Three essays on commodity markets

Ziran Li
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Three essays on commodity markets

by

Ziran Li

A dissertation submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Major: Economics

Program of Study Committee:
Dermot Hayes, Co-major Professor
Keri Jacobs, Co-major Professor
Gray Calhoun
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The student author and the program of study committee are solely responsible for the content of this dissertation. The Graduate College will ensure this dissertation is globally accessible and will not permit alterations after the degree is conferred.

Iowa State University
Ames, Iowa

2017

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To Family.
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CHAPTER 1

GENERAL INTRODUCTION

The three essays that constitute this dissertation aim to understand the role of agribusiness organizational structures in competition, the risk management practices of grain producers, and the characteristics of the U.S. corn harvest futures price.

The cooperative (co-op) model is held up as a novel solution to many kinds of market failures. It integrates the business successes and members’ utilities and provides a countervailing force to the market power of investor-owned firms (IOFs). A traditional cooperative business is characterized as being owned and controlled by its member-users, to whom benefits are intended to primarily accrue. The user-benefit principle has given rise to diverse assumptions regarding the objectives of co-ops in the existing literature. And the theoretical literature has yet to reconcile the extent to which operating objectives of a cooperative business deviate from profit maximization. Chapter 2 adds to the literature by developing a model of duopsony competition from which the strategic interactions of a cooperative firm and an investor owned firm (IOF) under output price uncertainty are interpreted. I analyze the way in which the market equilibrium varies as the co-op takes on different objectives.

Crop producers’ risk management practices have long been understood using either survey based data or aggregate trading data. These studies suggest there is limited relevance of Expected Utility (EU) optimal hedging theory as farmers may deviate from rationality. There are two impediments to this line of research. First, hedging theories that rely on alternative utility paradigms may be too complicated to test with data. Second, there is a lack of data on the actual hedging activities of producers. Chapter 3 provides a solution that partially overcomes these two problems. I investigate the role of reference-dependence, a central feature of most utility
paradigms other than EU in the optimal hedging theory under the EU framework. The theoretical predictions facilitate a direct comparison of optimal hedge ratios with and without a reference price. I then test the model’s results with a unique database of forward contracting transactions of Iowa corn producers over a five-year period. The corn producers’ hedging pattern indeed appears to be reference-dependent: more hedges are placed when futures prices rise above the recent price trend. This finding has important implications for future research on grain producers’ marketing practices because if the futures markets are efficient, price-based triggers as a motivation for hedging may not be beneficial to farm income.

A well-known phenomenon in the corn futures market, weather premium, suggests that producers may enhance their marketing strategies by forward contracting early in the season. This is because the commodity futures market for grain over-predicts the actual harvest price more often than not. Chapter 4 formally defines the weather premium, and recovers the potential weather premium in the corn futures market. I show theoretically that the size of weather premium depends on the expected supply at harvest, which consists of the carryout from last year and the expected new harvest. These two covariates partially explain the variation of the forecast error of the December futures contract price from 1968 to 2015. However, the existence of weather premium does not imply the biasness in the futures, i.e. risk premium. The Sharpe ratio of the passive strategy of routinely shorting the corn December futures in spring is too small to justify such an approach.
CHAPTER 2

DUOPSONY COMPETITION FOR GRAIN AND PRICING STRATEGIES OF AGRICULTURAL MARKETING COOPERATIVES

Abstract

This article reconciles theories of firm pricing behavior when a cooperative and non-cooperative firm engage in competition for grain. Standard theoretical models of firm behavior require the researcher to choose an operational motive for the firm, and a standard assumption is profit maximization. The unique governance and economic participation features of cooperatives suggest, however, that not only is profit maximization an unlikely objective, choosing a single objective for a cooperative firm may not be appropriate. A cooperative’s objective reflects the degree to which it is integrated with its member-owners, thus integrating buyers and sellers. We propose a theoretical model of duopsony under uncertainty from which the pricing behaviors and market share outcomes of cooperative firms with their investor-owned counterparts can be compared. Our specific focus is on characterizing the market equilibriums for the cooperative firm with varying degrees of integration with its member producers.
Introduction

Cooperatives are integrated with their customers (members) in a way that their non-cooperative counterparts are not. The hallmarks of a cooperative business are member-ownership, member-control, and benefits derived primarily to members on a proportional basis. That is, the cooperative firm is governed by and capitalized by those who use its services, and those who use it also share in the resulting profits and losses. This unique organizational structure suggest that cooperatives may not strictly seek profit maximization. Instead, the operating objectives of cooperatives are commonly perceived on a spectrum, from maximizing co-op’s profit (e.g. Sexton, 1990), to vertical integration with members (e.g. Sexton and Iskow, 1993), to maximizing members’ on-farm profit (e.g. Albæk and Schultz, 1998). Soboh et al. (2009) provide a recent review of cooperative objectives. Each of the candidate objectives reflects an aspect of cooperative’s business model, and the differences among them reside in the implicit assumption about the degree of interconnection between members and the cooperative.

The effects on prices and market shares of the integration with members and the co-op’s operational choices are best viewed through their strategic interactions with investor owned firms (IOFs). Previous literature has investigated interactions and outcomes of cooperative firms and IOFs. Karantininis and Zago (2001) and (Fulton and Giannakas (2001), study the role of the cooperative in promoting competition and Giannakas and Fulton (2005) identify the role of competition on innovation. A missing component in this literature is an explicit linkage of the degree of integration between the co-op and its producer-members with implications for equilibrium pricing and market shares of competing firms in an imperfectly competitive market.

In this article, we build a theoretical model that is sufficiently flexible to incorporate a range of operational objectives of a marketing cooperative, and then derive the equilibrium price and
market share outcomes under different objective regimes based on strategic interactions of a cooperative firm with an IOF. The competition environment is structured as a duopsony where two grain marketing firms – one a profit-maximizing IOF, the other an open-membership cooperative – engage in price competition to buy grain from a risk-averse representative member. The member is assumed to make a crop allocation decision by maximizing the expected utility of his crop sale. The member experiences some uncertainty in this decision because the cooperative profits or losses are part of his expected function, the IOF profits or losses are not.

Our analysis explicitly accounts for the level of integration of the cooperative and its members in two ways. First, we allow the cooperative’s objectives to vary along a spectrum: the co-op is considered more integrated as its objective moves towards maximizing the member’s on-farm profit, and less so if it behaves more like a profit-maximizing firm. Secondly, we assume a random final product price at the time firms make their input purchase decisions. The uncertain nature of the cooperative’s business has implications for the cooperative’s pricing behavior, but it also influences producers’ decisions about whether to transact with the cooperative because of the linkage between producers and cooperative through patronage. In other words, the relative risk appetite of cooperative to its member affects the equilibrium outcomes.

1 Open membership is standard assumption in cooperative literature; it presumes that the farmer-member is not committed to selling through the cooperative.
This game characterizes the situation in which a group of similar risk-averse farmers countervails the market power of an IOF through a marketing co-op, without the existence of which the IOF would be the only game in town and would pay zero dollar for members’ crop. Members in turn have to face the uncertainty of the cooperative’s profit, which they will receive later as patronage refunds. The model results shed lights on an important disconnect in the cooperative literature between theories and empirical observations. We shows that a co-op has a narrower margin and a smaller market share than the IOF regardless of how it weighs in its objective between the profit of the company and its members. This competitive disadvantage of cooperatives provides one explanation to the continuing restructuring of marketing/supply cooperatives in the U.S. as shown in figure 1. And despite its importance, the cooperative business model has yet to become the dominant form of agribusiness in the United States measured by the market share.

![Figure 1. Marketing Cooperatives: Sizes, Sales and Shares](image)

*Note.* The aggregate time series plots show the characteristic facts of the U.S. agricultural marketing cooperatives in 1951 - 2007. The number of co-ops series has the y-axis labeling on the left. The net sales series has the y-axis labeling on the right. The market share series has y-axis in the unit interval. Data Sources: United States Department of Agriculture, Rural Development, Cooperatives Historical Data; United States Department of Agriculture, Economic Research Service, Farm Income and Wealth Statistics.
Empirical literature aiming to explain the continuing restructuring of agricultural co-ops and the diminishing market share have focused on co-op’s operating inefficiencies (Sexton et al., 1989; Crooks, 2001; Hailu et al., 2007) and capital constraints (Featherstone et al., 1995; Chaddad et al., 2004; Soboh et al., 2012; Li et al., 2015). These literature often take an agnostic approach with regard to the relationship between the co-ops’ objectives and their operating decisions, which limits the implications of relative importance of different factors that contribute to the secular decline of the cooperative as a business model in the grain marketing industry.

In the next section, we describe the model framework that encompasses the price competition between a marketing co-op and an IOF, and the allocation decision of a representative producer under uncertainty. We then analyze the equilibrium outcome as the result of the strategic interactions among the firms and the producer, and how it changes as the co-op’s objective varies. Finally, we conclude the article by comparing our model to the Cook’s (1995) seminar paper that describes the challenges to traditional agricultural cooperatives via the neo-institutional approach.

**A Duopsonistic Market for Agricultural Output with a Cooperative and IOF**

We consider a three period model of duopsony in which two processors – a cooperative and an IOF – engage in price competition in the local market for producers’ grain. In the first period, the cooperative and the IOF simultaneously announce the price they will pay for any amount of grain. In the second period, the producer’s crop is allocated by selling a portion of grain to each processor. In the third period, the price of the processed commodity is revealed and the firms’ profits are realized. The important linkage in this model occurs between the cooperative processor and the producer. In this model, the cooperative’s profit is assumed to be
fully distributed to the representative producer who allocated grain to the cooperative in the second period. This linkage is understood when the crop allocation decision is made, and the producer incorporates uncertainty of the firm’s profits in his decision problem.

The model is solved by backwards induction. In what follows, we describe the producers’ allocation decision and the processors’ production technology. We derive the optimal pricing response functions for both firms when the unknown cooperative objective can include a spectrum of operational objectives ranging from maximizing the firm’s profits to maximizing the producers’ profitability.

**Producer’s crop allocation decision**

A representative producer makes his crop allocation decision in period two based on the announced prices of the two processors in period one. The producer can allocate any portion of his grain, \( \delta \), to the cooperative and receive price \( w_c \). He receives price \( w_p \) for the amount of grain, \( (1 - \delta) \), allocated to the IOF. Selling grain to the cooperative necessarily results in a return of the cooperative’s profitability in the form of patronage in the third period, which can be positive or negative. The producer maximizes his expected utility from allocating grain between the cooperative and IOF with uncertainty over the profit of the cooperative. We assume a constant absolute risk aversion (CARA) form of utility, \( u(y) = -e^{-\rho y} \), where \( \rho \) is the producer’s coefficient of absolute risk aversion. The producer’s income, \( y \), takes the form:

\[
y = (1 - \delta)w_p + \delta w_c + \beta \pi(w_c, \delta, p),
\]

where \( \pi(\cdot) \) is the cooperative’s profit and \( \beta \in [0,1] \) is the discount factor applied to the patronage received in the following period. The producer may not regard the cash payment and patronage refund as the same because of his time preference and risk attitude. Patronage refund today is
always better than tomorrow, because as long as it is at the co-op, it is at risk for the co-op’s profit is uncertain. The cooperative’s profit is a function of the grain price, its allocation of the farmer’s grain, and the price of the final product, \( p \). This function is detailed in the following section. We impose normality of the cooperative’s profit, implying the producer’s expected utility of income, \( E(u(y)) \), can be expressed \( E(y) - \frac{1}{2} \rho Var(y) \) where \( E(\cdot) \) and \( Var(\cdot) \) denote the mean and variance, respectively. The producer’s problem is:

\[
\max_{\delta \in (0,1)} \left\{ (1 - \delta)w_p + \delta w_c + \beta E \pi(w_c, \delta, p) - \frac{1}{2} \rho \beta^2 \text{var}(\pi(w_c, \delta, p)) \right\},
\]

Use \( \pi \) as the abbreviation for the cooperative’s profit function, the producer’s optimal allocation decision satisfies:

\[
w_c - w_p + \beta \partial E\pi / \partial \delta^* - \frac{1}{2} \rho \beta^2 \partial \text{var}\pi / \partial \delta^* = 0
\]

This modeling makes explicit that producers must invest in the cooperative to use it and incorporates Staatz’s (1987) recognition that the decision to use a cooperative deepens a producer’s financial commitment to a single firm rather than diversifying it. Specifically, the producer needs to tradeoff between the benefit of patronizing with the co-op and risk associated with the patronage refund. We describe a simple technology function in the next section for the marketing firm that will make this tradeoff mathematically straightforward.

**Processors’ production technology, profit and market shares**

Firms face identical production technologies that use grain and other inputs to produce a homogenous final product. We assume the firms’ production is Leontief with respect to grain and requires fixed proportions of all other inputs. Formally, the firms’ production is:
(4) \[ q = \min \{ x_0, f(x_1, \ldots, x_k) \}, \]

where \( q \) is output quantity, \( x_0 \) is the quantity of grain purchased from producers, and \( x_1, \ldots, x_k \) are other inputs purchased from competitive markets at exogenous prices. Firms optimally employ non-grain inputs such that \( x_0 = q \), and the firms’ cost functions are:

(5) \[ C(q, w_i) = w_i q + cq, \]

where \( i \) denotes cooperative or IOF and the constant \( c \) is the minimized cost of the inputs \( x_1, \ldots, x_k \) required to produce one unit of output. Constant marginal costs of storing, processing, and marketing the final product derives from homogeneity of the production function.\(^2\) The final good of each firm is homogeneous and sold in a competitive market at price, \( p \), and is assumed to follow a normal distribution, \( p \sim N(\bar{p}, \sigma^2) \). The price distribution is known by both firms but the price is not known until the third period. The resulting profit function is:

(6) \[ \pi_i(q, w_i, p) = (p - c - w_i)q, \quad s.t. \quad 0 \leq w_i \leq \bar{p} - c. \]

The non-negativity constraint on firms’ expected profits, regardless of their operational objectives, is necessary to ensure they do not operate with expected losses, which will result in shut down. It is

\(^2\) Suppose the production function is homogeneous of degree \( \gamma \) with respect to \( x_1, \ldots, x_k \). Denote \( \tilde{C}(q) = \min \sum_{i=1}^{k} x_i w_i \), subject to \( f(x_1, \ldots, x_k) \geq q \). The cost function satisfies \( \tilde{C}(q) = (q) = \tilde{C}(1) \frac{1}{q^\gamma} \). Denote \( c = \tilde{C}(1) \), which is a function of the exogenous input prices \( w_1, \ldots, w_k \), hence a constant.
possible that both firms realize negative profits without this constraint, however, because of bad realization of the output price.

Under the Leontief technology, firms’ output level is equal to the amount of grain they are able to buy from the producer, which is also their respective market shares as the total amount of grain is normalized to one. Thus we can write the expected profit and its variance as

\[ E[\pi(\delta, w_c, p)] = (\bar{p} - c - w_c)\delta, \text{ and } Var[\pi(\delta, w_c, p)] = \sigma^2\delta^2 \] respectively. Substitute the expected profit and variance of the profit of the cooperative firm into equation (3), we obtain the producer’s optimal allocation to the co-op:

\[ \delta^*(w_c, w_p) = \frac{w_c - w_p + \beta(\bar{p} - c - w_c)}{\rho \beta^2 \sigma^2} \]

and \(1 - \delta^*(w_c, w_p)\) is the market share for the IOF.

**IOF’s price response**

An IOF and cooperative that engage in price competition in the grain market do so by deriving response functions for grain prices they will announce to the producer. Both firms consider the other’s pricing decisions, and know the optimal allocation function of the producer conditional on the mill price offers. The risk neutral IOF chooses its grain price, \(w_p\), based on what it expects the cooperative firm’s grain price to be. The IOF’s expected profit maximization problem is:

\[ \max_{w_p} (\bar{p} - c - w_p)[1 - \delta^*(w_c, w_p)]. \]

The producer’s possible allocation scenarios are given in equation 7, and these will determine the extent of the market. Either the IOF receives all the grain (\(\delta^* = 0\)), the cooperative does (\(\delta^* = 1\)), or the two firms split the market (\(0 < \delta^* < 1\)). When the latter holds, the expected profit of
the IOF is differentiable and globally concave with respect to its price, \( w_p \), and the IOF’s price strategy is obtained from the first-order condition:

\[
(9) \quad w_p^* = a_p + b_p w_c,
\]

where \( a_p = \frac{1}{2} [(1 + \beta)(\bar{p} - c) - \rho \beta^2 \sigma^2] \),

\[
b_p = \frac{1}{2} (1 - \beta),
\]

s.t. \( w_c \in (\frac{\bar{p} - c - \frac{\rho \beta^2 \sigma^2}{1+\beta}}{w_c}, \frac{\bar{p} - c + \frac{\rho \beta^2 \sigma^2}{1+\beta}}{w_c}) \).

