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Numerical Investigations of Bio-Inspired Blade Designs to Reduce Broadband Noise in Aircraft Engines and Wind Turbines

Andrew Lee Bodling
Iowa State University

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Numerical investigations of bio-inspired blade designs to reduce broadband noise in aircraft engines and wind turbines

by

Andrew Lee Bodling

A thesis submitted to the graduate faculty in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

Major: Aerospace Engineering

Program of Study Committee:
Anupam Sharma, Major Professor
Alberto Passalacqua
Leifur Leifsson

Iowa State University
Ames, Iowa
2017
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DEDICATION

I dedicate this to my beautiful wife, Jindan Wang-Bodling, who has provided me with so much support throughout my studies. She is my favorite cheerleader who has always been by my side throughout the ups and downs of graduate school. She has helped me get back up during my biggest failures and cheered me on when I succeeded.

I also dedicate this to my family for really being my motivation to succeed.
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This work numerically models an airfoil geometry inspired by the downy coat of the barn owl and contrasts its aerodynamic and aeroacoustic performance with the baseline airfoil. The owl-inspired geometry simulated here was suggested by Clark et al. [2], which used “fences” near the trailing edge of the airfoil to simulate the canopy effect of the owl feathers. The simulated geometry is not an exact replica of the experimental model and the aerodynamics and aeroacoustics consequences of these differences are discussed. Implicit large eddy simulations are performed with low-pass filtering of the solution using the extensively validated, high-order accurate Navier Stokes solver FDL3DI. A baseline NACA 0012 airfoil is compared against the same model with an array of fences at the trailing edge. Both models are simulated with the flow conditions listed in Table 1.1 in the thesis.

The owl-inspired airfoil geometries do not significantly degrade the aerodynamic performance but lead to reductions in unsteady surface pressure by up to 4.0 dB compared to the baseline. Unsteady surface pressure is the primary aerodynamic noise source in this problem where the flow Mach number (0.2) is small. The fences give a reduction in the overall sound pressure level (OASPL) of unsteady surface pressure in the entire region where the fences are located, except around the fence leading edge where the OASPL is found to increase.

Results of noise source diagnostics are reported. Contours and profiles of turbulence kinetic energy obtained from the simulations clearly show that the turbulence is lifted off the surface and relocated above the fences. The observed surface OASPL reductions are believed to be due to the increased separation between the sound sources (turbulence) from the scattering surface, primarily the trailing edge of the airfoil.

It is hypothesized that the observed increase in surface pressure near the leading edge of the fences in the simulations is due to the differences in the modeling of the leading edge of the fences between the experiments and the simulations. The sharp leading edge of the fence is a singularity which is very effective at scattering hydrodynamic energy in the turbulence in the boundary layer.
into acoustics. There does not appear to be a significant difference in either the spanwise coherence or the far field predicted noise between the baseline and fence geometries. The lack of far field noise reduction and spanwise coherence reduction in the simulations suggests a direction for future work – removing the geometric differences in the experiments and simulations and repeating the analysis.
CHAPTER 1. INTRODUCTION

The continued increase in air travel and the recent exponential growth in wind energy is bound to exacerbate the noise pollution problem. The detrimental effects on the hearing health of humans due to aircraft noise \[4, 5\] is well known. The effects from wind turbine noise \[6, 7\] have also been investigated. The consequent economic implications are severe; e.g., the U.S. Department of Veterans Affairs spends over $1 billion per year for hearing loss! With noise affecting many peoples lives, reducing noise is no longer a luxury, but a critical technology that must be incorporated into the designs of wind turbines and aircraft in the future.

Airfoil self noise in wind turbines and aircraft is due to the interaction of flow unsteadiness with the airfoil surface. Airfoil “self noise” can be generated visa multiple mechanisms. As described in detail in Ref. \[8\], these are separation stall noise, laminar boundary layer-vortex shedding noise, tip vortex formation noise and trailing edge noise. In this thesis, we will focus on the reduction of airfoil trailing edge noise. Trailing edge broadband aerodynamic noise in an airfoil results from the interaction of the surface boundary layer turbulence with the trailing edge. The fluctuating eddies in turbulence are a source of noise by themselves, but it is their close proximity to an edge such as the airfoil trailing edge that amplifies the sound produced \[9\]. When the flow Mach number is small (\(\leq 0.2\)), unsteady surface pressure resulting from the interaction of turbulence with the trailing edge is the primary aerodynamic noise source.

Reducing trailing edge noise is an important problem for many different applications. Trailing edge noise is an important aspect of noise generation from civil aircraft while landing and approaching the ground. Controlling trailing edge noise is critical to achieving the long term goal of the aviation industry, which is to reduce aircraft noise by 20 dB \[10\]. It is important to control trailing edge noise from propellers and hydrofoils so that the stealthiness of surface and underwater craft
is increased [11]. The major noise generation mechanism for helicopters [12], and wind turbine blades [13] is trailing edge noise, which limits their use in urban areas.

A relevant research question is how do we reduce trailing edge noise? One interesting approach, known as biomimicry, is to create trailing edge noise reduction designs that emulate nature's time-tested patterns and strategies. A variety of engineering applications have used biomimicry to create many great innovations, [14] e.g., temperature-regulated buildings inspired by termite mounds, [15] self-cleaning paints using the lotus leaf effect, [16] etc. However, the flight of nocturnal owls is yet to find its due engineering application. The only bird known to man that is capable of almost silent flight is the owl. [17, 18] It can not be heard until it is within 3 meters of its prey. [1] Owls use the acoustic stealth to aurally locate their prey in the dark and also avoid aural detection by the prey. [19] One species of nocturnal owls - the barn owl (Tyto alba) - is particularly skilled at silent flight. Hereinafter, we shall refer to the barn owl as “the owl”.

During gliding flight the owl’s chord-based Reynolds number is between 50,000 – 90,000. A similar Reynolds number range is operated in by small-scale micro- and unmanned aerial vehicles (MAVs/UAVs). Figure 1.1 illustrates the range of Reynolds number over which various flying machines and animals, including the owl, operate. Bio-inspired designs based off of the owl anatomy would therefore apply to UAVs and MAVs where the flow is expected to be mainly laminar across the blades/wings. The objective of this thesis, however, is to start exploring if similar bio-inspired designs at much higher Reynolds numbers ($10^5 – 10^7$), where the flow is expected to be turbulent, will also lead to noise reductions. Therefore, knowledge gained from this thesis will assist in the development of trailing edge noise-quieting designs for applications with high Reynolds number regimes such as aircraft engines and wind turbines.

![Figure 1.1: Chord based Reynolds number ($Re_c$) of various species compared with different aircraft](image-url)
Previous investigations [17, 18, 1] have shown three key anatomical features unique to the nocturnal owl that play a role in reducing noise during flight:

1. Stiff comb-like structure at the leading edge (LE) of the wing,

2. Flexible fringe like structure at the trailing edge (TE) of the wing, and

3. Soft, thick downy coat on the flight feathers.

The stiff comb-like structures (referred as serrations) at the leading edge consists of the barbs that are extended from the 10th primaries of the owl. The fringes at the trailing edge, which are present on each primary feather, is formed by the barbs that extend from the posterior part of the vane. The downy coat, the layer of fine feathers found under the tougher exterior feathers, acts as a poroelastic surface. These features are visualized in Fig. 1.2 using images of barn owl wing specimens. Each of these features contributes towards making the owl flight silent. The owl hush kit refers to these three features of the owl [20].

Figure 1.2: The owl hush kit: unique feather adaptations that enable the owl to fly silently. Top: barn owl wing specimen. Bottom: Photographs through a microscope of (a) leading edge comb, (b) downy coat on flight feathers, and (c) trailing edge fringe. Images (b) and (c) are from Refs. [1, 2].
Owl-inspired LE and TE serrations have been extensively studied [21, 22, 23, 24, 25, 20], even on full-scale, field tests [26]. In this thesis, I will focus instead on the third owl feather feature in the list above – the downy coat. Geyer et al. [27] related the downy coat to the porosity of a wing. By doing a series of experiments that used airfoils made out of different porosity materials, they found that at the frequencies less than about 10 kHz porous airfoils were able to attenuate the trailing edge broadband noise by 10 dB and more. However, the aerodynamic performance decreased as the resistivity of the porous airfoils increased.

Jaworski and Peake [28, 29] analyzed the trailing-edge condition and found that the fifth-power dependency of the radiated acoustics of a trailing edge was weakened by both the porosity and flexibility. They however, did not look into the noise reduction mechanisms behind the fine hairs of the downy coat.

Clark et al. [2] found from microscope observations of owl flight feathers that hairs from owl feathers first rise vertically up and then plateau out, forming a canopy-like structure [2]. Such “canopy” structures are also found in forests. Fluid flow in such plant canopies has been investigated elsewhere [30]. Clark et al. [2] found that the owl feathers had an open area ratio of about 70% and suspend about 0.5 mm above the feather substrate. Based off of these microscope observations, they performed wall-jet wind tunnel experiments to examine the aeroacoustic effects of artificial canopies designed with the same open area ratio as the hairs from the owl feathers. They designed the canopies using a large number of parallel fibers made from the material used for wedding veils. These fibers were oriented in the flow direction and located just above the flow surface. The canopies were found to reduce the surface pressure fluctuations underneath them by as much as 30 dB as well as attenuate broadband roughness noise.

Their exciting discovery motivated them to create trailing edge noise reduction designs that applied the features of the owl canopy in a way that could be practically implemented in the industry field. Clark et al. [3] did this by attaching small structures at the trailing edge, which they called “finlets”. Figure 1.3 shows schematics of two finlet designs used in these experiments; Figure 1.3 (a) and (b) are the fence and rail configurations, respectively. Twenty different configurations were tested in the experiments by changing the height, spacing, thickness, and extension of the fences and rails. Compared to the unmodified (baseline) airfoil, these configurations were found to be effective at
reducing the broadband trailing edge noise by up to 10 dB [3]. However, the experiments alone are unable to identify the mechanisms behind the noise reduction. The measurements have a finite spatial and temporal resolution which limits the source diagnostics that can be performed in order to explain the observed noise reductions.

![Figure 1.3: Schematics of two finlet designs from Ref. [3].](image)

Based on the results of the different configurations tested, the finlets are believed to: (a) reduce spanwise coherence in the airfoil boundary layers, and (b) lift the energy containing turbulence eddies away from the surface and the airfoil trailing edge. These are the working hypotheses behind the observed noise reduction. High-fidelity aeroacoustics simulations can assist in understanding the mechanisms behind the observed noise reduction with the finlets and help bridge the gap between experimental and theoretical knowledge.

This thesis focuses on high-fidelity computational fluid dynamics simulations of an airfoil with fences applied to the trailing edge with the objective of supplementing the experimental results of Ref. [3]. We perform compressible large eddy simulations to shed light on the fluid dynamics phenomena that lead to noise reduction in these designs.

The remainder of the thesis is organized as follows. The second chapter describes the numerical methodology and the gridding techniques for modeling the finlet designs. In the same chapter, the computational fluid dynamics solver and the integral method for far field noise prediction
(Fowcs Williams-Hawkings or FW-H equation solver) are also explained. A verification study is carried out for the FW-H solver wherein comparisons with FDL3DI simulations of point sources in uniform meanflow are performed. The potential mechanisms of noise reduction that are investigated in later chapters are listed. The computational meshes used for the baseline and fence simulations are also described.

