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The electrical properties of a fluidized bed

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by

Eddie Keh-Chen Yuan

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INTRODUCTION

An electrofluid bed is a fluidized bed containing conducting particles with electrodes immersed in the particles. When current flows between the electrodes, direct electrical heating occurs. This type of reactor combines the advantages of a fluidized bed, with its high heat and mass transfer rates, and direct electrical heating, where no heating surfaces are necessary and the heating efficiency does not decline with increasing temperature.

One of the areas of current research sponsored by the Office of Coal Research of the Department of the Interior is hydrogen production by coal gasification in an electrofluid bed reactor. For producing hydrogen, conducting coal particles are used as the bed material and steam is used as the fluidizing gas. However, coal gasification in an electrofluid reactor is not yet fully developed or operated commercially. The Institute of Gas Technology in Chicago has built a pilot plant which includes an electrofluid reactor to produce hydrogen, which will then be used to manufacture natural gas.

Proper design of an electrofluid reactor requires information about the electrical characteristics of the bed. Work in this area has been carried out for several years at Iowa State University. The resistance of electrofluid beds
had been studied at room temperature (22, 27) as well as at elevated temperatures (37). The current research was a continuation of these electrical property studies, mainly in three areas, with the experiments being carried out at room temperature and atmospheric pressure.

A method for predicting the potential and current fields in electrofluid beds was developed by Knowlton (27). Essentially the same technique was extended to curved boundaries and employed in this work to analyze some of the systems investigated. Verification of the method was tested at voltage and current levels higher than those Knowlton used in his work.

Arcing in electrofluid beds has been observed by many researchers, and it may possibly cause electrode damage due to momentary local overheating. Therefore, an attempt was made to study the characteristics of arcing in the vicinity of a spherical electrode in an electrofluid bed. The effect of a number of parameters on arcing was examined such as electrode size, particle size, gas velocity, type of gas and applied voltage.

The interelectrode resistance of an electrofluid bed appears to be the sum of the contact resistance between the electrodes and the bed particles and the resistance between particles. General agreement has not been reached as to whether the contact resistance or the bed resistance is the
dominant component of the interelectrode resistance. Therefore, another interest in this work was to study and analyze these resistances. The interelectrode resistance, bed resistance, and contact resistance were measured in the same fluidized bed as it was felt that a better comparison could be made between the resistances when all the resistances were measured together in the same system. Furthermore, the contact resistance at the electrode was studied as a function of electrode material, bed material, current density and gas velocity.
LITERATURE REVIEW

Work on fluidized beds has been reviewed in several excellent books (30,32,40), and only the literature dealing with the properties of electrofluid beds will be reviewed here. This includes a discussion of applications of the reactor and the electrical properties of fluidized beds. Some of the literature dealing with arcing is also reviewed.

Applications

The electrofluid reactor was first proposed by Wickenden and Okell (38) in 1927 for making decolorizing carbon. A later patent was proposed by Winkler (39) in 1928 for producing water gas. Winkler performed his experiments in a coke bed fluidized with steam. Two electrodes immersed in the bed provided the heating. This process seemed more feasible than that Wickenden and Okell proposed, but was not operated commercially.

Electrofluid reactors were not developed further until the late 1950's. Subsequent patents have been issued to a group at Shingan Chemicals Limited of Canada for utilizing the electrofluid reactor to manufacture hydrocyanic acid (19, 23,24) carbon monoxide (18), carbon disulfide (17, 21) and titanium tetrachloride (20). Only hydrocyanic acid has been produced commercially in an electrofluid reactor. A mixture of ammonia and propane or methane passes through
a coke bed at a temperature of about 1500°F to produce hydrocyanic acid.

Recently, construction of a pilot plant to produce natural gas was started by the Institute of Gas Technology in Chicago (29). The process involves producing hydrogen in an electrofluid reactor. Hopefully, a commercial scale plant producing a gaseous fuel from coal can be developed in the near future. Atmospheric pressure reactors have been operated at Iowa State University to study the production of hydrogen from coal (2, 3, 33, 34).

Electrical Properties

Mechanism of current transfer in fluidized beds

Three possible mechanisms for current flow through the fluidized bed were suggested by Goldschmidt and LeGoff (12). These were (1) current flow along continuous chains of conducting particles, (2) a diffusion type of current flow where charge is shared between colliding particles, and (3) arcing between particles. Goldschmidt and LeGoff showed that mechanism (2) was not possible as the calculated resistivity by diffusion of charge was $10^7$ times greater than the measured value, and mechanism (3) was negligible when the system was operating at low voltage. Therefore, mechanism (1) was the primary method of current transfer through the bed at low operating voltages. At high voltages, current transfer was a
combination of mechanisms (1) and (3). These conclusions were confirmed by Graham and Harvey (14) as well as by Reed and Goldberger (35). Zheltov, et al. (41) proposed that high temperature photoionization between particles plays a considerable role in the conduction mechanism of electro-fluid beds. A further review of arcing is presented in a later section.

**Resistance in electrofluid beds**

The interelectrode resistance is the sum of the bed resistance and the contact resistance between the electrodes and the bed particles. Investigations have studied the total interelectrode resistance and its components, with the bed resistance having received more extensive study than the contact resistance.

*Interelectrode resistance* The effect of gas velocity on the interelectrode resistance was examined by Graham and Harvey (14) in a rectangular column with two graphite rods serving as electrodes. The resistance between electrodes increased from a minimum corresponding to a settled bed to a maximum value, slightly over the minimum fluidizing velocity, followed by a sharp decrease and finally a levelling off at high gas velocities. Graham and Harvey explained the decrease after the peak as being due to a decrease in the void fraction of the particulate phase caused by formation of gas
bubbles in the bed. This void fraction decrease facilitated
the current transfer from particle to particle. However,
no peak resistance above the incipient fluidization velocity
was observed by Goldschmidt and LeGoff (12) or Zheltov,
et al. (41). Instead, the resistance increased continuously
with gas velocity, with the highest increasing rate at the
velocity just exceeding the incipient fluidization velocity.

At constant relative gas velocity, the resistivity be­
tween two graphite electrodes in a graphite bed was found to
decrease with temperature by Zheltov, et al. (41). With
constant gas mass velocity, the interelectrode resistivity
of graphite particles increased, passed through a maxima,
and then decreased with increasing temperature. Similar
results were reported by Graham and Harvey (13), who also
studied the temperature effect with coke beds. The resis­
tivity of coke decreased and passed through a minimum at about
600°C, then increased for higher temperatures.

Knowlton (27) and Zheltov, et al. (41) have investigated
the effect of current density on interelectrode resistance
at room temperature, the resistance was found to decrease
sharply and nonlinearly with increasing current density.
However, at 2,000°C current density had little, if any effect
on the resistance as reported by Zheltov.

Graham and Harvey (14) examined the effect of particle
size on the interelectrode resistance. They stated that the
resistance between two graphite electrodes was lower for larger bed particles.

Zheltov, et al. (41) indicated the column diameter and the bed height also affected the interelectrode resistance of the fluidized bed, with larger column diameter or shallower beds giving lower interelectrode resistances. Moreover, Zheltov found that the effect of bed height on the resistance was more pronounced at low current densities than at high current densities.

Zheltov, et al. (41) also studied the changes in resistance caused by adding nonconducting solid particles to the fluidized bed. The experiments were carried out with graphite-alumina mixtures in a rectangular bed. The results showed that the addition of nonconducting alumina particles greatly increased the interelectrode resistance, with the increase being proportional to the concentration of alumina particles. Moreover, the finer the alumina particles, the greater their effect on the resistance. Zheltov observed that the addition of alumina particles greatly reduced the apparent viscosity of the bed, so it seemed that the greater fluidity of an alumina-graphite mixture meant greater resistance.
**Bed resistance**  Several investigators have used a four-terminal method to measure the bed resistance. The reason for using this method is that it measures only the resistance of the material, and excludes the contact resistance. Jones (22), Knowlton (27) and Smith (37) inserted two needle probes, while Reed and Goldberger (35) placed copper screens into a section of the fluidized bed with straight potential surfaces. The voltage drops between the probes or the screens were measured and then used to evaluate the bed resistivity with the relationship

\[ \rho = \frac{RA}{L} = \frac{\Delta V}{I \frac{A}{L}} \]

where

- \( \rho \) = fluidized bed resistivity
- \( R \) = resistance
- \( \Delta V \) = voltage difference
- \( I \) = current flow
- \( A \) = cross-section area
- \( L \) = length of current path

Reed and Goldberger (35), based on their experimental result with a rectangular fluidized bed of graphite particles, stated that the bed resistance increased linearly with length of current path and was also inversely proportional to the cross-sectional area for current flow. In other words, \( R = \rho \frac{L}{A} \) was a valid relation for the bed resistance of the
electrofluid reactor.

The effect of fluidizing gas velocity on the bed resistance has been studied by Jones (22). He performed his experiments in 2 in. and 4 in. diameter fluidized beds, and found that the fluidized bed resistivity of both graphite and coke particles increased rapidly with increasing gas velocity and passed through a maximum, then decreased to a minimum and began increasing again. However, in a rectangular bed, the fluidized bed resistivity of graphite increased continuously with gas velocity as observed by Reed and Goldberger (35).

Reed and Goldberger (35) found that the bed resistance of graphite beds decreased with increasing temperature, and they believed that the major reason for the bed resistance decrease was the softening of the particles under the high temperature which reduced the contact resistance between particles. Smith (37) has investigated the effect of temperature on the bed resistance of calcined coke and coal char. The results showed that the bed resistance decreased sharply as temperature increased from room temperature to about 700°F and continued to gradually decrease at higher temperatures. Smith proposed that the resistance of fluidized beds of calcined coke and coal char was characteristic of semiconducting materials.

The effect of current density on bed resistance was first
investigated by Reed and Goldberger (35). The bed resistance was found to be constant with current densities from 2.5 to 100 ma./sq. in. However, at current densities of 0.3 to 2.5 a./sq. in., the bed resistance decreased with current density. Knowlton (27) studied the resistance at even lower current densities, 0.1 to 4.0 ma./sq. in., and also indicated that bed resistance was constant with current density over this range. Smith (37) has examined the effect of current density at various temperatures, at high temperatures the effect of current density on the bed resistance was negligible. Smith concluded that the current density effect was diminished at high temperature because the heat generated between particles in a high-temperature bed by an incremental increase in current density did not decrease the bed resistivity significantly as it had with low bed temperatures.

Jones (22), as well as Reed and Goldberger (35), both showed that the fluidized bed resistivity decreased as particle size increased. The fluidized bed resistivity also showed an overall decrease with an increase in column diameter as observed by Jones (22) and Knowlton (27).

Studies have been carried out at Iowa State University (5, 36) on the effect of the addition of nonconducting particles to a conducting fluidized bed of graphite particles. The results showed that the bed resistance was greatly increased after 50 percent or more of nonconducting coke or
sand was added to the graphite bed. Two or three percent of superfine silica (Cab-O-Sil) in a graphite bed also had the effect of greatly increasing the bed resistivity. The researchers commented that the bed resistivity increased because the fine, nonconducting particles surrounded the graphite particles, breaking the conducting chains.

**Contact resistance**

No general agreement has been reached on the contact resistance between the electrode and bed particles. The contact resistance of a graphite electrode in a fluidized bed was investigated by Glidden and Pulsifer (11). They measured the contact resistance by placing four voltage point probes at various distances from the graphite center electrode. The voltage drops between the center electrode and the probes were measured and then plotted versus the distance from the electrode. The plot was extrapolated to zero distance, the voltage drop at the electrode was assumed due to the contact resistance. By applying Ohm's Law to this voltage drop, the contact resistance was calculated. From the experimental results, Glidden and Pulsifer claimed that the contact resistance of the graphite electrode was never more than one or two percent of the total inter-electrode resistance.

Knowlton (27) obtained the contact resistance by subtracting the bed resistance from the interelectrode resistance. The interelectrode resistances were measured experimentally,
and the bed resistances were evaluated from the voltage and current fields which were determined from a computer solution of the field equations. The contact resistance obtained with this method consisted of the contact resistance of the graphite center electrode and also the contact resistance of the brass screen wall electrode which Knowlton used. The results indicated that contact resistance was the major resistance in his electrofluid bed.

