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Static and dynamic fault tree analysis with application to hybrid vehicle systems and supply chains

Xue Lei
Iowa State University

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Static and dynamic fault tree analysis with application to hybrid vehicle systems and supply chains

by

Xue Lei

A thesis submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

Major: Industrial and Manufacturing Systems Engineering

Program of Study Committee:
   Cameron A. MacKenzie, Major Professor
   Chao Hu
   Mingyi Hong

The student author and the program of study committee are solely responsible for the content of this thesis. The Graduate College will ensure this thesis is globally accessible and will not permit alterations after a degree is conferred.

Iowa State University
   Ames, Iowa
   2017

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DEDICATION

I would like to dedicate this thesis to my parents without whose support I would not have been able to complete this work. I would also like to thank my friends for their loving guidance during the writing of this work.
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ABSTRACT

One of the most challenging parts of reliability analysis is building a reliability model of the system. Reliability block diagram, Markov models, and fault tree analysis are some of the most common techniques for constructing a reliability model. Fault tree analysis provides a way to combine components, which together can cause system failure. This research uses both static and dynamic fault trees to quantify the reliability of a hybrid vehicle system and to analyze supply chain risk. The hybrid vehicle combines a mechanical power source, such as the internal combustion engine (gasoline engine or diesel engine), and an electric power source (electric motor) to take advantage of two power sources and compensate from each source. The hybrid systems complexity and non-mature technology carry potential risks for the vehicle. This research uses a static fault tree to analyze the reliability of the 2004 Toyota Prius under different operational modes. We apply Bayesian analysis that combines survey data to estimate the reliability of the hybrid vehicles battery. Supply chain risk analysis is increasingly becoming an important field and supply chain risk models help identify significant risks that can occur and the consequences if those risks occur. We use dynamic fault trees, which are relatively new in reliability analysis, to understand the timing of potential failures in different types of supply chains. We estimate failure rates for each supply chain under different production scenarios and simulate delivery time for the supply chain.
CHAPTER 1. OVERVIEW OF STATIC AND DYNAMIC FAULT TREE ANALYSIS

We live in a world full of unknown and uncertainty. Many unexpected and uncontrollable things happen every day. In the field of engineering, failure is very common for all kinds of engineering systems. Different failures could lead to different consequences. Failures are caused by many factors, such like design errors, poor manufacturing techniques and lack of quality control, substandard components, lack of protection against over stresses, poor maintenance, aging, wear out and human factors (Verma et al., 2010). Most often, we already know in what stage the engineering system is. The first step of reliability analysis is exploring potential reasons which may give rise to failure. Based on the relationship of each component in the system, the reliability model of the system can be built to estimate the reliability. From the calculation results, we need to find which reason contributes to the failure. According to what we find and the current stage of the engineering system, some proper methods can be used to improve the reliability of the system.

The most challenging part of reliability analysis is building the reliability model of the system. Reliability block diagram, Markov models and fault tree analysis are the most common techniques for constructing reliability model. Reliability block diagram is a visual technique which use blocks to express logical relationship of the system. The reliability of system is calculated by analytical methods. The biggest disadvantage of the reliability block diagram is not considering conditions of the system, such like dependencies between components, repairable components, coverage factors, multiple
states. Markov models are developed to overcome these problems. But for complex and large system, Markov model could become too complicated (Fuqua, 2003). Fault tree analysis neatly sidesteps issues raised by Markov model by using diverse solutions.

Fault tree is based on the probability of individual components and logical relationship between different components. According to the fault tree analysis, we can easily identify the cause of failure and estimate the reliability information of a system. Fault tree analysis consists of static fault tree analysis and dynamic fault tree analysis. In static fault tree, the OR gate and the AND gate are often used to describe the failure situation. The failure expression of a static fault tree is represented by minimal cut set based on Boolean algebra. In dynamic fault tree, the priority and gate, the sequence enforcing gate, the spare gate and the functional dependency gate can be used to depict multiple failure modes in a single dynamic fault tree. The main methods developed to solve dynamic fault tree are Markov models, numerical method and simulation method. A dynamic fault tree usually consists of static gates and dynamic gates. The unique function of dynamic gates is depicting interactions in a complex system, which cannot be realized by static gates. In order to understand fault tree better, we apply static fault tree and dynamic fault tree in risk analysis of different areas.

The hybrid vehicle is becoming more popular since it was invented. The hybrid vehicle combines a mechanical power source, such as the internal combustion engine (gasoline engine or diesel engine), and an electric power source (electric motor) to take advantage of two power sources and compensate from each source. The hybrid systems complexity and non-mature technology carry potential risks for the vehicle. In Chapter 2, the reliability analysis of hybrid systems is conducted with application to the 2004 Toyota Prius. We calculates the reliability of the hybrid vehicles by building fault trees for different operation modes and applying Bayesian analysis that combine survey data to estimate the reliability of the battery. Although the focus of this study is the hybrid vehicle, the innovative Bayesian analysis that combines a prior probability distribution
with survey data of customers can be applied to other engineered components, especially new technology where reliability data is unavailable.

Supply chains are becoming more vulnerable and sensitive because of globalization, complexity, and occurrence of various risk events. Therefore, supply chain risk analysis is a significant field of supply chain risk management, which can help us recognize the reasons of risk occurring and figure out the main reasons to have mitigation strategies. In Chapter 3, we analyze supply chain risk by using dynamic fault tree. The reliability models for two typical supply chains are built by dynamic fault trees. Then the failure rates and delivery time for supply chains are estimated by simulation results under low volume production scenario and high volume production scenario. An innovative dynamic gate is designed for dynamic fault tree modeling.
CHAPTER 2. ASSESSING THE RELIABILITY OF HYBRID VEHICLE SYSTEM: APPLICATION TO THE 2004 TOYOTA PRIUS

2.1 Introduction

Interest in environmental issues, global climate change, and energy conservation has contributed to the development of alternatives to the traditional automobile internal combustion engine. The hybrid vehicle plays a pivotal role during a transitional period from the conventional vehicle to an electrical vehicle. From 2007 to 2015, 3,915,883 hybrid electric vehicles have been sold in the United States (AFDC, 2016). As hybrid technology matures and more hybrid cars are in use, the reliability of these cars becomes an important issue for owners who want to ensure they are purchasing vehicles that will last. Hybrid vehicles have great fuel economy, and some reports suggest the hybrid vehicle is more reliable than traditional automobiles (Haj-Assaad, 2014). However, a hybrid vehicle costs more and are heavier, and the battery replacement schedules are unknown. Cold weather may lead to more failures in the hybrid vehicle (Hunting, 2016). Although surveys of owners of hybrid vehicles suggest these vehicles are reliable, people may not be entirely truthful in surveys or accurately recall the reliability of their vehicles (Jensen, 2009). The variety of opinions demands a more careful analysis of the reliability of hybrid vehicles.

The existing literature on hybrid vehicles mainly focuses on designing control methodologies to improve the efficiency of energy use and the vehicles performance under different environmental conditions. Bizon (2011) proposes a topology method that improves the performance of the inverter system to increase the efficiency of operation and relia-
bility of the whole system. Meegahawatte (2010) prove that potential energy could be saved from hydrogen-powered fuel cells by analyzing a fuel cell series hybrid and comparing different fuels powered vehicles. Pourhashemi (2014) introduce a method for helping designers find an optimal design of a parallel hybrid electric vehicle. Panday (2015) show the performance and lifetime of vehicle are highly influenced by the variable temperature.

A large portion of the literature analyzes the effect of hybrid vehicles on the environment, the economy, and driving behavior. Kaushal et al. (2009) finds the factors which can minimize life cycle cost, petroleum consumption, and greenhouse gas emissions to obtain the optimal design of plug-in hybrids. Gallagher and Muehlegger (2011) present the popularity of hybrids may increase on account of sales tax waivers and higher fuel prices which could lead to the future fuel savings. Fontaras et al. (2008) find remarkable advantage of hybrids on fuel economy and air emissions. Some of the literature focuses on predicting or improving the reliability of different components in the hybrid vehicle. Hirschmann et al. (2007) predict the reliability of inverters in hybrid electrical vehicles by developing a simulation to estimate the temperature of a three-phase converter during long operations. Mirhakimi and Karimi (2014) recommend more redundancy within a hybrid vehicle. Allella et al. (2005) develop an optimization model to increase the reliability of the hybrid vehicles electric propulsion system. However, no study has attempted to model the reliability of the entire hybrid vehicle and analyze how the reliability changes under different operating modes.

The hybrid vehicle system is a complex system because it combines an internal combustion engine and electric battery. Often, more components in a system mean more potential for failure (Rausand et al., 2004), but it remains to be seen if this is true with the hybrid vehicle system. The hybrid vehicle has multiple operation modes, and each of these modes could fail. The propulsion system is composed of a prime motor, an electric motor with DC/DC converter, a DC/AC inverter, a controller, an energy storage system,
and a transmission system. This paper estimates the probability of failure for the main functional components and uses these failure probabilities to estimate the reliability performance of the hybrid system in distinct operation modes. Due to limited knowledge and data about the hybrid vehicles battery, we employ a Bayesian approach to estimate the reliability of this component. The innovative Bayesian analysis combines a prior probability distribution with survey data from owners of a hybrid vehicle to estimate parameters for a Weibull probability distribution. This method can be applied to new technology where reliability data might be limited or unavailable.

The Toyota Prius is one of the more popular hybrid vehicles on the market and represents the newest hybrid technology. The second generation Prius won the prestigious Motor Trend Car of the Year award and best-engineered vehicle of 2004 (Koraku, 2003). This paper assesses the reliability of the 2004 Toyota Prius although the model can be extended to other hybrid vehicle systems. The 2004 Toyota Prius uses the Toyota Hybrid System II (THS-II) hybrid system, which is equipped with a high voltage (HV) battery, engine, motor and generator, power control unit (PCU), and planetary gear unit. THS-II has both series and parallel system configuration.