The range of the co-op’s price offer is to ensure that the IOF’s price as it depends on that of the co-op, to be nonnegative and the resulting expected profit is also greater than or equal to zero. Specifically, if \( w_c < \underline{w_c} \), the IOF will capture the entire market share; if the co-op’s price offer is too high: larger than \( \bar{w}_c \), then the IOF will choose to exit the market. The complete solution to equation 8 is provided in the Appendix A. The corner solution of this model echoes the idea of entry deterrence in the endogenous market structure literature (Hueth and Moschini, 2014). For example, even if the co-op’s price offer is too low such that IOF becomes the only firm in the market, facing the threat of entry, IOF’s offer price must set the price greater than \( \beta (\bar{p} - c) + (1 - \beta) w_c \), namely the entry deterrence price.

The co-op’s role as a competitive yardstick is best seen when \( \beta = 1 \), the producer and co-op is vertically integrated. In this situation, the co-op is just an extension of members’ business as if there is only one decision maker. This special case is nested in our model framework, as show in equation 9, the IOF’s pricing strategy will not depend on the price offer by the co-op anymore and the producer’s per unit expected payoff is \( \bar{p} - c \), the marketing margin of the grain.
business. So the optimal pricing by the IOF is \((\bar{p} - c) - \frac{1}{2} \rho \sigma^2\), and substituting (9) into (7) yields the IOF’s market share \(\frac{1}{2}\).

However, a complete vertical integration assumption is neither realistic nor very useful in understanding the co-op’s operating behavior in relation to its IOF counterpart. Thus, in the rest of the article, we focus on the situation where \(\beta < 1\), that is the producer prefers the payment today than tomorrow. To further facilitate the remaining analysis, we impose the following parameter restriction: \((1 + \beta) (\bar{p} - c) = \rho \beta^2 \sigma^2\). This is a special case in the range of firm’s expected marketing margin that supports the interior solution. In the absence of it, most of our findings in the following sections still hold. However, this constraint is appealing for two reasons. First, the best response functions are linear and differentiable, which generates a unique, dominance solvable equilibrium. Second, the assumption implies that the IOF’s best response function evaluated at \(w = 0\) is zero, namely \(a_p = 0\). In other words, the possible entry deterrence price offer by the IOF is normalized to zero. Intuitively, when the co-op offers zero dollar for the producer’s crop, the IOF has no incentive to offer more. The slope \(b_p\) represents the sensitivity IOF’s best response to the co-op’s price offer, which is smaller than one.

**The cooperative’s price response**

The cooperative firm’s pricing strategy hinges on not only the price offer by the IOF, but also its own business objective. Standard modeling presumes the investor-owned firms – private or public – are risk neutral profit maximizers. However, a representative cooperative firm’s preferences and objectives are not easily pegged. Those who use it also control it, capitalize it through use, and share in its profitability. This suggests that the cooperative may not be a profit maximizing firm, but selects from a suite of objectives that range from maximizing producers’
profits to maximizing its own expected profits. Also, as agents for the producer, the cooperative may adopt either a neutral or averse risk appetite. Because a cooperative’s objectives are not transparent, a model that permits flexibility in considering any convex combination of cooperative business objectives in combination with risk preferences is necessary.

The cooperative’s objective function is given by:

\[
\max_{w_c} \alpha \delta^* w_c + (1 - \alpha) \delta^* [(\bar{p} - w_c - c) - \frac{1}{2} \theta \sigma^2 \delta^*],
\]

s.t. \(0 \leq \theta < \beta^2 \rho\), and \(\alpha \in \left[0, 1 - \frac{2}{(1-\beta)\zeta+4}\right]\).

\(\alpha\) weights the trade-off between the cash price paid to producers for grain and the profitability of the firm (future patronage refund paid to producers) and \(\theta\) is the cooperative’s absolutely level of risk aversion.

We impose \(0 \leq \theta < \beta^2 \rho\) to restrict attention to the case where the cooperative is less risk averse than the producer.\(^3\) Define \(\zeta = \theta / \beta^2 \rho\) as the relative risk aversion of the co-op to producers, and it follows from the restriction \(0 \leq \zeta < 1\). Note the producer’s risk aversion coefficient is adjusted for his time preference, which together reflect his attitude toward the risk associated with the patronage refund that will be distributed later in time.

The weighting parameter \(\alpha\) reflects the co-op’s balance between the producers’ cash payment and its own profitability. \(\alpha \in \left[0, 1 - \frac{2}{(1-\beta)\zeta+4}\right]\) is the necessary and sufficient condition for co-op’s objective function being concave.

\(^3\)The cooperative firm pools risk and has access to lower-cost technologies in storage, thus implying though a cooperative may be risk averse, it is less so than the representative producer.
A pair of $\alpha$ and $\theta$ describes a unique cooperative’s objective function that is observable to the producer. The cooperative firm’s altruism towards members is directly signaled via $\alpha$, while the upper bound of $\alpha$ depends on the producer’s time preference and the co-op’s risk attitude relative to the producer. We analyze in the next section how these signals would affect the market equilibrium.

Similar to the IOF’s price response, the co-op’s optimal price offer, $w^*_c$, based on IOF’s grain price is given by:

$$w^*_c = a_c + b_c \cdot w_p$$

(11)

where $a_c = \frac{(\beta-1)\alpha}{(1-\beta)[2(1-2\alpha)+(1-\alpha)(1-\beta)\zeta]}$ and

$$b_c = \frac{(1-2\alpha)+(1-\beta)(1-\alpha)\zeta}{(1-\beta)[2(1-2\alpha)+(1-\alpha)(1-\beta)\zeta]}$$

The varying cooperative’s objectives, governed by parameters $\zeta$ and $\alpha$, affects the market equilibrium prices through its impact on the cooperative’s best response function characterized by $a_c(\zeta, \alpha), b_c(\zeta, \alpha)$. The possible combinations of $\zeta$, $\alpha$ give rise to four distinct objective functions of the cooperative: 1) risk neutral profit maximizer ($\zeta = \alpha = 0$), 2) risk-averse profit maximizer ($\zeta > 0, \alpha = 0$), 3) risk-neutral altruistic firm ($\zeta = 0, \alpha > 0$), and finally the general case with a dual nature co-op, $\zeta > 0, \alpha > 0$. Thus, to analyze the equilibrium outcomes under different co-op objectives, we need to first understand the trajectory of co-op’s best response function as it moves along the objective spectrum.

The slope, $b_c$, represents the sensitivity of the co-op’s price offer to that of IOF. It is increasing in $\alpha$ as $\partial b_c / \partial \alpha = \frac{\zeta}{[2(1-2\alpha)+(1-\alpha)(1-\beta)\zeta]^2} > 0$, meaning that the co-op’s price decision becomes more sensitive to change in price offer made by the IOF counterpart as the
importance of the producer’s cash payment weighs more in the co-op’s objective. However $b_c(0, \alpha)$ does not vary with $\alpha$, meaning that the slope of the best price-response function of a risk neutral co-op is not affected by the objective-weighing parameter $\alpha$. On the other hand, the co-op will also be more responsive to the IOF’s price offer if $\alpha < 1/2$ as $\frac{\partial b_c}{\partial \zeta} = \frac{(1-\alpha)(1-2\alpha)}{[2(1-2\alpha)+(1-\alpha)(1-\beta)\zeta]^2}$. Thus, we can establish the following trajectory of the slope of the co-op’s BR as the co-op changes from being a risk-neutral profit maximizer to a risk-averse integrated firm:

$$b_c(0,0) = b_c(0, \alpha) \leq b_c(\zeta, 0) \leq b_c(\zeta, \alpha).$$

that is, caring about producer’s cash payment reinforces the positive impact of co-op’s risk aversion on how it responds to IOF’s price.

The intercept $a_c$ measures the co-op’s propensity to pay for the farmer’s crop. It is increasing in $\alpha$, $\frac{\partial a_c}{\partial \alpha} = \frac{(\overline{p}-c)(2-\beta\zeta)}{[2(1-2\alpha)+(1-\alpha)(1-\beta)\zeta]^2} > 0$, as the co-op would pay more for the farmer’s crop in cash when it integrates the farmer’s cash payment into its objective. To the contrary, the effect of risk aversion on the co-op’s propensity to pay is less negative as $\frac{\partial a_c}{\partial \zeta} = \frac{-(\overline{p}-c)(1-\alpha-\alpha\beta)}{[2(1-2\alpha)+(1-\alpha)(1-\beta)\zeta]^2} < 0$, as $\alpha < 1/2$. The risk aversion will render smaller propensity to pay for the crop as the co-op needs to be compensated by a higher margin, i.e. lower input cost, compared to the risk neutral co-op, for the risk associated with its output price. So this lead to the following trajectory of the intercept of co-op’s BR:

In the following section, we show that $\alpha < 1/2$ is the necessary condition for interior solution.
While the value of \(a_c(\zeta, \alpha)\) is less clear in comparison to \(a_c(0, 0)\), because the effects of altruism and risk aversion are pulling the co-op’s propensity to pay for the crop in the opposite directions.

Equilibrium

The equilibrium of this model is characterized by four components: mill prices, firms’ expected profit, market shares, and expected utility of the farmer from selling the crop. As demonstrated earlier, other equilibrium features will follow the determination of equilibrium prices through strategic interaction between the co-op and the IOF. So we will begin with the analysis of equilibrium prices, and follow by the discussion of the implications of varying co-op objectives on firms’ profit, and market share. We leave the analysis of the producer’s expected utility in the simulation given its complex analytic form.

The equilibrium price offers \((w_c^{**}, w_p^{**})\) obtained by solving the system of equations (9) and (11) have the following general form:

\[
(14) \quad w_c^{**} = \frac{a_c}{1-b_pb_c} \\
w_p^{**} = \frac{bp a_c}{1-b_pb_c}
\]

And the complete solution is given by:

\[
(15) \quad w_c^{**} = \frac{2(\alpha \beta+(1-\alpha)(1-\beta)\zeta + (1-2\beta))}{(1-\beta)(3-6\alpha+(1-\alpha)(1-\beta)\zeta)} (\overline{p} - c) \\
(15) \quad w_p^{**} = \frac{1-\beta}{2} w_c^{**}
\]
Substituting $w^*_c$ and $w^*_p$ into Equation (7) yields the equilibrium market share for a co-op:

$$\delta^{**} = \frac{(1-\alpha)-2\alpha\beta/(1+\beta)}{\beta(1-2\alpha)+(1-\alpha)(1-\beta)\zeta}$$

Equilibrium prices offered by both firms as shown above is proportional to the firms’ expected marketing margin. For a meaningful interior solution, the equilibrium prices and market shares should be both positive and the subsequent expected margins of two firms are non-negative (equation 6), i.e. $(w^*_c, w^*_p) \in [0, \overline{p} - c]$ and $\delta^{**} \in (0, 1)$. Mathematically, this means the ratio of co-op’s price offer to the expected profit margin is between zero and one, which leads to the following regularity conditions among parameters:

$$0 \leq \alpha \leq 1 - \frac{3-\beta}{4(1-\beta^2)\zeta}, \tag{17}$$

$$0 \leq \alpha \leq 1 - \frac{2(1-\alpha)\beta}{\beta(1-2\alpha)+(1-\alpha)(1-\beta)\zeta} \tag{18}$$

Inequality (17) restricts a range of values for the level of the co-op’s altruism to the producer that can support an interior solution. This is a narrower range for $\alpha$ than the concavity condition in expression 10, as the upper limit of $\alpha$ is no greater than $1/2$. This suggests that co-op cannot overweigh the cash price paid to producers against its own profitability under all values of $\beta$ and $\zeta$. As shown in (11), $\alpha < 1/2$ is a sufficient condition for the slope of the co-op’s BR to be positive. So the pricing strategies of two firms will always be complements to each other, i.e. as the price offer by co-op goes up, the IOF will also offer a higher price and vice versa. A direct implication of (14) is that co-op will pay a higher price for grain at equilibrium regardless of its objective, because $b_p = \frac{1-\beta}{2} < 1$. This means the co-op is less profitable, measured by the profit margin, $(\overline{p} - c - w)$, due to higher input cost. The intuition is that the producer demands risk premium for the uncertainty associated with his patronage refund. The relationship between the co-op’s objectives and the determination of the market equilibrium prices is illustrated in figure...
2. Moreover, it is straightforward to show that under (17) and (18), we have \( b_c b_p < 1 \), meaning that the equilibrium solution is stronger than a Nash equilibrium that it survives the iterated elimination of dominated strategies, and a game as such needs not to expand to a repeated game. See appendix B. for proof.

![Figure 2](image)

*Figure 2.* The best response functions and equilibrium prices

**Comparative statics**

In this section, we analyze the effects of risk aversion and altruism of the co-op on equilibrium prices and firms’ market shares. Because of strategic complementarity of firms’ pricing and symmetry in their market shares, we will only show the analysis on the co-op.

**Proposition 1.** A relative increase of co-op’s risk aversion to the producer reduces the price offers but increases the expected profit margin for both firms in the equilibrium.
The Proof is straightforward to show by taking the partial derivative of equilibrium price offer of the co-op with respect to the relative risk aversion coefficient:

\[
\frac{\partial \omega^*_c}{\partial \zeta} = \frac{2(\bar{p}-c)(1-\alpha)((\alpha-1)/(1+\beta)+2\alpha \beta)}{(3-6\alpha+(-1-\alpha)(-1-\beta)\zeta)^2} < 0.
\]

If \((\alpha - 1)(1 + \beta) + 2\alpha \beta < 0\), or \(\alpha < (1 + \beta)/(1 + 3\beta)\), that is the co-op is not too altruistic towards the member-producer, there is negative relationship between equilibrium price levels and co-op risk aversion relative to the producer. This is true under the regularity condition (17) which implies that \(\alpha < 1/2\), because \((1 + \beta)/(1 + 3\beta)\) is greater than \(1/2\) for \(\beta\) between zero and one.

So we can sign \(\frac{\partial \omega^*_c}{\partial \zeta}\) as negative. Trivially, the IOF’s price offer will also be lower as the co-op becomes more risk averse because the price offers are strategical compliments. Due to the inverse relationship between the firms’ expected profit margin and input cost, lower cost of grain leads to higher expected margins at the equilibrium. A direct corollary of the proposition 1, relating to the implication of co-op’s risk attitude on the firms’ market share is summarized below:

**Corollary 1.** A relative increase of co-op’s risk aversion to the producer reduces the co-op’s market share.

Because of symmetry, the IOF’s market share therefore will increase. There are two ways to prove the corollary 1. Mathematically, the partial derivative of the co-op’s equilibrium market share with respect to the relative risk aversion coefficient is given by:

\[
\frac{\partial \delta^*_c}{\partial \zeta} = \frac{(1-\alpha)(1-\beta)(-1+\alpha-\beta+3\alpha \beta)}{(1+\beta)(3-6\alpha+(-1+\alpha)(-1+\beta)\zeta)^2} < 0.
\]

This result can also be seen from the producer’s allocation function in equation (7), as \(\frac{\partial \delta^*_c}{\partial \omega_c} = \frac{1-\beta}{\rho \beta^2 \sigma^2} > 0\). As mentioned earlier, if the patronage refund kept at the co-op is not expected to
generate any return, the producer will prefer the cash now than later. But for a co-op that operates independently, it may also have the need for keeping the patronage refund, to compensate for the risk it is taking. The risk attitude analysis bears policy implications on the risk management of the co-op. In this article, risk refers to the exogenous volatility of the output price. However, nonsystematic risk may be diversified, and many shocks can be insured by participating the derivatives markets. Therefore, the degree of risk aversion is related to the extent to which the business risk can be hedged. If less risk aversion induces more desirable allocations for both the producers and the organization, managers who operate the co-op may pay more attention to the identification and hedging of the business risk.

**Proposition 2.** An increase in the co-op’s altruism towards the producer will lead to increase in the equilibrium price offers by both firms, but decrease in the firms’ profit margin.

Proposition 2 addresses the market equilibrium where the co-op takes into account the producer’s cash payment. The proof is straightforward:

\[
\frac{\partial \omega^*_c}{\partial \alpha} = \frac{2(\beta-c)(3-2\beta \zeta)}{(3-6\alpha+(\beta+\alpha)(-1+\beta)\zeta)^2} > 0.
\]

Intuitively, if co-op incorporates the producer’s payment into its objective, it will pay a higher price for the producer’s grain. Following the same logic of corollary 1, the effect of co-op’s altruism on the equilibrium market shares is given by

\[
\frac{\partial \delta^*}{\partial \alpha} = \frac{(1-\beta)(3-2\beta \zeta)}{[3(1-2\alpha)+(1-\alpha)(1-\beta)\zeta]^2(1+\beta)} > 0.
\]

We summarize this result in the following corollary:

**Corollary 2.** An increase in the co-op’s altruism towards the producer will lead to increase in the co-op’s market share.
The above results show that an increase in grain price offer by the co-op is always favorable to the producer as they would allocate more grain to the co-op. This is because the IOF will also have to up its bid, but by less amount. The impact of co-op’s objective changes on equilibrium prices can also be understood in terms of figure 2. According to (12) and (13), the propensity to pay by the co-op is increasing in the level of altruism, and the sensitivity of response to the IOF’s price offer is non-decreasing in $\alpha$. This means that for a higher $\alpha$ and given $\zeta$ the BR of the co-op will have a larger intercept and slope, both leading to a higher equilibrium price. On the other hand, the increasing co-op’s relative risk aversion to the producer has been shown to lead higher response sensitivity to the IOF’s offer, but less propensity to pay. However, (20) shows that the effect of propensity to pay dominates.