Reed and Goldberger (35) first noted that contact resistance decreased with increasing current density and was sensitive to the condition of the electrode surface. Glidden and Pulsifer (11) did a statistical study and reported that contact resistivity was a function of particle size, gas velocity and the interactions of (1) particle diameter with the square of the electrode diameter, (2) particle diameter with gas velocity, and (3) particle diameter with electrode diameter and gas velocity.

Knowlton (27) has done relatively extensive studies of contact resistance in a 6 in. diameter fluidized bed of calcined coke, with a carbon center electrode. As noted, he indicated that contact resistance appeared to be the major component of the interelectrode resistance and that it decreased with increasing current density. He also found that contact resistance was not evenly distributed over the center electrode surface, but rather varied along the length of the
electrode. Furthermore, Knowlton reported that contact resistance varied with relative gas velocity, current density and the interactions of (1) relative gas velocity and current density, (2) longitudinal position of the exposed center electrode area and current density, and (3) longitudinal position of the exposed center electrode area and relative gas velocity. The effect of the exposed area of the center electrode on the contact resistance was also tested by Knowlton. He concluded that contact resistance was a strong function of, but was not inversely proportional to, the exposed area.

Arcing

Arcing in electrofluid reactors has been observed by many researchers. In 1961, Johnson (16) reported that arcing between two immersed electrodes was visible from the top of the fluidized bed at a voltage gradient of 200 v./in. Reed and Goldberger (35) also observed small arcs within their fluidized bed at current densities above 2 a./sq. in. Evidence of arcing was reported by Ballain and Pulsifer (1). In their reactor, a sudden increase in current flow, which indicated a sharp drop in bed resistance, was observed at applied voltages of 180 to 210 volts, while the bed temperature was near 1,800°F. These observations indicated that extensive arcing existed in the electrofluid bed.
Lee et al. (29), felt that the mechanism for current flow in their high temperature and high pressure reactor was by spark discharge between particle gaps and believed this mechanism was a result of the particulate type of fluidization under high pressure. Zheltov, et al. (41) recently proposed that photoionization between particles may play an important role in the conduction mechanism of electro-fluid beds. They estimated that the temperature at the contact points between particles may reach 2,000°C and higher in beds which are at room temperature since heat is released in a very small volume. Vaporization of carbon at contacts under such a temperature would break the electrical circuit and cause a discharge in the particle gaps. They also indicated that the arc ignition voltage decreased almost linearly with increasing temperature (400° to 1,000°C) and, at 1,000°C, the arc ignition voltage decreased with increasing particle size.

So far there is a limited amount of information available concerning arcing in fluidized beds. However, there is much literature (4, 6, 7, 8, 9, 10, 15, 25, 26, 31) available describing the arcing phenomena at electrical contacts. Since the electrical circuit in a fluidized bed can be considered to be a number of conducting chains which are breaking and forming continuously, it seems worthwhile to review some of these articles.
The electrical breakdown of the gas in the gap between electrodes generally requires electron avalanches. This phenomenon is governed by Paschen's law which states that the breakdown voltage between two electrodes in a gaseous medium and a uniform electric field is a function of the product of gap length (d) and pressure (p). The breakdown voltage curve passes through a minimum breakdown voltage, and then increases again as the product, pxd, decreases. Paschen's law is based on the assumption that the probability of ionization of the gas per collision and the probability of producing electrons by ion collision with the electrode are both dependent on the kinetic energy of the electrons and ions. However, Boyle and Kisliuk (4), based on their experimental results, indicated that there are some failures in Paschen's law. A rather serious deviation from Paschen's law is noted for extremely small electrode separations; the results showed the breakdown voltage curve did not continue to increase with decreasing pxd after the minimum breakdown voltage, but decreased almost linearly after some increase from the "minimum breakdown voltage" of Paschen's law. Boyle and Kisliuk proposed that the process of gas breakdown in a small gap is primarily due to the space charge of a small number of ions which increases the field at the cathode. The steep increase in current with increase in the field is able to break down the gas even when the probability of any
particular electron having an ionizing collision is extremely small.

Germer (8, 9, 10) has investigated the arcing phenomena at an electrical contact on closure. He found that arcing can be activated by operating in some organic vapors such as benzene provided the electrode surfaces are also contaminated, such as with grease. He also reported that arc voltages were independent of current and ambient gas. The minimum arc voltage which Germer found for carbon was between 20 and 30 volts which was higher than the voltage needed for a noble metal electrode.
FIELD THEORY ANALYSIS OF FLUIDIZED BED RESISTANCE

A plot showing the electrical field in an electrofluid bed provides information about potential gradients, current densities and heat effects in various parts of the bed. This information is very valuable to studies of electrofluid reactors in areas such as arcing and electrode overheating problems.

Knowlton (27) developed a method for predicting the field and analyzing the resistance and other electrical characteristics of conducting fluidized beds based on the use of field theory. Field theory is used to describe the electrical field in the electrofluid bed assuming steady current flow and that the resistivity of any given fluidized bed is uniform and constant, which is tantamount to saying the bed is homogeneous and neither time nor current dependent. To provide convenient boundary conditions, it was assumed that the entire bed surface in contact with any given electrode is at a uniform potential and, concomitantly, non-electrode surfaces in contact with the bed are nonconducting.

Field theory was also applied in this research, with the same assumptions, but this method was extended to electrodes with curved surfaces. The bed resistance was calculated from the field plot with a method different from that
Knowlton used. The results of calculating the resistance by these two different methods were compared.

Method of Analysis

The determination of the fields was made utilizing field theory. The field equation applicable to the experimental system used in the investigation was Laplace's equation. A finite difference approximation method was employed by Knowlton (27) to numerically solve Laplace's equation on a digital computer. This same method was used, however, due to the curvature of the electrode surface, curved boundary conditions were involved.

Laplace's equation is

$$\nabla^2 p = \frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} + \frac{\partial^2 p}{\partial \theta^2} + \frac{\partial^2 p}{\partial z^2} = 0 \quad (1)$$

where $p$ is the potential function and $r$, $\theta$ and $z$ are the cylindrical coordinates. The center electrode was always placed along the vertical centerline of the bed which led to symmetry around the $\theta$ axis. Therefore,

$$\frac{\partial p}{\partial \theta} = \frac{\partial^2 p}{\partial \theta^2} = 0 \quad (2)$$

which reduced Equation (1) to:

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} + \frac{\partial^2 p}{\partial z^2} = 0 \quad (3)$$
The finite difference approximation method in Lapidus (28) involves covering the r-Z plane with a network of rectangular spaces. An initial estimated value is assigned to each intersection point. Then the new value of each point is recalculated from the values of the four adjacent points and itself. Many iterations are performed until all points are approximately the real solution. A typical example of an electrode system is given in Figure 1; a 0.5 in. diameter spherical electrode suspended by a 1/8 in. diameter insulated rod is the center electrode. Due to symmetry, only half of the bed is shown in the figure.

Before the finite difference approximation method is applied, the r axis is transformed to the R axis using

\[ R = K - r \]  

where \( K \) is the radius of the bed. Using this transformation, Equation (3) becomes:

\[ \frac{\partial^2 P}{\partial R^2} - \frac{1}{(K-R)} \frac{\partial P}{\partial R} + \frac{\partial^2 P}{\partial Z^2} = 0 \]  

Upon applying the finite difference approximation

\[ \frac{\partial^2 P}{\partial R^2} = \frac{1}{h^2} \{ (+1) -2 (+1) \} \]  

where the circles represent adjacent points in the R direction shown in Figure 1, (-2) indicates that the value of the point
Figure 1. Two-dimensional bed region over which LaPlace's equation was applied.
is multiplied by (-2), and \( h \) is the spacing between any two adjacent horizontal points. Also,

\[
\frac{\partial P}{\partial R} = \frac{1}{2h} \{ +1 \quad 0 \quad -1 \}, \quad \text{and} \quad (7)
\]

\[
\frac{\partial^2 P}{\partial Z^2} = \frac{1}{k^2} \{ -2 \} \quad \{ +1 \}
\]

where \( k \) is the spacing between two adjacent vertical points. For convenience, \( h \) and \( k \) are taken as equal in this case, thus Equation (5) becomes:

\[
1 - \frac{h}{2(K-R)} \quad -4 \quad 1 + \frac{h}{2(K-R)}
\]

The boundary conditions in this problem (Figure 1) are:

\[
P = 0 \quad \text{at} \quad R = R_3 \quad \text{and} \quad Z_1 \leq Z \leq Z_4 \quad (10)
\]

\[
\frac{\partial P}{\partial R} = 0 \quad \text{at} \quad R = R_2 \quad \text{and} \quad Z_3 \leq Z \leq Z_4 \quad (11)
\]

\[
\frac{\partial P}{\partial R} = 0 \quad \text{at} \quad R = R_1 \quad \text{and} \quad Z_1 \leq Z \leq Z_2 \quad (12)
\]

\[
\frac{\partial P}{\partial Z} = 0 \quad \text{at} \quad Z = Z_1 \quad \text{and} \quad R_1 \leq R \leq R_3 \quad (13)
\]
\[ \frac{\partial P}{\partial Z} = 0 \quad \text{at} \quad Z = Z_1 \quad \text{and} \quad R_2 \leq R \leq R_3 \quad (14) \]

\[ P = 1 \quad \text{on the sphere surface} \quad (15) \]

Equation (9) and the boundary conditions can be used to obtain the values at each intersection point on the R-Z plane, except points 1, 2, 4 and 5 in Figure 1 (point 3 is on the boundary). The values at points 1, 2, 4, and 5 are more difficult to calculate because the distances between the four adjacent points and the center point are no longer all h. For instance, points 2 and 4 are a distance less than h from the right of the spherical boundary. Equation (9) is not suitable in this case.

Fortunately, Taylor series can be employed to solve this kind of problem. A general case will be considered here, with a center point, C, at \((R_0, Z_0)\) and the four adjacent points: L (left), R (right), T (top) and B (bottom). The distances between these four points and the center point C are \(D_L h\), \(D_R h\), \(D_T h\) and \(D_B h\) respectively.
Upon applying the Taylor series and truncating after the second derivative:

\[
P_R = P(R_0 + D_R h, Z_0) = R(R_0, Z_0) + (D_R h) \frac{\partial P(R_0, Z_0)}{\partial R} + \frac{(D_R h)^2}{2} \frac{\partial^2 P(R_0, Z_0)}{\partial R^2}
\]

\[
P_C = P_L = \frac{\partial P_C}{\partial R} + \frac{(D_R h)^2}{2} \frac{\partial^2 P_C}{\partial R^2} \tag{16}
\]

\[
P_L = P(R_0 - D_L h, Z_0) = P_C - (D_L h) \frac{\partial P_C}{\partial R} + \frac{(D_L h)^2}{2} \frac{\partial^2 P_C}{\partial R^2}
\]

\[
P_T = P(R_0, Z_0 + D_T h) = P_C + (D_T h) \frac{\partial P_C}{\partial Z} + \frac{(D_T h)^2}{2} \frac{\partial^2 P_C}{\partial Z^2} \tag{17}
\]

\[
P_B = P(R_0, Z_0 - D_B h) = P_C - (D_B h) \frac{\partial P_C}{\partial Z} + \frac{(D_B h)^2}{2} \frac{\partial^2 P_C}{\partial Z^2} \tag{18}
\]

Using Equations (16) and (17) to solve for \( \frac{\partial P_C}{\partial R} \) and \( \frac{\partial^2 P_C}{\partial R^2} \),

and Equations (18) and (19) to solve for \( \frac{\partial^2 P_C}{\partial Z^2} \),

\[
\frac{\partial P_C}{\partial R} = \frac{1}{h} \left( \frac{D_R}{D_L (D_L + D_R)} P_L - \frac{1}{D_R (D_L + D_R)} P_R + \frac{(D_L - D_R)}{D_L D_R} P_C \right) \tag{20}
\]

\[
\frac{\partial^2 P_C}{\partial R^2} = \frac{2}{h^2} \left( \frac{1}{D_L (D_L + D_R)} P_L + \frac{1}{D_R (D_L + D_R)} P_R - \frac{1}{D_L D_R} P_C \right) \tag{21}
\]

and

\[
\frac{\partial^2 P_C}{\partial Z^2} = \frac{2}{h^2} \left( \frac{1}{D_T (D_T + D_B)} P_T + \frac{1}{D_B (D_T + D_B)} P_B - \frac{1}{D_B D_T} P_C \right) \tag{22}
\]

Substituting in Equation (5), it becomes,
For the normal case, where \( D_L = D_R = D_T = D_B = 1 \), Equation (23) reduces to Equation (9). For point 4 in Figure 1, which is an example of a point less than a distance \( h \) from the surface, \( D_R = D_T = D_B = 1 \), \( D_L = 2h(1-\cos \theta) \) and \( P_L = 1 \).