In the third chapter, results from two sets of wall-resolved large eddy simulations are presented: (a) NACA 0012 baseline geometry, and (b) baseline NACA 0012 airfoil with one of the fence designs of Ref. [2, 3]. The baseline and fence simulation is compared for three different flow conditions, as summarized in Table 1.1. These flow conditions are chosen to be similar to the experiments of Ref. [2, 3], with the exception of the Reynolds number, which is lower in the simulations. In simulations 1 and 2, the fences are applied to only the pressure side of the airfoil. The only difference between the first and second simulation is in the type of boundary layer transition: simulation 1 uses natural transition and simulation 2 uses forced transition. Between the second and third simulation, more than one parameter is changed and therefore, any conclusions for how one parameter effects the noise reduction can not be made. However, by changing more than one parameter, we can investigate whether a general noise reduction mechanism is present with the fences applied that is independent of the simulation parameters.

<table>
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<td>3°</td>
<td>Natural transition</td>
<td>Pressure side</td>
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</tbody>
</table>

It should be noted that the experiments in Ref. [2, 3] were conducted at much higher Reynolds numbers ($3 \times 10^6$) and with a different baseline airfoil (DU96-W-180); the intent here is to focus on the physical phenomena behind noise reduction rather than perform direct validation (one-to-one comparison) with experimental data. The low-$Re_c$ simulations presented in this thesis serves as a first step towards the ultimate goal of simulating the DU96-W-180 airfoil at higher $Re_c(\sim 10^6)$ to be more representative of the experiments of Ref. [2, 3]. The computational cost of resolving the turbu-
lence spectrum at the Reynolds number in the experiment is very high. Despite the differences in the airfoil geometry and $Re_c$ between experiments and simulations, qualitative comparisons between the predicted and measured far field noise and surface pressure reductions from baseline to finlet designs are drawn in this thesis. Numerical results for the baseline geometry are also compared against XFOIL (a panel method code coupled with boundary integral methods), as well as against existing experimental data [31].

The fourth and the final chapter presents some conclusions drawn from the study and lays a path forward for future research.
CHAPTER 2. NUMERICAL METHODOLOGY AND GRIDDING TECHNIQUES

In this chapter, the numerical methodology and the gridding techniques for modeling the finlet designs will be described. The computational fluid dynamics (CFD) solver and the integral method for far field noise prediction (Ffowcs Williams-Hawkings or FW-H equation solver) are also explained. A verification study is carried out for the FW-H solver wherein comparisons with FDL3DI simulations of point sources in uniform meanflow are performed. The potential mechanisms of noise reduction that are investigated in the next chapter is listed. The computational meshes used for the baseline and fence simulations are also described.

2.1 Computational Fluid Dynamics Solver Methodology

The compressible Navier-Stokes solver, FDL3DI [32], is used for the fluid flow simulations. FDL3DI solves the full unfiltered compressible Navier-Stokes equations on curvilinear meshes. The governing fluid flow equations (solved by FDL3DI), after performing a time-invariant curvilinear coordinate transform \((x, y, z) \rightarrow (\xi, \eta, \zeta)\), are written in a strong conservation form as

\[
\frac{\partial}{\partial t} \left( \mathbf{U} \right) + \frac{\partial \hat{F}_I}{\partial \xi} + \frac{\partial \hat{G}_I}{\partial \eta} + \frac{\partial \hat{H}_I}{\partial \zeta} = \frac{1}{Re} \left[ \frac{\partial \hat{F}_{v}}{\partial \xi} + \frac{\partial \hat{G}_{v}}{\partial \eta} + \frac{\partial \hat{H}_{v}}{\partial \zeta} \right],
\]

where \(J\) is the Jacobian of the coordinate transformation, \(\mathbf{U} = \{\rho, \rho u, \rho v, \rho w, \rho E\}\); the expressions for inviscid flux terms, \(\hat{F}_I, \hat{G}_I, \hat{H}_I\) and viscous flux terms, \(\hat{F}_{v}, \hat{G}_{v}, \hat{H}_{v}\) are provided in Ref. [32].

In our simulations, large eddy simulations (LES) mode is used. However, unlike the traditional LES approach, no additional sub-grid stress terms are added to the governing equations to model the dissipation of the small scale eddies. Instead, to dissipate the energy from the eddies not resolved by the grid, high-order (up to 10th order) low-pass spatial filters are applied to the conserved dependent variables after each timestep or subiteration. This procedure is referred to as Implicit LES. To discretize the spatial domain, up to sixth-order accurate compact finite difference schemes
are used. Further details about the filtering and compact finite difference schemes are given in appendix A. Time integration is performed implicitly using the second-order accurate Beam Warming scheme [33, 34]. Although not used in the simulations in this thesis, the code is set up to work with complex geometries by piecing together the fluid domain with multi-block overset (Chimera) meshes. The overset meshes communicate with high order interpolation stencils which allows the solver to keep its spectral-like accuracy. The spatial domain is decomposed and solved in parallel using the message passing interface (MPI) library. The structured meshes are divided into multiple overlapping sub-blocks that each have a 5-point overlap with one another. The 5-point overlap allows the meshes to communicate with each other via high-order (10th) interpolation. BELLERO is used to compute high-order accurate interpolation weights among the sub-blocks. Multiple OpenMP threads are used on multi-core machines to further enhance the parallel performance of the software.

2.2 Noise Prediction Solver Methodology

This section explains how the data from the CFD solver is used to predict the far field noise radiation. The noise prediction solver used is also verified.

2.2.1 Near and Far field Noise Radiation

A two-step approach is used for noise prediction. Fluid flow simulations are first carried out using the CFD solver FDL3DI and an integral method (acoustic analogy) is used with the CFD data to compute the radiated noise in the far field. The unsteady pressure data on the airfoil surface from the simulation is also used to find the surface pressure spectra. Surface pressure spectra is representative of surface noise as well as trailing edge noise. The numerical approach used here has been previously validated by Ref. [20] and utilized to assess noise reduction ability of leading edge serrations.

Far field sound propagation is performed using the Ffowcs Williams-Hawkings (FW-H) acoustic analogy [35]. By neglecting volume sources (non-negligible only at very high flow speeds), the
following integral equation is obtained for far field acoustic pressure, \( p' \) at location \( x \) and time \( t \):

\[
p'(x, t) = \frac{1}{4\pi|1 - Mr| |x|} \left[ \frac{\partial}{\partial t} \int \rho_0 u_i n_i + \rho'(u_i - U_i) n_i \, d\Sigma + \frac{x_i}{c|x|} \frac{\partial}{\partial t} \int [p'n_i + \rho u_i (u_j - U_j) n_j] \, d\Sigma \right].
\]  \((2.2)\)

Solving Eq. 2.2 requires integrating over a surface \( \Sigma \) that encloses all sound sources. In the above, \( n_i \) is normal to the surface \( \Sigma \), \( p' \) and \( \rho' \) are pressure and density fluctuations, \( \rho_0 \) is mean density, \( u'_i \) is perturbation flow velocity and \( U_i \) is the velocity of the surface \( \Sigma \). The source is at the origin, and \( x \) denotes the observer location. As illustrated in Fig. 2.1, we choose a “porous” surface around the airfoil defined by one of the gridlines (\( \xi = \text{constant} > 1; \xi = 1 \) is the airfoil surface) of the grid block. A frequency domain formulation [36] of the FW-H equation is used. The following sections will show verification results for canonical problems – radiation of sound field from point monopole, dipole, and quadrupole sources in a moving medium.

![Figure 2.1: “Porous” surface around the airfoil used by the FW-H solver.](image)

2.2.2 FW-H Validation: Point Source in a Moving Medium

The FW-H solver is verified against computational results for point sources in a moving medium. Single-frequency, harmonic sources are considered. The FDL3DI solver is used to obtain acoustic radiation field for point monopole, dipole, and quadrupole sources in a moving medium. The acoustic sources are specified using a vector source term \( S = [0, 0, 0, 0, S_5] \), which is added to the governing equations. The sources are centered at the origin and defined for monopole, dipole and quadrupole
respectively as:

\[ S_5(x, y, t) = \exp\left(-\ln(2) \frac{(x)^2 + (y)^2}{b^2}\right) \sin(\omega t) f(t), \]

\[ S_5(x, y, t) = \exp\left(-\ln(2) \frac{(x - x_c)^2 + (y)^2}{b^2}\right) \sin(\omega t) f(t) + \]
\[ \exp\left(-\ln(2) \frac{(x + x_c)^2 + (y)^2}{b^2}\right) \sin(\omega t + \pi) f(t), \]

\[ S_5(x, y, t) = \exp\left(-\ln(2) \frac{(x - x_c)^2 + (y - y_c)^2}{b^2}\right) \sin(\omega t) f(t) + \]
\[ \exp\left(-\ln(2) \frac{(x - x_c)^2 + (y + y_c)^2}{b^2}\right) \sin(\omega t + \pi) f(t) + \]
\[ \exp\left(-\ln(2) \frac{(x + x_c)^2 + (y + y_c)^2}{b^2}\right) \sin(\omega t) f(t) + \]
\[ \exp\left(-\ln(2) \frac{(x + x_c)^2 + (y - y_c)^2}{b^2}\right) \sin(\omega t + \pi) f(t), \]

where,

\[ f(t) = \min\left(1.0, \left(\frac{t}{t_0}\right)^3\right), \quad (2.3) \]

and \(x_c\) and \(y_c\) are the center of the poles, \(\omega\) is the angular frequency of the harmonic source, \(b\) is the scaling parameter, \(t_0\) is the ramp parameter, and \(f(t)\) is the function used to ramp the source at the beginning of the simulation. For the simulations, the following parameters are used: \(x_c = y_c = 0.01, b = 0.2, \omega = 10\pi, t_0 = 3\). The distance between the poles (2\(x_c\)) is equal to 1% of the acoustic wavelength to ensure source compactness.

The mesh used for the simulation, shown in Fig. 2.2, is a structured 2-D grid extending from \(-100 \leq \{x, y\} \leq 100\). The red inner circle in the figure is the “porous” surface that was used to predict the far field power spectral density (PSD) located at the blue outer circle. The radius of the “porous” surface is made large enough that the predicted value no longer changes as the radius is increased. The far field location is chosen so that it is at least 10 acoustic wavelengths away from the source. The domain is chosen large enough so that the outer boundaries do not influence the pressure fluctuations at the far field location. At the center of the sources the minimum mesh spacing is
\[ \Delta x = \Delta y = 0.01. \] The mesh is stretched with a hyperbolic tangent distribution that has a maximum mesh spacing rate of change of 0.6% at the outer boundaries. This slow growth rate is chosen to minimize any numerical dissipation at the acoustic far field location.

Figure 2.2: Structured grid used for validation of the FW-H solver; every 4th grid line is shown for clarity. The red inner circle is the FW-H integration surface; noise is predicted at the observer locations shown with the blue outer circle.

