s\text{-ctr}(t) \cdot \text{ENV}(S))$$

first of all decrements the counter which indicates the number of presently active tasks; and if the counter is nonzero, the current task is terminated. The argument $t$ satisfies the predicate "is-join-st" of (A22) in Table 5.1. The denotation of the associated counter is selected by "$s\text{-ctr}(t) \cdot \text{ENV}(S)$". The execution of the task termination primitive $\text{terminate}(ctr)$, where "ctr" is the counter's unique name, now performs the decision-making function. If the counter is zero, then the
int-goto-st(id) statement of (ST18) continues the execution of the current task at the label indicated by "id". An illustration of task termination is presented in Figure 5.7. Figure 5.7a displays the structure of the concurrent computations branch of the state of the STAL machine with the following assumptions.

1) Program P on page 65 is being executed.

2) The "join t,x" statement at lines 11, 17, and 19 has been interpreted by the int-join-st of (ST22) in the respective tasks n_j, n_j and n_k.

3) The unique name assigned to the variable "t" is "n_k".

Figure 5.7b gives a snapshot of the machine state after execution of terminate (n_k) in task n_i.

Execution of the int-lock-st(t),

(ST23) int-lock-st(t) = request (t·ENV(S)),

assures mutually exclusive access to the program code PC enclosed by the lock w and unlock w instructions discussed in Chapter II. Since exclusive access to PC by several tasks must be granted by a scheduler, the code PC is regarded as a resource; and the associated lock, whose identifier is specified as the argument t of the int-lock-st(t) in (ST23), is regarded as the associated resource use lock. The interpretation of a lock statement is then a request to use the resource PC, and its associated lock must indicate its status. The request primitive of Definition 4.3 performs this function when the unique name of the lock in statement t is
Figure 5.7a. State of the STAL machine after execution of the `int-join-st` in tasks $n_i$, $n_j$, and $n_k$.

Figure 5.7b. State of STAL machine after execution of the `terminate(n_k)` instruction in task $n_i$. 
passed as an argument. Since the abstract syntax of a lock statement consists of only the lock identifier, "t*ENV(S)" selects its unique name.

Execution of the int-unlock-st(t) instruction,

\[
\text{int-unlock-st}(t) = \text{release}(t*\text{ENV}(S))
\]

is interpreted as the release of the resource PC. Of course, during the time that the currently active task had possession of PC, many tasks may have requested it. Therefore since the release of PC must activate only one of the tasks awaiting use of PC, a scheduler must be invoked to allocate PC. The task synchronization primitive release of Definition 4.4 performs just this function. The unique name of the lock is selected by "t*ENV(S)", as in (ST23).

Assuming the assignment of the code PC (at line 8 of program P) to a task \( n_i \), the request for PC (at line 14 of program P) by task \( n_j \) effects the placement of \( n_j \) on the list of tasks waiting for PC. The sequence of Figures 5.8, 5.9, and 5.10 illustrate the request by \( n_j \) for PC, its waiting for access, and the release of PC by \( n_i \), respectively. Assume the unique name of the associated resource use lock \( w \) in \( n_m \).

The 24 interpreter instructions (ST1) through (ST24) of Appendix I and the 4 task primitive instructions of Chapter IV constitute a complete definition of the multitasking language STAL using the Vienna method. This definition assumes that programs are represented by an abstract syntax in an "intermediate language" that is independent of a specific linear representation but exhibits the operator operand structure of expressions.
Figure 5.8. State of the STAL machine just previous to execution of program code PC in task $n_1$.

Figure 5.9. State of STAL machine after request for program code PC by task $n_j$. 
The semantics is defined by specifying the state transformations to which source programs give rise when they are executed in a CONCOM.

Therefore, a CONCOM has been exhibited whose structure is sufficient to control the execution of concurrent computations specified via the control structures of STAL. In order to show the generality of the CONCOM, an additional example of its application is presented. Using the same task primitive instructions, the semantics of a simple parallel processing language SIMPAL is given.

SIMPAL, like STAL, is a block structured language. The syntactic structure of the two languages differs only in the multitasking control features available to the programmer. Therefore, the state transitions in the SIMPAL machine differ from those of the STAL machine only when
specifying the interpretation of task initiation, task termination, and task synchronization. The entire syntax of SIMPAL is presented in Table 5.2 and a complete definition of the SIMPAL machine is given in Appendix II.

The initiation of m tasks in SIMPAL is specified by the special bracket pair `parbegin` and `parend` surrounding the m concurrent computations which are statements in SIMPAL. This construct has been illustrated in Chapter II. The implementation of SIMPAL will accommodate the initiation of n parallel activities during interpretation of `parbegin`, whereas the STAL machine initiated only 1 additional task upon interpretation of a `fork`.