The unique contributions of this paper are the development of a fault-tree model to quantify the time-dependent reliability of the hybrid vehicle and using Bayesian analysis to estimate the probability the HV battery will fail. The Bayesian model relies on customer survey data, which we treat as interval data. To our knowledge, this paper represents the first overall model and analysis of the hybrid vehicle. Section 2 describes the fault-tree model and the Bayesian analysis for reliability. Section 3 applies the fault tree model and reliability analysis to the 2004 Toyota Prius and calculates time-dependent probabilities for the hybrid vehicle. Conclusions appear in Section 4.
2.2 Model

This paper models and calculates the reliability of the 2004 Toyota Prius by developing a fault tree for different operation modes and use typical functions for reliability. Most of the components reliability is described by an exponential function based on a components mean time to failure \( MTTF \). The hybrid vehicle battery's reliability is described by a Weibull distribution, and the parameters of this distribution are estimated using Bayesian analysis. The components reliabilities are used in the fault tree for different operation modes to calculate the probability of failure for the hybrid system.

2.2.1 Fault Tree

A fault tree is used to model the probability a system fails based on the probability failures of individual components. We can identify the cause of failure and obtain the reliability of a system from fault tree analysis. The fault tree allows us to determine the operational relationship among different components under different operation modes, and we use the fault tree to derive analytical expressions for the probability of failure.

2.2.2 Reliability Based on Exponential Distribution

The fault tree requires assessing the probability that each component will fail. Since the goal of this analysis is to determine the probability of failure at different points in time, we seek a method to evaluate the reliability of each component. Many components in an engineering system are standard components whose failure rates are known. We assume the reliability \( R(t) \) at time \( t \) of a standard component follows an exponential distribution \( Rausand et al., 2004 \):

\[
R(t) = P(T > t) = e^{-vt} \quad (t \geq 0)
\]
where $T$ is the random variable for the time of failure and $v > 0$ is the rate of failure for the exponential distribution. The $MTTF$ is:

$$MTTF = \int_0^\infty R(t)dt = \int_0^\infty e^{-vt}dt = \frac{1}{v}$$  \hspace{1cm} (2.2)$$

We use a components $MTTF$ to calculate $v$ and the exponential distribution to calculate the probability a component has failed by time $t$. The probability a component has failed within the time interval $[0, t]$ is:

$$P(T \leq t) = 1 - R(t) = 1 - e^{-vt}$$  \hspace{1cm} (2.3)$$

### 2.2.3 Reliability Based on Bayesian Analysis

As will be discussed in Section 3, new engineering systems will have new components whose reliability or $MTTF$ is unknown. We may have some information about the failure rate. This information could come from initial tests or, as is the case in this paper, from customer survey data. We consider that the distribution for the probability that the component fails within the time interval $[0, t]$ follows a Weibull distribution

$$P(T \leq t) = F(t|\beta, \lambda) = 1 - e^{-\lambda t^\beta}$$  \hspace{1cm} (2.4)$$

where $\lambda > 0$ is the scale parameter and $\beta > 0$ is the shape parameter for the Weibull distribution. The Weibull distribution provides greater flexibility to model the probability of failure than the exponential distribution. The Weibull distribution can model
hazard functions that are decreasing, increasing, or constant.

The probability density for the Weibull distribution is:

$$f(t|\beta, \lambda) = \lambda \beta t^{\beta - 1} e^{-\lambda t^\beta}$$  \hspace{1cm} (2.5)

Bayesian analysis requires prior probability distributions for $\lambda$ and $\beta$, and we assume each of these parameters follows a gamma distribution. Typically, the parameters for the gamma distribution are chosen so that the gamma distribution is “non-informative” and closely resembles a uniform distribution (Gelman et al., 2014). The goal of the Bayesian analysis is to use the known information to estimate posterior distributions for $\lambda$ and $\beta$.

The known information in this paper is derived from consumer survey data in which customers report time intervals in which the component has failed. If a consumer reports that a component fails within a time interval $[t_1, t_2]$, the likelihood of observing this result is:

$$P(t_1 \leq T \leq t_2) = F(t_2|\beta, \lambda) - F(t_1|\beta, \lambda)$$  \hspace{1cm} (2.6)

where $F(t_2|\beta, \lambda)$ is the Weibull cumulative distribution function from equation (2.4). Sometimes a consumer reports that he or she has used an engineered system for a length of time $t_3$ and the component has not failed within that time. This observation is typically called censored data because the observation has a lower bound but no upper bound. For this type of observation, the likelihood of observing that the component has not failed before $t_3$ is:

$$P(t_3 \leq T) = 1 - F(t_3|\beta, \lambda)$$  \hspace{1cm} (2.7)
Bayes rule allows us to use these likelihood functions with the prior distributions for $\beta$ and $\lambda$ to calculate a posterior distribution for these parameters:

$$g(\beta, \lambda | t) = \frac{L(t | \beta, \lambda)h(\beta)h(\lambda)}{p(t)}$$  \hspace{1cm} (2.8)

where $t$ is a vector of observations (intervals or censored values), $g(\beta, \lambda | t)$ is the posterior joint probability distribution for $\beta$ and $\lambda$ given the observations $t$, $L(t | \beta, \lambda)$ represents the likelihood of observing the interval or censored data as represented by equations (2.6) and (2.7), $h(\cdot)$ represents the gamma prior distribution, and $p(t)$ is the normalization constant.

Since the prior distributions are not conjugate with the likelihood distributions, an analytical solution for $g(\beta, \lambda | t)$ is impossible. The Gibbs sampler, a type of Markov Chain Monte Carlo simulation, can be used to estimate $g(\beta, \lambda | t)$.

The Gibbs sampler is used to estimate the posterior distributions for $\beta$ and $\lambda$. The Gibbs sampler requires distributions for each parameter conditional on the other parameters and the observations: $p(\beta | \lambda, t)$ and $p(\lambda | \beta, t)$. The algorithm for the Gibbs sampler is as follows:

1. Choose a set of initial values for the parameters $\beta_0$, $\lambda_0$
2. Generate $(\beta_1, \lambda_1 | \beta_0, \lambda_0)$ by sampling:
   - $\beta_1$ from $p(\beta | \lambda_0, t)$
   - $\lambda_1$ from $p(\lambda | \beta_1, t)$
3. Repeat step 2 $n$ times to obtain chain $\{\beta_0, \lambda_0; \beta_1, \lambda_1; \beta_n, \lambda_n\}$.

The results of the Gibbs sampler are convergent under some regularity conditions. The simulation can generate the conditional distributions $p(\beta | \lambda, t)$ and $p(\lambda | \beta, t)$, which are difficult to obtain from analytical calculation. WinBUGS (Lunn et al., 2000) is free
software that implements the Gibbs sampler in the Windows environment to simulate and calculate the posterior distribution.

Bayesian analysis for reliability with censored or interval data has seen a limited amount of research. Coolen (1997), Coolen (1996) developed an innovative model for Bayesian analysis of failure data and introduced a method to perform reliability analysis based on priors derived from an engineers experience and censored data. Van Dorp and Mazzuchi (2004) build a Bayes inference model and use Markov chain Monte Carlo methods for life testing. Fernandez (2000) applied a Bayesian approach for reliability analysis with censored data. Other papers use a Bayesian approach to incorporate censored data of different problems in different areas. Wong et al. (2005) use a Bayesian approach to analyze multilevel interval-censored data from a clinical dental study. Greco et al. (2016) investigation better methods based on Bayesian approach to handle a left-censored continuous biomarker in a family-based study.

2.3 Application to Hybrid System

We apply the fault tree and Bayesian analysis to the hybrid Toyota Prius. A hybrid system combines a mechanical power source, such as an internal combustion engine (gasoline engine or diesel engine) and an electric power source (electric motor). The hybrid system is designed to provide a smooth response and sufficient power while taking advantage of the two power sources by compensating from each source. The hybrid control system selects the best combination control mode of these two power sources depending on diverse driving conditions. When the car is running at low speeds (less than 40 mph), the electric power source is sufficient to provide power to the wheels, and the hybrid system only uses the HV battery. If extra power is needed for sudden acceleration, the hybrid system uses the engine and battery simultaneously. Although hybrid systems are equipped with an electric motor, the electric motors do not need external charging as in electric vehicles. In the 2004 and later Priuses, the traditional
brake booster is replaced by a new regenerative brake system to improve power efficiency. Depending on the motor type, the regenerative brake system can increase fuel efficiency by at least 20% (Ahn et al., 2009).

The automobile main components are the engine, automotive chassis, automotive body, and the electric system. We keep five main components which are critical to the operation of hybrid vehicle and leave subtle parts out of the analysis like joints, ball sockets, and hoses. The main components are:

1. HV Battery
2. Engine
3. Vehicle Electrical Equipment
   [Motor Generator 1 (MG1), Motor Generator 2 (MG2)]
4. Vehicle Power Control Unit
   [Power Control Unit (PCU)]
5. Mechanical System
   [Reduction Gear, Planetary Gear, Wheels]

The THS-II hybrid system in the Toyota Prius integrates the series hybrid system and parallel hybrid system together to achieve better performance by using the benefits of both systems. The system has two significant electrical devices: Motor Generator 1 (MG1) and Motor Generator 2 (MG2). MG1 and MG2 serve as both highly efficient alternating current generators and electric motors, and they provide extra power to assist the engine if needed. A planetary gear unit is a power splitting device. MG1 is connected to the sun gear, MG2 is connected to the ring gear, and the engine output shaft is connected to the planetary gear. The sun gear and ring gear belong to the planetary gear. These components are used to combine power delivery from the engine and MG2, and to recover energy to the HV Battery. A reduction gear is used to ensure extremely quiet operation. After simplification, the THS-II system can be drawn as Figure 2.1.
2.3.1 Fault Tree Model

Since the operation of a hybrid system depends on the driving conditions, the fault tree considers the different operational scenarios. The five operational scenarios are: starting, driving under normal conditions, sudden acceleration, deceleration and braking, and battery recharging (Koraku, 2003).

A functional block diagram describes the operational logic among different components and demonstrates how the components work together in series or in parallel. The functional block diagram is translated to a fault tree where components in series in the functional block diagram are connected via an OR gate in a fault tree and components in parallel are connected via an AND gate. The 8 main components with their abbreviations are listed in Table 2.1.