A computer program was written using Equations (9) and (23) along with the given boundary conditions. Through many iterations on an IBM 360/65 digital computer, values at each point on the R-Z plane of Figure 1 were found. The equipotential lines were then determined by linear interpolation.

The current flux lines were found from a solution of the following equation.

\[
\frac{\partial^2 C}{\partial r^2} - \frac{1}{r} \frac{\partial C}{\partial r} + \frac{\partial^2 C}{\partial z^2} = 0
\]  

(24)

Where \( C \) is the stream function. This equation is derived in Appendix A. Upon applying the finite difference approxi-
27

formation, an equation similar to Equation (9) is obtained.

\[ 1 + \frac{h}{2(K-R)} \]

\[ 1 - \frac{h}{2(K-R)} \]

\[ +1 \]

\[ -4 \]

\[ +1 \]

Since the current lines are always orthogonal to the potential lines, the boundary conditions for solving for the current flux lines are "orthogonal" to the boundary conditions for the potential lines. For example, the boundary conditions for the system in Figure 1 are

\[ C = 1 \quad \text{at} \quad R = R_2 \quad \text{and} \quad Z_3 \leq Z \leq Z_4 \quad (26) \]

\[ C = 1 \quad \text{at} \quad Z = Z_4 \quad \text{and} \quad R_2 \leq R \leq R_3 \quad (27) \]

\[ C = 0 \quad \text{at} \quad R = R_1 \quad \text{and} \quad Z_1 \leq Z \leq Z_2 \quad (28) \]

\[ C = 0 \quad \text{at} \quad Z = Z_1 \quad \text{and} \quad R_1 \leq R \leq R_3 \quad (29) \]

\[ \frac{\partial C}{\partial R} = 0 \quad \text{at} \quad R = R_3 \quad \text{and} \quad Z_1 \leq Z \leq Z_4 \quad (30) \]

\[ \frac{\partial C}{\partial n} = 0 \quad \text{on the surface of the sphere} \quad (31) \]

where \( n \) is the direction normal to the sphere surface.

The values at each intersection point on the \( R-Z \) plane of Figure 1 except points 1, 2, 3, 4 and 5 can be determined by using Equation (25) and the boundary conditions. The
problem encountered here in calculating the values at points 1, 2, 3, 4 and 5 is different from the potential field case. For example, point 4 in Figure 1 is supposed to be calculated using the four adjacent points, 3, 6, 7 and 8. However, point 7 is an imaginary point and, also, the value of point 9 is not specified by a boundary condition. Therefore, instead of using Taylor series which cannot be solved in this case, a polynomial in R and Z was assumed to fit C(R,Z).

\[ C(R,Z) = a_0 + a_1 R + a_2 R^2 + a_3 Z + a_4 Z^2 + a_5 RZ \]  

(32)

The coefficients of Equation (32), \( a_1, a_2, a_3, a_4 \) and \( a_5 \), are related to the values \( C_3, C_4, C_6, C_7 \) and \( C_8 \). Then, by applying the boundary condition (Equation (31)) at point 9,

\[ 0 = \frac{\partial C}{\partial R}_9 = \frac{\partial C}{\partial R}_9 \cos \theta + \frac{\partial C}{\partial Z}_9 \sin \theta, \]  

(33)

a function \( F(C_3, C_4, C_6, C_7, C_8) \) can be obtained. This function \( F \) is used to eliminate \( C_7 \) from Equation (25). In other words, a polynomial is assumed to fit the solution \( C(R,Z) \), and by using the curved boundary condition to generate another relation among \( C_3, C_4, C_6, C_7 \) and \( C_8, C_7 \) can be eliminated from Equation (25). Therefore, point 4 can be evaluated from the three nearest points 3, 6 and 8. A detailed derivation is not given since no general equation can be obtained equivalent to Equation (23) for the potential lines.

A program similar to the one used to solve for the constant potential lines was used to establish values of the
stream function at the grid points. The constant current flux lines were then determined by linear interpolation.

Calculation of Bed Resistance

The resistance between the center electrode and the wall electrode consists of the contact resistance between the electrodes and the bed particles and the resistance of the bed. Two methods have been developed to calculate the bed resistance, both requiring an experimentally determined value of the bed resistivity. One is based on use of both the potential and current fields, while the other only utilizes the potential field. Both of these methods are discussed below.

The equipotential and constant current flux lines form many curvilinear squares between the electrodes as shown in Figure 2. In a three dimensional sense, there are equipotential surfaces and surfaces representing constant values of the stream function in the bed region. These two sets of surfaces divide the bed region into a series of curvilinear sections. Since the same amount of current flows through each of these sections and the difference in potential across each section is the same, the resistance of each of these sections would be the same. Therefore, the collection of curvilinear sections in the bed can be viewed as a number of equal resistors in series and parallel so that the total resistance of the bed is related to the resistance of each single curvilinear section by,
Figure 2. An example of field plot
\[ R_b = \frac{m}{n} R_s \]  

(34)

where \( m \) is the number of curvilinear sections in series, and \( n \) is the number in parallel. For convenience, \( m \) and \( n \) can be chosen as equal, which makes the bed resistance equal to the resistance of a single section or

\[ R_b = R_s \]  

(35)

The problem of predicting the bed resistance, then, is reduced to that of predicting the resistance of a single curvilinear section. The particular section chosen would be one which had negligible field distortion and thus could be closely approximated by an annular ring of rectangular cross-section. Consequently, the resistance of this section can be calculated by the expression

\[
R = \rho \int_{r_i}^{r_0} \frac{dr}{2\pi l} = \frac{\rho}{2\pi l} \ln \frac{r_0}{r_i}
\]  

(36)

where \( r_i \) and \( r_0 \) are the inner and outer radii, respectively, of the annular ring, \( l \) is the average height of the ring, \( R \) is the bed resistance or the resistance of the ring, and \( \rho \) is the resistivity of the bed material.

The bed resistance of an electrofluid reactor also can be evaluated from the equipotential lines without using the current flux lines. With this method, the potential field is used to calculate the current flow in the bed which can then be used to obtain the bed resistance from Ohm's law. The
bed current is evaluated from the expression

\[ I = \int \int \hat{J} \cdot ds \]  \hspace{1cm} (37)

where \( I \) is the bed current, \( \hat{J} \) is the current density and \( S \) is the surface area perpendicular to \( \hat{J} \). It is important to pick a surface which completely surrounds the center electrode and over which it is easy to perform the integration in Equation (37). In the cylindrical system employed, an annular cylindrical surface having the height of the bed was chosen. With this surface, Equation (37) becomes

\[ I = \int_{0}^{L} \int_{0}^{2\pi} \hat{J}(r) d\phi \, dz \]  \hspace{1cm} (38)

where \( \hat{R} \) is the radial position of the annular cylinder, \( L \) is the bed height, and \( \hat{J}(r) \) represents the current density in the \( r \) direction. \( \hat{J}(r) \) can be expressed in terms of potential gradient in the \( r \) direction as:

\[ \hat{J}(r) = \frac{\hat{E}(r)}{\rho} = \frac{1}{\rho} \frac{\partial \hat{P}}{\partial r} \]  \hspace{1cm} (39)

where \( \hat{P} \) is the potential function. With the potential known at a series of grid points, the potential gradient in Equation (39) can be approximated by the expression

\[ \frac{\partial \hat{P}}{\partial r} = \frac{\Delta \hat{P}(r)}{\Delta r} \]  \hspace{1cm} (40)

where \( \Delta \hat{P} \) is the potential difference over the increment \( \Delta r \).
Upon substituting this expression for \( \mathbf{J}(r) \) back into Equation (37), it becomes

\[
I = \frac{2\pi R}{\rho} \int_{0}^{L} \frac{\Delta P(r)}{\Delta r} \, dz \tag{41}
\]

The bed resistance, \( R \), can be calculated from Ohm's law,

\[
R = \frac{V}{I} = \frac{V \rho}{2\pi R \int_{0}^{L} \frac{\Delta P(r)}{\Delta r} \, dz} \tag{42}
\]

where \( V \) is the applied voltage.

The remaining problem in this calculation is the integration of the denominator in Equation (42). With the potential field determined, the potential difference existing between the increments \( \Delta r \) at any bed height \( Z \) are known. These points are integrated numerically using Simpson's rule. The annular cylinder (\( \overline{R} \)) is located at a radial position where \( \Delta P(r) \) does not change dramatically with respect to \( Z \), as this should reduce the error in the numerical integration.

**Calculated Results**

Laplace's equation was used to predict the potential and current fields of several different shapes of electrodes. The effect of grid spacing (\( h \)) on the predicted field was examined. Also, the bed resistance was evaluated using the two methods previously described so as to compare them.
Predicted fields

Potential fields for five different types of electrodes were predicted from Laplace's equation. These electrodes, which have either a hemispherical or a spherical surface, are depicted in Figure 3. The purpose of selecting these electrodes was to find the electrode with a potential field which closely approximates the potential field for a spherical electrode in an infinite medium so that this electrode could be used to examine arcing in the bed. The first electrode (A) in Figure 3 consisted of a long, uniform cylindrical section joined to a tapered section followed by a hemispherical end. The second electrode (B) was an insulated rod, followed by a hemisphere of the same diameter. The third electrode (C) was a sphere. The fourth (D) and fifth electrodes (E) were spheres supported by thin, insulated rods. These electrodes were placed along the vertical centerline of a 6 in. diameter fluidized bed, with the knob end of each electrode located at the center of the 16 in. bed height. The inside wall of the column served as the other electrode.

The predicted potential field for these five electrodes were obtained and are shown in Figures 4, 5, 6, 7 and 8 respectively. Due to the symmetry of this field, only a half or quarter of the field is shown. The equipotential lines of electrode (A) (Figure 4) were uniformly spaced along the cylindrical section, but became closer together around
Figure 3. Five electrodes of potential interest
Figure 4. The potential field predicted for electrode A
Figure 5. The potential field predicted for electrode B
Figure 6. The potential field predicted for electrode C
Figure 7. The potential field predicted for electrode D
Figure 8. The potential field predicted for electrode E
the hemispherical end, indicating that a larger voltage
gradient existed around the hemisphere. As shown in Figure
5, the equipotential lines of electrode (B) were closer
together in the region of the hemisphere and the insulated
rod junction. The equipotential lines of electrode (C) in
Figure 6 are nearly a spherical curve, but are slightly
stretched vertically due to the shape of the bed. The
nearly spherical equipotential lines of electrode (D) were
slightly distorted by the insulated rod, as shown in
Figure 7. In Figure 8, the constant current flux lines and
nearly spherical equipotential lines of electrode (E) are
given.

Except for the imaginary case of electrode (C), the
equipotential lines of electrodes (D) and (E) are closest
to those of the ideal sphere. Therefore, this type of
electrode was used in the experiments for arcing detection
so that the voltage gradients could be calculated easily.
A more detailed description of this is shown in the arcing
section.

Grid spacing effect The effect of grid spacing (h) on
the calculated values of the stream function was tested for
one particular system. This system consisted of a 1 in.
diameter cylindrical center electrode immersed 11 in. in a
16 in. deep bed, the bed diameter being 6 in. The center
electrode was insulated except for 1 in. at the tip. Grid spacings of 1/2, 1/4, 1/8 and 1/16 in. were used. The results are given in Figure 9, which shows the values for the four different grid spacings at some of the common points. No significant differences were found in these values. However, larger errors probably would be introduced for the larger grid spacings during the linear interpolation between two adjacent points.