Only 1 task is to be assigned the computation specified by the program component following a `parend`. As in STAL any task (the last one to complete its computation) is a candidate to continue execution; but unlike STAL, no explicit `join` statement is specified for each computation. The burden of termination is placed, therefore, on the implementation.

The synchronization of tasks in SIMPAL is more implicit (natural) than STAL. The critical region structures discussed in Chapter II are implemented. The statement

```
region v do S
```

implies the lock `v` and the resource, statement `S`. The statement

```
region v when B do S
```

implies conditional execution of `S`. This implementation will accommodate
Table 5.2. Syntax of SIMPAL

<table>
<thead>
<tr>
<th>Rule</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A1')</td>
<td>is-program = is-block</td>
</tr>
<tr>
<td>(A2')</td>
<td>is-block = (<a href="">s-dec-pt:is-dec-pt</a>,<a href="">s-st-list:is-st-list</a>)</td>
</tr>
<tr>
<td>(A3')</td>
<td>is-dec-pt = (#{<a href="">id:is-attr</a></td>
</tr>
<tr>
<td>(A4')</td>
<td>is-attr = is-var-attr V is-proc-attr V is-lock-attr V is-label-attr</td>
</tr>
<tr>
<td>(A5')</td>
<td>is-var-attr = {INT,LOG}</td>
</tr>
<tr>
<td>(A6')</td>
<td>is-proc-attr = (<a href="">s-parm-list:is-id-list</a>,<a href="">s-st:is-st</a>)</td>
</tr>
<tr>
<td>(A7')</td>
<td>is-lock-attr = LOCK</td>
</tr>
<tr>
<td>(A8')</td>
<td>is-label-attr = LABEL</td>
</tr>
<tr>
<td>(A9')</td>
<td>is-st = is-lab-st V is-unlab-st</td>
</tr>
<tr>
<td>(A10')</td>
<td>is-lab-st = (<a href="">s-lab:is-ld</a>,<a href="">s-st:is-unlab-st</a>)</td>
</tr>
<tr>
<td>(A11')</td>
<td>is-unlab-st = is-assign-st V is-cond-st V is-proc-call V is-goto-st V is-par-st V is-crit-reg V is-cond-crit-reg V is-block</td>
</tr>
<tr>
<td>(A12')</td>
<td>is-assign-st = (<a href="">s-lpart:is-var</a>,<a href="">s-rpart:is-expr</a>)</td>
</tr>
<tr>
<td>(A13')</td>
<td>is-expr = is-const V is-var V is-bin</td>
</tr>
<tr>
<td>(A14')</td>
<td>is-const = is-log V is-integer</td>
</tr>
<tr>
<td>(A15')</td>
<td>is-var = is-id</td>
</tr>
<tr>
<td>(A16')</td>
<td>is-bin = (<a href="">s-op1:is-expr</a>,<a href="">s-op2:is-expr</a>,<a href="">s-op:is-op</a>)</td>
</tr>
<tr>
<td>(A17')</td>
<td>is-op = {+,&amp;}</td>
</tr>
<tr>
<td>(A18')</td>
<td>is-cond-st = (<a href="">s-expr:is-expr</a>,<a href="">s-thfn-cl:is-st</a>,<a href="">s-else-cl:is-st</a>)</td>
</tr>
<tr>
<td>(A19')</td>
<td>is-proc-call = (<a href="">s-id:is-id</a>,<a href="">s-arg-list:is-id-list</a>)</td>
</tr>
<tr>
<td>(A20')</td>
<td>is-goto-st = is-id</td>
</tr>
<tr>
<td>(A21')</td>
<td>is-par-st = is-st-list</td>
</tr>
<tr>
<td>(A22')</td>
<td>is-crit-reg = (<a href="">s-lock:is-id</a>,<a href="">s-st:is-st</a>)</td>
</tr>
<tr>
<td>(A23')</td>
<td>is-cond-crit-reg = (<a href="">s-lock:is-id</a>,<a href="">s-expr:is-expr</a>,<a href="">s-st:is-st</a>)</td>
</tr>
</tbody>
</table>
the evaluation of B and execution of S as a resource protected by lock "v". Repetitive protected testing of the value of B is specified since another computation may manipulate a value in the expression.