2.3.1.1 Start and driving at low speeds

When the hybrid vehicle is starting or moving at low speeds, the engine is not needed to provide power. The battery outputs electrical current to the PCU, and the MG2 serves
Table 2.1: The Abbreviations of Main Components

<table>
<thead>
<tr>
<th>Component</th>
<th>Abbreviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>HV Battery</td>
<td>H</td>
</tr>
<tr>
<td>Engine</td>
<td>E</td>
</tr>
<tr>
<td>MG1</td>
<td>M</td>
</tr>
<tr>
<td>MG2</td>
<td>N</td>
</tr>
<tr>
<td>PCU</td>
<td>P</td>
</tr>
<tr>
<td>Reduction gear</td>
<td>R</td>
</tr>
<tr>
<td>Planetary gear</td>
<td>G</td>
</tr>
<tr>
<td>Wheels</td>
<td>W</td>
</tr>
</tbody>
</table>

as a motor to generate power to the driving wheels. The MG1 rotates but it does not generate electricity. The functional block diagram shows the main components in series in Figure 2.2, which translates to a fault tree in which the components are connected via an OR gate in Figure 2.3.

![Functional Block Diagram for Starting](image)

Figure 2.2: Functional Block Diagram for Starting

The fault tree can be translated to a Boolean algebraic equation describing failure during starting $T1$ as failure in one of the five components:

$$T1 = H + P + N + R + W$$

(2.9)

2.3.1.2 Driving under normal conditions

During normal driving conditions (less than 40 mph), the engine runs and provides power. The mechanical power from the engine is divided by the planetary gear unit.
Some of the power drives MG2, and some of the power drives the wheels directly. During normal driving conditions, MG1 runs in the same direction to generate electrical power for MG2. MG2 starts and runs to provide an electric assist as a motor. The functional block diagram shows the main components in series and parallel in Figure 2.4, which translates to a fault tree in which the components are connected via an OR gate and an AND gate in Figure 2.5.

Boolean algebra reduces failure during normal driving conditions $T2$ to the failure of one of the four components failing:

$$T2 = E + G + W + R$$  \hspace{1cm} (2.10)
2.3.1.3 Sudden acceleration

Sudden acceleration or speeds over 100 mph require a sudden force which comes from the HV battery. The battery generates current going to the PCU which passes current to MG2. MG2 serves as a motor under this scenario. In order to ensure a smooth response for improving acceleration performance, the engine and the high-output motor should work together. During the sudden acceleration, the operation processes of engine is the same as driving under normal conditions. The functional block diagram shows the main components in series and parallel in Figure 2.6, which translates to a fault tree in Figure 2.7.

Failure during deceleration and braking $T3$ includes the redundancy between the engine and HV battery:

$$T3 = W + R + HE + PE + NE + GH + GP + GN$$  \hspace{1cm} (2.11)
2.3.1.4 Deceleration and braking

During the deceleration and braking process, the Toyota Prius uses a creative concept called regenerative braking. Regenerative braking converts kinetic energy to electrical energy which is stored in the HV Battery. MG2 works as a high-output generator, driven by the wheels. The functional block diagram shows the main components in series in Figure 2.8, which translates to a fault tree in Figure 2.9.

The fault tree can be translated to a Boolean algebraic equation describing failure during deceleration and braking $T_4$ as failure in one of the five components:

$$T_4 = W + R + N + P + H$$  \hfill (2.12)

2.3.1.5 Battery recharging

The Toyota Prius cannot be recharged from an external power supply like a plug-in hybrid vehicle. The HV battery has to maintain sufficient reserves to satisfy the driving requirements. The battery is recharged by the engine which drives the generator (MG1) when the battery level is lower than the standard level. Figure 2.10 depicts the functional block diagram, and Figure 2.11 depicts the fault tree.
Figure 2.7: Fault Tree for Sudden Acceleration
The fault tree can be translated to a Boolean algebraic equation describing failure during battery recharging $T5$ as failure in one of the five components:

$$T5 = E + G + M + P + H$$ \hspace{1cm} (2.13)

Because the vehicle needs to operate in each of the five driving scenarios in order for the vehicle to operate properly, the fault tree for the entire hybrid system connects the five operational modes via an OR gate, as depicted in Figure 2.12.

The fault tree means that the failure in the hybrid system $T6$ occurs if failure in one of the five modes occurs:

$$T6 = T1 + T2 + T3 + T4 + T5$$ \hspace{1cm} (2.14)
Inserting the failure modes for each of the five operational modes and eliminating similar terms via Boolean algebra, we arrive at the minimal cut sets for the total failure in hybrid system:

$$T_6 = H + P + N + R + W + E + G + M$$  \hfill (2.15)

The hybrid system fails if any one of the 8 main components identified at the beginning of this section fails. This result should not be surprising because automobile vehicles need all of their components to function in order to operate properly. Since the hybrid vehicle can provide power via two different modes, one could wonder if the vehicle can operate if only one of the power systems fail. Although the vehicle could accelerate
suddenly via either the engine or the HV battery, the vehicle requires all components to operate in order for all the operational modes to work correctly. The functional block diagram, fault tree, and Boolean algebra justify this conclusion.

2.3.2 Data Collection and Component Probability Estimation

2.3.2.1 Engine and Other Main Components

The reliability of the engine and the HV battery are based on the number of miles the vehicle travels. Since the reliability of other components are determined on the number of years, we need to translate failure in number of miles to failure in number of years. We calculate the average number of miles traveled per year in the United States. The U.S. Federal Highway Administration records the average annual miler per driver by age group (OHPI, 2016) (see Table 2.2). We weight the average number of miles driven by each age group by the proportion of the population in the United States according to age (Joyce A. Martin et al., 2015). Based on these two data sources, a car averages 12,826 miles per year.

The engine in the 2004 Toyota Hybrid is the Toyota 1NZ-FE/FXE engine. According to WikiMotors, the official life span of this engine is 120,000 miles (WikiMotors, 2016). Assuming 12,826 miles in a year, the $MTTF$ of the engine is $120000/12826 = 9.4$ years.
Table 2.2: Average Annual Miles per Driver

<table>
<thead>
<tr>
<th>Age</th>
<th>Male</th>
<th>Female</th>
<th>Total</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-19</td>
<td>8,206</td>
<td>6,873</td>
<td>7,624</td>
<td>7.10%</td>
</tr>
<tr>
<td>20-34</td>
<td>17,976</td>
<td>12,004</td>
<td>15,098</td>
<td>20.30%</td>
</tr>
<tr>
<td>35-54</td>
<td>18,858</td>
<td>11,464</td>
<td>15,291</td>
<td>27.90%</td>
</tr>
<tr>
<td>55-64</td>
<td>15,859</td>
<td>7,780</td>
<td>11,972</td>
<td>11.80%</td>
</tr>
<tr>
<td>65+</td>
<td>10,304</td>
<td>4,785</td>
<td>7,646</td>
<td>13.10%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Total=80.2%</td>
</tr>
<tr>
<td>Average</td>
<td>16,550</td>
<td>10,142</td>
<td>13,476</td>
<td>Weighted average=12,826</td>
</tr>
</tbody>
</table>

The MTTFs for PCU, reduction gear, planetary gear, MG1, and MG2 are calculated based on the MTTF of engine and data from Ping et al. (2010). The authors present the experimental data of mean time between failures (MTBF) of main components in the hybrid electric transit bus. We assume that proportional relation of MTBF between main components in the hybrid electric transit bus is the same with the proportional relation of MTTF of main components in the hybrid system we analyzed in this paper. For example, we can calculate the MTTF of PCU by using equation (2.16):

\[
\frac{\text{MTBF of Engine}}{\text{MTBF of PCU} \text{ (in hybrid electric transit bus)}} = \frac{\text{MTTF of Engine}}{\text{MTTF of PCU}} \text{ (in hybrid system of 2004 Toyota Prius)}
\]

From above method and assumption, we calculate the MTTF for the mechanical system, the electrical equipment, and the PCU in the 2004 Toyota Prius.

The mechanical system in this paper includes the wheels, planetary gear, and reduction gear; the electrical equipment includes MG1 and MG2. But Hu et al. only list the MTBF of the mechanical system and electrical equipment. We divide the MTTF of the mechanical system and electrical equipment to obtain the MTTF for each component. Except for the HV battery, the main components in this paper follow the exponential distribution. We assume a failure in one component in the mechanical system results in failure of the entire mechanical system. According to the property of exponential distri-
bution, the $MTTF$ of wheel, the $MTTF$ of planetary gear, and the $MTTF$ of reduction gear can be obtained from the equation (2.17).

$$\frac{1}{MTTF \text{ of wheel}} + \frac{1}{MTTF \text{ of planetary gear}} + \frac{1}{MTTF \text{ of reduction gear}} = \frac{1}{MTTF \text{ of mechanical system}}$$

(2.17)

We assume the $MTTF$ of each of these three components are identical. In a similar way, the $MTTF$ of MG1 and the $MTTF$ of MG2 can be derived.

### 2.3.2.2 HV Battery

The 2004 Toyota Prius battery is a nickel metal hybrid battery. To our knowledge, no official data on the lifespan of this HV battery exists. The Panasonic EV Energy Ni-MH handbook (Panasonic Corporation, 2016) claims this kind of battery can be recharged over 500 times, but translating this recharging information to the lifetime of the HV battery requires additional data, which is not available.

Given this lack of reliable data on the lifetime of the HV battery, we estimate the probability that the HV battery fails each year based on a survey of the number on the number of miles the HV battery lasts. An online poll conducted in PRIUSchat asked users how many miles their HV battery lasted (PriusChat, 2013). Although this survey is not scientific and there is no way to verify if the users are truthful, the survey provides some information that can be used to estimate the reliability of a HV battery. Table 2.3 shows the result of the survey of hybrid battery, and we use 12,826 miles per year to estimate the failure in terms of number of years.