The unique characteristic of this game is that the symmetry of firms’ objectives do not lead to symmetric equilibrium outcomes regarding price offers and market share. In particular, even when the co-op has the same objective as the IOF, it still follows the general model outcome that the co-op will pay a higher price but obtain a smaller market share. But in this case where co-op is a risk-neutral profit maximizer, its market share is $\delta^{**}(\zeta = 0, \alpha = 0) = 1/3$, unaffected by the producer’s preference.

We show that the only way for the co-op to increase market share is by incorporating the producer’s on-farm profit into its own objective. And the highest possible market share for the co-op is achieved at its maximum level of altruism, i.e. $\alpha^{max} = 1 - \frac{3-\beta}{4-\beta^2}\zeta$.

$$\delta_{max}^{**}(\alpha^{max}) = \frac{1}{2}.$$ 

So it is feasible for the co-op with dual nature to achieve a higher market share than the co-op just maximizing profit. However, it’s practically difficult to split the market evenly with the IOF which
reconstitute the standard prediction from a duopsony model, because in such case the co-op has to operate at average cost (break-even), i.e. \( w^*_c(\zeta, \alpha = \alpha^{max}) = \bar{p} - c \).

**Simulation**

Numerical simulation is aimed to shed lights on the general relationship of the cooperative’s objective to the producer’s expected utility at equilibrium, whose analytic solutions are too complicated. The numerical simulation can also serve to verify our comparative analysis in the prior section.

For simulation, we standardize the expected value of firms’ output price to be 100, and the processing cost 80. The producer’s time discount factor is chosen to be \( \beta = 0.95^{20} \), to reflect the long horizon before all patronage refund is distributed back to the producer. We evaluate the co-op’s equilibrium price, the co-op’s market shares and the producer’s expected utility at many possible co-op objectives that are defined by a pair of \{ \alpha, \zeta \}. The IOF’s price offer is just a constant fraction of that of the co-op, and the IOF’s market share is one minus the market share of the co-op. We choose 50 evenly spaced values of \( \alpha \) ranging from 0 to 0.5 and 100 evenly spaced values for \( \zeta \) range from 0 to 1. The admissible range of values generated by simulation for plots are those satisfy the non-negative price offer and non-negative expected profit constraints.
The figures 3 and 4 show the equilibrium price offer and market share of the co-op as a function of the co-op’s objective function respectively. It clearly verifies our comparative statics analysis that the both the co-op’s market share and price offer will increase in its level altruism towards the producer, but decrease in the relative level of risk aversion. Same can be said for the producer’s expected utility as shown in figure 5. That is, the producer will do better if the co-op can operate more in favor of the producer’s utility that leads to higher cash payment by both firms. Note that the uneven surfaces exhibited in plots when the value of \( \alpha \) is close to 0.5. This is because a) with \( \beta \) smaller than 1, \( \alpha \) will be strictly smaller than 0.5, and b) the admissible value of \( \alpha \) also depends on \( \zeta \), which at some level will result in some \( \alpha' \)'s not supporting the interior solution.

*Figure 3. The impact of change co-op’s objective on co-op’s equilibrium price offer*
Figure 4. The impact of change co-op’s objective on co-op’s market share

Figure 5. The impact of change co-op’s objective on the producer’s expected utility
Discussion and Conclusion

In this article, we analyze the implications of co-op’s organizational structure in a duopsony market under uncertainty. The model results shed light on an important empirical observation that the number and the market share of cooperative businesses have been declining in spite of the important role they play in the U.S. farm marketing industry. We find that the frictions to a complete vertical integration of the co-op and its members serve as competitive disadvantages to the co-op. These frictions echo some of the property rights constraints of agricultural cooperatives outlined in Cook’s (1995) seminar paper: portfolio problem, free rider problem, horizon problem, influence costs problem and control problem.

The portfolio problem justifies our analysis of a risk-averse co-op that is usually ignored in the literature. In our model, the portfolio problem faced by the member-producer is to allocate their crop between the IOF that is "risk-free" and the co-op, which is "risky" for the amount of patronage refund is uncertain. In reality, the members of a co-op may hold a suboptimal portfolio due to the illiquidity of their equity, which then results in the mismatch of the co-op risk attitude and that of members. We model explicitly how the co-op’s risk aversion relative to the producer’s affect the producer’s allocation decisions and equilibrium pricing outcomes.

With an open membership, our model takes into account an aspect of free rider problem pertaining to the cooperative business model, in which members have no commitment to patronize the co-op but rather to take advantage of its competitive yardstick role. We show when the member-producer do not treat the patronage refund the same as the cash payment, he will send more of his crop to the IOF, and co-op in turn will have a smaller market share at the equilibrium.

The horizon problem will further cause the member-producer to discount the patronage refund, which is paid out in the future because older members do not expect to fully benefit from the
co-op investment with his money. Li et al. (2015) show that the grain marketing co-ops in Iowa carry significantly less long-term debt than IOFs in the same industry. This indicates that co-ops may rely on equity financing for long-lived assets, which compounding with the horizon problem may adversely affect the longevity and growth of co-ops. We do not model the co-op’s investment decisions, but the producer’s preference of uncertain patronage refund is likely to be captured by his time value and risk attitude.

As for the agency problems, the heterogeneity among member-producers may cause damaging influence in determining the co-op’s policy (influence cost problem); on the other hand, the co-op may diverge from the interests of members (control problem). Our model captures the control problem through the weighting parameter in the co-op’s objective function. Conditional on the producer’s preference of cash payment over patronage, the co-op is getting bigger market share as it focuses more on the producer’s on-farm profit. However, if the co-op behaves like the profit-maximizing IOF, its market share will no longer be affected by its pricing policy. This empirical prediction may warrant further investigation as co-ops have grown in size and complexity, which furthers the control problem (Staatz, 1987). Our model framework assumes a representative producer and the objective weighting of the co-op is exogenous, thus not accounting the influence cost problem. Studying the impact of heterogeneous members on the co-op’s optimal weighting factor and subsequent objective is an interesting topic for future research.
A. Cooperative as an Entrant

The complete solution to the IOF’s problem, Equation (8), subject to non-negative profitability constraint is:

**Case 1** if \((1 - \beta)(\overline{p} - c) > \rho \beta^2 \sigma^2\),

\[
w^*_p = \begin{cases} 
\beta (\overline{p} - c) + (1 - \beta)w_c & 0 \leq w_c < w^*_c \\
 a_p + b_pw_c & w^*_c \leq w_c \leq \overline{p} - c 
\end{cases}
\]

**Case 2** if \((1 - \beta)(\overline{p} - c) \leq \rho \beta^2 \sigma^2 \leq (1 + \beta)(\overline{p} - c)\)

\[w^*_p = a_p + b_pw_c, \forall w_c \in [0, \overline{p} - c].\]

**Case 3**, if \(\rho \beta^2 \sigma^2 > (1 + \beta)(\overline{p} - c)\),

\[w^*_p = \max [a_p + b_pw_c, 0], \forall w_c \in [0, \overline{p} - c].\]

Considering the IOF as the only game in town before the establishment of a co-op, the corner solution of our model facilitates comparison of markets with and without the co-op, and is analogous to the idea of entry deterrence in the endogenous market structure (EMS) literature (e.g. Etro, 2007; Hueth and Moschini, 2014). The magnitude of the expected marketing margin, \(\overline{p} - c\), determines whether the IOF will play to deter the entry of the co-op. When the marketing margin is above the threshold \(\rho \beta^2 \sigma^2 / (1 - \beta)\) (case 1), the IOF will find it optimal to obtain the entire market as a monopolist when the co-op’s price offer is below \(w^*_c\). However, the IOF still faces entry threats by the formation of a producer’s co-op. The term \(\beta (\overline{p} - c) + (1 - \beta)w_c\)
captures how the IOF reacts to entry threats by the co-op, which would be the so called entry
deterrence price. The producers’ endowment value will be improved by that amount
consequently. As the marketing margin decreases, it’s no longer optimal for the IOF to be the
monopolist but rather accept the entry by the co-op as described in case 2 and 3. The possibly
excessive entry by other IOFs can be limited by imposing a large entry cost, which we do not
consider in this article as it has been discussed intensively in the I.O. literature (e.g. Mankiw and

B. Iterated Elimination of Dominated Strategies

Let the two best responses be

\[ w_p^* = a_p + b_p w_c, \forall w_c \in [0, \bar{p} - c], \]

\[ w_c^* = a_c + b_c w_p, \forall w_p \in [0, \bar{p} - c]. \]

where \( b_p > 0, b_c > 0, b_p b_c < 1. \)

Given \( w_c \in [0, \bar{p} - c], \) the IOF’s best response satisfies \( w_p \in [a_p, a_p + b_p(\bar{p} - c)]. \) So the first
round of elimination removes IOF’s strictly dominated pricing strategies in the set \([0, a_p) \cup (a_p + b_p w_c, \bar{p} - c]. \)

Given \( w_p \in [a_p, a_p + b_p(\bar{p} - c)], \) the co-op’s best response must be in the interval \( w_c \in [a_c +
\quad b_c a_p, a_c + b_c a_p + b_c b_p(\bar{p} - c)]. \) So the co-op’s pricing strategies outside that interval are
eliminated in the second round of elimination.

Given \( w_c \in [a_c + b_c a_p, a_c + b_c a_p + b_c b_p(\bar{p} - c)], \) the IOF’s best response satisfies \( w_p \in [a_p +
\quad b_p a_c + b_p b_c a_p, a_p + b_p a_c + b_p b_c a_p + b_p b_c b_p(\bar{p} - c)]. \) This further eliminates IOF’s
dominated strategies outside the best response interval.
As the iterations continue, the band width of the two bounds of the survived $w_p$ (and $w_c$ similarly) takes the form $(bpbc)^{(n-1)/2}b_p(\bar{p} - c)$, where $n$ is the number of iterations. Since $bpbc < 1$, we have $(bpbc)^{(n-1)/2}b_p(\bar{p} - c) \to 0$ as $n \to \infty$.

Thus the bounded $w_p$ converges to a single point:

$$a_p \sum_{n=1}^{\infty} (bpbc)^{n-1} + bpac \sum_{n=1}^{\infty} (bpbc)^{n-1} = \frac{1}{1-bpbc} a_p + \frac{1}{1-bpbc} b_p ac.$$

Similarly, $w_c$ also converges to

$$ac \sum_{n=1}^{\infty} (bpbc)^{n-1} + bca_p \sum_{n=1}^{\infty} (bpbc)^{n-1} = \frac{1}{1-bpbc} ac + \frac{1}{1-bpbc} b_c ap.$$

This is the unique strategy profile that survives the iterated elimination of dominated strategies.
References


CHAPTER 3
REFERENCE-DEPENDENCE HEDGING: THEORY AND EVIDENCE FROM IOWA CORN PRODUCERS

Abstract

We examine the role of reference-dependence in the optimal hedging theory under the expected utility framework. Our theory characterizes the ways in which producers’ optimal hedging differ with or without reference. We test our theoretical predictions with a unique database consisting of every forward contract written by a major cooperative over a five-year period to Iowa corn producers. Analysis of these data suggests that hedging activity is triggered when the current December futures price is higher than a reference price. A likely candidate for the reference price is a rolling average of the current futures price. Trading activity implied by the Iowa corn producers are then used to determine if producers benefit from the way they hedge. The evidence is mixed. The Iowa data is then compared to the only publically available data on producer hedging: The Commodity Futures Trading Commission Disaggregated Commitment of Traders Report (DCOT) for Short Hedgers. The hedge ratio constructed from the open interests in new futures contracts of the DCOT report is highly correlated with the producer hedge series in the Iowa data, indicating that DCOT data represents the actual farmers’ hedging behavior reasonably well. This has important implications for future research that continues to use the DCOT report.
Introduction

Agricultural economists often find limited relevance of Expected Utility (EU) optimal hedging theory to real-world marketing decisions, and there is no consensus on how useful other utility theory paradigms are as they apply to hedging. One reason for these different views is the lack of empirical evidence on what motivates producers’ hedging decisions. The only prior data on the use of pre-harvest hedges by crop growers has been survey based, and this literature shows widely different participation rates. However, there is great benefit to understanding actual hedging behavior of grain producers. First, it contributes to an understanding of why empirical observation does not corroborate EU theoretical prediction. Second, the impact of producers’ hedging practices on the stability of farm income is a crucial issue to both farmers and extension economists. Mechanical rules of hedging have been proposed under the perception that farmers may be worse off due to their behavioral bias in hedging. An empirical comparison of the actual average prices received by producers sheds light on the need for alternative hedging strategies. Third, the commercial hedgers (elevators) that provide forward contracting services will benefit from an understanding of how and when producers will hedge.

The goal of this paper is to investigate the effect on optimal hedging of introducing reference-dependence into a traditional EU framework. Reference-dependence is a central feature of most alternative utility theory paradigms to EU, positing that agent’s utility is derived from the relative gain and loss to his reference rather than his final wealth. Despite the recent effort of analyzing hedging under the alternative frameworks, the role of reference-dependence is not understood in ways that provide clear empirical guidance that can be used to test against the EU optimal hedging.
In this paper, we first develop a simple theoretical model that facilitates the comparison of optimal hedging with and without reference. We then empirically investigate the motivation of producers’ hedging utilizing a unique data set that consists of every forward contract written by a major grain cooperative for the period of January 2009 through August 2013. The theoretical model offers the null hypotheses that allows us to test whether the forward hedging behavior exhibited in the data violates the EU optimal hedging rules. If the EU optimal hedging is violated in the data, we identify the role of reference-dependence in producers’ hedging decisions. Trading activity implied by producers’ hedging patterns are also examined to determine if producers benefit from the way they hedge. Finally, we compare the actual hedging data against the Disaggregated Commitment of Traders Report (DCOT) provided by the Commodity Futures Trading Commission (CFTC).

Previous Work on Hedging

Under EU optimal hedging, the primary motive for producers’ hedging behaviors is risk aversion (Johnson 1960; Holthausen 1979; Feder, Just, and Schmitz 1980; Grant 1985; Castelino 1992; Lapan and Moschini 1994). Without basis risk and production risk, the EU optimal hedge ratio is constant and independent of the futures price if producers believe the futures price is unbiased. This result is robust to any concave utility function and incorporation of production decisions because of the separation theorem (Feder, Just, and Schmitz 1980). Adding basis risk, the optimal hedge ratio is less than a full hedge regardless of the farmer’s belief about the expected harvest price, which depends on the correlation coefficient between expected spot and forward prices. Further incorporating yield risk, Lapan and Moschini (1994) show that under mean-variance approach, the optimal hedge ratio is the regression coefficient of random revenue
on the futures price. In a dynamic setting, the optimal hedge ratio increases at the inverse of the interest rate and time to maturity (Myers and Hanson 1996). To the extent correlation between expected spot prices and forward prices vary overtime, the EU optimal hedge ratio can change. But McNew and Fackler (1994) show that with the GARCH model there is little variation in the case of corn.

In survey-based studies, Sartwelle et al. (2000) find no evidence that producers’ self-identified risk attitudes impact their hedging practices, while Goodwin and Schroeder (1994) find a negative impact of risk aversion on the use of forward contracts. On the contrary, Schroeder et al. (1998) find that both producers and extension economists perceive forward contracts as more price-enhancing than risk-reducing and believe in the existence of market timing strategies, even though these strategies have little support in the literature (Irwin et al. 2006). This rising number of empirical anomalies has led to exploration of alternative theoretical frameworks (e.g., Collins, Musser, and Mason 1991; Musser, Patrick, and Eckman 1996; Lien 2001; Mattos, Garcia, and Pennings 2008; Kim, Brorsen, and Anderson 2010). The central feature of most alternative utility theories is reference-dependence. This includes prospect theory (Tversky and Kahneman 1979), regret theory (Loomes and Sugden 1982) and expected target utility (Fishburn 1977). Instead of focusing on the utility of final wealth, reference-dependence implies that individuals generate utility from gains and losses measured relative to a reference point (Tversky and Kahneman 1992).

Research studying the role of reference-dependence in hedging suggests the potential for a price-based trigger for changes in hedging positions including both the price change from a prior period and the price level itself. Brorsen et al. (1995) find that when prices are low, there is a lack of interest in forward contracting. What defines a high crop price that would motivate
producers to hedge has yet to be identified. Using data from a hedging game, McNew and Musser (2002) find hedge ratios respond to changes in the futures price relative to last year’s high price. Kim, Brorsen, and Anderson (2010) show that under an expected target utility function, the producer will hedge more when prices rise above a targeted profit margin—the reference point. The motivation of this theoretical construct comes from the profit margin hedging strategy (e.g., Parcell and Pierce 2009) that is often suggested by market advisory services and anecdotal evidence that farmers forward sell a greater portion of their crop when prices are high. Mattos et al. (2016), in a recent experimental study, show that producers’ marketing decisions depend on the difference between the current futures price and the reference, which is influenced by the price trend and producers’ expectations.

