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Consideration behavior and design decision making

Minhua Long

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Consideration behavior and design decision making

by

Minhua Long

A dissertation submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

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2016

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NOMENCLATURE

\( u_{i,j} \) Utility of product \( j \) for individual \( i \)
\( \beta_i \) Part-worth coefficient vector for individual \( i \)
\( \epsilon_{i,j} \) Error term in utility of product \( j \) for individual \( i \)
\( P_{j}^{MNL} \) Choice probability of multinomial logit model
\( P_{j}^{RC} \) Choice probability of random coefficient logit model
\( P_{j}^{NML} \) Choice probability of nested logit model
\( P_{j|B_{k(i)}} \) Choice probability of product \( j \) conditional on nest \( B_k \)
\( \lambda_k \) Log-sum coefficient that indicates the correlation of unobservable factor between products within nest \( k \)
\( C_r \) Consideration set defined by screening rule labelled as \( r \)
\( s \) Consideration screening rule
\( x_j \) Feature vector of product \( j \)
\( X \) Feature matrix consisted of \( x_j \)
\( p_j \) Price of product \( j \)
\( p \) Price vector consisted of \( p_j \)
\( P_{CTC}^{CTC} \) Choice probability of consider-then-choose model
\( P_{j|C_r} \) Choice probability of product \( j \) conditional on consideration set \( C_r \)
\( P_{C_r} \) The probability that consideration set \( C_r \) is formed
\( \pi(.) \) Profit function in design optimization problem
\( c(.) \) Cost function in design optimization
\( c^E_j(.) \) Equality constraints in design optimization
\( c^I_j(.) \) Inequality constraints in design optimization
\( l \) Lower bounds of design variables
\( u \) Upper bounds of design variables
\( b \) Body style indices vector, size of \( J_f \)
\( c_b(.) \) Body-style specific unit cost function
\( g_b(.) \) Body-style specific fuel consumption function
\( L_{e,b} \) body-style specific fuel economy lower bound
\( U_{e,b} \) body-style specific fuel economy upper bound
\( L_{a,b} \) body-style specific acceleration lower bound
\( U_{a,b} \) body-style specific acceleration upper bound
\( \theta \) Coefficient vector in consumer models
\( \theta_e \) Part-worth coefficient of fuel economy
\( \theta_a \) Part-worth coefficient of acceleration time
\( \theta_p \) Part-worth coefficient of price
\( \theta_b \) coefficient of body style \( b \)
\( \theta_0 \) outside good coefficient
\( \alpha(s_r) \) The probability that an individual in the population has screening rule \( s_r \)
\( \Delta \) Binary coded genome consisted of body style feature vectors of a product line in genetic algorithm
\( I \) Number of individuals in Monte-Carlo sampling
\( J_f \) Number of products in the product line owned by firm \( f \)
\( J_m \) Number of products in choice set \( m \)
\( N_m \) Number of choice observations in choice set \( m \)
\( M \) Number of choice sets
\( KLD \) Kullback-Leibler divergence
\( n_b \) Number of vehicles that have body style \( b \) in a product line
\( \mathbf{v}_i \) Coefficient vector in the aspirational model sampled for synthetic individual \( i \)
\( \gamma_i \) Aspirational limit in aspirational model
\( \delta_i \) Binary screening rules in conjunctive+ screeening sampled for individual \( i \)
\( A \) Number of attributes in a product profile
\( K \) Number of acceptable attributes that a product should at least have to be considered
\( \mathbf{w} \) Coefficient vector in the estimated aspirational model
\( y_n \) Consider or reject response in observation \( n \)
\( ||.||^2 \) Two-norm
\( P_{LCL}^j \) Choice probability of latent class logit model
\( P_{LCL}^{j|q} \) Choice probability of product \( j \) conditional on class \( q \)
\( \alpha_q \) The probability that an individual belongs class \( q \)
\( P_{j}^{True} \) Choice probability generated with the pre-specified synthetic behavior model
$RL$  Relative likelihood

$DCI$  Degree of commonality index

$\Phi_k$  The number of immediate parents that feature level $k$ has in the DCI metric

$Y$  Design variable matrix
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ABSTRACT

Over the past decade, design engineering has developed a systematic framework to coordinate with consumer behavior models. Traditional consumer models applied in the past has mainly focused on the preference of compensatory trade-offs in the choice decisions. Recent marketing research has become interested in developing consumer models that are "representative" in that they reflect realistic human decision processes. One important example is consideration: the process of quickly screening out many available alternatives using non-compensatory rules before trading off the value of different feature combinations. This research investigates the impact of modeling consideration behavior to design engineering, aiming at constructing consideration models that can inform strategic decisions. The study includes several features absent in existing research: quantifying the mis-specifications of the underlying choice process, tailoring survey instruments for particular models, and exploring the models' strategic value on product profitability and design decisions.

First, numerical methods are explored to address the discontinuity in the profit-oriented optimization problem introduced by the consideration models. Methods based on complementarity constraints, smoothing functions, and genetic algorithms are implemented and evaluated with a vehicle design case study. Second, a simulation experiment based on synthetic market data compares consideration models and a variety of conventional compensatory choice models in model estimation and design optimization. The simulation finds that even when estimated compensatory models provide relatively good predictive accuracy, they can lead to sub-optimal design decisions when the population uses considerations; convergence of compensatory models to non-compensatory behavior is likely to require unrealistic amounts of data; and modeling heterogeneity in non-compensatory screening is more valuable than modeling heterogeneity in compensatory trade-offs. The synthetic experiment framework then further extends the comparison to include the survey design process guided by the different assumptions behind consideration models and traditional choice models. A product line design case study reveals
that even though both choice models and consideration models show robustness in profitability, using consideration models leads to optimal portfolios with higher feature diversity and reducing the risk of overestimating profits. Finally, the research explores how to use consideration models to analyze the market penetration of a new designed product in a hybrid vehicle adoption case study.

It is the hope that this research can arouse the attention of designers to the informative power of consideration models, expand the understanding of consumer behavior modeling from the predictive power to the strategic impacts to design decisions, and provide technical supports to the future application of consideration models in design engineering.
CHAPTER 1. INTRODUCTION

1.1 Design Decision, Consumer Behavior, and Energy Policy

Engineering designers are faced with growing challenges of coordinating consumer needs and policy regulations. The design activity in vehicle industry is a typical example. Since 1975, the energy regulation program Corporate Average Fuel Economy (CAFE) standards has served to maintain energy secure by penalizing the manufacturers who fail to meet minimum targets for sales-weighted average fuel economy. In addition, since becoming federal requirements in the late 1960s, the emission standards managed by Environmental Protection Agency (EPA) exert increasingly stringent limits on pollutant emissions based on Federal Test Procedure. Seemingly, both higher fuel economy and lower emission meet the consumers needs for a cleaner vehicle with lower fuel cost. Yet, the design decisions concerning these two development directions are not straightforward for two reasons. First, conflicts between these two development directions occur in the engineering constraints. Second, consumers also balance between mutually connected factors such as price, driving experience, body type, capacity, and safety. For example, diesel engines enjoyed the popularity of providing more torque and higher fuel economy than the gasoline engines. However, the cost of reducing the emission level to comply with the same standard level for diesel engines is higher than that needed for gasoline engines [Sanchez et al. 2012]. Gasoline engines, on the other hand, compete against diesel engines with lower pollutant emissions, with the additional help of the tax imposed on diesel fuel. As the new generation of hybrid powertrain technologies arises, the options for high fuel economy and low emission enter the sight of consumers. But the cost of the hybrid technologies and the cognitive barrier on the trust of the new technology are two drawbacks. In this illustrative example, vehicle manufacturers have to face the challenging design decisions such as making bets on engine types and the related technologies that reduce emission without sacrificing the performance on
fuel economy and other product features, so that they can attract consumers.

In fact, designers and policy makers have noticed the important role of consumer modeling - the achievement of the entrepreneur value such as profit closely relates to the purchase decision, and policy incentives or regulations cannot have meaningful impacts without acting through products adopted by consumers. In the past decades, significant attention has been paid on capturing the relationship between the demand and the product features, as well as consumer preferences on the tradeoffs among the product features. One of the popular tools to achieve these two goals is discrete choice analysis (DCA). DCA introduces the concept of choice probability, which describes the chance of a consumer to choose a particular product from a given set of alternatives. DCA formulates the choice probability as a function of product features and consumers’ demographic information. The convenience of translating such choice probability into the market share prediction makes DCA an important component when informing energy policy making and design decision making. In the past applications, DCA was employed to explain the vehicle market share changes after the energy policy applied[Boyd and Mellman 1980, Goldberg 1995; 1998, Train and Winston 2007, Beresteanu and Li 2011] by capturing important tradeoffs such as those between fuel economy, vehicle size and price. On the engineering design side, DCA served as the demand component in a profit maximization framework. Combining engineering models such as cost models, performance constraints, and technical bounds, designers were able to identify the technical barriers and strategic changes of the automakers under policy regulations[Michalek et al. 2004, Shiau et al. 2009a, Whitefoot et al. 2011].

1.2 Challenges from Reality: Complex Consumer Behaviors and Considerations

Two basic assumptions behind classic DCA models have permeated previous design engineering research: consumers evaluate a product with the holistic utility additively contributed by every product feature, and consumers make choice from a universal set of available products in the market. Concededly, these assumptions offer a foundation to capture consumer decisions on product feature trade-offs, as well as lead to some mathematical benefits during deriving
choice probability formulas. However, how consumers process product information in reality challenges these assumptions. Due to cognitive limitations, consumers may not carefully evaluate every piece of product information, and their perception of a feature may not quantitively match the actual performance. In vehicle markets, the perception of fuel economy is a typical example. Allcott and Wozny [2012] observed a discount rate when estimating the contribution of fuel costs saving to hybrid choice. The discount rate indicated that consumers "acted" as if they were only willing to pay $0.61 to reduce the future cost by one dollar. In a lab experiment, Larrick and Soll [2008] found that the fuel cost saving intuitively believed by the consumers was biased. They termed the phenomena as "MPG illusion" in which people misunderstood the fuel cost as linearly related to MPG. Empirical evidences also uncovered that attribute like fuel cost saving may not participate the careful trade-off process during the buying decision. In a survey sampling 57 households in California conducted by Turrentine and Kurani [2007], the respondents were found not tracking their gas expenditures in general. Even for those who owned a hybrid vehicle, their decisions were stated to be more about values such as protecting environment and being pioneers in new technology rather than caring in saving money on gas. Whether the benefit of tax credit is evaluated in the hybrid choice is also questionable. For example, according to the state-level hybrid sales statistics, in Colorado and Pennsylvania, there were over 30% eligible hybrid purchasers who did not claim their income tax credits [Gallagher and Muehlegger 2011], which made it less convincing that consumers would universally account for tax credits in the feature tradeoffs.

Consideration is an important type of complex behaviors that describes a frugal and quick screening process before the consumer will make careful evaluation and comparison among the remained products [Roberts and Lattin 1997, Hauser 2014]. Fig.1.1 illustrates this process with a vehicle purchase example: when facing with a large number of vehicle options, a consumer first narrows down to five options via the screenings of brand, body type, powertrain, and price; among the five options, the consumer then compares the utilities of price, head-room, quality rating, safety in an additive manner as assumed by the traditional DCA models. The consideration screening process differentiates itself from the comparison process in two aspects. First, consideration reflects non-compensatory decision making, which means that unacceptable
features cannot be compensated by other attractive features. Second, the non-compensatory screening leads to only a subset of products considered by the consumer instead of a universal choice set assumed by the conventional models.

Figure 1.1 Consideration decisions and compensatory comparisons

These two structural differences of consideration models potentially reshape the understanding of design decisions. For example, when a household with children searches for a primary vehicle with enough passage capacity for the whole family, then body types such as two-seaters will be excluded, even if it has high fuel economy that the household also favors. Without understanding the non-compensatory property, a firm may wrongly make investments on im-
proving performance in product series that consumers do not consider. With the assumption that consumers choose from a subset of products, consideration models predict market shares and substitution patterns in a different viewpoint. When using consideration models, decision makers expect sales only from the proportion of consumers who consider it, and foresee a product’s competition against the alternatives within the same consideration set. Ignoring considerations potentially over-simplifies market share and competition patterns, and eventually misleads the design decisions. In the policy making perspective, identifying the screening rules used during vehicle purchases enhances the effectiveness of energy policies. For example, for consumers who do not consider hybrid vehicle because of price, financial incentives may nudge their considerations. However, for those who reject hybrids because of vehicle engineering features, perhaps more effective strategy is encouraging vehicle industry to design hybrid vehicles without sacrificing performance features that consumers screen on. Policy makers can also find opportunity to enhance hybrid adoptions by directly changing the screening rules via informational strategies such as role models and social orientations.

1.3 Contributions

The dissertation aims to bridge the gap between the current understanding of consideration models and the practical usage of consideration models in design engineering and energy policy analysis:

First, optimization tools are provided to handle the numerical difficulty in integrating consideration models in the design framework. While consideration models capture more realistic non-compensatory screenings in decision process, the non-compensatory property introduces discontinuous choice probability into the optimization problem. Such discontinuity causes obstacles to the usage of derivative-based methods during searching for optimal solutions. This research offers treatments to get over these obstacles by implementing and testing two classes of methods - nonlinear programming methods and genetic algorithms.

Second, the impact of consideration models on design decisions is identified by comparing consideration models and traditional compensatory models. The model performance evaluation quantifies not only the predictive accuracy in consumer choices, but also the differences of design
features and the strategic value of the designs. The model comparison aims at communicating between designers and marketers on their understanding of model applications. For designers, it is the goal to show new opportunities of using more descriptive consumer models, and the consequences of using traditional models when consumers are actually using non-compensatory screenings. For marketers, this research serves to broaden their view of consumer modeling from predicting choice observations further to guiding strategic outcomes in design applications.

A simulation framework is constructed to investigate model mis-specifications, i.e. the inconsistency between the model assumptions and the behavioral mechanism behind the data. The real world datasets are unable to quantify mis-specifications, because the complete behavioral mechanism is unknown. This research creates a synthetic experimental environment where mis-specifications can be controlled to focus on the consumer behavior of interest. Compared to synthetic experiments in the past that only focused on model estimations, the experiments in this research particularly simulate the model application in the design process. The simulation also takes into account how the assumptions of a model affect the data collection process.

Finally, new design objective and new analysis tools are explored to utilize consideration stage information. In the conventional profit-driven optimization framework, it is natural to measure the strategic impact of a design as profitability. However, consideration models offer new information such as the consideration sets formed by different types of individuals in a population. One goal of this dissertation is to present new angles to analyze competitions and substitution patterns based on the consideration sets.

The remaining chapters are organized as follows: Chapter 2 reviews conventional discrete choice models, and their applications in the decision based design framework. The consideration models will be also introduced in a general manner. Chapter 3 reveals the numerical difficulty in applying the two-stage consider-then-choose models in design optimization problems. This chapter implements a variety of treatments and evaluates their performances. Chapter 4 constructs a synthetic data simulation to investigate the consequences of using mis-specified compensatory models and consideration models in the estimation and design processes. The investigation discusses multiple performance measures, including predictive power, design feature difference, and profitability. Chapter 5 extends the simulation to include the survey design
process tailored according to different model assumptions. Chapter 6 explores new design objectives and analysis approaches particularly for using consideration stage information with an illustrative case of enhancing hybrid vehicle adoptions. Chapter 7 concludes and discusses open questions.
CHAPTER 2. BACKGROUND REVIEW

2.1 Classic Discrete Choice Models

Classic discrete choice models describe the probability an individual consumer chooses a product from a set of available alternatives. Given the set of alternatives $\mathcal{J}$, the characteristics of the alternatives and the demographic information of the consumer, a random utility is assigned to each of the alternatives which consists a representative component and an unobserved disturbance. The utility of product $j$ for individual $i$ is modeled as:

$$ u_{ij} = \beta_i^T y_{ij} + \epsilon_{ij} $$

(2.1)

where $\beta_i$ are coefficients indicating the taste of the consumer, $y_{ij}$ are information of products attribute and the demographic information, for example $\frac{\text{price}_i}{\text{income}_i}$. In the representative component, product attributes are weighted and contribute to the utility in an additive form. Classic discrete choice theory assumes a product is chosen by an individual when its random utility is the maximum among all the available alternatives. The models that follow this assumption are called Random Utility Maximization models (RUM) [Block and Marschak 1960, Manski 1977]. From this assumption, together with the prior beliefs of the distribution of the random terms, a variety of formula of the choice probability can be derived. Three typical models are reviewed in the follows.

Multinomial Logit Model

When the disturbance terms are iid distributed extreme value over all individuals and all alternatives and the utility coefficients vector $\beta$ is identical across all individuals, it yields the
Logit model. The choice probability takes the form [McFadden 1974]:

\[
P_{j}^{MNL} = \frac{\exp u_j}{\sum_{k \in J} \exp u_k}
\]

Although Logit model was built for a market where consumers have homogeneous taste, it is still a popular model in practice because it has a close form of choice probability and its likelihood function is globally concave [Maddala 1983].

**Random Coefficients Model (Mixed Logit Model)**

In random coefficients model (RC), the utility coefficients are randomly distributed across individuals. The most commonly assumed distribution of the coefficients are normal and log-normal distribution. Given \( \beta \)’s density function \( f(\beta) \), the RC choice probability can be derived as the integral of logit choice probability over \( \beta \)’s distribution:

\[
P_{j}^{RC} = \int P_{j}^{MNL}(\beta) f(\beta) d\beta
\]

As this integral form does not have a close expression, Monte-Carlo sampling is needed to compute the choice probability. Despite of its computation complexity, mixed logit model allows high flexibility in taste variations, substitution patterns and correlation of unobserved disturbance over time. These advantages make mixed logit model popular in practice [Hensher and Greene 2003]. Furthermore, McFadden and Train [2000] have shown that this model can approximate any random utility model. In practice, however, the coefficients are typically assumed to be normally distributed [Nevo 2000].

**Nested Logit Model**

In a two-level nested logit model, alternatives are categorized into different predefined nests \( B_1, \ldots, B_K \), consumers are assumed to first pick one nest then choose a product from the nest. For example, suppose there are vehicle alternatives from brands Honda, Toyota, GM, Ford, if the first level is categorized by foreign or domestic cars, then we will have nest \( B_1 = \{ \text{Honda, Toyota} \} \) nest \( B_2 = \{ \text{GM, Ford} \} \), thus the consumer first pick a nest, for instance, domestic
cars, then choose between GM and Ford. In a nested model which presumes the nests are disjoint, the choice probability is the product of the nest level probability and the lower level choice probability within the nest:

\[ P_{j|NML} = P_{j|B_{k(j)}} \cdot P_{B_{k(j)}} \]  

(2.4)

where \( P_{B_{k(j)}} \) is the probability that nest \( B_k \) is chosen and \( P_{j|B_{k(j)}} \) is the conditional probability that product \( j \) is chosen within the nest.

Denote the characteristics used to identify the nests are \( z_{ik} \) and the characteristics within the nest level as \( w_{ij} \), the condition choice probability within nest is the logit formula with \( w_{ij} \)

i.e.

\[ P_{j|B_{k(j)}} = \frac{\exp u(w_{ij}|\beta)}{\sum_{h \in B_k} \exp u(w_{ih}|\beta)} \]  

(2.5)

The choice probability of the nest also takes the logit formula:

\[ P_{B_k} = \frac{\exp\{V_k\}}{\sum_{k=1}^{K} \exp\{V_k\}} \]  

(2.6)

with nest utility:

\[ V_k = \gamma^T z_{ik} + \lambda_k \log \sum_{h \in B_k} \exp u_h \]  

(2.7)

The log-sum term in this formula carries the lower level features information into the choice of the nest. \( \lambda_k \) is referred as the log-sum coefficients which indicates the correlations of the unobserved factor between products within nest \( k \), with lower \( \lambda \) reflecting higher correlation. The formula reviewed here is from Daly [1987]. Another alternative formula was given by McFadden [1978]. The difference between two formulations is that McFadden’s model has the lower level within nest utility normalized by the log-sum coefficient \( \lambda_k \). Dividing \( u_h \) by \( \lambda_k \) is required for consistency with RUM, but there is still debate about whether that is essential in the model [Train 2009].
2.2 Data Resource: Revealed Preference and Stated Preference

Discrete choice models can be estimated from two broad categories of data resources: revealed preference (RP) data and stated preference (SP) data. Revealed preference data come from the real purchase observations and sales record. Besides the resource from aggregated sales, revealed preference data can be collected to have more specific information. For example, in Berry et al. [2004], a survey conducted by General Motor was used, in which a sample of real purchasers for each vehicle in 1993 model year was drawn for the questionnaire about household attributes. This sampling survey data supplemented pure macro observation of market shares.

Stated preference data are collected from designed choice experiments. In stated preference data, a choice experiment generally takes the following steps [Hensher 1994]: (1) identify a set of attributes with specified measurement and decide the number and magnitude of attribute levels; (2) design the combinations of attribute levels; (3) translate the designed combinations (profiles) into a survey that respondents can comprehend for data collection.

Both data resources have well-known strengths and downsides. For example, revealed preference reflects realistic purchase habit but has difficulties in gathering sufficient information in product attributes and specific demographic information, while stated preference is important in obtaining information of attributes that are not available in the market and researchers can design and control the experiment specifically for the choice models being studied but the responses in stated preference may not closely match their actual preference [Wardman 1988]. To have the advantage of both RP and SP data in discrete choice modeling, there is a trend in combining both data resources in estimation. Adamowicz et al. [1994] has estimated a multinomial logit model from combined SP and RP data and found that combined information can overcome the preference inconsistency in independent usage of either data type. Empirical evidence also uncovered that the application of mixed logit model and the use of combined SP and RP data are mutually beneficial [Brownstone et al. 2000, Hensher 2008].

To collect stated preference data for discrete choice modeling, a choice or ranking survey is often used. The assumptions of the choice behavior not only influences the model structure, but also conduct the design of the discrete choice survey for estimation. One example is in the
evaluation of D-optimality, unlike the linear model where the efficiency only depends on the alternative features, the efficiency formula of the discrete choice models also depends on the choice probability [Kanninen 2002]. Therefore the prior beliefs of the estimated coefficients can influence the design of the survey [Sandor and Wedel 2001].

Desired properties of a discrete survey design include orthogonality (each attribute level appears an equal number of times in combination with all other attribute levels), level balance (each level within an attribute appears an equal number of times), minimal overlap (the alternatives are prevented to have the same level for a given attribute) and utility balance (alternatives within a choice set should be equally attractive to respondents) [Huber and Zwerina 1996, Johnson et al. 2013]. The efficiency of a choice survey is also quantified by criteria such as D-efficiency [Kuhfeld et al. 1994], which is connected to maximum-likelihood estimation in a way that by maximizing the determinant of the inverse of the variance-covariance matrix, the joint confidence sphere around the complete set of estimated model parameters is minimized.

In achieving these properties, survey design methods vary. Methods based on orthogonal arrays can achieve level balance and orthogonality by systematically shifting levels on the basis of orthogonal arrays [Chrzan and Orme 2000, Street and Burgess 2007]. Sawtooth software system uses levels selection strategies that directly target to achieve level balance and level minimal overlap [Saw 2013]. Methods constructed to optimize D-efficiency can use candidate profile exchange algorithm. The Fedorov algorithm, which commercial software SAS based on [Kuhfeld 2010], is in this category. D-efficiency can be also locally optimized with heuristic procedures such as cyclically generating alternatives, swapping and relabeling attribute levels, and co-ordinate exchange algorithm [Huber and Zwerina 1996, Sandor and Wedel 2001, Yu et al. 2009].

2.3 Decision-Based Design and the Integration of Consumer Models

The idea of decision-based design (DBD) framework stems from the need to coordinate design teams within the same project who usually handle different aspects of information or requirements [Hazeldigg 1998]. In its early proposal, the framework emphasized on unifying a normative objective for a design project, and functionally mapping design alternatives into such
objective, so that designers can reach consistent decisions via optimization. The construction
of framework adopted the views in information theory and engineering economics, in which the
mapping between design alternatives and the objective reflected the nature of risk and uncer-
tainty analysis in engineering and demand performances. The development of DBD actively
grows in the past two decades. Engineering community has enriched this decision making tool
in both case-specific applications and generalizable methodology study [Lewis et al. 2006]. One
important stream in DBD is to specify the objective of product design as business value such as
profit. In this interest, discrete choice models serve as a convenient tool in demand forecasting.
The flowchart in Figure 2.1 illustrates the interactions between the engineering components
and the market components in decision based design framework. The developments in DBD
research are shown in three aspects: consumer models, design scenarios and the corresponding
solving methods.