Due to the similarity of the machines, interpreter instructions (ST1) - (ST10) and (ST12) - (ST20) of the STAL machine specify the same state transitions as instructions (SII) - (SI10) and (SI12) - (SI20) of the SIMPAL machine, respectively. Therefore, specification of the SIMPAL machine begins with (SII), the interpretation of a statement by int-st,

\[\begin{align*}
\text{(STII)} & \quad \text{int-st}(t) = \\
& \quad \text{is-assign-st}(t) \rightarrow \text{int-assign-st}(t) \\
& \quad \text{is-cond-st}(t) \rightarrow \text{int-cond-st}(t) \\
& \quad \text{is-proc-call}(t) \& (\text{at} = \text{PROC}) \\
& \quad \rightarrow \text{int-proc-call}(t) \\
& \quad \text{is-goto-st}(t) \& (\text{lat} = \text{LABEL}) \\
& \quad \rightarrow \text{int-goto-st}(t) \\
& \quad \text{is-par-st}(t) \& \text{is-st-list}(s-st-list(t)) \\
& \quad \rightarrow \text{int-par-st}(t) \\
& \quad \text{is-crit-reg}(t) \& \text{is-st}(s-st(t)) \\
& \quad \& (\text{s-lock}(t)(\text{ENV}(S)) \& \text{s-at}(S) = \text{LOCK}) \\
& \quad \rightarrow \text{int-crit-reg}(t) \\
& \quad \text{is-cond-crit-reg}(t) \& (\text{s-lock}(t)(\text{ENV}(S)) \& \text{s-at}(S) = \text{LOCK}) \\
& \quad \& \text{is-st}(s-st(t)) \& \text{is-expr}(s-expr(t)) \\
& \quad \rightarrow \text{int-cond-crit-reg}(t) \\
& \quad \text{is-block}(t) \rightarrow \text{int-block}(t) \\
& T \rightarrow \text{error}.
\end{align*}\]

It is important to note that the abstract syntax representation of the construction
parbegin

st1;
st2;
.
.
.stn;
parbegin

specifies only a statement list. And the abstract syntax of a critical
region statement specifies two components, the lock and the statement;
while the conditional critical region statement specifies an expression
as a third component. The interpretation of each statement type is ini­
tiated only after each component of the statement has been validated
according to the syntax in Table 5.2.

The concurrent control statement in SIMPAL which creates new tasks
is a sequence of statements bracketed by parbegin and parend. The com­
 pound statement is regarded as a specification of parallel activities and
its structure satisfies the predicate "is-par-st". The assignment of a
unique task to execute each of the parallel statements is accomplished by
the int-par-st(t) instruction

(S121) \[
\begin{align*}
\text{int-par-st}(t) &= \text{terminate}(k); \\
&\quad \text{create-tasks}(t); \\
&\quad \text{initialize}(k, \text{length}(t)); \\
&\quad k: \text{un-name}.
\end{align*}
\]

In the following discussion the execution of the SIMPAL program Q,
is illustrated in Figures 5.11, 5.12, and 5.13. It is intended to specify the same concurrent computations as program P on page 65.

The environment and dump components of each task are taken as the environment and dump components of the initiating task and selected respectively by "ENV(S)" and "DUMP(S)". The m parallel activities require the creation and assignment of m-1 tasks to m-1 parallel activities and the assignment of the current task to the last activity. The unique names for the created tasks are generated by the un-name instruction.

Every task is a candidate to continue execution of the program after completion of all the m concurrent computations. Therefore, every control tree must contain the ensuing instructions. But the placement of the terminate(k) task primitive instruction (of Definition 4.2) on the
successor node of such instructions precludes all tasks executing the ensuing statements. With the value of the unique name $k$ set to the number of parallel activities, $\text{terminate}(k)$ assures that only the last task to complete its computation will execute the ensuing statements. Figure 5.11 displays the state of the SIMPAL machine after assignment of the 3 tasks when initiated from task $n_1$. (Note that the assignment of task names to tasks is arbitrary. This particular implementation has chosen to assign the parent (initiating) task to the last computation.)

![Diagram of SIMPAL machine state](image)

Figure 5.11. State of the SIMPAL machine after assignment of 3 tasks to the 3 concurrent computations of program $Q$
Creation of a denotation to contain the value of the number of parallel activities is accomplished, prior to task assignment, by invocation of \texttt{un-name}; and its initial value is set to the length of the parallel statement list by the instruction \texttt{initialize}(k,length),

\begin{equation}
\text{initialize}(k,l) = \ s-den: \ \mu(s-den(S);<k:l>).
\end{equation}

The \(m-1\) tasks are created and assigned by the \(m-1\) \texttt{init-task} instructions generated by the instruction \texttt{create-tasks},

\begin{equation}
\text{create-tasks}(stlist) = \null;
\text{init-task}(v_i, \\
\mu(\delta(S;ISEL\cdot CON(S)); \\
<ISEL\cdot CON(S): \\
\mu_0(<s-in:int-st>,<s-al:elem(i)\cdot stlist>)), \\
ENV(S),DUMP(S)); \\
v_i:un-name \mid 1<i<length(stlist)-1}, \\
int-st(elem(length(stlist))\cdot stlist).
\end{equation}