The existence of a reference effect is well documented in other markets. Babcock (2015) shows that cumulative prospect theory with insurance premiums as a reference can generate crop insurance purchase decisions that are consistent with observed low participation rates, a result that is anomalous to expected utility maximization. In a study of the stock market, Grinblatt and Keloharju (2001) find reference price effects in stock trading by individual investors, who have a higher propensity to sell if a stock rises above its high of the past month. In fact, individual investors in general are found to have greater tendency to sell stocks with positive returns than at losses (Shefrin and Statman 1985; Odean 1998). This is called the disposition effect and it cannot be reconciled with portfolio management or justified by the subsequent portfolio returns. The reference effect is also found in the real estate market. Those who move from expensive cities tend to rent more expensive apartments (Simonsohn and Loewenstein 2006), and home sellers use the original purchase price as a reference when setting asking prices (Genesove and Mayer 2001).
The role of basis in forward contracting

The unique characteristics of agricultural forward contracts, which are the most common marketing tools used by farmers in reality, have also been studied in the past literature (Musser, Patrick, and Eckman 1996, Garcia and Leuthold 2004). Nelson (1985) discusses in detail the differences between the forward and futures contracts. Among these is the fall basis, the difference between the December futures price and the local forward prices, which may reflect information about the expected harvest (Taylor et al. 2014). The relationship between the basis and hedging, however, is less clear because of two opposing forces. On the one hand, the basis represents the cost of hedging with forward contracts relative to the futures contracts. Thus, a narrowing of the basis may cause more hedging with forward contracts. However, Stringer and Sanders (2006) show that the corn basis in Illinois is very small, and in some counties may be negative. On the other hand, the basis is also co-determined by the level of hedging. A number of studies have shown that an increase in volume of forward contracting results in a lower cash price, namely a widening of basis (e.g., Elam et al. 1989; Elam 1992; Schreoder et al. 1993). The elevator may even be reluctant to provide the forward contracting service during periods of high price volatility, because of the difficulty of managing its own futures account due to margin risk (Mark et al. 2008). There are a few episodes where surge in hedging was accompanied by the widening in fall basis (e.g., the summers of 2010 and 2012).

Theoretical Framework

Suppose the objective of the corn producer that markets his crop is to maximize the expected utility of price received per bushel by determining his optimal hedge ratio, $h$:

$$1 \max_h Eu(p^s(1 - h) + p^f h),$$
where \( p^s \) is the harvest price that takes one of two possible values in relation to the futures price \( p^f \): 

\[
(2) \quad p^s = \begin{cases} 
    p^f + \epsilon, & \text{with probability } \pi \\
    p^f - \epsilon, & \text{with probability } 1 - \pi
\end{cases}
\]

The realized harvest price can be higher or lower than the current December futures price by \( 0 < \epsilon < p^f \), with probability \( 0 < \pi < 1 \) and \( 1 - \pi \) respectively. \( \pi \) represents the subjective belief of the producer about the likelihood of the price going higher. If \( \pi = 1/2 \), then the producer believes the futures price is unbiased. While this may seem unrealistic, the binomial model technique imposes no more distributional assumptions than a continuous price distribution.

Denote \( R \equiv p^s(1 - h) + p^f h \) as the final revenue per bushel received by the producer, and we consider the following utility function to study the impact of introducing reference dependence in EU on the producer’s optimal hedging decision,

\[
(3) \quad u(h; b, \alpha, \epsilon, p^f) = \begin{cases} 
    [R - b]^{1-\alpha} / (1 - \alpha) & \forall R \geq b \geq 0 \\
    -(b - R)^{1-\alpha} / (1 - \alpha) & \forall 0 < R < b
\end{cases}
\]

When \( b \), the reference price, is greater than zero, the producer’s utility is derived from the relative gain or loss. When \( b = 0 \), utility function is a power utility function with a constant relative risk aversion \( 0 < \alpha < 1 \). So the reference-dependent producer exhibits risk-aversion in gains but risk-seeking in losses.

The producer’s hedge ratio \( h \) is assumed to be bounded, with lower bound \( 0 \) and upper bound \( 1 + \frac{|p^f - b|}{\epsilon} \). A typical grain producer is unlikely to enter into a long futures or forward contract that will add to his existing long position. However, this lower limit does not prevent the producer from speculating, as he can choose to not fully hedge, or hedge slightly more than his
own harvest. The upper limit of the hedge ratio is to ensure that a reference-dependent producer’s utility function, equation 3, is meaningful for all possible value of the hedge ratio.

The deviation of the futures price from the producer’s reference is also assumed to be not too big, such that \(|p^f - b| < \epsilon\). This assumption keeps the reference-dependent producer’s objective function from degenerating to a globally concave function that will neglect the role of reference.

**Proposition 1.** If the producer believes the futures price is unbiased, his optimal hedge ratio is:

\[
(4) \quad h^* = \begin{cases} 
1 & \forall \ p^f > b \geq 0 \\
0 & \forall 0 < p^f < b . . . \\
[0,1] & p^f = b 
\end{cases}
\]

*Proof:* See appendix.

Equation 4 illustrates that for a reference independent producer that believes the futures is unbiased, his optimal hedging decision is to fully hedge. While for the producer that derives utility from the revenue level relative to his reference, he will only hedge when the futures price is higher than his reference. In other words, reference dependence causes the hedging decision to respond to the level of the futures price; whereas the EU optimal hedge ratio is a constant.

This simple model does not capture explicitly the dynamics of producers’ hedging during the pre-harvest season. In practice, the producer is unlikely to price all his crop early in the season, but instead increases the hedge gradually with the approach of harvest and uncertainty dissipating. If the producer’s utility does not depend on reference, we shall not observe much hedge ratio variation year-to-year nonetheless.

However, for a reference-dependent producer, there are two possible empirical implications of his hedging pattern from proposition 1:
(i) The percentage of total crop sold forward by producers before harvest will change from year to year.

(ii) The producer who gradually markets his crop would hedge more during times when the futures price is higher than their reference, while hedge little at the futures price below their reference.

The producer may change his hedging decision if he believes that the futures price is biased. A trivial extension of proposition 1 is when the futures price is equal to reference: if the producer believes the futures price is more likely to go down (up), he will (not hedge) fully hedge because the marginal expected utility of hedging is always positive (negative). The comparative statics analysis is not applicable in this special case because the initial optimal hedge ratio can take any value between zero and one.

It is also a salient observation in the literature that producers tend to hedge more as the futures price increases. The EU can explain the change in producer’s hedging decisions to the extent that he believes the futures price is biased. For example, if the futures price increases while the producer’s belief about the harvest price stays the same, then he may increase his short position as he speculates the futures price will fall in the future. However, this price-induced hedging response may differ with and without reference, as summarized in the following proposition:

**Proposition 2.** If producers believe the futures price is biased, then the marginal impact of his belief about the probability of the futures price going up in the second period is:

\[
(5) \quad \frac{\partial h^*}{\partial \pi} = \begin{cases} 
-2p^f/[g_1(\pi)\alpha\epsilon] & \text{if } b = 0 \\
-2(p^f - b)/[g_1(\pi)\alpha\epsilon] & \text{if } p^f > b > 0 \\
-2(b - p^f)/[g_2(\pi)\alpha\epsilon] & \text{if } b > p^f > 0 & g_3(\pi) < \epsilon/(b - p^f) \\
0 & \text{if } b > p^f > 0 & g_3(\pi) > \epsilon/(b - p^f) \\
\end{cases}
\]
where \( g_1(\pi) = \frac{[(1-\pi)\pi]^{1-\alpha/\alpha}}{\pi^{1/\alpha}+(1-\pi)^{1/\alpha}} \pi, \ g_2(\pi) = \frac{[(1-\pi)\pi]^{1-\alpha/\alpha}}{\pi^{1/\alpha}-(1-\pi)^{1/\alpha}} \pi, \ g_3(\pi) = \frac{\pi^{1/\alpha}+(1-\pi)^{1/\alpha}}{(1-\pi)^{1/\alpha}-\pi^{1/\alpha}}. \)

**Proof:** See appendix.

The first line in equation 5 describes that for a reference-independent farmer who thinks the current futures price is too low, he will reduce his optimal hedge ratio from a full hedge. Conversely, if the producer thinks the futures price is more likely to go down in the second period, he will increase his hedge ratio.

On the other hand, if the producer is reference-dependent and the futures price is above his reference, he will exhibit similar speculative behavior as described in the second line. However, as the third and fourth lines of equation 5 illustrate, if the futures price is below the reference, then the producer will not hedge unless he believes the probability of the harvest price ends up being lower than the current futures price is high enough such that \( \pi < g_3^{-1}(\frac{e}{b-p_f}). \) Note that \( g_3(\pi) \) is continuous and monotonically increasing when \( \pi \in [0,0.5] \). In other words, \( g_3(\pi) \) is invertible with \( g_3^{-1}(.) \) denoting its inverse function. The intuition is that in losses as the price below reference, the producer is risk-loving, and he will not choose to limit the upside of his revenue by hedging in case the good state is realized; unless he is so certain that the futures price will go down even more in the next period.

Also, the marginal effect on the hedge ratio of the producer’s subject probability is smaller when the futures price is below the reference as \( g_2(\pi) > g_1(\pi) \), not to mention when \( \pi > g_3^{-1}(\frac{e}{b-p_f}), \) the producer will not hedge even when he believes that the futures price has an upward bias. The intuition here is that when the price is below reference, the producer is risk loving, and will not choose to limit the upside of revenue by hedging unless he is certain that the futures price will go down even more in the next period.
To bring proposition 2 to data, we rely on a salient observation in the literature regarding the price – hedge ratio relationship. Short hedgers’ position of corn as reported in the Commodity Futures Trading Commission (CFTC) Disaggregated Commitment of Traders reports (DCOT) appears to increase as the futures price goes up. However, the economic argument behind this observation is not pegged due to several competing hypotheses. For example, the positive price - hedge ratio correlation is consistent with the reference-dependent hedging theory: as the futures price increases above the producer’s reference, he will hedge to lock in the gains. However, the EU optimal hedging without reference can also explain the producer’s price response in hedging if the futures price increases while the producer’s belief about the harvest price stays the same. Then there will be an increase in the speculative short position.

Resolving the differences among competing hypotheses hinges on whether researchers can observe the producers’ beliefs about the future spot price. This is only possible with the experimental data. As for the actual trades data, the proposition 2 implies a necessary empirical condition for the existence of reference-dependence without assuming the motivation for the price-induced hedging:

(iii) When the reference effect is removed, the response of the hedge ratio to price will be symmetric and independent of the level of the futures price. When the reference effect is introduced, the hedge ratio will only respond to price changes if the futures price is above the reference, this will lead to an asymmetric response.

In the empirical section that follows, we do not observe how the producer thinks—speculation is just one possible explanation as to why the producer’s hedge ratio responds to the price change. To the extent producers are reference-dependent, the asymmetric response
suggested by the model refers to the decision of whether to adjust hedge ratio, rather than the size of hedge ratio change.

Data

We analyze the hedging activities of Iowa corn producers using daily forward contract data from a major grain marketing cooperative for the period from January 2009 to August 2013. This firm has over 30 grain-receiving locations with an average annual total handle of more than 100 million bushels. The data include over 115,000 forward contracts for corn and contain the contract date, bushels contracted, and delivery date. We focus on forward contracting in the pre-harvest period from January 1 to August 31 as harvest grain bids are commonly available between January and the end of August (Mallory et al., 2015). Restricting our analysis in this way separates the decision to hedge anticipated production from storage hedges and post-harvest contracts used to lock in favorable prices or basis for planned deliveries.

The numerator of the hedge ratio at date \( t \) is measured as the total number of bushels contracted from January to date \( t \) for delivery in the period September 1 to August 31 of the following year. Because expected production is not observable, we use annual grain purchased (handled) by the cooperative as a proxy. We make the assumption of producer homogeneity in their hedging patterns as the data does not allow identification of individual producers, while the farm characteristics only affect the adoption decisions of futures and forward contracts by producers but not how they hedge (Katchova and Miranda, 2004).

Figure 1 shows that the proportion of corn that is hedged varies considerably from year to year. In particular, more than 20% of corn was forward priced in the high price years of 2010–2012, while only 3.75% was hedged at the same time in 2013, a year when prices fell
significantly from 2012 levels. This empirical observation is certainly consistent with the theoretical prediction of the hedging behavior of a reference-dependent producer.

*Figure 1.* The level of producer’s pre-harvest hedge ratio, 01/2009–08/2013

**Empirical Procedure**

The reference-dependent utility theory, as it applies to hedging, predicts that producers’ hedging behaviors may exhibit asymmetry depending on the level of the futures prices relative to their reference price. Figure 2 plots weekly change in hedge ratios within each crop year, and the producers do seem to hedge differently during some periods than others.
We do not observe producers’ reference prices directly, but may infer whether the current futures price is above his reference or not based on the observed hedging behavior using the Markov Switching regression model (Hamilton 1989). There are two states of producers’ hedging decisions: ‘to hedge’ or ‘not to hedge’, and the probability of each hedging state prevails is assumed to follow a first-order Markov process. The state of no hedging would correspond to the period when the futures price is below the producer’s reference, when a reference-dependent producer is reluctant to forward price his crop.

Our reference-dependence hedging theory prescribes two state-dependent variables, the effects of which on hedging vary across states. Let \( h_t \) denote the cumulative hedge ratio, the total amount of crop forward priced as a percentage of expected harvest up to date \( t \), and then \( \Delta h_t \) is the proportion of total harvest hedged in week \( t \). Proposition 1 implies that if the producer’s utility is

\[ \textit{Figure 2. Weekly change in producer’s pre-harvest hedge ratios, 01/2009–08/2013} \]
reference-dependent, more of the crop would be hedged in weeks when the futures price is above the reference:

\[
\Delta h_t = \begin{cases} 
\alpha_0, & p_t \geq R_t, \\
\alpha_0^\prime, & p_t < R_t, 
\end{cases}
\]

where \( p_t \) is the current futures price and \( R_t \) is the producer’s reference. \( \alpha_0 \) and \( \alpha_0^\prime \) is the average weekly portion of expected production forward priced during the pre-harvest season, conditional on the current futures price being above and below the reference respectively. Thus, the empirical objective is to compare the intercepts: whether \( \alpha_0 \) is significantly differing from \( \alpha_0^\prime \).

We are agnostic about the motivation for the potential hedging response to the futures price changes by the producer. However, as proposition 2 suggests, if the producer’s utility function is reference-dependent, the producer may hedge more as prices increase above the reference price. However, his hedging is much less responsive to price changes if the level of futures price is below reference:

\[
\Delta h_t = \begin{cases} 
\beta_0 \Delta p_t, & p_t \geq R_t, \\
\beta_0^\prime \Delta p_t, & p_t < R_t, 
\end{cases}
\]

where \( \Delta p_t \) is the weekly difference in the logged price of the December futures contract. To simplify the notation, let \( s \) denote the unobservable state variable. Accounting for the impact of \textit{time to harvest} and price volatility, we write the empirical specification as:

\[
\Delta h_t = \alpha_s + \beta_1 time + \beta_2 vol_t + \beta_s \Delta p_t + \varepsilon_t, \quad \varepsilon_t \sim iid N(0, \sigma_s^2).
\]

Two state-dependent coefficients in equation 8 are intercept \( \alpha_s \) and the responsiveness of hedge ratio change to price changes \( \beta_s \). We also allow the variance of residuals to differ between states, as the hedge ratio may exhibit larger variation in the state in which farmers are actively placing hedges. The variable \textit{time} measures the weeks left until the harvest in October, and producers are expected to hedge a greater proportion of new crop as harvest approaches due to resolving
uncertainty over yields. Price volatility $\text{vol}_t$, measured as the annualized standard deviation of the daily log returns for the December futures contract for each week (Irwin and Sanders 2012), is to control for the possibility that producers indeed hedge to reduce minimize their price risks. $\text{time}$ and $\text{vol}_t$ are control variables and are assumed to be state-independent.

Denote $P_t$ the two-by-two matrix that governs the transition of the states of the producer’s hedging, and states 1 and 2 correspond to the state of ‘no hedging’ and ‘hedging’ respectively. Then the $i, j$th element of the matrix, $p_{ij} = \Pr(s_t = j|s_{t-1} = i)$, represents the probability of state $j$ realizing in the current period $t$ given the last period’s state $i$. The transition matrix $P_t$ is unobservable, but we can infer from the producers’ hedging patterns, conditional on the explanatory variables specified in equation 8. The null hypotheses of no state-dependent hedging patterns are associated with the EU optimal hedging without reference; while evidence of state-dependent hedging would be consistent with the scenario in which the producer is reference-dependent. Statistically, if the difference between the state-dependent coefficients estimates are significant and the transition matrix of the Markov process is not singular and symmetric, then there is an empirical support that switching does occur.