Table 2.1 Decision based design practices with discrete choice models

<table>
<thead>
<tr>
<th>Literature</th>
<th>Choice model</th>
<th>Design scenario</th>
<th>Solving method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wassenaar et al. [2005]</td>
<td>Logit</td>
<td>Single product</td>
<td>Enumerate</td>
</tr>
<tr>
<td>Besharat et al. [2006b]</td>
<td>Logit</td>
<td>Single product, multiobjectives</td>
<td>Sequential iteration</td>
</tr>
<tr>
<td>Michalek et al. [2004]</td>
<td>Logit</td>
<td>Multiple firms</td>
<td>Equilibrium KKT constraints</td>
</tr>
<tr>
<td>Shiau et al. [2009b]</td>
<td>Logit</td>
<td>Multiple firms</td>
<td></td>
</tr>
<tr>
<td>MacDonald et al. [2010]</td>
<td>Latent Class Logit</td>
<td>Single product</td>
<td>Mesh adaptive direct search</td>
</tr>
<tr>
<td>Michalek et al. [2011]</td>
<td>Mixed Logit</td>
<td>Single firm product line</td>
<td>ATC</td>
</tr>
<tr>
<td>Morrow and Skerlos [2011]</td>
<td>Mixed Logit</td>
<td>Multiple firms pricing</td>
<td>Fixed-point iteration</td>
</tr>
</tbody>
</table>

Table 2.1 lists relevant DBD research literatures where discrete choice models are employed.
In the pioneer work of Wassenaar and Chen [2003] and Wassenaar et al. [2005] proposed a
framework to include discrete choice models as demand modelling for decision-based design
community, starting with the simple Logit model on a single product selection problem. Models
with more complexity are applied to meet the need of different market scenarios such as
consumer heterogeneity [Michalek et al. 2011, Morrow and Skerlos 2011] and hierarchical choice
[Hoyle et al. 2010].

The development of solving tools is often paired with the growing complexity of the design
scenario considered in the problems. Analytical target cascading (ATC) is successfully applied
in both single product and product line optimization problem [Michalek et al. 2011]. Design
under competitive market has aroused the study of more complex optimization approaches.
The associated solving approaches for multiple firms design under competitive environment
includes: iterative approach in which each firm will sequentially maximize profit according to the updating competitors' information until convergence [Michalek et al. 2004], equilibrium constraints approach where the KKT condition of the equilibrium is added in the optimization problem [Shiau et al. 2009b] and fixed point iteration approach where the equilibrium condition is formulated in a fixed-point form to prevent trivial solutions [Morrow and Skerlos 2011, Morrow et al. 2013].
2.4 Non-compensatory Behavior and Consideration

The classic compensatory models reviewed above shares the same assumption that decision makers will weigh and add all the alternatives information and maximize the utility. However, the concept of *satisficing* proposed by Simon in 1957 gave a different concept – a decision maker will stop searching for further options as soon as one alternative exceeds the minimum aspiration level. According to this view, decision makers need not examine all the options or examine all their features. This concept has been supported by abundant empirical studies in which respondents were revealed to use fast and frugal non-compensatory rules to eliminate options when faced with time pressure [Rieskamp and Hoffrage 2008], information cost [Bröder 2000] and memory requirement [Bröder and Schiffer 2003]. Decision makers can tailor different simple heuristics to adapt to different problem solving environment [Gigerenzer and Gaissmaier 2011]. The use of non-compensatory strategies such as conjunctive and disjunctive screening was captured with early protocol tracing techniques [Einhorn 1971]. Consideration strategies were found more likely to appear in some typical decision tasks. For example, Payne et al. recorded that compensatory process usually existed in the two-alternative comparison tasks, while the quick elimination of alternatives happened when the respondents were faced with multi-alternative task [Payne 1976]. Rieskamp et al. observed that under high time pressure, the participants are more likely to select a non-compensatory heuristic, particularly lexicographic screening [Rieskamp and Hoffrage 2008]. Bröder et al. validated that when decision tasks require memory search, respondents were more likely to use take-the-best strategy [Bröder and Schiffer 2003].

As an impact in marketing research, the awareness of the use of non-compensatory rules among consumers changed the traditional concept of choice set in discrete choice modelling. Instead of only assuming a universal choice set (with all the alternatives), the *Consideration Set* (a subset of the universal choice set) is studied [Roberts and Lattin 1997, Hauser et al. 2009]. Typical non-compensatory rules employed to form consideration set were discussed (see early concepts in [Dawes 1964, Einhorn 1970, Tversky 1972a]):

- *Aspirational*: the early idea was from [Simon 1972] who proposed the theory of bounded rationality, one of the examples was that decision makers will stop searching for options
once the benefit they obtain from an option exceed some “aspirational limit”. Later this idea was translated into consideration modeling, see work from [Gilbride and Allenby 2004]. In aspirational screening, consumers will consider a product if the product’s utility exceed some aspirational criteria.

- **Conjunctive**: a decision maker will consider an alternative only if all its features are acceptable. For example, If a consumer screens on hybrid and brand, a conjunctive rule will be "I will consider a vehicle if it is hybrid AND it is Toyota”.

- **Disjunctive**: a decision maker will consider an alternative as long as one of its features is acceptable. For example, "I will consider a Toyota OR a hybrid vehicle”.

- **Subset conjunctive**: a decision maker will consider an alternative if a certain number of its features are acceptable. For example, there are brand, price and hybrid three screening features and suppose the minimum number of acceptable features should be 2 for a vehicle to be considered, then a subset conjunctive rule can be stated as "I will consider a vehicle if the vehicle is a Toyota hybrid OR if it is a hybrid with price under $25,000 OR if it is a Toyota under $25,000”.

- **Disjunction of conjunctions**: a decision maker will consider an alternative if it satisfies one or more than one sets of conjunctive rules. For example, also screening on brand, price and hybrid, a disjunction of conjunctions rule can be ”I will consider a vehicle if it is a Toyota OR if it is a hybrid under $25,000”.

- **Elimination-by-Aspect**: a decision maker successively chooses unacceptable feature levels and eliminates alternatives with those levels. For example if one order screening aspects as ”hybrid → price → brand ”, then the consumer will first eliminate non-hybrid vehicles then compare the candidates on the price, if the price is a tie, then move on to the brand.

### 2.5 Consider-Then-Choose Models

A general model representing consider-then-choose decisions can be described as follows. Suppose the universal choice set is $J = 1, ..., J$. A consideration set indexed by $r = 1, ..., R$,
denoted as $C_r \subset \{1, \ldots, J\}$ is defined by a set of screening rules $s_r = [s_{r,1}, \ldots, s_{r,L_r}]$. For conjunctive rules, it can be written as:

$$C_r = \left\{ j \in \{1, \ldots, J\} : s_r(x_j, p_j, \gamma) \leq 0 \right\}$$

(2.8)

This definition means that a product needs to satisfy all the screening rules to be a member in the corresponding consideration set. An alternative form illustrating this is

$$C_r = \left\{ j \in \{1, \ldots, J\} : \max_l s_{r,l}(x_j, p_j, \gamma) \leq 0 \right\}$$

(2.9)

The screening rules depend on product features $x_j$, price $p_j$ and individual specific weights or criteria $\gamma$. For example, a consideration set $C_r = \{ \text{all products } j \in \{1, \ldots, J\} \text{ with price } p_j \text{ under 20,000 dollars AND fuel economy } e_j \text{ over 30mpg} \}$ can be defined by two screening rule functions $s_r = [s_{r,1}(x_j, p_j, \gamma_1) \leq 0; s_{r,2}(x_j, p_j, \gamma_2) \leq 0] = [p_j - 20,000 \leq 0; 30 - e_j \leq 0]$.

For disjunctions, e.g. $C_r = \{ \text{all products } j \in 1, \ldots, J \text{ with price } p_j \text{ under 20,000 dollars OR fuel economy } e_j \text{ over 30mpg} \}$, the consideration set can be defined as:

$$C_r = \left\{ j \in \{1, \ldots, J\} : \min_l \{s_{r,l}(x_j, p_j, \gamma_l)\} \leq 0 \right\}$$

(2.10)

Disjunctions of conjunctions can be formed by combining the min and max representations in Eqns. (4.2) and (4.3).

This structure is consistent with the forms used in marketing literatures. For example in Dzyabura and Hauser [2011], product features are binary coded, which can be transformed into the structure presented here.

Given a collection of screening rules and the associated consideration set, let the conditional probability that product $j$ is chosen within the set be $P_{j|C_r}$ and let the probability that the consideration set $C_r$ is formed by $P_{C_r}$, then the choice probability $P_j$ can take the form of sum up the joint probabilities cross all possible consideration sets.

$$P_j = \sum_r P_{j|C_r} \cdot P_{C_r}$$

(2.11)
Note that this structure is been found similar to a nested logit model. Relative research can be seen in Swait [2001b] where the generalized nested logit formulation is used to model consideration sets by specifying the possible consideration sets as “nests”. This work connected the consideration sets explosion models with the family of discrete choice models.

The conditional probability within the consideration set takes the form

\[
P_{j|C_r}(X, p) = \begin{cases} 
\frac{e^{u(x_j, p_j, \beta)}}{e^{u(x_j, p_j, \beta)} + \sum_{k \in C_r} e^{u(x_k, p_k, \beta)}} & \text{if } j \in C_r \\
0 & \text{if } j \notin C_r
\end{cases}
\] (2.12)

The utility coefficients \( \beta \) can be assumed to be homogeneous across population or take the random coefficients form to include heterogeneity (which requires a Monte-Carlo integral of the simple Logit form above).

The representation of consideration set probability \( P_{C_r} \) were developed into different specifications in previous marketing research. Swait and Ben-Akiva [1987] introduced a random component into the screening rules function, so that with some assumed distribution of the random component, \( P_{C_r} \) can be derived based on the probability of each alternative’s availability in this consideration set. Ben-Akiva and Boccara [1995] extended this random consideration set generation model by specifying the availability probability as Logit form. Instead of connecting \( P_{C_r} \) with parameterized screening rules, Chiang et al. [1999] assumed this consideration set probabilities have a Dirichlet distribution across the population. Gilbride and Allenby [2004] avoided the enumeration of consideration sets by using a reduced form choice probability which applied the Markov chain to identify the posterior distribution of the allowable screening criteria values.

The specifications reviewed above can work on choice data and can be estimated with classical tools such as Maximum Likelihood Estimation (MLE) and Bayesian approaches. However the exponential growth of the possible consideration sets makes consider-then-choose models difficult to estimate in practice. This challenge motivated researchers to gather ”consider” stage information other than just using the final choice observations. For example, Jedidi and Kohli [2005] used ”acceptable/unacceptable” response data to estimate subset-conjunctive rule where the probability that a particular attribute level is acceptable were estimated and each level is
assumed to be independently perceived; this treatment allowed MLE method operate on the likelihood of an alternative is considered. Another example came from Dzyabura and Hauser [2011] who also presumed the binomial distribution for the screening rules to infer the most likely consideration pattern from the stated consideration responses to the adaptive questions online.

2.6 Methods of Estimating Consideration Models

Tracing and protocol analysis were classic methods applied to detect non-compensatory rules [Payne 1976, Bettman 1980] in which respondents decision process were self-reported or tracked. Shortcoming was reported along with the application of these direct elicitation approaches, for example, inconsistency was observed between the stated screening criteria and the later choice made by the same individual [Green and Krieger 1988]. In spite of its imperfection, direct elicitation approach continued contributing to later research in two ways. First, with the improvement of experiment design, self-elicitation approach can achieve higher accuracy. An example was provided by Ding and Hauser [2011] where the incentive-aligned setting was used. Second, methods based on latent constructs can use self-reported screening aspects and criteria to pre-establish a two-stage model that requires less estimation complexity, for example, Swait [2001a] constructed a two-stage model in which the acceptability thresholds were directly reported from a survey. Their technique avoided enumerating all the possible consideration sets.

Estimation tools that are widely applied in classic conjoint analysis such as maximum likelihood and Bayesian methods were also useful in non-compensatory model parameter estimation where probabilities of screening and choice are correspondingly formulated. Jedidi and Kohli [1996] generalized conjunctive and disjunctive rules as a linear threshold model then operated maximum likelihood method only on the probability of accepting an alternative and the consideration stage data. In their later work in 2005, a more informative model was estimated where the probability that each aspects/levels is screened was also included [Jedidi and Kohli 2005]. Gilbride and Allenby [2004] measured conjunctive and disjunctive rules with Bayesian approach from choice data, their reduced form of consider-then-choose choice probability eased
the enumeration of consideration sets.

Recently developed machine learning techniques brought new inferring tools to this field. Yee et al. [2007] and Kohli and Jedidi [2007] adapted greedoid language into screening rules inferring. One advantage of greedoid algorithm is that it makes use of the relationship between profile ordering and screening aspects ordering to effectively update the screening pattern towards the best fit to the data. Dzyabura and Hauser [2011] developed an active machine learning method to adaptively select survey questions in which the profiles are configured to maximize the information on the decision rules according to posterior beliefs and prior responses. These machine learning techniques largely reduced the computation time compared to the traditional consideration sets exposure methods and thus enable the inferring of more possible screening aspects and levels.

### 2.7 Improvement of Predictive Power

Pair with methods to uncover non-compensatory rules, the improvement of choice model predictions has been observed in a variety of product categories when non-compensatory rules were modeled. Swait reported an multinomial logit model with soft cut-offs fit the choice data of rental cars better than a pure multinomial logit model [Swait 2001a]. In a smart phone study Yee et al. observed that lexicographic models inferred by greedoid method predicts better than Bayes-ranked logit model [Rossi and Allenby ] on hit rate for at least half of the population in ranking task samples [Yee et al. 2007]. Ding et al. reported an improvement of relative Kullback-Leibler divergence (relative to null model prediction) in cellphones data with screening rules from unstructured direct elicitation than Hierarchical Bayesian logit model [Ding and Hauser 2011]. Other examples in categories of cameras, batteries, laptops, GPS Units, vehicles [Hauser et al. 2009, Jedidi and Kohli 2005, Kohli and Jedidi 2007, Hauser and Toubia 2010, Dzyabura and Hauser 2011] are listed in Table 4.1.

Note that the validation of prediction power can only indicate that the model better describe the data as if the respondents are more likely to react as how these models describe. But they don’t necessarily prove that the respondents are actually using these procedures of these models.
Table 2.2: Recent consider-then-choose models constructed from stated preference data (cited from NSF Award NO.1334764). Abbreviations: HB - Hierarchical Bayes; MLE - Maximum Likelihood; HR - "hit rate" (frequency of correct prediction on hold-out samples); KLD - Kullback-Leibler Divergence; TAU - Kendall’s Tau [Hauser 1978]; LL - Log-likelihood (increase)

<table>
<thead>
<tr>
<th>Literature</th>
<th>Product</th>
<th>Compensatory Model</th>
<th>Consider-then-Choose Model</th>
<th>Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hauser et al. [2009]</td>
<td>Cameras</td>
<td>HB Logit</td>
<td>HB, Conjunctive screening</td>
<td>7.1% in HR</td>
</tr>
<tr>
<td>Jedidi and Kohli [2005]</td>
<td>Batteries</td>
<td>MLE Logit</td>
<td>MLE, Subset conjunctive</td>
<td>1.1% in HR</td>
</tr>
<tr>
<td>Kohli and Jedidi [2007]</td>
<td>Laptops</td>
<td>LP Logit</td>
<td>Greedy, Lexicographic</td>
<td>0% in TAU</td>
</tr>
<tr>
<td>Hauser and Toubia [2010]</td>
<td>GPS Units</td>
<td>HB Logit</td>
<td>Greedy, Lexicographic</td>
<td>4.5% in HR, 54.5% in KLD</td>
</tr>
<tr>
<td>Ding and Hauser [2011]</td>
<td>Cellphones</td>
<td>HB Logit</td>
<td>Unstructured Direct Elicitation</td>
<td>9.1% in KLD</td>
</tr>
<tr>
<td>Yee et al. [2007]</td>
<td>Smartphones</td>
<td>HB Ranked Logit</td>
<td>Lexicographic by aspects</td>
<td>8.7% in HR</td>
</tr>
<tr>
<td>Swait [2001a]</td>
<td>Rental Cars</td>
<td>MLE Logit</td>
<td>&quot;Cut-off rules&quot; (conjunctions)</td>
<td>14% in LL</td>
</tr>
<tr>
<td>Dayabura and Hauser [2011]</td>
<td>Vehicles</td>
<td>HB Logit</td>
<td>Adaptive question HB</td>
<td>44.1% in HR, 16.7% in KLD</td>
</tr>
</tbody>
</table>
CHAPTER 3. CONSIDERATION AND DESIGN OPTIMIZATION

3.1 Introduction

Generally applying consider-then-choose models in a design optimization framework still faces challenges. Specifically, the choice probabilities in consider-then-choose models can be discontinuous in continuous decisions. For example, suppose an individual can purchase a product with price \( p \) ($\) and “value” \( x \geq 0 \). If they buy the product they obtain a utility of 
\[
U(x, p) = u(x, p) + \mathcal{E}
\]
where \( u(x, p) = 1 - p + x \), and if they opt not to buy the product they obtain a utility of \( \Theta = 1 + \mathcal{E}_0 \) for i.i.d. extreme value errors \( \mathcal{E}, \mathcal{E}_0 \). However they will consider the product only if \( p \leq 1 \) -- a screening rule. This example models a situation where individuals only have so much to spend on a product, and can obtain utility from both ownership and the amount of money left over after purchase. Consider-then-choose choice probabilities are then given by

\[
P^C(x, p) = \begin{cases} 
  e^{1-p+x}/(e + e^{1-p+x}) & \text{if } p \leq 1 \\
  0 & \text{if } p > 1 
\end{cases} \tag{3.1}
\]

Observe that \( \lim_{p \uparrow 1} P^C(x, p) = e^x/(e + e^x) > 0 \) while \( \lim_{p \downarrow 1} P^C(x, p) = 0 \), and thus \( P^C(x, \cdot) \) is discontinuous at \( p = 1 \). See also Fig. 3.1.

Because of these discontinuities, optimization methods that rely on derivatives information cannot be assured to solve this type of problem. In this section, we present research that addresses this numerical issue. The contribution of this work is to explore efficient tools to support further application of consider-then-choose models. The outcomes of this research are:

- Numerical methods based on Nonlinear Programming (NLP) are explored. Three formulations of the optimization problem are derived: formulations based on constrained complementarity, penalized complementarity and smoothing.
• Genetic Algorithm (GA) is applied. Two classes of GA methods are implemented: GA with a penalty on constraints violation and a hybrid GA with constraints solved by NLP.

• The performance of the numerical methods are obtained in a demonstration case where a firm is designing a vehicle under consumers’ budget screening rules. The properties of five methods are shown by the optimality, feasibility and computation cost of their solutions.

In the remaining section, we will first introduce the general design optimization problem with consider-then-choose model in Section 3.2. The formulations of three NLP methods and two GA methods will be presented in Section 3.3 and Section 3.4. The demonstration example will be described in Section 3.4. Section 3.5 reports the performance of the methods as results. This discussion is based on work from Morrow et al. [2012] and Morrow et al. [pted], though we include more details than available in those publications.

3.2 The Optimization Problem

Suppose a firm is designing a product line with \( J \) products, each with engineering features vector \( \mathbf{x}_j \) and price \( p_j \). The engineering relationships among the features can be described in equality or inequality constraints. With the costs as the function of engineering features and the mapping from product attributes to predicted demand (i.e. choice probability), the general formulation the design optimization problem can be written as:

\[
\text{maximize} \quad \pi(\mathbf{X}, \mathbf{p}) = \sum_{j=1}^{J} P_j(\mathbf{X}, \mathbf{p})(p_j - c_j(\mathbf{x}_j))
\]

with respect to \( p_j \geq 0 \)

\( L_j \leq \mathbf{x}_j \leq U_j \)

subject to \( c^E_j(\mathbf{x}_j) = 0 \)

\( c^f_j(\mathbf{x}_j) \geq 0 \) (3.2)
where $c_j(.)$ is the cost function, $c^E_j(.)$ is equality constraints, $c^E_j(.)$ is inequality constraints, $L$ and $U$ are lower bounds and upper bounds of the features. When applying a consider-then-choose model, the choice probability under individuals sample size $I$ is specified as:

$$P_j(X, p) = \sum_{i=1}^{I} P_{ij}^{CTC}(X, p)$$  \hspace{1cm} (3.3)

If product $j$ is considered by individual $i$ (i.e. in consideration set $C_i$), it will be compared with other products in the same consideration set, otherwise it won’t be chosen. That is:

$$P_{ij}^{CTC}(X, p) = \begin{cases} \exp u_i(x_j, p_j) / \left( 1 + \sum_{k \in C_i} \exp u_i(x_k, p_k) \right) & j \in C_i \\ 0 & j \not\in C_i \end{cases}$$  \hspace{1cm} (3.4)

The consideration set $C_i$ is defined by a group of individual specific screening rules $s_i(x_j, p_j)$ such that

$$C_i = \{ j \in 1, \cdots, J : s_i(x_j, p_j) \leq 0 \}$$  \hspace{1cm} (3.5)

Note that the consider-then-choose (CTC) choice probability is discontinuous on the design space, because at the point where screening criteria is active, the product will "enter" or "leave" an individual’s consideration set. This discontinuous property is crucial for design optimization as taking derivatives with respect to the design variables. When multiple products and multiple screening rules are involved, this issue become particularly challenging not only because the number of possible consideration sets grows exponentially but also because the number of feasible combinations of the possible consideration set are relatively small. Method like enumeration the possible consideration pattern will suffer from explosive number of inefficient searches.

### 3.3 NLP Methods

To implement nonlinear programming (NLP) approaches for the consider-then-choose optimization problem, Morrow et al. [2012] proposed a relaxation for the CTC choice probability...
in Eqn. (3.4), the relaxed form is:

$$\hat{P}_{ij}^{CTC}(w, X, p) = \frac{\left(\prod_{r=1}^{R_i} w_{ijr}\right) \exp u_{ij}}{1 + \sum_{k=1}^{J} \left(\prod_{r=1}^{R_i} w_{ikr}\right) \exp u_{ik}}.$$  \hspace{1cm} (3.6)

This formula relaxed discontinuity by introducing slack variables $w_{i,j,r}$ to represent: if product $j$ satisfies individual $i$’s screening rule $r$ then $w_{i,j,r} = 1$, otherwise $w_{i,j,r} = 0$. To match the CTC structure, ideally we seek to construct a formula such that for all $i, j, r$, when screening rules $s_{i,r}(x_j, p_j) \leq 0$, $w_{i,j,r}$ strictly equals to 1, and when $s_{i,r}(x_j, p_j) > 0$, $w_{i,j,r}$ strictly equals to 0. We call this desired property as *strictly choice-consistency*.

![Figure 3.1](image)

**Figure 3.1** Two classes of NLP methods to handle discontinuous non-compensatory choice probabilities. *Left:* Discontinuous choice probabilities from the example at the start of this section. *Center:* “Smoothed” choice probabilities using Eqn. (3.6) and the smoothing model in Eqn. (3.7). *Right:* “Relaxed” smooth choice probabilities using Eqn. (3.6) and the complementarity constraints in Eqn. (3.9).

Two classes of NLP methods are implemented on this purpose: NLP with smoothing approach and NLP with complementarity constraints.
3.3.1 NLP with Smoothing

The smoothing approach is using a smoothed step function to approximate the desired relationship between \( w \) and \( s \). This step function is defined as:

\[
L_\eta(s) = (1 + e^{-\eta}) \left( \frac{e^{-\eta s}}{1 + e^{-\eta s}} \right)
\]  

(3.7)

for any \( \eta \geq 0 \) and \( s \in \mathbb{R} \). These functions define the following smoothed problem:

\[
\begin{align*}
\text{maximize} & \quad \bar{\pi}^C(w, X, p) \\
\text{with respect to} & \quad w_{i,j,r} \in \mathbb{R} \text{ for all } i, j, r \\
\text{subject to} & \quad l_j \leq x_j \leq u_j, \quad p_j \geq 0 \text{ for all } j \\
& \quad c^E_j(x_j) = 0, \quad c^I_j(x_j) \leq 0 \text{ for all } j \\
& \quad w_{i,j,r} - L_\eta(s_{i,r}(x_j, p_j)) = 0 \text{ for all } i, j, r 
\end{align*}
\]  

(3.8)

The smoothing factor \( \eta \) performs in a way that when \( \eta = 0 \), the optimization problem will collapse to the problem without consideration behavior and when \( \eta \uparrow \infty \), the step function will be approaching the clear "cut-off". The solve of Problem (3.2) takes the following procedures: solve Eqn. (3.8) with an initial small value of \( \eta \); If the solve is successful, increase \( \eta \) and resolve Eqn. (3.8) starting at the last solution. This process is repeated until either solving failure is encountered or the two successive solutions are sufficiently close.