We use the Bayesian approach described in Subsection 2.3 to estimate the reliability of the HV battery and assume a Weibull distribution for failure. The likelihood functions come from equations (2.6) and (2.7). If the battery fails before 100,000 miles (or 7.8 years), the likelihood function is $P(t \leq 7.8) = F(7.8|\beta, \lambda)$. If the battery fails between
Table 2.3: Survey of Battery Performance

<table>
<thead>
<tr>
<th>How is your Gen 2 Prius (2004-2009) Hybrid Battery Doing?</th>
<th>Vote(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Failed below 100,000 miles (7.8 years)</td>
<td>6</td>
</tr>
<tr>
<td>Failed between 100,000 and 150,000 miles (7.8-11.7 years)</td>
<td>8</td>
</tr>
<tr>
<td>Failed between 150,000 and 200,000 miles (11.7-15.6 years)</td>
<td>5</td>
</tr>
<tr>
<td>Failed at over 200,000 miles (15.6 years)</td>
<td>1</td>
</tr>
<tr>
<td>Has not failed below 100,000 miles (7.8 years)</td>
<td>42</td>
</tr>
<tr>
<td>Has not failed between 100,000 and 150,000 miles (7.8-11.7 years)</td>
<td>37</td>
</tr>
<tr>
<td>Has not failed between 150,000 and 200,000 miles (11.7-15.6 years)</td>
<td>19</td>
</tr>
<tr>
<td>Has not failed at over 200,000 miles (15.6 years)</td>
<td>8</td>
</tr>
</tbody>
</table>

100,000 and 150,000 miles (7.8 and 11.7 years), we use the likelihood function in equation (2.6) where \( t_2 = 11.7 \) and \( t_1 = 7.8 \). The formula is similar for when the battery fails between 150,000 and 200,000 miles (11.7 and 15.6 years). If the battery fails over 200,000 miles (15.6 years), we use the likelihood function in equation (2.7) where \( t_3 = 15.6 \). If the battery has not failed below 100,000 miles (7.8 years), we use equation (2.7) where \( t_3 = 7.8 \). If the battery has not failed between 100,000 and 150,000 miles (7.8 and 11.7 years), that means the battery has not failed before 11.7 years. Thus, we use equation (2.7) where \( t_3 = 11.7 \). Equation (2.7) is similarly used when the battery has not failed between 150,000 and 200,000 miles (11.7 and 15.6 years).

The prior distributions for \( \beta \) and \( \lambda \) are gamma distributions where the parameters for the gamma distributions are chosen so that the gamma distribution resembles a uniform distribution. We use WinBUGS to run the Gibbs sampler and run 3 separate chains each with a burn-in phase of 1500 samples. The burn-in phase means that first 1500 samples are discarded. After the burn-in phase, we record 1000 samples for each chain. Figure 2.13 depicts the 3 chains for (a) \( \beta \) and (b) \( \lambda \). With 3 chains, we have a total of 3000 samples. The different lines represent different chains or runs in the simulation.

The mean value for \( \beta \) is 2.808 with a standard deviation of 0.3487. The mean value for \( \lambda \) is \( 4.284 \times 10^{-4} \) with a standard deviation of \( 3.333 \times 10^{-4} \). We use the 3000 samples for \( \beta \) and \( \lambda \) with the Weibull distribution to simulate failure times for the battery. Figure 2.14
Figure 2.13: Gibbs sampler results for $\beta$ and $\lambda$

depicts the histogram of this simulation.

The \textit{MTTF} for this simulation is 16.2731 years. As Figure 2.14 shows, the HV battery has a 0.2647 probability of lasting more than 20 years and a 0.0247 probability of lasting more than 30 years. These results seem to overestimate the lifetime of a battery. Although a battery could last 20 years, it seems very unlikely that a battery will last 30 years or more. Thus, we desire a method that will use the survey and limit the results to lifetimes that appear more reasonable.

We set an upper limit during the Bayesian optimization by assuming that the battery never lasts longer than a predefined number of years. Most vehicles fail before 300,000 miles, and we set 300,000 miles, or 23.39 years, as the upper bound for the four likelihood equations without an upper limit. Running 3000 samples in the Bayesian optimization reveals that the mean value of $\beta$ is 3.845 with a standard deviation of 0.3571 and the mean value for $\lambda$ is $5.333 \times 10^{-5}$ with a standard deviation of $1.071 \times 10^{-4}$. Using the
simulated values of $\beta$ and $\lambda$ with the Weibull distribution reveals the follow histogram for the lifetime of the battery in Figure 2.15. The MTTF of this distribution is 13.70 years with only a 0.0637 probability the battery lasts longer than 20 years.

![Figure 2.14: Histogram of Failure Time](image)

We repeat the Bayesian optimization simulation with an upper limit of 250,000 miles (19.49 years) and 200,000 miles (15.59 years). Figure 2.16 depicts the time to failure for the battery using a limit of 19.49 years. Running 3000 samples in the Bayesian optimization reveals that the mean value of $\beta$ is 4.575 with a standard deviation of 0.4607 and the mean value for $\lambda$ is $1.028 \times 10^{-5}$ with a standard deviation of $1.226 \times 10^{-5}$. Using the simulated values of $\beta$ and $\lambda$ with the Weibull distribution predicts the distribution of the battery lifetime in Figure 2.16. The MTTF of this distribution is 13.11 years with only a 0.0150 probability the battery lasts longer than 20 years.

Figure 2.17 depicts the time to failure with a limit of 15.59 years. Running 3000 samples in the Bayesian optimization reveals that the mean value of $\beta$ is 5.671 with a standard deviation of 0.7739 and the mean value for $\lambda$ is $2.059 \times 10^{-6}$ with a standard deviation of $4.442 \times 10^{-6}$. Using the simulated values of $\beta$ and $\lambda$ with the Weibull distribution reveals the follow histogram for the lifetime of the battery in Figure 2.17. The
\(MTTF\) of this distribution is 12.06 years with a 0 probability the battery lasts longer than 20 years.

Table 2.4 displays the results on the reliability of the HV battery for different upper limits. We need to determine which probability distribution is the most reasonable. The average age of vehicles in the United States is 11.5 years (Gardner, 2015), which suggests that perhaps we should choose a smaller upper limit so that there is close to a 0.5 probability the battery fails before 11.5 years. However, the average age of the vehicle does not really explain how long a battery will last because many factors influence why a person disposes of a vehicle and purchases a new one.

Table 2.4: Probability that HV Battery Fail Before a Given Time Period

<table>
<thead>
<tr>
<th>Upper Bound</th>
<th>P(1 year)</th>
<th>P(5)</th>
<th>P(10)</th>
<th>P(15)</th>
<th>P(20)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No upper bound</td>
<td>0.0003</td>
<td>0.0207</td>
<td>0.1793</td>
<td>0.4473</td>
<td>0.7353</td>
</tr>
<tr>
<td>300,000 miles</td>
<td>0.0003</td>
<td>0.0167</td>
<td>0.1877</td>
<td>0.6150</td>
<td>0.9363</td>
</tr>
<tr>
<td>250,000 miles</td>
<td>0</td>
<td>0.0100</td>
<td>0.1780</td>
<td>0.7007</td>
<td>0.9850</td>
</tr>
<tr>
<td>201,000 miles</td>
<td>0</td>
<td>0.0060</td>
<td>0.2083</td>
<td>0.8860</td>
<td>1.0000</td>
</tr>
</tbody>
</table>
Selecting 250,000 miles, or 19.5 years, as the upper limit seems most reasonable to us. The MTTF is 13.1143 years and there is only a 0.0150 probability the battery will last longer than 20 years. (Even though the limit is 19.5 years, a battery can last longer than 19.5 years. The upper limit assumes that a battery has not lasted longer than 19.5 years, but it is still possible a battery could last longer than 19.5 years.)

2.3.3 Results

Due to limited data of the lifetime of the HV battery, we assess the reliability of the HV battery by using Bayesian approach. The Gibbs sampler allows us to estimate posterior the posterior distribution, which are used to model the failure of the HV battery. The probability of failure for the HV battery in a given year can be obtained from the simulation results. The other main components are standard components. We assume the reliability of a standard component at specific time follows the exponential distribution. The parameter of the exponential distribution is calculated from the MTTF of each component. The probability of failure of a standard component in a given year
Figure 2.17: Histogram of Failure Times With Upper Limit of 200,000 Miles

is calculated from equation (2.3). Table 2.5 shows the $MTTF$ and the probability of failure for each of the main components. $P(t)$ represents the probability a component fails within $t$ years.

The Boolean algebraic equations in Section 3.1 shows the logical relation for each operation modes failure. We use the probability of failure for the main components in Table 2.5 with the Boolean algebraic equation for each operation mode to calculate the probability of failure for different operation modes at specific times. Table 2.6 depicts the probability of operation failure at specific time. Figure 2.18 displays the probabilities of failure for the entire hybrid system.

These calculations estimate that a hybrid vehicle has a 0.9985 probability of failing within 5 years, and Figure 18 shows that the probability of failure increases dramatically in the first few years. From Table 5, the probabilities the HV battery and engine fail are smaller than the other main components. The PCU has highest probability of failure, and the $MTTF$ of the PCU is 2.682 years. The PCU consists of three inverters and
<table>
<thead>
<tr>
<th>Component</th>
<th>MTTF</th>
<th>1/MTTF</th>
<th>P(1 year)</th>
<th>P(5)</th>
<th>P(10)</th>
<th>P(15)</th>
<th>P(20)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HV Battery</td>
<td>13.5</td>
<td>0.0741</td>
<td>0</td>
<td>0.0103</td>
<td>0.178</td>
<td>0.7007</td>
<td>0.985</td>
</tr>
<tr>
<td>Engine</td>
<td>9.4</td>
<td>0.1064</td>
<td>0.1009</td>
<td>0.4125</td>
<td>0.6549</td>
<td>0.7972</td>
<td>0.8809</td>
</tr>
<tr>
<td>MG1(vehicle electrical equipment)</td>
<td>8.3614</td>
<td>0.1196</td>
<td>0.1127</td>
<td>0.4501</td>
<td>0.6976</td>
<td>0.8337</td>
<td>0.9085</td>
</tr>
<tr>
<td>MG2(vehicle electrical equipment)</td>
<td>8.3614</td>
<td>0.1196</td>
<td>0.1127</td>
<td>0.4501</td>
<td>0.6976</td>
<td>0.8337</td>
<td>0.9085</td>
</tr>
<tr>
<td>PCU(vehicle power control unit)</td>
<td>2.682</td>
<td>0.3729</td>
<td>0.3112</td>
<td>0.845</td>
<td>0.976</td>
<td>0.9963</td>
<td>0.9994</td>
</tr>
<tr>
<td>Reduction Gear</td>
<td>5.1273</td>
<td>0.195</td>
<td>0.1772</td>
<td>0.6229</td>
<td>0.8578</td>
<td>0.9464</td>
<td>0.9798</td>
</tr>
<tr>
<td>Planetary Gear</td>
<td>5.1273</td>
<td>0.195</td>
<td>0.1772</td>
<td>0.6229</td>
<td>0.8578</td>
<td>0.9464</td>
<td>0.9798</td>
</tr>
<tr>
<td>Wheel</td>
<td>5.1273</td>
<td>0.195</td>
<td>0.1772</td>
<td>0.6229</td>
<td>0.8578</td>
<td>0.9464</td>
<td>0.9798</td>
</tr>
</tbody>
</table>
Table 2.6: Probability of Operation Failure