It is suggested that a larger grid spacing be used for simple systems for economy in computation. With curved electrode systems, a smaller grid spacing probably should be employed to provide more information on the field in the region near the curved electrode.

Bed resistance

The bed resistance for several electrode geometries was calculated using the two methods previously described so as to compare them. The bed resistivity used in this calculation was 275 ohm in., which was experimentally determined by Knowlton (27).

In the first method the bed resistance was calculated using a single curvilinear square. The second method used the potential field to calculate the bed resistance by numerical integration. A comparison of values for the bed resistance determined by these two methods for two
Figure 9. Effect on field of changes in grid spacing
(grid spacing top to bottom, 1/16, 1/8, 1/4
and 1/2 in.)
<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.021</td>
<td>0.020</td>
<td>0.019</td>
</tr>
<tr>
<td>4</td>
<td>0.020</td>
<td>0.068</td>
<td>0.065</td>
</tr>
<tr>
<td>5</td>
<td>0.036</td>
<td>0.125</td>
<td>0.129</td>
</tr>
<tr>
<td>6</td>
<td>0.150</td>
<td>0.125</td>
<td>0.127</td>
</tr>
<tr>
<td>7</td>
<td>0.258</td>
<td>0.262</td>
<td>0.279</td>
</tr>
</tbody>
</table>

Note: The values in the table are not clearly visible due to the image quality.
different electrode configurations is given in Table 1. The resistance values are for a 6 in. diameter cylindrical bed with a 1 in. diameter center electrode immersed 10.75 in. in a 16 in. bed. The wall electrode ran the full height of the bed. The two different electrode configurations refer to the different uninsulated lengths of the electrode. These lengths were 1/4 in. and 10.75 in. measured from the tip. The values of bed resistance calculated by the second method at two radial locations are also given in Table 1.

<table>
<thead>
<tr>
<th>Calculation method</th>
<th>Resistance, ohms</th>
<th>Exposed electrode length, in.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.25</td>
</tr>
<tr>
<td>Curvilinear square (first method)</td>
<td>54</td>
<td>10.75</td>
</tr>
<tr>
<td>Potential field (second method)</td>
<td>53.25</td>
<td>6.7</td>
</tr>
<tr>
<td>at $\bar{R} = 2.125$ in.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>at $\bar{R} = 1.125$ in.</td>
<td>53.28</td>
<td>6.7</td>
</tr>
</tbody>
</table>

The results were almost identical for the two different methods of calculation. Therefore, a judgment as to superiority of either method should be based mainly on economy and convenience. The second method is cheaper and easier to use since the constant current flux lines are not needed in this method. Besides, the values of bed resistance
calculated by the second method at two different radial positions ($\bar{R}$) were the same, providing some evidence of the consistency of this calculation method. However, the first method not only gives the bed resistance, but also can provide information about the current distribution and power generation in the bed.
EXPERIMENTAL ANALYSIS

Potential Field

The potential field in a fluidized bed can be predicted mathematically with a Laplacian field model by assuming the bed is homogeneous as discussed in the previous section. Knowlton (27) used a voltage probing technique to compare the predicted potential fields with the actual fields for a concentric electrode system with different exposed lengths of the center cylindrical carbon electrode. At rather low current and applied voltage, Knowlton reported that the Laplacian field model appeared to accurately predict the voltage potential distribution throughout the bed, with the exception of the area immediately under the center electrode tip.

In this research, the potential fields around two sizes of spherical electrodes were probed. The fields for 1/2 in. and 1/4 in. diameter spherical electrodes were shown previously in Figures 6 and 7, respectively. Voltage probes were used to test these predicted potential fields at different current and voltage levels.

Equipment for determining the potential field

The equipment used in these experiments consisted of a gas recycle system, a test column and the electrical instrumentation necessary for determining the potential field. The
gas recycle system is described in Appendix B and was used throughout the experiments in this research. The test column and electrical instrumentation will be described individually.

**Column**  A 6 in. I.D. Plexiglas tube was used for the column, which consisted of two sections. The 4-1/2 ft. long upper section of the column was mainly used for minimizing the entrainment of the bed particles. The lower section of the column, shown in Figure 10, contained the bed particles which were fluidized when the gas passed through the porous plate at the bottom of the column. A 14 in. high brass screen was glued onto the inside wall of the column 2 in. above the porous plate to serve as a wall electrode. The center electrode was held by an electrode holder which sat on top of the lower section of the column. A 3 in. long electrode cap was used to connect the electrode holder and the center electrode. By adjusting the length of the center electrode in the electrode cap, the immersion of the center electrode could be varied within a range of about two inches.

Five voltage probes could be inserted into the column through a Plexiglas slab which was connected to the column wall and these were used to determine the potential field around the center electrode. The five probes were evenly spaced along the 16 in. fluidized bed and were located 3.33,
Figure 10. Detailed drawing of the 6 in. diameter fluidized bed test cell used to measure interelectrode resistance
5.67, 8.00, 10.33 and 12.67 in. above the porous plate. The voltage probe was made by soldering two different diameter brass rods together. The front portion of the voltage probe, which was inserted into the bed, was a 3-1/4 in. long, 1/16 in. diameter brass rod and was electrically insulated except at the tip of the rod by a piece of shrink tubing. The rear portion of the probe was a 1/2 in. diameter brass rod, which fitted rigidly in a hole in the Plexiglas slab to prevent vertical vibration of the probe tip in the fluidized bed. A brass fitting and rubber o-ring were used to hold the probe onto the slab at the desired position and to prevent gas leakage. A dial micrometer set on an aluminum support behind the voltage probe was used to measure the distance of the probe tip from the center electrode. A detailed drawing of this section of the column is given in Figure 11.

**Electrical instrumentation for determining potential field**

The electrical circuit for determining the potential field in the fluidized bed between the center and screen wall electrode is given in Figure 12. Two identical 0 to 75 V DC power supplies in series provided the voltage across the bed. An ammeter connected in series with the two electrodes was used to measure the current through the bed but, because of the violent motion of the fluidized particles in the bed, the interelectrode current fluctuated causing oscillation of
Figure 11. Detailed drawing of 6 in. diameter test cell used to determine the bed potential field
Figure 12. Schematic of electrical circuit used to measure bed potential distribution
the ammeter needle. Thus the reading from the ammeter was recorded by taking the average of the oscillations of the needle. A Hewlett-Packard 410C voltmeter was used to measure the voltage between the two electrodes, or the voltage between the center electrode and the voltage probe. There was essentially no current flow through the voltage measuring circuit due to the extremely high input impedance of the voltmeter. The voltage readings were recorded by taking the average of the relatively minor oscillations of the voltmeter needle.

**Procedure**

Spherical electrodes 1/4 in. and 1/2 in. in diameter were used in this experiment. Since both spherical electrodes were small compared to the 16 in. fluidized bed, only the center voltage probe of the five probes was used to determine the potential field between the center spherical electrode and the screen wall electrode. The proper immersion of the center electrode was obtained by adjusting the length of the insulated rod which supported the center spherical electrode in the electrode cap. The electrode cap was screwed onto the electrode holder. The center voltage probe was then pushed into the bed and positioned so that the tip of the voltage probe was just touching the surface of either the insulated rod or the spherical electrode, or was under
the center of the spherical electrode, depending upon the immersion of the center electrode. A dial micrometer was placed behind the voltage probe and set at zero.

Calcined coke particles then were charged to the column to obtain a 16 in. high fluidized bed. Nitrogen from the gas holder was circulated through the system to remove air. The system was then sealed and nitrogen was allowed to circulate for more than ten minutes in order to test for leaks and to remove moisture from the system in the silica gel dryer. The system was then ready for the measurements.

A desired gas flow rate was obtained by adjusting the by-pass valve, and the DC power supply was adjusted to give the required voltage between the electrodes. The voltage probe was pulled outward radially and the voltage drop at the location of the probe tip was measured. In the region near the center spherical electrode, measurements were taken at 0.1 in. intervals, while 0.2 in. intervals were used in the region away from the center electrode. Both the inter-electrode voltage and the voltage between the probe tip and the center electrode were measured by the Hewlett-Packard 410C voltmeter. The fraction of voltage drop at the tip of the voltage probe was evaluated by dividing the voltage between the center spherical electrode and probe tip by the inter-electrode voltage.
Results and discussion

The predicted potential fields of both a 1/4 in. and 1/2 in. diameter spherical electrode in a fluidized bed of carbon particles were analyzed by the Laplacian field equation. A number of measurements were taken to verify this model of the electrical characteristics.

A 0.5 in. diameter spherical electrode was first used as the center electrode, with both brass and stainless steel spheres being employed. Each sphere was suspended by a 1/8 in. diameter insulated rod in the middle of a 16 in. fluidized bed. Nitrogen at a relative gas velocity of 2 was used to fluidize the bed. All measurements were made at room temperature. The field was investigated at three different voltage levels and in two different horizontal planes.

In Figure 13, the results of probing the field in the horizontal plane passing through the center of the spherical electrode are shown. In the figure, the fraction of the interelectrode voltage drop is plotted versus the dimensionless radial position of the probe tip, the smooth curve representing the continuous values predicted by the Laplacian field equation. Deviations between the measured and predicted values are indicated in this figure in the region near the center electrode for both center electrode materials, with these deviations diminishing with increasing inter-electrode voltage. This phenomenon is more obviously shown
Figure 13. Experimental vs. theoretical bed voltage profiles in the horizontal plane passing through the center of the spherical electrode
in Figures 14 and 15 in which the average of the fractional voltage drops for the two identical runs is plotted versus the reciprocal of the radial position of the probe tip, with the solid line still representing the predicted values. Figures 14 and 15 not only show that the deviations existed near the stainless steel and brass center electrodes, but also in the region near the brass screen wall electrode. It is believed that the differences between the measured and predicted values in the regions near the center and wall electrodes were caused by the contact resistance between the bed particles and the electrodes. More work in the area of contact resistance has been done and will be discussed in a later section, but the results of this work also indicated a contact resistance existed between the bed particles and the stainless steel or brass electrodes.

The results of probing the field in the horizontal plane passing 0.5 in. under the center of the spherical electrode are given in Figure 16. It indicated that the measured voltage drops were higher than the predicted values in the region beneath the spherical electrode. Similar results were obtained by Knowlton (27) with a cylindrical electrode. These results were probably due to a stagnant region beneath the electrode which caused a higher current flow and higher voltage drop than predicted.

Voltage probing was also used to test the use of the
Figure 14. Experimental vs. theoretical bed voltage profiles in the horizontal plane passing through the center of the spherical stainless steel electrode
Figure 15. Experimental vs. theoretical bed voltage profiles in the horizontal plane passing through the center of the spherical brass electrode.
APPLIED BED VOLTAGE, \( V_B \)
- 5.5 V
- 8.8 V
- 13.5 V

RECIPROCAL OF RADIUS, in.\(^{-1}\)

FRACTION OF VOLTAGE DROP
- WALL ELECTRODE
- BRASS SPHERICAL ELECTRODE
Figure 16. Experimental vs. theoretical bed voltage profiles in the horizontal plane passing 0.5 in. below the center of the spherical electrode.
Laplacian field equation for the 1/4 in. diameter spherical electrode at higher interelectrode voltages. The probing was carried out in the horizontal plane passing through the center of the electrode. The results are plotted in Figure 17 and indicate that the measured voltage profiles increasingly deviated from the predicted voltage profile for the 1/4 in. diameter spherical electrode as the interelectrode voltage was increased. Superimposed upon this figure is the predicted voltage profile of a 1/2 in. diameter spherical electrode. This curve approximated the higher voltage profiles better than did the curve for the smaller diameter electrode.

