The mechanics of soil cutting equipment and emergence of seedlings

Ghulam Sarwar Sheikh
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The mechanics of soil cutting equipment
and emergence of seedlings

by

Ghulam Sarwar Sheikh

A Dissertation Submitted to the
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INTRODUCTION

The importance of developing the mechanics of mechanical equipment used to penetrate soil has long been recognized by engineers concerned with the problems of soil resistance encountered as a result of forces applied by such equipment. Problems related to the design, construction, and operation of such mechanical equipment, namely soil penetrating machines, tillage equipment, earth moving machines, and all other off-the-road locomotive equipment, involve the use of mechanics which provide a method for describing the application of forces to the soil and its consequent reaction. Accurate mechanics would provide a means by which the forces applied by such machines could be predicted and controlled by their design and construction.

The prediction of forces determined from the mechanics, for machine elements which are in contact with soil, would be a guide for improving the design of off-the-road locomotive equipment (which have been and still are developed largely by a trial and error process). In order to predict such forces, the knowledge of machines, principles of mechanics, design methods, and behavior of soil under dynamic situations is of foremost importance. Further, the prediction on such soil machine systems stresses the need for accurate measurement of forces applied by the machines, as well as the determination of soil parameters which characterize the dynamic behavior of soils.

The forces resulting from the application of the mechanical equipment, such as tillage and traction devices, affect the physical conditions of soil. Tillage is generally performed to fragment the soil to reduce soil
strength, reduce compaction, and allow the free movement of air and water in order to promote plant growth. Traffic by wheel and crawler tractors, on the other hand, cause soil compaction, which increases the resistance or physical impedance of soil and reduces soil's permeability to water and aeration of the soil. All of these factors may affect the quality and quantity of crops grown on the soil. Extensive investigations have yet to be carried out to describe the conditions of soil conducive to plant growth in physical, mathematical, and engineering terms. Relationships need to be established between the emergence of plants and the physical conditions of soil, as affected by the forces resulting from the application of the mechanical equipment. Optimum conditions of soil remain to be determined so that machine operations are carried out at proper times. The relative importance of the different environmental factors affecting the physical impedance of soil and the emergence of seedlings need to be studied. All of this information is needed as a basis for the development of farm equipment and their operation at proper times in order to enhance plant production.

For the purpose of studying the soil-machine-plant complex, this investigation was undertaken to analyze and blend the various aspects related to the forces applied to soils and the reaction of plants to such forces. Specifically speaking, this study measured the forces and developed and tested theoretical relationships for predicting the forces resulting from the application of the tillage, traction, and penetration equipment under different soil conditions and to investigate the effect of the physical conditions of soil, as affected by such forces, on the emergence of seedlings. The relative importance of the different soil variables
affecting such forces and the emergence of plants were also studied. Optimum conditions for emergence were determined by using the statistical and mathematical procedures.

The investigations have been presented here in two main parts. The first part deals with the measurement and prediction of forces applied to the mechanical equipment operated at different soil conditions. In the second part, the influence of the physical properties and the penetration resistance of soil on the emergence of seedlings has been studied.
OBJECTIVES

The broad and specific objectives of this study are outlined as follows:

Broad Objectives

To advance the understanding of those factors which influence the design and construction of soil engaging mechanisms of traction, tillage, and planting machines.

Specific Objectives

The specific objectives are as follows:

1. To present and experimentally test the mechanics of penetration, tillage, and traction equipment.
2. To study the emergence of seedlings under different soil conditions.
3. To establish a relationship between the resistance of soil offered to soil penetrating equipment and the emergence of seedlings under different soil conditions.
REVIEW OF LITERATURE

The literature for this study will be discussed under the following headings:

1. Mechanics of penetrating equipment
2. Mechanics of tillage equipment
3. Mechanics of traction equipment
4. Soil resistance and emergence of crops

Mechanics of Penetration Equipment

When a penetrating tool is forced vertically into soil, plastic failure of the soil occurs. Penetration is often termed cutting since it implies a localized soil failure in the neighborhood of the cutter. Two mathematical models have been used to represent the penetration behavior. One model indicates the force required to cause penetration of a static penetrometer pushed slowly into the soil. The other model measures the energy required to cause penetration. In this study, only static penetrometers will be reviewed.

Stone and Williams (1939) used a penetrometer to estimate the draft force of a plow in various soil conditions. They established an empirical relation between the penetrometer reading and the draft of a plow. McKibben and Hull (1940) used penetration resistance to predict rolling resistance of towed wheels.

Kostritsyn (1956), a Russian research worker, as reported by Gill and Vanden Berg in Agriculture Handbook No. 316 (1967) analyzed the forces on long soil cutting knives in uniform soil conditions. The equation developed shows that the cutting force is directly proportional to the area of the
wedge of the cutter, wedge angle, and coefficient of soil-metal friction.
He stated that the resultant force along the side of a pointed tool does not operate along the direction of movement or along the normal to the side surface but along a line inclined forward of the normal at an angle established by soil-tool friction. Gill and Vanden Berg further reported that another Russian author, Zelin (1950), developed empirical relations between the cutting force of a horizontal blade and physical conditions of the soil as measured by the penetrometer. He conducted a large number of experiments on horizontal cutters and measured the draft and depth of cutting. He observed a parabolic relationship between the draft and depth.

Morton and Buchele (1960) developed an upward traveling penetrometer to simulate a seedling and to provide a continuous record of soil resistance with depth for different soil conditions. The force-depth diagram plotted by the oscillograph gave a pictorial view for the variation of penetrating force with soil depth under drying and nondrying conditions. They also plotted penetrating resistance vs compaction force and energy of penetration vs compaction force graphs. They concluded that the penetrating force varied directly with compaction, initial moisture, and amount of soil surface drying. They, however, did not develop any theoretical equation from the standpoint of soil dynamics from which the force or energy expended by the penetrometer could be predicted.

Sheikh (1964) developed the mechanics of penetrating tools moving vertically upward in soil. He developed an equation for the force exerted by a penetrating device deduced from the principles of mechanics. While developing such an equation, he considered frictional resistance between the soil and the cutting tool and passive earth pressure developed by the
soil in response to the movement of the penetrometer. The final equation of cutting force was found to be a function of the geometry of the tool, soil-tool friction, bulk density, cohesion, and angle of internal friction of soil. The magnitude of force predicted by this equation was compared with the value determined experimentally. Large differences were found to occur between the two values at higher bulk densities.

Knight and Rula (1961) used a cone penetrometer to establish an index of soil trafficability. The penetrometer was pushed to a given depth, and the penetration resistance as a function of soil depth was recorded. After making penetrometer tests in a number of soil conditions and experimentally testing the vehicles for their trafficability, an index scale was developed to indicate the weight and number of vehicles that could travel over the soil with a given penetrometer value. The index gave a "go" or "not go" rating to any soil condition.

Gill (1968) observed the failure pattern of soil by moving a vertical penetrometer. The failure pattern indicated a logarithmic spiral near the bottom of the penetrometer tip and a linear failure of soil surface beyond the tip. Gill also used an equation for the resistance of the soil to its deformation caused by a cone shaped penetrometer. The soil resistance was found to be a function of soil-metal friction and diameter and cone angle of the penetrometer.

Turnage and Freitag (1969) performed experiments with vertical and horizontal penetrometers. Their data on fine grained soils showed that the penetration resistance of soil was proportional to velocity of penetrometer and inversely proportional to its cone diameter. An exponential equation
based on velocity-diameter ratio was developed to describe the interrelation. The exponent was found to change slightly with the type of soil.

Mechanics of Tillage Equipment

Soehne (1956) used the principles of soil dynamics to develop an equation for the force required to pull a simple inclined tool through the soil. The tool considered was a rectangular blade sharpened on the underside of the leading edge. It was positioned so that it lifted the soil in a manner similar to a plow share. He analyzed the action of such a tool and concluded that four simple behavior equations described the tillage action: soil-metal friction, shear failure, acceleration force, and cutting resistance. In his analysis, he disregarded the adhesion of soil on metal. From the analysis of forces, he developed an equation for the draft of tillage equipment. The equation contained soil parameters, geometry of tool, and angle of inclination and velocity of tool. He performed experiments in a soil bin to verify his equation for the draft of an inclined tool moving in a loam soil. The theoretical value of draft was considerably higher than the measured value. He, however, did not perform experiments over a wide range of speed, inclination, and depth of cutting tools to verify his equation.

Kawamura (1952) developed the mechanics of inclined tools using an approach different from Soehne, but, unfortunately, his work was not translated into English until after Soehne's work. He measured the draft force of an inclined tool operated at various depths and angles of inclination. He used the general procedures of plastic equilibrium in developing the mechanics for computing the draft force of plows. For a limited range of
angles of inclination of tools, his measured and computed values of draft did not vary much from each other. More data, however, was required to verify his procedure. Further, in his analysis, he did not take into account the effect of soil-metal friction and acceleration forces.

Payne (1956) applied the principles of soil mechanics for predicting the draft force and volume of soil disturbed by blades. He worked with vertical rectangular tines and was able to predict with some success the forces acting on the blades from measurements of soil shearing resistance and bulk volume weight of soil. His investigations showed that draft was most sensitive to a change in cohesion of soil and that soil-metal friction and angle of internal friction of soil are almost constant throughout a range of soil conditions under which machine operations are normally performed. He confined the speed of operation of the tools to less than one mile per hour with the objective of neglecting the effect of such a speed on draft and the effect of acceleration forces of soil on draft. It may, however, be noted that while Soehne did not include in his analysis the angle of the failure plane of soil, Payne did consider such an angle. A considerable discrepancy was, however, observed between the computed and measured values of draft.

Bockhop (1957) demonstrated, for the first time, the use of the principles of similitude in predicting the draft force for a disk plow operating under different soil conditions. He used three concave disks and four different types of soils for tillage experiments in model soil bins. He concluded that the principles of similitude could be effectively utilized in tillage investigations for determining the influence of soil and implement variables on tillage implements and that the forces acting on a disk
operated in the field could be predicted. He emphasized, however, that more work was required in order to develop a precise prediction equation for the resultant forces acting upon a disk tool. His tests indicated that the prediction equation became less precise as moisture content and clay percentage in soil increased.

Rowe (1959) modified Soehne's equation to include the adhesion of soil to metal parameter. He investigated the effect of speed on the draft of an inclined blade and determined that soil or blade variables were not affected by blade velocity. The draft force was found to increase with the speed of the blade.

McLeod (1959) modified and improved Bockhop's procedure of predicting draft force on a disk plow. He used Coulomb's equation in soil mechanics while evaluating soil properties. Further, he recognized distortion of the model system and calculated a prediction factor. He concluded that the model system developed for the disk plow would give reasonably precise and reliable prediction of force components. He added that the techniques of measuring soil variables should be developed further.

Siemens and his associates (1964) developed an equation for the tools inclined less than 70 degrees to the horizontal. They assumed that the failure surface of the soil was a plane making an angle of $45^\circ - \phi/2$ degrees with the surface of soil, where $\phi$ is angle of internal friction of soil. They concluded that the predicted tool forces were higher than the measured tool forces.

Larson (1964) used a model system for moldboard plows whose shapes were generated from the logarithmic spiral. Soil strength parameters, as described by Coulomb's equation, were obtained by means of a shear cylinder. The functional relationship developed in order to predict the draft forces
on plows was found to be valid for model plows of a given configuration operating in soils of different strength values.

Reece (1965) presented a generalized theory which could be applied to problems involving soil-implement mechanics. In his theory, he made use of dimensionless groups of parameters involving soil properties, soil-metal friction, and blade geometry for determining the draft force of wide blades.

Gupta and Pandya (1967) studied soil cutting tools under dynamic loading and concluded that the prediction of draft of tool required information on soil variables, tool geometry, and variables due to operating conditions. They studied the problem from the standpoint of rheology. The soil variables were specified by velocity of propagation of compression and shear wave in soil, coefficient of soil-metal friction, adhesive force, and yield stress of soil in compression and shear. The operating variables were denoted by the velocity of tool and its depth of operation. With the help of these variables, equations were derived for predicting the draft force on soil cutting tools.

Reaves et al. (1968) developed an equation to predict forces exerted by soil on vertical chisels. Their studies indicated that the dimensionless parameter for draft varied linearly with the second power of depth-width parameter. The resistance to penetration determined with a cone penetrometer was described as a better criterion for soil strength than determining cohesion for shear strength of soil with the help of an annulus ring torsional device.

Schafer et al. (1968) determined the effect of soil variables on the draft force of concave disks with the help of model studies. They concluded that the predicted value of the dimensionless term for draft tended
to be larger in Decatur soil and smaller in Norfolk soil than the measured value of draft as the dimensionless speed increased. The predicted and measured values, though not in exact agreement, did, however, show a great deal of promise.

Lantin (1968) developed a prediction equation for draft of a moldboard plow working in a submerged clay soil. The pertinent variables considered were soil properties, tool geometry, and operational variables. His studies indicated that the soil property defined by stress was more important than the gravitational force in predicting the value of draft.

Reaves et al. (1969) evaluated the performance of bulldozer blades with one geometrical shape. They concluded that the draft force on a blade increased exponentially with the distance of loading till it reached the maximum value. Beyond this stage, the draft force did not increase with further blade travel.

Mechanics of Traction Equipment

Traction is the force required to move a machine over a certain medium (usually soil). A traction machine converts the rotary motion obtained from an engine into linear motion. The machine that develops the desired traction at high efficiency may not be useful if it compacts the soil or ruts it so severely that excessive erosion, mechanical impedance, or poor aeration affects greatly the subsequent growth of crops.

Bekker (1960) reports that Micklethwait (1944) used Coulomb's equation, represented graphically by Figure 1, to predict the maximum tractive effort of a tracked vehicle as follows:
Figure 1. Coulomb's diagram

\[ S = c + N \tan \phi \]
\[ S = c + N \tan \phi \]
\[ A S = A c + A N \tan \phi \]
\[ \text{or } H = A c + W \tan \phi \]

where:

- \( S \) = shear stress of soil
- \( A \) = area of tractive device in contact with soil
- \( H \) = tractive effort
- \( c \) = cohesion of soil
- \( \phi \) = angle of internal friction of soil
- \( N \) = normal stress = pressure acting on tractive device
- \( W \) = weight on tractive device = \( A N \)

It was further proposed that drawbar pull could be obtained from the following equation:

\[ \text{DBP} = H - R \]

where:

- \( \text{DBP} \) = drawbar pull
- \( R \) = rolling resistance

Bekker (1960) developed an empirical relationship between load on a rectangular plate and its sinkage in soil. His equation is expressed as follows:

\[ p = \left( \frac{k_c}{b} + k_\phi \right) z^n \]

where:

- \( p \) = pressure acting on grousered plate or tractive device
- \( k_c \) = Bekker's modulus of deformation due to cohesion of soil
- \( k_\phi \) = Bekker's modulus of deformation due to friction of soil
b = smaller dimension of plate
z = sinkage of plate
n = Bekker's exponent of deformation

Bekker further used the following equation for the work done in making a rut of length L, width b, and depth z

\[ RL = bL \int_0^z p \, dz \]

where:

L = length of plate

The value of the rolling resistance, R, was derived from this equation by substituting the value of pressure, p. The drawbar pull of the vehicle was determined from the Equation \( DBP = H - R \).

Buchele (1963) derived a set of equations for vehicular mechanics relating soil strength to vehicle performance. The equations developed provide a means of utilizing soil values for predicting vehicular performance under various soil conditions.

Pierrot (1964) used the principles of similitude to determine the variables involved for studying soil-tire relationships in predicting rolling resistance. He concluded that the variables affecting rolling resistance are tire geometry, tire stiffness, velocity, load, and density of soil. He added that the tire stiffness could be completely described by one parameter, spring rate.

Reece (1964) developed a theory of soil vehicle mechanics for evaluating cross-country vehicles. He made several suggestions for improving the basic equation given by Bekker (1960). He emphasized that slip-sinkage factor be considered while developing the mechanics of soil vehicle systems,
which Bekker did not consider in his equation. His new sinkage equation, after experimental verification, was given as follows:

$$p = k_1 c + k_2 w / 2 + (k_c c + k_\varphi w b / 2) (z / b)^n$$

where:

- \(p\) = pressure acting on plate
- \(k_1, k_2\) = Reece's constants
- \(k_c\) = Reece's component of deformation due to cohesion of soil
- \(k_\varphi\) = Reece's component of deformation due to friction of soil
- \(w\) = weight of soil per unit volume
- \(b\) = width of plate
- \(z\) = sinkage of plate
- \(n\) = exponent of deformation

Reece and Adams (1966) suggested after experimental studies that Bekker's equation be treated with caution. They described the deformation constant as a function of the shape and size of the test device and also of the mean contact pressure used.

Riedy and Reed (1966) observed that sinkage in a submerged sandy soil increased with vehicle speed and that it was independent of track width.

Taylor, Vanden Berg, and Reed (1967) conducted experiments to determine the effect of diameter on the traction performance of powered wheel tractors. Their findings are summed up as follows:

1. At the same normal load and inflation pressure, increased diameter gave increased pull or tractive effort for pneumatic tires.
2. Steel wheels showed a definite increase in pull with increased diameter at the same normal load.
3. An increase in inflation pressure for constant normal load and diameter gave a decrease in pull.

4. Increasing the normal load on a tire or steel wheel increased the pull.

Konaka (1967) studied the slip-sinkage phenomenon in a clay soil using unpowered rectangular groused plates. The following conclusions were drawn from his study:

1. Slip-sinkage increased as vertical pressure increased.
2. Slip-sinkage increased as the width of plate decreased and the grouser height increased.
3. Slip-sinkage decreased with the inclination of plate.
4. The draft increased as the slip rate increased.

Ikeda and Persson (1968) studied the performances of seven different types of model track shoes. They observed large sinkage of triangular shoe models and concluded that the Bekker's Equation was not applicable to their data.

Clark and Liljedahl (1969) tested small garden tractor size tires in an artificial traction medium to determine the tractive performance of single, dual, and tandem wheel arrangements under various loading and soil conditions. The results of these tests were as follows:

1. Dual tires performed better than single tires having equal vertical wheel loads in the loose soil.
2. Dual tires provided an advantage in the firmer soils when the inflation pressure was reduced from 12 to six pounds per square inch.
3. Tandem tires were superior to equal size single tires in their tractive performance.

4. Tandem tires did not provide a consistent performance advantage over the low pressure dual tires.

5. Dual and tandem tires reduced wheel sinkage considerably for all soil conditions tested.

Soil Resistance and Emergence of Crops

Stout (1959) studied the effect of soil compaction on sugar beet seedling emergence. His data indicated that emergence decreased with the increase of bulk density.

Phillips (1959) performed laboratory experiments to study the growth of corn seedlings. His conclusions were as follows:


2. Needle penetrability is a more sensitive indicator of mechanical impedance of soil than is bulk density.

3. Needle penetration is a more sensitive index to final yields than is bulk density.

Bowen (1960) studied the effect of different factors on the physical impedance of soil. The following conclusions were drawn from his study:

1. Physical impedance generally increased as the soil dried.

2. Physical impedance increased with surface compaction.

Shinaishin (1960) studied the relationship between the resistance of soil determined by a penetrometer and the emergence of actual plant seed-
lings. He found that there was an indirect relation between the emergence of seedlings and the penetration resistance of soil.

Garner and Bowen (1963) determined that seedlings growing through uncompacted soil emerged more rapidly than seedlings growing through compacted soils. They found that there was no apparent difference in the rate of seedling emergence for the three levels of compaction of one, three, and five pounds per square inch.

Wittsell and Hobbs (1965) studied the effect of severe artificial soil compaction on the growth of common dry land crops. Their studies indicated that subsurface compaction reduced wheat yields more than surface compaction did. They found, however, that surface compaction reduced sorghum growth more than did subsurface compaction.

Wanjura et al. (1966) found a negative correlation between penetrometer readings and cotton seedling emergence. They stated that the penetrometer reading gave a precise index of soil strength.

Taylor et al. (1966) have proposed a general relationship between observed grass seedling emergence and penetrometer readings. Their data indicated that emergence was affected very little by penetration resistance up to 100 pounds per square inch. Above that value, emergence dropped off rapidly until no emergence occurred at 225 pounds per square inch. The general relationship was developed from the study of eight grasses, seven soil types, various temperature and moisture conditions, and different planting depths.

Bowen (1966) studied 18 distinct environmental situations affecting cotton seed germination and emergence. His pertinent conclusions follow:
1. Histories of only four environmental variables were needed to predict the degree of emergence resulting from various combinations of surface compaction and soil moisture content at planting time.

2. An unique physical environment resulted from each combination of soil moisture content, surface compaction at planting time, and weather pattern following planting.

3. A wide range in physical environmental histories could be predicted simply by varying moisture content and surface compaction at the time of planting.

4. The effects of planting operations on the physical environment were clearly evident throughout the period of emergence.

5. A stress of any kind increased the time for the emergence of seedlings.

Kollman (1968) performed laboratory experiments to investigate the effects of soil compaction, soil moisture, temperature, planting depth, and planting density. The following conclusions were drawn from his study:

1. An indirect relationship existed between emergence and physical resistance of soil as measured by a soil penetrometer.

2. The soil penetrometer reading was directly correlated with bulk density and indirectly with moisture level.

3. Seedling emergence was correlated indirectly with bulk density and directly with moisture.

4. Percentage emergence was directly related to planting density.

5. Emergence from high resistance soil decreased rapidly as planting depths increased.
Carter and Tavernetti (1968) studied the influence of precision tillage and soil compaction on cotton yields. They found that increases in cotton yield with precision tillage were proportionate to the average strength that existed before the precision tillage. Considering the speed and ease of obtaining measurements, they suggested the use of a penetrometer rather than bulk density for the evaluation of tillage under field conditions.

Several research workers have developed techniques for measuring the forces developed by the seedlings during emergence. Williams (1956) measured the forces developed by small seeded legumes. The forces for alfalfa, crimson clover, rose clover, and subterranean clover were found to be 15.2, 23.8, 24.1, and 60.0 grams force, respectively. His data showed a high correlation between the emergence force and the weight of seed.

Drew et al. (1965) observed forces developed by corn and cotton seedlings by growing them between two layers of soil and measured the force exerted against the upper layer. The maximum force exerted by corn and cotton seedlings was 0.6 and 0.5 pound, respectively.

Edwards (1966) determined the force required for the emergence of cotton seedlings. The mean force for eight seedlings was measured as 260 grams force each. He also observed that two seedlings emerging simultaneously exerted a force up to 590 grams (295 grams per seedling) and three seedlings up to 880 grams (293 grams per seedling). He concluded that planting cotton seed in hills would be beneficial in overcoming emergence difficulties.

Miles and Matthes (1969) constructed a plexiglass container which allowed a corn seedling to grow upward, forcing a plunger against a cantilever beam. They found that a double cross hybrid corn seedling exerted
a maximum force of 1.04 pounds after 130 hours of its placement in the seedling holder. It maintained such a force for 50 hours after which a slight decrease was observed.

Mayeux (1970) determined that the average force exerted by sugar cane sprouts while germinating in a clay loam soil compacted under three pounds per square inch was 1.75 pounds or 14 pounds per square inch.
DESCRIPTION OF EQUIPMENT

The equipment used by the author will be described in this chapter.

Shear Box

Values of angle of internal friction and cohesion of soil were determined by the shear box employing a hollow cylinder (Figure 2), used by Schafer (1961) and Larson (1964). The cylinder, with vanes on its inside surface, using an SR-4 strain gage bridge was pushed into the soil. The soil around the outside of the cylinder was carefully removed to a depth of 1/8-inch below the lower edge of the cylinder. This left a short circular column of soil below the shear cylinder. Weights were placed on a pan attached to the top of the center rod. The amount of weight added determined the level of normal stress on the shear plane. The soil column in the grip of the cylinder was allowed to fail by shear from torque through the handle. The maximum torque $M$ developed before failure of the soil was recorded by the Offner dynograph. From this maximum torque, the maximum shearing stress ($S$) of soil was calculated by using the following formula developed by Hvorslev (1939):

$$S = \frac{3M}{2\pi r^3}$$

where $r =$ radius of soil sample

Normal stress was calculated by dividing the weight applied on the rod by the inner cross-sectional area of the shear cylinder. A series of shear tests with increasing normal loads on the soil column were carried out. The values of the normal and shear stresses were then plotted against each
Figure 2. Shear box

Figure 3. Offner dynograph
other. The linear regression analysis of the shear and normal stresses gave the values of angle of internal friction and cohesion of soil.