3.3.2 NLP with Complementary Constraints

If adding complementary constraints to relate this slack variables \( w \) to the screening rule by the following mixed complementarity problem (MCP):

\[
0 \leq w \leq 1 \perp s \equiv \begin{cases} 
  w = 1 & \text{if } s \leq 0 \\
  w \in [0, 1] & \text{if } s \leq 0 \\
  w = 0 & \text{if } s > 0 
\end{cases}
\]  

(3.9)

the initial problem can be smoothen to be an MPCC problem. This MPCC relaxation
needs to be implemented explicitly for the NLP solver to handle. We use two implementations: a) adding constraints to reflect complementarity (C-MPCC); b) penalizing the violation of complementarity (P-MPCC).

**Constrained MPCC (C-MPCC):** With slacks variable $s_{i,j,r}^+$ and $s_{i,j,r}^-$ to capture the positive part and the negative part of screening rules value, adding complementary constraints defines the following problem:

$$\text{maximize} \quad \hat{\pi}_C^C(w, X, p) + \tau \sum_{i,j,r} w_{i,j,r}$$

with respect to $1_j \leq x_j \leq u_j, \ p_j \geq 0$ for all $j$

$$w_{i,j,r}, v_{i,j,r}, s_{i,j,r}^+, s_{i,j,r}^- \geq 0$$

subject to $c^E_j(x_j) = 0, \ c^I_j(x_j) \leq 0$ for all $j$

$$\sum_{i=1}^I \left( \prod_{r=1}^{R_i} w_{i,j,r} \right) \geq 1$$

$$s_{i,j,r}(x_j, p_j) - s_{i,j,r}^+ + s_{i,j,r}^- = 0$$

$$w_{i,j,r} + v_{i,j,r} = 1$$

$$w_{i,j,r}s_{i,j,r}^+ \leq 0$$

$$v_{i,j,r}s_{i,j,r}^- \leq 0$$

for all $i, j, r$ (3.10)

The term $\tau \sum w$ with $\tau > 0$ serves to enforce strictly choice consistency of $w$. This formulation will be solved with Sequential Quadratic Programming (SQP) solver SNOPT, for the reason that SQP relaxes the constraints during solving the subproblem, preventing the difficulty in an Interior point method where the feasible set of this problem does not have a topological interior.

**Penalized MPCC (P-MPCC):** Penalizing the violation of complementarity in the objective
function yields the following formulation:

\[
\text{maximize } \pi^C(w, X, p) + \tau \sum_{i,j,r} w_{i,j,r} - M \sum_{i,j,r} (w_{i,j,r}s_{i,j,r}^+ + v_{i,j,r}s_{i,j,r}^-)
\]

with respect to \( l_j \leq x_j \leq u_j, \ p_j \geq 0 \) for all \( j \)

\( w_{i,j,r}, v_{i,j,r}, s_{i,j,r}^+, s_{i,j,r}^- \geq 0 \) for all \( i, j, r \)

subject to \( c_j^E(x_j) = 0, c_j^I(x_j) \leq 0 \) for all \( j \) \hspace{1cm} (3.11)

\[
\sum_{i=1}^{I} \left( \prod_{r=1}^{R_i} w_{i,j,r} \right) \geq 1 \text{ for all } j
\]

\( s_{i,j,r}(x_j, p_j) - s_{i,j,r}^+ + s_{i,j,r}^- = 0 \) for all \( i, j, r \)

\( w_{i,j,r} + v_{i,j,r} = 1 \) for all \( i, j, r \)

for some \( M > 0 \). In practice, \( M \) is empirically chosen to be sufficiently large to enforce complementarity.

### 3.4 GA Methods

Two classes of genetic algorithm are applied to solve the multiple products design problem under consider-then-choose model: a penalty-based genetic algorithm (P-GA) that combines feasible solutions selection technique; a hybrid genetic algorithm that embeds an NLP solver to strictly handle design constraints (C-GA).

#### 3.4.1 Penalized GA (P-GA)

Penalty-based genetic algorithm is widely applied to incorporating constraints, but there are well-known difficulties associated with it. Specifically, the selection and the scaling of the penalty parameter highly affect the balance between optimizing the objective and achieving feasibility, and the appropriate value of penalty parameter is sensitive to problem features including the number of constraints and the nonlinear complexity of the constraint functions [Richardson et al. 1989]. These drawbacks are especially crucial when equality constraints are involved, as the feasible searching space has smaller dimensions than the variables space. For
our multiple products design problem, we have nonlinear equality constraints that describe the relationship between acceleration and fuel consumption for each vehicle, therefore methods that have less sensitivity of the penalty parameter to the constraints number and has demonstrates successful example of incorporating equality constraints are expected to have a better fit to our design problem features.

With the consideration of above, we implement the penalty-based method proposed by Deb [2000], given its previous evidence that it eases the selection of the appropriate penalty value and also has scheme of simplicity to implement. The scheme of the algorithm still follows the traditional iteration steps of generating population, evaluating fitness, selecting parents based on fitness and reproducing new generation with crossover and mutation. We describe each procedure implemented for the design problem (3.2) as follows.

1. **Population generation:** For a member \( n \) in the population, we represent its design and pricing solution for \( J \) product as real-coded \((X_n, p_n) = (x_1, ..., x_J, p_1, ..., p_J)\), where \( x_j = (a_j, g_j) \) \((p, a, g\) respectively represent price, acceleration and fuel consumption). Population members are randomly generated with their design and pricing variables uniformly distributed within their corresponding bounds.

2. **Fitness evaluation:** In order to allow searching space, we follow the strategy that the equality constraint \( c^E_j(x_j) = 0 \) is relaxed into inequality constraint \(|c^E_j(x_j)| - \epsilon \leq 0\) with small positive \( \epsilon \) as tolerance when judging whether a member is feasible or not. For notation convenience, we denote the whole set of inequality and relaxed equality constraint as \( \hat{c}_j(x_j) \leq 0 \). The fitness computation is distinguished between feasible members and infeasible members as:

\[
\text{fitness}_n = \begin{cases} 
\pi^C(X_n, p_n) & \text{if member } n \text{ is feasible, i.e. } \hat{c}_j(x_j) \leq 0 \text{ for all } j \\
-v(X_n) & \text{otherwise}
\end{cases}
\]  

(3.12)

where \( \pi^C(\cdot) \) is the profit (objective function) computed with Eqn. (3.4); \( v(\cdot) \) is some
norm of constraints violation, which is specified in our case as:

\[ v(X_n) = \max\{||\hat{c}_j(x_j)||\text{ over all } j \text{ such that } \hat{c}(x_j) > 0 \} \] (3.13)

3. **Parents selection:** Tournament technique is used in selecting parents, in which each member has equal chance to be picked to participate a tournament. The following criteria are enforced when deciding which member can win the tournament to be one parent: (1) a feasible member is always preferred to an infeasible member; (2) if two members are both feasible, select the one with higher objective value; (3) if two members are both infeasible, select the one with lower violation.

4. **Reproduction:** Crossover is operated on the real-coded variables of the selected parents, followed by the mutation process. In order to balance population diversity and convergence, dynamic mutation probability and elitism are applied during the evolution. The mutation probability starts with a relatively large mutation probability (e.g. 0.5 in our case) then decreases as the evolution continues. We empirically adjust the frequency of decreasing the mutation probability according to the number of constraints involved in the problem. In applying elitism, the top 10% of members (ranking based on fitness) are preserved in the new population.

3.4.2 **Constrained GA (C-GA)**

In the projection-based method, an NLP operator projects the population onto the feasible space by solving the following problem for each member: Given the current design solution of member \( n \) as \( (\bar{X}_n, \bar{p}_n) = (\bar{x}_1, ..., \bar{x}_J, \bar{p}_1, ..., \bar{p}_J) \)

minimize \[ \frac{1}{2}(||X_n - \bar{X}_n||^2 + ||p_n - \bar{p}_n||^2) \]

with respect to \( X_n, p_n \)

subject to \( \text{for all } j = 1, ..., J \)

\[ l_j \leq x_j \leq u_j, p_j \geq 0 \]

\[ c^l(x_j) \leq 0 \text{ and } c^E(x_j) = 0 \] (3.14)
The method takes steps as follows:

1. Randomly generate real-coded population.

2. Project the population onto feasible space by solving Eqn. (3.14).

3. Evaluate fitness by computing objective value of the feasible population.

4. If the population converges then stop; otherwise continue to Step 5.

5. Select parents through tournaments in which the member with higher fitness (therefor objective value) is selected from a pair of candidates.

6. Reproduce new population by operating crossover and mutation on the real-coded design and pricing variables of the parents.


In both GA implementations, the convergence criteria is: once the average fitnesses over the whole population in two successive iterations are sufficiently close (e.g. absolute difference smaller than $10^{-5}$), the iteration will stop.

### 3.5 Example

To demonstrate the application of numerical methods presented in Section 3.3 and 3.4, we study a vehicle portfolio design case where heterogeneous screening rules related to budgets are modelled.

#### 3.5.1 Choice model

Suppose there are $I$ individuals and each individual $i \in \{1, \ldots, I\}$ has an individual specific budget screening rule:

$$s_i(a, g, p) = R_i p + m_i p_i^G g - B_i$$

with

$$R_i = \frac{r_i (1 + r_i)^{n_i}}{(1 + r_i)^{n_i} - 1}$$

(3.15)

(3.16)
where $m$ is annual miles travelled, $p^G$ is gasoline price, $g$ is fuel consumption per mile, $r$ is annual interest rate for the loan and $n$ is the number of loan periods. With $s_i \leq 0$, it means that an individual only considers a vehicle when its annual owning cost and operating costs are within the budget. The annual miles driven is modeled based on data from the National Household Transportation Survey U.S. Department of Transportation, Federal Highway Administration [2009], assuming the following relationship with the income: $m_i = 6639i_1^{0.288}e^{0.253N_{m,i}}$ where $N_{m,i}$ is a sample from a standard normal distribution. The annual budget is assumed to relate to the income as $B_i = 0.5 + 0.06i_1 + 0.0512N_{B,i}$ with standard normal distributed sample $N_{B,i}$. The coefficients of the budget model are estimated from the Consumer Expenditure Survey [U.S. Department of Labor, Bureau of Labor Statistics 2006a]. Income $i_1$ is sampled from an empirical frequency distribution extracted from 2006 Current Population Survey [U.S. Department of Labor, Bureau of Labor Statistics 2006b] and interest rate $r_i$ is sampled from uniform distribution between 0.03 and 0.08.

In the compensatory stage, the each individual has utility:

$$u_i(a, g, p) = \beta_{p,i}p + \beta_{g,i}g + \beta_{a,i}\frac{a}{a} + \vartheta_i,$$

(3.17)

The coefficients in this equation are as follows: $\beta_{p,i} = -|4.591 + 0.1756/i_1 - 0.377N_{p,i}|$ where $i_1$ denotes household income; $\beta_{g,i} = -36.77 + 2.2N_{g,i}$; $\beta_{a,i} = 11.262 + 0.321N_{a,i}$; and $\vartheta_i = 23.178 + 0.5N_{0,i}$. The variables $N_{p,i}$, $N_{g,i}$, $N_{a,i}$, and $N_{0,i}$ each denote independent samples from a standard normal distribution.

- $\beta_{p,i} = -|4.591 + 0.1756/i_1 - 0.377N_i|$
- $\beta_{g,i} = -36.77 + 2.2N_i$
- $\beta_{a,i} = 11.262 + 0.321N_i$
- $\vartheta_i = 23.178 + 0.5N_i$

### 3.5.2 Engineering Model

In this vehicle portfolio design problem, the design variables are 0-60 acceleration time ($a$, ranging from 2.5 - 15s) and fuel consumption ($g$, in gallons per mile) and price ($p$, in
10,000 dollars). The engineering model is influenced by Whitefoot et al. [2011], giving the fuel consumption and acceleration time are inversely related by:

\[
G(a) = 0.035 + \frac{53.5 + 69.5e^{-a} - 1.8a^{1.4} + 106.9/a}{1000}
\]  

(3.18)

The unit cost is a function of acceleration:

\[
c(a) = e^{a/12} \left( 1.5 + 1.97e^{-a} - 0.04a + \frac{1}{a - 1.5} \right).
\]  

(3.19)

Finally, the design optimization problem for a product line with \( J \) products under heterogeneous budget screening rules and compensatory utilities can be written as:

\[
\begin{align*}
\text{maximize} \quad & \sum_{j=1}^{J} \left( \sum_{i=1}^{I} P_{i,j}^c(a, g, p) \right) (p_j - c(a_j, g_j)) \\
\text{with respect to} \quad & 2.5 \leq a_j \leq 15, \quad g_j \geq 0, \quad p_j \geq 0 \quad \text{for all} \quad j \\
\text{subject to} \quad & p_j - c(a_j, g_j) \geq 0 \quad \text{for all} \quad j \\
& g_j - G(a_j) = 0 \quad \text{for all} \quad j
\end{align*}
\]  

(3.20)

3.6 Results

3.6.1 Computational Details

Both NLP and GA methods described above are applied in the vehicle portfolio design example. Cases with multiple screening rules (\( I = 1, 5, 10, 20, 30, 40, 50 \)) and multiple products (\( J = 1, 2, 5, 10 \)) are tested. In all GA runs, we use initial mutation probability of 0.5 and population size of 50. For S-NLP method, smoothing factor \( \eta = 10 \) is used. For P-MPCC method, the penalty factor \( M = 10 \) is used. For all method except C-GA, we run 1000 trials by drawing random initial conditions. For C-GA, 100 trials are run, given this number is sufficient to capture the property of the C-GA in our test runs. Each method was written in C code, and all NLPs were solved using the SQP solver SNOPT [Gill et al. 2005a] with relative optimality and feasibility tolerances set to \( 10^{-6} \). All computations were undertaken on a single Mac Pro tower with 2, quad-core 2.26GHz processors and 32 GB of RAM running OS X (10.6.8).
The performance of the methods are compared in feasibility, optimality and computation costs (time and number of function evaluation spent on a single solve). The results are presented in illustrative cases in the follows.

### 3.6.2 Feasibility

In SNOPT solver, the feasibility of the problem is controlled by:

\[
\max_j \left\{ \frac{|g_j - G(a_j)|}{\max_j \left\{ \max \{|a_j|, |g_j|, |p_j|\} \right\}} \right\}.
\]

With our specified tolerance $10^{-6}$, when NLP solver terminated successfully, solutions in S-NLP, C-MPCC, P-MPCC and C-GA are guaranteed to achieve the relative constraint violation below this tolerance. For P-GA where the satisfaction of constraints depends on feasibility selection and random evolution, the constraint satisfaction is achieved in a looser level with relative violation controlled under $3 \times 10^{-3}$. Empirically one can tune the P-GA to obtain tighter feasibility, for example by restricting the violation criteria when selecting feasible parents and at the same time higher the initial mutation probability to avoid the population from being stuck in a point with high feasibility but low objective value too early. However, these tuning will sacrifice the optimality and computation time because the selection may get rid of members with high objective value but slightly lower feasibility and the high initial mutation probability will lead to slow convergence.

### 3.6.3 Optimality

In order to compare the performances on achieving high objective value in a normalized way, we measure the relative difference between a solution and the best-known empirical solution $\pi^C$. We find the $\pi^C$ by looking for the solution with the highest objective value over all trials of solves that have relative constraint violation under $10^{-4}$ in all methods. We capture the optimality using cumulative distribution functions (CDFs) of solution’s objective level found with each method over all trials normalized by $\pi^C$. Denote the optimal profit found by a
method in trial \( t \in 1, \ldots, T \) as \( \pi^C_t \), i.e. we plot

\[
D(\tau) = \frac{\left| \{ t : |\pi^C_t - \pi^C_*| \leq \tau \pi^C_* \} \right|}{T}
\]

The “shallower” the CDF \( D : [0,1] \rightarrow [0,1] \) is, the poorer a method is likely to perform for a randomly drawn initial condition. Figure 3.2 plots these CDFs for all five methods in two representative cases: \((J,I) = (5,10)\) (smaller problem size) and \((J,I) = (10,50)\) (larger problem size).

![Empirical CDF of solution profits found relative to the best-known profit in two cases: \((J,I) = (5,10)\) (left) and \((J,I) = (10,50)\) (right). Method abbreviations are as defined in the text.](image)

In both small and large size problems, C-GA has all solutions have objective level at least as 95% as high as the best-known solution and feasibility (with relative violation < \(10^{-4}\)) is guaranteed by the NLP projection operator. As for P-GA, over 95% of the solutions can achieve profit as high as 80% of the best-known profit, but its ability to achieve a better profit level (e.g. 90% as high as the best known profit) decreases as problem size increases. For example, in the \( I = 10, J = 5 \) case, there are over 90% of the solution observed to have a profit level of 90% as high as the best known profit, while in the \( I = 50, J = 10 \) case, only around 30% of solutions are observed.

Both P-MPCC and C-MPCC methods have good performance in small size problems, characterized by the portion of solutions that converge to the best-known profit and the portion of
solutions with profit as 80% high as the best-known profit. However this good performance of the constrained-NLP does not maintain in larger size problems. The growth of problem size affect P-MPCC in a way that solutions are highly attracted in a local optima that lies in a profit level of 80% 90% of the best-known solution. Among three NLP methods, the S-NLP formulation suffers the mostly from the portions of trivial solutions which result in zero profit; this shortcoming is more significant when problem size is larger.

Figure 3.3  Solution times for Eqn. (3.20) of various sizes. Means plotted with dashed lines, with solid lines illustrating one standard deviation above and below the mean.

The changes of computation time and function evaluation on a single solve with the growth of problem size are illustrated in Figure 3.3 and 3.4. Among all methods C-GA is the most expensive in both computation time and function evaluation in all problem sizes. C-MPCC and P-MPCC have comparable computation costs. When the problem size is small, P-MPCC can solve faster than P-GA method, while in larger size problem, P-GA has more advantage. We observe the computation time of both GA methods are not sensitive to the growth of number of screening rules $I$ but to the number of products $J$. For C-MPCC and P-MPCC methods, the increasing $I$ and $J$ can both have impact to the computation time.
Figure 3.4 Function evaluation counts for Eqn. (3.20) of various sizes. Means plotted with dashed lines, with solid lines illustrating one standard deviation above and below the mean.

3.7 Discussion

Our results has not poited to an absolution best method for design optimization with consider-then-choose model. C-GA is most reliable regarding feasibility and optimality but it is the most time consuming to solve. P-GA has the ability to compute solutions with good optimality with time efficiency in large size problem, however we need to accept a looser feasibility tolerance. P-MPCC has the best performance among three NLP formulations, and has more advantage in smaller size problem than P-GA, but increasing the problem size will undermine its time efficiency and optimality. In the future, we may further explore hybrid methods such that the advantage of MPCC and GA can be combined.

Based on the progress of this study, three directions will be in our future focus: First, more approaches will be explored to enhance global convergence of the design optimization problem. Second, design framework will be extended to include discrete product features which calls for further exploration of optimization tools dealing with mixed-integer variables. Third, more types of screening rules will be included in the design problem, requiring the corresponding formulations and numerical solving techniques.
CHAPTER 4. CONSIDERATION AND DESIGN INFORMED BY REVEALED PREFERENCE

4.1 Introduction

Conventional discrete choice models [Ben-Akiva and Lerman 1985b, Train 2009] have been applied in design for market systems [Wassenaar and Chen 2003, Michalek et al. 2004; 2005, Besharati et al. 2006b, Shiau et al. 2009b, Hoyle et al. 2010, MacDonald et al. 2010, Michalek et al. 2011] in the past decade. Generally, the choice model serves to forecast demand as a function of product features, thus enabling design decisions that maximize forecast profits. These conventional choice models share the assumption that individuals choose by processing and weighing all attributes, for all alternatives, when maximizing utility. According to this assumption choice is a compensatory decision making process where tradeoffs can take place across all features and all alternatives: in particular, shortcomings in one attribute can always be compensated by making others sufficiently attractive. Empirical studies have shown the opposite: people often use “fast and frugal” non-compensatory rules to eliminate options when faced with task complexity [Payne 1976], time pressure [Rieskamp and Hoffrage 2008], information cost [Bröder 2000] and memory requirements [Bröder and Schiffer 2003]. The use of such heuristics—decision rules that ignore information—is widespread and beneficial [Gigerenzer and Gaissmaier 2011]. This chapter investigates the importance of including consideration behavior when making design decisions.

The awareness of the use of non-compensatory rules among consumers has changed the traditional concept of the choice set in choice modeling [Shocker et al. 1991]. Instead of assuming only a universal choice set with all alternatives, consideration-sets [Roberts and Lattin 1997, Hauser et al. 2009] have become a topic of active research. Consideration-sets are subsets of the universal set that are chosen by individuals following internal, non-compensatory rules.
Building on early research on non-compensatory decision models [Dawes 1964, Einhorn 1970, Tversky 1972], non-compensatory rules proposed for consideration set formation include conjunctive, disjunctive, subset conjunctive, and even lexicographic rules; see Hauser [2014] for further background and examples. Accepting consideration implies that identification of the structure and distribution of screening rules is an important empirical task that much recent research addresses, as reviewed in Section 4.2.

But is modeling consideration important when making design decisions? Marketers have only shown the advantage of modeling consideration through improvement in model predictive accuracy, though this has been accomplished across a wide variety of product categories including cameras, batteries, automobiles, cellphones, and computers [Gilbride and Allenby 2004, Jedidi and Kohli 2005, Hauser et al. 2009, Ding and Hauser 2011, Yee et al. 2007]; e.g., see Table 4.1 below. Simulation experiments have illustrated the limits of classical compensatory models including the multinomial and random coefficient (Mixed) Logit models when modeling non-compensatory choice behavior [Johnson and Meyer 1984, Andrews and Manrai 1998, Andrews et al. 2008]. Existing engineering studies demonstrate how design can include consideration in choice model structure, and how this might affect decisions [Besharati et al. 2006a], but have not compared the performance of compensatory and non-compensatory models when both types of models are estimated on the same data with a comparable level of system knowledge. Even if compensatory models do not represent non-compensatory choice behavior well, could they still suggest product designs similar to designs that are optimal for true, non-compensatory behavior? If the non-compensatory behavior is modeled directly, how much closer could a firm get to true optimal designs? What is the difference of the value of the chosen designs, e.g. profits, between designs chosen using compensatory versus non-compensatory models?

We describe a simulation study that examines how well compensatory models perform in 1) recovering non-compensatory choice behavior, 2) suggesting design decisions near to ideal optimal decisions, and 3) suggesting designs that capture all potential profitability. Our “synthetic data” Dzyabura and Hauser [2011] simulation experiment has the following steps:

1: Define a synthetic population with known “true” choice behavior;

2: Simulate responses of this population to a sequence of “markets” with randomly generated
product profiles;

3: Estimate compensatory and non-compensatory models from the responses and validate predictive power;

4: Optimize design decisions with the estimated models and evaluate design profit using the “true” behavior.

Synthetic data experiment is an effective method for detecting choice model properties in specific situations or when testing the validity of an estimation approach [Andrews et al. 2008, Kohli and Jedidi 2007, Kropko 2011, Dzyabura and Hauser 2011]. We extend this paradigm to also include the quality and value of decisions made using estimated models, the ultimate goal of choice modeling within engineering design. The synthetic data experiment allows us to measure the divergence of design decisions and outcomes from ideal values that can be obtained only by knowing the true behavioral model. We describe an “econometric-style” (revealed preference) experiment that uses aggregate share data to estimate choice models. An alternative perspective, more common in marketing, samples the population for respondents to choice and/or consideration-based conjoint surveys (stated preference). Both perspectives have value, as is discussed in Train [2009], pg.152. Both types of models have also been used in design [Michalek et al. 2004, Wassenaar et al. 2005].

