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The determinants of portfolio selection by life insurance companies and the effects of monetary policy on their portfolios

by

Ikechukwu C. Nwobodo

A Dissertation Submitted to the Graduate Faculty in Partial Fulfillment of The Requirements for the Degree of DOCTOR OF PHILOSOPHY

Major: Economics

Approved

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CHAPTER I. INTRODUCTION AND GENERAL DISCUSSION OF THE TOPIC

The life insurance industry provides the means through which individuals and families offer financial protection to their dependents in case of the death of the policyholder. The industry also provides a channel through which individuals can save some of their surplus funds to provide for retirement and contingencies.

As financial intermediaries, life insurance companies help reduce the cost of capital. Because of the vast sums of funds they work with, they can reduce substantially the per unit cost of gathering and analyzing information on available direct securities by equipping and maintaining a staff of specialists that concerns itself with nothing but this aspect of their operations. Also, because of the large pool of funds at their command, they can reduce the per unit cost of transactions -- the cost of buying, holding and selling securities. This is so because they can spread costs over a larger volume of assets than can most individuals. For most individuals, the safety of their principal can be greatly enhanced if they can hold a variety of direct securities whose prices do not all move in a parallel way. They can reduce default risk by holding a variety of debt claims and ownership claims against several debtors. However, most
individuals simply do not have the large amounts of funds or expertise it would take to achieve the cost savings or effective diversification of their assets.

Also, it would be time consuming, inefficient and costly if such deficit units as large corporations that need vast sums of funds to finance their operations had to deal with millions of individuals each with only a small amount of funds to invest in any one period.

Thus, life insurance companies, along with other financial intermediaries, help bring together the surplus and deficit units or savers and dissavers in the economy. This helps maintain and promote a smooth and orderly functioning of the credit and capital market. Because they operate with vast sums of money, they can buy and hold a large number of different securities. They can buy each security in lots large enough to achieve lower transactions costs. Through expert management, they can diversify their assets in ways that would eliminate risk to a degree that would not be possible for most individuals who only have small sums of money to work with.

The objective of this study, is to isolate those primary factors that influence life insurance companies in their allocation of funds among the various competing assets and also to determine, in a limited way, the impact of monetary policy on the portfolio decisions of life insurance companies.
In general, it could be argued that the efficiency of the financial sector of the economy would have some influence on the savings-versus-consumption decisions of the public. It is thus of interest to try to shed some light on capital allocation by life insurance companies and the impact monetary policy has on this allocative process.

If monetary policy is to serve as an effective stabilization tool, different securities in the portfolios of life insurance companies and other financial intermediaries would be expected to be good substitutes for one another. For instance, if the Federal Reserve System (Fed) is pursuing a tight monetary policy by selling government securities in the open market, one can argue that this policy would not be a very effective stabilizing tool, unless government bonds and corporate bonds, as an example, are substitutes for one another in the portfolio.

Assuming rational economic behavior, it can be argued that capital should flow into areas with the highest rates of return. If two securities are good substitutes, a small change in the rate of return of one, should induce an adjustment in the relative quantities held of the two. For example, a small increase in the corporate bond rate relative to the mortgage rate should attract funds toward corporate bonds and corporate investments and away from mortgage lending and home building. If the two securities are poor substitutes or
independent, then it might take huge shifts in the rates of return to induce a desired flow of funds in the capital market. This implies inefficiency of the capital markets to allocate funds effectively and would impede the effectiveness of monetary policy as a stabilization tool.

Sources and Uses of Funds by Life Insurance Companies

Life insurance companies' funds are derived from three main sources (Life Insurance Association of America, 1962): new money derived from net savings by policyholders in life insurance companies; return flow from invested funds, which arises from amortization, maturities and other repayments of bonds and mortgages; outright sales of long-term assets from the portfolio or drawing down of cash and short-term security holdings.

Savings though life insurance companies differ fundamentally from savings through such deposit-type institutions as mutual savings banks and savings and loan associations in at least three important aspects (Life Insurance Association of America, 1962, pp. 8-9):

1. Life insurance savings are long-term and contractual in nature and therefore are more stable.

2. They are motivated primarily by the desire for family financial protection in the event of death.
3. They are ordinarily expected to be left intact until the death of the insured rather than withdrawn for some consumer expenditure.

**Investment Trend**

The long-term nature of life insurance contracts has led to preferred investments by life insurance companies largely in long-term fixed-income assets. This means that reserves have been placed primarily in investments bearing a fixed rate of return, regular payments of interest over the life of the loan, and scheduled repayment in a fixed number of dollars. This would provide a characteristic of stability over time paralleling the contractual obligations of life insurance companies to policyholders. Investments primarily in longs, also would avoid added transactions costs of reinvestments that would arise from investing in shorts and would avoid volatility and instability associated with shorter-term securities' rates.

Specifically, life insurance companies' reserves have been placed primarily in the following assets: mortgages, corporate bonds, and U.S. government securities. Other assets in their portfolios include stocks, real estate, and state, local, and foreign government securities.
Factors Governing Investment Policies

We assume that life insurance companies as an industry, are profit maximizers, that is, they strive for the highest possible rates of return. There are some constraints, however, foremost among which is the risk involved with holding each class of securities.

In addition to the strictly financial and economic motivation of profit, there also may be a public interest aspect to the investment policies of life insurance companies. By this is meant that the companies might wish to project the image of a good corporate citizen and portray themselves as serving the direct and immediate needs and interests of their policyholders by engaging in residential mortgage lending, housing projects and small business loans, even when these may not be the most profitable outlets for their funds. They also may buy an unusual quantity of government bonds during war times, for instance, as a patriotic gesture.

There is also the need to maintain a desired liquidity level by the companies. This need might be for an operating balance of cash and bank deposits, to guard against the possibility of unforeseen contingencies that might increase death benefits, for example, wars or epidemics; or also to remain sufficiently flexible to take advantage of worthwhile investment opportunities that may arise.
There are several statutory limitations on the investment policies of life insurance companies, and also regulations governing valuations of securities. These regulations vary from state to state. They are designed presumably to insure the diversification of life insurance company portfolios. Essentially, they require that investments in different areas meet certain criteria. For a detailed discussion of these regulations, see Life Insurance Association of America (1962).
CHAPTER II. A REVIEW OF RELATED STUDIES

The literature on portfolio selection is voluminous. Surprisingly though, studies on the portfolio behavior of life insurance companies or specifically studies on the factors determining portfolio choice by life insurance companies are almost nonexistent. This, despite the fact that life insurance companies are one of the largest financial intermediaries in terms of total assets.

The lack of studies on portfolio management by life insurance companies may be a result of the fact that the life insurance industry is highly regulated. The regulations vary from state to state and cover such areas as what assets may be held in the portfolio, and what percentage of the portfolio may be devoted to some of the assets. Thus, it might appear on the surface, given the myriad of regulations, that life insurance companies are virtually "locked in" in terms of what they can or cannot do, and that their portfolio behavior is determined just as much, if not more, by such institutional factors as regulations and changes in tax laws as by market returns on the various assets they hold in their portfolios.

Kenneth Kleefeld in a study of the postwar demand for financial assets, (Special Studies Paper, Number 33, July 17, 1973), included life insurance companies as one of the
investor classes he studied. His study, however, focussed on whether households and institutional investors reacted similarly to changes in market yields on time deposits, bonds and equities. He specified an aggregative model which determined endogenously the asset demand of the various investor classes as separate aggregates.

Kleefeld started with an investor's utility function, which is exponential in portfolio income during the investment period:

$$U(\pi) = e^{-b\pi};$$  \hspace{1cm} (2.1)

where

$\pi =$ portfolio income, and

$b =$ investor's coefficient of absolute risk aversion.

Assuming that the rates of return on $n$ alternative assets have a multivariate normal distribution, with a mean vector $\bar{r}$ and covariance matrix $S$, he asserted, without showing how, that maximizing expected utility would yield the following vector of "optimal" asset holdings:

$$A = -WH + \frac{1}{b}Gr,$$  \hspace{1cm} (2.2)

where

$A =$ $n$-order column vector of optimal asset holdings,

$W =$ investor's total assets \( \{ i = 1 \}^{n} A_{i} \)}
$$G = S^{-1} \frac{S_{ii}^{-1} S_{ii}}{i'S^{-1}i}$$

is symmetric matrix of asset rate of return coefficients.

$$H = \frac{S_{ii}^{-1}}{i'S_{ii}^{-1}}$$

and

$$i = n\text{-order column vector of ones.}$$

By assuming that the utility function is exponential in the portfolio rate of return, $\pi/W$, and substituting $b/W$ for $b$ in $A = -WH + (1/b)Gr$, we get

$$A = -WH + (W/b)Gr$$

(2.3)

Dividing both sides of (2.3) by $W$, yields

$$A/W = -H + (1/b)Gr$$

(2.4)

Equation (2.4) implies the demand equation for each asset is homogenous of degree one in total assets and linear in asset rates of return.

In his estimated model, the expected nominal rates of return on the nonmonetary assets and investor class holdings of each asset were determined within his model, while the total supply of each asset and the total assets of each investor class were exogenous.

In the present study, rates of return will be treated as exogenous and life insurance companies will then be expected
to react to them in terms of allocating their portfolio to the various assets. In this study, portfolio decision, as will be argued in the theory chapter, is determined by extrinsic asset yields and their associated risks and not by intrinsic qualities represented by corresponding utility function parameters as have been specified by Kleefeld.

Kleefeld, perhaps, was forced to resort to the use of a utility function because the focus of his study was a comparison between consumers' and institutional investors' reactions to changing yields. Consumers, it is generally argued, maximize utility, and for businesses, it is usually argued that underlying profit maximization is some utility function.

On a priori grounds, one would question the appropriateness of specifying a utility function exponential in the portfolio rate of return and deriving the asset demand equation from these for an institutional investor such as life insurance companies. In the actual estimation of the model, Kleefeld did not include mortgages as an asset held by life insurance companies. Mortgages averaged about 32 per cent of total life insurance companies' portfolios during the years covered by his study (Life Insurance Fact Book, 1974). The exclusion of such a major asset, would introduce serious bias to his simultaneous equation system.

In terms of empirical results, Kleefeld writes:
The least satisfactory model estimates were for the highly regulated life insurance sector, as (1) two of this sector's four nonmonetary asset demand equations have estimated own-rate coefficients which are both negative and significant at the .05 level in one-tailed tests... (Special Studies Paper, 33, p. 18).

The negative own-rate coefficient was one problem encountered in some of the estimated equations in the present study. Although that result is corroborated by Kleefeld's study, one is still left somewhat amiss trying to explain it.

Kleefeld also reported that cross-rate coefficient symmetry conditions did not hold in general. In other words, the estimated demand equation for security A may imply substitutability between A and B while the estimated demand for B may imply complementarity between the two. A similar problem was encountered in the present study with respect to the relationship between the smaller assets such as state and local government and U.S. government bonds with the major assets such as bonds and mortgages.

The negative own-rate coefficient and nonsymmetry problems appear rather pervasive in many of the recent institutional investor studies -- and are not limited to just life insurance companies.

Straszheim (1971), and Hendershott (1971) both reported such problems in their studies. Neither one could explain the cause of the problem.

William Silber, in his study of the portfolio behavior of
financial institutions, used a stock adjustment formulation for his asset demand equations (Silber, 1970). The form of his estimated equations is as follows:

\[ X_t = a(X_t^* - X_{t-1}) \]  

(2.5)

where

0 < a < 1;

\[ \Delta X_t = X_t - X_{t-1}, \]

refers to the flow into security X during time period t;

\[ X_t^* \]

represents the desired holdings of security X, and

\[ X_{t-1} \]

is the amount of X held at the end of the last period.

His general expression for \( X_t^* \) is:

\[ X_t^* = b_1 + b_j(i_j) + b_0 + A \]  

(2.6)

where

\( \{i_j\} \) is a set of interest rates that is relevant for portfolio choice,

and

\( A \) is the level of assets.

He estimated the demand for only three of the assets in the portfolios of life insurance companies: U.S. government bonds, corporate bonds and mortgages, on the rationale that these three were the major assets constituting more than 80 per cent of the total portfolio.

His hypotheses were tests of substitutability and comple-
mentarity between pairs of the securities. The substitute-complement relationships established by the results indicate that portfolio allocation by life insurance companies responds to interest rate changes. In estimating the demand for government bonds, the only interest rate variable he used was the own rate. Its t-value was insignificant. According to Silber, when other rates were included, all the t-values were less than .5 and some of the signs were incorrect. His demand equation for corporate bonds suggests they are substitutes both for governments and mortgages. The mortgage demand equation suggests that mortgages are substitutes for corporates and are complements with governments.
CHAPTER III. THEORETICAL FOUNDATION OF LIFE INSURANCE

COMPANY PORTFOLIO SELECTION

Securities and Attributes of Securities

Securities, in general, have two major attributes -- their rates of return or yields and the risk associated with each security. By the rate of return is meant that in each time period, the investor knows the current rate of return on each security and also has some concept of the probability distributions of expected rates of return over the desired holding period. As used in this study, security yields or rates of return represent the mean or expected value of the probability distribution of returns. The risk associated with holding each security will be a measure of the dispersion of outcomes around the expected value. When considering different alternatives, investors base their decisions on these expected returns and the risk of each security.

The overall risk of a portfolio can be reduced through diversification. While diversification can reduce risk, unfortunately, it can not eliminate risk completely because security returns are highly correlated. If the investor could find assets whose returns are perfectly negatively correlated, then he could eliminate risk entirely from his portfolio by holding only those assets. Unfortunately, since all securities are subject to the same common forces, albeit
in varying degrees, this is seldom, if ever, possible. At the other extreme, if the returns on all securities of similar maturities were perfectly positively correlated, that is if the rates of returns on all securities rose and fell by the same proportions at the same times, then diversification would not reduce risk at all. However, securities' returns, though highly correlated, are neither perfectly positively nor perfectly negatively correlated. Thus, it is possible to reduce, to some degree, the overall risk of a portfolio through diversification. To achieve this, investors would avoid those securities whose returns are highly positively correlated. However, there is a trade off between risk reduction and yield, that is, risk reduction may be sacrificed to some extent under the inducement of higher yields.

Uncertainties that Affect the Risk Rating of Various Securities

There are several factors that affect the degree of risk associated with each security. Tobin discussed four of these (Tobin, 1965):

1. Purchasing power risk -- Uncertainty about the purchasing power of the dollar affects securities with fixed face value in money terms.

2. Uncertainty about future interest rates -- Capital gains or losses will be made on interest-bearing bonds depending on whether future rates fall or rise.
3. Default risk -- This has to do with the ability of the issuing company to redeem debt claims against it.

4. Profitability risk -- private equities are subject to the specific risk of uncertainty regarding the earning power of a particular firm.

A fifth risk suggested by William Silber (Silber, 1970), is the marketability risk. If two securities are identical in all respects except that one has a well-organized secondary market and the other has a poor secondary market, investors in the latter run the risk of being able to liquidate their security holdings only at a depressed price compared with the price offered for the security with the better market.

The above risk classification can be used to analyze and contrast the different securities in the portfolios of life insurance companies. In general, securities with similar risks are more likely to be substitutes for each other in the portfolio while those with different or independent risks are more likely to be complements and can be used to diversify the portfolio.

If the risk components of different securities compensate for each other to a great extent, the diversification of the portfolio might actually result in a complementary relationship between these securities, that is, an increase in the yield on Security A, might increase the demand for Security B at the expense of another group of substitute assets.
Two commodities a and b are substitutes if \( \frac{\partial Q_a}{\partial P_b} > 0 \), i.e. \textit{ceteris paribus}, if an increase in price (P) of good b leads to an increase in the quantity demanded (Q) of good a (Henderson and Quandt, 1971).

Extending this definition to the securities market and keeping in mind that it is the expected yield on securities rather than their price which makes them more or less attractive to investors, two securities would be substitutes if

\[
\frac{\partial Q_a}{\partial r_b} < 0; \quad \text{where } r_b \text{ is the interest rate or yield on security B}
\]

and \( \frac{\partial Q_a}{\partial r_b} > 0 \) implies a complementary relationship.

Whereas consumer demand theory usually abstracts from the effect of expectations on the demand for different goods, expectations about future interest rates play an important role in securities demand.

Changes in current rates of interest influence expected future rates, which in turn, influence the current demand for securities. Current yields on the various securities, however, are good proxies for their expected yields.

We can classify the securities in the portfolios of life insurance companies according to maturity -- the distinction between short-term government bonds and long-term government bonds or, according to issuer -- the distinction between long-term government bonds and corporate bonds.

Of major concern in this study will be the degree of
substitutability of securities classified by issuer and not by maturity. In other words, we are restricting the investment process by life insurance companies. This restriction implies that if the portfolio manager is considering investing in long-term mortgages, the alternative is not negotiable certificates of deposits or treasury bills, but long-term government bonds or long-term corporate bonds. This is not to deny that life insurance companies buy and hold certificates of deposits and treasury bills. Rather, the restriction is meant to bypass the term structure of interest rates issue, since that is not the major focus of this study. One is fairly safe in imposing the above restriction because the obligations of life insurance companies are contractual in nature, long-term and actuarially predictable and as a result, most of their investments have traditionally been in long-term assets.