<table>
<thead>
<tr>
<th>Scenario of Failure</th>
<th>$P(1,\text{year})$</th>
<th>$P(5)$</th>
<th>$P(10)$</th>
<th>$P(15)$</th>
<th>$P(20)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start and low to mid-range speeds</td>
<td>0.5863</td>
<td>0.988</td>
<td>0.9999</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Driving under normal conditions</td>
<td>0.4992</td>
<td>0.9685</td>
<td>0.999</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Sudden acceleration</td>
<td>0.3373</td>
<td>0.9575</td>
<td>0.9989</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Deceleration or Braking</td>
<td>0.5863</td>
<td>0.988</td>
<td>0.9999</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Battery recharging</td>
<td>0.5479</td>
<td>0.9813</td>
<td>0.9997</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Hybrid System totally fails</td>
<td>0.7284</td>
<td>0.9985</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Two converters (Emadi et al., 2008). Song and Wang (2013) show that these sensitive power electronic component can influence the whole hybrid systems reliability. The links between circuit elements are the most vulnerable link. The redundancy of most converters is not considered in the PCU of the hybrids. If one of these converters fails to work, it could lead to the total failure of PCU. Therefore, in our results the reliability of the PCU may not be as good as other main components.

Table 2.6 shows the probabilities of failure under the different operational scenarios and for the entire hybrid system. Starting or driving at low to mid-range speeds and
Table 2.7: Probabilities of Operation Failure Due to the Engine or HV Battery

<table>
<thead>
<tr>
<th>Scenario of Failure</th>
<th>P(1 year)</th>
<th>P(5)</th>
<th>P(10)</th>
<th>P(15)</th>
<th>P(20)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start and low to mid-range speeds</td>
<td>0</td>
<td>0</td>
<td>0.0103</td>
<td>0.178</td>
<td>0.7007</td>
</tr>
<tr>
<td>0.985</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Driving under normal conditions</td>
<td>0</td>
<td>0.10092</td>
<td>0.412521</td>
<td>0.654869</td>
<td>0.797243</td>
</tr>
<tr>
<td>0.880884</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sudden acceleration</td>
<td>0</td>
<td>0</td>
<td>0.004249</td>
<td>0.116567</td>
<td>0.558628</td>
</tr>
<tr>
<td>0.867671</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deceleration or Braking</td>
<td>0</td>
<td>0</td>
<td>0.0103</td>
<td>0.178</td>
<td>0.7007</td>
</tr>
<tr>
<td>0.985</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Battery recharging</td>
<td>0</td>
<td>0.10092</td>
<td>0.418572</td>
<td>0.716302</td>
<td>0.939315</td>
</tr>
<tr>
<td>0.998213</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hybrid System totally fails</td>
<td>0</td>
<td>0.10092</td>
<td>0.418572</td>
<td>0.716302</td>
<td>0.939315</td>
</tr>
<tr>
<td>0.998213</td>
<td></td>
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</table>

deceleration or braking have the highest probability of failure because these modes rely on the PCU which has the single highest probability of failure. Sudden acceleration has the smallest probability of failure, because of the redundancy built into this operation. As can be seen from the functional block diagram (Figure 2.6) and fault tree (Figure 2.7), there are two power sources—electric power and mechanical power—that can drive the wheels. Even if one power source cannot provide power, the other one still can work.

2.3.4 Modified Reliability Model Based on HV Battery and Engine

The probability of failure of the hybrid vehicle is so high because the PCU has such a large probability of failure and because the model assumes that components are not fixed or replaced. Since the reliability of the hybrid vehicles electric and mechanical components are based on data from Ping et al. (2010) which was for a hybrid electric bus, it might not be accurate. The engine and the HV battery are the two most important components in the hybrid system and the power source of the hybrid vehicle. Since the accuracy of the failure of the other components is suspect, we next consider only the failure of the engine or the HV battery. Table 2.7 and Figure 2.19 depict the probability of failure under different operation modes if only the HV battery or engine fails.
If we assume the mechanical system, electrical equipment, and PCU, the probability the hybrid vehicle fails within 5 years is 0.42 compared with 0.999 in the original model. The probability the hybrid vehicle fails within 15 years is 0.94. Alternatively, this modified model could reflect the ability to repair or replace components in the electrical equipment, mechanical system, and PCU. It appears more reasonable that the hybrid vehicle will have a reliability of more than 0.5 during its first 5 years of its useful life.

### 2.4 Conclusions

Based on existing literature, this paper first presents an overall reliability model for the hybrid vehicle systems by constructing reliability block diagrams and fault trees for different operation modes, such as normal operation, sudden acceleration, braking, and battery recharging. We translate the fault tree to a Boolean algebraic equation describing failure for the different scenarios. The standard components reliabilities follow an exponential distribution, and we calculate their probabilities of failure based on the
Since the HV battery has limited data, we develop a unique Bayesian model to incorporate survey data to calculate the battery’s reliability.

Several other components in a vehicle in addition to the eight components examined in this paper could also fail. Other components may fail and need to be replaced, such as hoses, clamps, and brake pads. Proper maintenance can improve the reliability of a hybrid vehicle. These components are not included in our model. The better estimate of the reliability appears to come from the reliability model that only includes the HV battery and the engine, which raises doubts about the failure probabilities for the other components. Future work can undergo better data collection in order to obtain a better measure of reliability for all the components.

Other factors not considered in this paper may also impact a vehicle’s reliability. Temperature could influence the reliability of the HV battery. Tesla Motors has reported many incidents of spontaneous combustion in their vehicles due to unstable performance of battery (Lambert, 2016; Byrne, 2016). Extreme temperature environment can cause failure in battery operation. Moisture environment can result in electrical short circuit, which generate and release heat, burning the line in the PCU. Electrical components are small in size and highly sensitive to the environment factors like temperature, exposure thermal shock, and moisture exposure.

Despite these limitations and assumptions, the model presented herein provides a systematic framework for analyzing and estimating the reliability of a hybrid vehicle. The Bayesian analysis that integrates survey data to assess the probability of failure for the HV battery represents a unique method to measure the reliability. It appears that the HV battery is quite reliable and more reliable than the engine, whose lifespan is estimated at 120,000 miles. The reliability of the battery and engine lead to a vehicle whose reliability exceeds 0.5 for the first 5 years of its life and whose reliability is 0.28 in year 10. Future research can undertake more careful studies of the engine, the HV battery, and the other components to understand if the inputs used for this study are
accurate. A more complete model could also consider proper maintenance of parts and determine how that affects the vehicles reliability.
CHAPTER 3. SUPPLY CHAIN RISK ANALYSIS USING DYNAMIC FAULT TREE

3.1 Introduction

Supply chains are becoming more vulnerable and sensitive because of globalization, complexity, and occurrence of various risk events. There are several risk categories of a supply chain, such like disruptions, delays, systems risk, forecast risk, intellectual property risk, procurement risk, receivables risk, inventory risk and capacity risk (Chopra and Sodhi, 2004). Besides, there are a plenty of events could make threats happen, such as natural disaster, war, and terrorism, inflexibility of the supply source, information infrastructure breakdown and so forth. On March 2011, the earthquake and tsunami destroyed supply chains of over 27,000 businesses in Japan. Only a few of businesses recovered one year later (RebuildingTohoku, 2017). On March 17, 2000, a fire at a plant owned by Royal Philips Electronics was caused by lightning, which damaged millions of microchips. Ericsson, a major customer of Royal Philips Electronics, lost 400 million dollars due to the crisis. For another example, Boeing tries to reduce cost and expect to shrink the 787s development time by outsourcing. Superficially, outsourcing can reducing cost because of the low-cost labor. However, some components outsourced may not be assembled together. So the outcome is disappointing, and the development time of 787s is extended. Boeing does not win the market share from Airbus and spend more money (Denning, 2017). All these events demonstrate the importance of management of supply chain risk. The key to making strategies is having a comprehensive understanding and a thorough analysis of supply chain risk. By identifying and modeling risks, we can access
severity of risks. According to the results of assessment and various sources of risks, proactive and mitigating strategies can be made to response to potential risks in the future (Sodhi, 2014). Therefore, Supply chain risk analysis is a significant field of supply chain risk management, which enables us to recognize the reasons of risk occurring and find the main reasons. Finally, we can choose better strategies to reduce risk (Sheffi et al., 2005).

In general, to overcome vulnerability and increase the resilience of supply chain, one supply chain may have multiple suppliers. Under the future uncertainty, the cost objection function models for single, two, and multiple suppliers are developed (Parlar and Perry, 1996). Actually, the cost of supply chain is affected by the risk which may be reduced by using proper number of suppliers. The method used to choose the number of suppliers are designed to mitigate the risk of IT outsourcing (Currie, 1998). Besides having multiple suppliers in supply chain, inventory also can improve the resilience of supply chain. If agile supply chains try to compete in volatile markets, creating redundancy and responsiveness is very helpful (Christopher, 2000). In order to response to volatile markets, inventory should be concluded in supply chain. The accuracy of inventory information can impact the supply chain performance (Fleisch and Tellkamp, 2005), so the stochastic inventory systems and vendor managed inventory are proposed (Corbett, 2001; Waller et al., 1999; Dong and Xu, 2002). In a word, inventories play the role as assurance of a supply chain (Bogataj and Bogataj, 2007).