The data from Figure 17 is replotted as the fraction of voltage drop versus the reciprocal of the radial position of the probe tip in Figure 18. This figure clearly shows that the measured voltage drops were constantly lower than the predicted values for interelectrode voltages over 30V. This observation is contrary to that shown in Figures 14 and 15, where the measured voltage drop is higher than the predicted value. These low values of the measured voltage drop are believed to be caused by arcing near the center spherical electrode where a high voltage gradient existed. Arcing between the electrode and particles and between particles would reduce the resistance and would cause a lower voltage drop. A higher voltage gradient gives a higher probability of arcing and reduction in voltage drop.
Figure 17. Experimental and theoretical bed voltage profiles in the horizontal plane passing through the center of the spherical electrode.
FRACTION OF VOLTAGE DROP

THEORETICAL CURVE OF A 0.25-in. dia. SPHERE

THEORETICAL CURVE OF A 0.5-in. dia. SPHERE

V_{rel} = 2.0

APPLIED BED VOLTAGE, V_B

○ 15 V
■ 30 V
△ 50 V
▴ 60 V
□ 75 V
● 120 V

DIMENSIONLESS DISTANCE
Figure 18. Experimental vs. theoretical bed voltage profiles in the horizontal plane passing through the center of the spherical electrode.
Moreover, arcing near the center electrode seems to have the effect of enlarging the surface area of the center spherical electrode. Also, due to the irregularity of the arcing, the spherical electrode surface seems to be distorted as indicated by the fact the line is curved and not straight at high interelectrode voltages. Experiments for detecting arcing near a center spherical electrode were carried out and are described in the next section. The above interpretation seems valid since arcing was actually observed under conditions similar to those used in the experiments just described. Furthermore, Figure 18 also indicates contact resistance existed between the bed particles and the brass screen wall electrode as has been pointed out previously.

In conclusion, the Laplacian field equation is not adequate for predicting the potential field in a fluidized bed when there is a contact resistance at the electrode surfaces or arcing exists near the center electrode.

Detection of Arcing in an Electrofluid Bed

Arcing has been observed in electrofluid beds at high voltage gradients and current densities by many researchers. In the present study, a small spherical electrode was positioned in the middle of a 6 in. diameter, 16 in. high cylindrical bed and arcing was observed in the vicinity of
this electrode.

The technique used to detect arcing was based on the phenomena of sparking or light emission which usually accompanies arcing between electrodes. A fiber optics rod was used to transmit the light in the bed through a multiplier phototube to a counter. The number of counts shown on the counter in a given period of time was proportional to the intensity and number of the sparks. The effect of a number of parameters on arcing such as electrode size, particle size, gas velocity and type of gas were studied under this arrangement.

Equipment for arcing detection

A gas recycle system and test column along with the appropriate instrumentation for detecting arcing were used in this study. The gas recycle system is described in Appendix B.

Test column Two major modifications were made in the 6 in. diameter Plexiglas test column which had been used for determining the potential field. First, the center spherical electrode was inserted into the bed from the side rather than from the top. The center electrode was positioned 8 in. above the gas distributor and was supported on a brass rod, insulated with ceramic tubing, attached to a micrometer. This arrangement gave rigid support to the
electrode, preventing vibration caused by moving particles striking it. The spherical electrode could move a distance of 1 in. radially in the middle of the bed by twisting the micrometer. Ceramic, rather than plastic, insulation on the supporting brass rod was necessary in order to avoid problems with the insulation overheating.

A small 8-v light bulb was also inserted into the bed in place of the center spherical electrode for a few experiments. The light bulb was installed on a plastic tube attached to a micrometer.

The second modification was the replacement of the center voltage probe used in determining the potential field with a rod guide that held a 12 in. long, 1/8 in. diameter fiber optics rod. The tip of the fiber optics rod was positioned near the center spherical electrode or the light bulb surface, while the other end of the fiber optics rod could be viewed from outside the column. The exact distance between the fiber optics rod and the electrode or the light bulb was controlled by the micrometer. The front portion of the rod guide was a 3-1/4 in. long, 1/8 in. I.D. brass tube, which tightly held the fiber optics rod to prevent vertical movement. The 1-1/2 in. of the front portion of the rod guide that was in the column was electrically insulated. The rear portion of the rod guide was a 1/2 in. diameter brass tube which fitted snugly in the hole in the Plexiglas
slab. A rubber stopper was used to prevent gas leakage from the end of the rod guide. A drawing of the column used to detect arcing is given in Figure 19.

**Instrumentation for arcing detection** The apparatus and the circuit used in detecting arcing near the center electrode are shown in Figure 19. The fiber optics rod was used to transmit light caused by arcing near the center electrode to an RCA 6199 multiplier phototube. The rod was contained in two concentric copper tubes, which prevented exposure to light. The multiplier phototube was capable of multiplying photoelectric current produced at the cathode by light by a median value of 600,000 times when operated with a supply voltage of 1,000 volts. The phototube was connected to a Nuclear Chicago counter, which also provided a DC power supply for the phototube. An oscilloscope was connected to the counter. A pulse would be shown on the screen of the oscilloscope to identify a spark at the center electrode. A timer was used to control the exact period of time for each measurement.

A Heath integrating digital voltmeter was used to measure both the average of the interelectrode voltage and the voltage across a 0.1 Ω, 5 W resistor connected in series with the center electrode and the screen wall electrode. By knowing the voltage across the 0.1 Ω resistor, the inter-
Figure 19. Apparatus used to observe arcing near an electrode
electrode current was calculated.

A slight change was made for the measurements when the 8-v light bulb replaced the center spherical electrode. The voltmeter connected across the 0.1 Ω, 5W resistor was used to measure the current flow through the light bulb. The output power associated with the brightness of the light bulb was evaluated using the measured current and the resistance of the bulb.

**Experimental procedure for arcing detection**

A spherical electrode was first placed in the column 8 in. above the porous plate distributor. A 1/8 in. diameter fiber optics rod was inserted into the bed through the rod guide. One end of this rod was placed at the desired distance from the surface of the spherical electrode. Then the column was filled with calcined coke to obtain a 16 in. high fluidized bed, and the system was sealed and fluidized.

As sparking occurred near the center spherical electrode, the light was transmitted to the phototube through the fiber optics rod. From the phototube the signal was relayed to the Nuclear Chicago counter and then transferred to the oscilloscope where a pulse would be shown. The number of counts on the counter and the magnitude of the pulse on the oscilloscope depended upon the intensity and number of sparks. The number of counts shown on the counter was recorded for
three two-minute periods at any given interelectrode voltage. A picture was taken of the screen of the oscilloscope as needed. The interelectrode voltage and current were averaged and recorded by the Heath integrating digital voltmeter.

Essentially the same procedure was employed for the 8-v light bulb. The current through the light bulb was controlled by the DC power supply and was measured by recording the voltage across the 0.1 Ω resistor.

**Results and discussion**

**Use of light source** An 8-v light bulb was installed in the bed. The brightness of the light bulb was varied by controlling the current through the light bulb, with the distance between the fiber optics rod and the light bulb surface also being varied during the measurements. The light source was used to test the assumption that the number of counts shown on the counter in a given period of time was proportional to the intensity of the arcs near the center electrode.

The response of the number of counts per minute to changes in the distance, gas velocity and the brightness of the light bulb is shown in Figure 20. The results indicate that the number of counts per minute decreased sharply, especially for the lower gas velocity, as the distance between the fiber optics rod and the light bulb increased. Moreover,
Figure 20. Effect on counts per minute of distance between fiber optics rod and light bulb
\begin{center}
\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure}
\caption{Graph showing the relationship between average counts per minute and distance from the light bulb, for different power levels.}
\end{figure}
\end{center}

- \( V_{\text{rel}} = 2 \)
  - LIGHT BULB
  - POWER = 4.84 \text{ w}
- \( V_{\text{rel}} = 2 \)
  - LIGHT BULB
  - POWER = 60 \text{ w}
- \( V_{\text{rel}} = 3 \)
  - LIGHT BULB
  - POWER = 4.84 \text{ w}

The graph illustrates the decrease in average counts per minute as the distance from the light bulb increases.
the most significant information that Figure 20 shows is that a brighter light bulb caused by a higher current flow caused more counts per minute than a dimmer light bulb with a lower current flow. This observation indicates that the number of counts in a given period of time during the arcing experiments was proportional to the intensity of the sparks.

Arcing A 1/4 in. diameter spherical electrode was placed in the bed instead of the light bulb. The number of counts was measured as a function of interelectrode voltage and the distance between the fiber optics rod and electrode surface. The results, given in Figure 21, show that as the distance between the fiber optics rod and electrode increased, the number of counts decreased sharply. This indicates that the arcing was confined to a region very close to the spherical electrode. The distance between the fiber optics rod and electrodes was then fixed at 1/8 in. for the remaining experiments.

Using two different sizes of calcined coke (-48/+65 mesh and -100/+150 mesh), measurements of the arcing in the bed as a function of applied voltage were made for 1/4 in., 3/8 in., and 1/2 in. diameter spherical electrodes at relative gas velocities of 2 and 3. The fiber optics rod was positioned 1/8 in. from the surface of each electrode, and 900V was applied to the multiplier phototube. Three
Figure 21. Effect on arcing of distance between fiber optics rod and 1/4 in. electrode
INTERELECTRODE VOLTAGE
- 15V
- 25V
- 35V
- 45V
- 55V
- 65V

RELATIVE VELOCITY = 2
PARTICLE SIZE -100 + 150 mesh
NITROGEN

COUNTS PER MINUTE

DISTANCE FROM ELECTRODE SURFACE, in.
2 minute counts, taken for each applied interelectrode voltage, were averaged to get the counts per minute. The results are plotted as the logarithm of the counts per minute versus the logarithm of the applied voltage since this seemed to give a straight line. Figures 22 and 23 show the plots for -48/+65 mesh and -100/+150 mesh particle sizes, respectively.

With the data available from Figures 22 and 23, the voltage gradient and current density at the electrode surface were calculated at different degrees of arcing, represented by number of counts per minute. These calculated values are given in Table 2, where the current density was evaluated by dividing the bed current by the electrode surface area and the voltage gradient was approximated by a relationship for a spherical electrode in an infinite medium. This approximation was made because the potential fields for the two systems near the sphere surface are similar as shown in Appendix C. This relationship for calculating the voltage gradient is:

\[
V_G = \frac{VR}{r^2} = \frac{V}{R} \quad \text{(at electrode surface)} \tag{43}
\]

where

- \(V_G\) = voltage gradient
- \(V\) = applied voltage
- \(R\) = radius of electrode, and
- \(r\) = distance from electrode center to point
Figure 22. Effect of voltage and electrode diameter on amount of arcing in fluidized bed
Figure 23. Effect of voltage and electrode diameter on amount of arcing in fluidized bed
RELATIVE VELOCITY = 2.0

RELATIVE VELOCITY = 3.0

ELECTRODE DIAMETER
- 1/4 in.
- 3/8 in.
- 1/2 in.

PARTICLE SIZE
- 100 + 150 MESH

VOLTAGE APPLIED TO ELECTRODE, V

COUNTS PER MINUTE
<table>
<thead>
<tr>
<th>Electrode diameter, in.</th>
<th>Particle size, mesh</th>
<th>Relative gas velocity</th>
<th>V.G. (^a), C.D. (^b) V/in. A/in. at 0.167 counts per minute</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/4</td>
<td>-48+65</td>
<td>2</td>
<td>160 0.33</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>120 0.39</td>
</tr>
<tr>
<td>3/8</td>
<td>-48+65</td>
<td>2</td>
<td>160 0.38</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>112 0.42</td>
</tr>
<tr>
<td>1/2</td>
<td>-48+65</td>
<td>2</td>
<td>140 0.36</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>96 0.37</td>
</tr>
<tr>
<td>1/4</td>
<td>-100+150</td>
<td>2</td>
<td>40 0.047</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>40 0.064</td>
</tr>
<tr>
<td>3/8</td>
<td>-100+150</td>
<td>2</td>
<td>43 0.050</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>43 0.057</td>
</tr>
<tr>
<td>1/2</td>
<td>-100+150</td>
<td>2</td>
<td>44 0.051</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>36 0.049</td>
</tr>
</tbody>
</table>

\(^{a}\text{V.G.} = \text{the voltage gradient at electrode surface.}\)

\(^{b}\text{C.D.} = \text{the current density at electrode surface.}\)
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>at 8 counts per minute</td>
<td>at 21 counts per minute</td>
<td>at 33 counts per minute</td>
<td>at 84 counts per minute</td>
</tr>
<tr>
<td>200</td>
<td>0.46</td>
<td>240</td>
<td>0.59</td>
</tr>
<tr>
<td>160</td>
<td>0.59</td>
<td>192</td>
<td>0.82</td>
</tr>
<tr>
<td>179</td>
<td>0.46</td>
<td>203</td>
<td>0.57</td>
</tr>
<tr>
<td>139</td>
<td>0.57</td>
<td>160</td>
<td>0.70</td>
</tr>
<tr>
<td>164</td>
<td>0.44</td>
<td>180</td>
<td>0.51</td>
</tr>
<tr>
<td>116</td>
<td>0.48</td>
<td>130</td>
<td>0.56</td>
</tr>
<tr>
<td>59</td>
<td>0.082</td>
<td>76</td>
<td>0.112</td>
</tr>
<tr>
<td>48</td>
<td>0.076</td>
<td>64</td>
<td>0.102</td>
</tr>
<tr>
<td>64</td>
<td>0.082</td>
<td>80</td>
<td>0.097</td>
</tr>
<tr>
<td>56</td>
<td>0.075</td>
<td>75</td>
<td>0.099</td>
</tr>
<tr>
<td>64</td>
<td>0.080</td>
<td>80</td>
<td>0.107</td>
</tr>
<tr>
<td>52</td>
<td>0.069</td>
<td>68</td>
<td>0.092</td>
</tr>
</tbody>
</table>
The values listed in Table 2 provide some information about arcing in an electrofluid bed. For all cases, the number of counts per minute increased with increasing voltage gradient and current density. This behavior seems consistent because, as previously noted, arcing was confined to the region near the center electrode where the voltage gradient and current density were highest. As the interelectrode voltage increased, the voltage gradient and current density near the center spherical electrode also increased rapidly. Therefore, arcing existed not only at the electrode surface, but also among those particles near the electrode. At this stage, more flashes would be observed from the fiber optics rod by the counter.