Since the FDL3DI solution for this case is 2-D due to the 2-D grid used, and the FW-H code is three dimensional (solves the convected wave equation in 3-D), the following approach is used to essentially predict noise from a source that is effectively infinite along the axis \((z)\) not simulated. For any given \((x, y)\) source location and observer azimuth angle, noise is predicted at additional observer locations in the \(z\) direction and summed up in the frequency domain until additional observer locations no longer change the solution. This process is explained with the schematic in Fig. 2.3. Note that complex pressures are added during this summation to account for interference effects. This approach is equivalent to replicating sources along the \(z\) direction (simulating an infinitely long line source) and predicting noise at the original observer locations. The 3-D FW-H solution processed in this manner can then be directly compared with the 2-D FDL3DI solution.

Figure 2.4 shows the instantaneous dilatation fields for the monopole, dipole and quadrupole sources. The comparison of the predicted far field PSD from the FW-H solver to the actual PSD
values in the far field is shown in Fig. 2.5. All polar plots are shown at the frequency of the harmonic source. From the plots, we can see excellent agreement of the predicted and actual PSD values for all three sources except near the azimuth angles of 90° and 270° for the dipole and quadrupole sources. However, the sound at these observer angles are theoretical nulls for the dipole and quadrupole, and it is difficult to capture these null points with discrete computations. From the plots, we can also see that the convective amplitude is well captured by the FW-H solver.

2.3 Noise Reduction Mechanisms

There are currently two working hypotheses behind the observed noise reduction with the fences applied to the trailing edge. The fences are believed to: (a) reduce spanwise coherence in the airfoil boundary layers, and (b) lift the energy containing turbulence eddies away from the scattering bodies - the airfoil surface and the trailing edge.

The first hypothesis is important because Amiet’s theory [37] shows that the spanwise coherence length is directly proportional to the noise produced by the trailing edge of an airfoil. Therefore, calculating the spanwise coherence is useful for understanding how the fences can reduce trailing
edge noise. Spatial coherence between two points \( x \) and \( y \) is defined as

\[
\gamma^2_{xy}(\omega) = \frac{|S_{xy}(\omega)|^2}{S_{xx}(\omega)S_{yy}(\omega)},
\]  

(2.4)

where \( S_{xy}(\omega) \) is the cross-spectral density, \( S_{xx}(\omega) \) is the PSD evaluated at location \( x \) on the airfoil and \( S_{yy}(\omega) \) is the PSD evaluated at another point \( y \) on the airfoil.

The second hypothesis can be analyzed by investigating the turbulence kinetic energy (TKE) in the fence region. TKE is defined as

\[
TKE = 0.5(u_{rms}^2 + v_{rms}^2 + w_{rms}^2),
\]  

(2.5)

where \( u_{rms}, v_{rms}, w_{rms} \) are the root mean square of the perturbation velocities.

### 2.4 Computational Mesh for Fence Simulations

This section explains how the computational mesh is set up for the geometry of the baseline and fences. The particular mesh and grid metrics for each simulation are covered in chapter 3.

#### 2.4.1 Baseline Airfoil Mesh

The span length of the airfoil model in the simulations is 5.85% of the airfoil chord. For both the baseline and fence simulations, a very small computational non-dimensional timestep of \( \Delta \tau = \Delta t U_\infty / c = 0.00004 \) is used in order to provide sufficient temporal resolution of the fine-scale features. The choice of the first cell height gives an average \( y^+ < 1 \) for the baseline geometry. The O-grid topology is used for the mesh. Normal extrusion from the airfoil surfaces with a high degree of refinement near the walls is used to obtain the near-wall mesh.

Figure 2.6 shows close-up, cross-sectional views of the baseline grid. The outer computational domain boundary is kept approximately 110 chords away from the airfoil, with the grid deliberately stretched excessively away from the airfoil in order to completely dissipate all fluctuations before they reach the outer boundary. Freestream conditions are prescribed at the outer boundary. Spatially-periodic conditions were prescribed in both the spanwise and azimuthal directions using five-plane overlaps.
For the baseline grid that is compared to the one-sided fence, the grid is highly refined on the suction side of the airfoil to resolve the turbulent boundary layer on that surface. The grid is left relatively coarse on the pressure side to reduce computation time; for the prescribed conditions, the pressure-side boundary layer stays laminar and attached all the way to the TE. For the baseline grid that is being compared to the two-sided fence, the grid is highly refined on both the pressure and suction sides to resolve the turbulent boundary layer on both surfaces. The O-grid distributions along a z=constant plane in the baseline grids used in this study are similar to the grid used in Ref. [38], which was a LES of a pitching airfoil at $Re_c = 5 \times 10^5$. For those simulations, a grid resolution study was done and the grid resolution chosen here was found to be grid independent. [39].

2.4.2 Geometry and Mesh of the Fences

Ideally, one would choose to model the fences exactly as they were used in the experiments. However, it is nearly impossible to replicate an experimental setup in a simulation due to meshing constraints and solver stability concerns. Therefore, alternate configurations for which “practical” meshes (i.e., a mesh with manageable grid count and cell aspect ratios, skewness, etc. that do not render the numerical algorithm unstable) can be obtained, are attempted with the objective of identifying qualitative trends in noise reduction with the fences and rails.

Figures 2.7 and 2.8 show how the geometry of the fences is modeled in the simulations. The fences are made from the baseline grid by using NASA’s PEGASUS software to perform hole cutting (blanking out mesh nodes that represents the interior of the solid fence walls) on the regions occupied by the fences (defined by specifying ranges $\xi_1 - \xi_2$, $\eta_1 - \eta_2$, and $\zeta_1 - \zeta_2$) from the baseline grid; no-slip wall boundary condition is applied to the boundaries of the fences. Other than the holes introduced in the fence simulations, the grids between the baseline and fence cases are identical. The dimensions of the fences are selected to correspond to configuration # 13 (0.5 mm thickness, 4 mm height, 6 mm spacing, 10 mm extension, front of fences starting at $x/c = 0.875$) in the experiments of Ref. [3]. Since our simulations are performed at a $Re_c$ lower than that in the experiments, the height and spacing of the fences are scaled up by the ratio of the boundary layer displacement thicknesses ($\delta^*$) expected at the two different $Re_c$ ($\delta^*$ estimates are obtained using XFOIL). The effectiveness of
the fences in mitigating noise is expected to remain the same if the ratio of fence height to boundary layer displacement thickness is maintained.

The simulated geometry in the fence simulation is further different from the experiments in that the simulated height of the fences do not vary with downstream distance. An illustration of this is shown in Fig. 2.9. In the experiments, the fences starts nearly flush with the airfoil surface (zero fence height) at $x/c = 0.875$; the height of the fences increase linearly towards the TE. The simulated fences are of nearly constant height throughout. This poses a potential problem of introducing additional noise sources in the fence simulations - the sharp leading edge of the fence that is orthogonal to the incoming turbulent flow is efficient at scattering sound. The simulated geometry also does not include the support structures or the substrate used in the experiment, which are not expected to play a role in the observed noise reduction. As shown in Fig. 2.10, the fence grid is repeated in the span direction to simulate a wing with a span of 5.85% chord.

### 2.5 Computational Mesh for Finlet Rail Simulations

Preliminary attempts were made to model the finlet rails. The geometry and the mesh used to model the finlet rail is shown in Fig. 2.11. Using this meshing scheme resulted in the working simulation shown in Fig. 2.12. However, this effort was stopped after having difficulty properly load balancing the grids due to the irregular holes cut around the finlet rail. Irregular holes are troublesome with the FDL3DI solver because it requires an 11 point stencil in all three directions. Due to this, the finlet fence geometry was simulated instead and will be investigated in detail in this thesis. The finlet rail geometry can be revisited in the future. One potential solution to the load imbalance problem is to modify the solver to have it use lower order schemes (that require smaller stencils) in the sub-blocks that do not have an 11 point stencil due to the irregular hole cutting performed.
Figure 2.4: Dilatation fields for the monopole, dipole and quadrupole sources that are used for verification of the FW-H solver.
Figure 2.5: Directivity comparisons of PSD for a point source radiating in a moving medium between the FDL3DI predicted value (red lines) in the far field and the FW-H predicted value (purple lines). All polar plots are shown at the frequency of the harmonic source.
Figure 2.6: Baseline mesh using the O-grid topology. The trailing edge is rounded off and the mesh near the TE is shown in (b). Every 4th grid line is shown for clarity.

Figure 2.7: Cross sectional views of the computational mesh used to simulate the geometry of the fences. Every 4th grid line is shown for clarity.
Figure 2.8: Top views of the baseline and fence meshes to show how the geometry of the fences are modeled.

Figure 2.9: Comparisons of the fence leading edge used in the a) experiment and b) the fence simulation.
Figure 2.10: 3-D representation of the repeated fence geometry that will be simulated.

Figure 2.11: Geometry and meshing schemes used to model finlet rail geometry.
Figure 2.12: Inviscid flow over the finlet rail.
CHAPTER 3. RESULTS OF NUMERICAL INVESTIGATIONS AND ANALYSIS

In this chapter, results from two sets of wall-resolved large eddy simulations are presented: (a) NACA 0012 airfoil with no add-ons (baseline), and (b) NACA 0012 airfoil with one of the fence designs of Ref. [2, 3]. The baseline and fence simulation are compared for three different flow conditions summarized in Table 1.1. These flow conditions are selected to be similar to the experiments of Ref. [2, 3], with the exception of Reynolds number, which is much lower in the simulations.

In simulations 1 and 2, the fences are added only on the pressure side of the airfoil. The only difference between the first and second simulation is in the type of boundary layer transition: simulation 1 uses natural transition while simulation 2 uses forced transition. Between simulations 2 and 3, more than one parameter is changed and therefore, any conclusions for how one parameter affects noise can not be made. However, by changing more than one parameter, we can investigate whether a general noise reduction mechanism is present with the fences applied that is independent of the simulation parameters. Also these simulations are computationally very expensive, and it is not possible to perform a parametric variation/sensitivity study. Each simulation takes approximately two million core hours.

It should be noted that the experiments in Ref. [2, 3] were conducted at a much higher Reynolds number ($3 \times 10^6$) and with a different airfoil (DU96-W-180); the intent here is to focus on the physical phenomena behind noise reduction rather than perform a direct comparison with the experimental data. The low-$Re_c$ simulations presented in this thesis serve as a first step towards the ultimate goal of simulating the DU96-W-180 airfoil at a higher $Re_c$ ($\sim 10^6$) to be more representative of the experiments of Ref. [2, 3]. Despite the differences in the airfoil geometry and $Re_c$ between experiments and simulations, qualitative comparisons between the predicted and measured far field noise and surface pressure reductions from baseline to fence designs are drawn in this thesis. Numerical results
The accuracy of a simulation is best accessed by comparing with experimental data. However, since that experimental data does not exist at the flow conditions similar to the simulation, we have chosen to compare the predicted aerodynamic performance with the predictions from XFOIL. Such code-to-code comparison cannot be considered as validation, however it does provide confidence that the simulations are at least predicting a reasonable result. Therefore, in the three simulations presented here, the baseline simulation is compared to XFOIL wherever experimental data is not available. Since the grids of the baseline and fence geometries are identical (except for the holes cut out to model fences), we can compare the baseline simulation to the fence simulation to get qualitative trends of the effects of fences on acoustics. Having identical grids between baseline and fence geometries is helpful in that the trend prediction is not influenced by discretization errors.