Several observations are enabled by the experiment. As would be expected, modeling consideration with a non-compensatory model results in the best design and pricing decisions when the population exhibits matching non-compensatory behavior. Conventional compensatory models can reasonably support profitable design decisions, however, with several caveats: conventional models might require more data than is reasonably available to capture non-compensatory behaviors, can suggest simplistic product portfolios, can be sensitive to sample variance in the training data, and don’t forecast the value of design decisions well even if those decisions couldn’t be improved with a better model. Overall, modeling heterogeneity in the screening rules used to form consideration sets captures more value to design than modeling heterogeneity in the compensatory stage. A similar observation has been made by Andrews et al. [Andrews et al. 2008]. Finally, while assuming that better model predictive power implies better design
decisions is reasonable, it is not necessarily true: models with lower predictive power can suggest more profitable designs. We hope our case study will motivate market systems researchers to further examine what consideration behaviors exist in their product categories and how these behaviors might influence optimality of chosen designs.

The rest of this chapter is organized as follows: Section 4.2 reviews the consider-then-choose model construction and estimation studied in marketing research. Section 4.3 describes the simulation framework and synthetic data generation process. Section 4.4 and 4.5 respectively provides details of model estimation and design optimization. Section 6.3 presents our results, followed by discussion in Section 4.7. Section 4.8 concludes.

4.2 Consider-Then-Choose Models

A consider-then-choose model can be described as follows. Suppose the universal choice set is \( J = \{1, \ldots, J\} \). A consideration set indexed by \( r = 1, \ldots, R \), denoted as \( C_r \subset \{1, \ldots, J\} \) is defined by a set of screening rules \( s_r = [s_{r,1}, \ldots, s_{r,L_r}] \). For conjunctive rules, \( C_r \) can be written as:

\[
C_r(X, p) = \left\{ j \in \{1, \ldots, J\} : s_r(x_j, p_j) \leq 0 \right\}
\]

(4.1)

The screening rules depend on product features \( x_j \), price \( p_j \) as well as other rule-specific parameters. This definition means that a product needs to satisfy all the screening rules to be a member in the corresponding consideration set. An alternative form illustrating this is

\[
C_r = \left\{ j \in \{1, \ldots, J\} : \max_{l \in \{1, \ldots, L_r\}} s_{r,l}(x_j, p_j, \gamma_r) \leq 0 \right\}
\]

(4.2)

For example, the consideration set

\[
C_r(X, p) = \{ \text{all vehicles } j \text{ with price } p_j \text{ under } \$20,000 \text{ AND fuel economy } e_j \text{ over } 30 \text{ mpg} \} \]
can be defined by
\[ s_r(x_j, p_j) = \begin{pmatrix} p_j - 20,000 \\ 30 - e_j \end{pmatrix} \leq 0. \]

For disjunctions, e.g.
\[ C_r = \{ \text{all products } j \in 1, \ldots, J \text{ with price } p_j \text{ under 20,000 dollars OR fuel economy } e_j \text{ over 30mpg } \}, \]

the consideration set can be defined as:
\[ C_r = \left\{ j \in \{1, \ldots, J\} : \min_{l \in \{1, \ldots, L_r\}} s_{r,l}(x_j, p_j, \gamma_r) \leq 0 \right\} \quad (4.3) \]

Disjunctions of conjunctions can be formed by combining the min and max representations in Eqns. (4.2) and (4.3). This structure is consistent with the forms used in the marketing literature, although marketers often define screening rules in terms of indicators instead of inequalities. See, for example [Gilbride and Allenby 2004, Liu and Arora 2011, Dzyabura and Hauser 2011]. These representations can be transformed into the structure presented here.

Given a collection of screening rules and the associated consideration set, let the conditional probability that product \( j \) is chosen within the set be \( P_{j|C_r} \) and let the probability that the consideration set \( C_r \) is formed be \( P_{C_r} \). Then the choice probability \( P_j \) can be written as a weighted sum of the choice probabilities across all possible consideration sets:
\[ P_j = \sum_r P_{j|C_r} P_{C_r} \quad (4.4) \]

Hauser [2014] calls such models “consideration” or “choice set explosion” models, as they are subject to combinatorial explosion in the number of parameters needed to capture consideration set occurrence. Empirical methods estimate \( P_{C_r} \) directly, rather than uncovering structure behind screening by identifying the rules \( s_r \). Manrai and Andrews [1998] provide a thorough review of studies applying Eqn. (4.4) to scanner panel data. Note that Eqn. (4.4) can also be considered a type of random coefficients (Mixed) Logit model, though not one with normally distributed coefficients. This structure has also been found to be similar to a nested Logit model, as we detail in Sec. 4.4.5 below.
Preference-conditional choice probabilities then take the following form:

$$
P_{j|C_r}(X, p | \theta) = \begin{cases} 
e^{u(x_j, p_j, \theta)} & \text{if } j \in C_r \\ 
\frac{1}{1 + \sum_{k \in C_r} e^{u(x_k, p_k, \theta)}} & \text{if } j \not\in C_r 
\end{cases}$$

where utility $u(\cdot)$ is a function of product characteristics $x_j$ and price $p_j$ given coefficients $\theta$ that measure preferences. The utility coefficients $\theta$ can be assumed to be homogeneous across the population or take a random coefficients form to include heterogeneity (which requires a Monte-Carlo integral of the simple Logit form above). This formula can, in principle, be extended to capture heterogeneity across consideration sets by allowing a nontrivial joint distribution between coefficients $\theta$ and consideration sets.

Methods used in early studies to discover non-compensatory screening rules included tracing and protocol analysis [Payne 1976, Bettman 1980] in which respondents’ decision processes were self-reported or tracked. Shortcomings of this type of method have been reported and include inconsistencies between the stated screening criteria and observed choices from the same individual [Green and Krieger 1988]. More recent research shows that the accuracy of direct elicitation approaches can be improved by designing experiments that are incentive-compatible: for example, by participating in a survey in which respondents describe their screening rules for new vehicles, they have a decent chance of actually winning a vehicle described [Ding and Hauser 2011]. Estimation tools that are widely applied in traditional discrete choice analysis, e.g. maximum likelihood and Bayesian methods, can also be used in non-compensatory model parameter estimation [Jedidi and Kohli 1996; 2005, Gilbride and Allenby 2004]. These methods may, however, suffer from high computation costs due to exponential growth in the number of possible consideration sets as the number of attributes and/or attribute levels grows. Machine learning techniques have recently been adapted to circumvent this problem by applying greedy methods [Yee et al. 2007, Kohli and Jedidi 2007] or low-dimensional parameterizations of screening rule likelihood [Urban et al. 2010, Dzyabura and Hauser 2011]. Broadly speaking, marketing research has demonstrated predictive power improvement by modeling consideration; see Table 4.1.

In principle the specification reviewed above can be estimated from choice data with classical
Table 4.1 Recent consider-then-choose models constructed from stated preference data, compared to compensatory models estimated in the same study. Abbreviations: HB - Hierarchical Bayes; MLE - Maximum Likelihood; HR - “hit rate” (frequency of correct prediction on hold-out samples); KLD - Kullback-Liebler Divergence; TAU - Kendall’s Tau [Hauser 1978].

<table>
<thead>
<tr>
<th>Reference</th>
<th>Product</th>
<th>Compensatory Model</th>
<th>Consider-then-Choose Model</th>
<th>% Improvement</th>
<th>KLD</th>
<th>TAU</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kohli and Jedidi [2007]</td>
<td>Laptops</td>
<td>LP Logit</td>
<td>Greedy, Lexicographic</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jedidi and Kohli [2005]</td>
<td>Batteries</td>
<td>MLE Logit</td>
<td>MLE, Subset conjunctive</td>
<td>1.1%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hauser et al. [2009]</td>
<td>Cameras</td>
<td>HB Logit</td>
<td>HB, Conjunctive screening</td>
<td>7.1%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yee et al. [2007]</td>
<td>Smartphones</td>
<td>HB Ranked Logit</td>
<td>Lexicographic by aspects</td>
<td>8.7%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ding and Hauser [2011]</td>
<td>Cellphones</td>
<td>HB Logit</td>
<td>Unstructured Direct Elicitation</td>
<td>9.1%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yee et al. [2007]</td>
<td>Smartphones</td>
<td>HB Ranked Logit</td>
<td>Lexicographic by aspects</td>
<td>8.7%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ding and Hauser [2011]</td>
<td>Cellphones</td>
<td>HB Logit</td>
<td>Unstructured Direct Elicitation</td>
<td>9.1%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Swait [2001a]</td>
<td>Rental Cars</td>
<td>MLE Logit</td>
<td>“Cut-off rules” (conjunctions)</td>
<td>14.0%(a)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hauser and Toubia [2001]</td>
<td>GPS Units</td>
<td>HB Logit</td>
<td>Greedy, Lexicographic</td>
<td>4.5%</td>
<td>54.5%</td>
<td></td>
</tr>
<tr>
<td>Dzyabura and Hauser [2011]</td>
<td>Vehicles</td>
<td>HB Logit</td>
<td>Adaptive question HB</td>
<td>44.1%</td>
<td>16.7%</td>
<td></td>
</tr>
</tbody>
</table>

(a) Swait [2001a] characterized improvement with improvement in log-likelihood, which is proportional to KLD.

tools such as Maximum Likelihood Estimation (MLE) and Bayesian methods. To facilitate this, the representation of consideration set probability $P_c$ has taken different forms. Swait and Ben-Akiva [1987] introduced a random component into the screening rules so that with an assumed distribution $P_c$ can be derived based on the probability any alternative is acceptable.

Ben-Akiva Ben-Akiva and Boccara [1995] extended this random consideration set generation model by specifying the availability probability as Logit form. Instead of defining $P_c$, through parameterized screening rules, Chiang et al. [1999] assumed consideration set probabilities have a Dirichlet distribution across the population. Gilbride and Allenby Gilbride and Allenby [2004] avoided the enumeration of consideration sets by using a reduced form choice probability and Markov Chain Monte-Carlo methods to sample from the posterior distribution of the allowable screening criteria values. Exponential growth in the number of possible consideration sets and rules makes consider-then-choose models difficult to estimate in practice. This challenge motivated researchers to develop methods that apply to “consider” stage observations to infer screening rules with more attributes and complexity. For example, MLE methods have been used on “acceptable/unacceptable” responses to the profiles to estimate the probability that a particular attribute level is acceptable [Jedidi and Kohli 2005]. Dzyabura and Hauser [2011] model a case where capturing the distribution of screening rules would require $2^{53}$ parameters, too many for a direct estimation strategy. They develop an adaptive question survey strategy to estimate conjunctive screening rules by parameterizing screening rule likelihood presuming feature acceptability is independent, obtaining a model with only 53 parameters per respondent.
4.3 Case Study: Vehicle Design Under Body Style Screening

We simulate a stylized model of the new vehicle market with potential purchasers that screen over vehicle body style. Empirical studies have shown body style screening in both self-reported surveys [3] and statistical inferences Dzyabura and Hauser [2011]. Body style also significantly impacts the engineering relationships between other features in vehicle design. We often refer to the synthetic behavior described below as the “true” behavior. We do not use this terminology to suggest this is how households actually choose new vehicles. This is only a shorthand appropriate for the context of the simulation experiment.

4.3.1 Synthetic Behavior

Our population is a mix of groups that screen over \( B = 9 \) vehicle body styles listed in Table 4.2. Vehicles are described by fuel economy \((e)\), acceleration \((a)\), price \((p)\) and a \( B \)-element binary vector \( \delta \) for which \( \delta_b = 1 \) if, and only if, the vehicle has body style \( b \) (thus \( \sum_b \delta_b = 1 \)). Let \( s \neq 0 \) be a \( B \)-element binary vector defining which body styles are “acceptable” to a given individual in the population; we refer to these vectors succinctly as “screening rules.” Unlike \( \delta \), which can have only one element equal to 1, \( s \) can have any number of elements equal to 1. An individual with screening rule \( s \) considers only those vehicles with body styles \( b \) such that \( s_b = 1 \) or, equivalently, \( s^\top \delta \geq 1 \).

In the notation of Eqn. (4.1,4.5) we can index individuals by screening rules \( s \) and define

\[
C_s(\Delta) = \left\{ j \in \{1, \ldots, J\} : 1 - s^\top \delta_j \leq 0 \right\}
\]

(4.6)

where \( \Delta = (\delta_1, \ldots, \delta_J) \) is a matrix of binary body style vectors.

The fraction of individuals in the population with a particular screening rule \( s \) is given by a probability mass function \( \alpha(s) \) drawn from the results of the empirical study reported in [Urban et al. 2010, Dzyabura and Hauser 2011]. This study estimated conjunctive screening rules with a Bayesian adaptive question method for 874 respondents. More specifically, we take \( \alpha \) to be the empirical frequency distribution (over respondents) of the modal (most probable) rules \( s \) for the posterior distribution. Out of the 874 respondents, 219 distinct most-probable conjunctive
screening rules were estimated, and every body style is acceptable to some individual. See Table 4.2 for the aggregated acceptability of different body styles over the full respondent pool.

Table 4.2 Percentage of respondents accepting the given body style, as reported in Dzyabura and Hauser [2011].

<table>
<thead>
<tr>
<th>Body Style</th>
<th>Acceptability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sports car</td>
<td>16%</td>
</tr>
<tr>
<td>Hatchback</td>
<td>19%</td>
</tr>
<tr>
<td>Compact sedan</td>
<td>38%</td>
</tr>
<tr>
<td>Standard sedan</td>
<td>42%</td>
</tr>
<tr>
<td>Crossover</td>
<td>38%</td>
</tr>
<tr>
<td>Small SUV</td>
<td>39%</td>
</tr>
<tr>
<td>Full-size SUV</td>
<td>29%</td>
</tr>
<tr>
<td>Pickup truck</td>
<td>18%</td>
</tr>
<tr>
<td>Minivan</td>
<td>10%</td>
</tr>
</tbody>
</table>

Conditional on using a screening rule $s$, individuals choose from those vehicles in $C_s(\Delta)$ by maximizing the random utility:

$$U_j = \hat{u}(e_j, a_j, p_j; \hat{\theta}) + \mathcal{E}_j$$

$$\hat{u}(e, a, p; \hat{\theta}) = -\exp\{\hat{\theta}_p\}p + \frac{\hat{\theta}_e}{e} + \frac{\hat{\theta}_a}{a} + \hat{\theta}_0$$

for random coefficients $\hat{\theta}_l \sim \mathcal{N}(\hat{\mu}_l, \hat{\sigma}_l)$ ($l = p, e, a, 0$). $\mathcal{N}(\hat{\mu}, \hat{\sigma})$ is the normal distribution with mean $\hat{\mu}$ and variance $\hat{\sigma}$. The exponential in the price coefficient ensures that lower prices are preferred, all other things being equal. The errors $\mathcal{E} = (\mathcal{E}_0, \mathcal{E}_1, \ldots, \mathcal{E}_J)$ are i.i.d. extreme value variables mean-shifted towards zero. The resulting screening-conditional sub-populations thus follow a Mixed Logit model. There are no correlations between screening rules and preference over vehicle attributes and price, but there is heterogeneity in the population.

Table 4.3 Means and Variances of random coefficients ($\hat{\theta}$'s) in synthetic population utility function, Eqn. (4.7). $\mathcal{N}(\hat{\mu}, \hat{\sigma})$ refers to a normally distributed variable with mean $\hat{\mu}$ and variance $\hat{\sigma}^2$. Values based on the model from [?].

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Utility</th>
<th>Random Coefficient</th>
<th>Mean ($\hat{\mu}$)</th>
<th>Variance ($\hat{\sigma}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>$(p)$</td>
<td>$-\exp(\mathcal{N}(\hat{\mu}, \hat{\sigma}))p$</td>
<td>2.0</td>
<td>0.1</td>
</tr>
<tr>
<td>Fuel Economy</td>
<td>$(a)$</td>
<td>$\mathcal{N}(\hat{\mu}, \hat{\sigma})/e$</td>
<td>-36.8</td>
<td>2.2</td>
</tr>
<tr>
<td>Acceleration</td>
<td>$(e)$</td>
<td>$\mathcal{N}(\hat{\mu}, \hat{\sigma})/a$</td>
<td>11.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Constant</td>
<td>$(\cdot)$</td>
<td>$\mathcal{N}(\hat{\mu}, \hat{\sigma})$</td>
<td>-23.2</td>
<td>0.5</td>
</tr>
</tbody>
</table>
4.3.2 Market Share Simulation

We simulate sales data set that might be collected from multiple, separate new vehicle markets. The vehicles in each market form a “universal” choice set for the consumers in that market. A data set for estimation then consists of vehicle market shares in $M$ separate markets indexed by $m$. For each market, we draw a set of $J_m$ vehicles, denoted $J_m$. The profile of vehicle $j$ in market $m$ is given by drawing fuel economy ($e_{j,m}$), acceleration ($a_{j,m}$), price ($p_{j,m}$) and body style ($b_{j,m}$) from a uniform distribution respectively on intervals $[5, 50]$ (mpg), $[2, 15]$ (s) and $[10, 60]$ (10k$\$) and $\{1, \ldots, B\}$. An alternative consistent with our optimal design problem presented below would be to draw sets of vehicles that satisfy our assumed design constraints. This is possible, and better matches the stylized market modeling paradigm we employ. However random draws are likely to give us better information about choice behavior than correlated draws, and thus allow us to focus more completely on choice model quality. To investigate statistical properties of model estimation and use with stochastic data generation and choice outcomes, this process is repeated with different random seeds.

Given product profiles in market $m$, we draw $N_m$ choice observations in which individuals can purchase one of the vehicles or choose not to purchase any vehicle (choose the “outside good”). $N_m$ individuals are drawn from the synthetic population by drawing $N_m$ screening rules $s_i$ from the distribution $\alpha(s)$ along with associated random coefficients $\theta_i$. Shares $S_{j,m}$ for each vehicle $j$ in each market $m$ (and the outside good) are then generated by maximizing random utilities (utilities plus error term) foreach individual over their consideration set. See \[?] for an explicit algorithm.

(1) set $S_{j,m} \leftarrow 0$ for all $j \in J_m \cup \{0\}$

(2) for $i = 1, \ldots, N_m$

(2.a) set $C_i \leftarrow \{0\}$ (only contain “not buy” decision)

(2.b) for $j = 1, \ldots, J_m$, if $s_i^T \delta_j = 1$, $C_i \leftarrow C_i \cup \{j\}$

if $s_i^T \delta_j = 1$, $C_i \leftarrow C_i \cup \{j\}$

if $s_i^T \delta_j = 0$ (i.e. screening rule not satisfied)

- $S_{0,m} \leftarrow S_{0,m} + 1$ ("not buy" decision)
else $C_i \leftarrow C_i \cup \{j\}$

$\cdot \ C_i \leftarrow C_i \cup \{j\}$

(2.c) draw $\hat{\theta}_{i,l}$, ($l = p, e, a, 0$) in Eqn. (4.8) according to the distribution listed in Table 4.3;

(2.d) draw $\mathcal{E}_{i,j}$ in Eqn. (4.7) according to extreme value distribution for all $j \in C_i$

(2.e) evaluate $U_{i,j}$ for all $j \in C_i$ using Eqn. (4.7) and (4.8)

(2.f) find alternative $k = \arg \max_{j \in C_i} U_{i,j}$

(2.g) set $S_{k,m} \leftarrow S_{k,m} + 1$

(3) $S_{j,m} \leftarrow S_{j,m}/N_m$ for all $j \in J_m \cup \{0\}$

4.4 Choice Models

We examine four choice model specifications: Multinomial Logit (MNL), Random Coefficients Logit (RCL), Nested Multinomial Logit (NML) and Consider-Then-Choose Logit (CTC) models. We assume that all models incorporate the prior information that body style plays a role in consumer decision, but different model specifications incorporate this piece of information in different structures: MNL and RCL model assume the tradeoffs between body style and other attributes, NML uses nests that separate body styles, thus constructing a two-stage but yet compensatory process; CTC models the frequency of any possible consideration sets, based on body style, along with compensatory choices conditional on consideration set. Note that the true behavior of the synthetic population exhibits characteristics of both non-compensatory screening and heterogeneity in compensatory stage. Thus all the models are misspecified on at least one behavioral feature. The comparison between these models will thus illustrate the consequence of failing to capture different behavioral features.

Coefficients in all models are estimated by maximizing the log-likelihood with respect to the coefficients Train [2009]. For a general choice model with probabilities $P_{j,m}(\theta)$ for coefficients
$\theta$, this takes the form:

$$\maximize \sum_{m=1}^{M} \sum_{j \in J_m \cup \{0\}} S_{j,m} \log \left( P_{j,m}(\theta) \right)$$

(4.9)

with respect to $\theta$ (plus possible constraints)

where

$$\ell(\theta) = \sum_{m=1}^{M} \sum_{j \in J_m \cup \{0\}} S_{j,m} \log \left( P_{j,m}(\theta) \right)$$

(4.10)

We abbreviate this process by “MLE” and, for brevity, do not explicitly list each MLE problem below. Instead we define the choice probability model and list any constraints imposed on the coefficients as this is sufficient to recreate our process.

### 4.4.1 Multinomial Logit Model (MNL)

The MNL model takes the utility of product $j$ to be

$$u_{j,m}^{MNL}(\theta) = -\exp\{\theta_p\} p_{j,m} + \frac{\theta_e}{e_{j,m}} + \frac{\theta_a}{a_{j,m}} + \sum_{b=1}^{B} \theta_b \delta_{j,m,b} + \theta_0$$

(4.11)

giving choice probabilities

$$P_{j,m}^{MNL}(\theta) = \frac{\exp\{u_{j,m}^{MNL}(\theta)\}}{1 + \sum_{k \in J_m} \exp\{u_{k,m}^{MNL}(\theta)\}}$$

(4.12)

As with the true behavior, the “exp” term in the price coefficient ensures that the price coefficient is negative, and thus lower prices are preferred (all other attributes being equal). The “no buy” or outside good probability is $P_{0,m}^{MNL}(\theta) = 1 - \sum_{j \in J_m} P_{j,m}^{MNL}(\theta)$.

The maximum likelihood estimation problem is written as:

$$\maximize \ell^{MNL}(\theta)$$

with respect to $\theta$

(4.13)

subject to $\sum_{b=1}^{B} \theta_b = 0$
where
\[
\ell^{MNL}(\theta) = \sum_{m=1}^{M} \left( S_{0,m} \log \left( P_{0,m}^{MNL}(\theta) \right) 
+ \sum_{j \in J_m} S_{j,m} \log \left( P_{j,m}^{MNL}(\theta) \right) \right)
\] (4.14)

The coefficients over body styles are constrained to sum to zero because they are not independently identified from the constant \( \theta_0 \). Specifically, utilities are not changed if we add any number to all coefficients for all body style dummies and subtract the same number from the mean of the constant term. We could, equivalently, leave the constant term out of the specification. We prefer to include it as it allows us to capture only those body style specific variations in utility with the coefficients on the body style dummies. This type of representation is consistent with “effects coding” widely used in discrete choice modeling [Bech and Gyrd-Hansen 2005].

### 4.4.2 Random Coefficients Logit Model (RCL)

In the RCL model, choice probabilities \( P^{RCL}_{j,m} \) are defined for each vehicle \( j \) in each market \( m \) by
\[
P^{RCL}_{j,m}(\mu, \sigma) = \int P^{MNL}_{j,m}(\theta) \phi(\theta | \mu, \sigma) d\theta
\] (4.15)
for \( P^{MNL}_{j,m} \) as given in Eqn. (4.12). All random coefficients as written in Eqns. (4.12-4.15) are assumed to be normally distributed, \( \theta_l = N(\mu_l, \sigma_l) \) \( l = p, a, e, 1, ..., B \), with mean \( \mu_l \) and variance \( \sigma^2_l \). Note, however, that this implies that the price coefficient will be log-normal (e.g., Boyd and Mellman [1980], Berry et al. [2004]). The density \( \phi \) is thus a product of \( 4 + B \) independent normal densities each having two parameters. The RCL model thus has \( 8 + 2B \) coefficients we must estimate, two of which enter into the utility function nonlinearly.

Given synthetic revealed preference data we estimate the parameters \((\mu, \sigma)\) using simulated MLE Train [2009]. We perform Monte-Carlo sampling over random coefficients to obtain \( I \) samples \( \theta_i \sim N(\mu, \text{diag}(\sigma^2)) \) and simulated RCL choice probabilities
\[
\tilde{P}^{RCL}_{j,m}(\mu, \sigma) = \left( \frac{1}{I} \right) \sum_{i=1}^{I} P^{MNL}_{j,m}(\theta_i).
\] (4.16)
We use $I = 1,000$ Monte-Carlo samples throughout this study if not otherwise mentioned. Similar to the Logit model estimation, the perfect correlation between body style coefficients can lead to multiple estimators that give the same choice probability. Therefore the mean coefficients on body styles are also constrained so that their sum equals to zero. Note that this does not imply that $\sum_b \theta_b = 0$ with probability one, but only that $E[\sum_b \theta_b] = 0$.