The picture taken from the oscilloscope traces also indicated an increase in arcing with an increase in applied voltage. Figure 24 shows pictures of two traces, both taken at a relative velocity of 2 with the 1/4 in. diameter electrode. At the higher voltage, 50V, there was more arcing, as indicated by the increased number of jumps in the trace.

Table 2 also indicates that with the -48/+65 mesh calcined coke, the values of the voltage gradients and current densities at the same gas velocity for three different size electrodes were similar at low arcing rates, but were quite
Figure 24. Oscilloscope traces of signal from multiplier phototube, 1/4 in. diameter spherical electrode (relative velocity = 2, 10 mv/cm on ordinate)
\[ V_B \text{ (interelectrode voltage)} = 35 \text{ V} \]

\[ V_B = 50 \text{ V} \]
different at high rates. This observation apparently also means that arcing originated from the electrode surface in the first case, while the deviations in values of voltage gradient and current density at high arcing rates were due to arcing detected not only from the electrode surface, but also among particles. In the later case, a voltage gradient or current density based on the electrode surface area would not fully characterize the arcing process.

With -100/+150 mesh calcined coke, the voltage gradient and current density needed for the same number of counts are much smaller for the smaller particle size as shown in Table 2. This is probably because there are more particles surrounding the electrode surface for the smaller particle size, which provided more chances for arcing to occur between the gaps. Moreover, the differences in voltage gradients and current densities with the smaller particle size for the three electrodes were relatively small even at high arcing rates. The explanation of this observation may be that most of the arcing detected here was from the electrode surface because the voltage gradient and current density were too low to ignite arcs among particles.

The effect of gas velocity on arcing was more obvious with the smaller particle size (-100/+150 mesh) calcined coke. For a certain number of counts per minute, a higher voltage gradient and current density were required for a
relative gas velocity of 2 than for 3. This may have been due to increased breakage of conducting chains at the higher gas velocity.

Table 2 also shows that at the same current density, the voltage gradient had more effect on arcing than gas velocity. In other words, the voltage across the gap caused by breakage of the conducting chains was more important to arcing than was the frequency of breakage.

The effect of type of fluidizing gas on arcing was examined by conducting a series of experiments using nitrogen, argon, and helium with three sizes of spherical electrodes and relative gas velocities of 2 and 3 in a -100/+150 mesh calcined coke bed. The results of these measurements, shown in Figures 25 to 30, indicate that the degree of arcing did not seem to be influenced by the type of fluidizing gas used. This is somewhat surprising since the ionization potential of helium is almost twice that of nitrogen and argon. However, this observation may indicate that arcing in fluidized beds is essentially like the arcing in small gaps as noted by Kisliuk (25), the breakdown in such gaps is caused by emission of electrons from the electrode and not by an electron avalanche in the surrounding gas. Under this situation, the type of gas used should not have much effect on the breakdown.
Figure 25. Effect of voltage and type of fluidizing gas on amount of arcing in fluidized bed (log-log scale)
PARTICLE SIZE: -100 + 150 mesh
ELECTRODE DIAMETER = \( \frac{1}{2} \) in.
RELATIVE VELOCITY = 2
FLUIDIZING GAS:
- NITROGEN
- ARGON
- HELIUM

VOLTAGE APPLIED TO ELECTRODE, V
COUNTS PER MINUTE
Figure 26. Effect of voltage and type of fluidizing gas on amount of arcing in fluidized bed (log-log scale)
PARTICLE SIZE: -100 + 150 mesh
ELECTRODE DIAMETER = ½ in.
RELATIVE VELOCITY = 3
FLUIDIZING GAS:
- NITROGEN
- ARGON
- HELIUM

COUNTS PER MINUTE

VOLTAGE APPLIED TO ELECTRODE, V
Figure 27. Effect of voltage and type of fluidizing gas on amount of arcing in fluidized bed (log-log scale)
PARTICLE SIZE: -100 + 150 mesh
ELECTRODE DIAMETER = 3/8 in.
RELATIVE VELOCITY = 2

FLUIDIZING GAS:
- NITROGEN
- ARGON
- HELIUM

COUNTS PER MINUTE

VOLTAGE APPLIED TO ELECTRODE, V
Figure 28. Effect of voltage and type of fluidizing gas on amount of arcing in fluidized bed (log-log scale)
PARTICLE SIZE: -100 + 150 mesh
ELECTRODE DIAMETER = 3/8 in.
RELATIVE VELOCITY = 3

FLUIDIZING GAS:
- NITROGEN
- ARGON
- HELIUM

VOLTAGE APPLIED TO ELECTRODE, V
COUNTPS PER MINUTE
Figure 29. Effect of voltage and type of fluidizing gas on amount of arcing in fluidized bed (log-log scale)
PARTICLE SIZE: -100 + 150 mesh
ELECTRODE DIAMETER = 1/4 in.
RELATIVE VELOCITY = 2
FLUIDIZING GAS
NITROGEN
ARGON
HELUM

COUNTERS PER MINUTE

VOLTAGE APPLIED TO ELECTRODE, V
Figure 30. Effect of voltage and type of fluidizing gas on amount of arcing in fluidized bed (log-log scale)
PARTICLE SIZE: -100 + 150 mesh
ELECTRODE DIAMETER = \( \frac{1}{2} \) in.
RELATIVE VELOCITY = 3
FLUIDIZING GAS:

- NITROGEN
- ARGON
- HELIUM

COUNTS PER MINUTE

VOLTAGE APPLIED TO ELECTRODE, V
Electrical characteristics during arcing

As described previously in the literature review section, arcing in a fluidized bed was detected by Zheltov, et al. (41) using a double beam oscilloscope. They reported that the bed voltage dropped sharply and the bed current increased considerably at the moment of arc ignition between two spherical electrodes at a distance of 15 to 20 mm. apart. However, with the arrangement used in this experiment, the sharp changes in bed voltage and current were not observed during arcing. Instead, these changes occurred gradually. A typical example is given in Figure 31 where a 1/2 in. diameter spherical electrode served as the center electrode.

The differences between the two observations were probably caused by the different electrode arrangements. In Zheltov's experiment, two spherical electrodes were placed closely in the bed. High voltage gradients and current densities existed between the two electrodes where current traveled the shortest distance from one electrode to the other. Therefore, at the moment of arc ignition, arcing existed not only in the area near the electrodes but also among the particles between the two electrodes. The sharp changes of voltage and current observed would be due to the large drop in bed resistance. In this study, a small spherical electrode was placed in the middle of a 6 in. diameter and 16 in. high cylindrical bed. Therefore, a high voltage gradient and
Figure 31. Voltage-current plot of a fluidized bed
0.5-in. dia. SPHERICAL ELECTRODE

RELATIVE GAS VELOCITY = 3
current density existed only in the region near the spherical electrode, so arcs were ignited only in this rather small region and no large drop in bed resistance occurred. Of course, the sharp changes in voltage and current would not be observed in this case.

Bed and Contact Resistance Measurements

In a concentric system, the interelectrode resistance consists of the contact resistance between the center electrode and the bed, the bed resistance, and the contact resistance between the bed and the wall electrode. This relation may be expressed as:

$$R_I = R_{CE} + R_B + R_{CW}$$

(44)

where $R_I$ is the interelectrode resistance, $R_B$ is the bed resistance, and $R_{CE}$ and $R_{CW}$ are the contact resistances at the center electrode and wall electrode, respectively.

The interelectrode resistance, the bed resistance, and the contact resistance were measured in the same fluidized bed during a series of experiments. Since the resistivity of fluidized beds seems to be affected by time and treatment of the materials, it was felt that at least better relative values of the resistances could be obtained when all the resistances were measured together in the same system.
The bed resistivity and the contact resistance were both measured using voltage probes. The test cell containing the fluidized bed was set up to simulate an ideal concentric system with a completely immersed center electrode (the success of this simulation will be discussed in the Testing of method section) so that the fluidized bed resistivity could be calculated using the data obtained with the voltage probes using the following equation:

\[ R = \frac{V_2 - V_1}{I} = \rho \frac{\ln(r_2/r_1)}{2\pi L} \]  

where

- \( R \) = resistance
- \( V \) = voltage
- \( I \) = total current flowing
- \( r \) = radial position
- \( L \) = length of electrode

and

\( \rho \) = fluidized bed resistivity.

Moreover, since in an ideal concentric system the lines of constant voltage are vertical, straight lines parallel to the wall, the contact resistance at the wall was evaluated from the voltage drop at the wall. This was obtained by extrapolating the voltages measured at various radial positions in the bed to the wall. The interelectrode resistance
was calculated from the measured interelectrode voltage and current so that the contact resistance at the center electrode could be evaluated from the other resistances using Equation (44).

Measurements of the resistances were taken under this arrangement at two gas velocities, three electrode sizes, two current levels and with three electrode materials and two bed materials.

**Equipment**

The test column previously used to determine the potential field in a fluidized bed was modified to study the resistances. Also, the shape of the voltage probes was changed, but the gas recycle system was still the same as described in Appendix B. An integrating digital voltmeter was used to measure the voltage drops between the probes and the electrode.

A sketch of the 6 in. diameter Plexiglas test column, which contained a 16 in. high fluidized bed, is shown in Figure 32(a). A thin cylindrical stainless steel liner on the inside column wall served as the wall electrode, while a cylindrical rod with an insulated tip served as the center electrode. Both electrodes were 14.5 in. in length and were positioned 1-1/2 in. above the gas distributor. This experimental setup approximated the 14.5 in. high ideal concentric system shown in Figure 32(b).
Figure 32. Actual and idealized bed geometry
Nine voltage probes each inserted into the bed from the outside wall of the column through the stainless steel wall electrode, were located 3.33, 4.50, 5.67, 6.83, 8.00, 9.16, 10.33, 11.50 and 12.67 in. above the gas distributor. These probes could be moved in and out radially to measure the voltage at various points in the bed. The probes were constructed of 1/16 in. diameter brass rod with a 3/16 in. diameter disk attached to the end of each probe as shown in Figure 33. Each probe was insulated except for the disk which was added to the probe for better contact between the probe and the bed. A sketch of the test column and the measuring circuit is shown in Figure 34.

In the measuring circuit, a current regulator was used with a 0-75V DC power supply to maintain a constant current flow through the fluidized bed. The current was measured by recording the voltage across a 0.1 \( \Omega \) resistor in series with the two electrodes. The voltage difference between the electrodes, or any probe and an electrode, was obtained from the integrating digital voltmeter.