In addition, the information system also plays a significant role in a supply chain. The main work of information system is real-time sharing and processing production information within a supply chain. Information system realizes closer coordination between partners in supply chain (Wu et al., 2006). In the above Royal Philips Electronics example, another major customer, Nokia just had a little loss during the crisis due to the quick response capability. Initially, the information of order delayed were shown on the computer screens at headquarter of the Nokia. Later, managers of Nokia knew order
delayed and formulated solving methods to move chip orders at the first time. Once the information system breakdown occurs, the emergent information may not be captured and shown in time. Hence, more information sharing makes a great improvement of performance of the supply chain. The supply chain can seek lower risk by adjusting inventory level or coordinating different components (Yu et al., 2001; Lee et al., 2000). For further flexibility of supply chain, information technology and internet can be applied on information system design (Gunasekaran and Ngai, 2004; Pereira, 2009; Williamson et al., 2004).

In this paper, we analyze the main-backup supply chain and the mutual-assistance supply chain. The main-backup supply chain has a main supplier, a backup supplier, information system and inventory. When the main supplier operates normally, the backup supplier does not work until the main supplier fails. If both two suppliers cannot work, the inventory will be used. The information systems failure can lead to unavailability of the backup supplier. The mutual-assistance supply chain has two suppliers and information system, and does not consider the inventory. Different from the main-backup supply chain, two suppliers of this supply chain work simultaneously. If one supplier has a failure, the other supplier will help it out by increasing the production quantity or production rate. Once the information system fails to work, either one of two suppliers is unable to give assistance.

The fault-tree analysis is a useful technique of system reliability modeling, and it can show the logic relationship of the input events and output event. Fault-tree analysis is a classical tool for understanding operation processes and identifying failures in systems. For a low volume high value supply chain, a robust method is developed to reduce the likelihood of delays in material flow by representing the system of suppliers within a supply chain as a fault-tree and determining the proactive optimum mitigation strategy (Sherwin et al., 2016). However, the fault tree is unable to depict interplay between components in a supply chain. Modern supply chain becomes more and more compli-
Whether in low volume or high volume supply chain, the interaction between each component cannot be ignored. Therefore, dynamic fault tree (DFT) is constructed to overcome the limitation of the static fault tree.

This paper builds reliability models for two typical supply chains by using DFT. Then based on the function of each dynamic gate and realistic scenarios, we estimate failure rates for each supply chain under different production scenarios and simulate delivery time for the main-backup supply chain. Several unique contributions are made for future supply chain risk analysis. First, we use DFT to model supply chain. Most existing works of DFT mainly focus on reliability modeling for complex engineering system. Second, an innovative dynamic gate, mutual-assistance gate are created for DFT modeling. Third, for supply chain risk analysis, we calculate both failure rate and delivery time. Finally, two different production scenarios, low volume production and high volume production, are simulated by using different simulation methods.

The rest of this study is organized as following ways: Section 2 reviews the literature in supply chain risk analysis and DFT. Section 3 introduces five dynamic gates and presents the dynamic fault trees we build of two supply chains. Section 4 develops different simulation methods for simulating different scenarios of two supply chains. Then we show the simulation results. Finally, conclusions and future work are presented in Section 5.

3.2 Literature Review

The frequency of nature disasters and man-made accidents increases exponentially during the past decades in industrialized countries (Coleman, 2006). Nature disasters, terrorism and some unpredictable events all give rise to the risk of Supply chain (Stewart, 1995; Brown et al., 2006; Chopra et al., 2007). Under this background, supply chain risk has been extensively studied in the existing literature. The supply chain risk is usually analyzed from a qualitative view or a quantitative view. From a qualitative point of view,
the probability of occurrence for a risk event is assessed by different levels, such like rare event and likely event (Raj Sinha et al., 2004). The severity of risk also be evaluated from a qualitative view, such like low risk and high risk (Norrman and Jansson, 2004). Some strategies and approaches are developed to help decision making (Giannakis and Louis, 2011; Manuj and Mentzer, 2008). Meanwhile supply chain risk mitigation methods also are proposed (Giunipero and Aly Eltantawy, 2004; Christopher and Lee, 2004). For quantitative risk analysis, available past data is used to estimate the probability of risky events (Tuncel and Alpan, 2010; Kleindorfer and Saad, 2005). In addition to calculating probability, systematic strategies of managing and mitigating threats have been provided (Tomlin, 2006; Klibi et al., 2010). Other extensive works have been done in the area of management of disruption risk by inventory, facility location and empirical data (Cui et al., 2010; Schmitt and Singh, 2009; MacKenzie et al., 2014, 2012).

Not only reliability analysis of engineering system but also supply chain risk analysis can use fault tree (Aqlan and Lam, 2015). In the supply chain risk identification stage, most tools are qualitative. Fault tree analysis is not used a lot in this field. Some opportunities may exist (Hunter, 2009). In the area of predicting supply chain risks, “Data Mining” and “Failure Mode Effect Analysis (FMEA)” are popular methods. During FMEA processes, when some critical effects are found, fault tree analysis helps to analyze causes from the lowest level (Zsidisin and Ritchie, 2008). From existing literature, we only find Sherwin et al. (2016) represent a system of suppliers within a low volume high value supply chain as a fault tree to identify risks and make mitigation strategy. In their paper, when they build fault tree, they have not considered the dependency and interplay between basic events which triggers the risk of supply chain. However, in the modern supply chain, production information sharing and other interactions take place at any time. One static fault tree could not integrate diverse failure modes by itself. By comparison, DFT can expresses interplay which changes over time. However, a wide body of research using DFT focuses on reliability analysis of a complex engineering
system or computer system, such as aircraft power supply system, fault-tolerant flight control system, floating offshore wind turbine (Huang et al., 2012; Yiping and Minghua, 1999; Zhang et al., 2016). As a result, a potential research area of supply chain risk analysis using DFT exists. In this paper, we choose to use DFT to model supply chain risk.

There are two main methods used to solve dynamic fault trees, analytical method and simulation method. For analytical method, the Markov models are commonly used to solve DFT. Boudali et al. (2007) present how to use input/output interactive Markov chains to solve dynamic fault trees. However, the Markov model is complicated and time consuming when the number of basic events of DFT is growing. Because the number of states and transition rates increase exponentially when the number of basic events increase. Therefore, an efficient approximate Markov model is suggested (Yevkin, 2015). But this efficient method cannot ensure good accuracy of calculation. Some other analytical methods are developed to solve DFT. Generating the minimal cut set or sequence use a zero-suppressed binary decision diagrams (Tang and Dugan, 2004; Cui et al., 2013). Besides using minimal cute sets or sequence, a new Bayesian network approach and a tool which can translate DFT to Bayesian networks are created (Boudali and Duga, 2005; Montani et al., 2006). Compared with analytical method, simulation method can conquer all limitations of analytical method. Monte Carlo simulated-based approach is presented to solve DFT (Rao et al., 2009; Dai et al., 2011; Zhang and Chan, 2012).

Simulation method is utilized to solve the dynamic fault tree because of the following reasons. First, it is difficult to include test and maintenance information in Markov models. Second, when we generate minimal cut set for dynamic fault tree, we have to make independent assumption which is not accurate for complex system. Finally, simulation method can deal with non-exponential distributions for time to failure and repair of basic components. In order to simplify simulation processes, MatCarloRe, an integrated fault tree and Monte Carlo Simulink tool has been developed. But this tool only handles with
exponential distribution, Weibull distribution and constant distribution (Manno et al., 2012).

In this article, we employ the idea of reliability analysis by using DFT on supply chain risk analysis. Through building the DFT of supply chains, the logic relationship between suppliers, inventory and information systems is represented by dynamic gates. The probability of failing to produce product in supply chain and the actual delivery time are estimated by different simulation methods.

3.3 Model

This paper constructs different dynamic fault trees for two typical supply chains, the main-backup supply chain and the mutual-assistance supply chain by using four traditional dynamic gates and a new innovative dynamic gate. After this, the failure rate of each supply chain is calculated for two production scenarios, manufacturing one unit of product and manufacturing several units of product. We also obtain overall delivery time and the total units produced for each supply chain from simulation results.

3.3.1 Main-Backup Supply Chain

We consider a main-backup supply chain in which a single supplier provides product to a firm. During the production process, for the main supplier, it is inevitable to have a failure due to disruptions. Natural disasters, such as earthquakes or floods, or human errors, such as improper operations, may cause a disruption (Li et al., 2013; Rose et al., 2011; Staw, 1980). The model assumes an information system automatically relays the status of the main supplier to its customer, the firm. After the firm receives information that the supplier has production difficulties, the firm contacts a backup supplier who can deliver product. The backup supplier may also experience failures, however. The firm may also have inventory to help meet the supply difficulties. The information system could also fail to inform the firm of the main suppliers difficulties. If the information
system fails to delivery messages, the failure problems in supply chain will not be tackled.

This model can be applied to a low volume or high volume supply chain. In a low volume supply chain, such as airplane manufacturing or the nuclear power industry, the supplier only needs to produce a single unit of product. For a high volume supply chain, such as a food supply chain or the automobile industry, several units of product are required.

The static fault tree consists of AND, OR gates. Dynamic fault tree introduces dynamic gates in reliability modeling. Normally, a dynamic fault tree usually uses static gates and dynamic gates in combination like what we have done in this paper. The special use of dynamic gates is modeling interactions in a complex system for reliability analysis. Dynamic gates are the priority AND (PAND) gate, the sequence enforcing (SEQ) gate, the functional dependency (FDEP) gate and the spare (SPARE) gate. For example, it is not enough that all events fail together to make PAND gate fail. PAND gates failure is sequence-dependent. Figure 3.1 shows all dynamic gates (Rao et al., 2009).