### 4.4.3 Nested Multinomial Logit Model (NML)

We also examine a NML model in which vehicles with the same body style are assigned to the same nest. Suppose product $j$ in market $m$ belongs to nest $N_{b(j),m}$ where $b(j)$ is the body style of product $j$. The probability product $j$ is chosen in market $m$ is

$$P_{j,m}^{NML}(\theta) = P_{j|b(j),m}^{C} P_{b(j),m}$$

(4.17)

where $P_{b,m}^{N}$ is the probability that any product from nest $N_{b}$ is chosen in market $m$ and $P_{j|b(j),m}^{C}$ is the probability that product $j$ is chosen in market $m$, conditional on nest $b(j)$ being chosen. $P_{j|b(j),m}^{C}$ follows the logit formula in which only non-body style features are involved in the utility:

$$P_{j|b(j),m}^{C}(\theta) = \frac{\exp\{u_{j,m}^{NML}(\theta_p, \theta_e, \theta_a)\}}{\sum_{k \in N_{b(j),m}} \exp\{u_{k,m}^{NML}(\theta_p, \theta_e, \theta_a)\}}$$

(4.18)

with utility within the nest defined as:

$$u_{j,m}^{NML}(\theta_p, \theta_e, \theta_a) = - \exp\{\theta_p\} p_{j,m} + \frac{\theta_e}{e_{j,m}} + \frac{\theta_a}{a_{j,m}}$$

(4.19)

The choice of nest depends on the “nest utility”

$$V_{b,m}(\theta_p, \theta_e, \theta_a) = \log \left( \sum_{j \in N_{b,m}} \exp\{u_{j,m}^{NML}(\theta_p, \theta_e, \theta_a)\} \right)$$

(4.20)

and also takes the logit form

$$P_{b,m}(\theta, \lambda) = \frac{\exp\{\theta_0 + \theta_b + \lambda_b V_{b,m}(\theta_p, \theta_e, \theta_a)\}}{1 + \sum_{c=1}^{B} \exp\{\theta_0 + \theta_c + \lambda_c V_{c,m}(\theta_p, \theta_e, \theta_a)\}}$$

(4.21)
We again constrain the body style dummies coefficient to sum to zero, for the same reason as in MNL and RCL models.

This formulation follows Daly’s version of the NML Daly [1987], rather than the “Generalized Extreme Value” formulation given by McFadden McFadden [1978]. The difference between two formulations is that McFadden’s model uses

\[ \theta_0 + \theta_b + \lambda_b V_{b,m} \left( \frac{\theta_p}{\lambda_b}, \frac{\theta_e}{\lambda_b}, \frac{\theta_a}{\lambda_b} \right) \]

as the utility in Eqn. (4.21). This change is required for consistency with random utility maximization, but there is still debate about whether that is essential in the model Train [2009]. Both versions have similarities with consideration behavior, as discussed below.

4.4.4 Consider-Then-Choose Logit Model (CTC)

In the CTC model body styles are screened in the non-compensatory stage and do not enter the compensatory stage. Preference in compensatory stage is assumed to be homogeneous both among the population and across all consideration sets. The body styles screening rules are \( S = (s_1, \ldots, s_R) \) characterizing all \( R = 2^B - 1 \) possible consideration sets \( C_1, \ldots, C_R \), except the “null set” in which no body style is considered. Each screening rule is coded as \( B \)-element binary vector \( s_r = (s_{r,1}, \ldots, s_{r,B}) \) where \( s_{r,b} = 1 \) if body style \( b \) is acceptable, \( s_{r,b} = 0 \) otherwise. Thus

\[ C_{r,m} = \{ j : s_r^\top \delta_{j,m} \geq 1 \} \]

(4.22)

which means that a product will be considered as long as its body style is acceptable.

The choice probability for product \( j \) in market \( m \) is

\[ P_{j,m}^{CTC}(\theta, \alpha) = \sum_{r=1}^{R} \alpha_r \left( \frac{\exp\{u_{j,m}^{CTC}(\theta)\}}{1 + \sum_{k \in C_{r,m}} \exp\{u_{k,m}^{CTC}(\theta)\}} \right) \]

(4.23)

if \( j \in C_{r,m} \) and zero otherwise, where \( \alpha_r = \alpha(s_r) \) is an estimator of the probability that a randomly drawn individual in the population has screening rule \( s_r \) and utilities are defined by:
\[ u_{j,m}^{CTC}(\theta) = -\exp(\theta_p)p_{j,m} + \frac{\theta_e}{e_{j,m}} + \frac{\theta_a}{a_{j,m}} + \theta_0 \]  

(4.24)

We estimate this model with MLE, solving for both \( \theta \) and \( \alpha \in [0,1] \), \( \sum_r \alpha_r = 1 \). These constraints are required to ensure that \( \alpha \) is a probability mass function.

Note that we directly estimate consideration set probability \( P_{C_r} = \alpha(s_r) \) rather than estimating parameters of the distribution \( \alpha(s) \). Our case study is small enough to enable us to enumerate the consideration sets, requiring only \( R = 511 \) \( \alpha \) values to fully characterize the distribution of consideration sets. This formulation allows us to estimate from the same observed market share data using a MLE technique consistent with that employed for the MNL, RCL, and NML models. The CTC model we estimate does not, however, then reflect the level of generality and efficiency available in the applications we review above. This does not affect our main purpose, to demonstrate the impact on design of non-compensatory behavior.

### 4.4.5 Connecting Nested Logit with Consideration

A few comments regarding the connection between the NML and CTC models are required, motivated by the similarity in the choice probabilities in the CTC and NML models. If the consideration sets used in the population are disjoint, then Eqn. (4.4) describes the choice probabilities in a single-level nested Logit model whose nests are given by the consideration sets. However a NML would use the specific parameterization of \( P_{C_r} \) given in Eqn. (4.21). It is easy to see that any true value of \( P_{C_r} \) can be recovered in this parameterization by taking the nesting parameter \( \lambda_r \) to be 1 and choosing the right value of the coefficients for attributes that are constant over the consideration set (e.g., body style). Swait [2001b] has linked the generalized nested logit model to construct a general consideration set explosion model. Similarly, it is also easy to show that a single-level cross-nested Logit model [Bierlaire 2006] can realize choice processes as described in Eqn. (4.4).

Though these models are the mathematically similar their interpretations differ, which drives a non-trivial difference in formalization. The NML model pictures rational, compensatory individuals that might use any consideration set and choose any product, and models consideration set frequency as a function of the expected maximum utility of choosing from a
given consideration set [Train 2009]. While a NML can recover CTC behavior choosing the right parameters, it does not necessarily result in the same predictions as designs change because nest selection is a function of in-nest utilities. In contrast, the CTC model views individuals as drawn from a population with heterogeneous screening rules, and decouples consideration set frequency from utility. When consideration set occurrence is, completely or partially, independent of compensatory utilities, this distinction is meaningful.

4.5 Design Optimization

This section defines a single firm’s optimal vehicle design problem matching the stylized market model discussed above. The firm’s objective is to maximize the expected profit of its vehicle portfolio by deciding the number of vehicles \( J_f \) and choosing the body style, fuel economy, acceleration, and price for each vehicle. We allow firms to offer multiple vehicles of the same body style, as this is observed in real auto markets.

4.5.1 Engineering Model

Each vehicle is described by its 0-60 acceleration time \((a, \text{in s})\), fuel economy \((e, \text{in mpg})\), weight \((w, 10^3 \text{ lbs})\), body style \((b \in \{1, \ldots, B\})\), “technology content” \((t, \text{unitless})\), and price \((p, \$10^4)\). In the original model by ?, \(t\) is a continuous proxy for efficiency improvement through adoption of discrete technology content; this efficiency improvement can be directed towards either fuel economy or acceleration performance. To accomplish this, and to represent a physical connection between acceleration and fuel economy, \(e, a, w\) and \(t\) are related by a function \(g_b(e, a, t)\) as given in Eqn. (4.25):

\[
g_b(e, a, t) = \frac{1000}{e - 3.46} - \beta_{\text{const}}^g(b) - \beta_a^g(b) \exp(-a) - \beta_t^g(b)t \\
- \beta_{at}^g(b)a^2t - \beta_w^g(b)w - \beta_{wa}^g(b)wa
\]  

(4.25)

Eqn. (4.25) can be written as an equality constraint \(g_b(e, a, t) = 0\) on acceleration and fuel economy decisions. Unit costs are also a function of design variables expressed by the following
The body style specific coefficients in these models were estimated using detailed engineering simulations from AVL Cruise in conjunction with confidential technology production cost data provided to NHTSA by automakers in advance of the 2012 – 2016 fuel economy rule making [Whitefoot et al. 2013]. Table 4.4 and 4.5 summarizes body style specific coefficient values. In our case study we assume that vehicle weight and technology content for each vehicle are fixed, and thus do not include these as arguments in \( g_b \) or \( c_b \). We use the average curb weights listed in Table 4.6, based on 2005 model year vehicle data as reported in [Wenzel et al. 2010], and technology content \( t_b = 20 \).

### Table 4.4 Coefficients in fuel consumption function for each body style

<table>
<thead>
<tr>
<th>Body style</th>
<th>Coefficients</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \beta^c_{\text{const}} )</td>
<td>( \beta^c_a )</td>
<td>( \beta^c_t )</td>
<td>( \beta^c_w )</td>
<td>( \beta^c_{wa} )</td>
<td></td>
</tr>
<tr>
<td>Two-seat</td>
<td>20.8484</td>
<td>89.6806</td>
<td>-0.2049</td>
<td>0.0016</td>
<td>2.9159</td>
<td>0.028</td>
</tr>
<tr>
<td>Hatchback</td>
<td>10.792</td>
<td>69.5244</td>
<td>-0.2605</td>
<td>0.0013</td>
<td>12.9897</td>
<td>-0.5593</td>
</tr>
<tr>
<td>Compact sedan</td>
<td>10.792</td>
<td>9.5244</td>
<td>-0.2605</td>
<td>0.0013</td>
<td>12.9897</td>
<td>-0.5593</td>
</tr>
<tr>
<td>Standard sedan</td>
<td>11.5531</td>
<td>733.706</td>
<td>-0.0829</td>
<td>0.0002</td>
<td>8.668</td>
<td>-0.2954</td>
</tr>
<tr>
<td>Crossover</td>
<td>10.8515</td>
<td>452.552</td>
<td>-0.0794</td>
<td>0.0002</td>
<td>8.3583</td>
<td>-0.2539</td>
</tr>
<tr>
<td>Small SUV</td>
<td>11.5531</td>
<td>733.706</td>
<td>-0.0829</td>
<td>0.0002</td>
<td>8.668</td>
<td>0.2954</td>
</tr>
<tr>
<td>Full SUV</td>
<td>11.5531</td>
<td>733.706</td>
<td>-0.0829</td>
<td>0.0002</td>
<td>8.668</td>
<td>0.2954</td>
</tr>
<tr>
<td>Pickup truck</td>
<td>10.5185</td>
<td>2979.096</td>
<td>-0.089</td>
<td>0.0003</td>
<td>8.6604</td>
<td>-0.2225</td>
</tr>
<tr>
<td>Minivan</td>
<td>10.5185</td>
<td>2979.096</td>
<td>-0.089</td>
<td>0.0003</td>
<td>8.6604</td>
<td>-0.2225</td>
</tr>
</tbody>
</table>

### Table 4.5 Coefficients in cost function for each body style

<table>
<thead>
<tr>
<th>Body style</th>
<th>Coefficients</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \beta^c_{\text{const}} )</td>
<td>( \beta^c_a )</td>
<td>( \beta^c_t )</td>
<td>( \beta^c_w )</td>
<td>( \beta^c_{wa} )</td>
</tr>
<tr>
<td>Two-seat</td>
<td>0.3669</td>
<td>10.6686</td>
<td>0.0175</td>
<td>0.0002579</td>
<td>-0.0000082</td>
</tr>
<tr>
<td>Hatchback</td>
<td>0.78</td>
<td>1.9716</td>
<td>0.0016</td>
<td>0.000225</td>
<td>-0.0000123</td>
</tr>
<tr>
<td>Compact sedan</td>
<td>0.78</td>
<td>1.9716</td>
<td>0.0016</td>
<td>0.000225</td>
<td>-0.0000123</td>
</tr>
<tr>
<td>Standard sedan</td>
<td>0.554</td>
<td>24.3842</td>
<td>0.0054</td>
<td>0.0001963</td>
<td>-0.0000071</td>
</tr>
<tr>
<td>Crossover</td>
<td>0.4029</td>
<td>24.0527</td>
<td>0.0057</td>
<td>0.0002339</td>
<td>-0.0000069</td>
</tr>
<tr>
<td>Small SUV</td>
<td>0.02</td>
<td>92.3965</td>
<td>0.0038</td>
<td>0.000347</td>
<td>-0.0000108</td>
</tr>
<tr>
<td>Full SUV</td>
<td>0.02</td>
<td>92.3965</td>
<td>0.0038</td>
<td>0.000347</td>
<td>-0.0000108</td>
</tr>
<tr>
<td>Pickup truck</td>
<td>0.3025</td>
<td>160.56</td>
<td>0.0066</td>
<td>0.0002538</td>
<td>-0.0000055</td>
</tr>
<tr>
<td>Minivan</td>
<td>0.3025</td>
<td>160.56</td>
<td>0.0066</td>
<td>0.0002538</td>
<td>-0.0000055</td>
</tr>
</tbody>
</table>
Table 4.6  Vehicle curb weight for each body style

<table>
<thead>
<tr>
<th>Body style</th>
<th>Average weight (lbs)</th>
<th>Min weight (lbs)</th>
<th>Max weight (lbs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two-seat</td>
<td>2701</td>
<td>1873</td>
<td>3518</td>
</tr>
<tr>
<td>Hatchback</td>
<td>3128</td>
<td>2700</td>
<td>3709</td>
</tr>
<tr>
<td>Compact sedan</td>
<td>3128</td>
<td>2700</td>
<td>3709</td>
</tr>
<tr>
<td>Standard sedan</td>
<td>3258</td>
<td>2864</td>
<td>3818</td>
</tr>
<tr>
<td>Crossover</td>
<td>3909</td>
<td>3409</td>
<td>5391</td>
</tr>
<tr>
<td>Small SUV</td>
<td>3672</td>
<td>3182</td>
<td>3909</td>
</tr>
<tr>
<td>Full-size SUV</td>
<td>4985</td>
<td>3281</td>
<td>5727</td>
</tr>
<tr>
<td>Pickup truck</td>
<td>4651</td>
<td>3987</td>
<td>5754</td>
</tr>
<tr>
<td>Minivan</td>
<td>4264</td>
<td>3654</td>
<td>4813</td>
</tr>
</tbody>
</table>

4.5.2 Formulation and Solution

Given a portfolio with $J_f$ vehicles and body style vector $b$ the optimal choices of fuel economy, acceleration, and price for each vehicle are those that solve

$$\text{maximize} \quad \pi(p, e, a | J_f, b)$$

with respect to $\forall j, p_j \geq 0$

$$L_{e,b(j)} \leq e_j \leq U_{e,b(j)};$$

$$L_{a,b(j)} \leq a_j \leq U_{a,b(j)};$$

subject to $g_b(j)(e_j, a_j) = 0 \ \forall j$

where expected profits are

$$\pi(p, e, a | J_f, b) = \sum_{j=1}^{J_f} P_j(p, e, a, b)(p_j - c_j(e_j, a_j))$$

and $(L_{e,b}, U_{e,b}), (L_{a,b}, U_{a,b})$ are body-style specific lower and upper bounds on fuel economy and acceleration. Note that we are not specific about what probability model we use. Eqn. (4.27) is smooth for any of the models, because choosing prices, fuel economy, and acceleration does not affect screening in the CTC or, similarly, the nesting structure in NML.

The optimal number of vehicles, body styles, and associated designs and prices can be
obtained by solving

$$\text{maximize } \pi^*(J_f, b)$$

with respect to $J_f \in \{1, \ldots, B\}$, $b_j \in \{1, \ldots, B\}$ for all $j = 1, \ldots, J_f$ (4.29)

where $\pi^*(J_f, b)$ is the optimal value of Eqn. (4.27) for given $J_f$ and $b$. Note that we allow for multiple vehicles with the same body style. Because enumerating all the feasible choices of body styles $b$ for $1, \ldots, B$ vehicles is computationally prohibitive, we use a Genetic Algorithm (GA) to solve Eqn. (4.29).

### 4.5.3 Genetic Algorithm Scheme

The scheme of genetic algorithm in solving for optimal body styles combination:

**Generate members:** each member $n$ is assigned a binary coded genome $\Delta_n = (\delta_1, \ldots, \delta_J)$ where $\delta_j = (\delta_{j,1}, \ldots, \delta_{j,B})$ with $\delta_{j,b} = 1$ if $j$'s body style = $b$, $\delta_{j,b} = 0$ otherwise. If $\delta_j = 0$, it represents that product $j$ is taken away from the portfolio. Note that we force the condition $\sum_{b=1}^{B} \delta_{j,b} \leq 1$ to be satisfied (a product only has one body style, or the product is not launched) during randomly generating the population.

**Compute fitness:** For each member, we can formulate the problem in Eqn. (4.27) based on the coded information given in $\Delta$, i.e. number of vehicles decided by $J_f = \sum_{j=1}^{J} \max\{0, \sum_{b=1}^{B} \delta_{j,b}\}$, and the body style collection $b$ is coded in each nonzero vector $\delta_j$ as stated above. Fitness is defined as the optimal solution of Eqn. (4.27) whose solving process can be handled by NLP solver.

**Reproduce new generation:** the probability of member $n$ to be selected as a parent is computed by $\pi^*_n / \sum_{k \in \text{all members}} \pi^*_k$. When crossover takes place between a pair of parents, the parents will switch at least 1 but less than $J$ successive connected $\delta$, for example, crossover between member $n$ and $n'$ with crossover site $j$ with $j$ randomly chosen from 2 to $J$:

before crossover:

$\Delta_n = (\delta_1, \ldots, \delta_{j-1}, \delta_j, \ldots, \delta_J)$ and $\Delta_{n'} = (\delta_1', \ldots, \delta_{j-1}', \delta_j', \ldots, \delta_J')$

after crossover:

$\Delta_n = (\delta_1, \ldots, \delta_{j-1}', \delta_j', \ldots, \delta_J, \delta_{j+1}, \ldots, \delta_J)$ and $\Delta_{n'} = (\delta_1, \ldots, \delta_{j-1}, \delta_j, \ldots, \delta_J')$
\( \Delta_n = (\delta_1, ..., \delta_{j-1}, \delta'_j, ..., \delta'_J) \) and \( \Delta_{n'} = (\delta'_1, ..., \delta'_{j-1}, \delta_j, ..., \delta_J) \)

Mutation happens with probability \( q_{\text{mut}} \), mutation site is randomly selected among any \( (j, b) \in \{1, ..., J\} \times \{1, ..., B\} \), for the selected \( (j, b) \), we do the following to ensure \( \sum_{b=1}^{B} \delta_{j,b} \leq 1 \): if \( \delta_{j,b} = 1 \) then switch 1 to 0; otherwise find \( b' \) such that \( \delta_{j,b'} = 1 \) and make \( \delta_{j,b'} = 0, \delta_{j,b} = 1 \).

To enhance optimality, 50 trials have been done with different initial population. Among these trials, the best solution is reported.

### 4.6 Results

This section presents performance results pertaining to choice model accuracy or predictive power, design “error”, and profitability potential. To investigate how the amount of market information influences performance, we performed the simulation experiment with \( M = 10, 25, 50, 100, 200, 500, \) and 1000 markets. For each \( M \) we draw 20 separate sets of \( J_m = 5 \) profiles and \( N_m = 100 \) choice observations, estimate MNL, RCL, NML, and CTC models, and then use these models to design product portfolios obtaining 20 separate sets of model estimates and designs. Sampling different sets of share data for a given market size allows us to gauge the effect of sampling variance in the data on model predictions, design outcomes, and design value, while examining different numbers of markets allows us to assess the asymptotic properties of the estimated models and their associated designs. The MLE and design optimization routines were programmed in C language, and nonlinear programs involved in estimation and design optimization were solved with the sequential quadratic programming (SQP) solver SNOPT (version 7) Gill et al. [2005b]. All computations were undertaken on a single Mac Pro tower with 2, quad-core 2.26GHz processors and 32GB of RAM running OS X (10.6.8).

#### 4.6.1 Predictive Power

The predictive power of the estimated models is validated on a new data set that consists of \( M' \) markets, where each market \( m' = 1, ..., M' \) has a set of vehicles \( J_{m'} \). Kullback-Leibler Divergence \( ? \),

\[
KLD = \left( \frac{1}{M'} \right) \sum_{m'=1}^{M'} \sum_{j \in J_{m'}} P^T_{j,m'} \log \left( \frac{P^T_{j,m'}}{\hat{P}_{j,m'}} \right),
\]
captures how close the predicted choice probability distribution $P$ is to the actual choice probability distribution $P^T$ in the validation set. Predictive share errors are evaluated via Eqn. (4.30) using 1,000 markets of validation data different from the estimation data, but drawn using the same approach.

Fig. 4.1 plots the divergence between predictions for estimated models and the true behavior against the number of markets used to train the models. Increasing the amount of market data available for estimation reduces both expected prediction error and the variance of the error. Increasing the amount of data, however, does not result in traditional compensatory models that match the predictive power of the CTC model. For example, the divergence of the three traditional models’ predictions observing 1000 markets is larger than the CTC prediction observing only 10 markets. When observing more than 50 markets, the predictive power of RCL and NML models is generally between those of MNL and CTC with RCL predictions appearing to be slightly closer to the true behavior. However, when observing fewer than 10 markets the MNL model outperforms RCL and NML models. We believe designers should be particularly interested in performance when estimating models with relatively small amount of market data because real revealed preference market research often uses a very limited number of markets for estimation. For example, econometric new vehicle market models most often use fewer than 20 markets (marked with vertical line in Fig. 4.1) Berry et al. [1995], Goldberg [1995], Train [2009]. Our market simulation setting is not strictly comparable to these studies because of a difference in the number of vehicle-observations in each market, the complexity of real vehicle profiles, and the detail often given by population demographics. But these results suggest caution given the small number of markets usually used for model estimation in practice.

### 4.6.2 Decision Bias and Variance

We first define a “design error” metric to quantify how different portfolios chosen using an estimated model are from portfolios that would be chosen for the true behavior (perfect information). Comparing product portfolios is a complicated task, and we do not suggest we have a uniquely good metric for comparison. Essentially, our metric compares the relative
Figure 4.1 Kullback-Leibler Divergence (KLD) of predicted choice probability distribution from true behavior. Solid lines represent the range of observed values over 20 separate data sets while the dashed line represents the average value.

The specific numerical values of “design error” for any given choice model are less important than comparisons across the different choice model types we explore.

Suppose a portfolio has \( J_f \) vehicles, each with body style \( b_j \) and design \( x_j = (e_j, a_j) \). Denote the body style combinations of a portfolio as \( (n_1, n_2, ..., n_B) \) where \( n_b \) is the number of vehicles in the portfolio that have body style \( b \). We refer to the ideal portfolio as the optimal portfolio using the true behavior and denote the ideal portfolio with superscript “*”; multiplicity of ideal portfolios is addressed below. Our design error metric is

\[
d = \frac{1}{2} \left( \sum_{b=1}^{B} N_b + \max\{H^+, H^-\} \right)
\]  

(4.31)
where

\[ N_b = \begin{cases} |n_b - n_b^*|/n_b^* & \text{if } n_b^* > 0 \\ n_b & \text{if } n_b^* = 0 \end{cases} \] (4.32)

\[ H^+ = \max_{j:n_b^*(j) > 0} \left\{ \min_{k:b^*(k) = b(j)} \left\{ d_w(x_j, x_k^*) \right\} \right\} \] (4.33)

\[ H^- = \max_{j:n_b(k) > 0} \left\{ \min_{j:b(j) = b^*(k)} \left\{ d_w(x_j, x_k^*) \right\} \right\} \] (4.34)

\[ d_w(x, x^*) = \frac{1}{2} \left( \frac{|e - e^*|}{e^*} + \frac{|a - a^*|}{a^*} \right) \] (4.35)

The first term in Eqn. (4.31), Eqn. (4.32), captures differences in body style combinations by penalizing differences in the number of vehicles offered with each body style. The second term in Eqn. (4.31), composed of Eqns. (4.33-4.35), is a Hausdorff distance \[^7\] comparing sets of vehicles with the same body styles using the relative error metric in Eqn. (4.35). This portion of the metric is zero so long as the sets of vehicles offered are equivalent, even if offered in different multiplicities. If nonzero, this portion gives the relative error in the attributes of any vehicle offered when that vehicle shares a body style with a body style offered in the ideally optimal portfolio. Note that this distance measure gives an error only in engineering decisions, while pricing is obviously important to profitable product design. However it is plausible that “incorrect” prices could be corrected relatively quickly in the marketplace after offering a particular set of products, while errors in engineering features cannot be. Section 4.6.4 explores this in more detail. See \[^7\] for a generalization of Eqn. (4.31)-(4.35) that accounts for prices.