### 3.1 NACA 0012 Re = 300,000, AOA = 3°, M=0.2, Fence pressure side, Natural Transition

In simulation #1, we simulate the NACA 0012 cross section with and without fences applied near the trailing edge of the suction side of the airfoil. We compare the aerodynamic performance and acoustic effects of using the fences.

#### 3.1.1 Mesh Setup

Figures 3.1 and 3.2 show close-up, cross-sectional views of the grids for the baseline and one-sided fence geometries, respectively. Some lines are omitted for clarity. The grid is refined around the fence region, particularly at the leading edge of the fences. Tables 3.1 and 3.2 provide the corresponding grid metrics from the simulations. The $x^+$, $y^+$, and $z^+$ average values reported for the baseline grid are obtained by averaging over both the span and chordwise directions. The $x^+$, $y^+$, and $z^+$ values reported for the fence grid are obtained in the vicinity of a fence and are only averaged over the chordwise direction of the airfoil.
Figure 3.1: O-grid topology of the baseline mesh used in simulation #1. The trailing edge is rounded off and the mesh near the TE is shown in (b). Every 4th grid line is shown for clarity.

Table 3.1: Baseline grid metrics used for the $Re_c = 3 \times 10^5$, $AOA = 3^\circ$, natural transition simulation.

<table>
<thead>
<tr>
<th>$N_\zeta$</th>
<th>$N_\eta$</th>
<th>$N_\zeta$</th>
<th>avg $y^+$</th>
<th>avg $x^+$</th>
<th>avg $z^+$</th>
<th>max $y^+$</th>
<th>max $x^+$</th>
<th>max $z^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>410</td>
<td>1476</td>
<td>245</td>
<td>0.135</td>
<td>5.8</td>
<td>2.64</td>
<td>0.3</td>
<td>48.34</td>
<td>8.35</td>
</tr>
</tbody>
</table>

3.1.2 Aerodynamic Performance Assessment

Before accessing the acoustics, it is important to ensure that the fences do not degrade the aerodynamic performance of the airfoil. For aerodynamic performance assessment, the FDL3DI simulation results are averaged in time and along the span. Figure 3.3 (a) compares the predicted time- and span-averaged pressure coefficient ($C_p$) distributions over the airfoil with XFOIL predictions. The transition location is predicted at approximately the same chordwise position on the top surface by both methods, and the suction peak and the overall spatial distribution of $C_p$ is also in good agreement. Figure 3.3 (b) compares the FDL3DI predicted time- and span-averaged skin friction co-

Table 3.2: Metrics of the grid used for the $Re_c = 3 \times 10^5$, $AOA = 3^\circ$, natural transition fence simulations.

<table>
<thead>
<tr>
<th>$N_\zeta$</th>
<th>$N_\eta$</th>
<th>$N_\zeta$</th>
<th>avg $y^+$</th>
<th>avg $x^+$</th>
<th>avg $z^+$</th>
<th>max $y^+$</th>
<th>max $x^+$</th>
<th>max $z^+$</th>
</tr>
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<tbody>
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<td>2.95</td>
<td>48.31</td>
<td>3.27</td>
</tr>
</tbody>
</table>
Figure 3.2: Cross sectional views of the computational mesh that is used to simulate the one-sided geometry of the fences at $Re_c = 3 \times 10^5$, $AOA = 3^\circ$. Every 4th grid line is shown for clarity.

Efficient ($C_f$) distributions with the XFOIL prediction. At this $Re_c$, the boundary layer experiences a laminar separation and transition occurs in the separated shear layer. FDL3DI predicts a “two-stage” transition where $C_f$ increases rapidly after the laminar-separation induced transition, but plateaus, and then rises again to its peak before gradually decaying towards the trailing edge. This two-stage transition is not captured by XFOIL, which uses a simple linear stability criterion ($e^N$). However, the agreement between FDL3DI and XFOIL is quite good in the turbulent boundary layer region once the transition to turbulence is complete.

Aerodynamic differences between the baseline and fence geometries are compared in Fig. 3.4. The $C_p$ distribution is very similar between the two geometries. The predicted $C_f$ distributions are the same for the two geometries wherever the boundary layer is laminar. The boundary layer transition location is slightly altered due to the fences, likely because of subtle changes in the adverse pressure gradient. The transition point in the baseline simulation is at $x/c \approx 0.72$ while in the fence simulation it is located at $x/c \approx 0.78$. This variation in transition location leads to differences in the turbulent boundary layers near the trailing edge between the baseline and fence geometries, which is not desirable for evaluating effectiveness of fences in reducing trailing edge noise. The boundary layer is therefore tripped using a numerical “trip wire” for simulations 2 and 3 to avoid this problem.
With the results in Fig. 3.4, it is concluded that the addition of fences does not adversely affect the aerodynamic performance of the airfoil. Therefore, we now access the acoustics of the fences.

### 3.1.3 Surface Pressure Spectra

Surface pressure spectra is representative of surface noise as well as trailing edge noise. Unsteady surface pressure spectra are computed using Welch’s method [40]. The spectra are computed at the trailing edge ($x/c = 1$) and at the leading edge of the fences ($x/c = 0.875$). These are further averaged over the span of the model, and the results are plotted as sound pressure level (SPL) in Fig. 3.6. The SPLs for the fence geometry are higher than the baseline at the leading edge of the fences, over the entire frequency range (see Fig. 3.6 (a)). This undesirable increase in unsteady surface pressure is likely due to the sharp leading edge of the fences that is orthogonal to the incoming turbulence in the boundary layer, which is efficient at scattering sound (leading edge noise). An illustration of this is shown in Fig. 3.5. This undesirable noise source is present only in the simulations as the geometry of the fences in the experiments had a leading edge which is nearly parallel to the flow (see Fig. 2.9).

At the trailing edge however, a slight reduction in the SPL is observed at high frequencies (> 1500 Hz) with the fences. The surface pressure SPL reductions observed in the simulations with the fences agree qualitatively with the far field SPL reductions seen in the experiments from Ref. [3].
Figure 3.4: Time- and span-averaged $C_p$ and $C_f$ comparisons between baseline and fence simulations at natural transition, $Re_c = 3 \times 10^5$. Results shown are predictions from FDL3DI.

Qualitative agreement refers to the fact that the observed reductions in experiments were also at high frequencies; little to no change was observed at low frequencies.

To get a clearer picture of how the overall surface pressure spectra is changing along the chord through the fences, the overall sound pressure level (OASPL) defined as

$$OASPL = 10 \log_{10} \sum_{i=m}^{N} \left( \frac{p_{rms}}{p_{ref}} \right)^2$$

is computed. In Eq. 3.1, $m$ and $N$ correspond to the indices of the minimum and maximum frequency bands resolved by the time signal. However, $m$ and $N$ can be selected to obtain an integrated sound pressure level over any desired frequency band. While a precise terminology for such an integrated quantity would be ‘band pressure level’, we use OASPL here since the frequency band
is reasonably large. We integrate over the frequency range $2.5 \ kHz \leq f \leq 5 \ kHz$. The results of the integrated sound pressure level are plotted in Fig. 3.7. The fence geometry has slightly lower integrated surface pressure level from the trailing edge up to $x/c \approx 0.9$. Near the leading edge of the fences however, the integrated surface pressure levels of the fence geometry are greater. The largest OASPL reduction is about 1.8 dB.

3.1.4 Noise Reduction Mechanism

The fences are believed to reduce spanwise coherence in the airfoil boundary layers, thereby reducing far field noise radiation. Spanwise coherence along the trailing edge is plotted for the baseline and fence cases in Fig. 3.8. There does not appear to be a significant reduction in the spatial coherence with the fences.

3.2 NACA 0012 Re = 300,000, AOA = 3 °, M=0.2, Fence pressure side, Forced Transition

In simulation #1, the boundary layer transitions naturally from laminar to turbulent flow. In simulation #2, we use the same flow conditions and fence configuration but trip the boundary layer so that we have a larger region of turbulent flow that better represents the higher $Re_c$ flows in the experiments from Clark et al. [3]. A similar I-blanking method as used to model the fences, is used to
Figure 3.7: Compressible LES results for NACA 0012 cross section at natural transition, $Re_c = 3 \times 10^5$ showing OASPL throughout the fence region.

trip the boundary layer at $x/c = 0.05$. As shown in Fig. 3.9, the “trip wire” is defined by hole cutting (blanking out) the regions occupied by the trip wire (defined by specifying ranges $ξ_1 - ξ_2$, $η_1 - η_2$, and $ζ_1 - ζ_2$) from the baseline grid; no-slip wall boundary condition is applied to the “trip wire” boundaries.

The effect of tripping the boundary layer is shown with iso-surfaces of Q contours ($Q = 50$) shown in Fig. 3.10. Q contours are a three dimensional vortex criterion that is used to detect and visualize vortices. The scalar is defined by the equation below,

$$Q = \frac{1}{2} [||Ω||^2 - ||S||^2],$$

where $Ω$ is the vorticity tensor and $S$ is the rate of strain tensor. From the iso-surfaces of the Q contours, we see the large differences between the boundary layer flow in simulations #1 and 2. In simulation #1, there are spanwise coherent vortex structures well downstream; in simulation #2 these vortex structures are not present.
Figure 3.8: Compressible LES results for NACA 0012 cross section at natural transition, $Re_c = 3 \times 10^5$ showing a) spanwise coherence for baseline and b) spanwise coherence for fences.

3.2.1 Mesh Setup

The baseline and one-sided fence grid for simulation #2 was the same as the grids used for simulation #1 (see Figs. 3.1 and 3.2). However, since the boundary layer was tripped, the grid metrics were slightly different than in the natural transition case. These are listed in Tables 3.3 and 3.4 for the baseline and fence simulation, respectively. The $x^+$, $y^+$, and $z^+$ average values reported for the baseline grid are obtained by averaging over both the span and chordwise directions. The $x^+$, $y^+$, and $z^+$ values reported for the fence grid are obtained in the vicinity of a fence and are only averaged over the chordwise direction of the airfoil.

Table 3.3: Baseline grid metrics used for the $Re_c = 3 \times 10^5$, $AOA = 3^\circ$, forced transition simulation.

<table>
<thead>
<tr>
<th>$N_{\xi}$</th>
<th>$N_{\eta}$</th>
<th>$N_{\zeta}$</th>
<th>avg $y^+$</th>
<th>avg $x^+$</th>
<th>avg $z^+$</th>
<th>max $y^+$</th>
<th>max $x^+$</th>
<th>max $z^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>410</td>
<td>1476</td>
<td>245</td>
<td>0.142</td>
<td>7.85</td>
<td>4.88</td>
<td>0.3</td>
<td>52.07</td>
<td>13.77</td>
</tr>
</tbody>
</table>

Table 3.4: Metrics of the grid used for the $Re_c = 3 \times 10^5$, $AOA = 3^\circ$, forced transition fence simulations.

<table>
<thead>
<tr>
<th>$N_{\xi}$</th>
<th>$N_{\eta}$</th>
<th>$N_{\zeta}$</th>
<th>avg $y^+$</th>
<th>avg $x^+$</th>
<th>avg $z^+$</th>
<th>max $y^+$</th>
<th>max $x^+$</th>
<th>max $z^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>410</td>
<td>1476</td>
<td>245</td>
<td>0.408</td>
<td>7.29</td>
<td>1.24</td>
<td>3.27</td>
<td>48.31</td>
<td>3.26</td>
</tr>
</tbody>
</table>
Figure 3.9: Points that are blanked out to act as “trip wires”. As with the fences, the nodes adjacent to the holes are defined as no slip boundaries.