We can now examine the groups of securities in life insurance company portfolios, classified according to issuer, with respect to susceptibility to the various categories of risk. The discussion that follows parallels what will be stated more formally later as part of the hypotheses that will be tested.

In general, all bonds face the threat of inflation, whereas real assets or equity capital do not. Thus an in-
vestor can hedge against inflation or purchasing power risk by holding both equities and bonds.

Default risk differentiates between U.S. government securities and securities of other borrowers. This is because investors know the government will always meet its obligations to creditors. The government can always raise taxes or print new money to redeem its debt. This is not true of private debtors.

Marketability risk -- The existence of a well-organized secondary market is another measure of risk (Robinson, 1964). Robinson established U.S. government securities as having the best secondary market, with state-local government and corporate securities in a close tie for the next best secondary market and mortgages trailing.

On the basis of the discussion of the various risks and how they affect the various assets, one would hypothesize that equity capital and bonds would lend themselves to a complementary relationship in a portfolio, given the relative independence of their risks.

Mortgages, which are relatively illiquid and with a poor secondary market would tend to be complementary in a portfolio with government securities. Although both share similar risks because of inflation and the future course of interest rates, an increase in the yield of mortgages and thus the amount of mortgage holdings might lead to an increase
in holdings of governments to maintain a desired liquidity position, at the expense of a third group of securities. This third group might be corporate or state-local bonds, which have a poorer secondary market than governments but not so poor as mortgages.

Mortgages would tend to be substitutes for either corporate or state-local government bonds, since the liquidity distinction does not exist between mortgages versus corporates or versus municipals as it does between governments and mortgages.

Governments will most likely be substitutes for both state-local and corporates. All three share purchasing power risk and interest rate risk. The default risk distinction between governments and state-local and corporates can be suppressed as negligible by assuming that life insurance companies only consider high quality bonds for investment.

Corporates and state-local securities share similar risks and might serve the same purpose in a portfolio and thus would tend to be substitutes. However, the interest on state and local government bonds is tax free, which would tend to make them an attractive profitable investment. This feature of state and local government bonds should be reflected in their yields and should not make any difference with respect to the risk classification described earlier.
CHAPTER IV. MONETARY POLICY AND ITS EFFECTS
ON SECURITY PRICES AND YIELDS

Because the flow of life insurance company investments is distributed among several securities spanning virtually the entire national market for loanable funds, there are many points at which one would expect that Federal Reserve System (Fed) policy can exert influence.

As discussed earlier, life insurance company investments are mostly in long-term, fixed-income obligations, which are subject to wide swings in market prices occasioned by changes in long-term interest rates. Life insurance companies are competitive and seek the highest returns from their portfolios and thus would be sensitive to changing differentials in investment yields. An example of the effect of the Fed's policy might be as follows:

An easy money policy would increase the net free reserves of commercial banks. This would increase the availability of loanable funds and lower rates on loans. The policy, at least initially, could be expected to result in an increase in commercial banks' purchases of short-term government securities, resulting in a decline in yields of shorter-term government securities. The reduction in yield and the increased availability of loanable funds would act to bring down the entire structure of short-term loan rates.
As the rates on shorts decline, banks would tend to become more competitive in the long-term lending field. A reduction in short-term rates and a tendency for long-term rates to soften would encourage prospective long-term borrowers to make greater use of short-term commercial bank financing in anticipation of lower long-term borrowing costs.

One possible effect of all this could be a decline in the yield of new offerings of corporate bonds. If the yield on corporates declines, life insurance companies, given the risk differentiation above, could be expected to shift investment emphasis to mortgages, thus transmitting the credit-easing effect to the residential mortgage market by increasing availability of funds in that area.

The speed with which this can be done will be dictated by demand and supply and the expectations of investors. When the Fed creates expectations of credit ease, the increased willingness of investors to commit their funds while rates are still comparatively high should strengthen market forces toward ease and vice versa.

In summary, credit ease would tend to lead to a fully committed forward position and investment shifts to areas where yields are not declining as fast, e.g., from corporate bonds to residential mortgages, especially government-underwritten mortgages.
On the other hand, when the Fed shifts to a stringent monetary policy and tightens the reserve position of banks and thus the general availability of bank credit, to obtain funds for loan expansion, banks would dispose of shorter-term government securities. This would result in a rise in short-term interest rates. An expanding demand for business and industrial loans would also contribute to this rise.

Thus, with expanding loan demand and rising short-term interest rates, commercial banks would tend to reduce their term-lending and their purchases of state and local government bonds, thus contributing to lesser availability of long-term financing.

The rise in short-term rates, would tend to induce more demand for long-term borrowing, setting in motion a general rise of long-term rates. This rise will be reinforced and accelerated by the expectations of borrowers and investors.

When borrowers expect increasing rates, they would seek financing promptly to avoid expected higher rates. And as investors anticipate rising rates, they would be less willing to commit fully their anticipated cash flow in order to take advantage of expected higher rates in the future.

Thus, tightening the credit spigot would affect the long-term capital markets through arbitrage and expectations.

As yields on corporate bonds rise, life insurance
companies would tend to be less willing to assume a fully committed position with respect to their cash flow. Investment emphasis should shift to areas where yields are sensitive and rising and away from areas where yields are slower to change. Specifically, this would mean shifting to direct placements of corporate securities and industrial and commercial mortgages and away from government-insured and guaranteed mortgages, where interest rates are less flexible.

The rising rates, also might impair the ability of life insurance companies to generate cash flows for investment. Monetary policy affects the value of fixed-income assets. Tight money policy would affect the ability of the companies to raise investible funds through the sale of existing holdings.

As interest rates rise in response to market pressures and a restrictive credit policy, capital values of fixed-income obligations decline so that any effort to sell existing holdings would be at a loss. However, life insurance companies may still sell securities at a loss to raise funds with which to acquire higher-yielding securities or mortgages, if the higher return after taxes on the new securities will make up for the loss in a short time.

Policy loans might increase as a result of tight money. As interest rates rise, policyholders may find it advantageous
to borrow on their policies with fixed contractual rates, if that rate becomes lower than the rate charged by commercial banks. While this may not necessarily affect the cash flow of life insurance companies, it might result in a decline of cash flow for investments in bonds and mortgages. It could make companies more cautious about forward commitment and raise concern about their liquidity positions.

Tight money also might have some effect on mortgage repayments, which constitute an important part of the gross cash flow of life insurance companies. Regular amortization and partial prepayment would not be affected, but refinancing mortgages and thus unscheduled repayments might decline.

Federal debt management policy, though not of specific interest in this study, would affect life insurance companies in about the same ways as general policy measures by the Fed. By easing or restricting credit, debt management would result in changes in interest rates and expectations and thus investments. These, just as the actions by the Fed, would cause shifts in the direction of the flow of funds, affect the capital values of assets, result in changes in total cash flow of companies, and affect forward commitment policies.
CHAPTER V. PORTFOLIO ANALYSIS

It was stated earlier that the objective of life insurance companies is to maximize the rate of return on their portfolios subject to some constraints. To achieve this objective would require that the companies hold efficient portfolios at all times and adjust those portfolios as rates of return or risks change over time.

An efficient portfolio is one that has the maximum return in its risk class, or one that has the minimum risk in its return class, and is a legitimate portfolio, i.e. has no negative weights for some securities (Markowitz, 1959). To achieve this efficient portfolio means efficient diversification of the portfolio.

Markowitz efficient diversification involves combining securities whose rates of return are less than perfectly positively correlated in order to reduce risk in the portfolio without sacrificing any of the portfolio's return. In general, the lower the correlation of the rates of return of the assets in a portfolio, the less risky the portfolio will be, regardless of the risk associated with individual assets contained in the portfolio (Markowitz, 1959).

If the investment manager were operating in a world of certainty, and if perfect information were available in advance on the returns and risks of the various securities, he...
simply would invest his wealth in the one asset with the highest expected return in order to maximize his utility. However, due to uncertainties, he can only maximize what he expects utility to be, since he does not know what it will actually turn out to be. Thus risk enters the analysis.

The relationship between the investor's utility and his investments, can be specified as

\[ E(U) = f(E(r), \text{risk}) = f(E(r), \sigma), \quad (5.1) \]

where

- \( E(r) \) is the expected rate of return, and
- \( \sigma \), the standard deviation, is the measure of risk.

We define risk as the variability of expected returns from investments in the different securities. A more formal analysis of risk will focus on the probability distributions of rates of return.

We define the expected value of the probability distribution or the mean of returns as

\[ E(r) = \sum_{k} p_k r_k \quad (5.2) \]

where

- \( r_k \) is the kth outcome, and
- \( p_k \) is the probability of that outcome or return, and
- \( k = 1, \ldots, n = \) all possible outcomes or rates of returns.
The variance of returns measures the dispersion of the probability distribution and is defined as:

$$\sigma_k^2 = \sum_k p_k (r_k - \text{E}(r))^2.$$  \hspace{1cm} (5.3)

The standard deviation, which will be used as the measure of risk is defined as:

$$\sigma_k = \sqrt{\sigma_k^2} = \sqrt{\sum_{k=1}^{n} p_k [(r_k - \text{E}(r))^2] = \sqrt{\text{E}(r - \text{E}(r))^2}.}$$ \hspace{1cm} (5.4)

Standard deviation is the measure of risk that will be adopted in this study as the index of unpredictability or risk to measure the spread or dispersion of the probability distribution.

Its principal advantages are technical (Tobin, 1965, p. 17): a) If the central tendency of the probability distribution is described by the mathematical expectation, the standard deviation is, for reasons of probability theory, the natural measure of dispersion. b) The standard deviation of the return of a compound portfolio can be easily derived from the standard deviations, and correlations, of the returns on the constituent portfolios.

It was stated above that, in general, a portfolio will be less risky the lower the correlation of its constituent assets' rates of return. We define correlation as:
\[
\rho_{ij} = \frac{\text{cov}(r_i, r_j)}{\sigma_i \sigma_j} = \frac{\sigma_{ij}}{\sigma_i \sigma_j} = \frac{E\{(r_i - E(r_i))(r_j - E(r_j))\}}{\sigma_i \sigma_j} \tag{5.5}
\]

One can now proceed to construct a diversified efficient portfolio, with a knowledge of the correlation coefficients between the various assets' rates of return, i.e., after ascertaining if the assets are independent or share similar risks and thus will act as complements or substitutes in the portfolio.

A correlation coefficient can vary between \(-1 \leq \rho \leq +1\). An extreme \(\rho_{ij} \) of +1 indicates perfect positive correlation between the two assets' rates of return, and means that the two move in the same direction, by the same proportion at the same time. An \(\rho_{ij} \) of -1 indicates perfect negative correlation and the two securities' rates of return vary inversely.

Having determined the rates of return, the expected future rates and the risks associated with these expected rates, one can solve for the optimum proportion of each asset to be held in the portfolio.

Let \(w_i\) be the proportion of the \(i\)th security in the portfolio. A necessary constraint for a meaningful analysis
of the portfolio problem is
\[ \sum_{i} w_i = 1 \]

This means that the fractions invested in the different securities sum to one, and that all funds allocated for portfolio selection are accounted for.

If \( r_p \) represents actual return from a portfolio and \( E(r_p) \) its expected return, \( E(r_p) \) may be defined thus:

\[ E(r_p) = \sum_{i} w_i E(r_i) = \sum_{i} w_i \sum_{j=1}^{J} r_{ij} \mu_{ij} \]

where

\[ E(r_i) = \sum_{j=1}^{J} \mu_{ij} r_{ij} \]

\( r_{ij} \) = jth outcome on the ith asset and \( J \) is the number of possible outcomes.

This says that the expected return on a portfolio is the weighted average of the expected returns from the assets contained in the portfolio.

In general, the variance or variability of return of a portfolio can now be written as:

\[ \sigma_{r_p}^2 = \text{Var}(r_p) = E(r_p - E(r_p))^2 \]

For a two-security portfolio, the variance expression can be derived and expressed in the following way.
\[ \sigma_{r_p}^2 = \text{Var}(r_p) = E\{r_p - E(r_p)\}^2 \]

\[ = E[w_1 r_1 + w_2 r_2 - E(w_1 r_1 + w_2 r_2)]^2 \]

\[ = E[w_1 r_1 + w_2 r_2 - w_1 E(r_1) - w_2 E(r_2)]^2 \]

\[ = E[w_1\{r_1 - E(r_1)\} + w_2\{r_2 - E(r_2)\}]^2 \]

\[ = E[w_1^2\{r_1 - E(r_1)\}^2 + w_2^2\{r_2 - E(r_2)\}^2 + 2w_1w_2\{r_1 - E(r_1)\}\{r_2 - E(r_2)\}] \]

\[ = w_1^2 \text{Var}(r_1) + w_2^2 \text{Var}(r_2) + 2w_1w_2 \text{cov}(r_1r_2) \]

\[ = \sum_{i=1}^{n} w_i^2 \sigma_{ii} + \sum_{i<j}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij} \]

\[ \text{for } i \neq j \]

\[ = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij} \text{ in matrix notation.} \]

This expression says that the variance of a weighted sum is the sum of the weighted variances plus the covariance terms for all possible pairs of securities in the portfolio.

There are \( n \) expected returns for the \( n \) securities; \( n \) variances for the \( n \) expected returns, and \( (n^2-n)/2 \) covariances.
between all possible pairs of expected returns.

The utility function of the companies can be expressed as

\[ U = f(E(r_p), \sigma) \]

where

\[ E(r_p) = \text{Expected return from the portfolio} \]

and

\[ \sigma = \text{risk associated with the portfolio}. \]

The function is assumed positive with respect to returns and negative with respect to risk. The objective of the companies is to maximize the utility function subject to

\[ \sum_{i=1}^{n} w_i = 1 \]

which says that all of the investible funds are just accounted for or that the weights of the various assets in the portfolio sum to one. There are seven assets generally listed in the portfolios of life insurance companies (Life Insurance Fact Book, 1974): mortgages, corporate bonds, U.S. government bonds, state and local government bonds, stocks, real estate and foreign government bonds. The problem can be set up as a Lagrangian objective function:\(^1\)

\(^1\)Proof that the second order conditions exist to ensure a maximum is omitted here.
To get the first order conditions for an extremum, the partial derivatives of the function with respect to the seven assets and the constraint are set equal to zero.

\[
\frac{\partial Z}{\partial w_1} = \frac{\partial U}{\partial E(r_1)} E(r_1) + \frac{\partial U}{\partial \text{Var}} [2w_1 \sigma_{11} + 2w_2 \sigma_{12} + 2w_3 \sigma_{13} + 2w_4 \sigma_{14} + 2w_5 \sigma_{15} + 2w_6 \sigma_{16} + 2w_7 \sigma_{17}] + \lambda = 0
\]

\[
\frac{\partial Z}{\partial w_2} = \frac{\partial U}{\partial E(r_2)} E(r_2) + \frac{\partial U}{\partial \text{Var}} [2w_2 \sigma_{22} + 2w_1 \sigma_{12} + 2w_3 \sigma_{23} + 2w_4 \sigma_{24} + 2w_5 \sigma_{25} + 2w_6 \sigma_{26} + 2w_7 \sigma_{27}] + \lambda = 0
\]

\[
\frac{\partial Z}{\partial w_3} = \frac{\partial U}{\partial E(r_3)} E(r_3) + \frac{\partial U}{\partial \text{Var}} [2w_3 \sigma_{33} + 2w_1 \sigma_{13} + 2w_2 \sigma_{23} + 2w_4 \sigma_{34} + 2w_5 \sigma_{35} + 2w_6 \sigma_{36} + 2w_7 \sigma_{37}] + \lambda = 0
\]

\[
\frac{\partial Z}{\partial w_4} = \frac{\partial U}{\partial E(r_4)} E(r_4) + \frac{\partial U}{\partial \text{Var}} [2w_4 \sigma_{44} + 2w_1 \sigma_{14} + 2w_2 \sigma_{24} + 2w_3 \sigma_{34} + 2w_5 \sigma_{45} + 2w_4 \sigma_{46} + 2w_7 \sigma_{47}] + \lambda = 0
\]

\[
\frac{\partial Z}{\partial w_5} = \frac{\partial U}{\partial E(r_5)} E(r_5) + \frac{\partial U}{\partial \text{Var}} [2w_5 \sigma_{55} + 2w_1 \sigma_{15} + 2w_2 \sigma_{25} + 2w_3 \sigma_{35} + 2w_4 \sigma_{45} + 2w_6 \sigma_{56} + 2w_7 \sigma_{57}] + \lambda = 0
\]
On the assumption that the investors take the risk associated with each asset as given, it can be considered a constant and the system of n+1 equations can be expressed as a Jacobian matrix. In setting up the matrix, the current yields also are assumed to be good proxies of the expected yields, and therefore are treated as constants in trying to find the weights of the various assets that would give an efficient portfolio:

\[
\frac{\partial Z}{\partial w_6} = \frac{\partial U}{\partial E(r_7)} E(r_7) + \frac{\partial U}{\partial \text{Var}} [2w_6 \sigma_{66} + 2w_1 \sigma_{16} + 2w_2 \sigma_{26} + 2w_3 \sigma_{36} + 2w_4 \sigma_{46} + 2w_5 \sigma_{56} + 2w_7 \sigma_{67}] + = 0 \quad (5.14)
\]

\[
\frac{\partial Z}{\partial w_7} = \frac{\partial U}{\partial E(r_7)} E(r_7) + \frac{\partial U}{\partial \text{Var}} [2w_7 \sigma_{77} + 2w_1 \sigma_{17} + 2w_2 \sigma_{27} + 2w_3 \sigma_{37} + 2w_4 \sigma_{47} + 2w_5 \sigma_{57} + 2w_6 \sigma_{67}] + = 0 \quad (5.15)
\]

\[
\frac{\partial Z}{\partial \lambda} = w_1 - w_2 - w_3 - w_4 - w_5 - w_6 - w_7 - 1 = 0 \quad (5.16)
\]
The solution of the system will give the n+1 variables in the weight vector in terms of the expected rates of return on the assets. Let the coefficient matrix be C, the weight vector \( w \), and the vector of expected returns \( E \). The system now becomes \( Cw = E \), from which we can get:

\[
C^{-1}Cw = C^{-1}E \\
w = C^{-1}E
\]
The solution for the weights will be of the general form: \( w_i = c_i + k_i \frac{3u}{3E(r_i)}E(r_i) \), where \( c_i \) and \( k_i \) are some constants.