The execution of the \texttt{create-tasks} instruction generates \(2m-1\) vertices on the control tree of the current task. \(m-1\) of the vertices specify \texttt{un-name} to generate unique task names; and \(m-1\) vertices contain \texttt{init-task}(arg_1, arg_2, arg_3, arg_4) instruction for assignment of new tasks. The environment and dump components, "arg_3" and "arg_4", respectively, are those of the current task. "arg_1" is the unique name; and "arg_2" has the structure of a control tree specified in Definition 3.9. It is constructed
by attaching an \texttt{int-st}(elem(i)(t)) instruction -- to interpret statement i of the list of parallel statements -- to the current task's control tree. Thus we include the \texttt{terminate} instruction and the ensuing instructions of the program on the control tree of every task. An illustration of the control tree of the initiating task n_1 is given in Figure 5.12 after execution of \texttt{create-tasks} at line 2 of program Q on page 78. Note the assignment of the current task n_1 to the last element of the concurrent computations by \texttt{create-tasks}.

Note the use of \texttt{ISEL} from Definition 3.11 as the control selector of the currently executing instruction. Further, note the abbreviation 
"CON(S) = s-c\cdot s-task(S)\cdot s-cc(S)" to select the current control tree. The δ-operation deletes the current instruction; and then the attachment of \texttt{int-st}(elem(i)\cdot stlist) is performed. The illustration in Figure 5.11 displays the state of the SIMPAL machine after execution of the 3 \texttt{init-task} instructions, in Figure 5.12, in tasks n_1, n_2, and n_3.

The execution of a critical region, as specified by the instruction \texttt{int-crit-reg}(t) of (SI24), by a task implies exclusive access to the critical region. Therefore, the statement of the critical region is a resource protected by the lock of the critical region. Since the argument t satisfies the predicate "is-crit-region" of (A22') of Table 5.2, the statement will be selected by "s-st(t)" and the associated resource use lock will be selected by "s-lock(t)". Execution of the "\texttt{int-crit-reg}" instruction,
(S124) \[
\text{int-crit-reg}(t) = \\
\text{release}(s\text{-lock}(t) \cdot \text{ENV}(S)); \\
\text{int-st}(s\text{-st}(t)); \\
\text{request}(s\text{-lock}(t) \cdot \text{ENV}(S)),
\]

expands into the \text{int-st}(s\text{-st}(t)) instruction execution protected by the task synchronization primitives \text{request} and \text{release} to assure only one task is executing the critical statement at any one time. The execution of the control tree from its terminal vertices guarantees the request of the resource before its use.

Figure 5.12. Current control tree after execution of create-tasks and 2 un-name instructions in program Q
Execution of the \texttt{int-cond-crit-reg(t)} instruction,

\begin{align*}
\text{(SI25)} \quad \texttt{int-cond-crit-reg(t)} &= \\
&= \text{release}(s\text{-lock}(t)\cdot\text{ENV}(S)); \\
&\quad \texttt{int-st}(s\text{-st}(t)); \\
&\quad \text{test}(v,t); \\
&\quad v:\texttt{int-expr}(s\text{-expr}(t)); \\
&\quad \text{request}(s\text{-lock}(t)\cdot\text{ENV}(S)),
\end{align*}

where \(t\) satisfies the predicate

\begin{equation*}
\text{is-cond-crit-reg} = (<s\text{-lock}:\text{is-id}>, \\
<s\text{-expr}:\text{is-expr}>, \\
<s\text{-st}:\text{is-st}>),
\end{equation*}

is again interpreted as a request for resources. In this case the resources are the expression and the statement components. During evaluation of the expression and execution of the statement, the task executing the expression evaluation and the statement must have exclusive access to both the expression and the statement. But since the value of an expression cannot change unless modified by another task, whenever the value of the expression cannot be interpreted as true the resources are released and requested at a later time. The \texttt{request} and \texttt{release} tasking primitives

1) assure mutually exclusive access to the expression evaluation by the \texttt{int-expr} instruction,

2) the testing of the truth value of the expression by \texttt{test(value)} of (SI26), and

3) the execution of the statement by \texttt{int-st}.
The execution of \texttt{test}(value, conditional critical region statement),

\begin{align*}
\texttt{test}(value, t) = & \text{convert}(value, \text{LOG}) \rightarrow \text{PASS: } \land \\
& T \rightarrow \texttt{test}(v, t);
\end{align*}

converts the representation of "value" to type logical. If such a representation has truth value "1", the execution of the statement component of the conditional critical region is allowed by effecting no actions in \texttt{test}. If it does not, then the resources have to be released and requested at a later time. \texttt{"test"} accomplishes this by generating a 4-vertex control subtree whose terminal vertex specifies a \texttt{release} instruction. The resources are later requested at the leisure of the scheduler of the associated lock due to the presence of the \texttt{request} primitive at the immediate predecessor node. The generation of instructions to evaluate and test the value of the expression permits repetitive checking of the expression at the convenience of the scheduler.