**Explore the possible reference price**

To the extent producers’ hedging exhibits state-dependent patterns, the Markov Switching regression model may lack the insight regarding what triggers the transition between the states. In this section, we modify equation 3 to explore whether the reference-dependent hedging is associated with some of the candidate reference prices in the previous literature:

\[
\Delta h_t = \alpha_0 + \alpha_1 1_{\{p_t - R_t < 0\}} + \beta_1 \text{time} + \beta_2 \text{vol}_t + \beta_3 \Delta p_t + \beta_4 \Delta p_t 1_{\{p_t - R_t < 0\}} + \varepsilon_t.
\]
\[ 1_{\{p_t - R_t < 0\}} \] is a dummy variable equal to one if the current futures price is below the reference and zero otherwise. \( \alpha_0 \) estimates the average proportion of the crop hedged per week when the futures price is above the reference and \( \alpha_1 \) is the estimate of difference between the amount hedged when the futures price is above and below the reference price. The interaction term in equation 9 is to account for the potential asymmetric response of hedge ratios to price changes as discussed earlier. The error term, \( \varepsilon_t \), is an identically, independently and normally distributed shock, with mean zero and variance \( \sigma^2 \).

There are a large number of candidate references. The RMA projected harvest price that used to establish revenue guarantees in crop insurance and the estimated production cost per-bushel are thresholds that producers might use. The insurance price may serve as a lower-bound below which a producer has no incentive to enter a forward contract. Similarly, a producer may be reluctant to hedge when the futures price is below his production cost because it means they will be locking in a loss. Behaviorally, a producer may also refer to the last year’s average marketing price when making hedging decisions, or a past 30-day moving average price as a reference.\(^8\)

The statistical validity of regression 9 hinges on the assumption of normality of the error term. We check the validity of this assumption by examining the autocorrelation and variance of the realized residuals. To the extent this assumption is violated, the standard errors for estimates are calculated with Heteroskedastic-Autoregressive-Consistent (HAC) estimator. If the null hypothesis of no reference price effect is rejected, different candidate reference prices are then compared using goodness-of-fit. However, it is hardly possible to statistically identify which candidate reference price is better if the high and low price periods, defined relative to references, overlap. This issue pertains especially to non-dynamic references including production cost and
insurance. However, the reference price effect argument can be strengthened if the hedging response to prices are asymmetric (i.e., producers sell into price rallies only when the futures price is above the reference).

Results

Table 1 presents quasi-maximum log-likelihood estimation results for equation 8. Without loss of generality, we let state 1 be the state where hedging is less active and state 2 represents the active hedging state. The estimated time-to-harvest effect has the expected negative sign and is statistically significant; corn producers, on average, price more of their upcoming expected harvest as yield uncertainty diminishes towards harvest. The magnitude of the time-to-harvest effect is small however, one week closer to the harvest month leads an increase in the weekly hedges by around 0.01 percentage points of the total harvest. The other state-independent variable, price volatility in the preceding week, does not have a significant impact on the change in hedge ratios.

The state-dependent intercept \( \alpha_s \) estimates indicate that the weekly average portion of total harvest forward sold by producers in state 1 is 0.4 percentage points, while in state 2 the producers on average hedge 1.2 percentage points of their crop per week, three times more than in state 1. This clear asymmetry is also found in producers’ hedging response to the futures price changes. Producers in state 2 are increasing their hedge by 0.18 percentage points for a one percent increase in the futures price, which is six times greater than in state 1. Finally, the hedging pattern in state 2 exhibits higher variance is also as expected, as larger swings in hedge ratio changes are hard to capture. In this regard, we perform the Lagrange multiplier tests of autocorrelation and autoregressive conditional heteroscedasticity (Hamilton 1996), which
suggest that residuals from the estimation of equation 3 exhibit neither heteroscedasticity nor 
serial correlation.

Table 1. Quasi-Maximum Log-Likelihood Estimation of Parameters and Standard Errors Based 

Equation 8: $\Delta h_t = \alpha_s + \beta_1 t + \beta_2 \nu o l_t + \beta_3 \Delta p_t + \epsilon_t$ \hspace{1cm} $\epsilon_t \sim iid N(0, \sigma_{st}^2)$

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<tr>
<th>State Independent Variables</th>
<th>Markov Transition Probabilities</th>
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<tr>
<td>$\beta_1$</td>
<td>$p_{11}$ \hspace{0.5cm} 0.95</td>
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<td>$p_{12}$ \hspace{0.5cm} 0.05</td>
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<td>$\beta_2$</td>
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<td>$p_{22}$ \hspace{0.5cm} 0.75</td>
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<table>
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<tr>
<th>State Dependent Variables</th>
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<td>(0.0004)</td>
<td>(0.0015)</td>
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</table>

| Autocorrelation | 0.95 |
| ARCH Effect      | 2.67 |
| Log-likelihood   | 689  |
| AIC              | -1366|
| BIC              | -1317|

Note. Significance levels indicated as: * p<0.1; ** p<0.05; *** p<0.01. Standard errors in the parentheses. The join-significance of intercepts is examined with Wald test with p-value reported. Specification tests (Hamilton 1996) are performed and Lagrange multiplier statistic are reported. 5% critical value of F-distribution is 3.84.
Figure 3 plots the actual weekly hedge data against the smoothed probability that the producer is in the active hedging state ($p[s = 2]$) based on the full sample information.

![Figure 3. Weekly change in producer hedge ratios vs. the inferred probability that the producer was in an active hedging state during each week, 01/2009–08/2013](image)

The producer hedge series exhibits clear shifts between the states of hedging and not hedging as the smoothed probabilities, which rarely lie around 50%, offer a strong verdict about whether the producer is actively hedging. Consistent with intuition, the active hedging state, state 2, is associated with the periods where large weekly hedges took place, but the timing of which seems not to follow a consistent pattern year-to-year. In particular, the active hedging state can prevail close to harvest, as in 2009 and 2012; it can also prevail in early spring as in 2011, or does not appear at all during the entire growing season, as in 2013. More insight is gained from the estimated Markov transition probabilities, as reported in table 1. The producers have the tendency to stay in their current state in the subsequent period. There is a 95% probability state 1 will be followed by another realization of state 1; and if the producer starts in state 2, there is a
75% chance that they continue to actively hedge in the next period. This suggests that the producer’s hedging decision is unlikely to be altered, and shifts between the states of hedging may be triggered by some economic stimuli, which we explore with regression equation 9.

Table 2 presents the OLS estimates and model fit statistics for equation 9 and a base case regression where no reference price specification is included. Robust standard errors are used as the assumption of homoscedasticity of the residuals is rejected for all candidate reference prices at 5% significance level. This is in agreement with the result of the Markov Switching regression result where the sample variances differ across states. Unlike the Markov Switching where the probability of a state prevailing at each period is inferred from the data, regression 4 serves to test whether the asymmetry in producers’ hedging occur for a preset candidate reference. For this approach, we would ideally like to see the futures price oscillate around the reference price so as to generate enough observations in both high price and low price periods. The estimated production cost, however, is always below the December futures prices in our sample period. This prevents the empirical test of production cost as the reference as the regression will restore to base case with no reference.

Comparison between the reference price specification and the base case shows little improvement in model fit when the references are static. When the past 30-day average price is used as reference, adjusted $R^2$ is increased from 31% to more than 38%.
Table 2. *OLS Estimates of Parameters and Robust Standard Errors Based on Iowa Corn Producers’ Pre-Harvest Weekly Forward Contracting Data, 2009–2013.*

Equation 9: \( \Delta h_t = \tilde{\alpha}_0 + \alpha_1 \mathbb{1}_{(p_t - R_t < 0)} + \beta_1 \text{time} + \beta_2 \text{vol}_t + \beta_3 \Delta p_t + \beta_4 \Delta p_t \mathbb{1}_{(p_t - R_t < 0)} + \epsilon_t \).

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<th>Coefficient</th>
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<th>Last year average</th>
<th>30 day average</th>
<th>RMA projected price</th>
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<tr>
<td>( \beta_2 )</td>
<td>0.0019</td>
<td>0.0018</td>
<td>0.0010</td>
<td>0.0018</td>
</tr>
<tr>
<td>(0.0019)</td>
<td>(0.0018)</td>
<td>(0.0014)</td>
<td>(0.0018)</td>
<td></td>
</tr>
<tr>
<td>( \beta_3 )</td>
<td>0.0964***</td>
<td>0.0915***</td>
<td>0.1370***</td>
<td>0.0982***</td>
</tr>
<tr>
<td>(0.0241)</td>
<td>(0.0225)</td>
<td>(0.0411)</td>
<td>(0.0291)</td>
<td></td>
</tr>
<tr>
<td>( \beta_4 )</td>
<td>-0.0087</td>
<td>-0.1175***</td>
<td>-0.0203</td>
<td>-0.0203</td>
</tr>
<tr>
<td>(0.0433)</td>
<td>(0.0419)</td>
<td>(0.0395)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \tilde{\alpha}_0 + \alpha_1 )</td>
<td>0.0058***</td>
<td>0.0046***</td>
<td>0.0059***</td>
<td></td>
</tr>
<tr>
<td>(&lt;0.0001)</td>
<td>(0.0044)</td>
<td>(0.0001)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \beta_3 + \beta_4 )</td>
<td>0.0828**</td>
<td>0.0195*</td>
<td>0.0778**</td>
<td></td>
</tr>
<tr>
<td>(0.0269)</td>
<td>(0.0608)</td>
<td>(0.0109)</td>
<td></td>
<td></td>
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<tr>
<td>Adj-R(^2)</td>
<td>0.3109</td>
<td>0.3194</td>
<td>0.3754</td>
<td>0.3182</td>
</tr>
<tr>
<td>DW Test</td>
<td>0.2528</td>
<td>0.2644</td>
<td>0.1917</td>
<td>0.2568</td>
</tr>
<tr>
<td>BP Test</td>
<td>0.0011</td>
<td>0.0004</td>
<td>0.0002</td>
<td>0.0004</td>
</tr>
</tbody>
</table>

*Note.* Significance levels indicated as: * p<0.1; ** p<0.05; *** p<0.01. Standard errors in the parentheses. The joint-significance of intercepts is examined with a Wald test with p-value reported. Durbin-Watson test and Breusch-Pagan test are performed against autocorrelation and heteroscedasticity respectively, and p-values are reported.

The time-to-maturity effect is still robust and statistically significant across all reference prices. No significant impact of price volatility on hedging is found. In the base case, the weekly average hedge captured by the intercept estimate is 0.68 percent of the total harvest. A 1 percent weekly price increase will lead to an additional hedging of 0.1 percentage point of the crop. The estimates in the base case may represent the averaging parameters estimates depicting the producer’s hedging pattern above and below the references, if the asymmetry does exist.
Parameter $\alpha_0$ captures the weekly average hedge when the futures price is above the reference. Producers on average hedge 0.78 percent of their total harvest in weeks when the futures price is above the previous year average marketing price and the RMA projected harvest price. This is greater than the average weekly hedge of 0.7 percent in the base case scenario of no reference. However, a threshold response in the hedge ratio as captured by estimate $\alpha_1$ when the current December futures contract price is below the producers’ candidate reference prices is not statistically significant for these two static references. Similarly, RMA projected price and last year’s price do not seem to cause the asymmetric price response in hedging. The parameter estimate $\beta_4$ are not significant in these two situations, and estimated $\beta_3$ capturing the producer’s price response in hedging when the futures price is above reference are similar in magnitude as in the base case.

We do observe different hedging patterns when the futures price fluctuates around a dynamic reference price (i.e., the past 30-day average price of the December futures contracts). As shown in table 2, the difference in hedging dynamics appears in two ways. First, when the futures price is above the price trend, the weekly average hedge is about the same in the base case, around 0.7 percentage point of the crop. However, the weekly hedge on average is significantly reduced to 0.46 percentage points when the futures price is below trend. The difference in hedging response to the price change is also detected. In particular, a 1% increase in price when the price is above the past 30-day average results in a 0.14 percentage point increase in hedging, this much higher than in the base case. On the other hand, in weeks when the futures price is below the 30-day average price, producers barely increase their hedging position, as the joint parameter estimates, $\alpha_0 + \alpha_1$ and is statistically insignificant from zero at the 5% level.
The 30-day average contract price parameter is robust to adding longer-dated price changes such as a 60-day, 90-day, and 6-month old moving average. The results are also robust to small changes in the number of days used to calculate the moving average. Yearly seasonality may also play a role in explaining variations in producers’ hedging behaviors, since the uncertainty of harvest may be resolved at different paces in different crop years. We test for such seasonality by adding yearly dummies as well as interacting the yearly dummies with the *weeks to harvest*. The joint test cannot reject the null hypothesis of no yearly seasonality in producers’ hedging.

Besides providing more explanatory power than other candidate references, the implication from the 30-day average price as reference for producers’ hedging is also consistent with the finding of the Markov Switching regression—an asymmetric hedging behavior exhibited by the corn producer. Note that if the shifts between the hedging and non-hedging states are indeed triggered by the futures price moving across the 30-day average price, then the periods when the futures price is above the 30-day average price shall also be accompanied by high probabilities of state 2 as inferred from the Markov Switching model. We plot the smoothed probability that state 2 prevails in each week in figure 4 alongside the actual weekly hedge data, and also the price change from the 30-day average price. The probability is normalized by the maximum value of the weekly hedge ratio change to facilitate a visual comparison. Immediately apparent is the high correlation between the probability of state 2 and the price change from the past 30-day average, which strengthens the evidence of reference-dependent hedging by corn producers, with a 30-day moving average of the futures price the likely candidate reference price. Finally, figure 5 plots the 30-day moving averages of the December futures and the level of producer hedge ratio. It shows that in recent years, producers’ hedge ratio was increasing at a
much faster pace when the futures price is trending up rather than down, leading the producer to hedge a greater portion of their harvest at the end of the pre-harvest season.

**Figure 4.** Weekly change in producer hedge ratios vs. percent price changes for December futures from its past 30-day moving average, 01/2009–08/2013

**Figure 5.** Producer hedge ratios vs. the 30-day moving average of December futures, 01/2009–08/2013
Do the observed hedging patterns result in a higher price?

The previous results suggest that hedging behavior is related to price changes, which in turn suggests that hedging might be driven in part by an attempt to time the market. This raises the question as to whether this trading pattern is better than selling a fixed portion of the expected crop each month or all of their crop at harvest or at planting. The October and January average price of the December futures contract is used to approximate the harvest price and planting price respectively. If the producers’ hedging patterns result in a superior performance to the common alternative hedging strategies, the reference-price effect argument may be weakened as the EU hedging does permit the possibility of producers speculating with private information.

To facilitate the analysis, we ignore the transaction cost and basis risk, and assume that the producers adjust their hedge ratio weekly. We calculate the weighted average price per bushel received on the hedged crop $P$, using actual pre-harvest hedging. The weights correspond to the volume sold at the end of each week $t$ as a percentage of the total harvest, namely weekly change in hedge ratio $\Delta h_t$, normalized by the level of hedge ratio the end of pre-harvest season $T$, $h_T$:

$$P = \frac{\sum_{t=1}^{T} (\Delta h_t * p_t)}{h_T},$$

(10)

Table 3 shows the average prices per bushel sold using the producer hedge series, the futures price at harvest, the futures price at planting, and the average price of December futures contracts during the pre-harvest period. With only five years of data it is not advisable to use these results to draw strong conclusions. However, we can say that during this five-year period both farm-level hedgers obtained a better price than the strategy of selling equal amounts every month before harvest and the strategy of selling all of their expected production in January or
March. The dominant strategy during this somewhat unusual period was to wait until harvest to sell grain. This is due to large increases in corn prices during 2010 and 2012.

Table 3. Average Price Received by Producers before Harvest, in Cents.

<table>
<thead>
<tr>
<th></th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
<th>5-year average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sell equal amount monthly</td>
<td>402</td>
<td>395</td>
<td>642</td>
<td>609</td>
<td>538</td>
<td>517</td>
</tr>
<tr>
<td>Price at harvest</td>
<td>371</td>
<td>546</td>
<td>632</td>
<td>750</td>
<td>439</td>
<td>548</td>
</tr>
<tr>
<td>Price in January</td>
<td>435</td>
<td>413</td>
<td>569</td>
<td>567</td>
<td>585</td>
<td>514</td>
</tr>
<tr>
<td>Price in March</td>
<td>411</td>
<td>397</td>
<td>598</td>
<td>559</td>
<td>558</td>
<td>505</td>
</tr>
<tr>
<td>Price received by farm level hedger</td>
<td>415</td>
<td>400</td>
<td>657</td>
<td>680</td>
<td>549</td>
<td>540</td>
</tr>
</tbody>
</table>

Comparison to the DCOT new crop hedging data

The DCOT report has been criticized for the limitation in accurately classifying traders’ activities (CFTC 2006a), as it aggregates across all futures contracts, and thus the DCOT report will include storage hedges as well as spread trades. However, CFTC also separates the open interest into new and old futures in the DCOT report. The new futures open interest corresponds to the contracts maturing in the first contract of the next marketing year and later, which presumably contains information about producers’ pre-harvest hedging. We construct the commercial pre-harvest hedge ratio in a similar fashion as the producer hedge ratio, which is calculated as the ratio of the weekly short hedgers open positions in new futures contracts over the USDA’s final number of total U.S. corn harvest. Figure 6 plots the level of both producer and commercial hedge ratios during the pre-harvest period. It is immediately apparent that the two series are highly correlated, indicating the CFTC DCOT short hedgers’ position in new futures represents the actual producers’ hedging reasonably well. This makes sense as corn producers can hedge their expected crop either by taking a short position in the futures markets or by forward contracting with a local grain dealer or warehouse that in turn hedges this exposure on futures markets.
Figure 6. The level of hedge ratios, commercial vs. producer, 01/2009–08/2013
The DCOT data may be published with a lag. Grain elevators may accumulate forward contracts and lay off the risk in futures markets on a sporadic basis. There also might be lag in the reporting process from futures participants to the CFTC. This seems to be the case for corn. The weekly producer hedge ratio constructed according to the release dates of the DCOT reports leads the change in commercial hedge ratios as shown in figure 7. A hedge ratio constructed using three trading-days prior to the release date of the DCOT report, as plotted in figure 8, shows a noticeable degree of improvement in terms of the matching of the two series.