Using Cramer's rule, the general form of the solution for the weight of each asset also may be obtained by replacing the appropriate column in the coefficient matrix by the vector of constants and dividing the resulting matrix by the determinant of the original matrix of coefficients. For instance, the weight of security 1 in the portfolio can be found as follows:
\[ \frac{\partial U}{\partial E(r_i)} E(r_i) \]

| 2σ_{11} | 2σ_{12} | 2σ_{13} | 2σ_{14} | 2σ_{15} | 2σ_{16} | 2σ_{17} | 1 |
| 2σ_{21} | 2σ_{22} | 2σ_{23} | 2σ_{24} | 2σ_{25} | 2σ_{26} | 2σ_{27} | 1 |
| 2σ_{31} | 2σ_{32} | 2σ_{33} | 2σ_{34} | 2σ_{35} | 2σ_{36} | 2σ_{37} | 1 |
| 2σ_{41} | 2σ_{42} | 2σ_{43} | 2σ_{44} | 2σ_{45} | 2σ_{46} | 2σ_{47} | 1 |
| 2σ_{51} | 2σ_{52} | 2σ_{53} | 2σ_{54} | 2σ_{55} | 2σ_{56} | 2σ_{57} | 1 |
| 2σ_{61} | 2σ_{62} | 2σ_{63} | 2σ_{64} | 2σ_{65} | 2σ_{66} | 2σ_{67} | 1 |
| 2σ_{71} | 2σ_{72} | 2σ_{73} | 2σ_{74} | 2σ_{75} | 2σ_{76} | 2σ_{77} | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |

\[ \frac{\partial U}{\partial \text{Var}} = w_1 \]

\[ (5.18) \]
The weights of the other securities can be found in a similar way. The solutions exist provided the original coefficient matrix is nonsingular.

We have now devised a way that would permit us to determine monetary policy impact, if any, on the portfolio composition of life insurance companies. Securities in the portfolio are related to each other either as complements or substitutes. Given these relationships, if we can isolate periods of tighter and easier monetary policy, we can test the hypothesis that changes in the rates of return of the securities during these periods will induce portfolio adjustments resulting in different weights for the securities such as to maintain an efficient portfolio with the maximum return subject to risk considerations.

Before proceeding with the analysis, it should be pointed out that transactions costs and illiquidities would affect the willingness of the companies to revise their portfolios.

Asset exchange costs, illiquidities and irreversibilities impart some inertia and stability to portfolio choices, keeping the planned period for holding any portfolio from being infinitesimally short. Shifts involve some cost in time, effort and money and thus any new portfolio must promise enough advantage in return over the old to compensate for these costs (Tobin, 1965).

Tobin argues that innumerable portfolio sequences are
available to the investment manager as new information is gathered and expectations about rates of returns are modified. Each sequence specifies the quantities of all the assets to be held on every date. Each sequence implies different transactions costs. At one extreme are portfolio sequences involving no shifts and no exchange costs. At the other extreme are sequences involving daily or hourly shifts in response to small or temporary differences in asset prospects.

The impact of shifting costs on portfolio sequence choices, argued Tobin, depends on, among other things, the relation of the costs to a) the number of portfolio shifts, b) the number of assets involved in a shift, and (c) the total value of transactions (Tobin, 1965).

Costs of type (a) encourage infrequent but thorough portfolio revisions. Costs of type (b) are an incentive to minimize the number of assets involved in any portfolio shift and to concentrate on particular occasions the dealings in any one asset. To save costs of type (c), the investor will seek to keep the total value of transactions down; but in the absence of the other two relationships, he would not care whether shifts were frequent and small or infrequent and large.

How would these considerations affect the portfolio
The obligations of life insurance companies are long-term in nature. Thus, one would expect that except for purposes of maintaining some desired liquidity position, the assets in a typical life insurance company portfolio would be long-term and not short-term. This makes frequent portfolio adjustments infeasible. Huge shifts from one security into another could involve large losses when one tries to dispose of long-term securities in advance of their maturity dates.

However, the argument remains that shifts will be made and the portfolio adjusted, when such a move is deemed profitable.

A two-security portfolio is now analyzed to see how changes in the rates and expected rates of return will affect the weights of the various securities in the portfolio.

Let the first of the two securities be U.S. government securities \((g)\), and the second be all other securities combined \((o)\).

U.S. governments are chosen because the Fed has a direct control over them with respect to volume outstanding and yields, more so than any other security.

For this portfolio, the objective function to be maximized is:
\[ Z = U \left( \sum_{i=1}^{2} w_i E(r_i), \sum_{i,j} E_{ij} \right) + \lambda \left( \sum w_i - 1 \right) \]  

(5.19)

\[ = U \left( w_1 E(r_1) + w_2 E(r_2), w_g^2 \sigma_{gg} + w_o^2 \sigma_{oo} + w_g w_o \sigma_{go} \right) \]

\[ + \lambda (w_1 + w_2 - 1) \]

where

\[ w_g = \text{weight of government securities} \]

\[ w_o = \text{weight of all other securities} \]

\[ E(r_g) = \text{expected return on governments with variance } \sigma_{gg} \]

\[ E(r_o) = \text{expected return on others combined with variance } \sigma_{oo} \]

The constraint is the same as before: \( \sum w_i - 1 = 0 \). The partial derivatives of the function with respect to the weights and constraint are set equal to zero:

\[ \frac{\partial Z}{\partial w_g} = \frac{\partial U}{\partial E(r_g)} E(r_g) + \frac{\partial U}{\partial \text{Var}} \left[ 2 w_g \sigma_{gg} + 2 w_o \sigma_{go} \right] + \lambda = 0 \]  

(5.20)

\[ \frac{\partial Z}{\partial w_o} = \frac{\partial U}{\partial E(r_o)} E(r_o) + \frac{\partial U}{\partial \text{Var}} \left[ 2 w_o \sigma_{oo} + 2 w_g \sigma_{go} \right] + \lambda = 0 \]  

(5.21)

\[ \frac{\partial Z}{\partial \lambda} = w_1 + w_2 - 1 = 0 \]  

(5.22)

As before, the equations can be expressed as a Jacobian matrix:
The two weights can be solved for using Cramer's rule.

\[
\begin{bmatrix}
\frac{\partial U}{\partial \text{Var}} & \begin{bmatrix} 2\sigma_{gg} & 2\sigma_{go} & 1 \\ 2\sigma_{go} & 2\sigma_{oo} & 1 \\ 1 & 1 & 0 \end{bmatrix} \\
\end{bmatrix} \begin{bmatrix} w_g \\ w_o \\ \lambda \end{bmatrix} = \begin{bmatrix} \frac{\partial U}{\partial E(r_g)}E(r_g) \\ \frac{\partial U}{\partial E(r_o)}E(r_o) \\ 1 \end{bmatrix} \tag{5.23}
\]

Expanding the top and bottom matrices by their third columns using the formula \(|C_{ij}| = (-1)^{i+j}|M_{ij}|;\)

\[
w_g = \frac{\frac{\partial U}{\partial E(r_o)}E(r_o) - 2\sigma_{oo} - \frac{\partial U}{\partial E(r_g)}E(r_g) + 2\sigma_{go}}{4\sigma_{go} - 2\sigma_{oo} - 2\sigma_{gg}} \cdot \frac{\partial U}{\partial \text{Var}} \tag{5.25}
\]

The weight of all other securities combined, \(w_o\), can be solved for similarly, or, since \(w_g + w_o = 1\),
\[ \omega = 1 - \gamma. \]  

(5.26)

Having found these weights, the task now is to determine what, if anything, happens to them when the Fed moves, as an example, from a relatively tight money policy to a relatively easy money policy. A formal definition of tightness and ease will be given later.

A change in monetary policy would be expected to change the rate of return on U.S. government securities. This change would, in return, alter the weights of the two securities in the portfolio assuming the yields of the other securities stay the same and the risk associated with holding government securities does not increase with an increase in their returns.

Suppose \( r_g \) increased as a result of a change in monetary policy. Then, the weight of government securities would be expected to increase and the weight of the other securities, \( \omega \), would be expected to decline.

This relationship can be expressed more formally by taking partial derivatives of the weights from the two-security portfolio with respect to changes in the rate of return on government securities. A specific answer will not be provided here, but can be obtained through a simulated experiment with actual numbers plugged into the equation or through a
sign analysis. The general solution above for the weights provides the framework for the empirical section, which is the next chapter.
CHAPTER VI. DERIVATION OF THE ASSET DEMAND EQUATIONS

In Chapter V, it was shown how life insurance companies could go about the process of selecting an efficient portfolio and what proportion of total portfolio each asset would constitute. Although transactions costs and their impacts were discussed in Chapter V, it was assumed implicitly that transactions costs were zero. In the derivation in the present chapter of the form of the demand equations that will be estimated, it will be assumed that life insurance companies either hold or strive for efficient portfolios. However, transactions costs are explicitly taken into account, thus necessitating the use of a stock adjustment model.

The form of the asset demand equations that will be tested empirically in this study will be similar to Silber's, discussed in Chapter II but with modifications.

The liabilities of life insurance companies are of a long-term nature and the amount of disbursements to meet these liabilities are highly predictable. As a result, most investments by life insurance companies are long-term. Based on this premise, frequent portfolio adjustments in terms of already committed funds would seem unlikely under most normal circumstances.

Of major interest in this study is determining how the
companies allocate their net inflow of funds (premium income and income from matured investments) to the various assets in their portfolio.

Specifically, the form of the estimated demand equations can be derived as follows:

\[(S_i/A)_t = a_0 + a_i r_{it} + a_{i+1} A \] (6.1)

Equation 6.1 expresses the desired level of the ith asset, \(S_i\), (as a proportion or ratio of total assets at time \(t\)), as a function of a vector of the relevant yields \(r_{it}\) and the total volume of assets \(A\).

By arguing that the current value of the ith asset in the portfolio depends on its current desired value and past desired values, where past desired values are assumed to incorporate the values of the explanatory variables, we can write:

\[(S_i/A)_t = \lambda \sum_{i=0}^{n} (1-\lambda)^i (S_i/A)_{t-1} = \lambda (S_i/A)_t \]

\[+ \lambda \sum_{i=1}^{n} (1-\lambda)^i (S_i/A)_{t-1} \] (6.2a)

where

\[0 < \lambda < 1.\]

Equation 6.2a argues that current values of the ith asset in the portfolio are obtained by partially adjusting
the level of that asset in each period in an attempt to attain a desired level of that asset in the portfolio. Equation 6.2a can be expanded to get:

\[(S_i/A)_t = \lambda\{ (S_i/A)_t^* + (1-\lambda) (S_i/A)_{t-1}^* + (1-\lambda)^2 (S_i/A)_{t-2}^* + \ldots + (1-\lambda)^n (S_i/A)_{t-n}^* \} \] (6.2b)

By applying Koyck transformation (Koyck, 1954), both sides of 6.2b are lagged one period and multiplied by \((1-\lambda)\) to get:

\[(1-\lambda) (S_i/A)_{t-1} = \lambda\{ (1-\lambda) (S_i/A)_{t-1}^* + (1-\lambda) (S_i/A)_{t-2}^* + \ldots + (1-\lambda)^n (S_i/A)_{t-n}^* + (1-\lambda)^{n+1} (S_i/A)_{t-n+1}^* \}. \] (6.3)

Subtracting Equation (6.3) from (6.2b) yields:

\[(S_i/A)_t - (1-\lambda) (S_i/A)_{t-1} = \lambda (S_i/A)_t^* \] (6.4)

where the last term in Equation (6.3) is considered close enough to zero that it can be ignored.

Manipulation of Equation (6.4) yields:
\[(S_i/A)_t - (S_i/A)_{t-1} + \lambda (S_i/A)_{t-1} = \lambda (S_i/A)_t \quad (6.5)\]

Substituting \(\Delta (S_i/A)_t\) for \((S_i/A)_t - (S_i/A)_{t-1}\) and transferring \(\lambda (S_i/A)_{t-1}\) to the right, we get:

\[\Delta (S_i/A)_t = \lambda (S_i/A)_t - \lambda (S_i/A)_{t-1} \quad (6.6)\]

We can substitute Equation (6.1) for \((S_i/A)^*_t\) in Equation 6.6 to get

\[\Delta (S_i/A)_t = \lambda a_0 + \lambda \{a_{it} + \lambda^2 A_{i+1} - \lambda (S_i/A)_{t-1} \quad (6.7)\]

Equation 6.7 is the equation that will be estimated. It is consistent with the argument that it is not the total level of each asset at time \(t\) that is relevant to the portfolio manager, since part of that portfolio already is invested. The manager is more concerned with allocating net increases in reserves than with juggling or rearranging the portfolio. The task then is to try and isolate those factors that determine how much of the net change in total assets is allocated to each asset in the portfolio.

Using ordinary least squares estimation, the value of \(\lambda\) will be the coefficient on the lagged dependent variable. Once \(\lambda\) is obtained, the other coefficients can be recovered by dividing through by the value of \(\lambda\).

The usual classical assumptions about the disturbance or error term in ordinary least squares regressions are
assumed to hold:

$$\epsilon \sim N(0, \sigma^2); \ E(\epsilon_i \epsilon_j) = 0, (i \neq j)$$

If the Durbin-Watson statistic (Durbin and Watson, 1950, 1951) indicates that the residuals of an equation are autocorrelated, the autoregressive equation of the residuals will be estimated to get the coefficient \( \hat{\rho} \). This coefficient will then be used to transform all the variables in the original equation. The equation will be reestimated using the transformed variables. For example, if we assume that the disturbance \( U_t \) follows a first-order autoregressive scheme, \( U_t = \rho U_{t-1} + \epsilon_t \) will be estimated; \( |\rho| < 1 \) and \( \epsilon_t \) satisfies the assumptions:

$$E(\epsilon_t) = 0 \quad (6.9a)$$

$$E(\epsilon_t \epsilon_{t+s}) = \sigma^2 \quad s=0$$
$$= 0 \quad s \neq 0 \quad (6.9b)$$

The original equation will then be reestimated in the following form to remove the autocorrelation:

$$(S_i/A)_t - \rho (S_i/A)_{t-1} = a_0 (1-\rho) + a_i (r_{it} - \rho r_{it-1}) + \epsilon_t \quad (6.10)$$

Interest rates will be entered in the equations either as levels or as differentials. That is, instead of using the level of each interest rate as an explanatory variable,
the difference between the yield on the asset whose demand is being estimated and each of the other yields will be used. For example, if we estimate the demand for $X_1$ with the rates $r_1$, $r_2$ and $r_3$, expressing it with the rates as differentials, we get

$$X_1 = \alpha_1 (r_2 - r_1) + \alpha_2 (r_3 - r_1)$$

Interest rate levels tend to move together which may result in serious multicollinearity problems. So, the specification of the interest rate variables as differentials rather than levels might prove statistically helpful in terms of alleviating the multicollinearity problem, which could mask statistically significant regression coefficients.

Also, interest rate differentials seem more pertinent than absolute levels to the portfolio manager in his decision on the flow of investment funds in a given period. The absolute levels might be of prime concern if the decision were a question of how much cash to hold versus how much securities to purchase.