In order to illustrate the interpretation of critical regions, we exhibit the concurrent computations component of the SIMPAL machine in Figure 5.13 under the following assumptions.

1) Program Q on page 78 is being interpreted;

2) Interpretation has proceeded, starting from the state exhibited in Figure 5.11, with termination of task \( n_2 \) and execution of \texttt{int-crit-reg} in both tasks \( n_1 \) and \( n_3 \);
3) "@" is the unique name associated with the lock variable "w".

```
CON(S)
    n₁
      s-c
        exit
        int-st-list(REST)
        terminate(k)
        int-st-list(st4)
        release(£)
        int-st(PC)
        request($)
    n₃
      s-c
        exit
        int-st-list(REST)
        terminate(k)
        int-st-list(st2)
        release(£)
        int-st(PC)
        request($)
```

Figure 5.13. State of SIMPAL machine just prior to request for use of critical region PC

The interpretation of the critical regions now proceeds under the direction of the request and release primitives.

We have illustrated, in this chapter, two CONCOMs which define the semantics of two "simple" multitasking languages. The word "simple" is used to indicate that only those control structures of a language were implemented which seemed most pertinent in the discussion of concurrent control. Such things as input and output statements, iterative statements, and complex data types were not included as they would detract from the central theme of the dissertation. The implementation of such language features are discussed elsewhere (18) and can be implemented in a CONCOM.
The concurrent computations control structures discussed in Chapter II can all be implemented in a CONCOM, and they represent all the concurrent control structures known to the author. For example, the tasking facilities of PL/I have been implemented as a CONCOM. Its implementation defines the initiation of a procedure as a task whose

1) control component is an int-st(t), where t is the statement part of a procedure,
2) environment is that in which the procedure was declared,
3) dump component is null.

The implementation regards an event variable as a resource whose request implements a "wait" statement and whose release indicates the happening of the event.

Implementation of the counting semaphores of Dijkstra (6) require only the addition of a counter whose magnitude is the number of resources available for consumption (assignment to a task). The incrementation and decrementation of such a counter is protected from interruption by the request and release primitives.
CHAPTER VI.

DISCUSSION

The design goal of the class of abstract machines presented in this dissertation was that of conceptual clarity and not execution-time efficiency. Applicability to definition of multitasking programming languages has already been established and the Vienna definition method has been shown to be quite well suited to describe such implementations. It is therefore our purpose in this chapter to show that CONCOMs can be used as a basis for the study of computer operating systems whose function is to direct the execution of programs of many concurrent users.

As in all other areas of academic endeavor, the study of computer operating systems usually proceeds in both formal and pragmatic directions. We will discuss the validity of this model's conceptual clarity and its relationship to more efficient models in the development of both areas. CONCOMs will be applied to the five abstract areas of operating system study proposed by Denning (4).

The first area proposed is programming. This topic is concerned with language features and the operations they effect. Two implementations have been presented and it is felt that the concepts of concurrent control have been presented in a more concrete and understandable form than previous informal descriptions. CONCOMs are intended to be a framework within which to formally prove certain hypotheses about the implementation models.

The formal definition of a CONCOM in this dissertation presents a
model at a level of detail which lends itself to formal proofs of correctness of implementation, equivalence of implementations, and equivalence of interpreter models in general. We propose mathematical induction on the number of primitive task control instructions executed in a program structure as a natural method of proving correct assignment of tasks in both the STAL and SIMPAL machines. The McGowan Mapping Technique (20) seems applicable for proving equivalence of multitasking models whose concurrent control structures differ. The Twin Machine proof technique (23) certainly can be used to prove equivalence of implementation models of multitasking programming languages, such as complete to local environment, as already described by Wegner (23) for single task models. Since it is generally felt that conceptual models (like CONCOMs) of operating systems must evolve before the development of more complex and efficient models can proceed, equivalence of such models is quite important in the design of new systems.