Fig. 4.2 plots design error for optimal decisions based on the MNL, RCL, NML, and CTC choice models against number of markets observed. Two features are of interest: design bias refers to the difference between mean model-optimal designs and ideal optimal designs; design variance refers to the spread of designs that might be made given different observed markets used to estimate the choice models. With fewer than 25 markets the CTC model has the lowest design bias, consistent with the performance this model showed in predictive power. As the amount of market data grows, designs under the CTC model appear to be converging to ideal designs. Designs chosen using a NML model compare well to CTC designs when observing
50 or more markets. Designs chosen using a RCL model have the largest variation among the models, and this variation cannot be overcome by increasing the number of markets observed. Unlike RCL, NML and CTC models, using a MNL model suggests identical vehicles with the same body style should be produced; this is reflected as the high design error compared to the ideal design in which there is wide diversity among body styles.

![Graph showing design error for different models](image)

Figure 4.2 Design error, measured as in Eqn. (4.31), for all models over all numbers of markets observed. Solid lines represent the range of observed values over 20 separate data sets while the dashed line represents the average value.

In computing design error we presume that a computed ideal portfolio is a good representation of portfolios required to achieve optimality relative to the true behavior. If there are distinct locally optimal portfolios that achieve nearly globally optimal profits a different metric would be required. Similarly, if profits were "flat" near the ideal portfolio design error loses meaning. In the next section we discuss decision profitability, which is the ultimate metric of portfolio performance.

### 4.6.3 Decision Profitability

A decision that differs from an ideal decision is not necessarily un-profitable. The true profit of a product portfolio is its expected profits computed under the true behavior, rather than the estimated model. We compute true profit $\Pi$ as

$$
\Pi = \sum_{j=1}^{J_f} p_j^T(e, a, p, \Delta)(p_j - c_j)
$$

(4.36)
computing market share $P_j^T$ by computing the choice probability in the true behavior described in Section 4.3.1 instead of the sampling procedure described in Section 4.3.2. Here the Monte-Carlo sampling size used to approximate the compensatory stage random coefficients model was $I = 100,000$. We do not include competitive firms’ vehicles in this profit validation in order to be consistent with the design optimization problems, which do not include competition. Note that true profit for any model-optimal portfolio should be less than the true profit given by the optimal portfolio under the true behavior. We refer the the profit gained by optimal decisions under the true model as the ideal profit.

Fig. 4.3 plots the percentage of ideal profits that can be achieved by choosing designs and prices using an estimated model. CTC designs and prices are, not surprisingly, best able to capture true profits. Even when observing only 10 markets, CTC designs and prices can be expected to achieved at least 90% of the ideal profits with the worst designs achieving around 70% of the ideal profits. MNL designs and prices can be expected to obtain only 60% of the ideal profit due to a single body-style portfolio that lacks diversity. Estimating a model with limited amount of market data appears to affect the profitability of the RCL model designs the most out of all the models, consistent with our observations regarding design error variance. Even observing up to 50 markets it is possible for RCL designs and prices to recover less than 40% of the ideal profits, depending on sampling variance in the market data. More data results in RCL designs and prices within 90-95% of the ideal profits. Observing less than 25 markets NML designs capture approximately 20% less true profit than the CTC designs and also shows relatively high variation in true profits (facts obscured by the log₁₀-scale axis in Fig. 4.3). However like RCL, NML designs and prices can ultimately capture 90-95% of the true ideal profits when estimating the model with enough data.

4.6.4 Pricing-On-Offering

An additional test assesses the degree to which a model suggests unprofitable decisions simply because of a poor representation of preferences over prices. Prices can, in principle, be changed up until the point-of-sale while design decisions must often be fixed far in advance of sale. Thus it is reasonable to consider a case where firms learn more about preferences when
they offer the portfolio designed and exercise price flexibility to maximize profits. Suppose that the MNL, RCL, NML, and CTC models inform the design of the product portfolio but that prices can be changed even the product is offered (as in, e.g., Morrow 2014). How much more profits could the firm recover by using the true choice behavior in order to set optimal prices, for fixed designs? While the firm is not likely to actually know the true behavior, this value represents an upper bound on profitability of design decisions made using an estimated model when prices are flexible and determined when offering the portfolio.

Fig. 4.4 plots percent of ideal profits obtained using the vehicle portfolios suggested by the estimated models, but offered at prices determined by the optimizing profits for that portfolio under the true behavior. From this perspective the RCL, NML, and CTC models each have the potential to suggest nearly equivalently profitable design decisions. RCL and NML, in particular, can suggest much more profitable portfolios if we admit pricing flexibility than if we don’t, and thus RCL and NML capture pricing preferences more weakly than does the CTC model. Moreover, for intermediate numbers of markets (25, 50, 100, and 200), the NML model appears to suggest the most profitable portfolios by a small margin (less than 2.7%) that is exaggerated by the log10 axis scaling. Finally, even the best possible pricing strategy cannot increase the true profitability of the single body-style portfolio designed under the MNL model.

4.7 Discussion and Limitations

There are several observations for designers to take away from this exploratory simulation study.

First, conventional compensatory models can reasonably support profitable design decisions even when the population exhibits non-compensatory behavior with enough data. Designs based on estimated RCL and NML models were capable of obtaining above 90% of the ideal profits (Fig. 4.3); however this required roughly twice the amount of market data (50 markets) that might typically be available (20 markets) judging from the vehicle modeling literature. The RCL and NML models could suggest designs that obtain almost 100% when an ideal pricing strategy is followed (Fig. 4.4), but this would require learning preferences exactly when actually offering the vehicles designed (Fig. 4.4). This is practically impossible but does suggest that a significant
portion of the “error” made with conventional models pertains to pricing bias, not design bias. However designers should be aware of the possible side effects of different compensatory model structures: The MNL model suggests portfolios with identical body styles; the RCL model, estimated on limited amounts of aggregate market share data, is highly sensitive to sample error leading to large variations in optimal designs; and the NML model, while it might capture optimal designs very well if the nesting structure reflects consideration, suggests biased pricing decisions and thus cannot present accurate forecasts of design profitability. Designers also need to take into account the amount of information available to train their model when they decide what model to use. According to our simulation using the MNL might be more reliably profitable than using the RCL and NML models if market data are very limited, because noise in the data induces greater variance in designs suggested by RCL and NML models.

Second, modeling the heterogeneity in the screening rules may capture more value to design than modeling heterogeneity in the compensatory stage. This is most directly observed by comparing the CTC and RCL models. The RCL model ignores screening stage heterogeneity, and achieved only 30% of the ideal profit (on average) with 10 markets while displaying an unacceptably large sensitivity to sample variance with limited training data. The CTC model with only 10 marketsof training data gives a firm expected profit that is at least 80% as much as what they could get with perfect knowledge. Recall that the CTC model is mis-specified, in that it ignores compensatory stage heterogeneity. Other evidence comes from the NML. NML and CTC are similar in a two-stage modeling structure. In effect, our NML is a close approximation to a CTC model assuming that individuals consider one, and only one, vehicle body style. Decisions made based on the NML are most often more profitably than those made with the RCL model for all amounts of training data. This observation may be driven by limited amount of heterogeneity in our assumed true behavior (see Table 4.3), suggesting further research is required.

Note also that the CTC model has the potential to be seriously overfit. For example, the CTC model in with M = 50 has more than twice as many parameters (515) as observations (250), but is still the most predictively accurate model (Fig. 4.1, right) and results in the most profitable decisions (Fig. 4.3, right). Conventional wisdom would suggest that at least as
many observations as parameters are required for a valid model; i.e., the CTC model requires, at a minimum, \( M \geq 103 \) markets with 5 vehicles per market. We believe that the stability of estimated model predictions is more important than the ratio of parameters to observations; Fig. 4.1 shows that the predictive power of the estimated CTC model is as good as it can get with as few as 50 markets. However, overfitting effects may result in the difference between NML and CTC model performance when pricing on offering (Fig. 4.4): our CTC model presumes that all 511 screening rules may be in use by the population generating the data, while the NML approximates a CTC model with only \( B = 9 \) screening rules. Slightly better performance with the simpler model suggests some overfitting may be occurring, although any such overfitting could be easily corrected by restricting the number of nonzero \( \alpha \) coefficients in the CTC estimation.

Third, assuming that better predictive power indicates better design decisions is reasonable but not necessarily true. Pearson correlation coefficients are positive but weak: 0.62 between predictive power (divergence) and design error, 0.73 between predictive power and profit error (measured as error, not percent of ideal profits recovered), and 0.73 between design and profit errors. Fig. 4.5 scatters the average design error and average true profitability versus the average Kullback-Leibler divergence of four models estimated under two market information conditions: 10 and 1,000 markets; Here, the average is taken over different data sets with the same number of markets. While there is a general trend that lower divergence (better model predictive power) is consistent with lower design error, deviation from this trend is also observed. For example, NML has, on average, worse choice predictions but better designs than RCL for both 10 and 1,000 markets worth of data. Lower model divergence also generally indicates less loss of profit. However there are exceptions, such as the comparison between the NML and the RCL. Note also the difference in scales: the MNL model does not appear to predict that much worse than the NML or RCL models while suggesting designs that capture almost no profit relative to NML or RCL.

These results shows that the true profitability of designs made using traditional models cannot be judged from predictive power alone. While further investigation of the relationship between predictive accuracy and decision value across a range of design problems and market
conditions is needed, it is clear that choice models with structure representative of the under-
lying choice process are better for design even if they may not show significant benefits from
the perspective of modeling choice alone.

Important limitations of our study are as follows.

Our study has focused on the value of incorporating prior knowledge on screening without
demonstrating the process needed to obtain that knowledge. Nor has our study examined the
consequences of misspecified prior knowledge. The CTC model in this study is able to estimate
the distribution of the possible consideration sets from choice data given that the attributes
involved in the screening process are known and limited. Our presumed behavior—screening
over body style—is a reasonable prior for the case study and is represented in some form in
every model we tested. Aggregate share data is insufficient to infer what attributes and screens
are involved in the consideration stage. We are currently mirroring this simulation study within
the context of survey design for both choice-based conjoint and consideration-based questions
[Dzyabura and Hauser 2011] in which screening rules can be statistically inferred. Subjective
beliefs, however, often inform choice model construction; they underlie decisions about what
utility function to use and what distribution the error term takes (including heterogeneous
preferences and nesting structures). While we must assume that misspecification of screening
rules would impact design outcomes, this is a generic problem for choice modeling.

The design problem in our case study is also simplistic. The engineering model merely
focuses on a body style specific fuel consumption-acceleration relationship and cost function
that depends only on fuel economy and acceleration. The screening rules we used were indepen-
dent of continuous features such as price and fuel economy; in contrast, the study from which
we drew screening rules estimated rules over a body style, brand, fuel economy, price, quality,
safety, power, and powertrain [Dzyabura and Hauser 2011]. Future studies should include more
engineering model as well as complexities in screening.

Bayesian methods for model estimation were also not used in this study. An important fact
is that Bayesian methods provide an alternative path to estimate the parameters of a choice
model, not fundamentally different models. Theoretically speaking, maximum likelihood and
Bayesian estimators are often similar; in particular the posterior mean of a Bayesian estimator
is asymptotically equivalent to the maximum likelihood estimator [Train 2009]. Empirically Bayesian estimators have been reported to have better fit small-sample data but have predictive power and parameter recovery similar to maximum likelihood estimation [Andrews et al. 2002]. In the context of our study we might then expect Bayesian estimators to achieve larger-sample performance with fewer data, but not to qualitatively change the comparison between conventional compensatory models and non-compensatory models when consumers consider.

4.8 Conclusions

This chapter explores the impact of consideration behavior on optimal design for market systems models by presenting a simulation study of vehicle portfolio design for a population with heterogeneous screening over body style and heterogenous compensatory evaluations after screening. With synthetically generated aggregate marketshare data we estimate multinomial Logit, random coefficient Logit, nested multinomial logit, and consider-then-choose logit models. All four models contain some representation of screening, and all are misspecified in at least one dimension of the true behavior. We use the estimated models to optimize designs for a single model and compare model performance in terms of predictive power, design error, and profitability. We find that capturing heterogeneous consideration, when it exists, is more important than capturing heterogeneous tradeoffs. This can be accomplished with consider-then-choose Logit, but also with the right nested Logit model. Decisions made using Logit models are simplistic, suggesting portfolios with a single body style, and decisions made using random coefficients Logit models are noisy; with limited amounts of data, Logit models may often lead to more profitable decisions than random coefficients Logit. We also observe that higher model predictive power generally does imply a more profitable design decision, but that there are cases where poorer predictors can yield higher profits.
Figure 4.3  Percent of ideal profits obtained by designs and prices under true behavior recovery when choosing designs and prices with estimated models. Note the log$_{10}$ scale y-axis focuses on differences from 100%. Solid lines represent the range of observed values over 20 separate data sets while the dashed line represents the average value.

Figure 4.4  Percent of ideal profits obtained by designs chosen using estimated models, but optimizing prices for these designs with knowledge of the true choice behavior. Note the log$_{10}$ scale y-axis focuses on differences from 100%. Solid lines represent the range of observed values over 20 separate data sets while the dashed line represents the average value.
Figure 4.5  Expected design error (left) and expected Profit Error (right) versus Kullback-Leibler divergence of choice prediction. Completely recovery of the ideal profit yields 0% error while zero profit yields 100% error.
CHAPTER 5. CONSIDERATION AND DESIGN INFORMED BY STATED PREFERENCE

5.1 Introduction

In the past decade, preferential choice models have played an important role in product design by informing designers of how consumer respond to product features [Wassenaar and Chen 2003, Michalek et al. 2004, Shiau et al. 2009b, MacDonald et al. 2010, Hoyle et al. 2010, Morrow and Skerlos 2011]. Widely applied traditional choice models with additive utilities [Ben-Akiva and Lerman 1985a, Train 2009] have viewed the product evaluation as a compensatory process, meaning that consumers allow attractive features to compensate for undesired features. Derived from Payne’s analysis work on the information search in task complexity [Payne 1976], marketers have been refreshing choice modeling with the concept of "consideration", [Roberts and Lattin 1997], which describes a non-compensatory screening process in which consumers quickly eliminate a large number of products before careful trade-offs. It remains open questions whether using compensatory choice models versus non-compensatory consideration models will lead to different design decisions, particularly: 1) Will consideration models suggest different product line designs than traditional choice models? 2) Will consideration models and choice models lead to different strategic values, such as profitability? 3) Can the predictive power of the models indicate their strategic values? These questions appeal to product designers because the product development concerns not only how a model predicts consumer choices of the existing products, but also how a model directs the feature innovation of new products. For example, suppose a firm observed that a household with children excluded any two-seaters when purchasing a primary vehicle. A compensatory choice model may be able to capture this observation by using specific part-worths on the body type attribute such that the existing two-seaters have significantly low utilities. However, when it comes to designing a new product,
the compensatory structure may lead the designers to believe that increasing the utilities of attributes such as fuel economy to a higher new level can compensate for the body type. In contrast, if a consideration model explicitly identifies body type as a non-compensatory feature, it will lead the firm to develop body types with sufficient passenger capacity. Furthermore, consideration models hold a different perspective of profit predictions, because consideration models expect the market share of a product only from consumers who consider the product.

Motivated by the intuitions above, this chapter aims at comparing the performances of compensatory choice models and non-compensatory consideration models in the design perspective. The comparison requires a framework to handle three issues. First, the capabilities of the models need to be tested in an environment where the performance benchmark can be found. A synthetic data experiment is proposed to compare the performances of the models in scenarios where the synthetic consumers use complex screening rules. Second, the comparison needs to accounts for the fact that using different models often implies different data collecting and model estimating processes. Thus, our experiment tailors different surveys according to different assumptions behind choice models and consideration models. Third, the comparison requires applying the models in a design process and accessing the strategic value of the design outcomes. Therefore, our experiment simulates the design optimization process after model estimations and quantitatively compares design outcomes in terms of feature diversity and profitability.

This chapter proceeds as follows: Section 5.2 reviews the studies related to the comparison between consideration models and choice models in the past, as well as the consideration modeling methods in marketing research. Section 5.3 describes our synthetic simulation framework in detail, including the design of experiment, the synthetic data generation process, the model estimation methods, the product design problem, and the metrics to evaluate the model performances. Section 6.3 presents and discusses the results of the simulation. Section 6.4 highlights the managerial implications. Section 6.5 concludes.
5.2 Background and Literature Review

5.2.1 Consideration models versus choice models

Growing evidence has revealed that consumers often use frugal and simple screening rules to eliminate products before carefully making trade-off decisions due to factors such as evaluation complexity [Payne 1976], information cost [Bröder 2000] and time pressure [Rieskamp and Hoffrage 2008]. Previous marketing literature referred this screening process as "consideration" [Roberts and Lattin 1997]. Marketing research in the past has modeled a variety of consideration screening strategies:

Aspirational: Simon [1956; 1972] first proposed the theory of bounded rationality, in which decision makers will stop searching for options once the benefit they obtain from an option exceeds some "aspirational limit". Gilbride and Allenby [2004] implemented this idea into consideration modeling in the form that consumers will consider a product if the product’s utility exceeds some aspirational criteria.

Conjunctive: a consumer will consider a product only if all its screened attributes are acceptable. For example, "I will consider a vehicle if it is hybrid AND it is Toyota”.

Disjunctive: a consumer will consider a product as long as one of its screened attributes is acceptable. For example, "I will consider a vehicle if it is hybrid AND it is Toyota”.

Subset conjunctive: a consumer will consider a product if a certain number of its screened attributes are acceptable. For example, a consumer screens on brand, price and powertrain, and suppose a considered product must have at least two acceptable features, then a subset conjunctive rule may state "I will consider a vehicle if it is a Toyota hybrid OR if it is a hybrid with price under 25,000 OR if it is a Toyota under 25,000”.

Disjunction of conjunctions: a consumer will consider a product if it satisfies at least one sets of conjunctive rules. For example, if a consumer screens by brand, price and powertrain, a disjunction of conjunctions rule can be "I will consider a vehicle if it is a Toyota OR if it is a hybrid under 25,000”.

It should be clarified that choice models are not necessarily labeled as "compensatory”, since choice decisions can also include non-compensatory processes, such as "elimination-by-aspects” [Tversky 1972b]. When referring choice models, this chapter particularly emphasizes
the model structure that does not include non-compensatory screenings. Marketers have compared the non-compensatory consideration models and compensatory choice models in terms of their capabilities of predicting choices in empirical data. Table 1 provides a list of research practices where consideration models improved the predictive power in a variety of product categories. On the other hand, some investigations revealed that compensatory choice models could approximate non-compensatory considerations. For example, the nested logit model can approximate the consideration sets structure when applied to choice-based data [Swait 2001a]; the aspect-coded linear weighted additive model with a specific sequence of part-worths can recover conjunctive rules [Martignon and Hoffrage 2002]. However, the capability of choice models to mimic considerations has limitations. Specifically, Andrews and Manrai [1998] showed in a synthetic data experiment that the capability of multinomial logit models to accommodate considerations depend on conditions such as the degree of heterogeneity of the considerations and the sizes of consideration sets. Further experiments revealed that underspecifying considerations had higher impact to predictive errors than underspecifying compensatory heterogeneity [Abramson et al. 2000, Andrews et al. 2008].

The literatures of comparing consideration models and choice models are relatively sparse in the domain of engineering design. Existing applications included the usage of conjunctive and disjunctive models in a product selection system [Besharati et al. 2006b], and the prediction of consideration sets via network analysis [Wang and Chen 2015]. Yet, these studies did not focus on comparing the design outcomes. Our previous synthetic market data experiment suggested that consideration models had the advantage in design profitability compared to logit, mixed logit, and nested logit models [Long and Morrow 2015]. Shin and Ferguson performed a choice survey based synthetic experiment where the latent class logit model and hierarchical Bayes mixed logit model yielded similar optimal designs to those by using a conjunctive model Shi [2015]. However, neither of these two studies took into account how the data collections would differ according to the model assumptions, given that the consideration models and choice models in these two studies were estimated from the same discrete choice data. Before introducing our simulation framework that tailors separate surveys for consideration models and choice models, we review methods to model considerations in the following.
5.2.2 Consideration modeling methods

A classic stream of consideration modeling methods relied on choice panel data. These consideration models probabilistically captured the existence of a consideration set and the memberships of the consideration set. This formulation presented choice probabilities conditionally on the consideration set probabilities [Manrai and Andrews 1998]. Yet, the computation burden of this method grows exponentially as the number of the product alternatives grows. The parameterization of the attribute acceptance distribution relaxed this problem by reducing the estimation dimensions [Siddarth et al. 1995, Chiang et al. 1999, Jedidi and Kohli 2005].

Recent advanced methods not only inherited the treatment of parameterization, but also inferred screening rules more efficiently with consideration data. These advanced methods often corresponded to their own specific models, for example, greedoid algorithm for lexicographic models [Kohli and Jedidi 2007, Yee et al. 2007], Bayes theorem based adaptive learning for conjunctive screening models [Dzyabura and Hauser 2011], and support vector machine learning for aspirational screening models [Huang and Luo 2015]. The following section presents how the Bayes adaptive learning and the support vector machine learning are applied in our simulation framework.

5.3 Synthetic Experiment

Figure 5.1 shows the flowchart of our proposed synthetic experiment. The simulation started with generating a synthetic population with pre-specified behaviors that includes both consideration stage and choice stage decision parameters. The simulation queried the synthetic respondents in two independent sets of survey questions: adaptive consideration surveys, and discrete choice surveys. The consideration surveys infer individual specific consideration rules by adaptively combine questions selection and responses collection in an iterative process. The discrete choice surveys estimate choice models via pre-designed question sets. The output consideration models and choice models then informed the product line optimization separately. Eventually, the optimal portfolios were compared on their design strategies and profitability. The remaining of this section further explains the framework components in detail.
5.3.1 Pre-specified behaviors

The simulation describes a product profile with the attributes and levels given in Table 5.1. There are 8 attributes (including vehicle make) with a total of 53 attribute-levels. Throughout this chapter, the profile vector $\mathbf{x}$ is denoted as a binary string of length 53 with 1 representing that the profile has the feature of the corresponding level and 0 otherwise.

The experiment configured two scenarios: In scenario I, respondents used aspirational screening; in scenario II, respondents used subset-conjunctive screening. When being asked the "consider/reject" questions, the respondents answered according to their individual specific screening rules. When being asked the choice questions, the respondents first performed
the screening then made a choice according to their individual specific compensatory choice parameters. Throughout this chapter, "true behavior" is used to denote the pre-specified mechanisms that generate the responses in the simulation environment, which are described as follows.