In a similar fashion, coefficient of soil to metal friction was measured with the help of a flat circular steel plate attached to the shear box, as demonstrated by Rowe (1959). The circular plate was pressed into the soil under a vertical pressure. Frictional stress was obtained by using the same form of equation as for the shearing stress. Linear regression analysis of the frictional and normal stresses provided the numerical values for adhesion and coefficient of soil-to-metal friction corresponding to the regression coefficients.

Offner Dynograph

This instrument (Figure 3), manufactured by Beckman Instrument Inc., Chicago, was used for amplifying and recording the torque or force on an eight channel oscillograph. The instrument consists of preamplifiers, power amplifiers, writer element, and a regulated power supply unit. A strain gauge bridge on the load cell, as mentioned for the shear box, was connected to dynograph by means of an input coupler. The torque required to shear the soil could be measured with the help of the instrument at a desired amplifier sensitivity. The instrument was operated according to the instructions contained in the instruction manual provided by the manufacturer.

Hydraulic Press

For packing the soil, a hydraulic press (Figure 4) was used. The pressure on the soil was applied by means of a circular plate which was placed on the surface of soil contained in a soil box. The size of the
Figure 4. Hydraulic press

Figure 5. Electric sprayer
plate was such that it fitted closely the inside surface of the plastic box. The compaction pressure could be raised gradually by means of the manual operation of the press for obtaining the desired density of soil.

Electric Sprayer

For increasing the moisture content of soil, an electric sprayer (Figure 5) was used. With the help of this sprayer, water was evenly distributed on the surface of the soil. The fine mist of spray gave a fairly homogenous mixture of soil and water. Any clods formed during the process were broken down into finer material by rubbing with hands. The desired amount of water was added to the tank of the sprayer, and its discharge rate was controlled by the regulator on the sprayer. An additional allowable quantity of water was added to the tank before the operation of the sprayer so as to account for the loss of moisture due to evaporation during the misting process.

Penetration Equipment

This equipment (Figure 6) used by Johnson (1969) was modified to work as a penetrating device, in addition to its use as shear strength testing equipment. The penetrating tool attached to the device could be moved vertically up or down manually or with the help of an electric motor and gearing arrangement. The penetrating tool was made conical in shape at its tip in order to minimize adhesion when penetrated through the soil. The tool, usually called a penetrometer, was attached to the bottom of a proving ring employing a four strain gage bridge circuit for sensing the force exerted by the soil.
Figure 6. Penetrating equipment
MECHANICS OF EQUIPMENT

The mechanics for the following types of soil cutting equipment will be presented in this chapter:

1. Penetration equipment
2. Tillage equipment
3. Traction equipment

Penetration Equipment

Figure 7a represents a round shaped steel penetrometer actuated by a vertical force $F$, for entering the soil surface.

Skin friction

Consider a small slice of the cylindrical body of the penetrometer situated at a depth $h$ from the soil surface. It is laterally subjected to the earth pressure (passive) as shown in Figure 7b. A small segment of the slice subtending an angle $d\theta$ at the center of the penetrometer and having a circular arc $r \, d\theta$ is shown in the Figure 7b.

Frictional force between the segment of the slice and the soil is given by

$$dF = \mu \, p \, r \, d\theta \, dh$$

where:

- $dF$ = frictional force between the segment of slice and soil
- $\mu$ = coefficient of friction between soil and metal surface
- $r$ = radius of penetrometer
- $p$ = passive earth pressure
  $$= 2 \, c \, \tan (45 + \phi/2) + w \, h \, \tan^2 (45^\circ + \phi/2)$$
Figure 7a. Penetrometer moving vertically in soil

Figure 7b. Free body diagram
[from Lambe and Whitman (1969)]

c = cohesion of soil
r d\theta dh = surface area of the segment of slice
w = unit weight of soil

Total frictional force between the penetrometer and soil is given as follows:

Total frictional force between penetrometer and soil = \mu r \int_0^L \int_0^{2\pi} p \, dh \, d\theta

= 2 \pi \mu r \int_0^L p \, dh

= 2 \pi \mu r \int_0^L [2 c \tan (45^\circ + \phi/2) + w h \tan^2 (45^\circ + \phi/2)] \, dh

= \mu \pi D [2 c L \tan (45^\circ + \phi/2) + w L^2 \tan^2 (45^\circ + \phi/2)]

where:

D = diameter of penetrometer

**Point resistance**

A number of semi-empirical equations have been presented to predict the end bearing capacity (point resistance) of pile, the resistance of metals or soils to punching shear, and similar problems. These, for most part, derive from the analyses of Prandtl modified by Terzaghi and Taylor, as reported by Jumikis (1962). An equation by Terzaghi and Peck (1967) in current widespread use for circular end bearing pile is

\[ q = 1.2 c N_c + w L N_q + 0.3 w D N_w \]  \hspace{1cm} (2)

Since \[ F_t = q A = \frac{\pi}{4} q D^2 \]

\[ F_t = 0.94 c N_c D^2 + 0.79 w L D^2 N_q + 0.24 w D^3 N_w \]  \hspace{1cm} (3)
where:

\[ F_t = \text{end bearing load or point (tip) resistance} \]

\[ A = \text{bearing area} \]

\[ D = \text{diameter of pile or a footing} \]

\[ q = \text{end bearing capacity of pile} \]

\[ c = \text{cohesion of soil} \]

\[ w = \text{unit weight of soil} \]

\[ L = \text{depth of pile} \]

\[ N_c = \text{bearing capacity factor with respect to cohesion of soil} \]

\[ N_q = \text{bearing capacity factor with respect to surcharge} \]

\[ N_w = \text{bearing capacity factor with respect to weight of soil} \]

The coefficients \( N_c, N_q, \) and \( N_w \) are evaluated from the presumed geometry of the failure surface. These derivations assume that the soil is ideally plastic, i.e., noncompressible, and that failure is simultaneous throughout the shearing soil. For progressive failures, termed "local shear" by Terzaghi and Peck, arbitrary lower \( N \)-values are used.

A somewhat different approach is used by Housei, as reported by Spangler (1960), for the analysis of surface plate bearing capacity data and does not presume an incompressible soil. Housei considers the resistance to penetration of a plate as the sum of two components, one an area component proportional to the plate diameter squared and the other a perimeter shear component, proportional to the diameter:

\[ F = A n + P m \]

\[ F = \frac{A}{4} D^2 n + \pi D m \]  \hspace{1cm} (4)

where:
F = penetration resistance of plate (lb)
A = area of plate (in²)
D = diameter of plate or footing (in)
P = length of perimeter (in)
n = compressive stress on soil column directly beneath the plate (psi)
m = perimeter shear (lb/linear in)

Tillage Equipment

A tillage tool such as a plow shown in Figure 8a, when operated and fixed in soil resembles a retaining wall. The theory of retaining walls was, therefore, used in order to develop an equation for the draft of a tillage tool. Before developing the equation, the inertia force associated with the tillage tool was first evaluated.

Inertia force

In order to derive the equation for the inertia force (Q) associated with the inclined plane tillage tool, theorems of conservation of mass and momentum applied to matter within a control volume were utilized.

As the tillage tool moves forward with velocity \( V \), the soil mass undergoes a continuous movement attaining a velocity \( V_e \) at the tool interface as shown by Figure 8a. Consider a control volume of soil bounded by regions 1 and 4, as represented by Figure 8b.

According to the law of conservation of mass, for a control volume

\[
\frac{\partial}{\partial t} \int_{c.v.} dm + \int_{c.s.} \rho \vec{V}_r \cdot d\vec{A} = 0
\]

where:
Figure 8a. Soil reacting to tillage tool

Figure 8b. Control surface of soil bounded by regions 1 and 4
C.V. designates the control volume and c.s. the control surface

dm = elemental mass of soil inside the control volume

\( \rho \) = mass density of soil at a point inside the control volume

\( \vec{V}_r \) = velocity of matter at a point on the control surface relative to the control surface

\( \vec{dA} \) = element of area of control surface = vector \( \hat{n}dA \), where \( \hat{n} \) is the outwardly drawn unit vector

Since we have steady flow of soil, \( \int dm \) is constant and \( \frac{\partial}{\partial t} \int \frac{dm}{c.v.} \) is zero, giving

\[
\int \rho \vec{V}_r \cdot \vec{dA} = 0
\]

c.s.

For constant density of continuum of matter,

\[
\int \vec{V}_r \cdot \vec{dA} = 0
\]

c.s.

Considering a control surface in which area 1 is entrance and area 2 is exit,

\[
\int \vec{V}_r \cdot \vec{dA} = \int_{1} \vec{V}_r \cdot \vec{dA} + \int_{2} \vec{V}_r \cdot \vec{dA} = V(-DL) + V \sin \theta A_2 = 0
\]

\[ A_2 = \frac{DL}{\sin \theta} \]

where:

\( D \) = width of tool, \( L \) = depth of soil disturbed

Considering a control surface in which area 2 is entrance and area 2' is exit

\[
\int \vec{V}_r \cdot \vec{dA} = \int_{2} \vec{V}_r \cdot \vec{dA} + \int_{2} \vec{V}_r \cdot \vec{dA} = V \sin \theta \left( \frac{D}{\sin \phi} \right) + V_e \sin (\delta + \phi) \hat{A}_2 = 0
\]

\[ \hat{A}_2 = \frac{V D L}{V_e \sin (\delta + \phi)} \]
Since $A_2 = \dot{A}_2$

\[
\frac{D L}{\sin \Theta} = \frac{V D L}{V_e \sin(\beta + \Theta)}
\]

\[
\frac{V}{V_e} = \frac{\sin(\beta + \Theta)}{\sin \Theta}
\]

\[
V_e = \frac{V \sin \Theta}{\sin(\beta + \Theta)}
\]

Considering a control surface in which area 2 is entrance and area 3 is exit

\[
\int \overrightarrow{V_r} \cdot dA = \int_{2} \overrightarrow{V_r} \cdot dA + \int_{3} \overrightarrow{V_r} \cdot dA = V_e \sin(\beta + \Theta) \left[ \frac{-V D L}{V_e \sin(\beta + \Theta)} \right] + V_e A_3 = 0
\]

\[
A_3 = \frac{V D L}{V_e}
\]

Considering a control surface in which area 3 is entrance and area 4 is exit

\[
\int \overrightarrow{V_r} \cdot dA = \int_{3} \overrightarrow{V_r} \cdot dA + \int_{4} \overrightarrow{V_r} \cdot dA = V_e \left[ \frac{-V D L}{V_e} \right] + V_e \cos \beta A_4 = 0
\]

\[
A_4 = \frac{V D L}{V_e \cos \beta}
\]

From the theorem of momentum, the net force $\Sigma F$ on matter within control volume is given by Newton's law as

\[
\Sigma F = \frac{\partial}{\partial t} \int_{c.v.} \overrightarrow{V} dm + \int_{c.s.} \overrightarrow{V} \rho \overrightarrow{V_r} \cdot dA
\]

For steady state condition, $\frac{\partial}{\partial t} \int_{c.v.} \overrightarrow{V} dm = 0$

\[
\Sigma F = \int_{c.s.} \overrightarrow{V} \rho \overrightarrow{V_r} \cdot dA
\]

The Equation for $\Sigma F$ can be expressed by its following two components:
\[ \Sigma F_x = \int_{\text{c.s.}} V_x \rho \vec{V}_r \cdot dA \]
\[ \Sigma F_y = \int_{\text{c.s.}} V_y \rho \vec{V}_r \cdot dA \]

Considering control surface in which area 2 is entrance and area 4 is exit:

\[ \Sigma F_x = \int_{\text{c.s.}} V_x \rho \vec{V}_r \cdot dA = \int_{2} V_x \rho \vec{V}_r \cdot dA + \int_{4} V_x \rho \vec{V}_r \cdot dA \]

\[ = -V_e \rho \sin(\theta)\frac{DL}{\sin(\theta)} + (-V_e \cos(\theta)) \rho (V_e \cos(\theta)) \frac{V \cdot D \cdot L}{V_e \cos(\theta)} \]

\[ = \rho DL \sqrt{V_e}^2 - (V_e \cos(\theta)) \rho DL \]

\[ = \rho DL \sqrt{V_e}^2 - V \left[ \frac{V \sin(\theta)}{\sin(\beta + \theta)} \right] \cos(\beta) \rho (DL) \]

\[ = \rho DL \sqrt{V_e}^2 - \rho DL \frac{V^2 \sin(\theta)}{\sin(\beta + \theta)} \cos(\beta) \rho (DL) \]

\[ = \rho DL \sqrt{V_e}^2 \left[ \frac{1 - \sin(\theta) \cos(\beta)}{\sin(\beta + \theta)} \right] \]

\[ = \rho DL \sqrt{V_e}^2 \frac{\sin(\theta) \cos(\beta)}{\sin(\beta + \theta)} \]

\[ \Sigma F_y = \int_{\text{c.s.}} V_y \rho \vec{V}_r \cdot dA = \int_{2} V_y \rho \vec{V}_r \cdot dA + \int_{4} V_y \rho \vec{V}_r \cdot dA \]

\[ = 0 + (V_e \sin(\beta)) \rho (V_e \cos(\theta)) \frac{V \cdot D \cdot L}{V_e \cos(\theta)} \]

\[ = \rho DL \sqrt{V_e}^2 V_e \sin(\beta) = \rho DL \sqrt{V_e}^2 \frac{V \sin(\beta) \sin(\theta)}{\sin(\beta + \theta)} \]

\[ = \rho DL \sqrt{V_e}^2 \frac{\sin(\beta) \sin(\theta)}{\sin(\beta + \theta)} \]

\[ |\vec{F}| = (\Sigma F_x^2 + \Sigma F_y^2)^{1/2} \]

\[ = \rho DL \sqrt{V_e}^2 \left[ \frac{\sin^2(\beta) \cos^2(\theta)}{\sin^2(\beta + \theta)} + \frac{\sin^2(\theta) \sin^2(\beta)}{\sin^2(\beta + \theta)} \right]^{1/2} \]
\[
\frac{DLV^2 \sin \alpha}{\sin(\beta + \theta)} \left[ \cos^2 \theta + \sin^2 \theta \right]^{1/2} \\
= \frac{DLV^2 \sin \alpha}{\sin(\beta + \theta)} \\
= \frac{w \ DLV^2 \sin \alpha}{g \ \sin(\beta + \theta)}
\]

where:

\( w \) = unit weight of soil

\( g \) = acceleration of gravity

The inertia force \( Q \) is, therefore, given by

\[
Q = |\vec{\Sigma F}| = \frac{w \ DLV^2 \sin \alpha}{\sin(\beta + \theta)} \quad (5)
\]

The angle made by \( Q \) with the x-axis = \( \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) \)

\( = \tan^{-1}(\tan \theta) = \theta \)

The \( x \) and \( y \) components of \( Q \) are given as follows:

\[
Q_x = Q \cos \theta \\
Q_y = Q \sin \theta
\]

Draft

The equation for draft will be derived by considering a free body diagram (Figure 9b) for the forces acting on soil mass in front of an inclined tool shown in Figure 9a. Angle \( \theta \) equal to 45° - \( \phi/2 \) shown in Figure 9b gives the orientation of the shear surface resulting from the passive failure of soil.

According to Newton's Law, we have

\[
\Sigma F_y = \frac{\delta t}{\text{c.v.}} \int V \, dm + \int_{\text{c.s.}} V \, \rho \, \vec{V_r} \cdot \hat{cA}
\]
Figure 9a. An inclined tool

Figure 9b. A free body diagram for the forces acting on soil mass in front of an inclined tool

\[ \theta = 45 - \phi/2 \]
or \( W + P + F + (cA)_y = Q_y \)
- \( W - cA \sin \theta + P \cos(45^\circ + \phi/2) + P \cos(\theta + \delta) = Q \sin \theta \)
- \( W - (cD L \csc \theta) \sin \theta + P \cos(45^\circ + \phi/2) + P \cos(\theta + \delta) = Q \sin \theta \)
\( W + cD L - F \cos(45^\circ + \phi/2) - P \cos(\theta + \delta) \)
\( + Q \sin(45^\circ - \phi/2) = 0 \) \( \quad (6) \)

where:

\( W = \) weight of soil mass shown in Figure 8b
\( c = \) cohesion of soil
\( A = \) area of shear surface \( b_e = D L \csc \theta \)
\( D = \) transverse width of tool
\( L = \) depth of soil disturbed
\( Q = \) inertia force given by Equation 2
\( \phi = \) angle of internal friction of soil
\( \theta = \) angle of shear surface \( = 45^\circ - \phi/2 \)
\( \beta = \) angle of inclination of tool
\( \delta = \) angle of soil-metal friction

\[ \sum F_x = \frac{\partial}{\partial t} \int_{c.v.} V_x \, dm + \int_{c.s.} V_x \vec{V}_x \, dA \]

or \( \dot{W}_x + P_x + F_x + (cA)_x = Q_x \)

\( P \sin(\beta + \delta) - F \sin(45^\circ + \phi/2) - cA \cos \theta = Q \cos \theta \)
\( P \sin(\beta + \delta) - F \sin(45^\circ + \phi/2) - cD L \csc \theta \cos \theta = Q \cos \theta \)
\( P \sin(\beta + \delta) - F \sin(45^\circ + \phi/2) - cD L \cot(45^\circ - \phi/2) \)
\( - Q \cos(45^\circ - \phi/2) = 0 \) \( \quad (7) \)

From Equation 6, \( F = \frac{W \cos \theta - P \cos(45^\circ + \phi/2) - Q \sin(45^\circ - \phi/2)}{\cos(45^\circ + \phi/2)} \)
Substituting the value of $F$ in Equation 7,

$$P \sin(\beta + \delta) - [W + c D L - P \cos(\beta + \delta) + Q \sin(45^\circ - \phi/2)]$$

$$\left[\tan(45^\circ + \phi/2) - Q \cos(45^\circ - \phi/2) - c D L \cot(45^\circ - \phi/2)\right] = 0$$

$$P \left[\sin(\beta + \delta) + \cos(\beta + \delta) \tan(45^\circ + \phi/2)\right] = c D L \left[\tan(45^\circ + \phi/2) + \cot(45^\circ - \phi/2)\right] + Q \sin(45^\circ - \phi/2) \tan(45^\circ + \phi/2)$$

$$+ Q \cos(45^\circ - \phi/2) + W \tan(45^\circ + \phi/2)$$

where $Q$ is given by Equation 2.

The value of $W$ is determined from the weight of soil mass enclosed by area $abe$ having a transverse width $D$.

$$W = w D (\text{area of triangle abf} + \text{area of triangle bfe})$$

$$= w D \left[\frac{1}{2} L L \cot \beta + \frac{1}{2} L L \cot(45^\circ - \phi/2)\right]$$

$$W = \frac{w D L^2}{2} \left[\cot \beta + \cot(45^\circ - \phi/2)\right]$$

where:

- $w = \text{weight of soil per unit volume}$
- $D = \text{width of tool}$
- $L = \text{depth of tool}$

Draft = horizontal component of $P = P \sin(\beta + \delta)$

where $P$ is given by Equation 8.

**Traction Equipment**

A tractive device operating under pressure, such as a vertical groused plate (Figure 10a) used in a tracked vehicle, when fully rutted
Figure 10a. A vertical groused plate

Figure 10b. A free body diagram for the forces acting on soil mass in front of a vertical groused
in soil resembles a vertical retaining wall carrying surcharge. The method outlined by Lambe and Whitman (1969) will be used in order to develop an equation for the tractive effort of a tractive device.

A free body diagram for the forces acting on soil mass in front of a vertical grouser is shown in Figure 10b. The diagram is identical to that of Lambe and Whitman (1969), except for the addition of force \( W_1 \) due to surcharge. The value of \( W_1 \) is equal to \( p_1 A_1 / n_1 \), where \( p_1 \) is pressure acting on the plate with surface area, \( A_1 \), and number of grousers, \( n_1 \).

For equilibrium, sum up the forces in the vertical and horizontal directions.

\[ \Sigma F_y = 0 \]
\[ W_1 + W + C \sin \theta + T \sin \theta - N \cos \theta = 0 \]  \hspace{1cm} (11)

where:

\[ W = \frac{1}{2} wL^2 \cot \theta, \quad C = cL \csc \theta, \quad T = N \tan \phi \]

Substituting the values of \( W, C, \) and \( T \) in Equation 11,

\[ W_1 + \frac{1}{2} wL^2 \cot \theta + cL - N (\cos \theta - \sin \theta \tan \phi) = 0 \]

\[ N = \frac{\frac{1}{2} wL^2 \cot \theta + cL + W_1}{\cos \theta - \sin \theta \tan \phi} \]  \hspace{1cm} (12)

\[ \Sigma F_x = 0 \]

\[ P - N \sin \theta - T \cos \theta - C \cos \theta = 0 \]

\[ P - N \sin \theta - N \tan \phi \cos \theta - cL \csc \theta \cos \theta = 0 \]

\[ P = N (\sin \theta + \cos \theta \tan \phi) + cL \cot \theta \]  \hspace{1cm} (13)

Substituting the value of \( N \) from Equation 12,

\[ P = \left( \frac{\frac{1}{2} wL^2 \cot \theta + cL + W_1}{\cos \theta - \sin \theta \tan \phi} \right) (\sin \theta + \cos \theta \tan \phi) + cL \cot \theta \]  \hspace{1cm} (14)
where:

\[ w = \text{specific weight of soil} \]

\[ D = \text{width of plate} \]

\[ L = \text{sinkage of plate} \]

\[ W_1 = \text{weight of plate with single grouser} \]

\[ \theta = 45^\circ - \phi/2 \text{ degrees} \]

\[ \beta = 90 \text{ degrees} \]

\[ g = \text{acceleration of gravity} \]

The tractive effort for a plate with multiple grousers is obtained by multiplying Equation 14 by the number of grousers.
COMPUTER PROGRAMS

In this study, various computer programs were used for analyzing the experimental data. The description of these programs is given as follows:

1. Subroutine ME0226

This computer program was written by Mischke (1972) to perform computations with respect to the linear regression, \( y = a + b \times \). The routine accepts the monitor print signal, number of data pairs, column vector of \( x \) and \( y \), and values of \( t \) and \( F \) (statistic). The following pertinent quantities returned by the subroutine were used for interpreting the results of the data:

- i) intercept and slope of the regression line
- ii) standard deviation of intercept and slope
- iii) confidence limits on intercept and slope
- iv) confidence bounds of the regression line
- v) correlation coefficient
- vi) standard deviation of \( y \) on \( x \)

The complete program is listed in Appendix XVI.

2. Subroutine ME0227

This subroutine was written by Mischke (1972) to perform a linear least squares regression of \( y \) on \( x \) wherein the regression line contains the origin. The following quantities computed by the routine were of interest in this study:

- i) slope of the regression line
- ii) standard deviation of slope
- iii) confidence limits on slope
iv) confidence bounds on the regression line
v) standard deviation of \( y \) on \( x \)
vii) correlation coefficient

The complete program is listed in Appendix XVII.

3. Subroutine GOLD

This subroutine was written by Mischke (1967) to perform the Golden Section Search. The routine discovers the optimum (extreme) ordinate and the corresponding abscissa in case of a unimodal function. The complete subroutine is listed in Appendix XVIII.

4. Helarctos II

This computer program was written by Kennedy (1971) to perform multiple linear regression computations. The following quantities computed by the program were of interest in this study:

i) regression coefficients
ii) standard error of regression coefficients
iii) values of \( t \)-statistic
iv) standard error of the estimated value
v) correlation coefficient

5. Linear regression, \( y = a + b \, x \)

This program was written by the author to perform the linear regression analysis. The following quantities were computed from the program:

i) coefficient of variation for the variables
ii) intercept and slope of the regression line
iii) standard deviation of intercept and slope
iv) values of t-statistic

v) upper and lower confidence bounds (confidence belt) on
the regression line at three different levels of significance (90, 95, and 99%)

vi) standard deviation of y on x

The complete program is listed in Appendix XIX.