The concepts of the four remaining abstract areas of study proposed -- storage allocation, concurrency, resource allocation, and protection -- are discussed in the framework of a CONCOM. Some needed extensions to the model are quite straightforward and are presented. The following discussion brings to the fore those concepts of current interest and the required extensions to a CONCOM to study them.

Storage allocation concerns memory management, name management, and dynamic space management. Study in this area can proceed with the addition of segment descriptor table and known segment table components to a task structure. Additional immediate state components for the active
segment table and page tables would also be necessary.

Accommodation of concurrency of tasks has been illustrated and its application to operating systems description seems quite appropos in the light of the task independence achieved. Only one problem still exists. The present model is sequential in that only one processor is allowed. But true multiprocessing can be achieved by an additional immediate component for each processor. The specification of task selection must be from the list of tasks on the concurrent computations branch, less the tasks being executed by other processors.

Resource allocation has been discussed. The current model of a CONCOM allows sharing of information among tasks through common environments. It also provides allocation of mutually exclusive resources (multiplexing). The state of deadlock exists in a CONCOM when no tasks are on the concurrent computations component and at least one task is in the task table of a resource use lock, awaiting its allocation of the resource. The set of deadlocked tasks is the union of the sets of tasks in all resource use lock task tables in the system.

Prevention of the deadlock state can be accommodated with the addition of immediate state components for the maximum claim matrix, the allocation matrix, and the available resources vector. Request and release primitives, with an additional argument containing number of resources requested and released, can implement the selection of a safe sequence (Habermann (8)) and hence allocate a new safe state.

Protection can be accommodated with the addition of an access matrix component. Additional primitive instructions must be specified to alter
such a matrix.

It is felt that this dissertation contributes to the state of the art of computing in the following manner.

1) The formalism presents the concepts of concurrent control precisely and in an understandable manner.

2) It defines the relationships of the five areas above precisely.

3) Its conceptual simplicity permits extension of its structure.

4) The existence of a formalism invites attempts to find more systematic approaches to implementation.

5) The formal description encourages formal proofs of correctness and equivalence of concurrent computation models.
BIBLIOGRAPHY


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APPENDIX I

The STAL interpreter

(ST1) \( \text{int-prog}(t) = \text{int-block}(t) \)

(ST2) \( \text{int-block}(t) = \)

DUMP: \( \mu_0(<s\text{-env}:\text{ENV}(S)>, <s\text{-c}:\text{CON}(S)>, <s\text{-d}:\text{DUMP}(S)> \)

CON: exit;

\( \text{int-st-list}(s\text{-st-list}(t)); \)

\( \text{int-dec-pt}(s\text{-dec-pt}(t)); \)

\( \text{update-env}(s\text{-dec-pt}(t)) \)

(ST3) \( \text{update-env}(t) = \)

null;

\{ \text{update-id}(id, n);

\quad n; \text{un-name} \mid id(t) \neq A \}

(ST4) \( \text{un-name} = \)

PASS: \( n \quad s-n(S) \)

s-n: \( s-n(S) + 1 \)

(ST5) \( \text{update-id}(id, n) = \)

ENV: \( \mu(\text{ENV}(S); <id:n>) \)

(ST6) \( \text{int-dec-pt}(t) = \)

null;

\{ \text{int-dec}(id, id(t), s\text{-st-list}(t))

\mid id(t) \neq A \)

& \text{is-id}(id) \}
\[ \text{int-dec}(id, \text{attr}, st) = \]
\[ \text{is-var-attr}(\text{attr}) \rightarrow \text{s-at: } \mu(\text{s-at}(S); <id; \text{ENV}(S): \text{attr}>) \]

\[ \text{find}(id, st, i) = \]
\[ \text{length}(st) = i \lor \text{s-lab}: \text{elem}(i) \cdot st = id \]
\[ \rightarrow \text{PASS: } i \]
\[ T \rightarrow \text{find}(id, st, i+1) \]

\[ \text{update-lab}(n, id, st) = \]
\[ \text{s-at: } \mu(\text{s-at}(S); <id; \text{ENV}(S): \text{LABEL}>) \]
\[ \text{s-den: } \mu(\text{s-den}(S); \nu_0(<id; \text{ENV}(S); \nu_0(<\text{s-env}; \text{ENV}(S), <\text{s-dump}; \text{DUMP}(S)>, <\text{s-st-list}:
\[ \mu(\lambda: \{<\text{elem}(i): \text{elem}(j) \cdot st
\]
\[ \mid i < j \leq \text{length}(st) \land j > n})}))))) \]

\[ \text{int-st-list}(t) = \]
\[ \text{is-\Lambda}(t) \rightarrow \text{PASS: } \Lambda \]
\[ T \rightarrow \text{int-st-list}(\text{tail}(t)); \]
\[ \text{int-st}(\text{head}(t)) \]