Table 5.1 Attributes and levels for the new vehicle case study, following Urban et al. [2010] and Dzyabura and Hauser [2011]

<table>
<thead>
<tr>
<th>Attributes</th>
<th># of Levels</th>
<th>Level Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Body Style</td>
<td>9</td>
<td>Sports Car, Hatchback, Compact Sedan, Standard Sedan, Crossover, Small SUV, Full-size SUV, Pickup Truck, Minivan</td>
</tr>
<tr>
<td>Make</td>
<td>21</td>
<td>BMW, Buick, Cadillac, Chevrolet, Chrysler, Dodge, Ford, GMC, Honda, Hyundai, Jeep, Kia, Lexus, Lincoln, Mazda, Nissan, Pontiac, Saturn, Subaru, Toyota, VW</td>
</tr>
<tr>
<td>Price</td>
<td>7</td>
<td>$12K, $17K, $22K, $27K, $32K, $37K, $45K</td>
</tr>
<tr>
<td>Cylinders</td>
<td>3</td>
<td>4, 6, 8</td>
</tr>
<tr>
<td>Powertrain</td>
<td>2</td>
<td>Hybrid, Gasoline</td>
</tr>
<tr>
<td>MPG</td>
<td>5</td>
<td>15, 20, 25, 30, 35</td>
</tr>
<tr>
<td>Quality Rating</td>
<td>3</td>
<td>3, 4, 5</td>
</tr>
<tr>
<td>Crash Rating</td>
<td>3</td>
<td>3, 4, 5</td>
</tr>
</tbody>
</table>

**Scenario I: Subset-conjunctive Screening**

Individual i’s consideration set $C_i(X)$ is formulated as:

$$C_i(X) = \{ j : \delta_i^T x_j \geq N \}, N = 1, \cdots, A$$ (5.1)

The binary vector $\delta_i$ is an individual specific screening rule. An element of $\delta_i$ is one, if and only if the corresponding attribute-level of the element is acceptable. $A$ is the total number of attributes. Thus $K = A$ corresponds to the conjunctive model. $K = 1$ yields a disjunctive screening rule. Any K between these two values represents a subset-conjunctive rule. The subset-conjunctive is sampled such that the profiles considered by the respondents follow a log-normal distribution with mean value of 0.1. The sampling process is as follows: all possible rules (i.e. $\delta$ and $K$ combinations in Eqn.6.1) were enumerated to compute the corresponding proportion of profile considered. The rules were categorized into 100 bins (i.e. $0 - 0.01, 0.01 - 0.02, \cdots, 0.99 - 1$) according to the proportion of profile considered given by the rules. One of
Table 5.2  Part-worths structure of the pre-specified compensatory stage MNL model parameters for the synthetic population.

<table>
<thead>
<tr>
<th>Attributes</th>
<th>Part-worths Structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Body Style</td>
<td>Uniform random draw from [-1, 1]</td>
</tr>
<tr>
<td>Make</td>
<td>Uniform random draw from [-1, 1]</td>
</tr>
<tr>
<td>Price</td>
<td>Uniform random draw from [-1, 1], then order as $\beta_{812K} &gt; \beta_{817K} &gt; \beta_{822K} &gt; \beta_{827K} &gt; \beta_{832K} &gt; \beta_{837K} &gt; \beta_{845K}$</td>
</tr>
<tr>
<td>Cylinders</td>
<td>Uniform random draw from [-1, 1], with 0.5 chance $\beta_{4cyl} = \beta_{6cyl} = \beta_{8cyl}$ (indifference) and 0.5 chance $\beta_{4cyl} &lt; \beta_{6cyl} &lt; \beta_{8cyl}$ (prefer larger engine sizes)</td>
</tr>
<tr>
<td>Powertrain</td>
<td>Uniform random draw from [-1, 1], with 0.25 chance $\beta_{gas} &gt; \beta_{hybrid}$ (prefer gasoline) and 0.5 chance $\beta_{gas} = \beta_{hybrid}$ (indifference) and 0.25 chance $\beta_{gas} &lt; \beta_{hybrid}$ (prefer hybrid)</td>
</tr>
<tr>
<td>MPG</td>
<td>Uniform random draw from [-1, 1], then order as $\beta_{15mpg} &lt; \beta_{20mpg} &lt; \beta_{25mpg} &lt; \beta_{30mpg} &lt; \beta_{35mpg}$</td>
</tr>
<tr>
<td>Quality Rating</td>
<td>Uniform random draw from [-1, 1], then order as $\beta_{3star} &lt; \beta_{4star} &lt; \beta_{5star}$</td>
</tr>
<tr>
<td>Crash Rating</td>
<td>Uniform random draw from [-1, 1], then order as $\beta_{3star} &lt; \beta_{4star} &lt; \beta_{5star}$</td>
</tr>
</tbody>
</table>

the rules from the bin that $c_i$ fell in was randomly selected for individual $i$.

**Scenario II: Aspirational screening**

An individual $i$ will consider product $j$ if it achieves a utility value above some "aspirational" limit. The following formula defines individual $i$’s consideration set $C_i(X)$:

$$C_i(X) = \{j : v_i^T x_j \geq \gamma_i\}$$

(5.2)

where $v$ is individual specific part-worths and $\gamma$ is the aspirational limit. To enable the proportion of profiles considered by the individuals to follow flexible distributions, the simulation sampled the part-worths and aspirational limit using a scheme inspired by Jedidi and Kohli [1996]. For each individual, we first sampled the proportion of profiles considered $c \in [0, 1]$ from its distribution $f(c)$, and assigned the screening rule that led to such proportion. The partworths $\beta_i$ were randomly drawn from uniform distribution between $[-1, 1]$. The utilities of all possible profiles were then computed and sorted. Given the sampled proportion of profiles considered $c_i$, the same proportion of profiles that have the highest utilities were identified as
"considered". An aspiration limit $\gamma_i$ was assigned to this individual such that $\gamma_i$ was lower or equal to any utility of a considered profile but higher than any utility of a rejected profile. In both screening behaviors, the proportion of considered profiles followed a log normal distribution with mean of 0.1.

**Compensatory choice**

After screening, a respondent chose an option with maximum random utility within the consideration set, i.e. chose option $j$ such that $j = \arg \max_{k \in C_i} \beta_i^T x + \epsilon_{i,k}$ where $\beta_i$ is individual specific part-worths and $\epsilon_{i,k}$ is the random disturbance of the utility following extreme value distribution across the respondents and options. The sampling of individual specific parameter $\beta_i$ followed a preference structure shown in Table 5.2. The main rationale of this structure is that, during the compensatory comparison, the respondents prefer lower price than higher price, prefer higher MPG than lower MPG, prefer higher quality than lower quality and prefer higher crash safety rating than lower rating.

### 5.3.2 Consideration Models and Adaptive Surveys

The simulation investigated two consideration models - the conjunctive model estimated with Bayesian adaptive questions and the aspirational model estimated with support vector machine. Both models ignore choice stage part-worths estimation (see Table 5.3 for a summary of the mis-specifications, estimation methods, and survey properties of four estimated models).

**Conjunctive model**

Consideration criteria $\delta_i^T x_j \leq K$ in Eqn.(6.1) collapses to a pure conjunctive model when $K$ equals to the number of screened attributes. Dzyabura and Hauser [2011] proposed an adaptive machine learning algorithm to estimated the individual specific screening rule $\delta_i$ by parameterizing the screening rules as the acceptance probabilities of the attribute levels, and assuming the acceptance probabilities to be independent across all attribute levels. Based on the history of profile queries and responses, the algorithm updated the posterior acceptance probability with Bayes theorem. After each update, the next question was adaptively selected
to minimize the expected information entropy. The update and selection process iterated until the maximum number of questions were asked. The estimated posteriors were converted to the binary screening rules such that posteriors $< 0.5$ was interpreted as "unacceptable" on the corresponding attribute level, and "acceptable" otherwise.

**Aspirational model**

Given the aspirational screening structure $\mathbf{v}_i^T \mathbf{x}_j \leq \gamma_i$ in Eqn. (6.2), the training of screening rule $\mathbf{v}_i$ of individual $i$ requires solving a support vector machine problem [Bishop 2006]:

$$\text{minimize} \quad \frac{1}{2} \| \mathbf{w} \|^2$$

with respect to $\mathbf{w}, b$

subject to $y_n \cdot (\mathbf{w}^T \mathbf{x} + b) \geq 1$ for all $n = 1, \cdots, N$

where $N$ is the number of questions asked and $y_j$ is the response to profile $\mathbf{x}_j$ (value as 1 if "consider", -1 if "reject"). Huang and Luo [2015] combined this learning algorithm with the adaptive question strategy in which the profile with the smallest distance to the decision hyperplane $h(\mathbf{x}) = \mathbf{v}_i^T \mathbf{x} - \gamma_i$ was selected.

<table>
<thead>
<tr>
<th>Model</th>
<th>Miss-specifications</th>
<th>Estimation Method</th>
<th>Survey Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conjunctive screening</td>
<td>Ignores choice stage, simples screening</td>
<td>Bayesian adaptive questions</td>
<td>Consider (one profile), or not, adaptive questions</td>
</tr>
<tr>
<td>Aspirational screening</td>
<td>Ignores choice stage (only)</td>
<td>Adaptive soft-margin support vector machine</td>
<td>Consider (one profile), or not, adaptive questions</td>
</tr>
<tr>
<td>Multinomial logit</td>
<td>Ignore consideration and heterogeneity</td>
<td>Maximum likelihood</td>
<td>Choose among 3 or 4 alternatives, or none; fixed questions</td>
</tr>
<tr>
<td>Latent-class logit</td>
<td>Ignore consideration and simplifies heterogeneity</td>
<td>Maximum likelihood</td>
<td>Choose among 3 or 4 alternatives, or none; fixed questions</td>
</tr>
</tbody>
</table>
5.3.3 Choice Models and Discrete Choice Surveys

Multinomial logit (MNL) and latent-class logit (LCL) serve as two representatives of the compensatory choice models. Both models ignore consideration stage and can be estimated from traditional discrete choice survey with Maximum likelihood method (see Table 5.3 for a summary of model and survey properties). In the choice surveys, respondents were asked to choose one profile, or none, from a set of alternatives. 25 sets of surveys used in this experiment varied in the number of questions (from 10 to 35), the number of alternatives (3 and 4), and the generation methods (SAS macros and Sawtooth software schemes). SAS macros generated "D-efficient" surveys [Kuhfeld 2010]. A "D-efficient" survey consisted of multiple sub-surveys, and each respondent was randomly assigned with one of these sub-surveys. Two types of Sawtooth's discrete choice survey are generated: a "shortcut" survey and a "random" survey [Saw 2013]. In the "shortcut" scheme, each alternative profile is built by choosing attribute levels used least frequently in previous profiles. If there are more than one least frequently used levels, the one that has been used the least in the same question will be selected. In the "random" scheme, every level of the same attribute has equal chance to be assigned to the profile. Such structured randomness in these two schemes enhance the level balance and level minimal overlap in the questions [Huber and Zwerina 1996, Johnson et al. 2013]

Multinomial model

Multinomial logit model was one of the most established models in the maximum random utility theoretic framework [Ben-Akiva and Lerman 1985a, McFadden 1974], with the choice probability derived as:

\[
P_{j}^{MNL} = \frac{\exp \beta^T x_j}{\sum_{k=0}^{J} \exp \beta^T x_k}
\]

(5.4)

In the weighted additive utility \( \beta^T x_j \), part-worths vector \( \beta \) is homogeneous across the population and \( x_j \) is an attribute-level coded (or called aspect coding) feature vector of option \( j \). To be consistent with survey setting, in options 0, ⋅⋅⋅, \( J \), with "none" option is labeled as 0.
Latent class logit model

Latent class logit model [Swait 1994, Greene 2001, Greene and Hensher 2003] captures preference heterogeneity by sorting individuals into one of $Q$ classes with probability $\alpha_q$. The conditional choice probability of choosing option $j$ given the sorted class $q$ takes the logit formula:

$$P_{LCL}^{j|q} = \frac{\exp \beta_q^T x_j}{\sum_{k=0}^{J} \exp \beta_q^T x_k} \tag{5.5}$$

The likelihood for option $j$ to be chosen is the sum over the product of latent class probability and the conditional choice probability across all classes:

$$P_{LCL}^j = \sum_{q=1}^{Q} \alpha_q \cdot P_{LCL}^{j|q} \tag{5.6}$$

Both multinomial model and latent class model are fitted via maximum likelihood method by solving:

$$\maximize \sum_{m=1}^{M} \sum_{j=0}^{J} N_{j,m} \log P_{j,m}^{Model}(\theta)$$

with respect to $\theta$ subject to possible constraints \(\tag{5.7}\)

For the multinomial model related to Eqn. (5.4), the estimated coefficients are $\theta = \beta$, for latent class model related to Eqn. (5.5) and (5.6), $\theta = (\alpha, \beta_q)$ for $q = 1, \ldots, Q$. For both models, the part-worths of the same attribute are constrained to have a zero sum. The latent class model estimation, in addition, is also constrained to satisfy $\sum_{q=1}^{Q} \alpha_q = 1$.

5.3.4 Product Line Design

Suppose an automaker aims to design $J$ distinct products (products with non-identical engineering features) to maximize profits across all vehicles. With the brand fixed, a firm
solves the following optimization problem:

\[
\text{maximize } \pi_f = \sum_{j=1}^{J} P_{j}^{Model}(p, Y|f) \cdot (\hat{p}_j - c(y_j))
\]

with respect to \( p = (p_1, \cdots, p_J), Y = (y_1; \cdots; y_{Jf}) \)

subject to \( (p_j, y_j) \cdots (p_k, y_j^T)^T \leq \bar{A} - 1 \)

for any \( (j, k) \) such that \( (j, k) \in \{1, \cdots, J_f\} \times \{1, \cdots, J_f\} \) AND \( j \neq k \)

(5.8)

Here, \( p_j \) is the binary level coded vector of price attribute and \( y_j \) is the non-price attributes’ binary vector consistent with the levels in the survey experiment. \( \hat{p}_j \) represents the corresponding numeric value mapped from \( p_j \). The profit is formulated as the product of the predicted choice probability and markup. This study simplified the cost function to be constant. Suppose, excluding the make feature, the number of attributes is \( A \), the constraint in Eqn.(5.8) serves to ensure the distinction between two products in a product line. The choice probability \( P_{j}^{Model} \) applied the estimation result of models described in Section 5.3.2 and 5.3.3. Our multinomial model and latent class model directly provided choice probability formula. Lacking of choice stage information, our subset-conjunctive and aspirational models assigned equal choice probability to the products within the same consideration set. For a baseline comparison, the case study also included a null model (a zero information logit model), which gives all the alternatives equal choice probabilities regardless what features are in the designs. As the null model merely generates random combinations of vehicle features. Its performance was based on 100 design trials.

In this problem, the number of distinct product portfolios grows rapidly as the number of products increases. A genetic algorithm (GA) was used to search for optimal attribute level combinations. The genetic algorithm scheme is described as follows:

1. **Generate members:** each member \( n \) is randomly assigned a binary coded genome \( \bar{X}_n = (\bar{x}_1, \cdots, \bar{x}_{J_f}) \) where \( \bar{x}_j \) includes the binary coded price and other brand-excluded attributes.

2. **Compute fitness:** for each member, compute the fitness using the objective formula in Eqn. 5.8.
(3) Reproduce new generation: the probability of member \( n \) to be selected as a parent is
\[
\pi_n / \sum_{k \in \text{all members}} \pi_k
\]
When crossover takes place between a pair of parents, the parents will switch at least one successively connected attributes. Mutation happens with probability \( q_{m,ut} \), mutation site is randomly selected among any attribute of any product. For the selected attribute, the current level will be replace by a different level.

(4) Check uniqueness: for each member, the product profiles in the product line are scanned and any two members’ genomes are compared. If two profiles are found to have identical non-price attributes, repairing will be performed by randomly mutate one attribute in one of the profiles. After all possible pairs of profiles are checked, the scanning process is started over again to make sure that the repaired profiles in the last check do not conflict the uniqueness of the previously scanned profiles. This "scan and repair" process will be repeated until all the profiles in the product line are unique in their non-price attributes.

(5) Check convergence: convergence is determined by the improvement of the best fitness as well as the average fitness of population. If both the improvement of best fitness and average fitness of the population are sufficiently small to fall within a tolerance of 0.0001, the iteration will terminated, otherwise the iteration will be back to step (2).

50 trials have been run with different initial population of size \( N = 50 \). Among these trials, the solution with the highest objective value is reported.

5.3.5 Performance Measures

Predictive power

Relative likelihood (RL) quantifies the predictive power by measuring the best possible likelihood of a model relative to the likelihood of the true distribution:

\[
RL = \left( \prod_{m=1}^{M} \left( \prod_{j=0}^{J_m} \left( \frac{P^{Model}_{j,m}}{P^{True}_{j,m}} \right)^{P^{True}_{j,m}} \right) \right)^{1/M}
\]

In this formula, \( P^{Model}_{j,m} \) represents the predicted choice probability of alternative \( j \) (with \( j = 0 \) labeling the none option) in question \( m \), and \( P^{True}_{j,m} \) represents the choice probability computed
with the true behavior parameters. The higher value of relative likelihood indicates higher predictive power. The simulation generated 25 sets of surveys to calibrate the multinomial and latent class logit models coefficients. 25 new validation sets validated the predictive power of each set of the estimated coefficients. The validation sets shared the same variety as the calibration sets in the number of alternatives per question, the number of questions per survey, and the survey generation methods. A zero information logit model (null model) also served as a baseline comparison.

**Feature commonality**

One of the widely used metric to measure commonality of the product line is degree of commonality index (DCI) [Collier 1981], which is implemented in our case as:

\[
DCI = \frac{\sum_{k=J+1}^{J+d} \phi_k}{d}
\]  

(5.10)

In the computation, each product is labeled as 1, \cdots, J and distinct features are labeled as \(J + 1, \cdots, J + d\), with \(J\) as the number of products in the product line (corresponding to the end items in the original definition), and \(d\) is the number of distinct feature levels (referred as component items in the original definition). \(\phi_k\) is the number of immediate parents that feature level \(k\) has. In our case, this metric reflects the average number of common products that share a feature. For example, in a simple two products case in our context, see Figure 5.2, both product 1 and 2 are standard sedans, one with 30 MPG and one with 35 MPG. Item 3 and 5 each has one immediate parent item, respectively end products labeled as item 1 and item 2. For item 4, both item 1 and item 2 are its immediate parents. Thus, we have \(\phi_3 = 1, \phi_4 = 2, \phi_5 = 1\) with distinct feature number \(d = 3\). The DCI in this example is \((1+2+1)/3 = 4/3\). With this metric, a product line with totally identical features yields a DCI value of \(J\), and the lowest possible value is 1, when all the products have no feature in common.

**Profitability**

The evaluation distinguishes two types of profit - the profit predicted by the estimated models (denoted as predicted profit), and the profit validated in the synthetic population.
(denoted as actual profit). Given an optimal design solution, the predicted profit is computed as the objective function value in Eqn.\((5.8)\), while the actual profit is computed using the true behavior parameters. To detect whether a firm will actually achieve what it expects, we quantify the fraction of actual profit that exceeds the predicted profit as:

\[
\%\text{profit exceeded} = 100 \cdot \left(\frac{\text{profit}_{\text{actual}} - \text{profit}_{\text{predicted}}}{\text{profit}_{\text{predicted}}}\right) \quad (5.11)
\]

5.4 Results

This section compares the predictive power, design portfolios, and profitability of two consideration models (conjunctive model and aspirational model) and two choice models ( multinomial logit model and latent class logit model). Each model is examined in two scenarios. In Scenario I, the synthetic population uses subset-conjunctive screening, while in Scenario II, the synthetic population uses aspirational screening.

5.4.1 Predictive Power

Figure 5.3 shows the relative likelihood in four estimated models across 25 validation sets. The aspirational model has the highest predictive power and the smallest variations over the validation sets even when the respondents do not use aspirational screening. This result verifies the mathematical generality of the aspirational model. That is, the aspirational formula governs the subset-conjunctive screening rules when the parameters take specific integers.
The conjunctive model predicts better in Scenario I than in Scenario II. The explanation lies within its over-stringent screening structure. The conjunctive model tolerates no unacceptable attributes, while the synthetic respondents allow some unacceptable attributes. Thus, the conjunctive model predicts the product considerations more conservatively. In scenario II, where the respondents are more likely to allow unacceptable attributes, the conservative prediction diverges even further from the observations.

Both the multinomial logit model and the latent class logit model approximate the considerations better when the consumers use aspirational screening, which is observed in their higher predictive power in Scenario II. This is reasonable because the respondents in Scenario II evaluate the overall utility during screening, still allowing compensatory trade-offs among the attributes.

Figure 5.3 Predictive power measured by relative likelihood in 25 validation sets. The results are grouped by the number of alternatives per question in the validation sets. The solid center bars mark the mean over all estimates validated in all validation sets. The shaded boxes indicate the maximum and minimum values.

5.4.2 Optimal Portfolios

Using different models leads to different optimal portfolios. The optimal portfolio of Ford is reported as a representative case. Figure 5.4 and 5.5 depict the body styles, prices, powertrain, number of cylinders, and fuel economy of the optimal portfolios.
Design strategies

In scenario I (Figure 5.4), the optimal portfolio of the true behavior model covers all body styles except hatchback and pickup truck. The product line also offers various price levels, mpg levels and powertrains among the SUVs. In the same scenario, the portfolio of the conjunctive model focuses sports cars and hatchbacks by offering multiple price levels. The aspirational model has the widest coverage of body styles. Two choice models suggest similar body style coverage but different focuses. Half of the products recommended by latent class logit model are small hybrid SUVs. The multinomial logit model mainly focuses on standard sedans and small SUVs, with all vehicles having the gasoline powertrain.

In scenario II (Figure 5.5), the true behavior model suggests all body styles except standard sedan. Two compact sedans at two price levels are offered, and two crossovers with different powertrains are offered. The conjunctive model and aspirational model have higher diversity of body styles and mpg levels. In contrast, multinomial logit model suggests four hybrid full-size SUVs, three of which share the same MPG, and latent class logit model recommends exclusively SUVs.

In both scenarios, the product lines of consideration models have lower degree of commonality index than the choice models on average. This quantification result is consistent with the representative optimal portfolios described above. That is, the portfolios of the consideration models diverse in attribute combinations, while the portfolios of choice models tend to share certain attributes, such as the gasoline powertrain of the multinomial logit model in Scenario I, and the "8 cyl engine & 35mpg" combinations of the latent class logit model in Scenario II.

Pricing strategies

The highest price level ($45K) dominants the optimal portfolios, due to the fact that as price increases, the profit margin in Eqn.(5.8) increases faster than the decrease of the choice probability. Two influential factors may drive the optimization process towards the highest pricing level. First, if a simulated consumer does not strictly screen out the product at the highest price level, then the optimal portfolios can still attract the consumer to consider the pricy products by introducing other acceptable feature aspects. For example, when the sub-set
conjunctive screening is used, even if a consumer may label $45K as "unacceptable", he/she may still consider a product at $45K. The tolerance of high price is even more significant when the consumer uses aspirational screening.

Second, if the fraction of consumers who strictly reject the highest price level is trivial in the whole simulated population, then the optimal portfolios may prioritize the major fraction that tolerates the highest price. Moreover, this experiment only involves ten products, which is a relatively small number to cover a wide range of pricing levels. In this case, spreading the product line on attributes such as body styles may benefit the profit more than varying price levels, because the product line can satisfy more people who are compromised to pay more.

5.4.3 Profitability

Next, the actual profits of the designs produced by the estimated models are compared to the actual profit of the true behavior model (denoted as ideal profit). Both consideration models and choice models show robustness in achieving at least 80% of the ideal profit. In Scenario I, both consideration models outperform the multinomial logit model. In Scenario II, where the respondents use aspirational screening, the choice models have higher capability of achieving profits.

The predictive power does not necessarily indicate profitability. However, the higher predictive power benefits the accuracy of profit predictions, since the error in profit predictions is highly correlated to the error in market share predictions (see Figure 5.7). The aspirational model has the most accurate prediction of profits and shares, which is consistent to its predictive power presented in relative likelihood (shown in Figure 5.6).

Using the conjunctive model prevents over-predicting profit. The actual profit of the conjunctive model designs achieves at least 30% higher than the prediction in Scenario II, while all other models achieve at least 40% lower profit than their prediction in Scenario I, and 20% lower profit in Scenario II. These observations echo the conservative prediction of the conjunctive model.
5.5 Discussions

This study investigated the performance of consideration models and choice models in product design. Our synthetic data simulation estimated consideration models and choice models respectively from adaptive consideration surveys and discrete choice surveys. The experiment further applied the estimated models to a single firm product line optimization problem and compared the design outcomes.

The use of consideration models changed design decisions, reflecting on the attribute combinations and the product line diversity. On the philosophy of selecting what consumer models to use, product decision makers need to account for that the assumptions behind the models will drive the data collecting methods towards different manners and trigger the consumer to give different aspects of responses. Thus, different models capture and deliver different information for design optimizations, and eventually lead to different design strategies.

On comparing profitability, our simulation identified two important aspects of evaluation for product decision makers - the actual profit, and the discrepancy between the actual and predicted profits. Despite the robustness in achieving actual profits, the choice models exposed their weakness in profit prediction, i.e. expecting significantly higher profit than they actually obtained. This weakness stems from their model assumption of expecting market shares from the universal choice set rather than merely a subset of products. In contrast, the conjunctive model may miss some highly profitable attribute combinations due to its stringent screening rule structure. However, the conservative prediction of the conjunctive model prevents over-expecting profit. Product decision makers should balance the strength and weakness of these models, for example, valuing not only the compensatory trade-offs information during profit optimizations, but also the non-compensatory screening information in avoiding over-ambitious planning.