### 3.2.2 Aerodynamic Performance Assessment

Figure 3.12 compares $C_p$ and $C_f$ distributions between FDL3DI and XFOIL predictions for the baseline airfoil; FDL3DI results are time and span averaged. XFOIL simulations were tripped at the $x/c$ location where a numerical trip was employed in the FDL3DI simulation. The $C_p$ distribution matches well over the entire airfoil surface between the two prediction methods, except for the “notch” in FDL3DI results due to the trip wire. Larger deviations are observed in the $C_f$ distributions, which is expected. The boundary layer in FDL3DI does not transition immediately behind the numerical trip wire. As shown in Fig. 3.11, the trip wire causes the boundary layer to separate (hence $C_f$ drops below zero) and then transition occurs in the separated shear layer which reattaches to the airfoil surface further downstream. However, once the transition is complete, the $C_f$ distributions predicted by both methods agree well.

Figure 3.13 compares the $C_p$ and $C_f$ distributions over the airfoil surface between the geometries of the baseline and fences. The $C_p$ distribution stays identical except near the leading edge of the fences where there is a slight bump in $C_p$ for the fence geometry because of the stagnation region created by the normal leading edge of the fences in the simulations. The $C_f$ distribution is also identical between the two geometries except in the fence region. There is a sharp jump in $C_f$ at the
leading edge of the fences, but the rest of the fence region shows reduced $C_f$ compared to baseline. This is indicative of reduced wall normal velocity gradient near the wall in the fence region. This is supported by boundary layer velocity profiles (shown later). The results in Fig. 3.13 show that the addition of the fence does not adversely affect the aerodynamic performance of the airfoil.

### 3.2.3 Surface Pressure Spectra

Pressure spectra are computed using the Welch method at the trailing edge ($x/c = 1$) and at the front of the fences ($x/c = 0.875$). The spectra are averaged along the span, and the averaged results are plotted in Fig. 3.14. Similar to simulation #1, the fence is found to have higher unsteady surface sound pressure than the baseline for all frequencies at the leading edge of the fences. As explained
earlier, this is hypothesized to be due to the sharp edge at the front of the fences that sit normal to the incident boundary layer turbulence in the FDL3DI simulations. At the trailing edge however, reduction in SPL is observed with the fence geometry compared to the baseline at all frequencies. The experiments [3] did not report surface sound pressure measurements, but reported far-field sound pressure spectra. Since the primary noise source in this problem is the unsteady surface pressure on the airfoil (primarily the trailing edge), we compare the two to check if the simulations can predict the design trend correctly. Figure 3.15 shows that the surface SPL reductions observed in the simulations with the fences agree qualitatively with the far field SPL reductions seen in the measurements.

As done for the previous simulation, we sum the surface SPL over the frequencies $f = 2.5$ kHz to $f = 5$ kHz to obtain the OASPL, which is plotted in Fig. 3.16 as a function of chordwise distance. It is again observed that the fence has lower integrated surface pressure levels from the trailing edge up to $x/c \approx 0.88$. Near the leading edge of the fences, the fences integrated surface pressure levels are greater. The largest OASPL reduction occurs at the trailing edge for a decrease of about 4.0 dB.

3.2.4 Far field Aeroacoustics

This section shows how the unsteady surface pressure spectra influences the far field acoustics. Figure 3.18 shows the dilatation ($|\nabla \cdot \mathbf{v}|$) field for the baseline and fence simulations. The dilatation

Figure 3.12: Time- and span-averaged $C_p$ and $C_f$ comparisons between baseline and fence simulations with forced transition, $Re_c = 3 \times 10^5$. 
Figure 3.13: Time- and span-averaged $C_p$ and $C_f$ comparisons between baseline and fence simulations with forced transition, $Re_c = 3 \times 10^5$.

Field is representative of the instantaneous acoustic field. To calculate the far field noise, the FW-H solver is used with the integration surface taken to be a $\xi = \text{constant}$ gridline enveloping the airfoil (see Fig. 3.17). Figure 3.19 shows the far field acoustics for the baseline and fence at an azimuth angle of 90°. Despite the reduction seen in unsteady surface pressure spectra, no significant difference is observed in the far field noise between the baseline and fences.

### 3.2.5 Noise Reduction Mechanism

In this section we perform noise source diagnostics to analyze the noise reduction mechanisms. Spanwise coherence ($\gamma^2$) plots for the baseline and fence geometries at the trailing edge are plotted in Fig. 3.20. As for the previous simulation, there does not appear to be a significant difference in spatial coherence between baseline and fence geometries.

Another hypothesis for noise reduction is that the fence lifts the energy containing turbulence eddies away from the surface, thereby increasing source separation from the scattering body and hence reduces efficiency of noise production. Figure 3.21 shows the time averaged normalized turbulent kinetic energy ($TKE/u^2$) at different $x/c$ locations. The plots are taken at a) upstream of the leading edge of the fences at $x/c = 0.832$, b) at the front face of the leading edge of the fences at $x/c = 0.873$, c) middle of the fences at $x/c = 0.94$, and d) near the trailing edge of the airfoil at $x/c = 0.994$. We can see that as the flow progresses toward the trailing edge, the TKE concentrates...
above the fences while the TKE of the baseline stays near the airfoil surface. The span-averaged TKE plots in Fig. 3.22 show this development more clearly. Note in these figures the wall normal distance, \( y \), is normalized by the fence height, \( H \). The eddies relocating above the fences appears to be the reason for the reduction in the surface pressure spectra of the fences compared to the baseline. It is not yet clear as to why this did not result in far field noise reduction as hypothesized, but a possible reason is that the extraneous noise source (due to the sharp leading edge of the fences being normal to the incident turbulence), negated any benefit from the fences lifting the energy containing turbulence eddies away from the surface.

Figure 3.23 shows the time- and span-averaged \( x \)-component of the velocity at the same \( x/c \) locations that the TKE was found. From the airfoil surface to the top of the fences (\( y/H = 0 \) to \( y/H = 1 \)), there is a lower velocity gradient with the fences than with the baseline. Above the fence height (\( y/H > 1 \)), the velocity abruptly changes while the velocity gradient of the baseline remains smooth. This sudden inflection in velocity profile is typical of canopy flows. The reduced gradient near the wall is responsible for the reduced wall skin friction seen earlier in Fig. 3.13.
3.3 NACA 0012 Re = 500,000, AOA = 0°, M=0.2, Fence both sides, Forced Transition

In simulation #3, the main change from simulation #2 is that we increase the Reynolds number to \( Re_c = 5 \times 10^5 \) and apply the fences to both sides of the airfoil. We do this to have a closer \( Re_c \) as the experiments from Clark et al. [3] and to observe the effect of having the fences applied to both sides of the airfoil as was done in the experiments [3]. The angle of attack was changed to 0° to ensure a turbulent boundary layer flow on both sides of the airfoil.

3.3.1 Mesh Setup

Figures 3.24 and 3.25 show close-up, cross-sectional views of the baseline and two-sided fence grids, respectively. Some lines are omitted for clarity. Table 3.5 and Table 3.6 provides the metrics of the grid used for the baseline and fence simulation, respectively. The \( x^+, y^+, \) and \( z^+ \) average values reported for the baseline grid are obtained by averaging over both the span and chordwise directions. The \( x^+, y^+, \) and \( z^+ \) values reported for the fence grid are obtained in the vicinity of a fence and are only averaged over the chordwise direction of the airfoil.

Table 3.5: Baseline grid metrics used for the \( Re_c = 5 \times 10^5, AOA = 0^\circ, \) forced transition simulation.

<table>
<thead>
<tr>
<th>( N_\xi )</th>
<th>( N_\eta )</th>
<th>( N_\zeta )</th>
<th>( \text{avg } y^+ )</th>
<th>( \text{avg } x^+ )</th>
<th>( \text{avg } z^+ )</th>
<th>( \text{max } y^+ )</th>
<th>( \text{max } x^+ )</th>
<th>( \text{max } z^+ )</th>
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<tbody>
<tr>
<td>410</td>
<td>1937</td>
<td>101</td>
<td>0.279</td>
<td>10.87</td>
<td>7.91</td>
<td>0.46</td>
<td>16.71</td>
<td>14.10</td>
</tr>
</tbody>
</table>
Figure 3.16: Compressible LES results for NACA 0012 cross section with forced transition, $Re_c = 3 \times 10^5$ showing a) OASPL throughout fence region and b) OASPL near trailing edge.

Table 3.6: Metrics of the grid used for the $Re_c = 5 \times 10^5$, AOA = 0°, forced transition fence simulations.

<table>
<thead>
<tr>
<th>$N_\xi$</th>
<th>$N_\eta$</th>
<th>$N_\zeta$</th>
<th>avg $y^+$</th>
<th>avg $x^+$</th>
<th>avg $z^+$</th>
<th>max $y^+$</th>
<th>max $x^+$</th>
<th>max $z^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>410</td>
<td>1937</td>
<td>101</td>
<td>0.521</td>
<td>17.93</td>
<td>11.60</td>
<td>1.64</td>
<td>91.16</td>
<td>34.33</td>
</tr>
</tbody>
</table>

3.3.2 Aerodynamic Performance Assessment

Figure 3.26 (a) shows the time- and span averaged pressure distribution for the baseline, XFOIL and the NACA 0012 $Re_c = 2.8 \times 10^6$ experiment from Ref. [31]. Note that both the experiment from Ref. [31] and XFOIL is tripped at $x/c = 0.05$ as is done in the FDL3DI simulation. The $C_p$ distribution matches well over the entire airfoil surface between the two prediction methods, except for the “notch” in FDL3DI results due to the trip wire. Figure 3.26 (b) shows the time- and span averaged $C_f$ distributions for the FDL3DI and XFOIL predictions. Larger deviations are observed in the $C_f$ distribution, which is expected. These deviations are similar to those observed in simulation #2, which are due to the numerical trip wire used in FDL3DI. However, once turbulence transition is complete, the $C_f$ distributions predicted by both methods agree well.

Figure 3.13 compares the $C_p$ and $C_f$ distributions over the airfoil surface between the baseline and fence geometries. The same differences among the baseline and fences in simulation #2 are observed here. Aside from the drop within the fence region, the $C_p$ and $C_f$ distributions are quite...
Figure 3.17: Integration surface used to predict the far field noise with the FW-H solver.

similar between the baseline and fence cases. The results in Fig. 3.27 show that the addition of the fences does not adversely affect the aerodynamic performance of the airfoil.

3.3.3 Surface Pressure Spectra

The pressure spectra are computed at the trailing edge \((x/c = 1)\) and at the front of the fences \((x/c = 0.875)\). Removing the points that lie within the fences, the pressure spectra are averaged in the spanwise direction, resulting in Fig. 3.28.

As seen in Fig. 3.28 (a), the fence geometry causes an increase in the SPL compared to the baseline geometry. Figure 3.29 shows that the surface SPL reductions observed in the simulations with the fence geometry agree qualitatively with the far field SPL reductions seen in the measurements for all frequencies.