6. Linear regression, \( y = b \cdot x \)

This program was written by the author to perform linear regression computations wherein the regression line contains the origin.

The following quantities were computed from the program:

i) coefficient of variation for the variables

ii) slope of the regression line

iii) standard deviation of slope

iv) t-statistic

v) upper and lower confidence bounds (confidence belt) on
the regression line at three different levels of confidence (90, 95, and 99%)

The complete program is listed in Appendix XX.
MEASUREMENT AND PREDICTION OF PENETRATION RESISTANCE

In order to test the mechanics of the penetration equipment presented earlier, it was necessary to measure the penetration resistance of soil by a penetrometer under different environmental conditions for comparing it with the value predicted from the mechanics.

Description of Soil

The soil used for determining the resistance of soil was Colo clay loam and had the following mechanical composition by weight, as given by Larson (1964):

<table>
<thead>
<tr>
<th>Component</th>
<th>Percentage</th>
<th>Size Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand</td>
<td>25.9%</td>
<td>(0.050 to 2.0 mm)</td>
</tr>
<tr>
<td>Silt</td>
<td>41.3%</td>
<td>(0.002 to 0.050 mm)</td>
</tr>
<tr>
<td>Clay</td>
<td>32.8%</td>
<td>(0.002 mm)</td>
</tr>
</tbody>
</table>

The same soil was used in experiments on seedling emergence, as discussed in a later chapter.

Measurement of Soil Resistance

Air dried soil passed through a nine mesh (0.078-inch opening) sieve was weighed, and the necessary amount of moisture based on the dry weight of soil was mixed with it by means of an electric sprayer. After the soil was thoroughly mixed, it was placed in a container and covered to avoid loss of moisture by evaporation. The soil was then transferred to cylindrical plastic boxes (3 11/16 inch diameter and 5 1/2 deep) and compacted to a height of two inches by means of the hydraulic press.

In the nondrying phase of the experiment, lids were placed on the boxes, sealed with masking tape to avoid loss of moisture by evaporation,
and placed in a controlled temperature room at 20°C. At the time of the measurement of soil resistance, lids were removed from the boxes, one at a time, and a record of penetration resistance (force) with the change of depth of nondried soil was obtained on the dynograph, as represented by Figure 10a. It will be observed from the figure that as the penetrometer entered the surface of the soil, the resistance (force) increased until it reached a peak value at a certain depth of soil beyond which it stayed at that value for the remaining depth of soil.

In the drying phase, the boxes were kept with lids removed in the controlled temperature room (20°C) for a desired period of time to allow gradual loss of moisture by evaporation. The penetration resistance was recorded on the second, fourth, sixth, and eighth day on different soil boxes. The resistance (force) versus depth graph traced by the Offner dynograph for the dried soil is represented by Figure 10b. This figure indicates that for the first top layers of soil, the force increased rapidly reaching a maximum value at a shallow depth of soil; beyond this, there was a rapid decline followed by a gradual decrease to a constant force. For the last bottom layers of soil, the force attained almost a constant value. The reason for this peculiar shape of the graph is advanced as follows:

The soil under the air-soil interface dries first as the drying front passes downward through the soil during the drying process. Depending on the bulk density of the soil and previous history, the soil gains strength, the strength remains the same, or the strength decreases. Consequently, the top of the soil offers a great resistance when the penetrometer passes downward. When the crust is penetrated, a sharp decline in the penetration
resistance occurs. As the penetrometer moves down further into higher moisture soil, a gradual decrease in resistance is observed. The resistance becomes almost constant as the penetrometer reaches the bottom layers of soil where little drying has occurred.

Figures 11a and 11b are drawn in the traditional method of soil dynamics with the independent variable (depth) shown along the ordinate. The graphs are redrawn in Figure 12a and 12b with force shown along the ordinate.

Measurement of Soil Parameters

The following soil parameters were experimentally determined:

1. Cohesion and angle of internal friction
2. Soil-metal friction

The shear cylinder with an internal diameter of three inches was attached to the shear box used by Schafer (1961) in order to measure the cohesion and angle of internal friction of soil. Boxes of soil at 12, 16, 20, and 24 percent moisture contents were compacted to different bulk densities. The boxes were kept covered except during the test. The soil was then subjected to torque at different normal stresses. The maximum torque \( M_1 \) at the moment of failure of each soil was recorded on the dynograph. From this maximum torque, the shear stress was computed from the formula, \( S = \frac{3}{2} \frac{M_1}{\pi r_1^3} \), where \( r_1 \) is the internal radius of the shear cylinder or the radius of the soil sample. The values of normal and shear stresses are recorded in Appendices I and II for different moistures and densities of soil. The normal stress was obtained by dividing the weight applied on the cylinder by its inner cross sectional area. The linear regression analysis
Figure 11a. Force vs depth for nondried soil (depth shown along the ordinate)

Figure 11b. Force vs depth for dried soil (depth shown along the ordinate)
Figure 12a. Force vs depth for nondried soil (force shown along the ordinate)

Figure 12b. Force vs depth for dried soil (force shown along the ordinate)
conducted for each set of values of normal and shear stresses shown in the Appendixes provided the values for cohesion and angle of internal friction corresponding to the intercepts and slopes of the regression lines. The results for 20% moisture content and two different bulk densities of soil are shown graphically in Figure 13.

Soil-metal friction was measured in a similar fashion using the procedure demonstrated by Rowe (1959). A flat steel circular plate having a diameter of 2.5 inches was attached to the shear box. The circular plate was pressed into soil under a vertical pressure. The maximum torque applied through the handle of the box was recorded on the dynograph. Maximum frictional stress ($S_f$) was calculated from the Equation $S_f = 3 M_2/2\pi r_2^3$, where $r_2$ is the radius of the plate. The values of frictional stresses for different moistures and densities are shown in Appendix III corresponding to different normal stresses. The linear regression analysis performed on each set of values for normal and frictional stresses given in Appendix III provided the value for adhesion as intercept and soil-metal friction as slope of the regression line. Three values of soil-metal friction were obtained at each soil moisture and density, as demonstrated by Figure 14 drawn for 16% moisture content and 100 lb/cft density of soil. The values of soil-metal friction for different moistures and densities have been entered in Appendix III.

Results and Discussion

The typical results, as shown by Figures 11a and 11b, indicate that the penetration resistance of soil is not a linear function of depth of penetration and that it essentially becomes constant beyond a certain
Figure 13. Determination of soil strength parameters $c$ and $\phi$, used in Coulomb's equation ($S = c + N \tan \phi$)
16% moisture, 100 lb/cft

soil-metal friction

3 slopes give

3 values of $\mu$

Figure 14. Determination of soil-metal friction at 16% moisture and 100 lb/cft
depth. This does not agree with the pattern predicted from skin friction alone, Equation 1. Nor is this predicted by Terzaghi-Peck Equation 3.

**Effect of penetrometer size**

Appendix IV shows the values of force measured with different sizes of penetrometers moving through soil boxes of two-inch depth having an initial moisture content of 12% and compacted to different densities. Nondrying conditions were maintained by keeping the boxes covered except during the test. The penetrometers used were of 0.115, 0.238, 0.364, and 0.490 inch in diameter. The analysis of the data in Appendix IV yielded the following regression equations at 95% significance level:

**70 lb/cft**

\[ F = 24.01 D^2 \]  \hspace{1cm} (15)

confidence limits on slope = 22.76, 25.26
standard error of estimate = 0.283
correlation coefficient = 0.992

**80 lb/cft**

\[ F = 76.82 D^2 \]  \hspace{1cm} (16)

confidence limits on slope = 73.46, 80.18
standard error of estimate = 0.761
correlation coefficient = 0.994

**90 lb/cft**

\[ F = b_1 D + b_2 D^2 \]
\[ = 12.64 D + 137.18 D^2 \]  \hspace{1cm} (17)

confidence limits on \( b_1 \) = 2.43, 22.85
confidence limits on \( b_2 \) = 117.16, 157.2
standard error of estimate = 0.894

correlation coefficient = 0.998

where:

F = estimated force (lb)

D = diameter of penetrometer (inch)

Figures 15 and 16 exhibit the results graphically. The force exerted by the penetrometer is found to be a function of its diameter squared or area at bulk densities of 70 and 80 lb/cuft, as expressed by Equations 15 and 16. At 90 lb/cuft, Equation 17 holds good. The confidence limits for the regression lines have been shown in the figures.

The exponents of D in Equations 15 and 16 for 70 and 80 lb/cuft soil are predicted by the terms in the Terzaghi-Peck Equation 3. The data derived Equations 15, 16, and 17 also agree in form with Housel's Equation; the coefficients m and n for a soil moisture content of 12% are as given in Table 1. The values of m and n, respectively, have been obtained by dividing the coefficients of $D^2$ with $\pi/4$ and those of D with $\pi$ in Equations 15, 16, and 17.

From this analysis, the end area resistance seems most important, the perimeter shear term contributing only for higher density soil. The effect

| Table 1. Housel's coefficients m and n for different densities and 12% moisture content of soil |
|-----------------------------------------|--------|--------|--------|
| w (lb/cft) | 70  | 80  | 90    |
| m          | 0   | 0   | 4.02  |
| n          | 30.6| 97.8| 174.7 |
Figure 15. Effect of penetrometer diameter squared on measured force
12% MOISTURE, 90 lb/cft

Figure 16. Regression of measured force on penetrometer diameter
of density on Housel's $n$ may be seen from Figure 17, drawn with the help of the data given in Table 1. The following linear regression equation was obtained from the values plotted in Figure 17:

$$n = 7.20 w - 457.4$$  \hspace{1cm} (18)

where:

$n =$ compressive stress on soil column directly beneath the penetrometer (Housel's $n$), psi

$w =$ unit weight of soil, lb/cft

**Effect of density (nondried soil)**

Appendix V shows the maximum values of force measured by a conical shaped penetrometer having a diameter of 0.115 inch when moved through a two-inch depth of nondried soil kept in a constant temperature room of 20°C. Each value of the force recorded in the Appendix is the average of three values obtained on each of the three soil boxes (replications). The effect of soil density on Housel's $n$ has already been demonstrated in Figure 17. The regression equation $n = 7.2 w - 457$ or $w = 66 + 0.139 n$ was obtained from the data given in Table 1. Because of the limited data, the intercept in the latter equation has been taken as 62.4 in subsequent statistical treatment of the data, regressions being made against $\gamma = w - 62.4$. This was done so that the data could be made to lie close to the origin. (The term $w - 62.4$ is an expression for the submerged unit weight of a soil, if $w$ is a saturated unit weight.)

In order to determine the effect of soil density on the penetration resistance (force), regression analysis of the data shown in Appendix V was carried out. Since the range of soil density in the experiments varied
Figure 17. Effect of soil density on Housel's n

\[ n = 7.20 \omega - 457.4 \]
from 70 lb/cft to 100 lb/cft, it was decided to use the variable \( \gamma = w - 62.4 \) (where \( w \) is unit weight of soil and 62.4 refers to the unit weight of water) so as to draw the data close to the origin and facilitate its statistical analysis. The coefficient of variation for force (F) and \( \gamma \) was computed as a first step towards the regression analysis and is given in Table 2.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Moisture (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12</td>
</tr>
<tr>
<td>( \gamma = w - 62.4 )</td>
<td>0.517</td>
</tr>
<tr>
<td>Force (F)</td>
<td>0.628</td>
</tr>
</tbody>
</table>

From Table 2, it will be observed that the coefficient of variation for force is greater than or approximately equal to that of \( \gamma \) for moisture contents of soil ranging from 12% to 18%. For moisture contents of 20% or above, the coefficient of variation for force is low. Consequently, it was decided to perform the regression of force (F) on \( \gamma \) for moisture contents of 12 to 18%. For moisture contents of 20 to 24%, the regression of force (F) on \( \gamma \) and vice versa was carried out.

Figures 18 and 19 exhibit the linear regression of F on \( \gamma \) for 12% and 16% moisture contents, respectively. The confidence bands on the regression lines at 95% level have been shown in the figures. The regression lines have been erected through the origin, since their intercepts were detected nonsignificant. Figures 20 and 21 show the regression of F on \( \gamma \)
Figure 18. Effect of \( \gamma \) on measured force (12\% soil moisture)
Figure 19. Effect of $\gamma$ on measured force (16% soil moisture)
20% MOISTURE

\[ F = 0.330 + 0.038 \gamma \]

\[ \gamma = 6.682 + 24.743 F \]

CONFIDENCE BANDS (F on \( \gamma \))

CONFIDENCE BANDS (\( \gamma \) on F)

Figure 20. Effect of \( \gamma \) on measured force (20% soil moisture)
24% MOISTURE

\[ F = 0.314 + 0.029 Y \]

\[ Y = 7.899 + 31.553 F \]

CONFIDENCE BANDS (F on Y)

CONFIDENCE BANDS (Y on F)

Figure 21. Effect of Y on measured force (24% soil moisture)
and vice versa for 20 and 24% moistures. The final confidence bands in these figures have been shown by continuous solid curves. The regression equations determined at 95% significance level are expressed below for different moisture contents:

\[ F = 0.127 \gamma \]  
(12% moisture) \hspace{1cm} (19)

\[ F = 0.105 \gamma \]  
(14% moisture) \hspace{1cm} (20)

\[ F = 0.073 \gamma \]  
(16% moisture) \hspace{1cm} (21)

\[ F = 0.067 \gamma \]  
(18% moisture) \hspace{1cm} (22)

\[ F = 0.33 + 0.038 \gamma \]  
(20% moisture) \hspace{1cm} (23a)

\[ \gamma = -6.682 + 24.74 F \]  
(20% moisture) \hspace{1cm} (23b)

\[ F = 0.279 + 0.035 \gamma \]  
(22% moisture) \hspace{1cm} (24a)

\[ \gamma = -6.23 + 27.045 F \]  
(22% moisture) \hspace{1cm} (24b)

\[ F = 0.314 + 0.029 \gamma \]  
(24% moisture) \hspace{1cm} (25a)

\[ \gamma = -7.9 + 31.55 F \]  
(24% moisture) \hspace{1cm} (25b)

where:

- \( F \) = estimated force for nondried soil (lb)
- \( \gamma \) = density of soil - density of water = \( w - 62.4 \) (lb/cft)

The confidence limits on the regression coefficients, standard error of the estimated value, correlation coefficient, and statistic t-value for testing the significance of the regression coefficients are given in Table 3.

The foregoing results indicate that the penetration resistance increases linearly with the density of soil at all levels of moisture contents used in this study, under nondrying conditions. A high correlation exists between the variables (force and density) at all moistures, as indicated by Table 3.
Table 3. Confidence limits, standard errors, and correlation coefficients related to force measured under nondrying conditions

<table>
<thead>
<tr>
<th>Moisture (%)</th>
<th>Calculated t value for testing</th>
<th>Std. error Correlation estimate coeff.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Intercept</td>
<td>Slope</td>
</tr>
<tr>
<td>12&lt;sup&gt;a&lt;/sup&gt;</td>
<td>(1.988)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>23.828</td>
</tr>
<tr>
<td>14&lt;sup&gt;a&lt;/sup&gt;</td>
<td>(-1.161)</td>
<td>32.37</td>
</tr>
<tr>
<td>16&lt;sup&gt;a&lt;/sup&gt;</td>
<td>(0.595)</td>
<td>40.458</td>
</tr>
<tr>
<td>18&lt;sup&gt;a&lt;/sup&gt;</td>
<td>(1.032)</td>
<td>48.773</td>
</tr>
<tr>
<td>20&lt;sup&gt;a&lt;/sup&gt;</td>
<td>4.142</td>
<td>11.967</td>
</tr>
<tr>
<td>20&lt;sup&gt;c&lt;/sup&gt;</td>
<td>-2.562</td>
<td>11.967</td>
</tr>
<tr>
<td>22&lt;sup&gt;a&lt;/sup&gt;</td>
<td>4.040</td>
<td>12.710</td>
</tr>
<tr>
<td>22&lt;sup&gt;c&lt;/sup&gt;</td>
<td>-2.570</td>
<td>12.710</td>
</tr>
<tr>
<td>24&lt;sup&gt;a&lt;/sup&gt;</td>
<td>4.372</td>
<td>10.153</td>
</tr>
<tr>
<td>24&lt;sup&gt;c&lt;/sup&gt;</td>
<td>-2.482</td>
<td>10.153</td>
</tr>
</tbody>
</table>

<sup>a</sup>Signifies the regression of force on \( \gamma \).

<sup>b</sup>Calculated t value shown in parentheses indicates the nonsignificance of parameter at 95% level.

<sup>c</sup>Signifies the regression of \( \gamma \) on force.

Prediction from skin friction equation

In the early stages of the investigation, Equation 1 was used in an attempt to predict the force exerted by the penetrometer. The data shown in Appendices I, II, and III were used for calculating the values of force for nondried soil at different moistures and densities. The values predicted from Equation 1 for a penetrometer diameter of 0.115 inch and soil depth of 2 inches are shown in Appendix VI. Sample values for 12% moisture and 70 lb/cft density of soil are given below:

\[
\begin{align*}
  w &= 70 \text{ lb/cft} = 0.0405 \text{ lb/in}^3, \\
  c &= 0.87 \text{ psi}, \\
  \phi &= 22.5^\circ, \\
  \omega &= 0.362, \quad D = 0.115 \text{ in}, \quad \text{and} \quad L = 2 \text{ in}.
\end{align*}
\]
Substitution of these values in Equation 1 gave the calculated value of force as 0.706 pound.

The regression analysis using the calculated force and \( y \) as the variables was carried out. The coefficient of variation for \( y \) and calculated or predicted force from Appendix VI was computed and is given in Table 4.

Table 4. Coefficient of variation for \( y \) and calculated force

<table>
<thead>
<tr>
<th>Variable</th>
<th>Moisture (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12</td>
</tr>
<tr>
<td>( y = w - 62.4 )</td>
<td>0.517</td>
</tr>
<tr>
<td>Calculated force</td>
<td>0.451</td>
</tr>
</tbody>
</table>

It was decided to use the regression of force on \( y \) and vice versa. The effect of \( y \) on the calculated force at different moistures may be observed from Figures 22 to 25 which incorporate Figures 18 to 21 in order to make a comparison between the measured and calculated values of force. The confidence bands at 95% have been drawn in these figures. The calculated regression lines and their confidence bands for 12 and 24% moistures almost coincided with each other, since the coefficients of variation for force and \( y \) were almost equal. The regression equations at 95% significance level developed from the analysis of the values of force calculated from Equation 1 and recorded in Appendix VI are presented below:

12% moisture:

\[
F' = 0.068 y \tag{26}
\]

\[
y = 14.916 F' \tag{27}
\]
16% moisture:
\[ F' = b_0 + b_1 \gamma + b_2 (\gamma)^2 \]
\[ = -0.844 + 0.279 \gamma - 0.006 (\gamma)^2 \]
\[ (28) \]

20% moisture:
\[ F' = 0.961 + 0.042 \gamma \]
\[ \gamma = 12.362 F' \]
\[ (29) \]
\[ (30) \]

24% moisture:
\[ F' = 0.06 \gamma \]
\[ \gamma = 16.36 F' \]
\[ (31) \]
\[ (32) \]

where:

\[ F' = \text{calculated force (lb)} \]
\[ \gamma = \text{unit weight of soil - unit weight of water} \]
\[ = w - 62.4 \]

The calculated value of t-statistic for testing the significance of
the regression coefficients, confidence limits, standard errors of the
estimated values, and correlation coefficients relating to Equations 26 to
32 are given in Table 5.

A comparison of the values of measured force and those calculated from
Equation 1 was made in order to test the mechanics of the penetration
equipment presented earlier. Both the measured and calculated values of
force have been shown in Appendix VI. Graphical comparisons are made in
Figures 22 to 25. The regression lines for the measured and calculated
force were tested for the equality of their intercepts and slopes. The
confidence bands of the regression lines were also studied. The following
results were achieved:
Table 5. Calculated values of t-statistics, confidence limits, standard errors, and correlation coefficients relating to calculated force

<table>
<thead>
<tr>
<th>Moisture (%)</th>
<th>Calculated t value for testing</th>
<th>95% confidence limits for slope</th>
<th>Std. error of correlation estimate coeff.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Intercept</td>
<td>Slope</td>
<td>Upper</td>
</tr>
<tr>
<td>12\textsuperscript{a}</td>
<td>(1.612)\textsuperscript{b}</td>
<td>18.235</td>
<td>0.069</td>
</tr>
<tr>
<td>12\textsuperscript{c}</td>
<td>(-0.247)</td>
<td>17.50</td>
<td>16.793</td>
</tr>
<tr>
<td>16\textsuperscript{a}</td>
<td>-3.251</td>
<td>10.447</td>
<td>0.201</td>
</tr>
<tr>
<td>20\textsuperscript{a}</td>
<td>5.277</td>
<td>5.764</td>
<td>1.367</td>
</tr>
<tr>
<td>20\textsuperscript{c}</td>
<td>(-1.9786)</td>
<td>12.268</td>
<td>14.495</td>
</tr>
<tr>
<td>24\textsuperscript{a}</td>
<td>1.699</td>
<td>12.014</td>
<td>0.071</td>
</tr>
<tr>
<td>24\textsuperscript{c}</td>
<td>(0.424)</td>
<td>11.23</td>
<td>19.562</td>
</tr>
</tbody>
</table>

\textsuperscript{a}Signifies regression of force on $\gamma$.

\textsuperscript{b}t value in parentheses indicates nonsignificance of the parameter.

\textsuperscript{c}Signifies regression of $\gamma$ on force.

1. At 12% moisture content of soil, the two regression lines represented by Equations $F = 0.127 \gamma$ and $F' = 0.067 \gamma$ were found to differ in their slopes. The value of t-statistic for testing the difference between the slopes was computed as 9.234 for 22 degrees of freedom, indicating the inequality of the slopes. The confidence bands for the regression lines were found to diverge greatly from each other, as shown in Figure 22.

2. At 16% moisture, the measured force was given by a linear regression Equation $F = 0.073 \gamma$ passing through the origin, whereas the calculated force was governed by the quadratic Equation $F' = -0.844 + 0.279 \gamma - 0.006 (\gamma)^2$. The confidence bands for the two regressions did not contain each other as indicated by Figure 23.
12% MOISTURE

— MEASURED \( F = 0.127 \gamma \)

— CALCULATED \( F' = 0.067 \gamma \)

Figure 22. Comparison of measured and calculated forces at 12% moisture content
16% MOISTURE

MEASURED \( F = 0.073 \gamma \)

CALCULATED \( F' = -0.844 + 0.279 \gamma - 0.006 (\gamma)^2 \)

Figure 23. Comparison of measured and calculated forces at 16% moisture content
3. At 20% moisture, the regression lines represented by Equations $F = 0.33 + 0.038 Y$ and $F' = 0.961 + 0.042 Y$ were found to have equal slopes but differed in their intercepts. The values of $t$ for comparing the differences between the intercepts and slopes were computed as $-2.977$ and $-0.585$, respectively, for 20 degrees of freedom. The confidence bands for the two regression lines did not lie in the same domain as indicated by Figure 24.

4. At 24% moisture, the measured force was given by Equation $F = 0.314 + 0.029 Y$, whereas the calculated force was represented by Equation $F' = 0.06 Y$ passing through the origin. A divergence of the confidence bands for the two regression lines may be observed from Figure 25.

The above results show a disparity between the measured values and those predicted from skin friction Equation 1. The lack of agreement between the measured and calculated values of force suggest that Equation 1 cannot accurately predict the force exerted by a penetrometer. The predicted value of force has been found larger in magnitude than for the measured force at all levels of moistures except at the lowest level of 12% used in this study. The percentage difference between the measured and predicted values as reported in Appendix VI varied from 8% to 117% of the measured values. The mean value of all the percentage differences was, however, computed as $-27.88\%$, the negative sign indicating overprediction by Equation 1. The mean absolute difference was evaluated as 0.789 pound. The differences may be referred to the following:

1) The soil strength parameters ($c$, $\phi$ and $\mu$) were not determined under the same stress conditions as induced by the penetrometer.
Figure 24. Comparison of measured and calculated forces at 20% moisture content
Figure 25. Comparison of measured and calculated forces at 24% moisture content
ii) The force was calculated from Equation 1, using a maximum depth value of two inches of soil, whereas the measured force became almost constant beyond a certain depth, as shown by Figures 11a and 11b.

iii) The tip resistance was ignored in the prediction equation.