Figure 7. Weekly change in pre-harvest hedge ratios, commercial vs. producer series with matching date, 01/2009–08/2013
Figure 8. Weekly change in pre-harvest hedge ratios, commercial vs. producer series lagged by three trading-day, 01/2009–08/2013

Summary and Conclusions

This paper explores the role of reference-dependence in producer’s optimal hedging. We develop a theoretical framework that highlights the empirical implications that would violate EU optimal hedging, but are consistent with the hedging behavior of a reference-dependent producer. Empirically, we analyze a dataset of every forward contract for more than 30 locations over a five-year period. We compute a producer hedge ratio by comparing the proportion of corn that was forward contracted to the total amount delivered in that year to those elevators. The data show that in the period from 2010 to 2012, when prices trended up, as much as 24% of the crop was forward contracted. In 2013, when prices trended down only 4% was forward contracted.

This paper demonstrates that the asymmetric behavior in the Iowa corn producers’ hedging is clearly present. Hedgers respond to price changes, and in particular they sell more when the current futures price is above the monthly average. The tendency of hedgers to sell into a price rally, will in the short run, help to stabilize futures prices. However, the tendency to sell
more corn in drought years may exacerbate harvest time price volatility because a smaller proportion of the crop will be uncommitted at the end of the season. The reduction in hedging behavior in years when prices trend down is problematic from a risk management perspective. These are the years when hedging is needed to stabilize revenues. We also examine whether this price-induced hedging activity results in higher prices than less active hedging strategies. The evidence is mixed, in part because the database is too short to make a statistically valid conclusion. This is something that can be addressed for a range of commodities now that it has been shown that the DCOT data in agricultural commodities reflect actual producer behavior.
Appendix

Proof of Proposition 1.

The producer believes the futures is unbiased with equal probabilities of higher or lower (i.e., \( \pi = 1/2 \)) prices. If the producer is reference-independent, they will maximize the objective function with respect to the hedge ratio

\[ \max_h ER^{1-\alpha}/1-\alpha, \]

and the optimal hedge ratio \( h^* \) is given by the following:

I. \( (1-h^*)\epsilon = 0, \)

which suggests that under the unbiased futures price, the producer will fully hedge their crop regardless of the level of the futures price (i.e., \( h^* = 1 \)).

To solve the optimal hedging of a reference-dependent producer, three cases need to be considered.

Case 1. The futures price is above reference: \( p^f > b \) and the producer believes the futures price is unbiased, \( \pi = 1/2 \).

In this case, the producer will have expected gain:

II. \( \frac{1}{2} [p^f + (1-h)\epsilon - b + p^f - (1-h)\epsilon - b] = p^f - b > 0, \)

and by concavity of the utility function in gains:

III. \[ u(h = 1; b, \alpha, \epsilon, p^f) = \frac{(p^f - b)^{1-\alpha}}{1-\alpha} \]

\( \geq \frac{1}{2} \left[ \frac{(p^f + (1-h)\epsilon - b)^{1-\alpha}}{1-\alpha} + \frac{(p^f - (1-h)\epsilon - b)^{1-\alpha}}{1-\alpha} \right], \forall h > h \)
where $\hat{h}$ is a threshold such that $p^f - (1 - \hat{h})\epsilon - b = 0$. In this case, where the futures price is higher than the reference, it is still possible that the producer ends up with a utility loss if he hedges too little. However, this is never optimal for the producer as captured in the second inequality in expression III, because the marginal utility loss is always greater than the marginal utility of gain when the futures price is higher than the reference price. To see this, consider $h = \hat{h}$, such that the producer’s utility has a limited downside of zero and upside equal to

$$\frac{(p^f + (1-\hat{h})\epsilon - b)^{1-\alpha}}{1-\alpha}.$$ Then the marginal utility of gain by reducing hedges is $$(p^f + (1 - \hat{h})\epsilon - b)^{-\alpha} \epsilon,$$ while the marginal utility of loss is $$(p^f - (1 - \hat{h})\epsilon - b)^{-\alpha} \epsilon = 0^{-\alpha} \epsilon = -\infty.$$ Thus, a reference-dependent producer who believes that the futures price is unbiased will hedge all of the crop when the futures price is above the reference.

**Case 2.** The futures price is below reference: $p^f < b$ and the producer believes the futures price is unbiased, $\pi = 1/2$.

Now the producer will always have an expected loss as $\frac{1}{2} [p^f + (1 - h)\epsilon - b + p^f - (1 - h)\epsilon - b] = p^f - b < 0$. Given the convexity of the producer’s utility function in losses, we can establish the following:

IV. \quad u(h = 1; b, \alpha, \epsilon, p^f) = -\frac{(b - p^f)^{1-\alpha}}{1-\alpha}

$$\leq -\frac{1}{2} \left[ \frac{(b - p^f - (1-h)\epsilon)^{1-\alpha}}{1-\alpha} + \frac{(b - p^f + (1-h)\epsilon)^{1-\alpha}}{1-\alpha} \right], \quad \forall h > \hat{h}$$

$$\leq \frac{1}{2} \left[ \frac{(p^f + (1-h)\epsilon - b)^{1-\alpha}}{1-\alpha} - \frac{(b - p^f + (1-h)\epsilon)^{1-\alpha}}{1-\alpha} \right], \quad \forall h < \hat{h}$$
\[ \leq \frac{1}{2} \left[ \frac{(p^f + \epsilon - b)^{1-\alpha}}{1-\alpha} - \frac{(b - p^f + \epsilon)^{1-\alpha}}{1-\alpha} \right] = u(h = 0; b, \alpha, \epsilon, p^f) \]

The second inequality can be established in the same way as in case 1, whereas in the case of the futures price below reference, the marginal utility gain is always greater than the marginal utility loss from a reduction in hedging. So the reference-dependent producer will not hedge if the futures price is below the reference.

Case 3. The futures price is equal to reference: \( p^f = b \) and the producer believes the futures price is unbiased, \( \pi = 1/2 \).

The utility of the producer with reference-dependence who fully hedges at \( p^f = b \) is zero. For any deviation from the full hedge, the marginal utility gain and loss will cancel out because the utility function is symmetric with respect to the origin as shown in figure 1. Therefore, his expected utility is always zero.

Clearly, for the reference-dependent producer the hedging decision hinges on the level of futures price; whereas the EU optimal hedge ratio without reference does not.

Proof of Proposition 2.

Under EU optimal hedging without reference, \( b = 0 \), the producer’s optimal hedge ratio is

\[ V. \quad h^* = 1 - \frac{p^f}{\epsilon} \left[ \frac{\pi^{1/\alpha} - (1-\pi)^{1/\alpha}}{\pi^{1/\alpha} + (1-\pi)^{1/\alpha}} \right]. \]

So for a reference-independent EU maximizing producer, as long as he believes the futures price is biased, his optimal hedge ratio will deviate from the full hedge. In particular, the marginal effect of higher probability for the futures price to go up on the hedge ratio is:
VI. \[ \frac{\partial h^*}{\partial \pi} = - \frac{2p^f}{\alpha e} \frac{[(1-\pi)\pi]^{1-\alpha/\alpha}}{\left[\pi^{1/\alpha} + (1-\pi)^{1/\alpha}\right]^2} < 0. \]

As for a reference-dependent producer who believes that the futures price is biased, there are still two cases to be considered.

**Case 4.** The futures price is above reference: \( p^f > b \) and the producer believes that futures price is biased, \( \pi \neq 1/2 \).

Suppose the producer’s optimal hedge ratio in this case is greater than \( \hat{h} = 1 - \frac{p^f - b}{\epsilon} \), his utility function is concave. Then the expected utility maximizing hedge ratio takes a similar formulation as the reference-independent producer:

VII. \[ h^* = 1 - \frac{p^f - b}{\epsilon} \frac{\pi^{1/\alpha} - (1-\pi)^{1/\alpha}}{\left[\pi^{1/\alpha} + (1-\pi)^{1/\alpha}\right]}, \text{ s.t. } h^* > \hat{h} \]

Clearly, the constraint is satisfied as \[ \frac{\pi^{1/\alpha} - (1-\pi)^{1/\alpha}}{\pi^{1/\alpha} + (1-\pi)^{1/\alpha}} < 1 \]. So a reference-dependent hedger will still speculate when the futures price is higher than his reference, but the magnitude of speculation, \[ \frac{\partial h^*}{\partial \pi} = - \frac{2(p^f - b)}{\epsilon \alpha} \frac{[(1-\pi)\pi]^{1-\alpha/\alpha}}{\left[\pi^{1/\alpha} + (1-\pi)^{1/\alpha}\right]^2} \] is smaller as the producer will try to avoid utility loss.

**Case 5.** The futures price is below reference: \( p^f < b \) and the producer believes that futures price is biased, \( \pi \neq 1/2 \).

As shown in case 2, the producer will not hedge when the unbiased futures price is below reference. So a corollary of proposition 1 is:

*If the producer believes the futures price has a higher probability to go up than down, the producer will still not hedge.*
To see this, let us consider the marginal impact of hedging on his expected utility, which can be written as:

\[
\text{VIII. } -\pi \varepsilon (p^f - b + (1 - h)\varepsilon)^{-\alpha} + (1 - \pi)\varepsilon (b - p^f + (1 - h)\varepsilon)^{-\alpha} \leq 0
\]

The first term of the expression represents the marginal utility loss from hedging—by increasing the hedge ratio the producer locks in the price at a loss. The second term captures the marginal utility gain from hedging, as if the bad state is realized, the producers’ utility will be protected from going even more negative. Thus, the above equation is always non-positive for \( \pi \geq 1/2 \).

The only case for the producer to consider a hedge is when the probability of the futures price going down is greater than 50%, or \( \pi < 1/2 \). In this situation, the producer’s optimal hedge ratio is:

\[
\text{IX. } h^* = 1 - \frac{b-p^f}{\varepsilon} \left[ \frac{\pi^{1/\alpha} + (1-\pi)^{1/\alpha}}{(1-\pi)^{1/\alpha} - \pi^{1/\alpha}} \right] \geq 0.
\]

Clearly, for the probability of the downward movement, the price needs to be high enough such that \( \frac{b-p^f}{\varepsilon} \left[ \frac{\pi^{1/\alpha} + (1-\pi)^{1/\alpha}}{(1-\pi)^{1/\alpha} - \pi^{1/\alpha}} \right] < 1 \). This is a possibility without more parametric restriction. Thus, the hedging response in this case is:

\[
\frac{\partial h^*}{\partial \pi} = \begin{cases} 
-\frac{2(p^f-b)}{\varepsilon \alpha} \left[ (1-\pi)^{1/\alpha} - \frac{\pi^{1/\alpha} + (1-\pi)^{1/\alpha}}{(1-\pi)^{1/\alpha} - \pi^{1/\alpha}} \right]^2 & \text{if } \frac{b-p^f}{\varepsilon} \left[ \frac{\pi^{1/\alpha} + (1-\pi)^{1/\alpha}}{(1-\pi)^{1/\alpha} - \pi^{1/\alpha}} \right] < 1 \\
0 & \text{Otherwise.}
\end{cases}
\]
References


Endnote

1. Sartwelle et al. (2000) find that 70% of grain producers surveyed, 351 respondents in total, use forward contracts. Musser, Patrick, and Eckman (1996) and Schroeder et al. (1998) find similar adoption rates for forward contracts at 74% and 64%, respectively. Davis et al. (2005) and Velandia et al. (2009) find significantly smaller adoption rates at 30.8% and 38%, respectively. In a 1999 study, the USDA Agricultural Resource Management (USDA-ARMS 1999) found that out of 2,662 corn producers, only 12% used forward contracts. Other surveys find that farmers who utilize forward contracts use them to hedge 15%–40% of their harvest (Schroeder et al. 1998; Davis et al. 2005).

2. The correlation coefficient between the average basis for five cities where the co-op has a presence (data purchased from GeoGrain Inc.) against the change in hedge ratios between 2009 and 2013 is -0.09, suggesting a limited role of fall basis in hedging during this time period.

3. Adding loss aversion complicates the optimization because of preference reversal around the reference point, as the objective function is not globally concave. Mattos, Garcia, and Pennings (2008) show that loss aversion only matters to hedge ratios in the presence of probability weighting.

4. Grain marketed in September is likely crop from a prior year; the data do not permit us to accurately separate grain marketed from storage and new crop. Our estimation strategy relies on changes in the hedge ratio and price changes.

5. A potential source of bias in the level of hedge ratio is that the cooperative annual purchases could be correlated with prices. However, five-year data is limited to identify such a
correlation if any, while our paper is really focusing on the weekly change in the hedge ratio, which dominates the variation in the annual cooperative purchase.

6. The average December futures prices during the pre-harvest season are $4.02, $3.95, $6.42, $6.09, and $5.38 per bushel for the years 2009, 2010, 2011, 2012, and 2013, respectively.

7. The pre-harvest forward selling motivated by futures price movements shall not be confused with the law of supply because the crop supply is perfectly inelastic after the planting decision within a crop year.

8. We refer to the calendar day here; 30 calendar days are equivalent to 22 trading days.
CHAPTER 4
THE WEATHER PREMIUM IN THE U.S. CORN MARKET

Abstract

We show that the weather premium, an anecdotal phenomenon in the US corn futures market, can arise from a convex and asymmetric demand function. We also show that the magnitude of the weather premium will depend on the carry over and expected yield at harvest. We then use data from 1968 to 2015 to evaluate the accuracy of the December futures price as a forecast of the maturity price on this contract. We identify a predictable component in the forecast errors that is consistent with the existence of a time-varying weather premium. We show that a passive strategy of routinely shorting the corn December futures does not provide an attractive risk-adjusted return.
Introduction

Participants in the US corn futures market often refer to a “weather premium”. This premium, if it exists, suggests that the price of the December futures contract in spring will over predict the actual harvest price more often than not. Over the last 50 years, the spring December futures price on average overestimated the realized harvest price by 4 percent. A predominant argument for the motivation of weather premium is the supposition that a weather event that results in negative shocks to supply causes a larger increase in prices than does the price reduction associated with a weather event that causes a positive supply shock. To the best of our knowledge, the academic literature has yet to formally define and explore the weather premium.

The goal of this paper is to analyze whether and to what extent the weather premium exists in the corn futures market. We show theoretically that a convex demand curve alone will give rise to the weather premium, which we define as the difference between the expected harvest price and the price at expected harvest. This differs from the definition of the risk premium, which refers to the deviation of the futures price deviates from the expected harvest price.

We then explore the forces that should influence the weather premium. We show that the asymmetry of the demand curve for grains caused by storage causes the weather premium to be dependent on the level as well as the variance of expected supply at harvest. We test these theoretical predictions by studying the forecast errors of corn December futures from 1968 to 2015. A predictable component of the forecast error that varies overtime with the distribution of the expected supply is consistent with the existence of weather premium. We find that the carryout and recent yield realizations, the key determinants of the expected supply, do provide statistically significant explanatory power for the variations in the forecast errors. In an out-of-
sample forecast evaluation, the futures prices adjusted for the estimated weather premium performs better than the unadjusted future prices.

Finally, we evaluate several trading strategies that aim to take advantage of the weather premium. Routinely shorting the corn December futures that is often recommended by Commodity Trading Advisories does provide a positive expected return. However, we find that the Sharpe ratio of this passive strategy is too small to justify such an approach.

Previous Work

The previous literature examining possible bias in the grain futures markets can be categorized into two strands: 1) those based on the theory of storage, and 2) those based on the theory of normal backwardation.

The theory of storage assumes an implicit benefit of holding inventory, i.e. convenience yield (Working 1949), which is inferred from the temporal basis, i.e. the difference between the futures price and the cash price. The literature posits that the inventory level is negatively correlated with the convenience yield (Fama and French 1987 and Carbonez et al. 2009), but how it links to the expected premium, i.e. the bias in the futures price, is less clear. Fama and French (1987) show the temporal basis has little value in explaining the premium in the corn futures contracts over a variety of time horizons, if there is any. Gorton et al (2013) use actual inventory data to approach this problem and arrive at a similar conclusion.