As done for the previous simulation, we sum the surface SPL over the frequencies \(f = 2.5 \text{ kHz}\) to \(f = 5 \text{ kHz}\) to obtain the OASPL, which is plotted in Fig. 3.30 as a function of chordwise distance. It is again observed that the fences have lower integrated sound pressure levels from the trailing edge up to \(x/c \approx 0.88\). Near the leading edge of the fences, the fences have larger integrated sound pressure levels. The largest OASPL reduction occurs at the trailing edge for a decrease of about 3.0 dB.
3.3.4 Far field Aeroacoustics

Figure 3.31 shows the dilatation (|∇·v|) field for the baseline and fence simulations. Note that the simulation is tripped at $x/c = 0.05$. As is clearly seen in the dilatation field, the “trip wire” causes extraneous noise. However, as seen in the dilatation field in Fig. 3.18, the “trip wire” caused a weaker extraneous noise in simulation #2. Simulation #2 is expected to have a larger boundary layer thickness than simulation #3 due to the smaller Reynolds number. The same “trip wire” height is used for both simulations though. We can hypothesize that if the trip wire is scaled by the boundary layer thickness accordingly, this extraneous noise source may be eliminated. Nevertheless, for this simulation since the tonal frequency of the “trip wire” is much higher than the frequencies we are interested in, we can assume this extraneous noise source will not significantly influence the far field sound predictions.

To calculate the far field noise, the FW-H solver is used with the same integration surface used in simulation #2. Figure 3.32 shows the far field acoustics for the baseline and fences at an azimuth angle of 90°. As was seen in simulation #2, we see that despite the reduction seen in unsteady surface pressure spectra, no significant difference is observed in the far field noise between the baseline and fences. Potential reasons for this have been discussed in section 3.2.5.
Figure 3.19: Compressible LES results for NACA 0012 cross section with forced transition, $Re_c = 3 \times 10^5$ showing far field noise comparison for baseline and fence cases at an azimuth angle of 90 degrees.

### 3.3.5 Noise Reduction Mechanism

Spanwise coherence ($\gamma^2$) plots for the baseline and fence geometries at the trailing edge are plotted in Fig. 3.33. As for the previous simulations, there does not appear to be a significant difference in spatial coherence across all frequencies. Figures 3.34, 3.35, and 3.36 show similar TKE and velocity profiles as simulation #2. Similar behavior, previously in terms of lifting up of turbulence eddies from the airfoil surface, is observed for this simulation.
Figure 3.20: Compressible LES results for NACA 0012 cross section with forced transition, $Re_c = 3 \times 10^5$ showing a) spanwise coherence for baseline and b) spanwise coherence for fence.
Figure 3.21: Time-averaged normalized turbulent kinetic energy \( \frac{\text{TKE}}{u^2} \) for the baseline and fences in simulation #2 at a & b) \( x/c = 0.832 \), c & d) \( x/c = 0.873 \), e & f) \( x/c = 0.94 \), g & h) \( x/c = 0.994 \).
Figure 3.22: Span- and time-averaged, normalized turbulent kinetic energy ($TKE/u^*^2$) for the baseline and fences in simulation #2 at a) $x/c = 0.832$, b) $x/c = 0.873$, c) $x/c = 0.94$, d) $x/c = 0.994$. Note in these figures the wall normal distance, $y$, is normalized by the fence height, $H$. 
Figure 3.23: Span- and time-averaged, normalized $x$-component of velocity for the baseline and fence in simulation #2 at a) $x/c = 0.832$, b) $x/c = 0.873$, c) $x/c = 0.94$, d) $x/c = 0.994$. Note in these figures the wall normal distance, $y$, is normalized by the fence height, $H$. 
Figure 3.24: O-grid topology of the baseline mesh used in simulation #3. The trailing edge is rounded off and the mesh near the TE is shown in (b). Every 4th point is shown for clarity.

Figure 3.25: Cross sectional views of the computational mesh used to simulate the two-sided fence geometry at $Re_c = 5 \times 10^5$, $AOA = 0^\circ$. Every 4th point is shown for clarity.
Figure 3.26: Time- and span-averaged $C_p$ and $C_f$ comparisons between baseline and fence simulations with forced transition, $Re_C = 5 \times 10^5$.

Figure 3.27: Time- and span-averaged $C_p$ and $C_f$ comparisons between baseline and fence simulations with forced transition, $Re_C = 5 \times 10^5$. 
Figure 3.28: Spanwise-averaged SPL vs frequency comparison for the baseline and fence grid in simulation #3 at (a) the LE of the fences \((x/c = 0.875)\), and (b) at the TE of the airfoil \((x/c = 1)\).

Figure 3.29: The figure shows the qualitative comparison between the sound pressure spectra noise in the simulation and the far field spectra in the experiments from Ref. [3].
Figure 3.30: Compressible LES results for NACA 0012 cross section with forced transition, $Re_c = 5 \times 10^5$ showing a) OASPL throughout fence region and b) OASPL near trailing edge.

Figure 3.31: Compressible LES results for NACA 0012 cross section with forced transition $Re_c = 5 \times 10^5$ showing instantaneous acoustic field using fluid dilatation contours for the a) baseline airfoil and b) fence airfoil.
Figure 3.32: Compressible LES results for NACA 0012 cross section with forced transition, $Re_c = 5 \times 10^5$ showing far field noise comparison for baseline and fence cases at an azimuth angle of 90 degrees.

Figure 3.33: Compressible LES results for NACA 0012 cross section with forced transition, $Re_c = 5 \times 10^5$ showing a) spanwise coherence for baseline and b) spanwise coherence for the fences.
Figure 3.34: Time-averaged normalized turbulent kinetic energy ($k/u^2$) for the baseline and fence in simulation #3 at a & b) $x/c = 0.832$, c & d) $x/c = 0.873$, e & f) $x/c = 0.94$, g & h) $x/c = 0.994$. 
Figure 3.35: Span- and time-averaged, normalized turbulent kinetic energy ($k/u^*_{wall}^2$) for the baseline and fence in simulation #3 at a) $x/c = 0.832$, b) $x/c = 0.873$, c) $x/c = 0.94$, d) $x/c = 0.994$. Note in these figures the wall normal distance, $y$, is normalized by the fence height, $H$. 


Figure 3.36: Span-averaged time-averaged normalized x-component velocity ($\frac{k}{\bar{u}^2}$) for the baseline and fence in simulation #3 at a) $x/c = 0.832$, b) $x/c = 0.873$, c) $x/c = 0.94$, d) $x/c = 0.994$. Note in these figures the wall normal distance, $y$, is normalized by the fence height, $H$. 
CHAPTER 4. CONCLUSIONS & FUTURE WORK

The unique feather features of the night owl are responsible for its silent flight. One of the unique features is the soft downy coat found on its flight feathers. This thesis presents numerical investigations of airfoil geometries inspired by the soft downy coat of the owl. These airfoils were introduced by Clark et al. [2] which are characterized by the addition of “fences” and “rails” in the aft 13% of the airfoil chord. Implicit large eddy simulations are performed for baseline and owl-inspired airfoils (with fences) using a high-order accurate, compact finite-difference based flow solver. Three different flow conditions are simulated. The $C_p$ and $C_f$ values from XFOIL and experimental data show good agreement with the baseline geometry. Comparisons of $C_p$ and $C_f$ distributions over airfoil surfaces show that the owl-inspired airfoil geometries do not significantly degrade the aerodynamic performance.

Surface pressure spectra is compared between the baseline and airfoils with fences applied to the trailing edge. The fences yield reductions in the surface pressure spectra over most of the fence region, which is in qualitative agreement with the far field noise reductions observed in the measurements reported in Ref. [3]. However, near the leading edge of the fences, the surface pressure spectra for all three flow conditions are found to increase at all frequencies. This is attributed to the sharp leading edge of the fences, which are normal to the incident boundary layer turbulence (a classical leading edge noise source) in the simulations; experiments had highly slanted fence leading edges. Future simulations will attempt to model a more gradual curvature at the leading edge of the fences to reduce this extraneous noise source.

The sound pressure level of unsteady surface pressure integrated over only the high frequency range is labeled loosely as the overall sound pressure level (OASPL). The OASPL of the fences compared to the baseline is reduced by different amounts for each flow condition, with the one sided fence, $Re_c = 3 \times 10^5$, $AOA = 3^\circ$, forced transition flow condition giving the greatest reduction of 4.0
dB. The fences are found to give a reduction in the OASPL throughout most of the fence region, except right near the leading edge of the fences, where the OASPL increases.

The reasons behind far field noise reductions observed in the experiments with fences from Ref. [3] were investigated. For all three flow conditions, there does not appear to be a significant difference in the spanwise coherence between the baseline and fence geometries. Plots of the turbulent kinetic energy field show that the eddies are relocated to the top of the fences, thereby supporting the hypothesis that the fences lift the energy containing turbulence eddies away from the surface and the airfoil trailing edge. This is further suggested by the velocity and TKE profiles in the boundary layer, which display characteristics typical of canopy flows. This suggests that a mechanism of noise reduction could be the increased source-scattering surface separation due to the turbulence eddies moving away from the scattering body/edge.

A verification study is carried out for the FW-H solver wherein comparisons with FDL3DI simulations of point sources in uniform meanflow are performed. Far-field noise predictions are performed using the FDL3DI simulation data and the Ffowcs Williams-Hawkings analogy. However, no significant difference is observed in the far field noise between the fence and baseline geometries. Potential reasons for why the fences do not reduce the far field noise are identified to be:

1. The sharp fence leading edge normal to the incident flow is an extraneous noise source, which more than offsets the noise reduction obtained via the canopy effect.

2. The height of the fences is too small to decorrelate the turbulent structures along the span

3. In the simulations, the fences have a constant height that does not vary with downstream distance; this might be responsible for not decorrelating spanwise coherence.