**Prediction using Housel's approach**

Equations 19 and 22 may be used to determine Housel's n values by assuming \( m = 0 \) corresponding to zero intercepts and dividing \( F \) by the penetrometer end area, 0.0104 square inch, giving:

<table>
<thead>
<tr>
<th>moisture content (%)</th>
<th>12</th>
<th>14</th>
<th>16</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n/\gamma )</td>
<td>12.2</td>
<td>10.1</td>
<td>7.02</td>
<td>6.44</td>
</tr>
</tbody>
</table>

The intercepts at higher moisture contents given by Equations 23b, 24b, and 25b suggest that \( \gamma \) should be defined as approximately \( w - 56 \) for the regressions to include the origin. For example, Equation 23b can be written as \( w - 62.4 = 24.74 F - 6.882 \), or \( w - 55.7 = 24.74 F \). Denoting \( w - 55.7 \) by \( \dot{\gamma} \), the equation can be rewritten as \( \dot{\gamma} = 24.74 F \). An approximate method, however, is to assume that the regressions do pass through the origin, which gives \( n \) values based on slopes \( (\dot{\gamma}/x) \) as follows:

<table>
<thead>
<tr>
<th>moisture content (%)</th>
<th>20</th>
<th>22</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n/\gamma )</td>
<td>5.03</td>
<td>4.55</td>
<td>4.11</td>
</tr>
</tbody>
</table>

As the moisture content becomes sufficiently high, \( n/\gamma \) will approach zero because the soil will become liquid.

The data above for moisture and \( n/\gamma \) fit a logarithmic relationship:

\[
\log n/\gamma = 2.8404 - 1.627 \log M
\]

\((R = 0.992, S = 0.0248)\)
where:

\( n \) = compressive stress on soil column directly beneath the penetrometer

\( \gamma \) = unit weight of soil - unit weight of water = \( w - 62.4 \)

\( M \) = moisture content of soil (\%)

\( R \) = correlation coefficient

\( S \) = standard error of the estimated value

Equation 33 can be written as follows:

\[
\frac{n}{\gamma} = (10)^{2.8404 - 1.627 \log M}
\]

\[
\frac{n}{\gamma} \left[ (10)^{2.8404 - 1.627 \log M} \right]
\]

Using \( F = A n = 0.0104 n \), where \( A = 0.0104 \) square inch (area of penetrometer),

\[
F = 0.0104 \gamma \left[ (10)^{2.8404 - 1.627 \log M} \right]
\]

where:

\( F \) = predicted value of penetration resistance (lb)

\( M \) = moisture content of soil (\%)

A second order quadratic equation also was found to describe the data:

\[
n/\gamma = 0.0586 M^2 - 2.778 M + 37.122
\]

\( (R = 0.993, S = 0.435) \)

Equation 35 can be written as follows:

\[
n = \gamma \left( 0.0586 M^2 - 2.778 + 37.122 \right)
\]  \hspace{1cm} (36)

Using \( F = A n = 0.0104 n \), where \( A = 0.0104 \) square inch,

\[
F = 0.0104 \gamma \left[ (0.0586 M^2 - 2.778 + 37.122) \right]
\]  \hspace{1cm} (37)

Equation 37 has the disadvantage that extrapolation to higher moisture contents will increase \( n/\gamma \), whereas logically it should decrease. However, within the bounds of the experimental conditions, it is sufficiently pre-
cise, and the form is advantageous for calculations. The effect of $n/\gamma$ on the moisture content of soil has been depicted graphically in Figure 26.

A comparison between the measured values and those predicted from Equations 34 and 37 is presented in Appendix VI. The percentage difference between the measured force and that predicted from Equation 37 range from 0.33 to 64%. The mean value of all the percentage differences was, however, computed as -4.34%, the negative sign indicating overprediction by Equation 37. The mean absolute difference was determined as 0.213 pound.

**Effect of moisture (nondried soil)**

The regression procedure was again undertaken with a view to predict the effect of moisture on the force exerted by the penetrometer. The coefficient of variation for moisture and force was computed at different densities of soil, using the data of Appendix V and is given in Table 6.

Table 6. Coefficient of variation for force and moisture

<table>
<thead>
<tr>
<th>Variable</th>
<th>Density (lb/cft)</th>
<th>70</th>
<th>80</th>
<th>90</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moisture (M)</td>
<td>0.228</td>
<td>0.228</td>
<td>0.228</td>
<td>0.228</td>
<td></td>
</tr>
<tr>
<td>Force (F)</td>
<td>0.131</td>
<td>0.285</td>
<td>0.473</td>
<td>0.481</td>
<td></td>
</tr>
</tbody>
</table>

It may be observed from Table 6 that the coefficient of variation for force is generally higher than that for moisture. It was, therefore, decided to carry out the regression of force on moisture. The regression analysis of the data on moisture and force, recorded in Appendix V, yielded
Figure 26. Effect of $n/\bar{y}$ on soil moisture
the following results at 95% level of significance for different bulk densities of soil:

**70 lb/cft**

\[ F = b_0 + b_1 M \]

\[ = 0.818 - 0.015 M \]  \hspace{1cm} (38)

- confidence limits on intercept = 0.724, 0.909
- confidence limits on slope = -0.0196, -0.0096
- standard error of estimate = 0.043
- correlation coefficient = 0.818

**80 lb/cft**

\[ F = b_0 + b_1 M + b_2 M^2 \]

\[ = 4.97 - 0.35 M + 0.008 M^2 \]  \hspace{1cm} (39)

- confidence limits on \( b_0 \) = 3.9, 6.04
- confidence limits on \( b_1 \) = -0.474, -0.226
- confidence limits on \( b_2 \) = 0.004, 0.011
- standard error of estimate = 0.103
- correlation coefficient = 0.961

**90 lb/cft**

\[ F = b_0 + b_1 M + b_2 M^2 \]

\[ = 14.31 - 1.14 M - 0.025 M^2 \]  \hspace{1cm} (40)

- confidence limits on \( b_0 \) = 12.62, 16.0
- confidence limits on \( b_1 \) = -1.33, -0.95
- confidence limits on \( b_2 \) = -0.03, -0.02
- standard error of estimate = 0.159
- correlation coefficient = 0.99
100 lb/cft

\[ F = b_o + b_1 M + b_2 M^2 \]

\[ = 15.07 - 1.137 M + 0.024 M^2 \quad (41) \]

confidence limits on \( b_o \) = 13.18, 16.97

confidence limits on \( b_1 \) = -1.355, -0.92

confidence limits on \( b_2 \) = 0.018, 0.03

standard error of estimate = 0.182

correlation coefficient = 0.99

where:

\( F \) = estimated force for nondried soil (lb)

\( M \) = moisture content of soil (\%)

\( b_o, b_1, \) and \( b_2 \) are regression coefficients

The results are depicted diagramatically by the falling curves shown in Figures 27 to 30. The effect of moisture on force is found to be non-linear except at the lowest density of 70 lb/cft used in this study. The decrease in force with moisture may be attributed to the decrease in strength and viscosity of soil.

In Figures 27 to 30, the general prediction Equation 37 has been added and is represented by the dotted curve. At a lower density of 70 lb/cft, the general prediction equation does not fit the data. As the density increases, the predicted curve draws nearer the measured regression line.

Relation to \( c \) and \( \phi \)

The observed dependence of penetration resistance on soil density and moisture content may also be predictable from the soil shear strength parameters, cohesion \( c \), and angle of internal friction \( \phi \). This is indi-
Figure 27. Effect of moisture on force at 70 lb/cft
Figure 28. Effect of moisture on force at 80 lb/cft
Figure 29. Effect of moisture on force at 90 lb/cft
Figure 30. Effect of moisture on force at 100 lb/cft
cated by classical soil mechanics, which assumes that the soil is incompressible. A spring loaded penetrometer (pocket penetrometer) frequently used by soil engineers for cohesive soils is empirically calibrated directly in terms on unconfined compressive strength. If $\phi = 0$, $N_c$ in Equation 2 becomes 5.70, as obtained from the chart produced by Terzaghi and Peck (1967). This value of $N_c$, when substituted in Equation 2, gives $q = 6.8c$ plus a surcharge term. A linear regression analysis of measured force on c for the data shown in Appendix VI established no relationship between them. The correlation coefficient was as low as 0.098. Further, no relationship existed between the values of $q$ and $c$, as determined by the regression analysis. This may be due to neglecting $\phi$, the angle of internal friction in the equations for $q$ and $F$. Therefore, Bell's Equation reported by Jumikis (1962) and given below was used to predict the tip resistance or end bearing capacity, $F_t$:

$$F_t = \frac{\pi D^2}{4} \left[ wL \tan^4(45^\circ + \phi/2) + 2c \tan(45^\circ + \phi/2) \left[ \tan^2(45^\circ + \phi/2) + 1 \right] \right]$$

(42)

The data in Appendix VI was used to predict the bearing capacity from Equation 42. The values of $F_t$ ranged from a minimum of 0.076 pound for 70 lb/cft to a maximum of 0.385 pound for 100 lb/cft of soil density and 24% moisture, as shown in Table 8, far below the measured values of penetration resistance. Low values were also obtained by the use of Terzaghi-Peck Equation 3 and have been reported in Table 8. The values of the coefficients $N_c$, $N_q$, and $N_w$ used in Equation 3 were obtained from the chart produced by Terzaghi and Peck (1967) for different values of $\phi$ and have been given in Table 7. We may conclude that values of $c$ and $\phi$ measured by the shear box and used to find the tip resistance or skin friction do not
Table 7. Values of $N_c$, $N_q$, and $N_w$ given by Terzaghi and Peck (1967) for different values of angle of internal friction ($\phi$) of soil

<table>
<thead>
<tr>
<th>Moisture (%)</th>
<th>Density (lb/cft)</th>
<th>C (psi)</th>
<th>$\phi$ (degrees)</th>
<th>$N_c$</th>
<th>$N_q$</th>
<th>$N_w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>70</td>
<td>12</td>
<td>22.5</td>
<td>17.0</td>
<td>7.8</td>
<td>4.5</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>1.50</td>
<td>28.0</td>
<td>25.0</td>
<td>15.0</td>
<td>12.0</td>
</tr>
<tr>
<td>16</td>
<td>70</td>
<td>1.60</td>
<td>13.0</td>
<td>10.0</td>
<td>3.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>1.95</td>
<td>27.5</td>
<td>24.0</td>
<td>14.0</td>
<td>10.5</td>
</tr>
<tr>
<td>20</td>
<td>70</td>
<td>1.31</td>
<td>9.0</td>
<td>8.0</td>
<td>2.5</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>2.10</td>
<td>15.0</td>
<td>11.0</td>
<td>4.0</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>2.30</td>
<td>25.0</td>
<td>20.0</td>
<td>10.0</td>
<td>3.5</td>
</tr>
<tr>
<td>24</td>
<td>70</td>
<td>1.31</td>
<td>8.5</td>
<td>7.5</td>
<td>2.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>1.89</td>
<td>35.0</td>
<td>47.0</td>
<td>32.0</td>
<td>33.0</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>2.13</td>
<td>32.9</td>
<td>40.0</td>
<td>25.0</td>
<td>30.0</td>
</tr>
</tbody>
</table>

Table 8. Values of end bearing capacity or point resistance obtained from Terzaghi-Peck and Bell's equations

<table>
<thead>
<tr>
<th>Moisture (%)</th>
<th>Density (lb/cft)</th>
<th>C (psi)</th>
<th>$\phi$ (degrees)</th>
<th>Mean measured penetration resistance (lb)</th>
<th>Point resistance (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Mean measured penetration resistance (lb)</td>
<td>Point resistance (lb)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Terzaghi-Peck</td>
<td>Bell</td>
</tr>
<tr>
<td>12</td>
<td>70</td>
<td>0.87</td>
<td>22.5</td>
<td>0.590</td>
<td>0.191</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>1.50</td>
<td>28.0</td>
<td>1.834</td>
<td>0.484</td>
</tr>
<tr>
<td>16</td>
<td>70</td>
<td>1.60</td>
<td>13.0</td>
<td>0.611</td>
<td>0.201</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>1.95</td>
<td>27.5</td>
<td>1.178</td>
<td>0.596</td>
</tr>
<tr>
<td>20</td>
<td>70</td>
<td>1.31</td>
<td>9.0</td>
<td>0.533</td>
<td>0.132</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>2.10</td>
<td>15.0</td>
<td>1.045</td>
<td>0.291</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>2.30</td>
<td>25.0</td>
<td>1.522</td>
<td>0.583</td>
</tr>
<tr>
<td>24</td>
<td>70</td>
<td>1.31</td>
<td>8.5</td>
<td>0.444</td>
<td>0.124</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>1.89</td>
<td>35.0</td>
<td>1.222</td>
<td>1.150</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>2.13</td>
<td>32.9</td>
<td>1.300</td>
<td>1.100</td>
</tr>
</tbody>
</table>
reliably predict the penetration resistance. An important reason may be that stress conditions in the shear box do not closely simulate those induced by the penetrometer.

**Effect of drying**

Appendix VII shows the values of force measured by the penetrometer of 0.115 inch in diameter on different days when the boxes were kept uncovered in a constant temperature room of 20°C. One reading of force was taken from each box. The value shown against zero day in Appendix VII refers to the force measured under nondrying condition and has been drawn from Appendix V.

The analysis of the data in Appendix VII using the regression of days on force was accomplished, since the coefficient of variation for days was found higher than for force at all levels of moisture and density, as shown in Table 9. The results are depicted graphically in Figures 31 to 36. The regression lines along with their confidence bands at 95% level have been shown in these figures. The confidence limits, standard errors, and correlation coefficients relating to the regression equations have been shown in Table 10. The regression equations are as follows:

Table 9. Coefficient of variation for force measured under drying conditions

<table>
<thead>
<tr>
<th>Density (lb/cft)</th>
<th>Initial moisture (%)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12</td>
<td>16</td>
<td>20</td>
</tr>
<tr>
<td>70</td>
<td>0.341</td>
<td>0.450</td>
<td>0.695</td>
</tr>
<tr>
<td>80</td>
<td>0.344</td>
<td>0.569</td>
<td>0.635</td>
</tr>
<tr>
<td>90</td>
<td>0.382</td>
<td>0.577</td>
<td>0.653</td>
</tr>
</tbody>
</table>

| Coefficient of variation for days = 0.732. |
Table 10. Confidence limits, standard error, and correlation coefficient related to force measured under drying conditions

<table>
<thead>
<tr>
<th>Moisture (%)</th>
<th>Density (lb/cft)</th>
<th>Calculated t value for testing</th>
<th>95% confidence limits for Intercept</th>
<th>Std. error Correlation estimate coeff.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Upper</td>
<td>Lower</td>
</tr>
<tr>
<td>12&lt;sup&gt;a&lt;/sup&gt;</td>
<td>70</td>
<td>15.6</td>
<td>0.66</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>13.5</td>
<td>1.933</td>
<td>1.40</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>13.92</td>
<td>4.32</td>
<td>3.158</td>
</tr>
<tr>
<td>12&lt;sup&gt;b&lt;/sup&gt;</td>
<td>70</td>
<td>-7.909</td>
<td>-3.174</td>
<td>-5.56</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>-6.734</td>
<td>-2.87</td>
<td>-5.58</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>-7.85</td>
<td>-2.544</td>
<td>-4.476</td>
</tr>
<tr>
<td>16&lt;sup&gt;a&lt;/sup&gt;</td>
<td>70</td>
<td>8.68</td>
<td>0.857</td>
<td>0.516</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>4.01</td>
<td>1.426</td>
<td>0.427</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>5.34</td>
<td>2.754</td>
<td>1.166</td>
</tr>
<tr>
<td>16&lt;sup&gt;b&lt;/sup&gt;</td>
<td>70</td>
<td>-5.415</td>
<td>-1.41</td>
<td>-3.28</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>-2.690</td>
<td>-0.198</td>
<td>-1.812</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>-4.063</td>
<td>-0.472</td>
<td>-1.545</td>
</tr>
<tr>
<td>20&lt;sup&gt;a&lt;/sup&gt;</td>
<td>70</td>
<td>--</td>
<td>44.8</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>--</td>
<td>24.0</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>2.24</td>
<td>19.5</td>
<td>2.312</td>
</tr>
<tr>
<td>20&lt;sup&gt;b&lt;/sup&gt;</td>
<td>70</td>
<td>(-0.49)</td>
<td>32.847</td>
<td>1.184</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>(-1.221)</td>
<td>23.336</td>
<td>0.795</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>(-1.54)</td>
<td>31.721</td>
<td>0.458</td>
</tr>
</tbody>
</table>

<sup>a</sup> Indicates the regression of force on days.

<sup>b</sup> Indicates the regression of days on force.

<sup>c</sup> t value shown in parentheses indicates the nonsignificance of parameters at 95% level.

12% initial moisture:

\[ d = -4.366 + 7.86 \frac{F_d}{70 \text{ lb/cft}} \]  \hspace{1cm} (43)

\[ d = -4.255 + 2.688 \frac{F_d}{80 \text{ lb/cft}} \]  \hspace{1cm} (44)

\[ d = -3.51 + 0.981 \frac{F_d}{90 \text{ lb/cft}} \]  \hspace{1cm} (45)
16% initial moisture:
\[ d = -2.346 + 3.699 \, F_d \quad (70 \text{ lb/cft}) \]  
\[ d = -1.005 + 1.312 \, F_d \quad (80 \text{ lb/cft}) \]  
\[ d = -1.008 + 0.563 \, F_d \quad (90 \text{ lb/cft}) \]  

20% initial moisture:
\[ d = 1.112 \, F_d \quad (70 \text{ lb/cft}) \]  
\[ d = 0.729 \, F_d \quad (80 \text{ lb/cft}) \]  
\[ d = 0.430 \, F_d \quad (90 \text{ lb/cft}) \]

where:
- \( d \) = days of drying
- \( \, F_d \) = force measured under drying condition (lb)

The regression of Force on days using the data in Appendix VII, however, produced the following equations:

12% initial moisture:
\[ F_d = 0.58 + 0.121 \, d \quad (70 \text{ lb/cft}) \]  
\[ F_d = 1.667 + 0.348 \, d \quad (80 \text{ lb/cft}) \]  
\[ F_d = 3.737 + 0.980 \, d \quad (90 \text{ lb/cft}) \]  

16% initial moisture:
\[ F_d = 0.686 + 0.257 \, d \quad (70 \text{ lb/cft}) \]  
\[ F_d = 0.927 + 0.727 \, d \quad (80 \text{ lb/cft}) \]  
\[ F_d = 1.960 + 1.733 \, d \quad (90 \text{ lb/cft}) \]  

20% initial moisture:
\[ F_d = 0.90 \, d \quad (70 \text{ lb/cft}) \]  
\[ F_d = 1.338 \, d \quad (80 \text{ lb/cft}) \]  
\[ F_d = 1.177 + 2.098 \, d \quad (90 \text{ lb/cft}) \]
Equations 52 to 60 were used to include the effect of drying in Equation 36, used to determine Housel's n values. Considering only the slopes in these equations and dividing them by the area (0.0104 square inch) of the penetrometer, Housel's n as a function of drying time (d) was obtained. The ratio n/d thus obtained from Equations 52 to 60 was evaluated and has been shown in Table 11 for different soil moistures and densities.

Table 11. Values of n/d at different moistures and densities under drying conditions

<table>
<thead>
<tr>
<th>Moisture (%)</th>
<th>Y = w - 62.4 (lb/cft)</th>
<th>n/d = slope/area = slope/0.0104</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>7.6</td>
<td>11.63</td>
</tr>
<tr>
<td></td>
<td>17.6</td>
<td>33.50</td>
</tr>
<tr>
<td></td>
<td>27.6</td>
<td>94.0</td>
</tr>
<tr>
<td>16</td>
<td>7.6</td>
<td>24.7</td>
</tr>
<tr>
<td></td>
<td>17.6</td>
<td>69.5</td>
</tr>
<tr>
<td></td>
<td>27.6</td>
<td>166.6</td>
</tr>
<tr>
<td>20</td>
<td>7.6</td>
<td>86.5</td>
</tr>
<tr>
<td></td>
<td>17.6</td>
<td>128.5</td>
</tr>
<tr>
<td></td>
<td>27.6</td>
<td>202.0</td>
</tr>
</tbody>
</table>

The multiple regression of the data in Table 11 gave the following equation:

\[
\frac{n}{d} = 5.661 Y + 11.578 M - 194.124
\]

\(R = 0.973, S = 17.42\)

\[
Y = d \left(5.661 Y + 11.578 M - 194.124\right)
\]  

(61)

The function given by Equation 61 was added to Equation 36 in order to account for the effect of drying. This resulted in the following equation:
\[ n = \sqrt[3]{0.0586 M^2 - 2.778 M + 37.122} + d (5.661 \sqrt[3]{\gamma} + 11.578 M - 194.124) \] (62)

Since \( F = A n \), where \( A \) is the end area of the penetrometer

\[ F = A \sqrt[3]{0.0586 M^2 - 2.778 M + 37.122} \]
\[ + A d (5.661 \sqrt[3]{\gamma} + 11.578 M - 194.124) \] (63)

The prediction Equation 63 has been added to Figures 31 to 36. This equation has the disadvantage that the error from the prediction of the nondrying case is propagated to the prediction of the drying case and appears as an intercept error in Figures 31 to 36. Further, at 12% moisture and 70 lb/cft, Equation 63 results in the decrease of force with drying time, as may be observed from Figure 31. The following reasons may be advanced for the errors involved:

i) Error is inherited from the nondrying term used in Equation 63.

ii) Figure 27 prepared for 70 lb/cft when compared to Figures 28, 29, and 30 (drawn for higher densities), indicates that a different phenomenon may be involved at low density under nondrying conditions. A linear relationship exists between the force and moisture at 70 lb/cft (Figure 27), whereas a quadratic relationship holds good at higher densities (Figures 28, 29, and 30).

iii) Intercepts in Equations 23b, 24b, 25b, 52 to 57, and 60 were ignored while developing Equation 63.

iv) Equation 63 was developed using the value of \( \sqrt[3]{\gamma} = w - 62.4 \). The value of the constant in \( \sqrt[3]{\gamma} \) may, however, vary and, consequently, an error exists in Equation 63. This error in \( \sqrt[3]{\gamma} (= w - 62.4) \) may become less at larger values of \( w \) (density of soil).
Figure 31. Effect of drying time on force at 12% moisture (70 and 80 lb/cft)
Figure 32. Effect of drying time on force at 12% moisture (90 lb/cft)
Figure 33. Effect of drying time on force at 16% moisture (70 and 80 lb/cft)
16% MOISTURE
90 lb/cft

---

○ MEASURED
_ _ PREDICTED

CONFIDENCE BANDS
(MEASURED)

PREDICTED

Figure 34. Effect of drying time on force at 16% moisture (70 and 80 lb/cft)
Figure 35. Effect of drying time on force at 20% moisture (70 and 80 lb/cft)
Figure 36. Effect of drying time on force at 20% moisture (90 lb/cft)
The predicted values of force, as obtained from Equation 63, have been entered in Appendix VII. The difference between the measured and predicted values, as shown in the Appendix, range from 0.68 to 45%, excluding the low moisture (12%) and low density (70 lb/cft) data. The mean of all the percentage differences was computed as -5.62% of the measured values, the negative sign indicating overprediction by Equation 63. The absolute value of the mean difference was determined as 0.712 pound.