On the other hand, the theory of normal backwardation (Keynes 1930), also known as the hedging pressure hypothesis, postulates that risk-averse short hedgers will pay a premium to the speculator to bear the spot price risk. This phenomenon may only apply to markets in which there are majorities of hedgers with natural long positions. Among such markets are those for
agricultural commodities. However, there is little empirical support for normal backwardation (Frank and Garcia 2009 and Gorton et al. 2013). For example, among those that report to the CFTC, the performances of larger speculators is largely random (Hartzmark 1991), despite a small subset of speculators that appear to consistently outperform the markets (Aulerich, Irwin and Garcia, 2013).

Weather during the growing season is the most important determinant of the harvest price forecast error. The correlation between the year-to-year change in harvest prices and yields is -74% over the period 1968-2015. Figure 1 illustrates that the monthly average of December futures prices prior to July is about 4% higher than the final harvest prices, approximated by the October average price of the December futures. After July, the average for this difference goes to zero.

Figure 1. Density plots of monthly average of December futures price of corn minus the harvest price, expressed as a percentage of the harvest price, 1968-2015
Good’s (2016) work using data during the mid-1970s to mid-1990s showed that the tendency for the market to over predict the actual harvest price may be owed to the volatility in yield. In response, commodity trading advisories (CTAs) have advocated trading strategies to take advantage of this statistical phenomena (e.g. Till 2000, 2001 and 2005), and their central working hypothesis is that the market is loss-averse and is willing to pay a premium to insure against adverse weather shocks. In other words, a passive short strategy on average will yield a positive return.

There are other potential sources of expected premium in the futures markets. Under the capital asset pricing model (CAPM), the risk premium in futures markets is proportional to the covariance of the futures return with the return on the market portfolio. However, there is little empirical evidence that the agricultural futures returns are correlated with the equity returns (Jagannathan, 1985 and Bessembinder, 1992).

There is a vast literature on futures pricing models and how hedgers and speculators respond to futures prices and also the types of biases or pressures that exist that generate premiums for certain positions. Still, we do not have concrete evidence on the source of the bias in commodities futures markets, specifically for grain commodities. This article explores weather premium as a potential source of bias. Also, the empirical work to date have focused on the use of futures contracts to gauge the behavior of trading responses. We extend this to pay specific attention to the harvest futures contract.

Theoretical Framework

In this section, we formalize the concept of the weather premium, and derive the comparative statics regarding what influence the magnitude of the weather premium.
Suppose a deterministic demand function for corn $D(p_T)$ is convex in $p_T$, the harvest price of corn in year $T$, such that $D'(p_T) < 0$, and $D''(p_T) > 0$. Given the supply $S_T$ at harvest is perfectly inelastic consisting of both the new production, $z_T$ and carryout available at harvest $c_T$, the equilibrium price function, $p_T(S_T) > 0$ is also convex. For future time periods further from harvest (i.e. Spring in the case of corn in the Northern Hemisphere), $t < T$, there is uncertainty in the harvest size, so the only state variable is the carryout amount from the prior marketing year, $c_t$. By Jensen’s inequality:

\begin{equation}
E_t[p_T(S_T)] - p_T[E(S_T)] \geq 0,
\end{equation}

where $E_t$ is the expectation operator conditional on the information available at time $t$. The inequality suggests that the difference between the expectation of the price and the price at the expected harvest quantity is positive. We term this difference, $E_t[p_T(S_T)] - p_T[E(S_T)]$, as the weather premium because weather is the primary determinant of yield once a crop has been planted, and therefore also determines the harvest quantity. Graphically, the line segment between point $b$ and point $d$ in figure 2 traces out the weather premium. By a visual inspection, the weather premium is unlikely to be a constant from year-to-year as it depends on 1) the carryout and 2) the ex-ante market expectation of the new harvest quantity.
Denote the cost of carry, $g(c_T | c_t)$, as the cost of storing $c_T$ amount of corn from time $t$ to the harvest time $T$ with the initial carryout available $c_t$. According to the rational expectation competitive storage model, the cost of carry is equal to the difference between the expected harvest price $E_t[p_T(S_T)]$ and the spot price $p_t$, i.e., the temporal basis:

(2) \[ E_t[p_T(S_T)] - p_t = g(c_T | c_t). \]

Suppose the risk premium is zero, then the futures price $f_{t,T}$ is equal to the expected harvest price $E_t[p_T(S_T)]$, and equation 2 can be rewritten as:

(3) \[ f_{t,T} - p_t = g(c_T | c_t). \]

If the inverse of the cost of carry function $g(\cdot)$ exists, then we can express the carryout available at this year’s harvest as:

(4) \[ c_T = g^{-1}(c_t, f_{t,T} - p_t). \]
In other words, the ending stock from the last marketing year is dependent on a price-based signal. Thus, the temporal basis should contain information about the weather premium.

As shown in equation (4) that the inter-marketing year carry and the temporal basis are co-determined. In good harvest years, the temporal basis tends to be positive to incentivize the market to store forward; while in bad years the price may spike due to the market’s inability to borrow from the future. Deaton and Laroque (1992) shows that this dynamics will cause the demand curve to asymmetric, and there is a threshold level of supply, above which the equilibrium price becomes much less sensitive to further increases in supply. This characteristic of the grain equilibrium price function implies that the weather premium will decrease as the expected supply at harvest increases.

To show this mathematically with a continuous demand function, we assume a negative third derivative, \( D'''(p_T) < 0 \). This ensures the negative slope of the demand curve holds for all possible values of the price. For analytic tractability, further assume that the realization of harvest follows an i.i.d. binominal distribution:

\[
(5) \quad z_T = \begin{cases} 
\bar{z} + \epsilon, & \text{with probability } \frac{1}{2}, \\
\bar{z} - \epsilon, & \text{with probability } \frac{1}{2}.
\end{cases}
\]

where \( \bar{z} \) is the market’s expectation about the size of harvest. In this way, the realized harvest amount is random and conditional on \( \bar{z} \), either higher or lower by \( 0 < \epsilon < \bar{z} \), with equal probabilities.

Given (1) - (5), it is straightforward to show how the weather premium will vary with the expectation of the supply at harvest and its associated variation, which we summarize into the following comparative statics:
Comparative statics 6 show that an increase in expected supply at harvest, due to increases in either the expected harvest quantity or the carryout, will have a negative impact on the weather premium. On the other hand, an increase in the variability of the harvest as captured by $\epsilon$ causes the weather premium to increases as shown in the expression 7. Figure 3 is a visual description of this result. As the harvest variability increases from $\epsilon$ to $\epsilon'$, holding the mean of the total supply constant, the weather premium increases.

**Figure 3.** Graphic demonstration of weather premium.
The weather premium is a phenomenon that arises from the property of the demand curve without implying a biased futures price. Deaton and Laroque (1992) demonstrate that with a convex demand curve, the equilibrium price distribution is skewed to the right with the left tail ‘thinning out’ because inventory holders buy at low prices. Inventory is limited in its ability to reduce the right tail because the reduction in supply due to a lower than expected harvest can overwhelm the existing size of inventory. In other words, the mean of this price distribution is greater than the median. Our theoretical analysis demonstrates a similar point, but complements the work of Deaton and Laroque (1992) by formally defining weather premium, which takes place within a marketing season in the harvest futures prices, whereas Deaton and Laroque focus on the year-to-year variation of cash prices.

From the perspective of risk aversion, a common narrative also consistent with the comparative statics (6) states that increasing carryout, which serves as a buffer for the adverse weather shocks, reduces the weather premium. The remaining carryout of corn is often very small as compared to the size of expected new crop at the time of harvest, so a negative supply shock can cause the price to spike, regardless. In other words, if the weather premium is indeed a risk premium such that $f_{t,T} > E_t[p_T(S_T)]$, the difference between the futures price and expected harvest price, i.e. the premium, should not vary year over year with the carryout alone.

Empirical Procedure

In the previous section, we show how the expected supply at harvest, including both the carryout and the expected size of the new harvest, might affect the weather premium. To bring the theoretical framework to the data, we focus the empirical analysis on the forecast errors of the December futures contract of corn as a predictor of the realized harvest price. The logic is
that before the growing season of each year, the futures market will build in the weather
premium according expression (1), but when an average or above weather condition is realized,
the spring futures price will then over predict the actual harvest price, i.e. a positive forecast
error. In other words, our identification of the existence of weather premium depends on to what
extent the forecast error variation can be captured by the variables describing the distribution of
the expected harvest size, which we discuss in the next subsection. Quantify the magnitude of the
weather premium is beyond the scope of this paper.

**Forecast errors**

We evaluate the monthly average price of the corn December futures contract traded on
the Chicago Mercantile Exchange (CME) as the forecast price for the U.S. corn harvest price,
covering the first six months of each calendar year from 1968 to 2015. Following the procedure
used by the United State Department of Agriculture (USDA) Risk Management Agency (RMA),
we use the average of the December futures prices during October as the approximate harvest
price. The forecast error is just the difference between the log forecast price and the log harvest
price. Figure 4 shows that the forecast errors in the December futures price appear random
between January and June.
Figure 4. Forecast errors by months from January to June, 1968-2015
Table 1 presents the summary statistics of the forecast errors of each month from January to September over the period of 1968 to 2015. The logarithm difference between the spring corn futures price and the final harvest price ranges from 0.030 to 0.044. Interestingly, the monthly forecast errors increase in magnitude during the spring season. The standard deviations of the forecast errors decline rapidly from July to September, but remain remarkably stable over the spring at approximately 20% of the harvest price. Notice the forecast errors before June are skewed to the left, but change in July from negative to positive skewness. This indicates that the information content in July, on average, pushes the forecast errors towards zero. Also, the change in kurtosis in July indicates a heavier tail in the forecast error distribution, which is often accompanied with the new information shocks.

<table>
<thead>
<tr>
<th></th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.030</td>
<td>0.036</td>
<td>0.041</td>
<td>0.041</td>
<td>0.036</td>
<td>0.044</td>
<td>0.026</td>
<td>0.015</td>
<td>0.003</td>
</tr>
<tr>
<td>Std. Dev</td>
<td>0.205</td>
<td>0.203</td>
<td>0.203</td>
<td>0.205</td>
<td>0.197</td>
<td>0.191</td>
<td>0.134</td>
<td>0.097</td>
<td>0.071</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.724</td>
<td>-0.661</td>
<td>-0.574</td>
<td>-0.517</td>
<td>-0.526</td>
<td>-0.222</td>
<td>0.343</td>
<td>0.067</td>
<td>0.659</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>-0.032</td>
<td>-0.057</td>
<td>-0.384</td>
<td>-0.165</td>
<td>-0.277</td>
<td>0.338</td>
<td>1.521</td>
<td>1.571</td>
<td>4.156</td>
</tr>
</tbody>
</table>
Carryout and temporal basis

On a year-to-year basis, carryout is the sum of last marketing year’s ending stock and harvest published in USDA WASDE reports. We detrend the carryout by its past 5-year average to obtain a unit-free measure of normal inventory level. This initial level of carryout together with the temporal basis, defined as the logarithm of the difference between the December futures price and the price on the nearby futures contract, determines the inventory to be carried to the next harvest. The continuous nearby futures price is constructed by rolling into the next nearby futures contract on the last trading day of the expiring contract. As demonstrated earlier, ceteris paribus, the weather premium should fall as carryout increases.

Figure 5 plots the normalized carryout, showing a wide range of year-to-year change from the past trend with a 36% increase in 1980 and a negative 20% in 2013. We also add the monthly basis series in figure 4, and it is shown to be highly correlated with the level of carryout.

*Figure 5. Normalized carryout versus the monthly temporal basis between January and June, 1968-2015*
**Past yield realization**

The other important determinant of the weather premium is harvest size, which is joint determined by the acreage planted and yield. Planting area is a choice variable by producers that is mainly explained by the futures price, cash price and government programs (Chavas, 1983), but not the weather. Therefore, we focus on the role of yield variability, a factor most influenced by weather. To measure the relative level and variability of yield, we propose two variables: the log change of last year’s crop from its 5-year trend yield, and the 5-year mean-normalized yield, respectively. Denoting $y_t$ as the corn yield in year $t$, the variability of the yield realizations, $v y_t$ is measured as the standard deviation of the rolling 5-year yield normalized by its mean, calculated by:

$$(8) \quad v y_t = \sqrt{5 \sum_{i=t-5}^{t-1} (y_i - \sum_{i=t-5}^{t-1} y_i)^2 / \sum_{i=t-5}^{t-1} y_i}.$$  

The level of the yield realization is the logarithm change of last year’s crop yield from a trend yield of past five years:

$$(9) \quad d y_t = \log(y_t) - \log(\sum_{i=t-5}^{t-1} y_i).$$

Intuitively, to the extent that the recent yield realization influences market participants’ subjective yield distribution, it may also impact the weather premium. Figures 6 shows that the yield variability has declined since the late 1990s, but that weather still prevails as the dominant factor: the yield dropped 20 percent from the trend yield when the drought hit the corn harvest in U.S. in 2012.
Figure 6. Last year’s yield realization from 5-year trend vs. the standard deviation of the yield realization in the past five years, 1968-2015.
Empirical specification

To understand the role of these factors, we estimate the following simple empirical

\[ e_{t,j} = \beta_0 + \beta_2 \text{carry}_{t-1} + \beta_3 \text{carry}_{t-1}^2 + \beta_4 \text{vy}_{t-1} + \beta_5 \text{dy}_{t-1} + \gamma_1 \text{basis}_{t,j} + \rho_j + \xi_{t,j} \]

where the forecast error in year \( t \) of month \( j \), \( e_{t,j} \equiv \log(f_{t,j}) - \log(s_t) \), the monthly December futures price of year \( t \) in month \( j \) minus the actual harvest price in year \( t \), depends on last year’s normalized carry from last year, \( \text{carry}_{t-1} \), relative yield realization \( \text{dy}_{t-1} \) as well as yield variability \( \text{vy}_{t-1} \). Carryout enters with a quadratic term to account for its potential nonlinear relationship with the weather premium. This year’s explanatory variable is a price-based signal \( \text{basis}_{t,j} \). Unobserved monthly fixed effects, \( \rho_j \)’s, are the difference in the conditional average expected premium in month \( j \) from the average of other months. The remaining disturbance term, \( \xi_{t,j} \), satisfies the i.i.d. assumption.

While a simple linear regression, a nice feature of regression equation (10) is that from a forecasting efficiency perspective, it encompasses the futures price and, by extension, all information embedded in the futures price. To see this, denote \( \hat{s}_{t,j} \) the model forecast of the harvest price in month \( j \), which is the average December futures price adjusted for estimated weather premium in that month:

\[ \hat{s}_{t,j} = f_{t,j} - \bar{e}_{t,j} \]

Then the model forecast error can be calculated as the difference between the forecast error of unadjusted futures prices and the estimated premium:

\[ \bar{s}_{t,j} - s_t = \bar{e}_{t,j} - e_{t,j}. \]
Notice that the term on the right-hand side of equation (12), $e_{t,j} - \tilde{e}_{t,j}$, is the negative residual in equation (10), $\xi_{t,j}$, and by construct, is not correlated with the model fitted values, namely the estimated weather premium, $\tilde{e}_{t,j}$.

For practitioners, our proposed empirical strategy is only useful if it provides a better out-of-sample forecasting performance. We compare the predictive success of different model specifications in terms of the reduction in mean squared prediction error (MSPE) from the no-change forecast. The reduction in MSPE is the ratio of the MSPE of the model-adjusted futures price to the MSPE of the no-change forecast (Baumeister and Kilian, 2016). Thus, any model with MSPE ratio greater than one is inadmissible as it adds more noise than explanatory power. The statistical significance of the MSPE reduction is assessed using a test proposed by Clark and West (2007) that accounts for the noise introduced by additional explanatory variables.

Results

As reported in table 2, the Breusch and Pagan Lagrangian multiplier test does not detect significant variation across months, i.e. there is no panel effect. Therefore, the parameters are estimated using OLS regression of the pooled sample and robust standard errors are reported.

Column 1 and 2 in table 2 provide results on whether the forecast errors of the December corn futures prices can be explained by the temporal basis and carryout respectively. The temporal basis by itself does not appear to contain any predictive information and the $R^2$ is close to zero. The normalized carryout explains some variation in the expected premium with $R^2 = 0.07$. The marginal effect is statistically significant and negative when evaluated compared to the carryout at the preceding 5-year-trend. This supports the theoretical prediction derived from the comparative statics equation (6): an increase in expected supply at harvest, due to increases in
either the expected harvest or the carryout, will have a negative impact on the weather premium. The regression results including both temporal basis and carryout are reported in column 3, this model specification results in an improved $R^2$ of 0.11. The magnitude of improvement is also statistically significant as suggested by the F-test with p-value 0.0003. In addition, the marginal effect of carryout remains negative, while the parameter estimate on the temporal basis becomes statistically significant at the 5% level. This empirical observation strengthens the need to consider both temporal basis and carryout. Intuitively, the temporal basis only tells us how much the market is compensating for the carryout of inventory, and it needs to be combined with the initial carryout level to determine the level of inventory still available at the upcoming harvest, which plays a direct role in determining the weather premium.