\[ \text{int-st}(t) = \]
\[ \text{is-assign-st}(t) \rightarrow \text{int-assign-st}(t) \]
\[ \text{is-cond-st}(t) \rightarrow \text{int-cond-st}(t) \]
\[ \text{is-proc-call}(t) \land (at_t = \text{PROC}) \rightarrow \text{int-proc-call}(t) \]
\[ \text{is-goto-st}(t) \land (lat_t = \text{LABEL}) \rightarrow \text{int-goto-st}(t) \]
\[ \text{is-fork-st}(t) \land (lat_t = \text{LABEL}) \rightarrow \text{int-fork-st}(t) \]
is-join-st(t) & (lat_t = LABEL)

& (s-ctr(t)(ENV(S)) • s-at(S) = INT)

→ int-join-st(t)

is-lock-st(t) & (at_t = LOCK)

→ int-lock-st(t)

is-unlock-st(t) & (at_t = LOCK)

→ int-unlock-st(t)

is-block(t) → int-block(t)

(ST12) int-assign-st(t) =

is-var-attr(n_t • s-at(S))

→ assign(n_t, v);

v : int-expr(s-rpart(t))

T → error

(ST13) assign(n, v) =

s-den → μ(s-den(S);

<n!convert (v, n(s-at(S)))>

where "convert(n,v)" is a VDL primitive function which converts the value v to the representation specified in the attribute table for the unique name n

(ST14) int-expr(t) =

is-bin(t) → int-bin-op(s-op(t), a, b);

a : int-expr(s-op1(t)),

b : int-expr(s-op2(t))

is-var(t) &

is-var-attr(n_t • s-at(S))

→ PASS: n_t • s-at(S)

is-const(t) → PASS: val(t)

T → error
(ST15) \[ \text{int-bin-op}(op, opl, op2) = \]
\[ \begin{align*}
  & op = '+' \rightarrow \text{PASS: } opl + op2 \\
  & op = '&' \rightarrow \text{PASS: } opl \& op2
\end{align*} \]

(ST16) \[ \text{int-cond-st}(t) = \]
\[ \text{branch}(v, s\text{-then-cl}(t), s\text{-else-cl}(t)); \]
\[ v: \text{int-expr}(s\text{-expr}(t)) \]

(ST17) \[ \text{branch}(v, st1, st2) = \]
\[ \text{convert}(v, \text{LOG}) \rightarrow \text{int-st}(st1) \]
\[ T \rightarrow \text{int-st}(st2) \]

(ST18) \[ \text{int-goto-st}(t) = \]
\[ \begin{align*}
  \text{ENV: } & \text{lenv} \\
  \text{DUMP: } & \text{ldump} \\
  \text{CON: } & \mu_0(<\text{s-in:exit},) \\
                & <\text{suc}(l):) \\
                & \mu_0(<\text{s-in:int-st-list},) \\
                & <\text{s-al;}:\mu_0(<\text{elem}(l):\text{s-st-list\cdot lab}>)>)\]

(ST19) \[ \text{int-proc-call}(t) = \]
\[ (\text{length}(\text{arglist}) = \text{length}(\text{plist})) \]
\[ \rightarrow \]
\[ \begin{align*}
  \text{DUMP: } & \mu_0(<\text{s-env:ENV(S)},) \\
                & <\text{s-c:CON(S)},) \\
                & <\text{s-d:DUMP(S))}> \\
  \text{ENV: } & \mu(\text{env;}) \\
              & {<\text{elem}(i)(\text{plist})} \\
              & :\text{elem}(i)(\text{arglist})(\text{ENV(S)})} \\
              & | \ l<i< \text{length}(\text{plist})}} \]
CON: exit;
    int-st(st)
T \rightarrow error

(ST20) exit =
    ENV: s-env(DUMP(S))
    CON: s-c(DUMP(S))
    DUMP: s-d(DUMP(S))

(ST21) int-fork-st(t) =
    init-task(v,v_0(<s-in:exit>,
        <succ(1):v_0(<s-in:int-st-list>,
            <s-al:st>)>),
        lenv,ldump);
    v:un-name

(ST22) int-join-st(t) =
    int-goto-st(s-lab(t));
    terminate(s-ctr(t)\cdot ENV(S))

(ST23) int-lock-st(t) =
    request(t\cdot ENV(S))

(ST24) int-unlock-st(t) =
    release(t\cdot ENV(S))
APPENDIX II

The SIMPAL interpreter

(SII1) - (SII10) and (SII2) - (SII20) are the same as (ST1) - (ST10) and (ST12) - (ST20) in Appendix I.