The simulation identified the positive correlation relationship between the predictive power in the validation surveys and their profitability on average. However, the simulation signaled the product decision makers to consider other evaluation perspectives when selecting what models to use. Because the predictive power in the validation surveys is unable to indicate the design strategies suggested by the models and the impacts of these strategies to a firm’s further
practices. This limitation arouses the challenge of finding metrics to signal and quantify these strategic impacts outside of a synthetic simulation framework.

The implications of this study should account for the simplistic engineering models and the randomly sampled synthetic consideration rules. More complex engineering constraints and cost functions may reshape the optimal portfolio, as the engineering constraints may reduce the feasible design space and the cost functions may counteract the choice probability in some regions of the design space. Despite this limitation, this study provides a comparison reference for future research to distinguish the impact of complex engineering models.

5.6 Conclusions

This study investigated the performance of consideration models and choice models on product line design strategies. The simulation tailored survey instruments for particular models, allowing us to comment on how the model assumptions sequentially resulted in differences in survey designs, response collections, and product designs. The findings suggest that product decision makers question the assumptions behind the consumer models deployed during the product development and expand the performance evaluation from the traditional predictive power to the potential strategic impacts.
Figure 5.4 The degree of commonality indices (DCI) of the optimal portfolios designed using different models in Scenario I and the corresponding representative optimal portfolios of Ford. The central bar represents the mean value of DCI across 21 brands. The shaded box represents the minimum and maximum values.
Figure 5.5  The degree of commonality indices (DCI) of the optimal portfolios designed using different models in Scenario II and the corresponding representative optimal portfolios of Ford. The central bar represents the mean value of DCI across 21 brands. The shaded box represents the minimum and maximum values.
Figure 5.6 Percentage of ideal profit versus relative likelihood compared across four estimated models, pre-specified "true" behavior model and null model. The width of the shaded box indicates the range between the minimal and maximal relative likelihood. The height of the shaded box indicates the range between the minimal and maximal percent of ideal profit.

Figure 5.7 Percentage of model predicted profit exceeded by actual profit (y-axis) correlates to the model predicted market share exceeded by actual market share (x-axis). Each scattered point represents the product line of one brand, 21 brands in total. Negative values indicate over-estimated profits or market shares.
CHAPTER 6. ENHANCING HYBRID VEHICLE ADOPTIONS WITH CONSIDERATION MODELS

6.1 Introduction

Hybrid vehicles increasingly capture attentions in vehicle markets due to their fuel efficiency and advantages in emission reduction. Since the first surge of sales in 2005 when governmental incentives such as tax credits, sales tax waiver and access to high-occupancy vehicle lane applied, the hybrid vehicles still experience slow growth in the light-duty vehicle market. Making design decisions of hybrid vehicle involves understanding what factors motivate consumers to consider a hybrid vehicle. Consumer surveys among hybrid users [Ozaki and Sevastyanova 2011] identified several important factors that influenced the consumers: the performance related to driving experience, the fashion appearance or space utility, the cost benefits from fuel saving or government incentives, the brand/manufacturer’s reliability and trust, the awareness of environment impact, the interest of new technology, and the social orientation towards compliance with the value of a community.

These factors have different implementations in hybrid design strategies. For consumers motivated by factors such as the awareness of environment impact, the interest of new technology, and social orientation, the feature “hybrid powertrain” directly drives the acceptance of the hybrid products. For those who concern more on performance or financial benefits, enhancing hybrid vehicle adoptions requires accounting for whether the hybrid powertrain conflicts other “must-have” features. For example, if a group of households specifically look for family traveling vehicles, then placing a hybrid powertrain in a two-seat car will not win the purchases of this group, even if fuel economy may also be of their interest. Or, if a household plans to buy a hybrid vehicle, merely aiming at financial savings from lower fuel cost and tax returns, but it turns out that the pricing of a hybrid counteract the financial benefit they perceived for
the future, then the hybrid product may also encounter failure in this group of households.

Consideration models have advantages in making analysis in this context. Explicitly modeling consideration sets allows identifying the acceptance of the attributes and thus placing hybrid vehicle design with optimal feature combination to target specific population groups. Traditional compensatory models may reflect the acceptance of the hybrid powertrain feature by modeling the tradeoffs between hybrid powertrain and other features. However, using consideration models has benefit of analyzing the impact of a newly introduced hybrid product to particular groups of consumers. Also, consideration models specifically identify the competitors rather than simplifying the substitution pattern by assuming that a product universally competing with all available products.

This chapter demonstrates the use of consideration models in a hybrid vehicle design problem with an simulation example. The example shows how a firm’s design maximizes the consideration of the hybrid vehicles of its own brand and detects the impact of such product to both the firm and its competitors. In the perspective of policy makers, the example illustrates how consideration models identify the population who cannot be influenced by a firm’s design activity and simulate the effect if a policy can use persuasive informative campaigns to enhance the hybrid powertrain acceptance directly. This chapter is organized as follows. Section 6.2 describes the configuration of the simulation example, including the construction of competing market, the consideration models applied, and the optimization problem of consideration maximization. Section 6.3 presents the results, followed by Section 6.4 discussions and Section 6.5 conclusions.

6.2 Maximizing Consideration in the Competing Market

6.2.1 Synthetic Market

To investigate how the consideration information will impact the design strategies of a firm, the simulation built a synthetic market with competing brands based on real U.S. market data - with the vehicle feature data collected from WardsAuto (U.S. Car and Light Truck Specifications and Prices, ’14 Model Year), and the monthly sales data collected from Auto News database (U.S. Total Vehicle Sales by Make, Jan.2010 - Mar.2014). The synthetic market
included 18 brands, in total 131 series that were still offered in 2014, with particular focus on vehicle series with sales over 10,000 sales during 2013 & 2014, and vehicles that were newly offered during 2013 & 2014. The vehicle feature information also combined crash rating from the test report of National Highway Traffic Safety Administration’s 5-Star Safety Ratings Program (NHTSA Vehicle Crash Test Database), which evaluated crash rating on the overall safety performance on frontal crash, side crash and rollover tests. Quality ratings were summarized through Initial Quality Study conducted by J.D. Power Research (J.D.Power 2014), which measured new vehicles quality after the first 90 days of ownership.

6.2.2 Consideration Models

The exploration continues the interest of two consideration behaviors studied in Chapter 5 - subset-conjunctive behavior and aspirational behavior. In subset-conjunctive screening, an individual will consider a product if the number of acceptable attributes exceeds a criteria number, which yields the consideration set expression:

\[
C_i(X) = \{j : \delta_i^T x_j \geq N\}, N = 1, \cdots, A
\]  

(6.1)

where \(A\) is the total number of attributes of a profile. For some a binary consideration rule \(\delta\). An element of \(\delta\) is 1 if, and only if, that attribute-level is acceptable. When \(N = A\), it corresponds to the conjunctive model, which means that all attributes of a considered profile must have acceptable levels. When \(N = 1\), the inequality is satisfied if, and only if, at least one attribute has an acceptable levels, giving a disjunctive screening rule. The value of \(N\) between these two extremes gives a subset conjunctive rule. In aspirational screening, an individual will consider a product if the holistic utility of the product exceeds some aspirational level, with the consideration set expressed as:

\[
C_i(X) = \{j : v_i^T x_j \geq \gamma_i\}
\]  

(6.2)

for some "part-worths" \(v\) and an aspiration level \(\gamma\). Given the goal focusing the impact of the models rather than the estimation methods and the influence of bias or mis-specification in the
estimations, the simulation inherits the consideration behavior parameters used in generating the synthetic population in Chapter 5 (see sampling process described in Section 5.3.1).

6.2.3 Maximize the Consideration of Hybrid Vehicles

Suppose a firm is planning a "remove add" strategy - remove one vehicle from its existing product line and add one newly designed vehicle. The firm aims at maximizing the proportion of hybrids it owns in a consideration set averaging over the sampled population. The firm solves the following optimization problem:

$$\max \left( \frac{1}{I} \sum_{i} \frac{\text{Number of hybrid vehicles considered by individual } i \text{ AND offered by the firm}}{\text{Total number of vehicles considered by individual } i} \right)$$

(6.3)

Ford is used as a case study. The removals are enumerated over each vehicle in the existing product line. After removing a vehicle, the optimization problem of Eqn.6.3 is solved. The "remove & add" strategy with the highest objective value from these removal enumerations will be selected as the optimal strategy.

6.3 Results

6.3.1 Impact of the Optimal Strategy

In the case of subset-conjunctive screening, the optimal strategy is removing Focus Titanium from the existing product line and at the same time introducing a hatchback at $17k, with 30mpg, and both quality and crash rating as 4, see Table 6.1. Figure 6.1 shows how the considerations of Ford and hybrid vehicles change after introducing the optimal strategy. The individuals are sorted into five categories: those who consider Ford and do not consider any hybrids (represented with Region A), those who consider hybrids and do not consider any Ford vehicles (Region B), those who consider both Ford non-hybrid vehicles and hybrids of other brands but do not consider Ford hybrids (Region C), those who consider Ford hybrids (Region D), and those who consider neither Ford vehicles nor hybrids (Region E). The new designed hybrid vehicle expands the Ford hybrid population by impacting four categories of individuals. First, the new product attracts the Ford considerers - for 7 out of 45 Ford non-
hybrid individuals, the new Ford hybrid enters their consideration sets (see Region $D_a$). Second, the new product increases the consideration of Ford among the hybrids considerers - for 38 individuals who consider hybrid but do not consider any Ford products, there are 19 of them include this new Ford hybrid in their consideration sets (See Region $D_b$). Third, the new design significantly impacts those who consider Ford and also consider hybrids but do not consider any Ford hybrid - 38 out of 113 of those individuals now consider this new vehicle. Fourth, previously there are 20 individuals who consider neither Ford nor hybrids, but now 3 of them consider the new Ford hybrid.

There are 22 individuals who previously do not consider Ford, but due to this new hybrid product, now their consideration sets include a Ford product (Region $D_b$ and $D_e$ in Figure 6.1). Interestingly, 14 out of these 22 individuals even have the brand "Ford" as an unacceptable product, thus the new hybrid vehicle is actually taking the advantage of the subset-conjunctive behavior to make them consider a Ford. The implication for this result is that, for consumers who have hybrid powertrain as an acceptable feature, a company is able to not only promote hybrid among its old customers, but also benefit the brand consideration with an optimally designed hybrid vehicle.

In the case of aspirational screening behavior simulation, the optimization results in a different strategy - removing Escape SE from the existing product line and adding a hybrid pickup truck at price level of $22k with 20 mpg, see Table 6.2. Figure 6.2 shows the impact of the optimal strategy in the aspirational screening population. The newly added vehicle gains the consideration of 44 individuals who did not consider Ford hybrid even though they considered both Ford and hybrids of other brands. This impact is similar as in the subset-conjunctive behavior. In terms of attracting Ford non-hybrid customers, the new design has slightly stronger impact than that in the subset-conjunctive population - attracting about 20% of those customers compared to attracting 15% in the subset-conjunctive population. On the other hand, the new design has a weaker impact to those who consider hybrids but did not consider any Ford vehicles - attracting 25% of them compared to 50% in the subset-conjunctive population.
Table 6.1 The optimal removed and added vehicle portfolios in the consideration maximization problems with subset-conjunctive screening population

<table>
<thead>
<tr>
<th>Removed</th>
<th>Added</th>
</tr>
</thead>
<tbody>
<tr>
<td>Body Style</td>
<td>Price Level</td>
</tr>
<tr>
<td>Hatchback</td>
<td>$25K</td>
</tr>
<tr>
<td>Hatchback</td>
<td>$17-22K</td>
</tr>
</tbody>
</table>

Table 6.2 The optimal removed and added vehicle portfolios in the consideration maximization problems with aspirational screening population

<table>
<thead>
<tr>
<th>Removed</th>
<th>Added</th>
</tr>
</thead>
<tbody>
<tr>
<td>Body Style</td>
<td>Price Level</td>
</tr>
<tr>
<td>Small SUV</td>
<td>$28K</td>
</tr>
<tr>
<td>Pickup Truck</td>
<td>$22-27K</td>
</tr>
</tbody>
</table>

6.3.2 Substitution Pattern

Figure 6.3 presents the statistics of the changes of the consideration set sizes in the population. Potential positive impact of hybrid sales comes in three different ways: (1) for individuals who already had hybrids in their consideration sets, remove one gasoline vehicle from their consideration sets (labeled as "only gas removed"); (2) for those whose consideration sets did not include the removed gasoline vehicle, add the new hybrid vehicle into their consideration sets (labeled as "only hybrid added"); (3) both remove an gasoline vehicle and add a new hybrid (labeled as "gas replaced with hybrid"). All these three changes can lead to increasing the proportion of hybrids in the consideration sets, which can potentially increase the purchase of hybrids, not necessarily benefiting certain brands’ hybrids, but growing the sum of choice probability of hybrids as a whole category. In both subset-conjunctive screening population and aspirational screening population, the "remove add" strategies are able to impact over 57% of the population in the three ways stated above. There remains approximately 43% of the population whose consideration sets either (1) do not change, or (2) did not include any hybrids thus removing a gasoline vehicle does not impact the sales of hybrids from them.

The "remove add" strategy impacts not only Ford’s product line but also those of other brands. In the subset-conjunctive screening case, where Focus Titanium is removed, the competitor products, such as Sonic LTZ offered by Chevrolet, will potentially benefits from the
removal of Focus Titanium among 46 consumers who previously consider both of these vehicles. That’s because those 46 consumers will have a higher chance to purchase a Sonic. On the other hand, Sonic LTZ may lose sales among 48 people who previously consider it. The reason is that the new designed vehicle enters the consideration sets of these people, which will distract the choice probabilities of the products in the same consideration sets. Some products may be more significantly affected by the new designed product rather than benefit from the removal of its competitor. Kia Rio LX will have around 36% of its previous considerers distracted by the new hybrid hatchback, while only around 14% will increase choice probability of Rio LX because the removal of the old hatchback. In the simulation, except hybrids offered by BMW, existing hybrids in the market generally have at least as twice as many considerers who are distracted by the new hybrid than those who have increased choice probabilities due to the removed product. As for Ford C-max series, there are around four times as many considerers who are distracted than increase purchase chances. Thus, Ford needs to be aware of the competition between its own existing hybrids series and the newly introduced hybrid product. In the aspirational screening case, the removal of Escape SE benefits competitors, such as Hyundai Sante Fe, Kia Sedona EX, Nissan Rogue, and Volkswagen Touareg, in a way that there are more individuals who have higher chance to choose these competing products because of the removal than those who have lower chance to choose these products because of added new vehicle.

6.3.3 Changing the Consideration Rules

The optimization problem above is solved in the condition that the screening rules of the population are unchanged. For the results shown in Section 6.3, we are able to see the impact of the firm activity alone. In the subset-conjunctive screening case, it is observed that, before Ford offers its optimal new hybrid, there are 65 sampled individuals (13% of the population) who do not consider hybrids of any brands (see Region A and E in 6.1). And Ford’s new hybrid vehicle turns 10 of those people to start considering a hybrid (see Region $D_a$ and $D_e$ in Figure 6.1). In a policy maker’s perspective, there are still 55 individuals (11% of the population) who still reject a hybrid and are not attracted by this new designed hybrid (see Region A’ and
E’ in Figure 6.1). Thus, we investigate what if the policy makers conduct campaigns that can change consumers’ screening rules. For the proportion of population who cannot be influenced by the firms optimal design activity, we aim to see the effects of changing their consideration rules.

Specifically, for all these 55 people, their screenings on the feature "hybrid" are switched from "rejected" to "accepted", as if ideally there are some campaigns successfully persuade them to regard hybrid as an acceptable feature. In this case, 20 of them will consider at least one hybrid in the existing market. However there still remain 35 of them who reject any hybrids in the market. We speculate two reasons for this. First, they are more picky consumers. On average, they screen out 95% of products in the market, while others screen out around 70%. Therefore the hybrids available in the market cannot satisfy them on other features even if hybrid is an acceptable feature. Second, their screening rules may contradict those of many other individuals. Thus, satisfying their screening may results in trading off the consideration of majority of others, which could explain why Ford is not able to capture this proportion of population with its optimal product.

### 6.4 Discussions

As in product planning, consideration model provides another perspective in optimization. A firm is faced with different optimization objective when using consideration information. Different from the compensatory models, which views a product potentially competing with all available products in the whole market, consideration optimization drives the design process towards specific customer groups. Our simulation demonstrates that consideration models enable the observation that which customers are attracted or distracted from what products by investigating the changes of the consideration sets. This design perspective is important in two ways. First, the changes of consideration sets explicitly show how the change of some attribute(s) can pull the adoption on other attribute(s). In this simulation, for example, the new designed hybrid vehicle of Ford gains the consideration from the group that considers a hybrid but never considers a Ford. Second, the consideration sets specifically identify the competing products. The impact of the new product or removed product to market is tracable
by investigating which products are in the same consideration sets of the new product.

In the policy-making perspective, the simulation illustrates that tracing the consideration pattern can identify the segment of population that cannot be affected by the firm’s optimal strategy. This provides better targets for future campaigns to change the consideration rules of particular groups. Further research should be conducted to quantify and compare the influence on hybrid vehicle adoption exerted by changing consideration rules and that by optimizing products.

The limitation of the optimization framework in this study is that the simulation has not included the preference trade-offs in the choice stage decision. Thus, the profitability cannot be assessed as an evaluation of a design as in the traditional profit-oriented optimization framework. This limitation, however, reflects the reality that real world’s product development often place detail pricing and marketing strategies in a different stage that the design decisions of technology or physical features. In this perspective, the process of maximizing consideration is reasonable and practical for making design decisions such as whether a firm needs to invest on energy efficient technology in a relatively long term planning. For example, a firm may not be able to predict the profitability but can prevent product being excluded as a result of consumers’ budgets. We call for further research that can fill this gap between the consideration maximization and the final design/pricing decision refinement related to profit analysis.

6.5 Conclusions

This chapter has explored the usage of consideration models in a product design problem with the goal of enhancing hybrid vehicle adoptions. The optimization example demonstrates that analyzing the memberships of the consideration sets and the changes of the consideration sets provides insights to brand promotion opportunity and competition pattern related to specific product features and specific consumer groups. The simulation also shows the potential for policy makers to target specific consumer groups to increase hybrid vehicles adoption by influencing their consideration rules.
Figure 6.1  The change of consideration of Ford hybrids in subset-conjunctive screening population after adding the new design.

Figure 6.2  The change of consideration of Ford hybrids in aspirational screening population after adding the new design.
Figure 6.3 The impact of "remove and add" strategy to the substitution pattern of hybrid vehicles.
CHAPTER 7. CONCLUSIONS AND FUTURE WORK

7.1 Conclusions

This dissertation offers three major contributions to the application of consideration models in design engineering: providing numerical methods to incorporate consideration models into the design optimization framework; identifying the impact of consideration models to design decisions in terms of design features and strategic values; exploring new analysis approaches regarding consideration maximization.

Chapter 3 provides the numerical tools to handle the challenge of discontinuity when applying the consider-then-choose models in design optimization problems. The chapter investigates two classes of methods - methods based on nonlinear programming (NLP) and methods based on genetic algorithms (GA). In the NLP methods, smoothing techniques and complementarity constraints are explored to enable the derivatives in NLP. In the genetic algorithms, where the engineering quality constraints increases the difficulty in balancing between optimality and feasibility, penalized GA and constrained GA are implemented to ensure feasibility. The chapter evaluates different methods on optimality, feasibility, and computation burden.

Chapter 4 and 5 utilize synthetic experiments to simulate the successive processes of estimating the consumer models and applying the consumer models in design. The experiments provide insights into the consequences of mis-specifying consideration behaviors. Two chapters focus on two different sources of data. In Chapter 4, market based revealed preference data are used. Synthetic markets are generated where purchase activities with consideration behaviors are simulated. Four models (multinomial logit, random coefficient logit, nested logit and consider-then-choose) are compared in the their predictive power, design decision and profitability. The experiment reveals that modeling considerations have benefits in achieving higher decision accuracy and profitability, especially under the small-data condition.
Chapter 5 focuses on survey based stated preference data. Unlike in Chapter 4 where mis-specified models are estimated from the same set of market based data, this chapter takes into account that different assumptions of models actually lead to different data collecting and estimating processes. Therefore two types of survey are investigated, the one designed to capture the consideration information (adaptive consideration experiment) and the other designed to capture the choice information (traditional discrete choice experiment). The simulation has found that using consideration models increases the product line diversity and prevents over-predicting profits.

Chapter 6 conducts an explorational case study of hybrid vehicle adoption to investigate new analysis approaches based on consideration models. The case study demonstrates that a firm can use the consideration information to observe how the new product will penetrate the consideration sets of different categories of consumers. The example also illustrates how policy makers can identify the proportion of consumers whose consideration of hybrids cannot be influenced by the design activity but can be changed if campaigns exist to persuade them to accept the hybrid powertrain.

In the simulation experiments, the model comparisons do not argue for an absolute best model for designers to use, but enrich the field of design engineering by introducing consideration models and their important informational role in design decisions. In our findings, explicitly modeling consideration changes designs and maintains robustness in profitability. Strategic value of a model, such as profitability, is not necessarily consistent with the predictive power of the model. This finding should bring cautions to researchers when evaluating consumer models in their application, since outside of the synthetic experiment, predictive power is often the only indicator used in model selection. As a future direction on this issue, we advocate for various perspectives and metrics analyzing the application value of consumer models.

The study allows us to comment on model complexity. Model complexity does not necessarily imply higher benefit in design application. This conclusion comes in several perspectives. First, increasing the data cannot guarantee the increase of estimation accuracy due to the existence of mis-specification. The mis-specification influences not only the estimation process
but also the data collecting process. Second, models with simple structure may still capture the essential information needed to make good design decisions. Moreover, simplified model structure potentially reduces the quantity of data and computation burden during estimation.

7.2 Open Questions

In Chapter 4, the simulation has simplified the engineering models of the cost and performance constraints. The simulation Chapter 5 has not considered the influence of engineering constraints. Whether engineering models will significantly change the impact of consideration models still need further validation, since engineering constraints will reshape the feasible space of the design optimization problem. One possibility is that, if engineering constraints or cost models strongly restrict the profitable and feasible design space, the optimal solutions will be dominated by the engineering constraints instead of by the consumer models. Thus, the optimal solutions may share higher similarity even using different consumer models.

Further research questions arise regarding how compensatory models and consideration models deliver information. For compensatory models, information is presented as aggregate preferences, even if the data are collected on an individual level. For example, logit model with the assumption of homogeneous preference uses the same taste coefficients to aggregate represent a population. The latent class model represents heterogeneity by sorting consumers into different segments, but the distribution of the latent class membership is still modeled in an aggregate manner. On the other hand, the conjunctive model and the aspirational model deliver individual specific information by explicitly describing the screening rules each individual uses. It remains an open question whether it has benefits to abstract information from individual-specific screening rules and translate the individual-specific information into an aggregate style model before the design application. For example, when the number of individuals become large, it is possible to categorize major types of consumers to determine market segmentations before designing the products targeting for each segment.

The illustrative case in Chapter 6 discusses the opportunity of directly encouraging the acceptance of hybrid powertrain in order to increase the consideration of hybrid vehicles. The discussion serves to arouse similar strategic analysis for pro-environment products. Further
study is needed to quantify and compare the effectiveness of structural and informational strategies on the acceptance of these products. For example, the effectiveness comparison can be based on the proportion of population that can be attracted to the products via design & pricing activities versus the proportion that can be persuaded to consider the products by directly making the consumers to accept the pro-environment features. Relevant cost analysis on these strategies should be also conducted. For example, which strategy would cost more given the same level of impact - regulating/compensating the product manufactures, financially stimulating the consumers, or organizing informational program to psychologically change the consumers’ acceptance?

The complexity of consideration rules deserves further investigation. The sampling of synthetic population in the simulation has not fully reflected consideration rules in real life. The distribution of the acceptance on particular features, and the correlation of the acceptance have not been taken into account. For example, there may exist that the rejection of low mpg relates to the acceptance of hybrid. More realistic consideration rules can enable more conclusive findings and provide practical guidances for vehicle design and energy policy in the future.
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