**Experimental procedure**

A cylindrical electrode of the desired material and diameter and the stainless steel wall electrode were first installed in the column 1-1/2 in. above the gas distributor. The voltage probes were positioned so that the disk of each probe was just touching the center electrode surface, and the
Figure 33. Probe used to measure voltage at various points in a fluidized bed
Figure 34. Schematic of electrical circuit used to measure interelectrode and bed resistances
length of each probe outside the column, referred to as zero position, was measured with a micrometer. Then the column was charged with particles of either calcined coke or graphite to obtain a 16 in. high fluidized bed and was sealed and fluidized with nitrogen.

The interelectrode voltage and current were measured with the integrating digital voltmeter to allow calculation of the interelectrode resistance. Each reading shown on the integrating digital voltmeter was a 10 sec. average, with ten of these readings being averaged to give the values for calculating the resistance. Somewhat different procedures were used to measure the bed resistivity and the contact resistance, as described below.

**Bed resistivity** The equipotential lines were straight in this system, and only a measurement of the voltage drop between two radial positions was necessary in order to evaluate the bed resistivity using Equation (45). The voltage difference between the two radial positions was not measured merely with two probes, but with all nine probes. The odd numbered probes were positioned at an inner radius, and the even numbered probes were positioned at an outer radius. The distance between the two radii was either 2 in. or 1.5 in. The voltage differences between probes 1 and 2, 2 and 3, 3 and 4, etc., were measured consecutively with the
integrating digital voltmeter. Each individual reading on the voltmeter was a 10 sec. average, with five of these readings being averaged to give a voltage drop between two probes. The eight voltage drops thus obtained were again averaged to give the final value for the voltage drop between the two radial positions to be used in Equation (45).

Contact resistance It was necessary to know the experimental potential field in the fluidized bed in order to obtain the voltage drop at the wall which was used to evaluate the contact resistance by Ohm's law. Therefore, essentially the same procedure was employed here as was described previously for determining the potential field of the fluidized bed.

Using all nine probes, the voltage differences between the center electrode and the probes were measured by the integrating digital voltmeter. The probes were initially set 1/8 in. from the center electrode surface, then were pulled out in steps with the voltage drops between the center electrode and the probes being taken after each step. The voltage drop measured by the nine probes, located at different heights in the bed, were somewhat different but were averaged to give a single value representing the voltage drop at each particular radial position. Detail of how these voltage drops varied with the location of the probes will be shown in the Testing of Method section.
In order to obtain the contact resistance, these measured voltage drops were plotted versus the logarithm of radius, yielding a straight line which could be extrapolated to the surface of the wall electrode. The voltage drop at the wall, which was assumed to be due to the contact resistance, was then used to calculate the contact resistance by dividing the wall voltage drop by the bed current flow.

Testing of method

Voltage probe The voltage probe used to measure the voltage drop for the calculation of bed resistivity and contact resistance was previously shown in Figure 33. The 3/16 in. diameter round disk added to the probe was to give better contact with the bed, since without the disk the voltage drop from the center electrode to the probe and from the probe to the wall electrode did not add up to the inter-electrode voltage.

Since the disk was placed vertically in the bed, it was hoped that the influence on the fluidization would be negligible. To prove this point, the voltage drop was measured with the center voltage probe with and without the presence of the other probes. The results indicated that
the measured voltage drop was not being affected by the presence of the other probes.

**Voltage drop** The values of the voltage drops measured between the center electrode and the probes were somewhat different (Figure 35) for the nine probes. Usually the probe having a deeper immersion showed a higher voltage drop. This observation indicated that the fluidized bed was not in a stage of perfect homogeneity with a greater degree of fluidization in the upper section of the bed. The voltage drops from the nine probes were averaged to give a single value representing the voltage drop at any particular radial position during the process of calculating the contact resistance. It was believed that this averaged value represented the fluidized bed better than using the value from any single probe.

**System** The validity of assuming that the experimental system behaved as an ideal concentric system was examined by comparing the calculated voltage and current fields of the actual and ideal electrode systems; the field for the experimental system (Figure 32(a)) being determined from a computer solution of the field equation and the ideal concentric system case (Figure 32(b)) being solved analytically. The fields, shown in Figure 36, are quite similar for the two
Figure 35. Experimental and calculated voltage profiles in fluidized bed
Figure 36. Comparison of fields for actual and ideal electrode configurations
systems, with the potential lines being almost coincidental. A higher percentage of the current flow is toward the bottom of the bed in the test cell. The resistance of the bed was calculated for both cases, assuming the fluidized bed resistivity, $\rho$, was 275 ohm in. The values were almost the same for the experimental and ideal systems, 5.66 and 5.40 ohms, respectively.

Since the calculated resistances and field plots were so similar, Equation (45) was used in calculating the fluidized bed resistivity for the experimental system from the voltage difference measured by the probes. Furthermore, by the same argument, the bed resistance also was evaluated from the measured bed resistivity using Equation (45), but with $r_2$ and $r_1$ in this case being the radius of wall electrode and center electrode, respectively.

**Results and discussion**

The contact and bed resistances were measured under a number of different conditions, including use of 0.5, 1.0 and 1.5 in. diameter graphite electrodes, 1 in. diameter brass and stainless steel electrodes, currents of 0.8 and 1.2 amps, and relative gas velocities of 2 and 3. Calcined coke particles were used in all cases except one, in which graphite particles were used. The results are shown in Figures 37-45, and the calculated resistances are summarized in Table 3.
Figure 37. Semi-ln plot of voltage profile in bed for 0.5 in. graphite electrode
Figure 38. Semi-ln plot of voltage profile in bed for 0.5 in. graphite electrode

Figure 39. Semi-ln plot of voltage profile in bed for 1 in. graphite electrode
CALCINED COKE

\[ V_{rel} = 3 \]

CURRENT

\( \bigcirc \) 1.2 amp.
\( \square \) 0.8 amp.

WALL ELECTRODE

CENTER ELECTRODE

PERCENTAGE OF VOLTAGE DROP

RADIAL POSITION, in.

100
90
80
70
60
50
40
30
20
10
0

-1.5 -1.1 -0.7 -0.3 0.1 0.5 0.9 1.3

CALCINED COKE

\[ V_{rel} = 2 \]

CURRENT

\( \bigcirc \) 1.2 amp.
\( \square \) 0.8 amp.

WALL ELECTRODE

CENTER ELECTRODE

PERCENTAGE OF VOLTAGE DROP

RADIAL POSITION, in.

100
90
80
70
60
50
40
30
20
10
0

-0.7 -0.5 -0.3 -0.1 0.1 0.3 0.5 0.7 0.9 1.1
Figure 40. Semi-ln plot of voltage profile in bed for 1 in. graphite electrode

Figure 41. Semi-ln plot of voltage profile in bed for 1.5 in. graphite electrode
CALCINED COKE
$V_{rel} = 3$
CURRENT
○ 1.2 amp.
□ 0.8 amp.
WALL ELECTRODE

CALCINED COKE
$V_{rel} = 2$
CURRENT
○ 1.2 amp.
□ 0.8 amp.
WALL ELECTRODE

PERCENTAGE OF VOLTAGE DROP

RADIAL POSITION, in.
Figure 42. Semi-ln plot of voltage profile in bed for 1.5 in. graphite electrode

Figure 43. Semi-ln plot of voltage profile for 1 in. brass electrode
Figure 44. Semi-ln plot of voltage profile for 1 in. stainless steel electrode

Figure 45. Semi-ln plot of voltage profile for 1 in. graphite electrode
<table>
<thead>
<tr>
<th>Center electrode</th>
<th>Electrode diameter, in.</th>
<th>Bed material</th>
<th>Relative gas velocity</th>
<th>Current A</th>
<th>Bed resistivity, Ω in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graphite</td>
<td>1.5</td>
<td>Calcined coke</td>
<td>2</td>
<td>1.2</td>
<td>269</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.8</td>
<td>275</td>
</tr>
<tr>
<td>Graphite</td>
<td>1.5</td>
<td>Calcined coke</td>
<td>3</td>
<td>1.2</td>
<td>190</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.8</td>
<td>194</td>
</tr>
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<td></td>
<td></td>
<td>0.8</td>
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<td></td>
<td></td>
<td></td>
<td>0.8</td>
<td>209</td>
</tr>
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<td>1.2</td>
<td>293</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td>0.8</td>
<td>305</td>
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<td>Calcined coke</td>
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<td>1.2</td>
<td>210</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.8</td>
<td>222</td>
</tr>
<tr>
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<td>1</td>
<td>Calcined coke</td>
<td>2</td>
<td>1.2</td>
<td>297</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.8</td>
<td>311</td>
</tr>
<tr>
<td>Stainless Steel</td>
<td>1</td>
<td>Calcined coke</td>
<td>2</td>
<td>1.2</td>
<td>287</td>
</tr>
<tr>
<td>Steel</td>
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<td>1.2</td>
<td>307</td>
</tr>
<tr>
<td>Graphite</td>
<td>1</td>
<td>Graphite</td>
<td>2</td>
<td>1.2</td>
<td>39.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.8</td>
<td>43.5</td>
</tr>
<tr>
<td>Bed resistance, resistance at wall, $\Omega$</td>
<td>Contact resistance at center electrode, $\Omega$</td>
<td>Contact resistance, measured inter-electrode resistance, $\Omega$</td>
<td>Sum of resistance components, $\Omega$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>--------------------------------------------</td>
<td>-----------------------------------------------</td>
<td>-------------------------------------------------</td>
<td>--------------------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.0</td>
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<td>-0.13</td>
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<td>4.7</td>
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<tr>
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<td>4.9</td>
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<tr>
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<td>0.20</td>
<td>3.6</td>
<td>3.5</td>
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<tr>
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<td>3.9</td>
<td>3.8</td>
<td></td>
</tr>
<tr>
<td>5.5</td>
<td>0.69</td>
<td>-0.17</td>
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<td>6.1</td>
<td></td>
</tr>
<tr>
<td>5.5</td>
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<td>-0.12</td>
<td>6.1</td>
<td>6.4</td>
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<tr>
<td>3.8</td>
<td>0.70</td>
<td>0.08</td>
<td>4.6</td>
<td>4.6</td>
<td></td>
</tr>
<tr>
<td>4.0</td>
<td>0.68</td>
<td>-0.04</td>
<td>4.6</td>
<td>4.6</td>
<td></td>
</tr>
<tr>
<td>7.9</td>
<td>0.72</td>
<td>-0.21</td>
<td>8.4</td>
<td>8.6</td>
<td></td>
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<tr>
<td>8.2</td>
<td>0.70</td>
<td>-0.18</td>
<td>8.7</td>
<td>8.9</td>
<td></td>
</tr>
<tr>
<td>5.7</td>
<td>0.65</td>
<td>0.08</td>
<td>6.4</td>
<td>6.6</td>
<td></td>
</tr>
<tr>
<td>6.0</td>
<td>0.70</td>
<td>0.14</td>
<td>6.8</td>
<td>6.9</td>
<td></td>
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<tr>
<td>5.7</td>
<td>0.67</td>
<td>0.42</td>
<td>6.8</td>
<td>6.7</td>
<td></td>
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<tr>
<td>6.0</td>
<td>0.76</td>
<td>0.48</td>
<td>7.2</td>
<td>7.1</td>
<td></td>
</tr>
<tr>
<td>5.6</td>
<td>0.67</td>
<td>1.28</td>
<td>7.6</td>
<td>7.8</td>
<td></td>
</tr>
<tr>
<td>5.9</td>
<td>0.74</td>
<td>2.04</td>
<td>8.7</td>
<td>8.6</td>
<td></td>
</tr>
<tr>
<td>0.76</td>
<td>0.04</td>
<td>0.05</td>
<td>0.85</td>
<td>0.86</td>
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<tr>
<td>0.84</td>
<td>0.02</td>
<td>0.07</td>
<td>0.93</td>
<td>0.96</td>
<td></td>
</tr>
</tbody>
</table>
The contact resistance at the center electrode listed in Table 3 was calculated by subtracting the other resistances from the measured interelectrode resistance (Equation (44)). This contact resistance can also be obtained by extrapolating the lines in Figure 37-45 to the surface of the center electrode as was done in determining the wall contact resistance. For comparison, the measured interelectrode resistance and the sum of the calculated resistance components are given in the last two columns of Table 3. In all cases there is less than 5 percent difference between the two, which lends some confidence to the experimental method.