![Dynamic Gates](image)

Figure 3.1: Dynamic Gates

In this model, we consider one main supplier, one backup supplier, information system, and inventory in the supply chain. In Figure 3.2, a dynamic fault tree of this model has one PAND gate, which has two basic events, the information systems failure and the main suppliers failure. The trigger event of functional dependency gate (FDEP) is the information systems failure, and the dependent event is the backup supplier. The principal component is the main supplier, and the spare component is the backup supplier in
The three basic events for the sequence enforcing gate (SEQ) are the main suppliers failure, the backup suppliers failure and inventory’s failure. Event A represents the information system’s failure. Event B represents the main supplier’s failure. Event C represents the backup supplier’s failure. Event D represents the inventory’s failure.

The PAND gate captures a failure of the output event when all basic events have failed in a pre-assigned order (from left to right in graphical notation). In the supply chain model, if the information system fails (event A) before the main supplier fails (event B), the information system will not alert the firm of the main suppliers difficulty, which means the backup supplier will not be alerted to replace the production. However, if the main supplier fails before the information system fails, the information system will function correctly to alert the firm and the backup supplier. The PAND gate captures this relationship because the PAND gate only induces failure if A fails before B, but
the system functions if B fails before A. The FDEP gate depicts when the trigger event happens the dependent events are supposed to occur. In this model, if the A happen, the backup supplier will fail to be activated by A. The backup suppliers failure (event C) is triggered by occurrence of A, which relationship is represented by FDEP gate. The SPARE gate can fail only when the number of surviving components is less than the minimum required (Manno et al., 2012). The SPARE gate models one or more principal components that can be replaced by one or more redundant components. In this supply chain, when information systems work normally, if the main supplier fails to produce product, the backup supplier will be activated to work to replace the main supplier. If both the main supplier and the backup supplier cannot produce production, the supply chain will get a failure. At least one supplier can work, then there is no failure in supply chain. The SPARE gate captures this relationship of the main supplier and the backup supplier. Therefore, all of three dynamic gates, the SPARE gate, the FDEP gate and the PAND gate, have failed simultaneously, which will give rise to supply failure, so there is a AND gate connected this three dynamic gates. In reality, the inventory is always used as a redundant supplier in a supply chain. We use SEQ gate to represent that all basic events have to fail in a particular order. Other different failure sequence could never happen. In this model, if the main supplier fails, and the information system still works, then the backup supplier will begin to work to support the supply chain. Only after the backup supplier fails, we could use inventory to provide products to the end of supply chain. No matter whether the AND gate fails or the SEQ gate fails, there must be failure in the supply chain. There is OR gate connecting the AND gate and the SEQ gate in the dynamic failure tree.

3.3.2 Mutual-Assistance Supply Chain

Companies often have two suppliers to manufacture the same product simultaneously (Sculli and Wu, 1981; Chung et al., 2010). If one supplier fails, the other supplier may be
able to increase its production quantity or production rate. In a closely integrated supply chain, an information system could provide information about the status of each supplier to the firm and to each other. We name this relationship a mutual-assistance supply chain. The unique relationship of two suppliers is mutual help and simultaneous work. The mutual-assistance supply chain could also apply to two facilities owned by the same company that produce the same product. Since a single company directs both facilities, if one facility encounters disruption difficulties, the other facility could quickly be alerted and increase its production. In this supply chain, we do not know the failure sequence between two suppliers. Because the specific failure sequence is randomly generated from simulation. It is hard for us decide a particular order of SEQ gate when two suppliers and inventory are considered. We simplify the problem in the mutual-assistance supply chain by not including inventory.

If both suppliers manufacture a single unit for a low volume supply chain, the two suppliers work independently to produce same product, but each supplier might have a different due date. If one supplier fails, the second supplier will not be able to change its production plan if the information system fails to relay the failure of the first supplier to the second supplier. The dynamic fault tree for the mutual assistance supply chain is constructed according to different manufacturing scenarios and the structure of supply chain.

Within a dynamic fault tree, the SPARE gate is used to model the relationship between the main supplier and the backup supplier. In the prior model of the main-backup supplier, if the main supplier fails, the backup supplier will start to work, but the two suppliers in the mutual-assistance supply chain work simultaneously. Other dynamic gates cannot model the relationship between the two suppliers either. We design a new dynamic gate, named the mutual-assistance gate (MA), to represent the relationship between suppliers. The MA gate is shown in Figure 3.3.

The MA gate can fail only when both basic events fail. When one of basic event
fails, the likelihood of failure of the other basic event will increase because the supplier is increasing its production quantity or production rate.

In this model, we consider two suppliers and information system in the supply chain. In Figure 3.4, the dynamic fault tree has one PAND gate with three basic events; the failure of the information system (event A), the failure of one supplier (event B), and the failure of the other supplier (event C). Similar to the main-backup supplier model, if the information system fails before either supplier fails, then the operating supplier will not receive the updated status of the failed supplier. This is modeled using the PAND gate, but since both suppliers are working simultaneously, two suppliers are connected via an OR gate. The FDEP gate is triggered by the information systems failure, and the dependent events are two suppliers failures. These two suppliers are the backup suppliers to each other in MA gate. All three dynamic gates, the MA gate, the FDEP gate and the PAND gate, need to fail in order for the firm to fail to receive its product, and an AND gate connects these three dynamic gates. In Figure 3.4, event A represents the information system’s failure. Event B represents one supplier’s failure. Event C represents the other one supplier’s failure.

3.4 Illustrative Example

As is typical with dynamic fault trees, we use simulation to measure the reliability of the supply chain. The number of times an event happens is counted to measure the reliability (Verma et al., 2010). We use different simulation methods for the two different
3.4.1 Simulation Methods for Main-Backup Supply Chain

A state time diagram helps to illustrate how failure occurs for the main-backup supply chain producing one unit of product. The time between each failure for the supply chain can be calculated from the state time diagram. The state time diagram is generated from the dynamic fault tree and failure rate of each basic event in the fault tree. If the supply chain produces several units of product, we use the simulation to calculate the number of units of product that are provided and the frequency in which the product satisfied requirements. We also calculate the time to delivery in order to capture situations in which a supplier fails and recovers but may deliver product late.

3.4.1.1 Manufacturing One Unit of Product

In the main-backup supply chain, the state time diagram illustrates when failure occurs in the supply chain. Available and unavailable status of each component in
supply chain can be visually depicted by up and down states in state time diagram. In our model, if the information system (A) fails before the main supplier (B) fails, it will cause the failure of the PAND gate. Otherwise, it will not cause any failure. The state time diagram of the PAND gate is depicted in Figure 3.5.

In the SPARE gate, for the active component, the main supplier, we generate time to failure and time to repair according to probability distributions. The spare component, the backup supplier, has different failure rates depending on the state of the main supplier. When the main supplier does not fail, the failure rate of the backup supplier is not affected by main supplier. If the main supplier fails, the backup supplier will be activated, and it could fail due to attempting to increase its production rate. We assume the probability of failure of the backup supplier (C) follows an exponential distribution with a failure rate equal to $\alpha \lambda$ ($0 < \alpha < 1$) given the main supplier (B) operates normally. The failure rate of the backup supplier equals $\lambda$ given the failure of the main supplier. The failure rate of the backup supplier increases if the main supplier fails. The state time diagram of the SPARE gate is depicted in Figure 3.6.

In the FDEP gate, the trigger event is information systems failure (A), and the dependent event is the backup suppliers failure (C). When the information system is in the down state, the backup supplier must be in the down state. If the trigger event does
not occur, the state of the dependent event cannot affect the trigger event. The state time diagram of the FDEP gate is depicted in Figure 3.7.

In the SEQ gate, the basic events must fail in a specific order. In our model, the main supplier fails first (B), the backup fails second (C), and inventory fails third (D). The simulation uses the failure rate $\lambda$ to generate times to failure based on the exponential distribution. First we generate time to failure ($TTF_B$) and time to repair ($TTR_B$) of the main supplier. If the main supplier fails, the backup supplier begins production for the firm. We generate time to failure ($TTF_C$) and time to repair ($TTR_C$) of the backup supplier. If the backup supplier fails, inventory could be used. We generate time to
Table 3.1: Simulation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>Total simulation time</td>
<td>86400 hours (= 10 years)</td>
</tr>
<tr>
<td>Mean time to failure of each component</td>
<td>200 hours</td>
</tr>
<tr>
<td>Mean time to repair of each component</td>
<td>48 hours</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.5</td>
</tr>
</tbody>
</table>

failure ($TTF_D$) and time to repair ($TTR_D$) of inventory. If the three basic events of the SEQ gate are in down states, it will a failed state of the SEQ gate.

The dynamic fault tree of the main-backup supply chain connects the SPARE gate, the FDEP gate, and the PAND gate via an AND gate. A failure only occurs if there is a failure in PAND, the FDEP, and the SPARE gates. As depicted in Figure 3.2, the OR gate connects failure from the information system and backup supplier relationship with the SEQ gate that orders the main supplier, backup supplier, and inventory (Figure 3.8). Since this is a notional example to demonstrate the applicability of the dynamic fault tree to supply chain risk, we assume the probability of failure of each component in supply chain follows identical exponential distributions. Table 3.1 shows simulation parameters for each component in supply chain.

From the simulation, the supply chain exhibits complete failure 21 times over these 10 years. There are 10 times caused by AND gate. The AND gate connects PAND gate, FDEP gate and SPARE gate. The AND gate fails, which means all there dynamic gate have failures at the same time. There are 11 times caused by SEQ gate. The SEQ gate has a failure, which means three basic events have common failure time. Figure 3.9 depicts a histogram of time to failure in this main-backup supply chain.

The mean time to failure of the main-backup supply chain is 3968 hours which is 166 days. The standard deviation is 3341 hours which is 140 days. The shortest time to failure is 9 hours which is less than one day. The longest time to failure is 11540 hours which is 481 days.
3.4.1.2 Delivery Time of Manufacturing One Unit of Product

We are interested in using this simulation to calculate the delivery time for the product. Since inventory is not delivered, we ignore the role of inventory in this section. We only consider the failure of suppliers and the failure of the information system, so we only use a part of the dynamic fault tree for analyzing, as depicted in Figure 3.10. Event A represents the information system’s failure. Event B represents the main supplier’s failure. Event C represents the backup supplier’s failure.