The use of differentials also implies that if some factor were added to each rate, the distribution of funds among the various securities would be unaffected. The use of differentials, however, imposes a restriction on the sign of the own rate of return. For example, if we estimate $(S_i/A)_t = $
\[ a_1(r_j - r_1) + a_2(r_k - r_1) \]: where \( r_j \) is the rate on the jth security; \( r_k \) is the rate on the kth security, and \( r_1 \) is the own rate for \( S_1 \), the sign of \( r_1 = -(a_1) \) plus \(-a_2\) or the negative of the sum of the coefficients attached to \( r_j \) and \( r_k \) respectively. Due to this restriction on the sign of the own rate, the interest rate variables also will be specified as levels in some cases.

It should be pointed out that expected future rates were not built in specifically as arguments in the equations. However, the presence of a lagged dependent variable as an explanatory variable, incorporates a particular weighted average of past rates and all other explanatory variables that may have been left out. The lagged dependent variable is the usual proxy for expected rates (De Leeuw, 1965). Also, it was argued earlier that current yields are good proxies for expected future rates.

In this study, only the demand for the major assets in the portfolio of life insurance companies will be estimated. There are some assets that are a very small proportion of total assets or that have changed very little over the sample period. The demand for this group of assets, which includes real estate holdings, foreign government bonds and "other assets", will not be estimated.

Since the estimation technique that will be used in this
study is single-equation, ordinary least squares, it does not matter much to the results that will be obtained that these items will be left out. However, even in a simultaneous equation approach, the problems of not estimating the demand for this group of assets could be solved by simply imposing a balance sheet identity on the portfolio of the companies. Given n assets in the portfolio, and the size of the portfolio, only n-1 of those assets can be functionally independent. The group of assets considered as residuals can then be determined by the balance sheet identity. This procedure, while not necessary for single equation estimation, is implicit in the approach.
CHAPTER VII. ESTIMATION OF THE DEMAND EQUATIONS, HYPOTHESES TESTING AND RESULTS

The estimated demand equations cover the period of January 1963 through March 1974, i.e., 1963-I through 1974-I. Most of the data used were obtained from the Federal Reserve Bulletin, which provides a comprehensive set of tables on the flow of funds through the economy. The data on the assets of life insurance companies were reported as seasonally unadjusted monthly series. Quarterly averages, created from the monthly series, are used in this study.

Interest rates are expressed in percentages, that is, an interest rate of 5 per cent is written as 5.0. The following is a brief description of the yield rates that were used:

- Government Bond rate: the rate on long-term government bonds
- Treasury Bill rate: the rate on three-month treasury bills
- Corporate Bond rate: the yield on long-term corporate bonds prepared by Moody's investor service on Baa corporate bonds
- Earnings/Price: a measure of the returns from common stocks
- Mortgage rate: the yield on conventional first mortgages
- State-Local government bond rate: the yield on 15 high-grade municipals (Standard and Poor's averages)
- GNP Deflator: obtained from the Survey of Current Business
The performance of the estimated equations will be evaluated by looking at the signs of the coefficients to see if they support the a priori specification of either a substitute or complementary relationship among securities. The postulation of substitutability or complementarity between any two securities will be based on the discussion in Chapter III of the risk attributes of the various securities. The sign of the own rate of interest is expected to be positive—directly so, when interest rates are entered as levels and implicitly so when they are entered as differentials. The overall reasonableness of the estimated demand equations will be determined, in addition, by looking at the correlation coefficients and the standard errors of the estimates.

To test whether two securities are complements, substitutes or independent, assume that the demand for security $X_1$ is estimated with the interest rate term $b_1(r_2-r_1)$. If $b_1$, the coefficient, has a positive sign, and is significant at the 5 per cent level on a one-tailed $t$ test, security $X_2$, whose rate is $r_2$, and security $X_1$, whose demand is estimated, will be considered complements. If $b_1$, on the other hand, has a negative sign and is significant, then $X_2$ and $X_1$, will be considered substitutes. If, however, the ratio of the coefficient to its standard error ($t$-value) were smaller than about .5, this would be considered an indication that the
two securities are independent in the portfolio regardless of the sign of the coefficient. This means that if the t-value of a coefficient is not significant at the 5 per cent level and, in addition, is less than .5 in absolute value, this will be considered an indication that the demand for one is unaffected by changes in the yield for the other. The calculated t-value of .5 is an arbitrary cutoff and there is nothing theoretically magic about it.

In addition to testing the substitutability and complementarity of the securities, the individual demand equations will be checked against each other for consistency. For example, one would expect a symmetrical relationship between any two equations. If the equation for security A, for example, implies that securities A and B are substitutes, then the equation for security B, also would be expected to give the same relationship between A and B.

The estimated equations are presented below using the following notations:

\[ \begin{align*}
A &= \text{total assets} \\
\Delta A &= A_t - A_{t-1} = \text{change in level of assets} \\
C &= \text{corporate bonds}
\end{align*} \]

\footnote{For a discussion of consistency checks, see Brainard and Tobin, 1968, pp. 99-122).}
Each variable takes on a value of 1 for the quarter represented by its subscript and is zero elsewhere. The dummy variables are included to account for any independent seasonal patterns that may exist in the demand for securities.

The Hypotheses that will be Tested

1. Equity Capital and all bonds are complements in the portfolio.

2. Mortgages and U.S. government bonds are complements.

3. Mortgages and corporate bonds are substitutes.

4. Mortgages and state and local government bonds are substitutes.
5. U.S. government bonds and state and local government bonds are substitutes in the portfolio.

6. Corporate bonds and U.S. government bonds are substitutes.

7. Corporate bonds and state and local government bonds are substitutes.

The Demand for Mortgage Holdings

The demand for mortgage holdings by life insurance companies was estimated with the following models:

\[ \Delta(M_A) = \alpha_0 - \alpha_1(r_c - r_m) + \alpha_2(r_g - r_m) - \alpha_3(M_A t-1) + \alpha_4D_1 + \alpha_5D_2 + \alpha_6D_3 \]  
(7.1)

\[ \Delta(M_A) = \alpha_0 + \alpha_1r_m - \alpha_2r_c + \alpha_3r_g - \alpha_4(M_A t-1) + \alpha_5D_1 + \alpha_6D_2 + \alpha_7D_3 \]  
(7.2)

\[ \Delta(M) = \alpha_0 - \alpha_1(r_c - r_m) + \alpha_2(r_g - r_m) + \alpha_3A - \alpha_4M_t-1 + \alpha_5D_1 + \alpha_6D_2 + \alpha_7D_3 \]  
(7.3)

\[ \Delta(M) = \alpha_0 + \alpha_1r_m - \alpha_2r_c + \alpha_3r_g + \alpha_4A - \alpha_5M_t-1 + \alpha_6D_1 + \alpha_7D_2 + \alpha_8D_3 \]  
(7.4)

The results are presented in Tables 1a and 1b.
### Table 1a. Regression coefficients and goodness of fit statistics for Equations 7.1 through 7.4

<table>
<thead>
<tr>
<th>Equation 7.1: Coefficient of</th>
<th>Dependent variable</th>
<th>Constant</th>
<th>$r_c - r_m$</th>
<th>$r_g - r_m$</th>
<th>$(M/A)_{t-1}$</th>
<th>$D_1$</th>
<th>$D_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta (M/A)$</td>
<td>.00984</td>
<td>-.00384</td>
<td>.00186</td>
<td>-.02503</td>
<td>-.00019</td>
<td>-.00047</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.0974)(^c)</td>
<td>(-5.7549)</td>
<td>(1.6819)</td>
<td>(-1.1438)</td>
<td>(-.5269)</td>
<td>(-1.3650)</td>
<td></td>
</tr>
<tr>
<td>Equation 7.2: Coefficient of</td>
<td>Dependent variable</td>
<td>Constant</td>
<td>$r_m$</td>
<td>$r_c$</td>
<td>$r_g$</td>
<td>$(M/A)_{t-1}$</td>
<td>$D_1$</td>
</tr>
<tr>
<td>$\Delta (M/A)$</td>
<td>-.00128</td>
<td>.00284</td>
<td>-.00504</td>
<td>.00346</td>
<td>-.0129</td>
<td>-.00016</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-.1264)</td>
<td>(2.2325)</td>
<td>(-5.6899)</td>
<td>(2.5533)</td>
<td>(-.6101)</td>
<td>(-.4655)</td>
<td></td>
</tr>
<tr>
<td>Equation 7.3: Coefficient of</td>
<td>Dependent variable</td>
<td>Constant</td>
<td>$r_c - r_m$</td>
<td>$r_g - r_m$</td>
<td>$A$</td>
<td>$M_{t-1}$</td>
<td>$D_1$</td>
</tr>
<tr>
<td>$\Delta M$</td>
<td>318.1106</td>
<td>-626.3427</td>
<td>361.0157</td>
<td>.008</td>
<td>-.00986</td>
<td>-200.1155</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.4044)</td>
<td>(-4.1492)</td>
<td>(1.8181)</td>
<td>(1.5857)</td>
<td>(-.4686)</td>
<td>(-3.1154)</td>
<td></td>
</tr>
<tr>
<td>Equation 7.4: Coefficient of</td>
<td>Dependent variable</td>
<td>Constant</td>
<td>$r_m$</td>
<td>$r_c$</td>
<td>$r_g$</td>
<td>$A$</td>
<td>$M_{t-1}$</td>
</tr>
<tr>
<td>$\Delta M$</td>
<td>255.8689</td>
<td>308.8265</td>
<td>-641.8792</td>
<td>472.3372</td>
<td>.007989</td>
<td>-.0212</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.3153)</td>
<td>(1.4438)</td>
<td>(-4.0870)</td>
<td>(2.0558)</td>
<td>(1.5219)</td>
<td>(-.8722)</td>
<td></td>
</tr>
</tbody>
</table>

\(a\) is the adjusted coefficient of multiple determination.

\(b\) is the Durbin-Watson statistic.

\(c\) The t-values of the coefficients are reported in parentheses directly below each coefficient.
<table>
<thead>
<tr>
<th>$D_3$</th>
<th>$\frac{R^2}{R_2^a}$</th>
<th>$b$</th>
<th>$\hat{\rho}$</th>
<th>$F$</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>.00057</td>
<td>.45</td>
<td>2.1249</td>
<td>.4617</td>
<td>5.9408</td>
<td>.000002441</td>
</tr>
<tr>
<td>(1.6414)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$D_1$</th>
<th>$D_3$</th>
<th>$\frac{R^2}{R_2}$</th>
<th>$d$</th>
<th>$\hat{\rho}$</th>
<th>$F$</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>-.00041</td>
<td>.00061</td>
<td>.51</td>
<td>2.1844</td>
<td>.4338</td>
<td>6.4548</td>
<td>.0000023</td>
</tr>
<tr>
<td>(-1.2085)</td>
<td>(1.7900)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$D_2$</th>
<th>$D_3$</th>
<th>$\frac{R^2}{R_2}$</th>
<th>$d$</th>
<th>$\hat{\rho}$</th>
<th>$F$</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>-81.5129</td>
<td>166.1811</td>
<td>.85</td>
<td>2.1756</td>
<td>.2972</td>
<td>31.514</td>
<td>68445</td>
</tr>
<tr>
<td>(-1.3127)</td>
<td>(2.6613)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_3$</th>
<th>$\frac{R^2}{R_2}$</th>
<th>$d$</th>
<th>$\hat{\rho}$</th>
<th>$F$</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>198.1636</td>
<td>-81.4302</td>
<td>166.4774</td>
<td>.84</td>
<td>2.2115</td>
<td>.3263</td>
<td>26.038</td>
<td>68793</td>
</tr>
<tr>
<td>(-3.1080)</td>
<td>(-1.3224)</td>
<td>(2.6884)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table lb. Disentangled values of the coefficients of:

<table>
<thead>
<tr>
<th>Equation</th>
<th>Dependent Variable</th>
<th>Constant</th>
<th>( r_c - r_m )</th>
<th>( r_g - r_m )</th>
<th>( (M/A)_{t-1} )</th>
<th>( D_1 )</th>
<th>( D_2 )</th>
<th>( D_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.1</td>
<td>( \Delta (M/A) )</td>
<td>.3929</td>
<td>-.1535</td>
<td>.0742</td>
<td>-.0250</td>
<td>-.0079</td>
<td>-.0188</td>
<td>0.0228</td>
</tr>
<tr>
<td>7.2</td>
<td>( \Delta (M/A) )</td>
<td>-.0992</td>
<td>.2199</td>
<td>-.3908</td>
<td>.2686</td>
<td>-.0129</td>
<td>-.0127</td>
<td>-.0317</td>
</tr>
<tr>
<td>7.3</td>
<td>( \Delta M )</td>
<td>32265.45</td>
<td>-63528.95</td>
<td>36617.24</td>
<td>.8109</td>
<td>-.0099</td>
<td>-20297.4</td>
<td>-8267.72</td>
</tr>
<tr>
<td>7.4</td>
<td>( \Delta M )</td>
<td>12061.99</td>
<td>14558.49</td>
<td>-30259.04</td>
<td>22266.6</td>
<td>.3766</td>
<td>-.0212</td>
<td>-9341.7</td>
</tr>
</tbody>
</table>

*The only difference between Tables Ia and Ib is that in Ib, the values of the coefficients presented in Table Ia have been disentangled. Each coefficient in Table Ia is divided by \( \lambda \), the value of the lagged dependent variable.*
The demand for mortgages (Equation 7.1) suggests that mortgages and corporate bonds are substitutes in the portfolios of life insurance companies. The coefficient of \((r_c - r_m)\) has a negative sign and is significantly different from zero at the 5 per cent level using a one-tailed t test. This is as was hypothesized. The estimated equation also suggests that U.S. government bonds and mortgages are complements in the portfolio as was hypothesized. The coefficient for \((r_g - r_m)\) is positive and significantly different from zero also at the 5 per cent level. It was pointed out earlier that mortgages are highly illiquid. It would appear, based on the results, that to compensate for the illiquidity resulting from an increase in the holdings of mortgages, the companies tend to increase their demand for highly liquid U.S. government bonds. U.S. government bonds have a highly well-organized secondary market and can be sold any time with very little difficulty to meet cash needs should the companies be strapped for liquidity.

The implicit own rate for mortgages has a positive sign \((+.00384 - .00186 = +.00198)\), which is what we would expect.

The speed of adjustment implied by the coefficient of the lagged dependent explanatory variable is that only about 3 per cent of the deficiency between desired and
actual holdings is made up after one quarter. This implies a very long lag in adjustment.

When the interest rate variables were entered as levels instead of differentials (Equation 7.2), the results were roughly similar with those of Equation 7.1. These results suggest a strong substitute relationship between mortgages and corporate bonds and a strong complementary relationship between mortgages and government bonds. The speed of adjustment implied by 7.2 was even slower. Less than 2 per cent of the discrepancy between actual and desired holdings is made up in one quarter.

When the mortgage demand was estimated as a level and not as a proportion of total assets (Equation 7.3), the same significant relationships existed between mortgages and corporate bonds and mortgages and U.S. government bonds. The implicit own rate is also positive as in 7.1. In Equation 7.4, all variables -- dependent and independent -- were entered as levels. Results here are similar to those in the preceding three equations. The speed of adjustment again is very slow with only 2 per cent of any deficiency corrected in the first quarter.

All four equations tend to support the hypothesized relationships between mortgages and the other major assets in the portfolios of life insurance companies. However, the explanatory power of Equations 7.3 and 7.4, as measured
by \( R^2 \), the adjusted coefficient of multiple determination, were higher than those of Equations 7.1 and 7.2. Total assets were entered as a variable in 7.3 and 7.4. The t-values of its coefficients in both equations were almost significant at the 5 per cent level. This is as expected, because over the years mortgages have constituted a major proportion of the total portfolio.

Life insurance companies' mortgage holdings increased by $4.4 billion during 1973 to a total of $81.4 billion or 32.2 per cent of total assets at the end of that year (Life Insurance Fact Book, 1974). The asset level, however, was not included in the final forms of Equations 7.1 and 7.2. It was tried, but perhaps, because these two equations were expressed as a proportion of total assets, the result was insignificant.

The Demand for Corporate Bonds

The following models were fitted for the demand for corporate bonds:

\[
\Delta C = \alpha_0 + \alpha_1 r_c - \alpha_2 r_g - \alpha_3 r_m - \alpha_4 \dot{p} + \alpha_5 A - \alpha_6 C_{t-1} \\
+ \alpha_7 D_1 + \alpha_8 D_2 + \alpha_9 D_3
\]

(7.5)
\[ \Delta(C/A) = \alpha_0 - \alpha_1 (r_g - r_c) - \alpha_2 (r_m - r_c) - \alpha_3 \frac{\Delta p}{p} + \alpha_4 A \]

\[- \alpha_5 (C/A)_{t-1} + \alpha_6 D_1 + \alpha_7 D_2 + \alpha_8 D_3 \tag{7.6} \]

\[ \Delta C = \alpha_0 - \alpha_1 (r_g - r_c) - \alpha_2 (r_m - r_c) - \alpha_3 \frac{\Delta p}{p} + \alpha_4 A \]

\[- \alpha_5 C_{t-1} + \alpha_6 D_1 + \alpha_7 D_2 + \alpha_8 D_3 \tag{7.7} \]

The results are presented in Tables 2a and 2b.