(SII1) \[ \text{int-st}(t) = \]
\[ \text{is-assign-st}(t) \rightarrow \text{int-assign-st}(t) \]
\[ \text{is-cond-st}(t) \rightarrow \text{int-cond-st}(t) \]
\[ \text{is-proc-call}(t) \land (a_t = \text{PROC}) \rightarrow \text{int-proc-call}(t) \]
\[ \text{is-goto-st}(t) \land (l_{at} = \text{LABEL}) \rightarrow \text{int-goto-st}(t) \]
\[ \text{is-par-st}(t) \land \text{is-st-list}(s-st-list(t)) \rightarrow \text{int-par-st}(t) \]
\[ \text{is-crit-reg}(t) \land \text{is-st}(s-st(t)) \land (s-lock(t)(\text{ENV}(S)) \cdot s-at(S) = \text{LOCK}) \rightarrow \text{int-crit-reg}(t) \]
\[ \text{is-cond-crit-reg}(t) \land (s-lock(t)(\text{ENV}(S)) \cdot s-at(S) = \text{LOCK}) \land \text{is-st}(s-st(t)) \land \text{is-expr}(s-expr(t)) \rightarrow \text{int-cond-crit-reg}(t) \]
\[ \text{is-block}(t) \rightarrow \text{int-block}(t) \]
\[ T \rightarrow \text{error} \]

(SII21) \[ \text{int-par-st}(t) = \]
\[ \text{terminate}(k); \]
\[ \text{create-tasks}(t); \]
\[ \text{initialize}(k, \text{length}(t)); \]
\[ k:\text{un-name} \]
(SI22) \textbf{initialize}(k,l) = \\
\quad s\text{-den}: \mu(s\text{-den}(S);<k:l>)

(SI23) \textbf{create-tasks}(stlist) = \\
\quad \text{null;} \\
\quad \text{init-task}(v_i, \\
\quad \quad \mu(\delta(S;\text{ISEL} \cdot \text{CON}(S)); \\
\quad \quad \quad \langle \text{ISEL} \cdot \text{CON}(S); \\
\quad \quad \quad \quad \mu_0(<s\text{-in}\text{-}int-st>, <s\text{-al}:\text{elem}(i)\cdot stlist>)>, \\
\quad \quad \quad \text{ENV}(S),\text{DUMP}(S)); \\
\quad \quad v_i: \text{un-name } | 1<i<\text{length(stlist)}-1), \\
\quad \quad \text{int-st}(\text{elem(\text{length(stlist)})\cdot stlist})

(SI24) \textbf{int-crit-reg}(t) = \\
\quad \text{release}(s\text{-lock}(t)\cdot \text{ENV}(S)); \\
\quad \text{int-st}(s\text{-st}(t)); \\
\quad \text{request}(s\text{-lock}(t)\cdot \text{ENV}(S))

(SI25) \textbf{int-cond-crit-reg}(t) = \\
\quad \text{release}(s\text{-lock}(t)\cdot \text{ENV}(S)); \\
\quad \text{int-st}(s\text{-st}(t)); \\
\quad \text{test}(v,t); \\
\quad \quad v:\text{int-expr}(s\text{-expr}(t)); \\
\quad \quad \text{request}(s\text{-lock}(t)\cdot \text{ENV}(S))

(SI26) \textbf{test}(value,t) = \\
\quad \text{convert}(value,\text{LOG}) \rightarrow \text{PASS}: \quad \wedge \\
\quad \quad T \rightarrow \text{test}(v,t); \\
\quad \quad \quad v:\text{int-expr}(s\text{-expr}(t)); \\
\quad \quad \quad \text{request}(s\text{-lock}(t)\cdot \text{ENV}(S)); \\
\quad \quad \quad \text{release}(s\text{-lock}(t)\cdot \text{ENV}(S)))


Hollmann, K. H., and J. M. Verley. 1967. La regression de la glande mammaire a l'arret de la lactation. II. Etude au microscope electronique. Z. Zellforschung 82: 222-238.


Mao, Peter, and Alfred Angrist. 1966. The fine structure of basal (Myoepithelial) cells of human prostate. Lab. Invest. 15: 1113.


Meites, Joseph, and C. W. Turner. 1942c. Studies concerning the mechanism controlling the initiation of lactation at parturition. III. Can estrogen account for the precipitous increase in the lactogen content of the pituitary following parturition? Endocrinology 30: 726-733.


Richards, R. C., and G. K. Benson. 1971c. Involvement of the macrophage system in the involution of the mammary gland in the albino rat. J. Endocrin. 51: 149-156.


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