**Effect of different variables on resistance**

The method of multiple regression was employed to estimate the penetration resistance of soil considering the effects of moisture, density, days of drying, and the interaction between them. The equation developed from the values of measured force shown in Appendix VII is presented below, based upon the highest multiple correlation coefficient and the least standard error of the estimated value:

\[
F_d = 0.134 w - 0.802 M - 0.033 M^2 - 5.46 d \\
- 0.005 M w + 0.1 M d + 0.06 w d
\]

(64)

where:

- \(F_d\) = estimated force (lb)
- \(w\) = unit weight or density of soil (lb/cft)
- \(M\) = moisture of soil (%)
- \(d\) = days of drying

The multiple correlation coefficient was computed as 0.976 and the standard error of the estimate 0.975. The value of \(t\) for testing the regression coefficient of each variable and the associated standard error is given in Table 12.
Table 12. Values of t and standard error for the regression coefficient associated with different variables affecting resistance

<table>
<thead>
<tr>
<th>Variable</th>
<th>Standard error</th>
<th>t value (d.f. = 127)</th>
</tr>
</thead>
<tbody>
<tr>
<td>density = w</td>
<td>0.02</td>
<td>6.617</td>
</tr>
<tr>
<td>moisture = M</td>
<td>0.388</td>
<td>-2.07</td>
</tr>
<tr>
<td>(moisture)^2 = M^2</td>
<td>0.013</td>
<td>2.493</td>
</tr>
<tr>
<td>day = d</td>
<td>0.322</td>
<td>-16.962</td>
</tr>
<tr>
<td>moisture x density = M x w</td>
<td>0.001</td>
<td>-4.781</td>
</tr>
<tr>
<td>moisture x day = M x d</td>
<td>0.008</td>
<td>11.86</td>
</tr>
<tr>
<td>density x day = w x d</td>
<td>0.004</td>
<td>16.444</td>
</tr>
</tbody>
</table>

Table 12 suggests that the drying time (day) has a clear effect on the resistance of soil, since the absolute value of t-statistic for the regression coefficient associated with the time (day) variable is very high. Similarly, the initial moisture content of soil may have less effect since the corresponding absolute value of t-statistic is the lower.
PREDICTION OF DRAFT

The data on draft for tillage equipment collected by other research workers was utilized for testing the mechanics for tillage equipment. The measured draft was compared with the value predicted from the tillage mechanics presented earlier. The data of Rowe (1959), Larson (1964), and McLeod (1959) on moldboard and disk plows were considered.

Rowe's Work

Rowe measured in model soil bins the draft force for a moldboard plow inclined at an angle of 25° to the horizontal. The blade was 4 inches wide, 2 inches long, and 1/8 inch thick. It was operated at one inch depth on masonry sand, Ida silt loam, Colo silty clay loam, and Luton silty clay soils.

Soil parameters $\epsilon$, $\phi$, and $\delta$ were determined with the help of the shear box. The maximum torque required to twist off the vertical column of soil was recorded on the Oscillograph. The soil parameters were evaluated by using the regression analysis on the normal and shear or frictional stresses.

The draft or the horizontal force was measured by fastening the tool to a force sensing assembly. Four SR-4 strain gages mounted on the proving ring constituted a Wheatstone bridge. The bridge was connected to Brush recording Oscillograph for amplifying and recording the force signals. The values of soil parameters and measured draft are shown in Appendix VIII.
Larson's Work

Larson measured the draft for different sizes of moldboard plows whose shapes were generated from the logarithmic spiral. The plows were inclined at an angle of 20° to the horizontal and were operated at different velocities. Soil parameters \( c \), \( \phi \), and \( \delta \) were determined by means of the shear box. The draft was measured by strain gages connected to an eight channel Offner dynograph. The data obtained on Colo clay loam soil for three- and four-inch moldboard plows are shown in Appendices IX and X.

McLeod's Work

McLeod (1959) measured the draft force for disk plows in masonry sand, Luton silty clay, and Colo silty clay loam soils. The disks were kept vertical (tilt angle = 0) and operated at an angle of approach \( (\varphi) \) of 40 degrees. The width of cut was maintained at 1.375 inch and a depth of 1.5 inch.

The shear box was used to determine the values of cohesion and angle of internal friction of soil. The draft was measured by strain gages connected to a Brush recording oscillograph. The data obtained for a six-inch diameter disk are shown in Appendix XI. The value of \( \delta \) was considered equal to \( \phi \) in this Appendix.

Comparison

Equation 10 in conjunction with Equations 5, 8, and 9 was used to calculate the value of draft using the data of Rowe (1959), Larson (1964), and McLeod (1959). The results are entered in Appendices VIII, IX, X, and XI.

Figure 37 makes a comparison of the measured and calculated drafts for a three-inch moldboard plow. The values of drafts have been drawn from
Figure 37. Comparison of measured and calculated drafts for three-inch plow
Appendix IX and correspond to the tool velocity of about 0.88 ft/sec. The coefficients of variation for the drafts and $\gamma$ are given below:

measured draft ($F_d$) = 0.395

calculated draft ($F'_d$) = 0.186

$\gamma = (\text{density} - 62.4) = 0.119$

The regression of draft on $\gamma$ yielded the following linear regression equations:

$F_d = -118.076 + 5.742 \gamma$ (measured) \hspace{1cm} (65)

$F'_d = 1.443 \gamma$ (calculated) \hspace{1cm} (66)

where:

$F_d = \text{measured draft force (lb)}$

$F'_d = \text{calculated draft force (lb)}$

$\gamma = \text{density} - 62.4 \text{ (lb/cft)}$

The confidence limits on parameters and other information is given in Table 13. The confidence bands on the two regression lines have been drawn in Figure 37. With the increase of density, these bands diverge from each other.

Figure 38 makes a comparison of the drafts for a four-inch moldboard plow operated at different velocities on soil having a density of 87 lb/cft, as mentioned in Appendix X. The coefficients of variation are given below:

measured draft ($F_d$) = 0.213

calculated draft ($F'_d$) = 0.019

$\gamma = \text{density} - 62.4 = 0.652$

The regression of draft on velocity and vice versa gave the following results:
Table 13. Confidence limits on parameters, standard error, and correlation coefficient relating to draft

<table>
<thead>
<tr>
<th>Name of tool</th>
<th>Calculated Std. of tool</th>
<th>Intercept Slope</th>
<th>Upper Lower</th>
<th>Upper Lower</th>
<th>Std. error</th>
<th>Correlation estimate coeff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3&quot; plow</td>
<td>Measured:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Intercept: -4.3</td>
<td>Slope: 16.2</td>
<td>1.497</td>
<td>-237.6</td>
<td>9.74</td>
<td>1.75</td>
</tr>
<tr>
<td></td>
<td>Calculated:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Intercept: (0.62)</td>
<td>Slope: 19.62</td>
<td>--</td>
<td>--</td>
<td>5.4</td>
<td>-1.50</td>
</tr>
<tr>
<td>4&quot; plow</td>
<td>Measured:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Intercept: 35.0</td>
<td>Slope: 19.5</td>
<td>39.12</td>
<td>32.61</td>
<td>5.05</td>
<td>3.63</td>
</tr>
<tr>
<td></td>
<td>Calculated:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Intercept: 24.5</td>
<td>Slope: 8.44</td>
<td>59.33</td>
<td>57.81</td>
<td>0.603</td>
<td>0.273</td>
</tr>
<tr>
<td></td>
<td>Calculated:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Intercept: 8.2</td>
<td>Slope: 8.44</td>
<td>-78.4</td>
<td>-178.2</td>
<td>3.02</td>
<td>1.366</td>
</tr>
<tr>
<td>Disk plow</td>
<td>Measured:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Intercept: (-0.64)</td>
<td>Slope: 55.7</td>
<td>--</td>
<td>--</td>
<td>0.44</td>
<td>0.392</td>
</tr>
<tr>
<td></td>
<td>Calculated:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Intercept: 5.68</td>
<td>Slope: (-0.42)</td>
<td>2.79</td>
<td>2.434</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

- Indicates the regression of draft on \( \sqrt{y} \) or velocity.
- \( t \) value in parentheses shows the nonsignificance of the parameter.
- Indicates the regression of draft on \( \sqrt{y/velocity} \) or vice versa.

measured:

\[
F_d = 35.86 + 4.34 V
\]  
\[
V = -8.165 + 0.229 F_d
\]

calculated:

\[
F'_d = 58.57 + 0.438 V
\]  
\[
V = -128.27 + 3.019 F'_d
\]

where:

\( V \) = velocity of plow (ft/sec)
4-INCH PLOW

○ MEASURED \( F_D = 35.861 + 4.341 V \)

△ CALCULATED \( F'_D = 58.573 + 0.438 V \)

95% CONFIDENCE BANDS

Figure 38. Comparison of measured and calculated drafts for four-inch plow
The linear regression lines and their confidence bands using the regression of draft on velocity and vice versa almost coincided with each other both for the measured and calculated values. The intercepts and slopes of the measured and calculated regression lines represented by Equations 67 and 69 were found to differ from each other. The values of $t$ for testing the differences between the intercepts and slopes were computed as 21.6 and 17.1, respectively, for six degrees of freedom, indicating the inequality of the parameters. The confidence bands for the measured and calculated regression lines, as shown in Figure 38, for the most part do not contain each other. The statistical information connected with the regression equations may be obtained from Table 13.

Figure 39 makes a comparison of the measured and calculated drafts obtained from Appendix XI for the disk plow operated in Ida silt loam at a velocity of 1.49 ft/sec. The coefficients of variation are as follows:

- measured draft ($F_d$) = 0.179
- calculated draft ($F'_d$) = 0.045
- $\hat{\gamma} = \text{density} - 62.4 = 0.161$

The following equations were obtained from the relevant data:

\[
F_d = 0.416 \hat{\gamma} \quad \text{(measured)} \quad (71)
\]
\[
F'_d = 2.611 \quad \text{(calculated)} \quad (72)
\]

The two lines represented by the above equations are different in form, one passing through the origin and the other possessing zero slope. Their confidence bands drawn in Figure 39 do not lie in the same domain. The confidence limits, standard error, and correlation coefficient relating to the regression equations are given in Table 13.
Figure 39. Comparison of measured and calculated drafts for disk plow
A general review of the Figures 37 to 39 reveals a lack of agreement between the measured and theoretical drafts. The differences between the measured and theoretical drafts for the moldboard and disk plows, as listed in Appendices VIII, IX, X, and XI, are given in Table 14.

Table 14. Differences between measured and calculated drafts for different types of plows

<table>
<thead>
<tr>
<th>Appendix</th>
<th>Plow</th>
<th>Difference (%)</th>
<th>Mean difference (%)</th>
<th>Mean absolute difference (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>From</td>
<td>To</td>
<td></td>
</tr>
<tr>
<td>XIII</td>
<td>4-inch, 25° (moldboard)</td>
<td>0.46</td>
<td>40.62</td>
<td>-2.2</td>
</tr>
<tr>
<td>IX</td>
<td>3-inch, 20° (moldboard)</td>
<td>1.28</td>
<td>41.04</td>
<td>+21.22</td>
</tr>
<tr>
<td>X</td>
<td>4-inch, 20° (moldboard)</td>
<td>1.47</td>
<td>46.00</td>
<td>+3.62</td>
</tr>
<tr>
<td>XI</td>
<td>1.375 inch (disk)</td>
<td>0.11</td>
<td>113.6</td>
<td>-20.95</td>
</tr>
</tbody>
</table>

The differences shown in Table 14 may be referred to the following:

i) Two-dimensional rather than three-dimensional stress approach was undertaken while developing the prediction equation.

ii) The prediction equation was developed from the considerations of steady and incompressible flow of soil. The condition of soil is, however, presumed to vary with the operation of the tillage tool.

iii) The soil strength parameters were not determined under the same stress conditions as occurring in front of the tool.
PREDICTION OF TRACTION

The data of Konaka (1967) obtained on vertical grousered plates operating under pressure were used in order to test the mechanics of traction equipment.

Konaka's Work

Konaka performed tests on Luton silty clay soil in the model tillage/traction laboratory of the Department of Agricultural Engineering of Iowa State University, Ames, Iowa. Movable soil bins and soil renovation equipment were used to provide desired soil conditions. The grousered plates used were held rigid similar to those used in a tracked vehicle and were constructed from cold rolled steel.

The values of the angle of internal friction and cohesion of soil were determined by using the shear box. The tractive effort, or the horizontal thrust, was measured and recorded with the Offner dynograph.

The grousered plates of different lengths and widths were operated at different magnitudes of sinkages in two soil conditions. The tractive effort was measured at different vertical pressures and speeds of the plates.

The data for vertical grousered plates with medium vertical pressures and sinkage were selected to avoid the effect of slip-sinkage phenomenon and are recorded in Appendices XII and XIII for two different soil conditions.
Calculated Tractive Effort

Equation 14 was used to predict the tractive effort for the vertical grouser. The results were multiplied by the number of grousers to obtain the tractive effort for the whole grousered plate. The values of the calculated tractive effort are entered in Appendices XII and XIII.

Comparison

A review of the Appendices XII and XIII reveals that a wide range of values for the variables is not available for making the proper regression analysis and comparison of the measured and calculated tractive efforts. Effort was made to convert the data into dimensionless parameters in order to reduce the number of variables and facilitate the regression analysis. The measured and calculated tractive efforts were expressed as a function of the different variables as follows:

\[ T = f_1(L_1, D, p, L_2, w, c, \phi) \]

\[ T' = f_2(L_1, D, p, L_2, w, c, \phi) \]

The dimensions and nomenclature of the variables used in the above equations are given below:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T ) measured tractive effort (lb)</td>
<td>( F )</td>
</tr>
<tr>
<td>( T' ) calculated tractive effort (lb)</td>
<td>( F )</td>
</tr>
<tr>
<td>( L_1 ) length of plate (in)</td>
<td>( L )</td>
</tr>
<tr>
<td>( L_2 ) sinkage of plate (in)</td>
<td>( L )</td>
</tr>
<tr>
<td>( D ) width of plate (in)</td>
<td>( L )</td>
</tr>
<tr>
<td>( p ) pressure on plate (lb/in^2)</td>
<td>( FL^{-2} )</td>
</tr>
<tr>
<td>( w ) unit weight of soil (lb/in^3)</td>
<td>( FL^{-3} )</td>
</tr>
</tbody>
</table>
c = cohesion of soil (lb/in^2)

\( \phi = \text{angle of internal friction of soil (radian)} \)

Number of Pi terms \( \geq \) number of variables - number of basic dimensions

The Pi terms were built and expressed into functional relationships as follows:

\[
\frac{T}{pL_1^2} = f_3(D/L_1, L_2/L_1, c/wL_1, c/p, \phi) \tag{73}
\]

\[
\frac{T'}{pL_1^2} = f_4(D/L_1, L_2/L_1, c/wL_1, c/p, \phi) \tag{74}
\]

Table 15 showing the values of Pi terms \( T/pL_1^2 \) and \( T'/pL_1^2 \), in terms of \( L_2/L_1 \) and \( c/p \), was prepared from the data of Appendix XIII, keeping the other Pi terms constant. The values of Pi terms kept constant and which went into the production of Table 15 were as follows:

\( D/L_1 = 0.2, c/wL_1 = 2.487, \) and \( \phi = 0.14 \) radian

Table 15. Values of \( T/pL_1^2 \) and \( T'/pL_1^2 \) in terms of \( L_2/L_1 \) and \( c/p \)

<table>
<thead>
<tr>
<th>( T/pL_1^2 )</th>
<th>( T'/pL_1^2 )</th>
<th>( L_2/L_1 )</th>
<th>( c/p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.076</td>
<td>0.150</td>
<td>0.175</td>
</tr>
<tr>
<td>0.1</td>
<td>0.078</td>
<td>0.158</td>
<td>0.175</td>
</tr>
<tr>
<td>0.1</td>
<td>0.087</td>
<td>0.188</td>
<td>0.175</td>
</tr>
<tr>
<td>0.098</td>
<td>0.073</td>
<td>0.070</td>
<td>0.35</td>
</tr>
<tr>
<td>0.098</td>
<td>0.082</td>
<td>0.087</td>
<td>0.35</td>
</tr>
<tr>
<td>0.098</td>
<td>0.095</td>
<td>0.108</td>
<td>0.35</td>
</tr>
<tr>
<td>0.118</td>
<td>0.078</td>
<td>0.040</td>
<td>0.70</td>
</tr>
<tr>
<td>0.118</td>
<td>0.106</td>
<td>0.064</td>
<td>0.70</td>
</tr>
</tbody>
</table>
From Table 15, Figure 40 was prepared indicating a plot of $T/pL_1^2$ and $T'/pL_1^2$ against $L_2/L_1$. This figure is represented by the following relations:

\[ \frac{T}{pL_1^2} = 0.114 \text{ (measured)} \]  \hfill (75)

\[ \frac{T'}{pL_1^2} = 0.087 \text{ (calculated)} \]  \hfill (76)

Equations 75 and 76 suggest that $T/pL_1^2$ and $T'/pL_1^2$ are not functions of $L_2/L_1$. The slopes of the measured and calculated regression lines were found nonsignificant with their respective $t$ values computed as -1.78 and -0.3. The confidence bands for the lines, as observed from Figure 40, are separated from each other.

Next, the effect of $c/p$ on $T/pL_1^2$ and $T'/pL_1^2$ was studied. The following relations were obtained from the values given in Table 15:

measured:

\[ \frac{T}{pL_1^2} = 0.09 + 0.036 \frac{c}{p} \]  \hfill (77)

confidence limits on intercept = 0.082, 0.098

confidence limits on slope = 0.017, 0.055

standard error of the estimate = 0.0045

correlation coefficient = 0.884

calculated:

\[ \frac{T'}{pL_1^2} = 0.076 \]  \hfill (78)

confidence limits on intercept = 0.067, 0.086

standard error of estimate = 0.01

Figure 41 makes a comparison of the regression lines expressed by Equations 77 and 78. The two lines are different in forms, one having a distinct slope and intercept and the other possessing zero slope. Their confidence bands as indicated by Figure 41 do not fall within each other.
Figure 40. Comparison of measured and calculated Pi terms for traction, using $\frac{L_2}{L_1}$ as abscissa.
Figure 41. Comparison of measured and calculated Pi terms for traction, using $c/p$ as abscissa.
The effect of soil properties (density, cohesion, and angle of internal friction) on traction could not possibly be studied since they were kept constant in Konaka's work as reported in Appendices XII and XIII. Further, it was difficult to draw any logical conclusions regarding the effect of $c/wL_1^2$ on $T/pL_1^2$ and $T'/pL_1^2$, because a maximum of three points was available for graphical presentation and statistical analysis.

Appendices XII and XIII were studied for the percentage differences between the measured and calculated tractive efforts. The range of percentage differences drawn from the two Appendices is given below:

- soil condition 1 (Appendix XII) -- 5.512% to 95.961% (mean = -9.8%)
- soil condition 2 (Appendix XIII) -- 0.33% to 105.335% (mean = -1.5%)

The lack of agreement between the measured and calculated tractive effort parameters as suggested by Figures 40 and 41 and the differences reported above may be attributed to the following:

1. Soil strength parameters were not determined under exactly the same stress conditions as experienced by the tractive device.
2. Metal-soil interface friction was ignored while developing the prediction equation.
3. Two-dimensional rather than three-dimensional stress approach was undertaken while developing the prediction equation.
EMERGENCE OF SEEDLINGS

In previous chapters, efforts were made to measure and predict the forces applied to soils. These forces may affect the soil environment to such an extent that the processes of germination and growth related to plants could seriously be hampered. The use of heavy tractors and other equipment, for example, may produce soil compaction that could greatly upset the balance between the air, solid, and water components of soil. The reduction in the aeration of soil resulting from compaction may obstruct the metabolic activities of the roots. The compaction may increase the strength of soil so that root growth could be impeded. Further, compacting operations are needed to gain good contact between seeds and the soil during planting. In this case, the increase in strength may be detrimental while the added contact is desirable. The amount of compaction or resistance to penetration of the soil for the purpose of plant growth is a subject requiring immediate attention.

The physical environment of soil brought about by the mechanical equipment is, therefore, of great importance in understanding the soil-machine-plant complex. The study of the soil conditions that plants need would help the engineers in designing and developing the proper equipment and practices that would apply proper forces and create those conditions. All of these factors may affect the quality and quantity of food and fibre grown on the soil. This part of the study was, consequently, directed towards investigating the influence of the physical conditions and the resistance of soil on the emergence of plants.
Factors Affecting Emergence

Investigations were carried out to determine the effect of various factors on the emergence of seedlings under controlled laboratory conditions. In a later investigation, as discussed in the following chapter, this information was used to establish a relationship between the soil resistance, as determined by a penetrometer, and the emergence of seedlings.

The important factors which may affect the emergence of seedlings are as follows:

**Seed factors**

1. Variety
2. Quantity planted
3. Viability
4. Size
5. Physiological capacity

**Soil factors**

1. Type
2. Structure
3. Fertility
4. Moisture
5. Density
6. Temperature
7. Crust
8. Drainage
9. pH value
10. Cohesion
11. Angle of internal friction
12. Particle size distribution

Operating factors
1. Equipment used for seedbed preparation
2. Depth of plowing
3. Depth of planting
4. Row width
5. Type of seed covering device
6. Degree of compaction of soil
7. Time of planting
8. Skill of operator

Environmental factors
1. Temperature of atmosphere
2. Humidity
3. Light
4. Diffusion of air and water

This investigation was limited to the determination of the effect of soil factors on emergence. It will be discussed under the following headings:

1. Variables
2. Effect of soil environment
3. Emergence on different days
4. Effect of planting depth
Variables

Laboratory investigations were carried out to evaluate the effects of the following independent variables on the emergence of soybean seedlings, while holding the other variables (soil type, seed, etc.) constant:

1. Bulk density
2. Moisture content
3. Temperature
4. Planting depth

The values used for these independent variables were as follows:

- **Bulk density**: 70, 80, 90, and 100 lb/cft
- **Moisture content**: 16, 20, and 24 percent (dry weight basis)
- **Temperature**: 15, 20, 25, 30, and 35°C
- **Planting depth**: 0.5, 1.0, 1.5, and 2.0 inch

**Effect of Soil Environment**

The effect of soil bulk density, temperature, and moisture content will be described in this section.

**Experimental procedure**

Air dried soil (Colo clay loam) passed through a nine mesh sieve was weighed and the necessary amount of moisture was mixed with it by means of an electric sprayer. Soil was placed in the cylindrical plastic boxes (3 11/16 inch diameter and 5½ inch deep) and compacted to a one-inch depth of soil at a desired bulk density level by means of the hydraulic press. On this compacted soil, ten soybean seeds were planted. The seeds before planting had been passed through a set of screens to obtain uniformity in size. Soil was placed over these seeds and recompacted to ½ inch at the
same bulk density level. Lids were placed on the boxes and sealed with masking tape to avoid loss of moisture by evaporation. The boxes were then placed in five equispaced temperature chambers. Lids were removed daily from the boxes for a short interval for recording emergence and to allow exchange of air. A maximum of 12 days was allowed to complete the emergence process. The experiment was repeated for various levels of initial moisture content and bulk density.

Figure 42 exhibits the seedlings emerging from soil containing an initial moisture content of 20% and compacted to a bulk density of 100 lb/cft. From this figure, a cone of soil may be seen to be pushed upward by the effort of the seedlings. Figure 43 shows some of the seedlings which emerged after eight days from soil with an initial moisture content of 20% and a bulk density of 80 lb/cft. The boxes shown in the Figures 42 and 43 were kept at 30°C.

**Effect of temperature**

Tables 16, 17, and 18 illustrated by Figures 44, 45, and 46 represent a summary of results for the emergence of soybean seedlings. Each recorded value of emergence is the average of values taken from three boxes after 12 days of planting. Complete results are shown in Appendix XIV.