As discussed earlier, we measure the time to failure and time to repair for the two suppliers and the information system. If a supplier fails after the standard delivery time, the actual delivery time is equal to the standard delivery time. If a supplier fails and then recovers, we assume the supplier can increase its production speed in order to make up for lost time. After a failure occurs, a supplier will increase the production speed.
Figure 3.9: Histogram of Simulated Time to Failure

Table 3.2: Simulation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total simulation time</td>
<td>86400 hours (= 10 years)</td>
</tr>
<tr>
<td>Mean time to failure of each component</td>
<td>200 hours</td>
</tr>
<tr>
<td>Mean time to repair of each component</td>
<td>48 hours</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.5</td>
</tr>
<tr>
<td>Standard delivery time</td>
<td>200 hours</td>
</tr>
<tr>
<td>$k$</td>
<td>0.5</td>
</tr>
</tbody>
</table>

The parameter $k$, where $0 < k < 1$, is used to represent that the delivery time will be shortened after the supplier recover. If the time to failure of a supplier is less than the standard delivery time, the actual delivery time is calculated by equation (3.1).

$$
\text{actual delivery time} = \text{time to failure} + \text{time to repair} + k \times (\text{standard delivery time} - \text{time to failure})
$$

(3.1)

Table 3.2 shows simulation parameters for each component in supply chain. We set the standard delivery time is 200 hours. During the total simulation time, we can get a set of actual delivery time for each supplier. Figure 3.11 and Figure 3.12 are the
The mean actual delivery time of the main supplier is 195 hours which is 8 days. The standard deviation is 44 hours. The mean actual delivery time of the backup supplier is 192 hours which is 8 days. The standard deviation is 54 hours. The simulated overall delivery time is shown in Figure 3.13. The mean actual overall delivery time is 189 hours which is 8 days. The standard deviation is 40 hours.

3.4.1.3 Manufacturing Several Units of Product

For manufacturing several units of product, we use a different simulation method to estimate the failure rate of the supply chain. The total simulation time is 43,200 hours or 5 years. The duration of each trial is 720 hours, which is equivalent to one month. The goal of this main-backup supply chain is to manufacture 1000 units, and the main supplier can produce a maximum of 1000 units. The following steps show the logic of a single trial:

We generate the time to failure of the information system ($TTF_A$) and the time to failure of the main supplier ($TTF_B$).
Figure 3.11: Simulated Actual Delivery Time of Main Supplier

Figure 3.12: Simulated Actual Delivery Time of Backup Supplier
(1). If $TTF_B >$ one month, the main supplier will produce 1000 units.

(2). If $TTF_B <$ one month,

(i) If $TTF_A > TTF_B$, the backup supplier will work immediately when the main supplier fails. If the main supplier recovers within a month, it produces 500 units. If the main supplier does not recover within one month, the amount the main supplier can produce is uniformly distributed between 300 and 500. We generate the time to failure of the backup supplier ($TTF_C$).

(a) If $TTF_B + TTF_C <$ one month, the backup supplier will fail within one month, and the number units produced by the backup supplier is uniformly distributed between 200 and 500 units. If the backup supplier fails, inventory can be used. We generate the time to failure of the inventory ($TTF_D$). If $TTF_B + TTF_C + TTF_D >$ one month, the firm can rely on 300 units of inventory. If $TTF_B + TTF_C + TTF_D <$ one month, the inventory may fail within one month, and the firm will have less than 300 units of inventory. (b) If $TTF_B + TTF_C >$ one month, the backup supplier will not fail, and it can manufacture 500 units in a month. Inventory will not be used.
(ii) If $TTF_A < TTF_B$, then the information system fails to alert the firm about the main suppliers' difficulty. There is a delay of 24 hours before the backup supplier works to meet the order unfulfilled by the main supplier. The length of delay is 24 hours. If the main supplier can recover in a month, it produces 500 units. Otherwise, the units of product produced by the main supplier is uniformly distributed between 300 and 500. We generate $TTF_C$. From here, the logic is identical to that of (i)(a) and (i)(b).

We calculate the total units from the main supplier, the backup supplier, and inventory in each trial. If the total units of product less than 1000 units, the supply chain cannot achieve the goal. The simulation parameters are shown in Table 3.3. The histogram of the total units produced is depicted in Figure 3.14. The average number of total units is 1137 units, and the standard deviation is 87 units. Among all trials we simulate, five trials total units do not meet the requirement. Eight percent of trials fails to satisfy the production goal. This simulation demonstrates how a firm can use this dynamic fault tree to understand how likely it is that the combination of suppliers and inventory will fail to meet the requirement of 1000 units.

3.4.2 Simulation Methods for Mutual-Assistance Supply Chain

In the mutual-assistance supply chain, we use different simulation methods for the low-volume supply chain (one unit) and the high-volume supply chain (hundreds of units). In the low-volume supply chain, we measure the frequency of failure and also calculate whether or not the supply chain can meet the due date. A firm wants to know
when it will receive a product. If a supplier misses the due date, the supplier has failed.

3.4.2.1 Manufacturing One Unit of Product

As depicted in Figure 3.4, the dynamic fault tree for the mutual-assistance supply chain employs the new MA gate and does not include the SPARE and SEQ gate. The basic events are the main suppliers failure, information systems failure and the backup suppliers failure. The state time diagram of the MA gate has some unique properties. In the mutual-assistance supply chain, the two suppliers are both actively producing for the firm. At the beginning of the simulation, the two suppliers have the same failure rate. If one supplier fails, the other suppliers failure rate will change due to assistance relationship between two suppliers. We assume the probability of failure of each supplier follows an exponential distribution, and the failure rate of one supplier is $\lambda$ given the other supplier is operating normally. The failure rate of a supplier is $\lambda/\beta$ ($0 < \beta < 1$) given the other supplier fails. The state time diagram of the MA gate is depicted in Figure 3.15. Event B represent one supplier’s failure. Event C represents the other one supplier’s failure.
Under the situation of manufacturing one unit of product, combining state time diagrams of the PAND gate and FDEP gate and MA gate, we can draw the compound state time diagram of each supplier. From the compound state time diagram of a supplier, we obtain the time to failure and time to repair. On the basis of the standard delivery time, the time to failure and the time to repair, we estimate the actual delivery time by using equation (3.1). By comparing the actual delivery time and upper bound of delivery time, we judge whether one unit of product can be delivered on time and identify failures of suppliers. Finally, based on the number of failures and duration of simulation time, the failure rate is calculated. Table 3.4 shows the simulation parameters.

The simulated actual delivery time of B supplier is shown in Figure 3.16.

The simulated actual delivery time of C supplier is shown in Figure 3.17.

The mean of actual delivery time of B supplier is 153 hours which is 6 days. The standard deviation is 40 hours. The total failure times of B supplier is 29. The mean of
Figure 3.16: Histogram of Simulated Actual Delivery Time of B Supplier

Figure 3.17: Histogram of Simulated Actual Delivery Time of C Supplier
actual delivery time of C supplier is 155 hours which is 6 days. The standard deviation is 43 hours. The total failure times of C supplier is 35. The actual overall delivery time of the supply chain is depicted in Figure 3.18. The mean of actual overall delivery time is 143 hours which is 6 days. The standard deviation is 27 hours.

### 3.4.2.2 Manufacturing Several Units of Product

Under the situation of manufacturing several units of product, we take the following simulation method. As in the previous main-back supplier model, the total simulation time is 43,200 hours, or 5 years, and the duration of each trial is 720 hours, or one month. If there is no failure, a supplier produce one unit of product per hour. According to the compound state time diagram of a supplier, we calculate the duration of down state. Besides, we know the duration of periods when one supplier is assisting the other one supplier. We assume that a supplier does not produce any product in down state. When a supplier is increasing production to give assistance, this supplier produced 1.5 units of product per hour. In every trial, the total units of product produced by a supplier is
Table 3.5: Simulation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total simulation time</td>
<td>43200 hours (= 5 years)</td>
</tr>
<tr>
<td>The number of trials</td>
<td>60</td>
</tr>
<tr>
<td>The duration of each trial</td>
<td>720 hours</td>
</tr>
<tr>
<td>Mean time to failure of each component</td>
<td>120 hours</td>
</tr>
<tr>
<td>Mean time to repair of each component</td>
<td>24 hours</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.5</td>
</tr>
</tbody>
</table>

calculated by equation (3.2).

\[
\text{total units} = \text{1 unit/hour} \times \text{duration of each trial} - \text{1 unit/hour} \times \text{down time} + 0.5 \times \text{1 unit/hour} \times \text{assistance period} \tag{3.2}
\]

Figure 3.19 represents equation (3.2).

Table 3.5 shows the simulation parameters. The simulation results for two suppliers are shown in Figure 3.20. The mean of total units produced by two suppliers is 1318. The standard deviation is 105 units. If the goal is manufacturing 1200 units per month for two suppliers, 8 trials will not meet the goal. Thirteen percent of trails fail to satisfy the production goal.
3.5 Conclusions

This paper constructs dynamic fault trees for supply chain risk analysis. We analyze the main-backup supply chain and the mutual-assistance supply chain. In order to depict complex relationship between every component of supply chain, the PAND gate, the FDEP gate, the SPARE gate and SEQ gate are used. A key aspect of this supply chain is the information system that can quickly relay that the main supplier is having production difficulties. We also create a new dynamic gate, the MA gate, for the mutual-assistance supply chain. The models are illustrated using simulation for a low-volume supply chain and high-volume supply chain. Some simple examples are presented to illustrate simulating process of each simulation way.

Since this paper represents the first representation of supply chain risk using dynamic fault trees, we may make assumptions that do not accurately represent real supply chains. We do not include all kinds of supply chains in our study, so some more complex relationship between basic components of supply chain may be ignored. Despite these limitations, our study provides the dynamic model for supply chain risk analysis. Based
on simulation results, supply chain managers can obtain helpful information for making better production strategies or taking some proactive work to avoid supply chain breakdown.

For future research, we can search some real cases and apply dynamic fault trees and simulation methods on these cases. More suppliers and more complex supply chain can be analyzed by using our methods. By reviewing more real supply chains, we can design some innovative dynamic gates for different interactions in supply chains. Additionally, we can build a Simulink library and form some blocks which represent different dynamic gates, which could make simulation more concise and efficient.
BIBLIOGRAPHY