The relationships between corporate bonds and the other securities implied by the estimated corporate bond demand equations presented above are inconclusive.

In Equation 7.5, the own rate of return is positive but insignificant. The coefficient for government bonds is negative, suggesting a substitute relationship between corporate bonds and U.S. government bonds. However, it is insignificantly different from zero and, thus, does not support the hypothesis of substitution between the two.

The coefficient for mortgage rates is negative and significantly different from zero at the 10 per cent level but not at the 5 per cent level. However, to the extent that the sign is negative and the t test nearly significant at the 5 per cent level, it is consistent with the result of the mortgage demand equation. Thus, the symmetry condition holds. Both the mortgage and corporate bond demand equations
Table 2a. Regression coefficients and goodness of fit statistics for Equations 7.5 through 7.7

<table>
<thead>
<tr>
<th>Equation 7.5:</th>
<th>Coefficient of</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent Constant variable</td>
<td>$r_c$</td>
</tr>
<tr>
<td>$\Delta C$</td>
<td>589.49 (0.8461)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation 7.6:</th>
<th>Coefficient of</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent Constant variable</td>
<td>$r_d-r_c$</td>
</tr>
<tr>
<td>$\Delta (C/A)$</td>
<td>0.0351 (1.1557)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation 7.7:</th>
<th>Coefficient of</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent Constant variable</td>
<td>$r_d-r_c$</td>
</tr>
<tr>
<td>$\Delta C$</td>
<td>-843.4377 (-0.8988)</td>
</tr>
</tbody>
</table>

*The value of $\hat{p}$ is given only for the equations whose variables were transformed.*
<table>
<thead>
<tr>
<th>C_{t-1}</th>
<th>D_1</th>
<th>D_2</th>
<th>D_3</th>
<th>R^2</th>
<th>d</th>
<th>\hat{\beta}</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>-.1174</td>
<td>-65.1531</td>
<td>88.2533</td>
<td>-130.4283</td>
<td>.70</td>
<td>1.7947</td>
<td>-a</td>
<td>81682</td>
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<tr>
<td>(-1.3871)</td>
<td>(-.8266)</td>
<td>(1.1612)</td>
<td>(-1.6180)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>D_1</th>
<th>D_2</th>
<th>D_3</th>
<th>R^2</th>
<th>d</th>
<th>\hat{\beta}</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>.0005</td>
<td>-.0005</td>
<td>-.0009</td>
<td>.29</td>
<td>1.8814</td>
<td>.2952</td>
<td>.000002</td>
</tr>
<tr>
<td>(1.4160)</td>
<td>(-1.3199)</td>
<td>(-2.3464)</td>
<td></td>
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<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>D_1</th>
<th>D_2</th>
<th>D_3</th>
<th>R^2</th>
<th>d</th>
<th>\hat{\beta}</th>
<th>SE</th>
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</thead>
<tbody>
<tr>
<td>-41.003</td>
<td>118.9456</td>
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<td>1.77</td>
<td>.3797</td>
<td>92473</td>
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<td>(-5.487)</td>
<td>(1.6775)</td>
<td>(-1.1997)</td>
<td></td>
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</tr>
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</table>
Table 2b. Disentangled values of the coefficients of:

<table>
<thead>
<tr>
<th>Equation</th>
<th>Dependent variable</th>
<th>Constant</th>
<th>( r_c )</th>
<th>( r_g )</th>
<th>( r_m )</th>
<th>( \dot{p}/P )</th>
<th>( A )</th>
<th>( C_{t-1} )</th>
<th>( D_1 )</th>
<th>( D_2 )</th>
<th>( D_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.5 ( \Delta c )</td>
<td>5021.2</td>
<td>62.5</td>
<td>-1336.5</td>
<td>-2657.7</td>
<td>-2090.3</td>
<td>.5291</td>
<td>-.1174</td>
<td>-555.2</td>
<td>752.0</td>
<td>-1111.4</td>
<td></td>
</tr>
<tr>
<td>7.6 ( \Delta(C/A) )</td>
<td>.3186</td>
<td>.0085</td>
<td>.0052</td>
<td>-.0133</td>
<td>.0000004</td>
<td>-.1101</td>
<td>.0045</td>
<td>.0041</td>
<td>-.0077</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.7 ( \Delta C )</td>
<td>-2135.5</td>
<td>1559.1</td>
<td>314.4</td>
<td>-1189.3</td>
<td>.4086</td>
<td>-.3013</td>
<td>-136.1</td>
<td>394.8</td>
<td>-297.3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
suggest that the two assets are substitutes in the portfolios of life insurance companies.

The rate of inflation variable, $\dot{P}/P$, is significant at the 10 per cent level and is negative. This is as expected. The rationale is that since life insurance companies can hedge against inflation by buying stocks, for instance, they will do so in the face of rising price levels. They will get out of, or at least not commit as much of their funds to such assets as bonds, which are susceptible to a loss in purchasing power at maturity due to inflation. Under such circumstances, more funds would be channelled into stocks under the presumption that rising price levels in general will mean rising business incomes and profits and thus rising value of stocks.

When the corporate demand equation was estimated as a proportion of total assets with interest rate variables entered as levels and not as differentials, only one explanatory variable, the lagged dependent variable, was significant. Many of the signs were incorrect. A reestimation of the same equation using rate differentials produced a result that is only slightly better (Equation 7.6).

The implicit own rate in 7.6 is negative. The coefficient of $r_m - r_c$ is positive, contrary to the hypothesis and the result from the mortgage demand equation which
suggests that mortgages and corporate bonds are substitutes. However, the t test on the coefficient is insignificant, indicating that corporate bonds and mortgages may be independent. This is an unlikely result, considering that corporate bonds and mortgages are the two biggest components of life insurance companies' portfolios.

The inflation variable was stronger in 7.6 than in 7.5. It was significant at the 5 per cent level in 7.6 and at the 10 per cent level in 7.5 and had a negative sign in both, suggesting, as hypothesized, that the companies would move away from bonds when the general price level in the economy is rising.

Equation 7.6 also implies that U.S. government bonds and corporate bonds are complements. The sign on the coefficient of \( r_g - r_c \) is positive. However, the t test is insignificant at both the 5 and the 10 per cent level. This result is contrary to the finding in 7.5, which suggests that the two are substitutes.

Both Equations 7.5 and 7.6 imply very long lags or slow speeds of adjustment. Both equations indicate that only 11 per cent of the deficiency is made up within the first quarter. Overall, the explanatory power of 7.6 is rather low.

When equation 7.5 was reestimated with interest rates in differential form rather than levels, 7.7, the result
was slightly better than 7.6, but not as good as 7.5. The implicit own rate in 7.7 is wrong, just as in 7.6. However, the t tests on the coefficients of 7.7 are more definitive than those of 7.6. The t score on the coefficient of $r_g - r_c$ is significant at the 5 per cent level indicating a complementary relation between corporate bonds and government bonds. The coefficient of $r_m - r_c$ is positive, but the t test is highly insignificant implying that mortgages and corporate bonds are independent -- again an unlikely event.

The inflation variable $\hat{P}/P$ was strongest in 7.7. The speed of adjustment implied by 7.7 is that 30 per cent of the deficiency between actual and desired levels of corporate bonds is corrected within one quarter. This is considerably faster than in the first two equations.

In all three equations, the asset level was correctly signed and significantly different from zero at the 5 per cent level on a one-tailed t test. This is not surprising. Over the sample period, corporate bonds averaged about 36 per cent of the total portfolio. At the end of 1973, they constituted 36.4 per cent of all assets.

Overall, the estimated demand equations for corporate bonds gave inconsistent and unsatisfactory results.
The Demand for Stocks

The following models were fitted for the demand for common stocks by life insurance companies:

\[
\Delta(K/A) = a_0 + a_1(r_g - E/P) - a_2(r_m - E/P) + a_3\Delta A
- a_4(K/A)_{t-1} + a_5 D_1 + a_6 D_2 + a_7 D_3 \quad (7.8)
\]

\[
\Delta(K/A) = a_0 + a_1 E/P - a_2 r_m + a_3 r_g + a_4\Delta A
- a_5(K/A)_{t-1} + a_6 D_1 + a_7 D_2 + a_8 D_3 \quad (7.9)
\]

\[
\Delta K = a_0 + a_1(r_g - E/P) - a_2(r_m - E/P) + a_3\Delta A
- a_4 K_{t-1} + a_5 D_1 + a_6 D_2 + a_7 D_3 \quad (7.10)
\]

\[
\Delta K = a_0 + a_1 E/P = a_2 r_m + a_3 r_g + a_4\Delta A
+ a_5 K_{t-1} + a_6 D_1 + a_7 D_2 + a_8 D_3 \quad (7.11)
\]

The results are presented below in Tables 3a and 3b.
Table 3a. Regression coefficients and goodness of fit statistics for Equations 7.8 through 7.11

### Equation 7.8:

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Constant</th>
<th>( r_{g-E/P} )</th>
<th>( r_{m-E/P} )</th>
<th>( \Delta A )</th>
<th>((K/A)_{t-1})</th>
<th>( D_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta (K/A) )</td>
<td>0.0023</td>
<td>0.0024</td>
<td>-0.010</td>
<td>0.000003</td>
<td>-0.0823</td>
<td>0.0092</td>
</tr>
<tr>
<td></td>
<td>(0.8668)</td>
<td>(2.0168)</td>
<td>(-0.9046)</td>
<td>(6.8866)</td>
<td>(-4.9045)</td>
<td>(1.8368)</td>
</tr>
</tbody>
</table>

### Equation 7.9:

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Constant</th>
<th>( E/P )</th>
<th>( r_m )</th>
<th>( r_g )</th>
<th>( \Delta A )</th>
<th>((K/A)_{t-1})</th>
<th>( D_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta (K/A) )</td>
<td>0.0014</td>
<td>-0.0013</td>
<td>-0.0011</td>
<td>0.0026</td>
<td>0.000003</td>
<td>-0.088</td>
<td>0.0009</td>
</tr>
<tr>
<td></td>
<td>(0.3816)</td>
<td>(-3.0440)</td>
<td>(-0.9200)</td>
<td>(1.9826)</td>
<td>(6.1569)</td>
<td>(-3.61119)</td>
<td>(1.8397)</td>
</tr>
</tbody>
</table>

### Equation 7.10:

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Constant</th>
<th>( r_{g-E/P} )</th>
<th>( r_{m-E/P} )</th>
<th>( \Delta A )</th>
<th>( K_{t-1} )</th>
<th>( D_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta K )</td>
<td>-108.27</td>
<td>530.8466</td>
<td>-256.2717</td>
<td>0.7447</td>
<td>-0.0496</td>
<td>213.2363</td>
</tr>
<tr>
<td></td>
<td>(-0.1927)</td>
<td>(2.1588)</td>
<td>(-1.1022)</td>
<td>(8.6424)</td>
<td>(-4.3839)</td>
<td>(2.1020)</td>
</tr>
</tbody>
</table>

### Equation 7.11:

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Constant</th>
<th>( E/P )</th>
<th>( r_m )</th>
<th>( r_g )</th>
<th>( \Delta A )</th>
<th>( K_{t-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta K )</td>
<td>-307.33</td>
<td>-256.0785</td>
<td>-261.9621</td>
<td>556.9064</td>
<td>0.7602</td>
<td>-0.0535</td>
</tr>
<tr>
<td></td>
<td>(-.3576)</td>
<td>(-2.8285)</td>
<td>(-1.1090)</td>
<td>(2.1181)</td>
<td>(7.5534)</td>
<td>(-3.1493)</td>
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</tbody>
</table>

*The variables in this table were not transformed.*
<table>
<thead>
<tr>
<th>$D_2$</th>
<th>$D_3$</th>
<th>$\overline{R}^2$</th>
<th>$d$</th>
<th>$F$</th>
<th>$\hat{\beta}$</th>
<th>$SE$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-.00026</td>
<td>-.00076</td>
<td>.67</td>
<td>2.2466</td>
<td>13.46</td>
<td>-a</td>
<td>.000003</td>
</tr>
<tr>
<td>(-.5742)</td>
<td>(-1.6400)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-.0003</td>
<td>-.0008</td>
<td>.66</td>
<td>2.2530</td>
<td>11.499</td>
<td>-a</td>
<td>.000003</td>
</tr>
<tr>
<td>(-.5792)</td>
<td>(-1.6106)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>-36.8121</td>
<td>-165.5502</td>
<td>.76</td>
<td>2.2337</td>
<td>20.288</td>
<td>-a</td>
<td>123150</td>
</tr>
<tr>
<td>(-.4004)</td>
<td>(-1.7632)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>216.9016</td>
<td>-38.257</td>
<td>-164.601</td>
<td>.75</td>
<td>2.2405</td>
<td>17.2670</td>
<td>-a</td>
</tr>
<tr>
<td>(2.0971)</td>
<td>(-.4104)</td>
<td>(-1.73)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 3b. Disentangled values of the coefficients of:

<table>
<thead>
<tr>
<th>Equation</th>
<th>Dependent variable</th>
<th>Constant</th>
<th>$r_{g-E/P}$</th>
<th>$r_{m-E/P}$</th>
<th>$\Delta A$</th>
<th>$(K/A)_{t-1}$</th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.8</td>
<td>$\Delta (K/A)$</td>
<td>.0279</td>
<td>.0295</td>
<td>-.0126</td>
<td>.00004</td>
<td>-.0823</td>
<td>.0112</td>
<td>-.0032</td>
<td>-.0093</td>
</tr>
<tr>
<td>7.9</td>
<td>$\Delta (K/A)$</td>
<td>.0159</td>
<td>-.0148</td>
<td>-.0122</td>
<td>.0293</td>
<td>-.0003</td>
<td>-.088</td>
<td>.0107</td>
<td>-.003</td>
</tr>
<tr>
<td>7.10</td>
<td>$\Delta K$</td>
<td>-2191.5</td>
<td>10694.3</td>
<td>-5162.8</td>
<td>15.0</td>
<td>-.0496</td>
<td>4295.8</td>
<td>-741.6</td>
<td>-3335.1</td>
</tr>
<tr>
<td>7.11</td>
<td>$\Delta K$</td>
<td>-5745.2</td>
<td>-4785.3</td>
<td>-4895.2</td>
<td>10406.7</td>
<td>14.2</td>
<td>-.0535</td>
<td>4053.2</td>
<td>-714.9</td>
</tr>
</tbody>
</table>


All the estimated demand equations for stocks presented imply that stocks and government bonds are complements in the portfolio while stocks and mortgages are substitutes. The significance levels vary, however, in each equation. In all the equations estimated, the sign on the coefficient of the own rate is negative. Earnings-price ratio was used as the proxy of the return from stocks.

In all the equations, the t test on the coefficient of \( r_g \) was positive and significantly different from zero at the 5 per cent level indicating that stocks and U.S. governments are complements. This could be for reasons of liquidity. The results imply that stocks are bought not for speculative purposes but as long-term investment. The purchase of government bonds is thus increased with any increase in the purchase of stocks to compensate for the loss in liquidity.

All the equations suggest that mortgages and stocks are substitutes although none of the t tests is significant. However, the t scores are big enough that one would be reluctant to conclude that stocks and mortgages are independent assets in the portfolio.

Equations 7.8 and 7.9 imply a speed of adjustment of about 8 per cent while 7.10 and 7.11 indicate yet slower adjustment of only about 5 per cent of any discrepancy between desired and actual levels being corrected within
The level of total assets was tried but was insignificant. However, when it was entered as a flow, change in assets (ΔA), it was significantly different from zero in all the estimated equations. This is not surprising. Stocks make up a small portion of total assets, but during the period covered by this study, they more than doubled as a per cent of the portfolio. In 1963, stocks made up 5 per cent of total assets and continued to grow to a high of 11.2 per cent of the portfolio in 1972 before declining to 10.3 per cent in 1973 (Life Insurance Fact Book, 1974, p. 68).

The major fault of the estimated equations is the negative sign of the own rate of return. This, however, may be attributable to the fact that legal restrictions on investments in stock by life insurance companies are quite specific. They vary from state to state, but in general limit the proportion of the portfolio that may be held in stocks and usually also the type of stocks that may be held.

Stocks have not been heavily used as a major investment medium for funds backing life insurance policies because of the contractual guarantees for specified dollar amounts in these policies.

It just may be that regardless of the yield from holding stocks, that life insurance companies held the maximum
amount allowed them by law for purposes of diversification and as a hedge against inflation.