The following equations showing the relationship between temperature \( t \) and percentage emergence \( E \) of the seedlings at different moistures and densities were developed from the data shown in Appendix XIV and reported graphically in Figure 44, 45, and 46:
Figure 42. A cone of soil pushed upward by soybean seedlings after six days of planting (soil initial moisture = 20% and bulk density = 100 lb/cuft)

Figure 43. Soybean seedlings emerged from soil (80 lb/cuft, 20% initial soil moisture) after eight days of planting
Table 16. Percentage emergence at 16% initial moisture content (recorded after 12 days of planting)

<table>
<thead>
<tr>
<th>Bulk density (lb/cft)</th>
<th>Temperature (°C)</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td></td>
<td>10</td>
<td>53</td>
<td>70</td>
<td>53</td>
<td>40</td>
</tr>
<tr>
<td>80</td>
<td></td>
<td>0</td>
<td>30</td>
<td>67</td>
<td>47</td>
<td>33</td>
</tr>
<tr>
<td>90</td>
<td></td>
<td>0</td>
<td>23</td>
<td>56</td>
<td>40</td>
<td>20</td>
</tr>
<tr>
<td>100</td>
<td></td>
<td>0</td>
<td>10</td>
<td>50</td>
<td>26</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 17. Percentage emergence at 20% initial moisture content (recorded after 12 days of planting)

<table>
<thead>
<tr>
<th>Bulk density (lb/cft)</th>
<th>Temperature (°C)</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td></td>
<td>10</td>
<td>70</td>
<td>90</td>
<td>57</td>
<td>53</td>
</tr>
<tr>
<td>80</td>
<td></td>
<td>0</td>
<td>63</td>
<td>90</td>
<td>60</td>
<td>50</td>
</tr>
<tr>
<td>90</td>
<td></td>
<td>0</td>
<td>53</td>
<td>90</td>
<td>67</td>
<td>30</td>
</tr>
<tr>
<td>100</td>
<td></td>
<td>0</td>
<td>50</td>
<td>83</td>
<td>67</td>
<td>30</td>
</tr>
</tbody>
</table>

Table 18. Percentage emergence at 24% initial moisture content (recorded after 12 days of planting)

<table>
<thead>
<tr>
<th>Bulk density (lb/cft)</th>
<th>Temperature (°C)</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td></td>
<td>40</td>
<td>90</td>
<td>93</td>
<td>60</td>
<td>66</td>
</tr>
<tr>
<td>80</td>
<td></td>
<td>40</td>
<td>90</td>
<td>93</td>
<td>80</td>
<td>66</td>
</tr>
<tr>
<td>90</td>
<td></td>
<td>30</td>
<td>90</td>
<td>93</td>
<td>80</td>
<td>50</td>
</tr>
<tr>
<td>100</td>
<td></td>
<td>20</td>
<td>83</td>
<td>93</td>
<td>66</td>
<td>40</td>
</tr>
</tbody>
</table>
Figure 44. Effect of temperature on emergence (recorded after 12 days of planting) at 16% initial moisture content of soil and 1/2-inch planting depth
Figure 45. Effect of temperature on emergence (recorded after 12 days of planting) at 20% initial moisture content of soil and ½-inch planting depth
Figure 46. Effect of temperature on emergence (recorded after 12 days of planting) at 24% initial moisture content of soil and ½-inch planting depth
16% moisture:

\[ E = -225.63 + 22.15 t - 0.419 t^2 \] (70 lb/cft) \hspace{1cm} (79)

\[ (R = 0.93, S = 8.58) \]

\[ E = -231.53 + 21.19 t - 0.39 t^2 \] (80 lb/cft) \hspace{1cm} (80)

\[ (R = 0.934, S = 8.6) \]

\[ E = -224.86 + 20.66 t - 0.39 t^2 \] (90 lb/cft) \hspace{1cm} (81)

\[ (R = 0.892, S = 10.3) \]

\[ E = -190.67 + 17.4 t - 0.333 t^2 \] (100 lb/cft) \hspace{1cm} (82)

\[ (R = 0.81, S = 12.156) \]

20% moisture:

\[ E = -909.5 + 111.15 t - 4.033 t^2 + 0.046 t^3 \] (70 lb/cft) \hspace{1cm} (83)

\[ (R = 0.974, S = 6.97) \]

\[ E = -852.71 + 99.93 t - 3.45 t^2 + 0.038 t^3 \] (80 lb/cft) \hspace{1cm} (84)

\[ (R = 0.96, S = 10.1) \]

\[ E = -382.96 + 35.75 t - 0.686 t^2 \] (90 lb/cft) \hspace{1cm} (85)

\[ (R = 0.983, S = 6.347) \]

\[ E = -359.25 + 33.439 t - 0.638 t^2 \] (100 lb/cft) \hspace{1cm} (86)

\[ (R = 0.944, S = 11.211) \]

24% moisture:

\[ E = -917.8 + 121.0 t - 4.7 t^2 + 0.057 t^3 \] (70 lb/cft) \hspace{1cm} (87)

\[ (R = 0.88, S = 11.864) \]

\[ E = -605.36 + 77.32 t - 2.755 t^2 + 0.031 t^3 \] (80 lb/cft) \hspace{1cm} (88)

\[ (R = 0.971, S = 5.53) \]

\[ E = -279.72 + 29.648 t - 0.581 t^2 \] (90 lb/cft) \hspace{1cm} (89)

\[ (R = 0.907, S = 12.79) \]
\[ E = -789.6 + 95.42\, t - 3.3\, t^2 + 0.036\, t^3 \quad (100\, \text{lb/cft}) \quad (90) \]

(R = 0.957, S = 9.589)

where:

- \( E \) = estimated percentage emergence
- \( t \) = temperature (°C)
- \( R \) = correlation coefficient
- \( S \) = standard error of the estimated value

The above results indicate that a curvilinear relationship exists between the temperature and emergence of seedlings at all moistures and densities considered in the experiments. The emergence increases with the rise of temperature to a certain limit beyond which it begins to decline.

**Optimum temperature**

To determine the optimum temperature for emergence, Figures 44, 45, and 46 and Equations 79 to 90 were studied for the type of functions involved. Each graph in the figure and its equation were found to represent a unimodal function.

Computer subroutine GOLD written by Mischke (1967) was employed to determine the optimum temperature for emergence from the unimodal functions represented by Equations 79 to 90 corresponding to different moistures and densities. The results are shown in Table 19. The table shows that the optimum temperature for emergence varies from about 22°C to 27°C over the range of soil moisture and density conditions studied. The extreme percentage emergence corresponding to the optimum temperature at different moisture and density has also been shown in the table.
Table 19. Optimum temperature for emergence

<table>
<thead>
<tr>
<th>Moisture (%)</th>
<th>Density (lb/cft)</th>
<th>Optimum temperature (°C)</th>
<th>Extreme emergence (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>70</td>
<td>26.45</td>
<td>67.2</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>27.16</td>
<td>56.3</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>26.45</td>
<td>48.3</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>26.10</td>
<td>36.4</td>
</tr>
<tr>
<td>20</td>
<td>70</td>
<td>23.06</td>
<td>84.3</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>24.02</td>
<td>84.2</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>26.08</td>
<td>83.1</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>26.21</td>
<td>78.8</td>
</tr>
<tr>
<td>24</td>
<td>70</td>
<td>21.50</td>
<td>94.6</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>23.23</td>
<td>96.5</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>25.50</td>
<td>98.5</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>23.30</td>
<td>94.7</td>
</tr>
</tbody>
</table>

Effect of density

Figure 47 shows the effect of density on the emergence of seedlings at different moisture conditions, when the temperature was maintained at 20°C. The values of percentage emergence have been drawn from Appendix XIV. The coefficient of variation for each variable is shown in Table 20.

Table 20. Coefficient of variation for $\bar{y}$ and emergence (%) at 20°C and different moistures

<table>
<thead>
<tr>
<th>Variable</th>
<th>Moisture (%)</th>
<th>16</th>
<th>20</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density - 62.4 = $\bar{y}$</td>
<td></td>
<td>0.517</td>
<td>0.517</td>
<td>0.517</td>
</tr>
<tr>
<td>Emergence (%) = E</td>
<td></td>
<td>0.611</td>
<td>0.168</td>
<td>0.095</td>
</tr>
</tbody>
</table>
20°C, 1/2-INCH PLANTING DEPTH

- 16% MOISTURE
- 20% MOISTURE
- 24% MOISTURE

Figure 47. Effect of soil density on emergence (recorded after 12 days of planting) at 20°C and different soil moistures
The regression lines in Figure 47 are represented by the following equations:

\[
\begin{align*}
E &= 60.05 - 1.367\, \hat{y} \quad \text{(16% moisture)} \\
\hat{y} &= 79.51 - 0.962\, E \quad \text{(20% moisture)} \\
E &= 92.853 \quad \text{(24% moisture)}
\end{align*}
\]

The confidence limits on parameters, standard error, and correlation coefficient relating to Equations 91, 92, and 93 are given in Table 21.

Figure 47 and Equations 91 and 92 indicate that at moisture contents of 16 and 20% and at a temperature of 20°C, the emergence of seedlings decreases linearly with the density of soil. At 24% soil moisture, the emergence is high and slightly affected by the soil density as may be concluded from the figure.

Table 21. Confidence limits, correlation coefficient, and standard error related to emergence and density

<table>
<thead>
<tr>
<th>Moisture (%)</th>
<th>Calculated value for testing</th>
<th>95% confidence limits for Intercept</th>
<th>95% confidence limits for Slope</th>
<th>Std. error of correlation estimate</th>
<th>Correlation coeff.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Intercept Slope</td>
<td>Upper Lower</td>
<td>Upper Lower</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16(^a)</td>
<td>11.11 -6.37</td>
<td>72.1 48.01</td>
<td>-0.889 -1.845</td>
<td>8.307</td>
<td>0.896</td>
</tr>
<tr>
<td>20(^b)</td>
<td>6.26 -4.54</td>
<td>107.8 51.22</td>
<td>-0.489 -1.434</td>
<td>7.00</td>
<td>0.820</td>
</tr>
<tr>
<td>24(^a)</td>
<td>16.96 (-0.92)(^c)</td>
<td>98.14 87.56</td>
<td>-- --</td>
<td>2.40</td>
<td>--</td>
</tr>
</tbody>
</table>

\(^a\)Indicates the regression of emergence (%) on \(\hat{y}\).

\(^b\)Indicates the regression of \(\hat{y}\) on emergence (%).

\(^c\)t value given in parentheses indicates the nonsignificance of the parameter.
Emergence on different days

Tables 22 and 23 show some of the results of cumulative percentage emergence on different days after planting. The complete results are shown in Appendix XV. Each recorded value is the average of values taken from three boxes. The coefficient of variation for the variables (day and emergence) pertaining to the selected data is given in Tables 24 and 25. Since the coefficient of variation for emergence was higher than for days, as observed from Tables 24 and 25, the regression of emergence on days was carried out.

The data given in Tables 22 and 23 are shown diagramatically by Figures 48a and 48b, wherein the plotted points have been joined by straight lines. The results have been reproduced in Figures 49a and 49b, which show the best lines of fit passed through the plotted points. The following regression equations were obtained from the data given in Tables 22 and 23 and represented by Figures 49a and 49b, using the regression of days on emergence:

Table 22. Cumulative percentage emergence (16% moisture and 80 lb/cft)

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>20</td>
<td>-</td>
<td>-</td>
<td>3</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>25</td>
<td>13</td>
<td>30</td>
<td>40</td>
<td>60</td>
<td>67</td>
</tr>
<tr>
<td>30</td>
<td>-</td>
<td>10</td>
<td>30</td>
<td>40</td>
<td>47</td>
</tr>
<tr>
<td>35</td>
<td>10</td>
<td>20</td>
<td>33</td>
<td>33</td>
<td>33</td>
</tr>
</tbody>
</table>
Table 23. Cumulative percentage emergence (24% moisture and 25°C)

<table>
<thead>
<tr>
<th>Bulk density (lb/cft)</th>
<th>Days after planting</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4</td>
</tr>
<tr>
<td>70</td>
<td>13</td>
</tr>
<tr>
<td>80</td>
<td>20</td>
</tr>
<tr>
<td>90</td>
<td>10</td>
</tr>
<tr>
<td>100</td>
<td>27</td>
</tr>
</tbody>
</table>

Table 24. Coefficient of variation for day and emergence at different temperatures (16% moisture and 80 lb/cft)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20</td>
</tr>
<tr>
<td>day = d</td>
<td>0.648</td>
</tr>
<tr>
<td>emergence (%) = E</td>
<td>1.652</td>
</tr>
</tbody>
</table>

Table 25. Coefficient of variation for day and emergence at different densities (24% moisture and 25°C)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Density (lb/cft)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>70</td>
</tr>
<tr>
<td>day = d</td>
<td>0.648</td>
</tr>
<tr>
<td>emergence (%) = E</td>
<td>0.702</td>
</tr>
</tbody>
</table>
16% MOISTURE, 80 lb/cft

○ 20°C
△ 25°C
□ 30°C
× 35°C

(1/2- INCH PLANTING DEPTH)

Figure 48a. Effect of time on emergence at 80 lb/cft and 16% soil moisture (points joined by straight lines)
Figure 48b. Effect of time on emergence at 24% moisture and 25°C (points joined by straight lines)
Figure 49a. Effect of time on emergence at 80 lb/cft and 16% soil moisture (best lines of fit shown)
Figure 49b. Effect of time on emergence at 24% soil moisture and 25°C (best lines of fit shown)
16% moisture, 80 lb/cft:

\[ E = 0.151 d^2 \quad (20^\circ\text{C}) \]  
\[ E = 5.433 d \quad (25^\circ\text{C}) \]  
\[ E = 0.359 d^2 \quad (30^\circ\text{C}) \]  
\[ E = 3.194 d \quad (35^\circ\text{C}) \]  

24% moisture, 25^\circ\text{C}:

\[ E = 8.739 d \quad (70 \text{ lb/cft}) \]  
\[ E = 8.194 d \quad (80 \text{ lb/cft}) \]  
\[ E = 7.794 d \quad (90 \text{ lb/cft}) \]  
\[ E = 8.122 d \quad (100 \text{ lb/cft}) \]  

The statistical information related to Equations 94 to 101 is given in Table 26.

Table 26. Confidence limits, standard error, and correlation coefficient related to emergence at different soil conditions

<table>
<thead>
<tr>
<th>Soil condition</th>
<th>Calculated t value for testing</th>
<th>Confidence limits for slope</th>
<th>Std. error of estimate</th>
<th>Correlation coeff.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Intercept</td>
<td>Slope</td>
<td>Upper</td>
<td>Lower</td>
</tr>
<tr>
<td>16% m.c.; 80 lb/cft:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20^\circ\text{C}</td>
<td>(-1.35)(^b)</td>
<td>4.91</td>
<td>0.231</td>
<td>0.072</td>
</tr>
<tr>
<td>25^\circ\text{C}</td>
<td>(-1.2)</td>
<td>20.2</td>
<td>6.125</td>
<td>4.741</td>
</tr>
<tr>
<td>30^\circ\text{C}</td>
<td>(-0.238)</td>
<td>13.45</td>
<td>0.427</td>
<td>0.290</td>
</tr>
<tr>
<td>35^\circ\text{C}</td>
<td>(-0.204)</td>
<td>14.0</td>
<td>3.78</td>
<td>2.61</td>
</tr>
<tr>
<td>24% m.c.; 25^\circ\text{C}:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>70 lb/cft</td>
<td>(-0.026)</td>
<td>10.9</td>
<td>10.798</td>
<td>6.68</td>
</tr>
<tr>
<td>80 lb/cft</td>
<td>(-0.476)</td>
<td>21.0</td>
<td>9.20</td>
<td>7.20</td>
</tr>
<tr>
<td>90 lb/cft</td>
<td>(-1.008)</td>
<td>13.45</td>
<td>9.282</td>
<td>6.307</td>
</tr>
<tr>
<td>100 lb/cft</td>
<td>(-0.537)</td>
<td>22.8</td>
<td>9.04</td>
<td>7.21</td>
</tr>
</tbody>
</table>

\(^a\) m.c. stands for moisture content.

\(^b\) t value shown in parentheses indicates the nonsignificance of parameter.
Equations 94 to 101 and Figures 48a to 49b indicate that emergence is a function of time. Most of the equations represent linear increase of emergence with time for the selected data reported in Tables 22 and 23.

**Effect of different variables on emergence**

The method of multiple regression was used to estimate the emergence of seedlings, considering the effects of moisture, density, temperature, and time. The regression equation developed from the data shown in Appendix XV is presented below, based upon the highest multiple correlation coefficient and the least standard error of the estimated value:

\[
E = -251.18 + 3.404M - 0.498w + 15.77t + 9.623d - 0.289t^2 \quad (102)
\]

where:

- \( E \) = estimated emergence (\%)
- \( M \) = moisture of soil (\%)
- \( w \) = unit weight or density of soil (lb/cft)
- \( t \) = temperature of environment (°C)
- \( d \) = number of days

The multiple correlation coefficient was computed as 0.857 and the standard error of the estimated value 15.30. The value of \( t \)-statistic and the standard error of the regression coefficient associated with each variable are given in Table 27.

From Table 27, it may be concluded that temperature and moisture have clear effect on emergence, since the associated values of \( t \)-statistic are very high.

It is worth mentioning here that planting depth could not be considered as a variable in the multiple regression analysis because of the
Table 27. Value of t and standard error for the regression coefficient associated with different variables affecting emergence

<table>
<thead>
<tr>
<th>Variable</th>
<th>Standard error</th>
<th>t-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>moisture = M</td>
<td>0.279</td>
<td>12.18</td>
</tr>
<tr>
<td>density = w</td>
<td>0.089</td>
<td>-5.588</td>
</tr>
<tr>
<td>temperature = t</td>
<td>1.139</td>
<td>13.851</td>
</tr>
<tr>
<td>day = d</td>
<td>2.272</td>
<td>4.235</td>
</tr>
<tr>
<td>(temperature)^2 = t^2</td>
<td>0.023</td>
<td>-12.683</td>
</tr>
</tbody>
</table>

limited data collected on the effect of the depth of planting on emergence. The effect of planting depth was, therefore, considered separately and has been discussed in the section that follows.

Effect of Planting Depth

Investigations were carried out to study the effect of depth of planting on the emergence of soybean seedlings. Controlled laboratory conditions, as adopted for the previous experiments, were maintained to study such an effect. Experiments were performed on the same soil as used before. The depth of planting was varied from 0.5 to 2.0 inches. The results are entered in Table 28.

The following equations showing the relationship between the emergence and depth of planting were obtained using the data of Table 28:

\[ E = 100.46 - 12.248 L^2 \]  
\[ (70 \text{ lb/cft}) \] 
\[ (R = 0.833, S = 12.63) \]
Table 28. Effect of planting depth on emergence at two different densities (20% moisture and 25°C)

<table>
<thead>
<tr>
<th>Density planting (lb/cft)</th>
<th>Depth of planting (in)</th>
<th>Emergence after 12 days (%)</th>
<th>Mean emergence after 12 days (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Box 1</td>
<td>Box 2</td>
<td>Box 3</td>
</tr>
<tr>
<td>70</td>
<td>0.5</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>20</td>
<td>60</td>
</tr>
<tr>
<td>90</td>
<td>0.5</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>50</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

\[ E = 105.08 - 23.153 L^2 \quad (90 \text{ lb/cft}) \quad (104) \]

\( (R = 0.931, S = 14.15) \)

where:

- \( E = \) emergence (%)
- \( L = \) depth of planting (in)
- \( R = \) correlation coefficient
- \( S = \) standard error of the estimated value

The above results are shown graphically by the falling curves in Figure 50. The emergence decreases with the depth of planting in accordance with the Equations 103 and 104 corresponding to different densities of soil.
Figure 50. Effect of planting depth on emergence (recorded after 12 days of planting)
SOIL RESISTANCE AND EMERGENCE OF SEEDLINGS

In the preceding chapter, it was demonstrated that the seedling emergence is related to an array of variables such as soil moisture, density, temperature, time, and depth of planting. In this chapter, the effect of soil resistance on seedling emergence will be discussed.

The resistance of soil is usually determined by means of a penetrometer giving an indication of soil strength and the physical impedance that is encountered by an emerging seedling. The preceding chapter presented the results on seedling emergence under different soil conditions. Experiments were performed on penetration resistance under similar conditions with a view to establish a relationship between the emergence of seedlings and the resistance of soil.

Effect of Density

In the preceding chapter, Figure 47 based upon the data drawn from Appendix XIV was presented to illustrate the effect of density at different moistures on the emergence of seedlings recorded after 12 days of planting from soil boxes maintained at 20°C and having $\frac{1}{2}$-inch depth of planting. To study the effect of density on soil resistance under the same conditions, boxes of soil with $\frac{1}{2}$-inch depth were prepared at different densities and moisture and kept covered in a constant temperature chamber of 20°C. After 12 days, the force reading was taken from these boxes with the help of a penetrometer of 0.364-inch in diameter, equal to the soybean's cotyledon size. The value of force was divided by the area of the penetrometer to obtain the value of the pressure. The values of pressure thus obtained are recorded in Table 29 along with the corresponding values of percentage.
Table 29. Pressure and emergence after 12 days (20°C and ½-inch depth)

<table>
<thead>
<tr>
<th>Moisture (%)</th>
<th>Density (lb/cft)</th>
<th>Pressure (psi)</th>
<th>Emergence (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Box 1</td>
<td>Box 2</td>
</tr>
<tr>
<td>16</td>
<td>70</td>
<td>60.1</td>
<td>60.1</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>90.55</td>
<td>87.34</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>139.4</td>
<td>165.1</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>267.6</td>
<td>270.8</td>
</tr>
<tr>
<td>20</td>
<td>70</td>
<td>52.9</td>
<td>36.9</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>68.1</td>
<td>68.9</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>102.6</td>
<td>90.55</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>138.6</td>
<td>137.8</td>
</tr>
<tr>
<td>24</td>
<td>70</td>
<td>32.86</td>
<td>36.86</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>43.27</td>
<td>39.74</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>56.09</td>
<td>69.9</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>78.53</td>
<td>73.72</td>
</tr>
</tbody>
</table>

emergence drawn from Appendix XIV. Each value of pressure recorded in the table is the average of three values taken from each box.

Figure 51 illustrates the effect of density on the pressure applied to the penetrometer at different moistures. The coefficients of variation for \( \dot{Y} \) and pressure were computed to be:

- pressure at 16% moisture ---- 0.577
- pressure at 20% moisture ---- 0.421
- pressure at 24% moisture ---- 0.305
- \( \dot{Y} \) at all moistures ---- 0.517

The following regression equations were obtained from the data plotted in Figure 51 and recorded in Table 29:
Figure 51. Effect of soil density on pressure applied to penetrometer at different soil moistures
16% moisture:
\[ p = 6.415 \dot{Y} \]  
\( (R = 0.96, S = 23.87) \)  \( (105) \)

20% moisture:
\[ p = 17.297 + 3.097 \dot{Y} \]  
\( (R = 0.984, S = 6.947) \)  \( (106) \)
\[ \dot{Y} = -4.678 + 0.313 p \]  
\( (R = 0.984, S = 2.211) \)  \( (107) \)

24% moisture:
\[ p = 22.408 + 1.347 \dot{Y} \]  
\( (R = 0.976, S = 3.661) \)  \( (108) \)
\[ \dot{Y} = -14.79 + 0.707 p \]  
\( (R = 0.976, S = 2.653) \)  \( (109) \)

where:
- \( p \) = pressure (psi)
- \( \dot{Y} \) = density - 62.4
- \( R \) = correlation coefficient
- \( S \) = standard error of the estimate

It may be noted that the regression lines shown in Figure 51 using the regression of pressure on \( \dot{Y} \) and vice versa coincided with each other for 20 and 24% moisture contents.

To visualize the effect of density on both the emergence of seedlings and pressure applied to the penetrometer at the same time, Figures 47 and 51 were combined and reproduced in Figure 52 for 16 and 20% moisture contents. At these moistures, an increase of pressure and decrease of emergence with the density may be observed.
Figure 52. Effect of density on emergence and pressure applied to penetrometer
Figure 53 prepared from Table 29 shows a relationship between the pressure (soil resistance) and the emergence of seedlings. The coefficients of variation for pressure and emergence were computed and are given in Table 30.

Table 30. Coefficient of variation for pressure and emergence at different moistures

<table>
<thead>
<tr>
<th>Variable</th>
<th>Moisture (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>16</td>
</tr>
<tr>
<td>pressure (psi) = p</td>
<td>0.576</td>
</tr>
<tr>
<td>emergence (%) = E</td>
<td>0.611</td>
</tr>
</tbody>
</table>

The following linear regression equations were obtained from the data on pressure and emergence recorded in Table 29 and plotted in Figure 53:

\[
E = 54.017 - 0.174 p \quad (16\% \text{ moisture}) \\
E = 81.422 - 0.246 p \quad (20\% \text{ moisture}) \\
p = 243.67 - 2.61 E \quad (20\% \text{ moisture}) \\
E = 95.6 \quad (24\% \text{ moisture})
\]

It may be noted that the regression line for 20% moisture drawn in Figure 53 represents Equation 112, showing the regression of pressure on emergence. The confidence limits on the regression coefficients, standard error of the estimated value, and correlation coefficient referring to Equations 110 to 113 are given in Table 31.