Geometric dissimilarities between the experiments and simulations are identified to be the key issue that should be rectified in the near future. Meshing of such complex geometries is a big challenge. One of the approaches to make the mesh is to use a different block for each fence, where the fence is geometrically resolved rather than simulated via cutting holes in the mesh. Hole cutting does not allow for a precise definition of the geometry of the fences. Even in this case, there will be significant challenges in obtaining good grid metrics, which are necessary for a high-order solver such as FDL3DI to be robust and give accurate results. Another approach is to use a solver that allows
hybrid unstructured/structured grids and use unstructured grids in the region around each fence. Once these geometric differences are resolved, the precise mechanisms behind observed noise reductions can be identified. Nevertheless, even with the simplified geometric modeling approach adopted here, the results reported here have clearly demonstrated that increased source separation distance is a key mechanism of noise reduction with fence geometries.
BIBLIOGRAPHY


APPENDIX A. HIGH ORDER FILTERING AND COMPACT FINITE DIFFERENCING IN FDL3DI

A.1 Theory and Implementation of Filtering Schemes

A.1.1 Spectral Function of the Interior Filters

The FDL3DI code uses a low pass filter to remove the energy in the sub-grid scales. For the interior nodes, it uses the following stencil,

$$\alpha_f \hat{\phi}_{i-1} + \phi_i + \alpha_f \hat{\phi}_{i+1} = \sum_{n=-N}^{N} a_n (\phi_{i-n} + \phi_{i+n}) / 2,$$  \hspace{1cm} (A.1)

where $\alpha_f$ is the free parameter that provides some control on the "degree" of filtering, $\phi$ is the solution before filtering, $\hat{\phi}$ is the filtered solution, $a_n$ is the set of coefficients for a given order of accuracy filter and $2N$ is the order of accuracy of the filter which has a stencil of $2N + 1$ points. In practice, $0.3 < \alpha_f < 0.5$ and for a low quality mesh $\alpha_f \approx 0.1$. To get the spectral function, the following Fourier transform identity relating the shift in the spatial domain to the shift in the frequency domain is used:

$$F \{ f(x_i + m\Delta x) \} = F(w) e^{jwm\Delta x},$$ \hspace{1cm} (A.2)

where $m$ is the shift in the spatial domain and $w$ is the reduced wave number defined as,

$$w = \frac{2\pi k}{N},$$

where $k$ is the physical wave number, and $N$ is the number of intervals. In applying the fourier transform identity in equation (A.2) to the stencil in equation (A.1), $m$ is either $\pm n$ or $\pm 1$, $f(x_i)$ is
\( \hat{\phi}_i \) and \( \phi_i \) and for a computational domain, \( \Delta x = 1 \). Taking the Fourier transform of the stencil in equation (A.1) we get,

\[
\alpha_f F_O(w)e^{-jw} + F_O(w) + \alpha_f F_O(w)e^{jw} = \frac{F_I(w)}{2} \sum_{n=0}^{N} a_n(e^{-jwn} + e^{jwn}).
\]

Factoring out \( F_O \) in the left hand side we get,

\[
F_O(w)[1 + \alpha_f(e^{-jw} + e^{jw})] = \frac{F_I(w)}{2} \sum_{n=0}^{N} a_n(e^{-jwn} + e^{jwn}). \tag{A.3}
\]

Now using the following identity,

\[
2 \cos(x) = e^{-jx} + e^{jx}, \tag{A.4}
\]

where \( x = w \) on the left hand side, and \( x = wn \) on the right hand side, this can be used in equation (A.3) to arrive at,

\[
F_O(w)[1 + 2\alpha_f \cos(w)] = F_I(w) \sum_{n=0}^{N} a_n \cos(wn).
\]

Using the definition of the spectral function

\[
SF(w) = \frac{F_O(w)}{F_I(w)}, \tag{A.5}
\]

the spectral function becomes,

\[
SF(w) = \frac{\sum_{n=0}^{N} a_n \cos(wn)}{1 + 2\alpha_f \cos(w)}. \tag{A.6}
\]

Because of the centered nature of the stencil in equation (A.1), \( SF(w) \) is real and the filter is purely dissipative i.e., it alters only the magnitude of the input signal without introducing additional dispersion errors.
A.1.2 Derivation of the Coefficients of the Interior Filters

Equation (A.1) has $N+2$ unknowns $\alpha_f$, $a_0$, $a_1$, ... $a_N$. To derive the coefficients, we first insist that the highest frequency mode be eliminated by enforcing $SF(\pi) = 0$. Imposing this on the spectral function eliminates any odd-even decoupling. Using this in equation (A.6) for $N = 2$ (4th order filter), the following equation results,

$$a_0 - a_1 + a_2 = 0. \quad (A.7)$$

The remaining $N+1$ additional equations are derived by using the filter stencil, equation (A.1), with $N = 2$, expanding $\phi$ and $\hat{\phi}$ about point "i" using a Taylor series, and then matching the Taylor series coefficients of equal order terms in the left and right sides. This results in,

$$\hat{\phi}_i(2\alpha_f + 1) + \hat{\phi}_i''\alpha_f \Delta x^2 + \phi_i'' \frac{2}{4!} \alpha_f \Delta x^4 + O(\Delta x^6)$$

$$= \phi_i(a_0 + a_1 + a_2) + \phi_i'' \left( \frac{a_1}{2!} + \frac{2^2 a_2}{2!} \right) \Delta x^2 + \phi_i'' \left( \frac{a_1}{4!} + \frac{4^2 a_2}{4!} \right) \Delta x^4 + O(\Delta x^6).$$

We can achieve 4th order accuracy by neglecting $\Delta x \geq \Delta x^4$ terms. This results in,

$$\hat{\phi}_i(2\alpha_f + 1) + \hat{\phi}_i''\alpha_f \Delta x^2 = \phi_i(a_0 + a_1 + a_2) + \phi_i'' \left( \frac{a_1}{2!} + \frac{2^2 a_2}{2!} \right) \Delta x^2.$$ 

Now matching the taylor series coefficients of equal order terms on the left and right hand side, we get the following $N$ additional equations,

$$2\alpha_f + 1 = a_0 + a_1 + a_2 \quad (A.8)$$

$$\alpha_f = \frac{a_1}{2} + 2a_2. \quad (A.9)$$

These $N$ additional equations above as well as equation (A.7) allow $a_0$, $a_1$, ... $a_N$ to be solved in terms of the free parameter $\alpha_f$. For a 4th order interior filter, $a_0 = \frac{5}{8} + \frac{3\alpha_f}{4}$, $a_1 = \frac{1}{2} + \alpha_f$, $a_2 = \frac{\alpha_f}{4} - \frac{1}{8}$. A table including interior filter coefficients for orders of accuracy 2, 4, 6, 8, and 10 can be found in Ref. [41].
A.1.3 Spectral Response of the Interior Filters

The ideal case for a filter is to have it attenuate the signal as least as possible so that the highest accuracy can be achieved. However there is some attenuation needed to remove the energy from the sub-grid scales and suppress any instabilities in the solution. Therefore, when choosing the correct filter for a problem, there is a balance between accuracy and stability.

The spectral responses of the interior filters are given in Figures A.1 – A.5. These figures give much insight into how the filter responds across all reduced wave numbers and $\alpha$ values given a step input. It is apparent that for a given value of $\alpha$, as the order of the interior filter and/or $\alpha$ is increased, the filter attenuates the input signal less. Since there is only a real term in the spectral function, the response never causes any amplification of the input signal.
A.1.4 Spectral Function of the Boundary Filters

For the points near the boundary, the FDL3DI code uses different stencils than in the interior nodes. Near the boundary it uses the following stencil,

\[
a_f \phi_{i-1} + \phi_i + a_f \phi_{i+1} = \sum_{n=1}^{N+1} a_n \phi_n,
\]

where \( N \) is the order of the filter and is a \( N + 1 \) point stencil. Note here that \( i \) is the node number that the filter is based on. For \( i = 1 \) the \( \phi_{i-1} \) term is not used. Writing out the summation on the right hand side for \( N = 2 \),

\[
RHS = a_1 \phi_1 + a_2 \phi_2 + a_3 \phi_3,
\]

we can see that each \( \phi \) term is shifted from point \( i = 1 \) by a different amount than the other \( \phi \) terms. \( \phi_1 \) has a \( m = (n - i) = (1 - 1)\Delta x = 0 \) shift from point \( i = 1 \), \( \phi_2 \) has a \( m = (n - i)\Delta x = (2 - 1)\Delta x = \Delta x \) shift from point \( i = 1 \) and \( \phi_3 \) has a \( m = (n - i)\Delta x = (3 - 1)\Delta x = 2\Delta x \) shift from point \( i = 1 \). Therefore the shift in \( \phi \) from point \( i \) can be written in a form applicable to all \( \phi \) terms in the summation using \( (n - i)\Delta x = m\Delta x \). Using \( n = i + m \) we can rewrite both sides of the near-boundary stencil equation (A.10) in terms of \( i \),

\[
a_f \phi_{i-1} + \phi_i + a_f \phi_{i+1} = \sum_{i+m=1}^{i+m=N+1} a_{i+m} \phi_{i+m}.
\]

Seeing that the left hand side is identical to the interior coefficients case, we can directly write,

\[
F_O(w)(1 + 2a_f \cos(w)) = \sum_{i+m=1}^{i+m=N+1} a_{i+m} \phi_{i+m}.
\]

The Fourier identity relating the shift in the spatial domain to the shift in the frequency domain is repeated below for convenience:

\[
F \{ f(x_i + m\Delta x) \} = F(f) e^{jum\Delta x}.
\]
We can apply this Fourier identity to the right side of (A.11), where $f(x_i)$ is $\phi_i$ and for a computational domain, $\Delta x = 1$ to get the following,

$$F_O(w)(1 + 2\alpha_f \cos(w)) = F_I(w) \sum_{i+m=1}^{i+m=N+1} a_{i+m} e^{iwm}.$$  \hspace{1cm} (A.12)

Now using Euler's formula below,

$$e^{iwm} = \cos(wm) + j \sin(wm),$$

equation (A.12) becomes,

$$F_O(w)(1 + 2\alpha_f \cos(w)) = F_I(w) \sum_{i+m=1}^{i+m=N+1} a_{i+m}[\cos(mw) + j \sin(mw)].$$

Inserting back in the relation $m = n - i$ the equation above becomes,

$$F_O(w)(1 + 2\alpha_f \cos(w)) = F_I(w) \sum_{n=1}^{n=N+1} a_n[\cos((n - i)w) + j \sin((n - i)w)].$$

Using the previous definition for the spectral function from equation (A.5),

$$SF(w) = \frac{F_O(w)}{F_I(w)},$$

we get,

$$SF(w) = \frac{\sum_{n=1}^{n=N+1} a_n[\cos((n - i)w) + j \sin((n - i)w)]}{1 + 2\alpha_f \cos(w)}. \hspace{1cm} (A.13)$$

**A.1.5 Derivation of the Boundary Filter Coefficients**

The method for obtaining the near-boundary filter coefficient is very similar to the way the interior filter coefficients were found. For a 2nd order filter ($N = 2$) at point $i = 1$, again we force $SF(\pi) = 0$ to get the following equation,

$$a_0 - a_1 + a_2 = 0. \hspace{1cm} (A.14)$$
It can be shown that you will always arrive at the equation above independent of what node number the filter is based on. Similar to the interior filter, we can again expand $\phi$ and $\hat{\phi}$ about point "i" using a Taylor series, and then match the Taylor series coefficients of equal order terms on the left and right sides. First expanding $\phi$ and $\hat{\phi}$ about point $i = 1$ results in,

$$
\hat{\phi}_1(1 + \alpha) + \hat{\phi}_1' \alpha \Delta x + \phi''_1 \frac{\Delta x^2}{2!} = \phi_1(a_1 + a_2 + a_3) + \phi'_1(a_2 + 2a_3) \Delta x + \phi''_1(a_2 + 2^2a_3) \frac{\Delta x^2}{2!} + O(\Delta x^3).
$$

We can achieve 2nd order accuracy by neglecting $\Delta x \gg \Delta x^2$ terms. This results in,

$$
\hat{\phi}_1(1 + \alpha) + \hat{\phi}_1' \alpha \Delta x = \phi_1(a_1 + a_2 + a_3) + \phi'_1(a_2 + 2a_3) \Delta x.
$$

Now matching the taylor series coefficients of equal order terms on the left and right hand side, we get the following $N$ additional equations,

\begin{align*}
1 + \alpha_f &= a_1 + a_2 + a_3 \quad (A.15) \\
\alpha_f &= a_2 + 2a_3. \quad (A.16)
\end{align*}

These $N$ additional equations above as well as equation (A.14) allow $a_1, a_2, ... a_{N+1}$ to be solved in terms of the free parameter $\alpha_f$. For a near-boundary point filter at the first node with 2nd order accuracy, $a_1 = \frac{3}{4} + \alpha_r, a_2 = \frac{1}{2} + \frac{\alpha_r}{2}, a_3 = \frac{\alpha_r}{4} - \frac{1}{4}$. A table including near-boundary point filter coefficients for nodes 1 – 5 at orders of accuracy 1 – 10 can be found in Gaitonde (1998).