The Demand for U.S. Government Bonds

The following models were used to estimate the demand for U.S. government bonds:

\[ A(G/A) = a_0 - a_1(r_c - r_g) + a_2(r_m - r_g) + a_3\Delta A - a_4(G/A)_{t-1} \]
\[ + a_5D_1 + a_6D_2 + a_7D_3 \]  
(7.12)

\[ A(G/A) = a_0 + a_1(r_m - r_g) - a_2(E/P - r_g) + a_3\Delta A - a_4(G/A)_{t-1} \]
\[ + a_5D_1 + a_6D_2 + a_7D_3 \]  
(7.13)

\[ A(G/A) = a_0 + a_1r_g - a_2E/P + a_3\Delta A - a_4(G/A)_{t-1} \]
\[ + a_5D_1 + a_6D_2 + a_7D_3 \]  
(7.14)

\[ A(G/A) = a_0 + a_1r_g - a_2\frac{\hat{p}}{P} + a_3\Delta A - a_4(G/A)_{t-1} \]
\[ + a_5D_1 + a_6D_2 + a_7D_3 \]  
(7.15)

The results are presented in Tables 4a and 4b below:
Table 4a. Regression coefficients and goodness of fit statistics for equations 7.12 through 7.15

<table>
<thead>
<tr>
<th>Equation 7.12:</th>
<th>Coefficient of</th>
<th>( \Delta G(A) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent variable</td>
<td>Constant ( r_c )-g ( r_m )-g ( \Delta A ) (G/A) ( t-1 ) D1</td>
<td></td>
</tr>
<tr>
<td>( \Delta G(A) )</td>
<td>.0015 ( (1.1160) )</td>
<td>-.0031 ( (1.1299) )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation 7.13:</th>
<th>Coefficient of</th>
<th>( \Delta (F/A) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent variable</td>
<td>Constant ( r_m )-g ( E/P )-g ( \Delta A ) (G/A) ( t-1 ) D1</td>
<td></td>
</tr>
<tr>
<td>( \Delta (F/A) )</td>
<td>.0016 ( (.0338) )</td>
<td>-.000096 ( (-.2467) )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation 7.14:</th>
<th>Coefficient of</th>
<th>( \Delta G(A) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent variable</td>
<td>Constant ( r ) ( E/P ) ( \Delta A ) (G/A) ( t-1 ) D1</td>
<td></td>
</tr>
<tr>
<td>( \Delta G(A) )</td>
<td>.0012 ( (.0276) )</td>
<td>.0002 ( (.6905) )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation 7.15:</th>
<th>Coefficient of</th>
<th>( \Delta G(A) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent variable</td>
<td>Constant ( r ) ( P/P ) ( \Delta A ) (G/A) ( t-1 ) D1</td>
<td></td>
</tr>
<tr>
<td>( \Delta G(A) )</td>
<td>.0004 ( (.1568) )</td>
<td>.0002 ( (.6891) )</td>
</tr>
</tbody>
</table>

\*The variables in this table were not transformed.*
<table>
<thead>
<tr>
<th>$D_2$</th>
<th>$D_3$</th>
<th>$\bar{R}^2$</th>
<th>$d$</th>
<th>$F$</th>
<th>$\hat{\beta}$</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>.0002</td>
<td>.0001</td>
<td>.38</td>
<td>2.2913</td>
<td>4.7723</td>
<td>-a</td>
<td>.0000004</td>
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<td>(1.1177)</td>
<td>(.6726)</td>
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</table>

<table>
<thead>
<tr>
<th>$D_2$</th>
<th>$D_3$</th>
<th>$\bar{R}^2$</th>
<th>$d$</th>
<th>$F$</th>
<th>$\hat{\beta}$</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>-.0002</td>
<td>.00008</td>
<td>.38</td>
<td>2.3886</td>
<td>4.8233</td>
<td>-a</td>
<td>.0000004</td>
</tr>
<tr>
<td>(1.2336)</td>
<td>(.5011)</td>
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<td></td>
<td></td>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$D_2$</th>
<th>$D_3$</th>
<th>$\bar{R}^2$</th>
<th>$d$</th>
<th>$F$</th>
<th>$\hat{\beta}$</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>.0002</td>
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<td>-a</td>
<td>.0000004</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>$D_2$</th>
<th>$D_3$</th>
<th>$\bar{R}^2$</th>
<th>$d$</th>
<th>$F$</th>
<th>$\hat{\beta}$</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>.0002</td>
<td>.0001</td>
<td>.37</td>
<td>2.3312</td>
<td>4.6189</td>
<td>-a</td>
<td>.0000004</td>
</tr>
<tr>
<td>(1.1919)</td>
<td>(.7604)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 4b. Disentangled values of the coefficients of:

<table>
<thead>
<tr>
<th>Equation</th>
<th>Dependent</th>
<th>Constant</th>
<th>( r_{c-g} )</th>
<th>( r_{m-g} )</th>
<th>( \Delta A )</th>
<th>( (G/A)_{t-1} )</th>
<th>( D_1 )</th>
<th>( D_2 )</th>
<th>( D_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.12</td>
<td>( \Delta (G/A) )</td>
<td>0.0379</td>
<td>0.0078</td>
<td>-0.0092</td>
<td>-0.000005</td>
<td>-0.0395</td>
<td>-0.0172</td>
<td>0.0045</td>
<td>0.0028</td>
</tr>
<tr>
<td>7.13</td>
<td>( \Delta (G/A) )</td>
<td>0.0335</td>
<td>-0.0020</td>
<td>-0.0033</td>
<td>-0.000004</td>
<td>-0.0477</td>
<td>-0.0013</td>
<td>0.0041</td>
<td>0.0017</td>
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<tr>
<td>7.14</td>
<td>( \Delta (G/A) )</td>
<td>0.0267</td>
<td>0.0038</td>
<td>-0.0035</td>
<td>-0.000002</td>
<td>-0.0448</td>
<td>-0.0138</td>
<td>0.0044</td>
<td>0.0017</td>
</tr>
<tr>
<td>7.15</td>
<td>( \Delta (G/A) )</td>
<td>0.0087</td>
<td>0.0038</td>
<td>-0.0045</td>
<td>-0.000003</td>
<td>-0.0459</td>
<td>-0.0139</td>
<td>0.0042</td>
<td>0.0028</td>
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</table>
The estimated equations for the demand for government bonds did not establish consistent significant relationships. The explanatory power of the presented equations is low.

Equation 7.12 implies that government bonds and corporate bonds are complements, although the t test on the coefficient is not significant, but is large enough to discount independence of the two assets in the portfolio. This tends to support the result obtained in the estimated demand equations for corporate bonds.

The t test on the coefficient of \((r_m - r_g)\) is insignificant but has a negative sign suggesting the two assets -- mortgages and U.S. government bonds may be substitutes in the portfolio. This is contrary to the hypotheses and the significant complementary relationships found between the two in the equations for mortgages.

In 7.13, the coefficient of \((r_m - r_g)\) also is negative but this time is highly insignificant, suggesting independence rather than the substitute relationship implied by 7.12.

The mortgage rate was not included in Equations 7.14 and 7.15.

The signs of the own rate of interest were correct in all the equations except in 7.13. However, the coefficients of \(r_g\) in 7.14 and in 7.15, where the rates were entered as levels, were insignificant.
Equations 7.13 and 7.14 imply that government bonds and stocks may be substitutes. The signs of the coefficients of \((E/P - r_g)\) and \(E/P\) in 7.13 and 7.14 are negative. However, the t tests on these coefficients are close to but not quite significant at the 10 per cent level. If they had been significant, one could surmise that life insurance companies do hedge against inflation by buying stocks and shunning government bonds when prices in the economy are rising since stock yields generally move in phase with rising prices.

The inflation variable \(\hat{P}/P\) was tried in Equation 7.15. Its sign is negative as expected, but the t test is insignificant.

Total assets were entered in all equations as a flow (change in assets). Its sign in all the equations is negative. In 7.12, the t test is significantly different from zero at the 5 per cent level. In Equations 7.13 and 7.14, significance is at the 10 per cent level. In 7.15, the test is close to but not significant at the 10 per cent level.

The negative sign of \(\Delta A\) implies that for the period estimated, government bonds were an inferior asset. Life insurance companies decumulated their holdings of U.S. government bonds over this period.
The speed of adjustment implied by the four equations is that only 4 to 5 per cent of any discrepancy between actual and desired levels is corrected within one quarter.

One possible explanation of the poor performance of the estimated equations is that government bonds may have been held in the portfolio more for liquidity and risk hedging than for yields. They make up a very small fraction of the total portfolio. At year end 1973, they amounted to $4.3 billion or less than 2 per cent of the total portfolio. Its size in the portfolio has decreased steeply since the end of World War II. In periods such as the two World Wars and the depression years of the 1930s, life insurance funds were heavily channelled into United States government securities. At other times, investments are ordinarily directed more into the private sector (Life Insurance Fact Book, 1974, pp. 74-75). At the end of 1945, for example, U.S. governments made up an extraordinary 45 per cent of the total portfolio. The continuous decumulation since then suggests that perhaps their primary function today is for portfolio balance. In other words, funds are taken out of U.S. governments and used to make up deficiencies in other assets in the portfolio as needed or alternatively, temporarily idle funds are put into U.S. government bonds instead of being held as idle money balances and are then liquidated as needed.
The Demand for State-Local Government Bonds

The following equations were fitted for the demand by life insurance companies for state-local government bonds:

\[
\Delta(S/A) = \alpha_0 + \alpha_1 r_g + \alpha_2 A - \alpha_3 (S/A)_{t-1} \tag{7.16}
\]

\[
\Delta(S/A) = \alpha_0 - \alpha_1 (r_m - r_g) - \alpha_2 (r_g - r_s) - \alpha_3 (r_c - r_g) + \alpha_4 A - \alpha_5 (S/A)_{t-1} \tag{7.17}
\]

The results are presented in Tables 5a and 5b below:
Table 5a. Regression coefficients and goodness of fit statistics for Equations 7.16 and 7.17

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Coefficients of</th>
<th>$R^2$</th>
<th>$d$</th>
<th>$F$</th>
<th>$\beta$</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta$(S/A)</td>
<td>$r_s$ $\Delta A$ $(S/A)_{t-1}$</td>
<td>.00046</td>
<td>.0006</td>
<td>- .00000007</td>
<td>-.0497</td>
<td>.50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.5043)</td>
<td>(.6301)</td>
<td>(-1.3805)</td>
<td>(-2.1425)</td>
<td></td>
</tr>
</tbody>
</table>

Equation 7.17:

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Coefficients of</th>
<th>$R^2$</th>
<th>$d$</th>
<th>$F$</th>
<th>$\beta$</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta$(S/A)</td>
<td>$r_{s-m_s}$ $r_{g-r_s}$ $r_{c-r_s}$ $\Delta A$ $(S/A)_{t-1}$</td>
<td>.0006</td>
<td>-.0058</td>
<td>.0123</td>
<td>.0080</td>
<td>-.000002</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.7933)</td>
<td>(-1.0953)</td>
<td>(1.2994)</td>
<td>(1.3349)</td>
<td>(-1.9274)</td>
</tr>
</tbody>
</table>

Table 5b. Disentangled values of the coefficients of:

<table>
<thead>
<tr>
<th>Equation:</th>
<th>Dependent Constant</th>
<th>$r_s$ $\Delta A$ $(S/A)_{t-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.16</td>
<td>$\Delta$(S/A)</td>
<td>.0107</td>
</tr>
<tr>
<td>7.17</td>
<td>$\Delta$(S/A)</td>
<td>.0159</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation:</th>
<th>Dependent Constant</th>
<th>$r_{s-m_s}$ $r_{g-r_s}$ $r_{c-r_s}$ $\Delta A$ $(S/A)_{t-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.16</td>
<td>$\Delta$(S/A)</td>
<td>.0107 $r_s$ $.0012$ $-.000001$ $-.0465$</td>
</tr>
<tr>
<td>7.17</td>
<td>$\Delta$(S/A)</td>
<td>.0159 $r_{s-m_s}$ $.0058$ $r_{g-r_s}$ $.0123$ $r_{c-r_s}$ $-.000002$ $-.0376$</td>
</tr>
</tbody>
</table>
State and local government bonds are a very small fraction of the total portfolio of life insurance companies. They accounted for just over 1 per cent of the portfolio in 1973. All through the study period, the quantity of state and local government bonds in the portfolio was fairly stable, averaging $3.4 billion and ranging from $3.14 billion to $3.85 billion, and for all practical purposes could be considered constant during the study period.

Only two of the attempts to estimate the demand for state and local government bonds were presented.

In Equation 7.16, the own rate is the only interest rate variable. It has the correct sign but is insignificant. The change in asset, ΔA, has a negative coefficient and the t test on the coefficient is significant at the 10 per cent level, indicating that as a proportion of total assets, state and local government bonds have been declining.

Equation 7.17, implies that mortgages and state-local government bonds are substitutes. However, the t test on the coefficient of \((r_m - r_g)\) is not quite significant at the 10 per cent level. The equation implies U.S. government bonds and state and local government bonds are complements -- an unlikely result since the two classes of securities share very similar risks. The equation also implies complementarity between corporate securities and state and local government
bonds. It is difficult to explain why this would be so. Again, both classes of securities are subject to the same sorts of risks especially if one believes that life insurance companies deal mostly in high grade corporate securities where default risk is minimal.

The speed of adjustment implied by both equations is that about 5 per cent of any adjustment is completed within one quarter.

The poor performance of the estimated equations is not surprising given that state and local government bonds made up a very small fraction of the portfolio and was nearly constant over the sample period.

One would suspect, in light of the insignificance of state and local government bonds in the portfolio, that in the short run the prime motivation for holding them may not be profit maximization so much as trying to project the image of a good corporate citizen by providing funds for local school and city projects. In the long run, such "benevolence" might prove profitable in helping them attract new funds.

The estimated equations are reexamined below in terms of the interest-elasticity of the demand for the various assets. The mean elasticities are presented here. They differ from the usual definition of elasticity in that the means of the dependent and independent variables, and not their levels, are used in the calculations.
The results are to be interpreted in the usual manner, that is, a per cent change in the independent variable causes an \( n \) per cent change in the dependent variable.

The elasticities presented in the tables below are long-run elasticities, calculated from the disentangled values of the equations. They are presented here merely to amplify the results discussed above from the estimated equations.

Table 6. The mean elasticity of \( \Delta(M/A) \)

<table>
<thead>
<tr>
<th>With respect to:</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_m )</td>
<td>-14.3868</td>
</tr>
<tr>
<td>( r_c )</td>
<td>42.2643</td>
</tr>
<tr>
<td>( r_g )</td>
<td>-28.0683</td>
</tr>
<tr>
<td>( r_c - r_m )</td>
<td>-3.6997</td>
</tr>
<tr>
<td>( r_g - r_m )</td>
<td>10.0270</td>
</tr>
</tbody>
</table>
Table 7. The mean elasticity of $\Delta(K/A)$

With respect to:

<table>
<thead>
<tr>
<th>Term</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E/P$</td>
<td>-5.5227</td>
</tr>
<tr>
<td>$r_m$</td>
<td>-5.7695</td>
</tr>
<tr>
<td>$r_g$</td>
<td>9.5231</td>
</tr>
<tr>
<td>$r_g - E/P$</td>
<td>-1.2916</td>
</tr>
<tr>
<td>$r_m - E/P$</td>
<td>-1.1901</td>
</tr>
</tbody>
</table>

Table 8. The mean elasticity of $\Delta(C/A)$

With respect to:

<table>
<thead>
<tr>
<th>Term</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_g - r_C$</td>
<td>4.3739</td>
</tr>
<tr>
<td>$r_m - r_C$</td>
<td>-1.2566</td>
</tr>
<tr>
<td>$\dot{P}/P$</td>
<td>4.1612</td>
</tr>
<tr>
<td>$r_C$</td>
<td>0.0534</td>
</tr>
<tr>
<td>$r_g$</td>
<td>-0.8723</td>
</tr>
<tr>
<td>$r_m$</td>
<td>-2.4365</td>
</tr>
</tbody>
</table>

Table 9. The mean elasticity of $\Delta(S/A)$

With respect to:

<table>
<thead>
<tr>
<th>Term</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_m - r_S$</td>
<td>1.6927</td>
</tr>
<tr>
<td>$r_g - r_S$</td>
<td>-0.4678</td>
</tr>
<tr>
<td>$r_C - r_S$</td>
<td>-1.3832</td>
</tr>
<tr>
<td>$r_S$</td>
<td>-0.8453</td>
</tr>
</tbody>
</table>
Table 10. The mean elasticity of $\Delta(G/A)$

With respect to:

\[
\begin{align*}
    r_m - r_g &= 0.3580 \\
    E/P - r_g &= 0.1849 \\
    r_g &= -1.3890 \\
    E/P &= 1.4745 \\
    P/P &= 0.3154 \\
    r_c - r_g &= -0.7805
\end{align*}
\]

The results should be read as follows: The mean long-run elasticity of the change in the demand for mortgages $\Delta(M/A)$ with respect to a one percentage change in the mortgage yield, $r_m$, is 14.4 per cent.

The results show the demand for mortgages as being highly sensitive to the interest rate variables both as levels and differentials, but less so when the rates were entered as differentials. The same results hold for stocks.