It may be observed that the emergence of seedlings decreases with the pressure (soil resistance) at 16 and 20% moisture contents as suggested by Figure 53 and Equations 110 and 111. At 24% soil moisture, the emergence
Figure 53. Effect of pressure (soil penetration resistance) on emergence after 12 days
Table 31. Confidence limits, standard error, and correlation coefficient between pressure and emergence

<table>
<thead>
<tr>
<th>Moisture (%)</th>
<th>t value for Intercept</th>
<th>Conf. limits on Intercept</th>
<th>Conf. limits on Slope</th>
<th>Std. error of estimate</th>
<th>Correlation coeff.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Upper</td>
<td>Lower</td>
<td>Upper</td>
<td>Lower</td>
</tr>
<tr>
<td>16&lt;sup&gt;a&lt;/sup&gt;</td>
<td>8.144</td>
<td>-4.28</td>
<td>68.8</td>
<td>39.24</td>
<td>-0.834</td>
</tr>
<tr>
<td>20&lt;sup&gt;a&lt;/sup&gt;</td>
<td>14.94</td>
<td>-4.24</td>
<td>93.6</td>
<td>69.29</td>
<td>-0.117</td>
</tr>
<tr>
<td>20&lt;sup&gt;b&lt;/sup&gt;</td>
<td>6.5</td>
<td>-4.25</td>
<td>327.3</td>
<td>160.1</td>
<td>-1.239</td>
</tr>
<tr>
<td>24&lt;sup&gt;a&lt;/sup&gt;</td>
<td>10.99 (-0.87)&lt;sup&gt;c&lt;/sup&gt;</td>
<td>100.89</td>
<td>90.31</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

<sup>a</sup>Indicates the regression of emergence on pressure.

<sup>b</sup>Indicates the regression of pressure on emergence.

<sup>c</sup>t value in parentheses indicates the nonsignificance of parameter.

may be slightly affected by pressure (soil resistance) as concluded from the figure.

**Effect of Depth**

In the preceding chapter, Table 28 supported by Figure 50 was presented to illustrate the effect of planting depth at 20% soil moisture on emergence recorded after 12 days of planting on soil boxes maintained at 25°C and having two different densities of 70 and 90 lb/cft. To study the effect of soil depth on the resistance of soil under similar conditions, boxes of soil with 70 and 90 lb/cft density and having a moisture content of 20% were prepared and kept covered in a constant temperature chamber of 25°C. The depth of soil was varied from 0.5 to 2.0 inches in an increment of 0.5 inch. After 12 days, the pressure reading was obtained from these
boxes with the 0.364-inch diameter penetrometer. The values of pressure thus obtained have been tabulated in Table 32. Each value of the pressure recorded in the table is the average of three values taken from each box.

Table 32. Effect of soil depth on pressure at two different densities (20% moisture and 25°C)

<table>
<thead>
<tr>
<th>Density (lb/cuft)</th>
<th>Depth of soil (in)</th>
<th>Pressure (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Box 1</td>
</tr>
<tr>
<td>70</td>
<td>0.5</td>
<td>60.10</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>87.34</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>96.95</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>100.96</td>
</tr>
<tr>
<td>90</td>
<td>0.5</td>
<td>120.99</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>155.44</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>144.23</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>160.26</td>
</tr>
</tbody>
</table>

Figure 54 prepared from Table 32 shows the nonlinear effect of soil depth on the resistance (pressure) of soil at two different densities. The following regression equations refer to the results shown in Table 32 and Figure 54:

70 lb/cuft:

\[ p = 123.52 L - 38.23 L^2 \]  
\( (R = 0.994, S = 4.728) \)

90 lb/cuft:

\[ p = 226.08 L - 81.12 L^2 \]  
\( (R = 0.95, S = 20.67) \)

where:
Figure 54. Effect of soil depth on pressure applied to penetrometer after 12 days
\[ p = \text{pressure (psi)} \]
\[ L = \text{depth of soil (in)} \]
\[ R = \text{correlation coefficient} \]
\[ S = \text{standard error of estimate} \]

To visualize the effect of depth on both the emergence of seedlings and the resistance (pressure) of soil simultaneously, Figures 50 and 54 were combined and reproduced in Figure 55 for two different densities. It may be observed from Figure 55 that the emergence decreases with the planting depth following the relationships given by Equations 103 and 104 corresponding to two different densities; whereas the resistance of soil increases with the depth of soil according to Equations 114 and 115.
Figure 55. Effect of depth on emergence and pressure (soil penetration resistance) recorded after 12 days.
SUMMARY

This study consisted of the following phases of investigations:

I. To measure the penetration resistance of soil under different soil conditions.

II. To present and test the mechanics of soil cutting equipment.

III. To study the seedling emergence and its relationship with the resistance of soil.

To measure the resistance of soil, penetration tests were carried out in Colo clay loam soil at different moistures and densities with penetrometers of different diameters. The resistance increased linearly with the density of soil at different moistures and decreased with the moisture of soil generally following a quadratic relationship at various densities under nondrying conditions. Further, the penetration resistance of soil increased with the area of the penetrometer and drying time.

To present the mechanics, a two-dimensional stress approach was undertaken for the following types of soil cutting equipment:

1. Penetration equipment
2. Tillage equipment
3. Traction equipment

The prediction equation for the soil penetration resistance, derived from the analysis of forces acting on the penetrometer, was found to be a function of penetrometer diameter, depth of penetration, soil density, cohesion, angle of internal friction, and soil to metal friction. The soil strength parameters (cohesion, angle of internal friction, and coefficient of soil to metal friction) were determined with the help of the shear box.
The parameters were substituted in the prediction equation to predict the resistance of soil. The measured and predicted (skin friction) regression lines generally did not lie within 95% confidence bands. However, the percentage differences between the values ranged from 8 to 117%, with the mean value computed as -27.9% of the measured values.

A general prediction equation, employing Housel's approach, was developed to describe the penetration resistance of soil within the measured ranges and is given below:

\[ F = A \sqrt{0.0586 M^2 - 2.778 M + 37.12} + A d (5.66 \sqrt{\gamma} + 11.578 M - 194.1) \]

where:

- \( F \) = penetration resistance of soil (lb)
- \( A \) = area of penetrometer (in\(^2\))
- \( \sqrt{\gamma} \) = unit weight of soil - unit weight of water (lb/cft)
- \( M \) = moisture content of soil (%)
- \( d \) = number of days of drying

The first term in this equation represents the penetration resistance of soil under nondrying conditions, and the second term takes into account the effect of air drying. This equation was found least accurate for low moisture and low density, and the measured and predicted values for the most part did not lie within 95% confidence bands. However, the percentage differences between the measured and predicted values ranged from 0.68% to 45%, excluding the low moisture and low density data. The mean of all the percentage differences was found as -5.62%.

The prediction equation for the draft of tillage equipment was tested by studying the regression lines for the measured and predicted values drawn with respect to the density of soil and the velocity of tool. The
regression lines mostly did not lie within 95% confidence bands. The percentage differences between the measured and predicted drafts, however, varied from 0.112% to 113.5%, having a maximum mean value of 21% of the measured values, over the range of data studied. The differences were referred to the following:

i) Two-dimensional rather than three-dimensional stress approach was undertaken in developing the prediction equation.

ii) The soil strength parameters were not determined under the same stress conditions as experienced by the tool.

iii) Prediction equation was developed from the standpoint of steady and incompressible flow of soil.

The predicted values for the tractive effort of traction equipment were compared with the measured values to test the mechanics of traction equipment. The data were converted into dimensionless parameters in order to reduce the number of variables. The measured and the predicted values did not lie within 95% confidence bands. The differences, however, ranged from 0.33 to 105% with a maximum mean value of -9.8%. The differences were attributed to the following:

i) Soil strength parameters not determined under similar conditions as occurring beneath the tractive device.

ii) Two-dimensional approach undertaken in presenting the mechanics.

iii) Soil-metal interface friction ignored in the prediction equation.

To study the emergence of seedlings and its relationship with the resistance of soil, tests were carried out on soybean seeds in Colo clay loam soil at different soil moisture, temperature, density, and depth of
planting under nondrying conditions. The penetration tests were carried out under similar conditions. The following results were obtained:

i) A curvilinear relationship existed between the temperature and emergence of seedlings. The optimum temperature for emergence varied from about 22°C to 27°C for different soil moistures and densities.

ii) At lower moisture contents, the emergence of seedlings decreased linearly with the density of soil. At higher moisture content, the emergence was high and slightly affected by the density of soil.

iii) An indirect relationship existed between the soil resistance and the emergence of seedlings at lower moisture contents. At higher moisture content, the emergence was slightly affected by the penetration resistance of soil.

iv) The emergence decreased and the soil resistance increased with the depth of planting in a nonlinear fashion.
The following conclusions were drawn from this study:

1. The penetration resistance of soil increased linearly with its density at different moistures and decreased with its moisture generally following a quadratic relationship at different densities under nondrying conditions.

2. The resistance of soil increased with the area of the penetrometer and drying time.

3. A general equation developed to describe the penetration resistance of soil within the ranges measured was given by:

   \[ F = A \gamma (0.0586 M^2 - 2.778 M + 37.122) + A d (5.661 \gamma + 11.578 M - 194.1) \]

   where \( F \) is penetration resistance of soil (lb), \( A \) is area of the penetrometer (in\(^2\)), \( M \) is initial moisture of soil (%), \( d \) is drying time (days), and \( \gamma \) is unit weight of soil minus unit weight of water (lb/cft). The first term represents the resistance under nondrying conditions and the second term incorporates the effect of air drying. This equation was least accurate for low moisture contents and low density, and the measured and predicted values for the most part did not lie within 95% confidence bands. However, the error of prediction ranged from about 0.68% to 45% (with a mean of -5.6%) excluding the low moisture-low density data.

4. Soil mechanics using \( c \) and \( \phi \) values from the shear box tests did not reliably predict the measured penetration resistance, probably
due to different stress conditions between the shear box and penetration tests.

5. The measured and predicted drafts for the most part did not lie within 95% confidence bands. The differences, however, varied from 0.11% to 114%, with the maximum mean value of 21% for the data relating to different types of tillage tools considered in the study.

6. The measured and predicted tractive efforts for the groused plates did not fall within 95% confidence bands. However, the differences varied from 0.33% to 105% with the maximum mean value of 10% of the measured values for the data considered in the study.

7. A curvilinear relationship existed between the temperature and emergence of seedlings under nondrying conditions. The optimum temperature for the emergence of soybean seedlings varied from about 22°C to 27°C over the range of soil conditions studied.

8. Under nondrying conditions, the emergence of soybean seedlings in Colo clay loam soil decreased with the density of soil at lower moisture contents. At higher moisture content, the emergence was high and slightly affected by the density of soil.

9. Temperature and moisture had clear effect on the emergence of seedlings.

10. At lower moisture contents, the emergence of seedlings decreased linearly with the penetration resistance of soil. At higher moisture content, the emergence was slightly affected by the resistance of soil.
11. The soil resistance increased, and the emergence of soybean seedlings decreased in a nonlinear fashion with the depth of planting under nondrying conditions.
SUGGESTIONS FOR FUTURE RESEARCH

The following suggestions are advanced for future investigation:

1. The prediction equations for the forces applied to the soil cutting equipment be developed from the three-dimensional stress approach and tested over a wide range of soil conditions.

2. Unsteady and compressible state of flow of soil be considered in the development of prediction equations.

3. The penetration resistance, draft, and traction be measured for a wide range of soil, equipment, and operating conditions.

4. To be most useful, field measurements on the full size of soil cutting equipment be carried out and compared with the predicted values for testing the mechanics.

5. The soil strength parameters be determined under similar soil conditions as occurring in front of tools.

6. Investigations for the emergence of seedlings be carried out under drying conditions.

7. Extensive investigations be undertaken to predict the effect of depth of planting on the emergence of seedlings.

8. Optimum conditions of soil moisture, density, and temperature be determined for the emergence of seedlings on different soils under field conditions.

9. Investigations be undertaken to determine the effect of the type and shape of traction device on the emergence of seedlings.
LITERATURE CITED


Edwards, F. E. 1966. The factors affecting uniform emergence of cotton seedlings in Houston clay soil. Mississippi Agricultural Experiment Station Information Sheet 953.


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The following facts are worth mentioning in connection with the preparation of this manuscript:

1) Experiments were planned and conducted and research investigations carried out under the active guidance of Dr. W. F. Buchele, Professor of Agricultural Engineering.

2) Statistical analysis of the data, computer work, and incorporation of theoretical aspects related to the mechanical equipment used in soil were performed under the able guidance of Dr. C. R. Mischke, Professor of Mechanical Engineering.
3) Soil mechanics aspects related to the soil cutting equipment were incorporated under the helpful guidance of Dr. R. L. Handy, Professor of Civil Engineering.

4) Valuable advice in the work related to the emergence of seedlings was received from Dr. J. S. Burris, Professor of Botany.

5) The liaison between the studies on the forces applied to the mechanical equipment and emergence of seedlings was established under the directions of Dr. L. C. Peters, Professor of Mechanical Engineering.

6) Overall encouragement and moral support were graciously provided by Dr. C. W. Bockhop, Head of Agricultural Engineering Department. Lastly, the author wishes to express his appreciation to his wife for her patience, unselfish help, and encouragement during the course of this investigation.
Appendix I. Values of Cohesion (c) and Angle of Internal Friction (\(\phi\)) of Colo Clay Loam Soil at Soil Densities of 70, 80, and 90 lb/cft

<table>
<thead>
<tr>
<th>Moisture (% )</th>
<th>Normal Density (lb/cft)</th>
<th>Normal Load (lb)</th>
<th>Torque (in lb)</th>
<th>Normal Stress (psi)</th>
<th>Shear Stress (psi)</th>
<th>c (psi)</th>
<th>tan (\phi) (degrees)</th>
<th>(\phi) (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>70</td>
<td>10</td>
<td>9.3</td>
<td>1.41</td>
<td>1.31</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>70</td>
<td>20</td>
<td>15.5</td>
<td>2.82</td>
<td>2.18</td>
<td>0.87</td>
<td>0.43</td>
<td>23.0</td>
</tr>
<tr>
<td>12</td>
<td>70</td>
<td>30</td>
<td>21.7</td>
<td>4.23</td>
<td>3.06</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>70</td>
<td>40</td>
<td>21.7</td>
<td>5.64</td>
<td>3.06</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>80</td>
<td>10</td>
<td>15.5</td>
<td>1.41</td>
<td>2.18</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>80</td>
<td>20</td>
<td>21.7</td>
<td>2.82</td>
<td>3.06</td>
<td>1.50</td>
<td>0.53</td>
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<tr>
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<td>80</td>
<td>30</td>
<td>27.8</td>
<td>4.23</td>
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Appendix VII. Force Measured and Predicted for Drying Conditions
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Appendix IX. Draft for 20° and Three-inch Moldboard Plow
($\phi = 20^\circ$, width of plow = $D = 3$ inches,
deepth of operation = $L = 1.88$ inches)
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Appendix X. Draft for $20^\circ$ and Four-inch Moldboard Plow
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Appendix XI. Draft for Disk Plow
($\beta = 40^\circ$, $D =$ width of cut = 1.375 inches,
$L =$ depth of cut = 1.5 inches)
| Type of soil | Density (lb/cft) | Moisture (%) | \(c\) (psi) | \(\phi\) (deg) | \(V\) (ft/sec) | Measured draft (lb) | Calculated draft (lb) | Difference (measured-predicted) (lb) | Percent difference | Moisture (%) | \(c\) (psi) | \(\phi\) (deg) | \(V\) (ft/sec) | Measured draft (lb) | Calculated draft (lb) | Difference (measured-predicted) (lb) | Percent difference |
|-------------|-----------------|-------------|------------|--------------|--------------|-----------------|-----------------|-------------------------------|-----------------|-------------|------------|--------------|--------------|-----------------|-----------------|-------------------------------|-----------------|-------------|------------|--------------|--------------|
| Colo | 68.1 | 25.0 | 1.49 | 23.4 | 0.99 | 5.474 | 5.467 | +0.006 | +0.112 | 25.0 | 1.49 | 0.99 | 5.474 | 5.467 | +0.006 | +0.112 |
| Silty | " | " | " | " | 1.49 | 6.384 | 5.491 | +0.893 | +13.989 | 25.0 | 1.49 | " | " | 1.49 | 6.384 | 5.491 | +0.893 | +13.989 |
| Clay | " | " | " | " | 0.98 | 5.210 | 5.467 | -0.257 | -4.941 | 25.0 | 1.49 | " | " | 1.49 | 5.865 | 5.491 | +0.374 | +6.374 |
| Loam | 68.5 | 25.0 | 1.34 | 22.2 | 0.98 | 4.900 | 4.761 | +0.139 | +2.840 | 25.0 | 1.12 | 0.99 | 4.803 | 4.111 | +0.692 | +14.412 |
| Silty | " | " | " | " | 1.49 | 5.102 | 4.136 | +0.966 | +18.934 | 25.0 | 1.12 | " | " | 1.49 | 5.102 | 4.136 | +0.992 | +19.435 |
| Clay | " | " | " | " | 0.98 | 5.102 | 4.136 | +0.992 | +19.435 | 25.0 | 1.12 | " | " | 1.49 | 5.438 | 4.136 | +1.302 | +23.939 |
| Luton | 70.4 | 21.0 | 1.08 | 22.4 | 1.49 | 3.925 | 3.919 | +0.005 | +0.136 | 21.0 | 1.08 | 1.49 | 3.925 | 3.919 | +0.005 | +0.136 |
| Silty | 61.6 | 26.0 | 1.73 | 11.9 | 1.00 | 3.788 | 4.568 | -0.779 | -20.575 | 26.0 | 1.73 | 1.00 | 3.788 | 4.568 | -0.779 | -20.575 |
| Clay | 55.4 | 25.0 | 1.55 | 15.9 | 1.00 | 4.003 | 4.563 | -0.560 | -13.988 | 25.0 | 1.55 | 1.00 | 4.003 | 4.563 | -0.560 | -13.988 |
| Masonry | 94.0 | 2.0 | 0.28 | 19.1 | 1.49 | 0.870 | 1.102 | -7.593 | -87.326 | " | " | 0.28 | 19.1 | 1.49 | 0.870 | 1.102 | -7.593 | -87.326 |
| Sand | 94.0 | 2.0 | 0.47 | 17.1 | 1.50 | 0.752 | 1.606 | -0.854 | -113.552 | " | " | 0.47 | 17.1 | 1.50 | 0.752 | 1.606 | -0.854 | -113.552 |
| Ida | 95.5 | 6.0 | 0.57 | 15.5 | 1.50 | 0.979 | 1.823 | -0.844 | -86.271 | " | " | 0.57 | 15.5 | 1.50 | 0.979 | 1.823 | -0.844 | -86.271 |
| Silt | 95.5 | 6.0 | 0.53 | 17.2 | 1.50 | 1.074 | 1.793 | -0.718 | -66.866 | " | " | 0.53 | 17.2 | 1.50 | 1.074 | 1.793 | -0.718 | -66.866 |
| Loam | 67.2 | 15.0 | 0.66 | 22.7 | 1.49 | 2.142 | 2.481 | -0.339 | -15.822 | 15.0 | 0.66 | 1.49 | 2.142 | 2.481 | -0.339 | -15.822 |
| Loam | 67.5 | 15.0 | 0.61 | 22.5 | 1.49 | 2.142 | 2.294 | -0.075 | -3.360 | 15.0 | 0.61 | 1.49 | 2.142 | 2.294 | -0.075 | -3.360 |
Appendix XII. Tractive Effort for Vertical Groused Plates in Soil Condition 1

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\phi = 11.0^\circ, \text{ number of grousers} = 7\)
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<th>Plate sinkage L (inch)</th>
<th>Velocity V (ft/sec)</th>
<th>Measured effort (lb)</th>
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Appendix XIII. Tractive Effort for Vertical Grousered Plates in Soil Condition 2
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**Note:** The table provides data on plate dimensions, pressure, weight, plate sinkage, velocity, measured and calculated efforts, along with the difference and percent difference.
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Appendix XIV. Percentage Emergence Recorded after 12 Days of Planting (¼-inch planting depth)

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# Appendix XV. Cumulative Percentage Emergence on Different Days

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Appendix XVI. Main Program and Subroutine ME0226
C......MAIN PROGRAM CALLING SUBROUTINE ME0226
C
C......Y= A+ B * X ...........
C
DIMENSION A3(30),A4(30),B7(30),B8(30),B9(30),B10(30),B11(30),
$B12(30)
1 READ(5,10,END=40) N
10 FORMAT(I2)
   READ(5,20) (A3(I),I=1,N), (A4(J),J=1,N)
20 FORMAT(8F10.4)
   READ(5,25) A5,A6
25 FORMAT(2F10.4)
   CALL ME0226(I,N,A3,A4,A5,A6,B1,B2,B3,B4,B5,B6,B7,B8,B9,B10,B11,
$B12,B13,B14,B15,B16,B17)
GO TO 1
40 STOP
END

SUBROUTINE ME0226(MON,N,X,Y,TSTAT,FSTAT,A,B,XBAR,YBAR,SA,SB,UP,
$DOWN,CUP,CDOWN,FUP,FDOWN,R,SYX,SYBARX,SYI,SFUT)
  STRAIGHT LINE REGRESSION OF Y ON X, Y=A+B*X
  THIS SUBROUTINE PERFORMS A LINEAR LEAST SQUARES REG. OF Y ON X
  THE ROUTINE ACCEPTS MONITOR PRINT SIGNAL, NUMBER OF DATA PAIRS, CO
  VECTORS OF DATA, T-STATISTIC AND F-STATISTIC FOR CONFIDENCE BOUNDS
  THE ROUTINE RETURNS INTERCEPT AND SLOPE OF REGRESSION LINE, DATA
  CENTROID COORDINATES, STANDARD DEVIATION OF INTERCEPT AND SLOPE,
  UPPER AND LOWER CONFIDENCE BOUNDS POINT-BY-POINT, UPPER AND LOWER
  CONFIDENCE BOUNDS FOR THE LINE-AS-A-WHOLE,
  CONFIDENCE BOUNDS FOR A FUTURE POINT, CORRELATION COEFFICIENT
  STANDARD DEVIATION OF Y ON X, YBAR ON X, YBARI AND FUTURE
SEE BASIC STATISTICAL METHODS BY NEVILLE AND KENNEDY (1964)

CALLING PROGRAM REQUIREMENTS

PROVIDE THE EQUIVALENT TO THE FOLLOWING DECLARATION:

DIMENSION A3(I2),A4(I2),B7(I2),B8(I2),B9(I2),B10(I2),B11(I2),B12(I)
B16(I2),B17(I2)

CALL LIST ARGUMENTS

I1=0 MONITOR AND PLOTTING TABULATION DOES NOT PRINT
I1=1 MONITOR AND PLOTTING TABULATION PRINTS ON SEPARATE PAGE
I2=NUMBER OF DATA POINT PAIRS
A3=COLUMN VECTOR OF DATA X-COORDINATES
A4=COLUMN VECTOR OF DATA Y-COORDINATES
A5=T-STATISTIC T(I2-2) FOR 1-ALPHA/2, ALPHA = SIGNIFICANCE LEVEL
A6=F-STATISTIC F(2,I2-2) FOR 1-ALPHA, ALPHA = SIGNIFICANCE LEVEL
B1=ORDINATE INTERCEPT OF REGRESSION LINE
B2=SLOPE OF REGRESSION LINE
B3=X-COORDINATE OF DATA CENTROID
B4=Y-COORDINATE OF DATA CENTROID
B5=STANDARD DEVIATION OF ORDINATE INTERCEPT OF REGRESSION LINE
B6=STANDARD DEVIATION OF SLOPE OF REGRESSION LINE
B7=COLUMN VECTOR UPPER CONFIDENCE BOUND POINT-BY-POINT
B8=COLUMN VECTOR LOWER CONFIDENCE BOUND POINT-BY-POINT
B9=COLUMN VECTOR UPPER CONFIDENCE BOUND LINE-AS-A-WHOLE
B10=COLUMN VECTOR LOWER CONFIDENCE BOUND LINE-AS-A-WHOLE
B11=COLUMN VECTOR UPPER CONFIDENCE BOUND FOR FUTURE POINT
B12=COLUMN VECTOR LOWER CONFIDENCE BOUND FOR FUTURE POINT
B13=CORRELATION COEFFICIENT
B14=STANDARD DEVIATION Y ON X, SYX
B15=STANDARD DEVIATION YBAR ON X, SYBARX
B16=COLUMN VECTOR OF STANDARD DEVIATION YBARI ON X, SYBARI
C  B17=COLUMN VECTOR OF STANDARD DEVIATION YFUTURE ON X, SYFUT