The fluidized bed resistivity data were similar to those collected by Knowlton (27). In each case, the resistivity decreased sharply as the relative gas velocity was increased from 2 to 3. Also, the resistivity seemed to increase slightly, less than 10%, with an increase in the diameter of the center electrode or a decrease in the current flowing.

The contact resistance at the wall electrode should not be affected by the use of center electrodes of different sizes or materials. A list of averaged values of the contact resistances at the outside wall for calcined coke under the same applied current and gas velocity is given in Table 4.

Table 4 indicates that the contact resistance at the wall electrode was not affected by changing the relative gas velocity from 2 to 3, but decreased by about 8% when the
Table 4. Contact resistance at outside electrode for calcined coke (average values from Table 3)

<table>
<thead>
<tr>
<th>Relative velocity</th>
<th>Current, amps.</th>
<th>Individual contact resistance at wall, ohms</th>
<th>Average contact resistance at wall, ohms</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.2</td>
<td>0.63, 0.69, 0.72, 0.67, 0.67</td>
<td>0.676</td>
</tr>
<tr>
<td>2</td>
<td>0.8</td>
<td>0.72, 0.70, 0.70, 0.76, 0.74</td>
<td>0.722</td>
</tr>
<tr>
<td>3</td>
<td>1.2</td>
<td>0.62, 0.70, 0.65</td>
<td>0.657</td>
</tr>
<tr>
<td>3</td>
<td>0.8</td>
<td>0.77, 0.68, 0.70</td>
<td>0.717</td>
</tr>
</tbody>
</table>

current flow was increased from 0.8 amps to 1.2 amps. This behavior was similar to Reed and Goldberger's (35) report that the contact resistance decreased with increasing current density.

The contact resistance at the electrode was strongly influenced by the electrode material used. Essentially no contact resistance was observed with graphite electrodes. This is similar to the findings of Glidden and Pulsifer (11) that contact resistance at a graphite electrode was only 1-2 percent of the interelectrode resistance. Contact resistances were noted for brass and stainless steel electrodes, with stainless steel giving the higher values.
When stainless steel was used for both the center and wall electrodes, the contact resistance at the wall was much less than that at the center electrode, even though the current density was higher at the center electrode because of the small surface area. Knowlton (27) reported that contact resistance was a strong function of the exposed area of the electrode. In fact, in this case it was a stronger function of exposed area than of current density on the electrode surface.

The bed resistivity of graphite was less than that of calcined coke as expected. The contact resistances at the wall and the center electrode were also much less when graphite was used. Examination of the calcined coke and graphite particles (Figure 46) showed them to be of similar shape. Therefore, some other property such as hardness must have accounted for the differences in contact resistance when graphite was used.
Figure 46. Electron microscope photographs of graphite (a) and calcined coke (b) particles (600X)
CONCLUSIONS AND RECOMMENDATIONS

Conclusions

Electrical field

1. The potential and current field in the vicinity of a curved electrode in a fluidized bed can be found from a solution of the electrical field equations.

2. Given a solution to the field equations, the fluidized bed resistance can be calculated using either of two different methods with the results being identical. The first method uses a single curvilinear square from the current and voltage field, while the second method utilizes the potential field to calculate the bed resistance by numerical integration. The second method is cheaper and easier while the first provides information about the current distribution and power generation in the bed.

3. The Laplacian field model is not adequate for predicting the potential field in a fluidized bed when there is contact resistance at the electrode surfaces or arcing exists near the electrode.

Arcing

4. Arcing in a fluidized bed originates from the center electrode where the voltage gradient and current density are highest, then extends to the region near the center electrode as the interelectrode voltage increases. At this stage,
arcing appears to have the effect of enlarging the surface area of the center electrode.

5. At the same interelectrode voltage, a higher gas velocity or a smaller particle size increases the arcing in fluidized beds. Furthermore, the voltage gradient has more effect on arcing than the gas velocity.

6. Arcing in a fluidized bed appears not to be affected by the type of fluidizing gas used.

**Contact resistance**

7. Contact resistance at the electrode is not affected by changing the relative gas velocity from 2 to 3, but decreases with increasing current density.

8. Contact resistance at the electrode is strongly influenced by the electrode material used. In a calcined coke bed, essentially no contact resistance was observed with graphite electrodes, but was noted for brass and stainless steel electrodes. Furthermore, the contact resistance at brass and stainless steel electrodes were much less in a graphite bed than in a calcined coke bed. Contact resistance also appears to be a strong function of the exposed area of the electrode.
Recommendations

1. Future investigations on contact resistance over a wider range of electrode materials, current densities, gas velocities and bed materials are suggested.

2. It is recommended that the investigation of arcing and contact resistance in a fluidized bed should be extended to elevated temperatures.
LITERATURE CITED


41. Zheltov, A. I., Zabrodsky, S. S. and Borodulya, V. A. Electrical properties of fluidized and settled beds of graphite particles at temperatures up to 2500°C. Presented at the Session on Fluidization Fundamentals, 72nd National Meeting of AIChE, St. Louis, Missouri. May 21, 1972.
ACKNOWLEDGMENTS

The author would like to express his appreciation to Dr. A. H. Pulsifer and Dr. T. D. Wheelock who suggested this project and provided valuable guidance throughout the investigation. Appreciation is also expressed to Dr. D. D. Robb for his timely advice and assistance.

The Office of Coal Research is also gratefully acknowledged for its financial support of this investigation.
APPENDIX A

Derivation of Stream Function

The electric field in a fluidized bed assuming there are no sources or sinks and no rotation of the field is solenoidal and irrotational, which leads to

\[ \nabla \cdot \mathbf{E} = 0 \]  
(46)

and

\[ \nabla \times \mathbf{E} = 0 \]  
(47)

where \( \mathbf{E} \) is the electric field. \( \nabla \cdot \mathbf{E} \) can be expressed in cylindrical coordinates, as

\[ \nabla \cdot \mathbf{E} = \frac{1}{r} \frac{\partial}{\partial r} (r E_r) + \frac{1}{r} \frac{\partial E_\theta}{\partial \theta} + \frac{\partial E_z}{\partial z} = 0 . \]  
(48)

Because of axial symmetry, \( \frac{\partial E_\theta}{\partial \theta} = 0 \), and Equation (48) becomes

\[ \frac{1}{r} \frac{\partial}{\partial r} (r E_r) + \frac{\partial E_z}{\partial z} = 0 \]  
(49)

or

\[ \frac{\partial}{\partial r} (r E_r) = - \frac{\partial E_z}{\partial z} . \]  
(50)

This is the necessary condition for

\[ (-r E_z) \, dr + (r E_r) \, dz \]  
(51)

to be an exact differential.
Furthermore, we assume

$$C(r, Z) = C_0$$  \hspace{1cm} (52)

where $C_0$ is an arbitrary constant, and

$$dC(r, Z) = (-rE_Z)dr + (rE_r)dz = 0 \, .$$  \hspace{1cm} (53)

Also,

$$dC = \frac{\partial C}{\partial r} dr + \frac{\partial C}{\partial Z} dz.$$  \hspace{1cm} (54)

By comparing Equations (53) and (54), we get,

$$\frac{\partial C}{\partial r} = -rE_Z$$  \hspace{1cm} (55)

and

$$\frac{\partial C}{\partial Z} = rE_r$$  \hspace{1cm} (56)

so that

$$E_Z = -\frac{1}{r} \frac{\partial C}{\partial r}$$  \hspace{1cm} (57)

and

$$E_r = \frac{1}{r} \frac{\partial C}{\partial Z} \, .$$  \hspace{1cm} (58)

From Equation (47)

$$\nabla \times E = \begin{vmatrix} \frac{\partial}{\partial r} & \frac{\partial}{\partial Z} \\ \frac{\partial}{\partial Z} & E_r \\ E_r & E_Z \end{vmatrix} = 0$$  \hspace{1cm} (59)

By substituting Equations (57) and (58) into Equation (59)
\[ \nabla \times \mathbf{E} = \begin{vmatrix} \frac{\partial}{\partial r} & \frac{\partial}{\partial z} \\ -\frac{1}{r} \frac{\partial C}{\partial z} & \frac{1}{r} \frac{\partial C}{\partial r} \end{vmatrix} = \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial C}{\partial r} \right) + \frac{\partial}{\partial z} \left( \frac{1}{r} \frac{\partial C}{\partial z} \right) = 0 \] 

(60)

or

\[ \frac{\partial^2 C}{\partial r^2} - \frac{1}{r} \frac{\partial C}{\partial r} + \frac{\partial^2 C}{\partial z^2} = 0 \] 

(61)

Equation (61) is used for finding the current stream lines in fluidized beds. For steady flow of current, lines of constant C are the curves along which the current flows.
APPENDIX B
Gas Recycle System

The gas recycle system flow sheet is shown in Figure 47. The purpose of the system was to provide gas to fluidize the bed and also to recover the gas in order to minimize the cost of operation.

Gas from the gas holder was compressed to about 15 lb./sq. in. gauge (psig) by the compressor. The gas then passed through a felt-element oil trap and into a finned-tube heat exchanger which removed the heat of compression and the heat generated in the bed. A silica gel dryer placed after the heat exchanger removed moisture from the gas. Following the dryer was a by-pass line. The by-pass line was needed to return excess gas not used to fluidize the bed to the gas holder. Gas sent to the column passed through either a large or small rotameter. The gas flow rate was controlled by means of the by-pass valve and measured by one of the two rotameters. From the rotameter, the gas was introduced to the bottom of the column through a porous plate diffuser. After leaving the column, the gas passed through the filter which removed particles elutriated from the bed and then returned to the gas holder.
Figure 47. Gas recycle system flowsheet
APPENDIX C

Electrical Field for a Spherical Electrode

For a spherical electrode situated in an infinite medium, Laplace's equation can be solved to give the electrical field surrounding the sphere. Laplace's equation in spherical coordinates is:

\[
\nabla^2 P = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial P}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial P}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 P}{\partial \phi^2} = 0
\]

where \( P \) is the potential function and \( r, \theta, \) and \( \phi \) are the characteristic spherical coordinates. Since the system is symmetrical with respect to \( \theta \) and \( \phi \),

\[
\frac{\partial^2 P}{\partial \phi^2} = \frac{\partial^2 P}{\partial \theta^2} = 0
\]

and Equation (62) reduces to

\[
\frac{\partial}{\partial r} \left( r^2 \frac{\partial P}{\partial r} \right) = 0
\]

(63)

The boundary conditions for the solution of Equation (63) are

\[ P = -V = \text{a constant at } r = R \]

and

\[ P = 0 \text{ at } r = \infty \]
where \( V \) is the electrode voltage and \( R \) is the radius of the sphere.

Solving Equation (63) with these boundary conditions gives:

\[
P = -\frac{R}{r} V
\]  

(64)

The voltage gradient at any point can be found by differentiating \( P \) with respect to \( r \).  

Voltage gradient = \( V_G = \frac{\partial P}{\partial r} = \frac{R}{r^2} V \)

At the surface of the electrode,

\[
\left(\frac{\partial P}{\partial r}\right)_{r=R} = \frac{V}{R}
\]

When a spherical electrode is suspended in a fluidized bed, the infinite medium assumption is not fulfilled because of the presence of the column wall. The effect of this was checked by using the computer to obtain the lines of constant potential from Laplace's equation for a 1/2 in. diameter sphere suspended by a 1/8 in. diameter insulated rod in a 6 in. I.D. column. The results of this calculation are compared with the infinite medium solution in Figure 48. The solid lines represent the infinite medium solution, while the dashed lines were obtained from the computer calculations. Near the sphere surface the two solutions are similar, allowing use of the simpler infinite medium equations. Further from the sphere surface, the results of the two
calculations are quite different.

It should be noted that in applying Laplace's equation to a fluidized bed, it is assumed that the fluidized bed is homogeneous with respect to its resistivity and that the voltage potential is constant over the electrode surface.
Figure 48. Comparison of potential lines for a 0.5 in. diameter spherical electrode with 1/8 in. diameter insulator rod in a 6 in. diameter column and an ideal 0.5 in. diameter spherical electrode in an infinite medium.