The demand for corporate bonds, however, is inelastic with respect to the rate on government bonds, $r_g$, and its own rate, $r_c$, when entered as levels, but show some sensitivity when these rates were entered with other rates in differential form. This is not surprising. The elasticities simply reflect the results of the estimated demand equations. The t tests on the coefficients of $r_g$ and $r_c$, as levels, were
The elasticities of the demand for U.S. government bonds and state and local government bonds with respect to the interest rates on the various securities lends credence to the already discussed result that these two assets may be held for reasons other than short-run profitability. According to Tables 9 and 10, the demand for these two assets were, in the main, interest-inelastic. Where they showed any sensitivity to interest rates, the change was just over 1 per cent.

These results on interest elasticities of the demand for the various assets parallel those presented earlier and are merely a different way of presenting and looking at the same thing.

Policy Loans

Policy loans are life insurance company loans to their policyholders against the cash values of those policies. These loans are not determined by life insurance company portfolio decisions, but by the desires of policyholders to borrow against the accrued cash value of their policies. The interest rate charged on policy loan is a fixed contractual rate. The rate remained at 5 per cent during most of the period covered by this study. It changed to 6 per cent in January 1972. For the purposes of this study,
however, it will be assumed that the 5 per cent rate was the actual rate all through the study period. It is doubtful that those whose policies were issued at 6 per cent since January 1972 would have built up enough cash value against which to borrow.

The low contractual rate at which policyholders could borrow from their life companies would make these loans very attractive during high interest rate periods. Individuals who borrow against the cash value of their policies do not have to repay these loans or the interest. However, the amount of the policyholder's protection is reduced by the loan and delinquent interest payment outstanding.

Although policy loans outstanding are not determined by the life companies, changes in these loans are important since an increase in the amount outstanding decreases the amount the companies will have available for other higher-yielding investments. The loans represent financial dis-intermediation peculiar to life companies. Companies are forced to maintain enough cash or other highly liquid assets to meet the demand for these loans. If the companies underestimate demand for the loans, this could mean having to liquidate some other assets to generate the cash needed to meet the loan demand.

The form of the equation that will be used to explain the behavior of policy loans will be the same as those
specified for the portfolio assets.

The hypotheses being tested are that the stock of policy loans will vary directly with (a) the level of interest rates, (b) the level of unemployment and (c) the cash surrender value (CSV) of policies in force.

The cash surrender value of policies in force establishes a ceiling on the maximum volume of policy loans that could be made by life companies in any period. Outstanding policy loans may be expected to rise as this ceiling rises.

Empirically, cash surrender value will be proxied by the difference between total assets and outstanding stock of policy loans at the beginning of each quarter. A better proxy, perhaps, would be the level of reserves less outstanding policy loans, where reserves are total assets less net worth of the companies. However, the data could not be obtained.

The yield on three-month treasury bills \( r_{tb} \) and the yield on long-term government bonds \( r_g \) will be used to represent interest rate levels.

The behavior of policy loans is estimated for the period from first quarter 1963 through the third quarter 1974 (1963-I to 1974-III) using the following additional notations:
Δ(PL/A): change in policy loans as a proportion of total assets

\( r_{tb} \) : yield on three-month treasury bills

\( r_g \) : yield on long-term government bonds

\( U \) : unemployment rate

CSV : cash surrender value of policies in force.

The Demand for Policy Loans

The following models were used to estimate the behavior of policy loans outstanding:

\[
\Delta(PL/A) = \alpha_0 + \alpha_1 r_{tb,t-1} - \alpha_2 (PL/A)_{t-1} + \alpha_3 U_{t-1} + \alpha_4 D_1 \\
+ \alpha_5 D_2 + \alpha_6 D_3 
\] (7.18)

\[
\Delta(PL/A) = \alpha_0 + \alpha_1 r_g t_{t-1} - \alpha_2 (PL/A)_{t-1} + \alpha_3 U_{t-1} + \alpha_4 D_1 \\
+ \alpha_5 D_2 + \alpha_6 D_3 
\] (7.19)

\[
\Delta(PL) = \alpha_0 + \alpha_1 CSV_t + \alpha_2 r_{tb,t} - \alpha_3 PL_{t-1} + \alpha_4 U_{t-1} \\
+ \alpha_5 D_1 + \alpha_6 D_2 + \alpha_7 D_3 
\] (7.20)

\[
\Delta(PL/A) = \alpha_0 + \alpha_1 CSV_t + \alpha_2 r_{tb,t-1} - \alpha_3 (PL/A)_{t-1} + \alpha_4 U_{t-1} \\
+ \alpha_5 D_1 + \alpha_6 D_2 + \alpha_7 D_3 
\] (7.21)

The results are presented in Tables 11a and 11b below.
Table 11a. Regression coefficients and goodness of fit statistics for Equations 7.18 through 7.21

Equation 7.18:

| Dependent variable | Coefficient of | 
|--------------------|----------------|----------------|
| Δ(PL/A)           | Constant       | $r_{tb_{t-1}}$ | (PL/A)$_{t-1}$ | $U_{t-1}$ | $D_1$ | $D_2$ | $D_3$ |
| .0004             | .0007          | -.0336         | -.0002         | .0002     | .0003 | -.00014 |
| (.4218)           | (5.7852)       | (-2.1721)      | (-.9316)       | (2.8699)  | (3.2209) | (-1.5925) |

Equation 7.19:

| Dependent variable | Coefficient of | 
|--------------------|----------------|----------------|
| Δ(PL/A)           | Constant       | $r_{g_{t-1}}$ | (PL/A)$_{t-1}$ | $U_{t-1}$ | $D_1$ | $D_2$ |
| -.0006            | -.0969         | -.0969         | -.0003         | .0002     | .00015 |
| (-.5350)          | (5.3544)       | (-3.5870)      | (-1.6391)      | (2.0714)  | (1.7303) |

Equation 7.20:

| Dependent variable | Coefficient of | 
|--------------------|----------------|----------------|
| ΔPL                | Constant       | $r_{tb_{t}}$ | (PL/A)$_{t}$ | $U_{t-1}$ | $D_1$ |
| 1229.0223          | -.0113         | 95.5334       | .0883          | -93.2955  | 17.1597 |
| (2.7717)           | (-3.1058)      | (5.0576)      | (3.3419)       | (-2.9675) | (.9812) |

Equation 7.21:

<p>| Dependent variable | Coefficient of |
|--------------------|----------------|----------------|
| Δ(PL/A)           | Constant       | $r_{tb_{t-1}}$ | (PL/A)$<em>{t-1}$ | $U</em>{t-1}$ | $D_1$ |
| .0008             | -.0000001      | .0007          | -.0060         | -.0002    | .0003 |
| (.8667)           | (-.9662)       | (6.0323)       | (-.1907)       | (-1.2560) | (2.8633) |</p>
<table>
<thead>
<tr>
<th>$\bar{R}$</th>
<th>d</th>
<th>F</th>
<th>$\hat{\beta}$</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>.73</td>
<td>1.8293</td>
<td>18.109</td>
<td>.5553</td>
<td>.00000012</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$D_3$</th>
<th>$\bar{R}$</th>
<th>d</th>
<th>F</th>
<th>$\hat{\beta}$</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>- .00003</td>
<td>.68</td>
<td>1.7841</td>
<td>14.571</td>
<td>.6242</td>
<td>.00000018</td>
</tr>
<tr>
<td>(-.3647)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$D_2$</th>
<th>$D_3$</th>
<th>$\bar{R}$</th>
<th>d</th>
<th>F</th>
<th>$\hat{\beta}$</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.3348</td>
<td>-1.3269</td>
<td>.93</td>
<td>1.9837</td>
<td>75.72</td>
<td>.3107</td>
<td>5891.4</td>
</tr>
<tr>
<td>(1.4194)</td>
<td>(-0.0743)</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$D_2$</th>
<th>$D_3$</th>
<th>$\bar{R}$</th>
<th>d</th>
<th>F</th>
<th>$\hat{\beta}$</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>.0003</td>
<td>-.0001</td>
<td>.75</td>
<td>1.8287</td>
<td>18.162</td>
<td>.5012</td>
<td>.0000002</td>
</tr>
<tr>
<td>(3.1668)</td>
<td>(-1.6227)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 11b. Disentangled values of the coefficients of Equation Dependent Constant \( \Delta(PL/A) \)

<table>
<thead>
<tr>
<th>Equation</th>
<th>Dependent Variable</th>
<th>Constant</th>
<th>( r_{t-1} )</th>
<th>( (PL/A)_{t-1} )</th>
<th>( U_{t-1} )</th>
<th>( D_1 )</th>
<th>( D_2 )</th>
<th>( D_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.18</td>
<td>( \Delta(PL/A) )</td>
<td>0.0119</td>
<td>0.0199</td>
<td>-0.0336</td>
<td>-0.0051</td>
<td>0.0073</td>
<td>0.0082</td>
<td>-0.0042</td>
</tr>
<tr>
<td>7.19</td>
<td>( \Delta(PL/A) )</td>
<td>-0.0670</td>
<td>0.0176</td>
<td>-0.0969</td>
<td>-0.0032</td>
<td>0.0018</td>
<td>0.0015</td>
<td>-0.0003</td>
</tr>
<tr>
<td>7.20</td>
<td>( APL_t )</td>
<td>13918.7123</td>
<td>-1.279</td>
<td>1082.4764</td>
<td>0.083</td>
<td>-1057.119</td>
<td>194.4341</td>
<td>287.0651</td>
</tr>
<tr>
<td>7.21</td>
<td>( \Delta(PL/A) )</td>
<td>0.1333</td>
<td>-0.000002</td>
<td>0.1126</td>
<td>-0.0060</td>
<td>-0.0375</td>
<td>0.0420</td>
<td>0.0463</td>
</tr>
</tbody>
</table>
In the equations presented above, the interest rate variables, both the long-term government and the 90-day treasury bill yields, were statistically significant. In all four equations, they were positive as specified and significantly different from zero at the 1 per cent level using a one-tailed t-test.

CSV was entered in Equations 7.20 and 7.21. Contrary to the hypothesis, its coefficient has negative signs in both equations and is significantly different from zero at the 1 per cent level in 7.20 and insignificant in 7.21. The negative sign could mean that CSV is not a good proxy of the ceiling on the maximum volume of policy loans. The result cannot be easily explained, however, since policy loans showed an almost uninterrupted increase as did assets over the study period. One plausible explanation may be that the growth rate in policy loans exceeded the growth rate in total assets.

Another surprising result is the negative sign of the coefficient of the unemployment variable in all four equations presented. The t-test on the coefficient is significant at the 1 per cent level in 7.20; at the 10 per cent level in 7.19 and quite strong although not significant at the 10 per cent level in 7.21 and 7.18.

One possible explanation of this result could be that an increase in unemployment usually affects unskilled labor first. This group may not have that much cash value built
up against which to borrow. If they did, they might be loath to jeopardize their future security by drawing from their employment benefits. Those that are still employed when unemployment starts rising may see increasing unemployment as a threat to their future security. Instead of drawing down the value of their policies, they may tend to increase it instead while they still are employed and are in the position to do so.

It could very well be that during the periods when unemployment was rising, that interest rate levels were low or declining. If this were the case, it seems plausible that low or declining rates would dominate the level of unemployment as an explanatory variable for the behavior of policy loans. It makes sense to argue that if the general interest rate levels decline and approach the contractual rate on policy loans, the unemployment rate is more likely to borrow, if they need to, from their policies, especially if they expect the cut of a future rate.

As a proportion of policy loans increased from 4.7 per cent (base line) to 8 per cent (197 million) in 1973.

From casual observations and through personal interviews.
employment are not all that surprising. Many companies suffered huge drains in the form of policy loans when inflation and high interest rates were all the rage through most of 1974. Toward the end of the year and the beginning of 1975, the increase in policy loans had slowed as interest rates started declining even though unemployment was hitting higher levels each month.

Overall, it would appear that the major factor determining the stock of policy loans outstanding is high interest rates. Since life insurance policies are a form of savings, individuals, sophisticated in the financial market, would more than like draw down the accrued cash values of their policies at a relatively low interest cost when money market rates increase relative to the rates on long-term financial securities. They can invest the loans in fairly liquid debt instruments yielding a high rate of return. When these instruments mature, they can pay back the loan and interest and have some income left. Such a behavior, indeed, would be optimal for individuals seeking the highest return for their funds. Others may simply borrow against their policies during high interest periods because they need to borrow an amount that cannot be afforded to pay the high rate, or they might be individuals trying to buy homes when mortgage funds are scarce and mortgage rates are high.
Such persons may borrow on their policies for the down payment on their homes.

Francis Schott (1971), in a similar study, tried to explain variations in policy loans between 1965 and 1970. He used simple and multiple regression to analyze net policy loan increases at 15 leading companies which he said consistently accounted for about 55 per cent of the industry's total assets. He tested the hypotheses that (1) interest rate variations, (2) monetary ease or restraint, (3) credit availability, and (4) rising prices, were the primary factors influencing policy loans.

In terms of results, interest rate variations were the most important single influence on policy loans. Rising consumer price and restraint on liquidity growth were less significant when credit availability was found insignificant. Although most of his influential variables were different from those used in the present study, the impact of interest rate variations on policy loans was established by both his and the present study.
CHAPTER VIII. IMPACT OF THE FEDERAL RESERVE SYSTEM MONETARY
POLICY ON LIFE INSURANCE COMPANIES' PORTFOLIOS

The issue this chapter seeks to explore is what direct impact, if any, monetary policy has on the portfolios of life insurance companies. That is, does it make any difference in the way life insurance companies manage their portfolios if the Federal Reserve System is pursuing an easy or a tight monetary policy. As the Fed shifts gears from monetary ease to tightness, or vice versa, relative rates on securities will be affected. This may influence the direction of flow of funds by life insurance companies. It can be argued, a priori, that an increase in interest rates resulting from a tight monetary policy would reduce the inflow of funds which the companies would have available for investments. This could come about in at least two ways. One is that rising rates would increase policy loans. The other is that rising rates would reduce the willingness of life companies to augment the inflow of investable funds by liquidating long-term securities due to the high probability that such liquidation could mean capital losses. Also, there are such assets in the portfolio as VA and FHA-guaranteed mortgages with ceiling rates imposed by the government. When rising interest rates increase the spread between the ceiling rate on these government-backed mortgages and other mortgages, life
companies may be expected to commit less of their funds to
the lower-yielding VA and FHA mortgages. The risk considera-
tion might be outweighed by the higher yields possible with
the nongovernment-backed mortgages.

One problem with the present study is that it would be
impossible to get at the issue of whether any relative
shifts occur from one group of mortgage holdings to another
since all mortgage holdings by life insurance companies are
grouped together without classification by subgroups.

Another interesting point, which also would be im-
possible to get at, is the breakdown between short-term
and long-term assets held by the companies during tight and
easy money periods. During tight money periods and high
interest rates, life companies may be expected to reduce their
cash and bank deposits as a result of the higher yields on
long-term securities. This effect, however, may be offset
by net additions to holdings of short-term securities, which
respond positively to the downward shift in the long-short
yield differential. An increase in the holdings of shorts
also could reflect the companies' concern with the loss of
liquidity resulting from the decline in the market value of
the major portion of their portfolios, which is mostly
long-term.

As was pointed out earlier, the securities in this
study are considered homogeneous with respect to maturity
and differ only by issuer. This specification would make it impossible to determine any shifts in the composition of the portfolio between shorts and longs or government-backed and nongovernment-backed securities during periods of easy or tight money.

For the purposes of this study, therefore, it is the contention that any differential effect of easy or tight money on the portfolio composition of life insurance companies will be through a change in the relative yields on competing assets of similar maturity structure.

The hypothesis that will be tested is that monetary policy will not have any effect on portfolio management by life insurance companies. Due to the actuarially predictable nature of their obligations to their policyholders, most of their assets are long-term. It is thus doubtful that life companies would get out of longs into shorts during periods of rising yields. This could involve capital losses. It is equally doubtful that under any circumstance, say easy money periods, they would hold most of their assets in shorts. If they are already holding most of their assets in long-term securities, a switch to shorts simply because of ease in monetary policy would seem out of the question.

The problem is to come up with an acceptable definition and measure of monetary tightness and ease. It is not the purpose of this study to dabble into the issue of monetary
targets, instruments and indicators.\footnote{For a discussion of these topics, see Starleaf and Stephenson, 1969; Poole, 1970; and Havrilesky, 1972.}

A recent study by Havrilesky et al. (1974), has delineated the period covered in the present study into tight and easy money periods, and will be adopted for our immediate purposes.