Column 4 of table 2 shows that the $R^2$ increases to 0.15 when past yield realizations are included in the model. Consistent with the theoretical predictions from comparative statics, increasing yield volatility tends to be followed by an increase in the weather premium. The level of last year’s yield relative to the trend has an expected negative impact on the weather premium. This may be due to a behavioral mindset among commodity traders who over estimate expected yield volatility in years after a weather shock.

Figure 7 plots the time series of the predicted log forecast errors in February from the regression equation (10). As discussed earlier, the predictive component in the forecast errors of the December futures related to the carryout and yield realizations is consistent with the existence of the weather premium. The time series in figure 6 shows negative model forecasts in some years while in theory the weather premium should be nonnegative. In this unconstrained estimation, the model forecast would pick up random variations in the actual forecast errors other than the weather premium.
Table 2. Pooled OLS Estimation of Parameters and Robust Standard Errors Based on the Corn December futures forecast errors, 1968–2015.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Basis only</td>
<td>Carryout only</td>
<td>Basis + Carryout</td>
<td>Full model</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.0379***</td>
<td>-0.1346</td>
<td>0.4751***</td>
<td>-0.9327***</td>
</tr>
<tr>
<td></td>
<td>(0.0117)</td>
<td>(0.1178)</td>
<td>(0.0899)</td>
<td>(0.1085)</td>
</tr>
<tr>
<td>Basis</td>
<td>0.0929</td>
<td>0.5512***</td>
<td>0.6185***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.1407)</td>
<td>(0.0526)</td>
<td>(0.0588)</td>
<td></td>
</tr>
<tr>
<td>Carryout</td>
<td></td>
<td>0.7406***</td>
<td>2.0349***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.2542)</td>
<td>(0.2036)</td>
<td>(0.2477)</td>
</tr>
<tr>
<td>Carryout²</td>
<td>0.5343***</td>
<td>-0.1795</td>
<td>-1.0715***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.1341)</td>
<td>(0.1169)</td>
<td>(0.1390)</td>
<td></td>
</tr>
<tr>
<td>Last year’s yield deviation from trend</td>
<td></td>
<td></td>
<td></td>
<td>-0.5577***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.0267)</td>
</tr>
<tr>
<td>Recent yield variability</td>
<td></td>
<td></td>
<td></td>
<td>0.6503***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.0227)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.0020</td>
<td>0.0720</td>
<td>0.1135</td>
<td>0.1531</td>
</tr>
<tr>
<td>BP test for Random Effect</td>
<td>0.0912</td>
<td>0.0880</td>
<td>0.1382</td>
<td>0.1546</td>
</tr>
<tr>
<td>BP test for Heteroskedasticity</td>
<td>0.0250</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>F-test</td>
<td></td>
<td></td>
<td>0.0003</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>Marginal Impact of Carryout at trend</td>
<td>-0.3279***</td>
<td>-0.5742***</td>
<td>-0.1081***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0263)</td>
<td>(0.0546)</td>
<td>(0.0368)</td>
<td></td>
</tr>
</tbody>
</table>

Notes. Estimated robust standard errors are given in parentheses. Asterisks denote significance levels as follows: * 10 percent; ** 5 percent; and *** 1 percent significance. Breusch-Pagan test are performed against random effect and heteroskedasticity respectively, and p-values are reported. The p-values of F-test that comparison the model specification 2 and 3, 3 and 4 are reported.
Though we can never observe the true weather premium, the model estimate of an empirical weather premium can offer some insights into its characteristics. First, contrary to the risk premium paradigm, the estimated weather premium shows considerable year-to-year variations. Moreover, the average of the estimated weather premium is about 4% of the actual harvest prices, which is very close to the arithmetic average of the forecast errors as shown in table 1. In other words, the difference between the spring December futures prices and the actual harvest prices can be largely accounted for by the covariates specified in equation 10.

Figure 8 plots the forecast errors of the December futures with and without adjusting for the estimated weather premium, and the former does offer a superior forecast from a visual inspection.

*Figure 7. Estimated logarithm of the weather premium in February, 1968-2015*
To gain further understanding about the properties of weather premium, we plot actual harvest prices with the model predicted price for the latest 10 years in figure 9. The model forecast prices appears to be no higher than the December futures price before the summer growing season. Thus in years when the futures price overestimates the actual harvest prices, the weather-premium-adjusted futures price performs better in terms of resulting in smaller forecast error.

**Out-of-sample performance**

We first examine whether the December futures price of corn, adjusted or not, contains additional information about the upcoming harvest price in comparison to a no-change forecast from last year. We use data from, 1996 to 2015 as our forecast evaluation period and calculate the initial parameter estimates based on data from 1968 to 1995. Table 3 presents the one-step
forward recursive estimations of MSPE ratios separated by months. The MSPE ratio of unadjusted futures price is presented in column 1. In column 2 we present the MSPE ratio of the model with carryout as explanatory variables only, and progressively add yield level, yield variability as well as temporal basis into the model, with the corresponding MSPE ratios presented in columns 3, 4 and 5 respectively. This is a simple procedure for model selection, which is important for out-of-sample forecast because of the bias-variance tradeoff (Kilian et al 2015): some variables may add more variance to forecasting in spite of their in-sample explanatory power.

Figure 9. Actual harvest price, futures prices and model forecast, 2006-2015

<table>
<thead>
<tr>
<th>Month</th>
<th>(1) Unadjusted futures</th>
<th>(2) Carryout only</th>
<th>(3) Carryout + yield level</th>
<th>(4) Carryout + yield level/variability</th>
<th>(5) Full model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>0.754</td>
<td>0.749</td>
<td>0.752</td>
<td>0.812</td>
<td>0.777</td>
</tr>
<tr>
<td>Feb</td>
<td>0.825</td>
<td>0.806</td>
<td>0.807</td>
<td>0.870</td>
<td>0.865</td>
</tr>
<tr>
<td>Mar</td>
<td>0.962</td>
<td>0.934</td>
<td>0.938</td>
<td>0.998</td>
<td>1.028</td>
</tr>
<tr>
<td>Apr</td>
<td>1.073</td>
<td>1.045</td>
<td>1.043</td>
<td>1.104</td>
<td>1.184</td>
</tr>
<tr>
<td>May</td>
<td>1.026</td>
<td>1.008</td>
<td>0.996</td>
<td>1.051</td>
<td>1.177</td>
</tr>
<tr>
<td>Jun</td>
<td>1.107</td>
<td>1.102</td>
<td>1.089</td>
<td>1.139</td>
<td>1.253</td>
</tr>
</tbody>
</table>

Note. West and Clark (2007) test statistic is reported, which follows the student t-distribution with critical values of 1.96 and 2.33 at 5% and 1% significance level respectively.

The t-statistics of West and Clarks (2007) shows that the out-of-sample forecast performances of all forecasts from January to June on average are significantly superior to a no change forecast. The month-to-month comparisons of forecasts need to be interpreted with caution, given for each month we have only 20 observations for forecast evaluation. That said, there is a strong pattern of deteriorating predictive success of all forecasts from January to June, this renders those after March inadmissible because they result in higher variances of prediction errors than a no-change forecast. This pattern might be due to the increasing trading in the December futures contracts as the planting and growing season progresses. As discussed earlier, there appears to be an increase in the weather premium in June, this is the time of the year the market focuses on weather. But the increase in weather premium in June does not seem to be
justified by the subsequent realizations of harvest price in most years. This leads to the
underperformance in June forecasts in comparison to other months.

Table 4 presents the rolling estimations of MSPE ratios, which might be preferable in the
presence of unknown structural breaks. However, the MSPE ratios from the rolling estimation
are very similar to that of the recursive forecast. This in part justifies our use of a parsimonious
regression model, which implicitly assumes that the dynamic structure of the weather premium
described by the carryout and realizations of yield in the past is time-invariant

Table 4. Out-of-Sample Forecast Comparisons to No-Change Forecast from Last Year’s Harvest

<table>
<thead>
<tr>
<th>Month</th>
<th>Unadjusted futures</th>
<th>Carryout only</th>
<th>Carryout + yield level</th>
<th>Carryout + yield level/variability</th>
<th>Full model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>0.754</td>
<td>0.755</td>
<td>0.758</td>
<td>0.868</td>
<td>0.852</td>
</tr>
<tr>
<td>Feb</td>
<td>0.825</td>
<td>0.804</td>
<td>0.808</td>
<td>0.921</td>
<td>0.9147</td>
</tr>
<tr>
<td>Mar</td>
<td>0.962</td>
<td>0.923</td>
<td>0.931</td>
<td>1.038</td>
<td>1.086</td>
</tr>
<tr>
<td>Apr</td>
<td>1.073</td>
<td>1.032</td>
<td>1.037</td>
<td>1.141</td>
<td>1.268</td>
</tr>
<tr>
<td>May</td>
<td>1.026</td>
<td>0.993</td>
<td>0.990</td>
<td>1.086</td>
<td>1.277</td>
</tr>
<tr>
<td>Jun</td>
<td>1.107</td>
<td>1.073</td>
<td>1.072</td>
<td>1.144</td>
<td>1.281</td>
</tr>
<tr>
<td>WC-test for MSPE ratio</td>
<td>4.790</td>
<td>3.850</td>
<td>3.775</td>
<td>3.824</td>
<td>3.261</td>
</tr>
</tbody>
</table>

Note. West and Clark (2007) test statistic is reported, which follows the student t-distribution
with critical values of 1.96 and 2.33 at 5% and 1% significance level respectively.
To further evaluate the model forecast accuracy, we construct the ratios of MSPE of model forecasts over the unadjusted February futures price, which is adopted by the USDA RMA as the market projected harvest prices. The column 1 of table 5 presents the MSPE of unadjusted futures price, the February average futures price outperforms all other months except for January. The pattern of increasing forecast error variances from January to June is also observed for adjusted futures prices. However, the out-of-sample forecast errors both generated by futures prices adjusted for last year’s carryout, and the futures price adjusted for carryout plus the normalized yield level of last year are smaller in dispersions than that of the unadjusted futures price in every month, such that the six months’ average MSPE reduction are statistically significant at 5% level.


<table>
<thead>
<tr>
<th>Month</th>
<th>Unadjusted futures</th>
<th>Carryout only</th>
<th>Carryout + yield level</th>
<th>Carryout + yield level/variability</th>
<th>Full model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>0.914</td>
<td>0.908</td>
<td>0.911</td>
<td>0.984</td>
<td>0.987</td>
</tr>
<tr>
<td>Feb</td>
<td>1.000</td>
<td>0.977</td>
<td>0.978</td>
<td>1.055</td>
<td>1.057</td>
</tr>
<tr>
<td>Mar</td>
<td>1.166</td>
<td>1.132</td>
<td>1.137</td>
<td>1.210</td>
<td>1.246</td>
</tr>
<tr>
<td>Apr</td>
<td>1.301</td>
<td>1.267</td>
<td>1.265</td>
<td>1.339</td>
<td>1.473</td>
</tr>
<tr>
<td>May</td>
<td>1.243</td>
<td>1.222</td>
<td>1.207</td>
<td>1.274</td>
<td>1.487</td>
</tr>
<tr>
<td>Jun</td>
<td>1.342</td>
<td>1.336</td>
<td>1.320</td>
<td>1.380</td>
<td>1.502</td>
</tr>
<tr>
<td>WC-test for MSPE ratio</td>
<td>1.758</td>
<td>2.100</td>
<td>0.593</td>
<td>0.387</td>
<td></td>
</tr>
</tbody>
</table>

*Note.* West and Clark (2007) test statistic is reported, which follows the student t-distribution with critical values of 1.64 and 1.96 at 5% and 1% significance level respectively.
Trading strategies using corn December futures

Suppose an investor wants to take advantage of the weather premium in the corn futures market. One strategy as recommended by CTAs (e.g. Till, 2000) is to passively shorting in the December futures contracts in spring and cover in October when the market take all the weather premium out. Panel A in table 6 presents the return and variation of this strategy for each month. The speculator’s return is measured as the ratio of negative price change of the December futures from the spring months to October, over the spring price of the December futures, at which he enters the trade. It appears that the best month to execute this trade is June, which produces an average return of 2.5% and standard deviation of 19.2%. If we use breakeven as the benchmark, then the Sharpe ratio of this passive strategy is 0.13. This is much lower than that of S&P 500 index that during the same time period has produced a Sharpe ratio of 0.46.

An alternative strategy is to utilize the correlation between the weather premium and the level of carryout: short the December futures in spring and cover in October, only when the carryout level is below the preceding 5-year average. The results are presented in Panel B of table 6. The Sharpe ratio of this active short strategy is highest in January equal to 0.62, or 9.5% return with 15.5% standard deviation, this is much better than the passive short program. The Sharpe ratio of this strategy is gradually declining throughout the growing seasons from January to June. However, this strategy should be used with caution for there are only 14 years when the carryout falls below trend.

<table>
<thead>
<tr>
<th>Strategies</th>
<th>Arithmetic Average</th>
<th>Standard Deviation</th>
<th>Sharpe Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Short in spring and cover in October</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jan</td>
<td>0.008</td>
<td>0.221</td>
<td>0.037</td>
</tr>
<tr>
<td>Feb</td>
<td>0.015</td>
<td>0.216</td>
<td>0.070</td>
</tr>
<tr>
<td>Mar</td>
<td>0.020</td>
<td>0.212</td>
<td>0.092</td>
</tr>
<tr>
<td>Apr</td>
<td>0.020</td>
<td>0.213</td>
<td>0.093</td>
</tr>
<tr>
<td>May</td>
<td>0.016</td>
<td>0.205</td>
<td>0.079</td>
</tr>
<tr>
<td>Jun</td>
<td>0.025</td>
<td>0.192</td>
<td>0.132</td>
</tr>
<tr>
<td><strong>Panel B: Short in spring and cover in October, when carryout below trend</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jan</td>
<td>0.094</td>
<td>0.152</td>
<td>0.619</td>
</tr>
<tr>
<td>Feb</td>
<td>0.095</td>
<td>0.155</td>
<td>0.612</td>
</tr>
<tr>
<td>Mar</td>
<td>0.093</td>
<td>0.164</td>
<td>0.567</td>
</tr>
<tr>
<td>Apr</td>
<td>0.090</td>
<td>0.171</td>
<td>0.529</td>
</tr>
<tr>
<td>May</td>
<td>0.076</td>
<td>0.186</td>
<td>0.407</td>
</tr>
<tr>
<td>Jun</td>
<td>0.082</td>
<td>0.180</td>
<td>0.459</td>
</tr>
<tr>
<td><strong>Panel C: Other alternatives</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Buy in Jan and Cover in June, when carryout above trend</td>
<td>0.029</td>
<td>0.157</td>
<td>0.185</td>
</tr>
<tr>
<td>S&amp;P 500 Index</td>
<td>0.078</td>
<td>0.165</td>
<td>0.476</td>
</tr>
</tbody>
</table>

*Note.* Results of panel A are based on 48 years of data, from 1968 to 2015. Results of Panel B are based on the 14 years of data when the carryout is below the average of the past 5 years. Panel C result is based on the 34 years of data, when the carryout is above the past trend. Data source of historical S&P 500 Index: https://finance.yahoo.com/quote/%5EGSPC/history?p=%5EGSPC.

Finally, the opposite of taking an active short strategy would be to go long the December futures when the carryout is above the trend. In particular, the speculator will initial the long
position in January and cover at the end of June. The logic of this trade is that an above average carryout shifts the mean of the supply distribution to right, corresponding the part of demand curve that is very elastic. The downside price movement in this case is limited even when the weather is good, while the upside price movement is significantly larger should bad weather outcomes prevail. The market will price in this asymmetry into the December futures until July when the weather condition is realized. Panel C of table 6 shows that the average return of this strategy is 3% with 16% standard deviation. Though measured by the Sharpe ratio this strategy did not appear to beat the active short strategy during this particular period, but it performs better than the passive short program.

Conclusion

We define the weather premium as the difference between the expected harvest price and the harvest price at expected supply. The weather premium should be nonnegative in theory, and its magnitude will vary with the carryout and the yield distribution because of the convex and asymmetric demand function of grain crops. Our empirical analysis shows a time-varying predictable component of the forecast errors of the December corn futures whose properties is consistent with those of the weather premium. Adjusting the spring prices of December futures by the model estimates of weather premium leads to significant out-of-sample reduction of MSPE.

Any model forecast claims to perform better than the market-based futures price should be treated with skepticism. The out-of-sample evaluations are generally sensitive to the length of forecast window. In this paper, we use the window of the most recent 20 years. Our empirical results suggest that in years when the adverse weather outcomes do not occur, the magnitude of
decrease in December futures from spring to the harvest is partially explainable by the carryout and yields.

Given the limited number of observations, it is impossible to measure whether and to what extent the market overestimates the probability of adverse weather outcomes. The forecast errors on average greater than zero may just be a historical aberration given the short time series. However, the fact that the weather premium is present in the data despite numerous commodity trading strategies designed to take advantage of this premium is telling. This finding has important implication for corn producers engaged in pre-harvest hedging activities. They would be better off to hedge significant bigger portion of their expected harvest to take advantage of the premium, especially in years when the carryout is smaller than average.
References