### A.1.6 Spectral Response of the Near-Boundary Filters

For this discussion, the spectral responses of the near-boundary filter at point $i = 1$ is not shown since the values of $\phi$ are explicitly specified through the boundary conditions and is not filtered.

The spectral responses of the near-boundary filter at point $i = 2$ is given in Figures A.6 – A.10. In these figures, $\alpha_r$ and $\alpha_i$ corresponds to only the real component and imaginary component, respectively. When interpreting the plots, note that to ensure stability, $Real(SF) \leq 1$ and $Im(SF) = 0$ is desirable. Having a $Real(SF) \leq 1$ means there is no amplification in the input signal. Having
a $Im(SF) = 0$ means that there is no dispersion error. This means that all disturbances, including localized ones, propagate without change of shape. Numerical dispersion often takes the form of so-called 'spurious oscillations'. This is usually depicted using an exact and computed step input. In the computed step input, there will be small oscillations near the step input. These spurious oscillations can cause problems in the CFD simulations. For example, if the step change was in a variable that can only be between 0 and 1, these oscillations can lead to unphysical values.

We can notice some important things about the near-boundary filter at this point. The effect of increasing $\alpha$ and the order of accuracy on the output signal are opposite. As $\alpha$ is increased, the amount of amplification in the real and imaginary components decrease. As the order of accuracy of the filter is increased, the amount of amplification in the input signal is increased. Therefore, as higher order filters are used, a higher value of $\alpha$ is needed to still have a stable output signal. According to the spectral responses for the near-boundary point filter at this node number, to ensure stability of the output a filter of order four with a $\alpha$ value of at least $\alpha \approx 0.4$ is recommended.

The spectral responses of the near-boundary filter at point $i = 3$ is given in Figures A.11 – A.14. Once again, changing $\alpha$ and the order of accuracy have opposite effects. Comparing to the filter at $i = 2$, we see that the filter at $i = 3$ is more stable. This allows a higher order filter to be used. According to the spectral responses for the near-boundary point filter at this node number, to ensure stability of the output a filter of order five or six with a $\alpha$ value of at least $\alpha \approx 0.4$ is recommended. Note that although increasing $\alpha$ past 0.4 will make the filter more accurate, it will also start to reach a point where the input is not attenuated enough and the output becomes unstable. This is why $\alpha$ is interpreted as a tuning parameter that can be optimized for the particular flow field that is being solved.

The spectral responses of the near-boundary filter at point $i = 4$ is given in Figures A.15 – A.16. Looking at the plots, it is clear now that as the filter node moves farther away from the boundary node, the filter becomes more stable and thus, higher order filters can be safely used. Even at low $\alpha$ values of 0.1, there is still no amplification in the output signal. For the filter at node $i = 4$, a filter of order 8 with a $\alpha$ value of at least $\alpha \approx 0.4$ is recommended.

The spectral responses of the near-boundary filter at point $i = 5$ is given in Figures A.17 – A.18. For the filter at node $i = 5$ a filter of order at least 8 with a $\alpha$ value of at least $\alpha \approx 0.4$ is recommended.
In summary, higher values of $\alpha$ causes a more stable output, higher orders of filter accuracy cause a less stable output and as the filter node moves away from the boundary the output is more stable. When choosing the order of accuracy and $\alpha$ value for a certain near-boundary filter, it is recommended that you start with a 0-4-6-8-8 order filter for points 1-2-3-4-5 with a $\alpha \approx 0.4$ and fine tune your filters by increasing/decreasing $\alpha$ and the order of accuracy at each near-boundary point. Note that this is only a general guideline since the spectral functions plotted have been obtained under the implicit assumption that the filter formulas will be applied at each point in the domain. However, in practice they are only applied at a limited number of points.

A.2  Compact Finite Differencing Derivations

A.2.1  Boundary Point Finite Difference Scheme

This section will explain how the coefficients are derived for the implicit compact finite difference formulations at different boundary nodes and of different orders of accuracy. We will first start with the formulations for boundary point 1. The general formula for the second order derivative at boundary point 1 is:

$$\phi'_1 = \frac{a \phi_1 + b \phi_2}{\Delta x} - \alpha \phi'_2 + O(\Delta x^2). \quad (A.17)$$

Note that the derivative is on both the left and right hand side. This is why it is called *implicit* compact finite differencing. When $\alpha = 0$, it is called *explicit*. Upon inserting Taylor series approximations about point 1, and matching coefficients of equal order terms, a sequence of equations is obtained who solution yields the coefficients listed in Table 2.2 of [41]. To show this procedure, we will expand the Taylor series about boundary points 2 for both $\phi$ and $\phi'$. This is shown below,

$$b \phi_2 = b(\phi_1 + \Delta x \phi'_1 + \frac{\phi''_1 \Delta x^2}{2!} + \frac{\phi'''_1 \Delta x^3}{3!} + O(\Delta x^4)), \quad (A.18)$$

$$\alpha \phi'_2 \Delta x = \alpha(\phi'_1 + \Delta x \phi''_1 + \frac{\phi''''_1 \Delta x^2}{2!} + \frac{\phi'''''_1 \Delta x^3}{3!} + O(\Delta x^4)), \quad (A.19)$$
Figure A.6: 4th Order Near-Boundary Filter at Node 2

Figure A.7: 5th Order Near-Boundary Filter at Node 2

Figure A.8: 6th Order Near-Boundary Filter at Node 2

Figure A.9: 7th Order Near-Boundary Filter at Node 2

Figure A.10: 8th Order Near-Boundary Filter at Node 2
Figure A.11: 5th Order Near-Boundary Filter at Node 3

Figure A.12: 6th Order Near-Boundary Filter at Node 3

Figure A.13: 7th Order Near-Boundary Filter at Node 3

Figure A.14: 8th Order Near-Boundary Filter at Node 3
Figure A.15: 7th Order Near-Boundary Filter at Node 4

Figure A.16: 8th Order Near-Boundary Filter at Node 4

Figure A.17: 9th Order Near-Boundary Filter at Node 5

Figure A.18: 10th Order Near-Boundary Filter at Node 5
Using $a\phi_1 + \text{Eq. A.18} - \text{Eq. A.19}$

$$a\phi_1 + b\phi_2 - \alpha \Delta x \phi_1' = \phi_1(a + b) + \Delta x(b - \alpha)\phi' + \frac{\phi''_1 \Delta x^2(b - 2\alpha)}{2!} + O(\Delta x^3). \tag{A.20}$$

Dividing by $\Delta x$,

$$\frac{a\phi_1 + b\phi_2}{\Delta x} - \frac{\alpha \phi_2'}{\Delta x} = \frac{\phi_1(a + b)}{\Delta x} + \frac{\Delta x(b - \alpha)\phi'}{\Delta x} + \frac{\phi''_1 \Delta x(b - 2\alpha)}{2!} + O(\Delta x^2). \tag{A.21}$$

To get into form of Eq. A.17,

$$0 + a + b = 0, \tag{A.22}$$

$$-\alpha + 0 + b = 1, \tag{A.23}$$

$$-2\alpha + 0 + b = 0. \tag{A.24}$$

Solving these series of equations arrives at the values for $a$, $b$ and $\alpha$.

To arrive at the coefficient for third order accuracy at boundary node 1 a similar procedure is followed where we start at the general derivative formulation below,

$$\phi_1' = \frac{a\phi_1 + b\phi_2 + c\phi_3}{\Delta x} - a\phi_2' + O(\Delta x^3). \tag{A.25}$$

If we expand about node 3 we get,

$$c\phi_3 = c(\phi_1 + 2\Delta x \phi_1' + \frac{2^2 \phi''_1 \Delta x^2}{2!} + \frac{2^3 \phi'''_1 \Delta x^3}{3!} + O(\Delta x^4)). \tag{A.26}$$

Using $a\phi_1 + \text{Eq. A.18} + \text{Eq. A.26} - \text{Eq. A.19}$ we arrive at,

$$a\phi_1 + b\phi_2 + c\phi_3 - \alpha \Delta x \phi_2' = \phi_1(a + b + c) + \Delta x(b + 2c - \alpha)\phi' + \frac{\phi''_1 \Delta x^2(b + 2^2 c - 2\alpha)}{2!} + \frac{\phi'''_1 \Delta x^3(b + 2^3 c - 3\alpha)}{3!} + O(\Delta x^4).$$
We need to divide by $\Delta x$ to get Eq. A.25,

$$\frac{a\phi_1 + b\phi_2 + c\phi_3}{\Delta x} - a\phi_2' = \frac{\phi_1(a + b + c)}{\Delta x} + (b+2c-a)\phi' + \frac{\phi^\prime\prime\Delta x(b+2^2c-2\alpha)}{2!} + \frac{\phi^\prime\prime\prime\Delta x^2(b+2^3c-3\alpha)}{3!} + O(\Delta x^3).$$

(A.28)

For third order accuracy we neglect the third order terms. To get into form of Eq. A.25 the following constraints must be made,

$$0 + a + b + c = 0,$$

(A.29)

$$-\alpha + 0 + b + 2c = 1,$$

(A.30)

$$-2\alpha + 0 + b + 4c = 0,$$

(A.31)

$$-3\alpha + 0 + b + 8c = 0.$$

(A.32)

Solving these series of equations arrives at the values for $a$, $b$, $c$ and $\alpha$. This procedure is followed for orders of accuracy up to 6th order and for boundary points 2, N-1 and N. Note that the reason it is called a "compact" finite difference schemes is because it allows us to use the values at one less node than "non-compact" formulations. The result of this is a more efficient algorithm.

**A.2.2 Interior Point Finite Difference Scheme**

For interior points, a centered formula is employed:

$$\alpha \phi'_{i-1} + \phi'_{i-1} + \alpha \phi'_{i+1} = b\frac{\phi_{i+2} - \phi_{i-2}}{4} + a\frac{\phi_{i+1} - \phi_{i-1}}{2},$$

(A.33)
where \( \alpha, a, \) and \( b \) are constants which determine the spatial properties of the algorithm. By choosing certain coefficients, up to sixth order accuracy can be achieved. Following the same procedure as the previous section where Taylor series approximations are made about point \( i \) and inserted in Eq. A.33, the following equations result,

\[
-2\alpha + a + b = 1, \tag{A.34}
\]

\[
-6\alpha + a + 4b = 0, \tag{A.35}
\]

\[
-10\alpha + a + 16b = 0, \tag{A.36}
\]

where only the first equation is solved for second order accuracy, the first and second equations are solved for fourth order accuracy and all three equations are solved for sixth order accuracy. Note that the "compact" finite difference formulation here allows us to use one less node than would be required for a "noncompact" formulation.
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