Havrilesky et al. in their study, estimated the influence of the state of the economy on the policy actions of the Fed in times of announced ease and tightness from July 1962 to October 1973. They used the Federal funds rate as their policy-control variable or instrument. They used a control or policy period of just one month, justified by the fact that the Federal Open Market Committee, the monetary policy arm of the Fed, meets approximately once a month to determine the direction of monetary policy. Using ordinary least squares, they estimated:

\[ FF_r = \alpha_0 + \alpha_1 (U_{t-1}, P_{t-1}, FX_{t-1}, BCP_{t-1}, M) \]  \hspace{1cm} (8.1)

where

- \( FF_r \) = federal funds rate
- \( U \) = unemployment rate
- \( P \) = wholesale price index
FX = exchange rate on Deutsche Mark as a barometer of the international position of the dollar

BCP = bank credit proxy

M = currency + demand deposits.

Based on the Federal Funds rate they cataloged several periods into tight and easy money periods and then estimated the policy reactions of the Fed to the price level, the rate of unemployment, a measure of the country's international economic position and the rates of growth of "key" monetary aggregates. Their classification is as follows:

<table>
<thead>
<tr>
<th>Tight Money Periods</th>
<th>Easy Money Periods</th>
</tr>
</thead>
<tbody>
<tr>
<td>January 1963 to December 1966</td>
<td>January 1967 to December 1967</td>
</tr>
<tr>
<td>January 1968 to June 1968</td>
<td>July 1968 to December 1968</td>
</tr>
<tr>
<td>July 1971 to September 1972</td>
<td>October 1972 to December 1973</td>
</tr>
<tr>
<td>January 1974 to September 1974</td>
<td></td>
</tr>
</tbody>
</table>

Adoption of their classification of tight and easy money periods is not meant to endorse it as a fait accompli. One can seriously question their use of the Federal funds rate

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1 Their study period did not include 1974. This period is included as tight based on casual empiricism on the economy during 1974.
"as the single variable among all possible candidates that the Fed is most likely actually to have used as a control-variable." Arguments can be made for the use instead of banks' free reserves or the growth rate in the money supply or other monetary aggregates. However, as a first "ad hoc" approximation of an answer to the issue of portfolio management by life insurance companies during tight and easy money periods, their classification above will suffice.

Having adopted the classification by Havrilesky et al., a Chow test (Chow, 1960) will be used to determine if portfolios were managed differently in the two subperiods. Statistically, this amounts to testing whether the two subgroups of observations can be regarded as belonging to the same regression model.

The mechanics of the test involves grouping all the data for the tight and easy money periods, respectively, separately. The model is estimated for each of the two subperiods to obtain: the residual sum of squares, $(X)$, for the first subgroup with $n$ observations, $n-p$ degrees of freedom; the residual sum of squares, $(Y)$, for the second subgroup with $m$ observations and $m-p$ degrees of freedom; and the residual sum of squares $(Z)$, for the full model with $n+m-p$ degrees of freedom. $P$ is the number of explanatory variables plus an intercept.
The Chow test performs an analysis of covariance to determine if the two subgroups belong to the same regression model.

The ratio \( \frac{(Z-Y-X)/P}{(Y+X)/(n+m-2p)} = A \), will be distributed as \( F(p, n+m-2p) \) under the null hypothesis that both groups belong to the same regression model or that there is no significant structural change between the two subgroups.

The 5 per cent level of significance will be used to evaluate the results. This means that to conclude there was a significant structural change between the two groups of observations or that they don't belong to the same regression model, the calculated \( F \) value or the ratio \( A \) will have to be at least equal to the tabular \( F \).

The models on which the Chow test was performed are presented below along with the results.

\[
\Delta(S/A) = \alpha_0 + \alpha_1 (r_{S}, \Delta A, \{S/A\}_{t-1}); \quad p = 4; \quad (n+m-2p) = 36 \\
A = 3.5063; \quad F_{05}(4, 36) = 5.73 \tag{8.2}
\]

\[
\Delta(S/A) = \alpha_0 + \alpha_1 [(r_{m}-r_{S}), (r_{g}-r_{S}), (r_{C}-r_{S}), \Delta A, \{S/A\}_{t-1}] \\
p = 6; \quad n+m-2p = 32; \quad A = 1.6633; \quad F_{05}(6, 32) = 3.79 \tag{8.3}
\]
\[ \Delta C = \alpha_0 + \alpha_i (r_c, r_g, r_m, \frac{\hat{p}}{P}, A, C_{t-1}, D_1, D_2, D_3) \]

\[ p = 10; \quad n + m - 2p = 24; \quad A = 2.7171, \]

\[ F_{0.05}(10, 24) = 2.74 \]  \hspace{1cm} (8.4)

\[ \Delta (C/A) = \{(r_g - r_c), (r_m - r_c), \frac{\hat{p}}{P}, A, (C/A)_{t-1}, D_1, D_2, D_3\} \]

\[ p = 9, \quad n + m - 2p = 26; \quad A = 0.5901; \]

\[ F_{0.05}(9, 26) = 2.84 \]  \hspace{1cm} (8.5)

\[ \Delta (G/A) = \alpha_0 + \alpha_i \{r_g, E/P, \Delta A, (G/A)_{t-1}, D_1, D_2, D_3\} \]

\[ p = 8, \quad n + m - 2p = 28; \quad A = 1.0049; \]

\[ F_{0.05}(8, 28) = 3.09 \]  \hspace{1cm} (8.6)

\[ \Delta (G/A) = \alpha_0 + \alpha_i \{r_m - r_g, E/P - r_g, \Delta A, (G/A)_{t-1}, D_1, D_2, D_3\} \]

\[ p = 8; \quad n + m - 2p = 28; \quad A = 0.9447; \]

\[ F_{0.05}(8, 28) = 3.09 \]  \hspace{1cm} (8.7)

\[ \Delta (M/A) = \alpha_0 + \alpha_i \{r_m, r_c, r_g, (m/a)_{t-1}, D_1, D_2, D_3\} \]

\[ p = 9, \quad n + m - 2p = 26; \quad A = 1.5712; \]

\[ F_{0.05}(9, 26) = 2.88 \]  \hspace{1cm} (8.8)
\[ \Lambda(M/A) = \alpha_o + \alpha_i \{ (r_c - r_m), (r_g - r_m), (M/A)_{t-1}, D_1, D_2, D_3 \} \]

\[ p = 8; \quad n+m-2p = 28, \quad A = 1.5996; \]
\[ F_{05}(8, 28) = 3.09 \] (8.9)

\[ \Delta(K/A) = \alpha_o + \alpha_i \{ E/P, r_m, r_g, \Delta A, (k/a)_{t-1}, D_1, D_2, D_3 \} \]

\[ p = 9; \quad n+m-2p = 26, \quad A = 1.2706; \]
\[ F_{05}(9, 26) = 2.88 \] (8.10)

\[ \Delta(K/A) = \alpha_o + \alpha_i \{ (r_g-E/P), (r_m-E/P), \Delta A, (K/A)_{t-1}, D_1, D_2, D_3 \} \]

\[ p = 8; \quad n+m-2p = 28, \quad A = 1.3087; \]
\[ F_{05}(8, 28) = 3.09 \] (8.11)

\[ \Delta(PL/A) = \alpha_o + \alpha_i \{ r_{tb_{t-1}}, (PL/A)_{t-1} U_{t-1}, D_1, D_2, D_3 \} \]

\[ p = 7; \quad n+m-2p = 32, \quad A = 0.7760; \]
\[ F_{05}(7, 32) = 3.37 \] (8.12)

\[ \Delta(PL/A) = \alpha_o + \alpha_i \{ r_{gt-1}, (PL/A)_{t-1} U_{t-1}, D_1, D_2, D_3 \} \]

\[ p = 7; \quad n+m-2p = 32, \quad A = 1.0372; \]
\[ F_{05}(7, 32) = 3.37 \] (8.13)
\[ \Delta(PL/A) = \alpha_0 + \alpha_i (CSV_t, R_{tb,t-1}, (PL/A)_{t-1}, U_{t-1}, D_1, D_2, D_3) \]

\[ p = 8; \ n+m-2p = 30; \ A = 1.00; \]

\[ F_{0.05}(8, 30) = 3.08 \]  \hspace{1cm} (8.14)

The results of the Chow test indicates there was no statistically significant change in structural relationships between tight and easy money periods in the management of portfolios by life insurance companies. Even at the 10 per cent level of significance, only Equation 8.4, the demand for corporate bonds, had a statistically significant result suggesting a difference between the two subperiods.

Based on these results, one would have to say that for the period covered in the study it made no difference to life insurance companies whether the Fed was pursuing a tight or easy money policy so far as portfolio management was concerned.

However, before one can conclude definitely that Fed policies, in general, do not affect portfolio management by life insurance companies, one would have to examine the Havrilesky et al. study more critically. Another problem could be the highly aggregative nature of the present study. It may be that the only effect Fed monetary policy has on life company portfolios is through a change in relative rates, which would change the direction and amount of commitments.
of new funds into the various assets. If life companies invest mostly in long-term securities, then, indeed, the result that there was no significant change in structural relationship between tight and easy money periods would not be questionable. However, even given that they invest mostly in longs, monetary policy could still influence portfolios in certain ways. One would be that the mix between cash and other short-term liquid assets and long-term assets could change. For example, they may hold less cash, less treasury bills and more U.S. government bonds during tight money periods. Another would be that the mix between government-guaranteed and nongovernment-guaranteed mortgages could change during the two periods.

In general, the specification that securities were of similar maturity structure precludes any attempt to even speculate on the effects of expectations about future rates on the direction of flow of funds, when monetary policy changes direction.

Overall, however, based on the present effort, the conclusion is that monetary policy had no direct effect on portfolio management by life insurance companies.
CHAPTER IX. SUMMARY, CONCLUSIONS AND SUGGESTIONS FOR FURTHER STUDIES

This study sought to depict the major determinants of portfolio management by life insurance companies and in a limited way the impact of monetary policy on the portfolios.

The study was done within the framework of a theory of portfolio selection. Ordinary least squares were used to estimate the demand for the major assets held by life insurance companies.

The results suggest that, at least for the major assets, changes in relative rates of returns were the major factors determining the flow of investment funds.

The estimated equations for the demand of corporate bonds and mortgages, which together make up about 70 per cent of total assets, yielded consistent results. Each equation established the other asset as a substitute in the portfolio. The estimated demand equations for state-local government bonds and United States government bonds however, had incorrect signs and, in general, were inconsistent. The result was not surprising, however, since the two classes of assets remained fairly constant all through the study period and actually declined as a proportion of total assets.

Overall, the stock adjustment model used to specify the
demand equations appears a reasonable approximation of the demand for securities by life insurance companies. The speeds of adjustment implied by the various equations were very slow, indicating that only a very small fraction of any deficiency between actual and desired holdings of any asset was corrected within one quarter.

The interest-elasticities of the demand for the various assets were fairly high. This result suggests that the capital markets can and do allocate funds into different investment categories in a way that would not result in any major distortion in one specific area of the market. For example, the demand for mortgages was highly sensitive to changes in the yields of bonds. This implies it would take only a slight change in the yield of bonds relative to mortgages to induce a flow of funds into or out of the corporate bond market. If the demand for mortgages, on the other hand, had been inelastic with respect to the yield on bonds, then it would require a major change in the yield of bonds, relative to mortgages, to get a flow of funds into or out of bonds. The high interest-elasticity thus suggests capital market efficiency in the allocation of funds.

Policy loans, although not determined by the life insurance companies themselves, represent financial disintermediation peculiar to the life insurance industry. The volume of policy loans outstanding varies directly with
interest rates and adversely affects the ability of the companies to take advantage of higher-yielding investments during periods when interest rates are high or rising.

In terms of any direct impact of monetary policy on portfolio management, the "tentative" result is that it has no effect. The word tentative is used to indicate that the specifications and assumptions of the present study could very well have masked any direct impact of monetary policy.

It is rather difficult to say precisely how good or bad the results of the present study are considering the exploratory and apparently pioneering nature of the study. There are no known similar studies against which to compare the present results. Silber (1970) did a similar but not directly comparable study. For one thing, the specification of his demand equations is different from that used here. For another, he estimated the levels of assets in the portfolio. It was argued in this study that the portfolio manager would be more concerned with investing net inflows of funds than with rearranging the existing or already invested portfolio. As a result, the focus of this study was to estimate quarterly changes in each asset. However, although not reported, results similar to Silber's were obtained when the levels of the various assets were estimated.
A major problem with the present study and the related studies cited in Chapter II would appear to be their highly aggregative nature. The aggregation problem encompasses several levels. In the first place, all life insurance companies were lumped together. Life insurance companies vary in size from the very small with assets in the millions of dollars to the very large with assets in the billions of dollars. It is conceivable that portfolio management could be as disparate as the number and sizes of companies in the industry. Management could vary from conservative to aggressive. In the second place, the industry is highly regulated and in some cases there are as many variations in the regulations as there are states. Thirdly, yields on assets were taken as given in the present study and some composite index of an average rate of return for each asset was used. This could be a source of problems. Within each class of assets and its corresponding rate of return, there could be substantial divergencies. For example, within corporate stocks and bonds held by life insurance companies, yields vary depending on the issuing company. Fourthly, the yields were treated as exogenous variables and ordinary least squares were used to estimate the demand for the assets. This assumes that rates of return and the risks associated with those rates and expected rates are independent of the investor. This seems unlikely. It is more likely, given the size of life insurance
companies, that their combined demand or lack of demand for some asset will affect the price and yield of that asset. This suggests that perhaps a system where the rates are endogenously determined would be more appropriate.

Another problem, pointed out earlier, is that there may exist some fairly strong but numerically immeasurable determinants of portfolio management such as good will and the need to project the image of a good corporate citizen. Such motives could lead to greater profitability in the long run, but could mean sacrificing some higher returns in the short run.

All the problems discussed above are candidates for further research. Another important subject for further research would be the forward commitment process used by life insurance companies to extend commitments to those seeking mortgage and bond financing. Life insurance companies use the forward commitment process rather extensively. The impact of the determinants of portfolio management, perhaps, may be felt more at the commitment instead of the acquisition stage of investment. If that indeed is the case, then it would be fruitful to obtain forecasts of future availability of funds and then to delineate the factors that determine how, why and how much of these funds are committed to the various assets. This would provide an opportunity to build expectation and its role into the model. A study of forward commitment and the
role of expectations in this process, would afford a better insight on the effect of monetary policy on portfolio management. Monetary policy changes may have their greatest impact on portfolio management by changing expectations about future yields and thus the direction and amount of funds committed to the various assets in the portfolio.

Another suggestion would be the disaggregation of the topic. The result may not be generalizable, but it might be fruitful to find an "average" life insurance company in terms of the size of its assets and to study its portfolio management. This could give some useful insights into the overall portfolio management problems of the entire industry, especially with respect to institutional constraints and the nonprofit motives that help, in addition to profit considerations, to determine the flow of funds into the various assets.
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APPENDIX

Sources of Data

1. Data on the assets -- mortgage holdings, corporate bonds, corporate stocks, U.S. government bonds, state-local government bonds and "policy loans" -- were obtained from various issues of the Federal Reserve Bulletin. Quarterly averages used in the study were created from the monthly series reported in the Bulletin.

2. The interest rates used in the study also were quarterly averages of monthly data obtained from several issues of the Federal Reserve Bulletin, except where indicated below.


   b. Treasury Bill rate: the yield on three-month treasury bills.

   c. Corporate Bond rate: the yield on long-term corporate bonds prepared by Moody's investor service on Baa corporate bonds.

   d. Earnings-Price ratio: a measure of the returns from common stocks.

   e. Mortgage rate: the yield on conventional first mortgages.
f. State-local government bond rate: the yield on 15 high-grade municipals (Standard and Poor's averages).

g. GNP deflator: the implicit price deflator was obtained from various issues of the Survey of Current Business and from this series the rate of inflation (rate of change in the GNP deflator) was created using the formula \[
\frac{P_{t+1} - P_t}{P_t} = \frac{\Delta P}{P_t}.
\]

Variable Dictionary

\begin{align*}
A & = \text{total assets} \\
\Delta A & = A_t - A_{t-1} = \text{change in level of assets} \\
C & = \text{corporate bonds} \\
G & = \text{U.S. government bonds} \\
S & = \text{state-local government bonds} \\
M & = \text{mortgages} \\
K & = \text{stocks} \\
r_g & = \text{interest rate on U.S. government bonds} \\
r_{tb} & = \text{interest rate on three-month treasury bills} \\
r_s & = \text{interest rate on state-local government bonds} \\
r_m & = \text{mortgage yield} \\
r_c & = \text{interest rate on corporate bonds} \\
E/P & = \text{earnings-price ratio on common stocks}
\end{align*}
\( \frac{\Delta P}{P} \) = rate of change in the GNP deflator

\( D_i \) = seasonal dummy variables. \( i = 1,2,3 \). Each variable takes on a value of 1 for the quarter represented by its subscript and is zero elsewhere. The dummy variables are included to account for any independent seasonal patterns that may exist in the demand for securities.

\( \Delta (PL/A) \) = change in policy loans as a proportion of total assets

\( r_{tb} \) = yield on three-month treasury bills

\( r_g \) = yield on long-term government bonds

\( U \) = unemployment rate

\( CSV \) = cash surrender value of policies in force