C  PREEMPTED NAMES

C  NONE

C  SIZE

DIMENSION X(1), Y(1), UP(1), DOWN(1), CUP(1), CDOWN(1), FUP(1), FDOWN(1),
1  SY(1), SFUT(1)

GO TO 100

1  SUMX=0.
  SUMY=0.
  SUMXX=0.
  SUMYY=0.
  SUMXY=0.
  SUM1=0.
  SUM2=0.
  SUM3=0.
  SUM4=0.

DO 2  I=1,N
  SUMX=SUMX+X(I)
  SUMY=SUMY+Y(I)
  SUMXX=SUMXX+X(I)*X(I)
  SUMYY=SUMYY+Y(I)*Y(I)
  SUMXY=SUMXY+X(I)*Y(I)

2  CONTINUE

XBAR=SUMX/FLOAT(N)
YBAR=SUMY/FLOAT(N)
ANUM=SUMXX*SUMY-SUMX*SUMXY
BNUM=FLOAT(N)*SUMXY-SUMX*SUMY
DENOM=FLOAT(N)*SUMXX-SUMX*SUMX
IF(DENOM.NE.0.) GO TO 1070
WRITE(6,106)
106  FORMAT(' +++ERROR MESSAGE SUBROUTINE ME0226+++++, /,
1* INTERCEPT AND SLOPE UNDETERMINANT, CHECK DATA*)
RETURN
A=ANUM/DENOM
B=BNUM/DENOM
DO 3 I=1,N
SUM1=SUM1+(X(I)-XBAR)*((X(I)-XBAR)
SUM2=SUM2+(Y(I)-YBAR)*((Y(I)-YBAR)
SUM3=SUM3+(X(I)-XBAR)*((Y(I)-YBAR)
SUM4=SUM4+(Y(I)-A-B*X(I)**2
3 CONTINUE
IF(SUM1.NE.0.) GO TO 108
WRITE(6,107)X(I)
107 FORMAT(***ERROR MESSAGE SUBROUTINE ME0226****,/,1'ALL X-OBSERVATIONS SAME,'G15.7,'CHECK DATA')
IERROR=1
108 IF(SUM2.NE.0.) GO TO 110
WRITE(6,109)Y(I)
109 FORMAT(***ERROR MESSAGE SUBROUTINE ME0226****,/,1'ALL Y-OBSERVATIONS SAME,'G15.7,'CHECK DATA')
IERROR=1
110 IF(IERROR.NE.0) RETURN
R=SQR((SUM2-SUM4)/SUM2)
SYX=SQR(SUM4/FLOAT(N-2))
SYBARX=SYX/SQR(FLOAT(N))
SB=SQR(SYX*SYX/SUM1)
S2A=SYX*SYX(1./FLOAT(N)*XBAR*XBAR/SUM1)
SA=SQR(S2A)
AUP=A+TSTAT*SA
ALLOW=A-TSTAT*SA
BUP=B+TSTAT*SB
BLOW=B-TSTAT*SB
YUP=YBAR+TSTAT*SYBARX
YLOW=YBAR-TSTAT*SYBARX
DO 4 I=1,N
YHAT=A+B*X(I)
ARG2=SYX*SYX(1./FLOAT(N)+(X(I)-XBAR)**2/SUM1)
SYI(I)=SQR(ARG2)
UP(I)=YHAT+TSTAT*SYI(I)
DOWN(I)=YHAT-TSTAT*SYI(I)  
CUP(I)=YHAT+ SQRT(2.*FSTAT)*SYI(I)  
COWN(I)=YHAT- SQRT(2.*FSTAT)*SYI(I)  
ARG3=SYX*SYX*(1.+1./FLOAT(N)+(X(I)-XBAR)**2/SUM1)  
SFUT(I)=SQRT(ARG3)  
FUP(I)=YHAT+TSTAT*SFUT(I)  
FDOWN(I)=YHAT-TSTAT*SFUT(I)  
CONTINUE
IF(MON.EQ.0) GO TO 99
WRITE(6,5)
FORMAT(*1',//,  
1' MONITOR AND PLOTTING DISPLAY IOWA CADET SUBROUTINE ME0226 MISCHK  
2E*,//,  
3' X Y Y Y Y Y' Y',//,  
4 Y Y Y Y Y Y' Y',//,  
5' DATA DATA REGRESSION UPPER BOUND LOWER BOUND UPPER BOUND LOWER BOUND',//,  
6UND UPPER BOUND LOWER BOUND UPPER BOUND LOWER BOUND',//,  
7' LINE POINT-BY-POINT POINT-BY-  
8POINT LINE-AS-WHOLE LINE-AS-WHOLE FUTURE POINT FUTURE POINT',//,  
DO 6 I=1,N  
YR=A+B*X(I)  
WRITE(6,9) X(I),Y(I),YR,UP(I),DOWN(I),CUP(I),COWN(I)  
FUP(I),FDOWN(I)  
9 FORMAT(*9IX,G13.6I)  
CONTINUE
WRITE(6,7)A,AUP,ALOW,B,BUP,BLOW,XBAR,YBAR,YUP,YLOW,TSTAT,FSTAT,R,  
1 SUMX,SUMY,SUMXX,SUMYY  
7 FORMAT(/,  
1' ORDINATE INTERCEPT A OF REGRESSION LINE .................',G14.7',/  
2' UPPER CONFIDENCE LIMIT ON ORDINATE INTERCEPT A .........',G14.7',/  
3' LOWER CONFIDENCE LIMIT ON ORDINATE INTERCEPT A .........',G14.7',/  
4' SLOPE B OF REGRESSION LINE ................................',G14.7',/  
5' UPPER CONFIDENCE LIMIT ON SLOPE B .........................',G14.7',/  
6' LOWER CONFIDENCE LIMIT ON SLOPE B ........................',G14.7',/  
7' X-COORDINATE OF DATA CENTROID ............................',G14.7',/  
8' Y-COORDINATE OF DATA CENTROID ............................',G14.7',/  
9'
1* UPPER CONFIDENCE BOUND ON YBAR 160
1* LOWER CONFIDENCE BOUND ON YBAR 161
2* T-STATISTIC T(N-2) FOR (1-ALPHA/2), USER SUPPLIED 162
3* F-STATISTIC F(2,N-2) FOR (1-ALPHA), USER SUPPLIED 163
4* CORRELATION COEFFICIENT R 164
5* SUM OF X 165
6* SUM OF Y 166
7* SUM OF X**2 167
8* SUM OF Y**2 168
WRITE (6,8) SUMXY, SUM1, SUM2, SUM3, SUM4, SYX, SYBARX, SA, SB 169
8 FORMAT (170
1* SUM OF XY 171
2* SUM OF (X-XBAR)**2 172
3* SUM OF (Y-YBAR)**2 173
4* SUM OF (X-XBAR)*(Y-YBAR) 174
5* SUM OF (Y-YHAT)**2 175
6* UNBIASED ESTIMATOR STANDARD DEVIATION Y ON X, SYX 176
7* UNBIASED ESTIMATOR SYBARX 177
8* UNBIASED ESTIMATOR OF STANDARD DEVIATION OF A, SA 178
9* UNBIASED ESTIMATOR OF STANDARD DEVIATION OF B, SB 179
1**
99 RETURN 180
C 181
......PROTECTION...... 182
100 IERROR=0 183
 IF(MON.EQ.0 .OR. MON.EQ.1) GO TO 102 184
 WRITE (6,101) MCN 185
101 FORMAT (**ERROR MESSAGE SUBROUTINE ME0226****',/, 186
 1* ARGUMENT I1,*,I15,*, NOT EQUAL TO ZERO OR ONE') 187
 IERROR=1 188
102 IF(N.GT.2) GO TO 104 189
 WRITE (6,103) N 190
103 FORMAT (**ERROR MESSAGE SUBROUTINE ME0226****',/, 191
 1* ARGUMENT I2,*,I15,*, NOT GREATER THAN TWO') 192
 IERROR=1 193
104 IF(TSTAT.GT.0 .AND. FSTAT.GT.0.) GO TO 1050 194
 WRITE (6,105) TSTAT, FSTAT 195
105 FORMAT('*****ERROR MESSAGE SUBROUTINE ME0226****',/,
1 ', ARGUMENT A5 OR A6,G16.7,G16.7, NOT GREATER THAN ZERO')
ERROR=1
1050 IF(EQERROR,NE,0) RETURN
ERROR=0
GO TO 1
END
Appendix XVII. Main Program and Subroutine ME0227
C... MAIN PROGRAM CALLING SUBROUTINE ME0227
C
C Y = B * X
C
DIMENSION A3(30), A4(30), B5(30), B6(30), B7(30), B8(30), B9(30), B10(30)
      1 READ(5, 10, END=40) N
10 FORMAT(I2)
      READ(5, 20) (A3(I), I=1,N), (A4(J), J=1,N)
20 FORMAT(8F10.4)
      READ(5, 20) A5
      CALL ME0227(1, N, A3, A4, A5, B1, B2, B3, B4, B5, B6, B7, B8, B9, B10, B11, B12,
$813, B14, B15)
      GO TO 1
40 STOP
END

SUBROUTINE ME0227(I1, I2, A3, A4, A5, B1, B2, B3, B4, B5, B6, B7, B8, B9, B10, B11, B12, B13, B14,
$ B15, TSTAT, 8, XBAR, YBAR, SB, UP, DOWN, CUP, CDOWN,
1, FUP, FDOWN, R, SYX, SYBARX, SYBAR, SYI, SFUT)
C STRAIGHT LINE REGRESSION OF Y ON X, Y = B * X Mischke

C THIS SUBROUTINE PERFORMS A LINEAR LEAST SQUARES REG. OF Y ON X
C THE ROUTINE ACCEPTS MONITOR PRINT SIGNAL, NUMBER OF DATA PAIRS, CO
C VECTORS OF DATA, T-STATISTIC FOR CONFIDENCE BOUNDS
C THE ROUTINE RETURNS SLOPE OF REGRESSION LINE, DATA
C CENTROID COORDINATES, STANDARD DEVIATION OF SLOPE,
C UPPER AND LOWER CONFIDENCE BOUNDS POINT-BY-POINT, UPPER AND LOWER
C CONFIDENCE BOUNDS FOR THE LINE-AS-A-WHOLE,
C CONFIDENCE BOUNDS FOR A FUTURE POINT, CORRELATION COEFFICIENT
C STANDARD DEVIATION OF Y ON X, YBAR ON X, YBAR AND FUTURE
C
C SEE BASIC STATISTICAL METHODS BY NEVILLE AND KENNEDY (1964)
CALLING PROGRAM REQUIREMENTS

Provide the equivalent to the following declaration:

```
DIMENSION A3(I2),A4(I2),B5(I2),B6(I2),B7(I2),B8(I2),B9(I2),B10(I2),
B14(I2),B15(I2)
```

CALL LIST ARGUMENTS

- \( I1 = 0 \): Monitor and plotting tabulation does not print
- \( I1 = 1 \): Monitor and plotting tabulation prints on separate page
- \( I2 \): Number of data point pairs
- \( A3 \): Column vector of data \( x \)-coordinates
- \( A4 \): Column vector of data \( y \)-coordinates
- \( A5 \): T-statistic \( T(I2-1) \) for \( 1 - \alpha/2 \), \( \alpha = \) significance level
- \( B1 \): Slope of regression line
- \( B2 \): \( x \)-coordinate of data centroid
- \( B3 \): \( y \)-coordinate of data centroid
- \( B4 \): Standard deviation of slope of regression line
- \( B5 \): Column vector of upper confidence bound point-by-point
- \( B6 \): Column vector lower confidence bound point-by-point
- \( B7 \): Column vector upper confidence bound line-as-a-whole
- \( B8 \): Column vector lower confidence bound line-as-a-whole
- \( B9 \): Column vector upper confidence bound for future point
- \( B10 \): Column vector lower confidence bound for future point
- \( B11 \): Correlation coefficient
- \( B12 \): Standard deviation \( y \) on \( x \), \( SYX \)
- \( B13 \): Standard deviation \( ybar \) on \( x \), \( SYBARX \)
- \( B14 \): Column vector of standard deviations \( ybari \) on \( x \), \( SYBARI \)
- \( B15 \): Column vector of standard deviation \( yfuture \) on \( x \), \( SYFUT \)

Preempted Names

None
C
SIZE
DIMENSION X(1), Y(1), UP(1), DOWN(1), CUP(1), CDOWN(1), FUP(1), FDOWN(1),
1 SYI(1), SFUT(1)
GO TO 100
1 SUMX=0.
SUMY=0.
SUMXX=0.
SUMYY=0.
SUMXY=0.
SUM1=0.
SUM2=0.
SUM3=0.
SUM4=0.
DO 2 I=1,N
SUMX=SUMX+X(I)
SUMY=SUMY+Y(I)
SUMXX=SUMXX+X(I)*X(I)
SUMYY=SUMYY+Y(I)*Y(I)
SUMXY=SUMXY+X(I)*Y(I)
2 CONTINUE
XBAR=SUMX/FLOAT(N)
YBAR=SUMY/FLOAT(N)
IF(SUMXX.GT.0.) GO TO 111
WRITE(6,112)X(I)
112 FORMAT(' '****ERROR MESSAGE SUBROUTINE ME0227****',/,
1 ' COLUMN VECTOR A3,'G16.7',' IS A NULL VECTOR')
RETURN
111 B=SUMXY/SUMXX
DO 3 I=1,N
SUM1=SUM1+(X(I)-XBAR)*(X(I)-XBAR)
SUM2=SUM2+(Y(I)-YBAR)*(Y(I)-YBAR)
SUM3=SUM3+(X(I)-XBAR)*(Y(I)-YBAR)
SUM4=SUM4+(Y(I)-B*X(I))**2
3 CONTINUE
108 IF(SUM2.NE.0.) GO TO 110
WRITE(6,109)Y(I)
**ERROR MESSAGE SUBROUTINE ME0227**

1. ALL Y-OBSERVATIONS SAME, G15.7, CHECK DATA

IF(ERROR.NE.0) RETURN

R = SQRT((SUM2-SUM4)/SUM2)
SYX = SQRT(SUM4/FLOAT(N-1))
SYBARX = SYX/SQRT(FLOAT(N))
SB = SQRT(SYX*SYX/SUMXX)
BUP = B + TSTAT*SB
BLOW = B - TSTAT*SB
YUP = YBAR + TSTAT*SYBARX
YLOW = YBAR - TSTAT*SYBARX

DO 4 I = 1, N
YHAT = B*X(I)
SYI(I) = ABS(X(I)*SB)
UP(I) = YHAT + TSTAT*SYI(I)
DOWN(I) = YHAT - TSTAT*SYI(I)
ARG3 = X(I)*X(I)*SB*SB + SYX*SYX
SFUT(I) = SQRT(ARG3)
FUP(I) = YHAT + TSTAT*SFUT(I)
FDOWN(I) = YHAT - TSTAT + SFUT(I)
CUP(I) = UP(I)
CDOWN(I) = DOWN(I)

CONTINUE

IF(MON.EQ.0) GO TO 99
WRITE(6,5)

5 MONITOR AND PLOTTING DISPLAY IOWA CADET SUBROUTINE ME0227 MISCHK

WRITE(6,5)

1 X           Y           Y             Y
2 DATA        DATA        REGRESSION  UPPER BOUND  LOWER BO
3 UND         UPPER BOUND  LOWER BOUND  UPPER BOUND  LOWER BOUND
4 POINT        LINE-AS-WHOLE  LINE-AS-WHOLE  FUTURE POINT  FUTURE POINT
5       DO 6 I = 1, N

CONTINUE
YR = B*X(I)
WRITE(6,9) X(I),Y(I),YR,UP(I),DOWN(I),CUP(I),CDOWN(I)
1,FUP(I),FDOWN(I)
9 FORMAT(9(1X,G13.6))
6 CONTINUE
WRITE(6,7) BUP,BLOW,XBAR,YBAR,YUP,YLOW,TSTAT,R,
1 SUMX,SUMY,SUMXX,SUMYY
7 FORMAT(/,
4' SLOPE B OF REGRESSION LINE .........................',G14.7/,
6' UPPER CONFIDENCE BOUND ON SLOPE B ..................',G14.7/,
7' LOWER CONFIDENCE BOUND ON SLOPE B ..................',G14.7/,
8' X-COORDINATE OF DATA CENTROID .....................',G14.7/,
9' Y-COORDINATE OF DATA CENTROID .....................',G14.7/,
1' UPPER CONFIDENCE BOUND ON YBAR ....................',G14.7/,
1' LOWER CONFIDENCE BOUND ON YBAR ....................',G14.7/,
2' T-STATISTIC T(N-1) FOR (1-ALPHA/2), USER SUPPLIED ....',G14.7/,
4' CORRELATION COEFFICIENT R ..........................',G14.7/,
5' SUM OF X ..........................................',G14.7/,
6' SUM OF Y ..........................................',G14.7/,
7' SUM OF X**2 ........................................',G14.7/,
8' SUM OF Y**2 ........................................',G14.7/
7 FORMAT(/,
1 SUM OF X ..........................................',G14.7/,
2 SUM OF (X-XBAR)**2 ..................................',G14.7/,
3 SUM OF (Y-YBAR)**2 ..................................',G14.7/,
4 SUM OF (X-XBAR)*(Y-YBAR) ............................',G14.7/,
5 SUM OF (Y-YHAT)**2 ..................................',G14.7/,
6 UNBIASED ESTIMATOR STANDARD DEVIATION Y ON X, SYX ....',G14.7/,
7 UNBIASED ESTIMATOR SYBARX ...........................',G14.7/,
9 UNBIASED ESTIMATOR OF STANDARD DEVIATION OF B, SB ....',G14.7/,
1')
99 RETURN
C PROTECTION
100 IERROR=0
IF(MON.EQ.0.OR.MON.EQ.1) GO TO 102
WRITE(6,101) MCN
101 FORMAT(' *****ERROR MESSAGE SUBROUTINE ME0227****',/,
1' ARGUMENT I1','I15',' NOT EQUAL TO ZERO OR ONE')
    IERROR=1
102 IF(N.GT.1) GO TO 104
WRITE(6,103)N
103 FORMAT(' *****ERROR MESSAGE SUBROUTINE ME0227****',/,
1' ARGUMENT I2','I15',' NOT GREATER THAN ONE')
    IERROR=1
104 IF(TSTAT.GT.0.) GO TO 1050
WRITE(6,105)TSTAT
105 FORMAT(' *****ERROR MESSAGE SUBROUTINE ME0227****',/,
1' ARGUMENT A5','G16.7',' NOT GREATER THAN ZERO')
    IERROR=1
1050 IF(IERROR.NE.0) RETURN
    IERROR=0
GO TO 1
END
Appendix XVIII. Subroutine GOLD
SUBROUTINE GOLD (K,XL,XR,F,MERIT,YBIG,XBIG,XL1,XR1,N)

THIS SUBROUTINE WILL SEARCH OVER A ONE-DIMENSIONAL UNIMODAL FUNCTION AND REPORT THE EXTREME ORDINATE FOUND, ITS ABSCISSA, FINAL ABSCISSAS BOUNDING THE INTERVAL OF UNCERTAINTY, AND THE NUMBER OF FUNCTION EVALUATIONS EXPENDED DURING THE SEARCH.

THE SUBROUTINE REQUIRES THE SPECIFICATION OF THE PRESENT INTERVAL OF UNCERTAINTY, THE FRACTIONAL REDUCTION REQUIRED IN THE INTERVAL, AND WHETHER OR NOT A CONVERGENCE MONITOR PRINTOUT IS REQUIRED.

CALLING PROGRAM REQUIREMENTS

PROVIDE A SUBROUTINE A5(X,Y) WHICH RETURNS THE ORDINATE Y WHEN THE ABSCISSA X IS TENDERED.

PROVIDE THE EQUIVALENT OF THE FOLLOWING STATEMENT:

EXTERNAL A5

CALL LIST ARGUMENTS

I1=0 CONVERGENCE MONITOR WILL NOT PRINT.
I1=1 CONVERGENCE MONITOR WILL PRINT.
A2=ORIGINAL LEFTHAND ABSCISSA OF INTERVAL OF UNCERTAINTY.
A3=ORIGINAL RIGHTHAND ABSCISSA OF INTERVAL OF UNCERTAINTY.
A4=FRACTIONAL REDUCTION IN INTERVAL OF UNCERTAINTY DESIRED.
A5=NAME OF THE ONE-DIMENSIONAL UNIMODAL FUNCTION.
B1=EXTREME ORDINATE DISCOVERED DURING SEARCH.
B2=ABSIISSA OF EXTREME ORDINATE.
B3=FINAL LEFTHAND ABSCISSA OF INTERVAL OF UNCERTAINTY.
C.......B4=FINAL RIGHHAND ABSCISSA OF INTERVAL OF UNCERTAINTY.
C.......J5=NUMBER OF FUNCTION EVALUATIONS EXPENDED DURING SEARCH.
C
    GO TO 100
C
C......PRINT CONVERGENCE MONITOR HEADINGS IF REQUIRED ......
C
111  IF(K)32,31,32
    32  WRITE(6,33)
        33  FORMAT('CONVERGENCE MONITOR SUBROUTINE GOLD ',//,
                        ' N     Y1    Y2    X1    X2'//)
C
C......INITIALIZE ........
C
31  N=0
    XLEFT=XL
    XRIGHT=XR
13  SPAN=XR-XL
    DELTA=ABS(SPAN)
14  X1=XL+0.381966*DELTA
    X2=XL+0.618034*DELTA
    CALL MERIT1(X1,Y1)
    CALL MERIT1(X2,Y2)
    N=N+2
C
C......PRINT CONVERGENCE MONITOR IF REQUIRED ........
C
3  IF(K)34,9,34
    34  WRITE(6,35)N,Y1,Y2,X1,X2
        35  FORMAT(I5,4(I1X,E15.7))
C
C............. IS SEARCH COMPLETE? ........
          9  IF(ABS(XL-XR)-ABS(F*SPAN))4,4,8
C
C............. CONTINUE GOLDEN SECTION SEARCH ........
C
8 DELTA=0.618034*DELTA
   IF(Y1-Y2)1,10,2
1  XL=X1
   X1=X2
   Y1=Y2
   X2=XL+0.618034*DELTA
   CALL MERIT1(X2,Y2)
   N=N+1
   GO TO 3
2  XR=X2
   Y2=Y1
   X2=X1
   X1=XL+0.381966*DELTA
   CALL MERIT1(X1,Y1)
   N=N+1
   GO TO 3
4 IF(Y2-Y1)5,5,6
5  YBIG=Y1
   XBIG=X1
   GO TO 7
6  YBIG=Y2
   XBIG=X2
7  XL1=XL
   XR1=XR
   GO TO 39
10 XL=X1
    XR=X2
    DELTA=XR-XL
    GO TO 14

C
CIF CONVERGENCE MONITOR PRINT REQUIRED, PRINT SUMMARY....
C
39 IF(K)40,40,37
37 WRITE(6,38)XLEFT,XRIGHT,F,YBIG,XBIG,XL1,XR1,N
38 FORMAT(/,1'E LEFTHAND ABCISSA OF INTERVAL OF UNCERTAINTY............',E15.7,/)
2' RIGHTHAND ABSCISSA OF INTERVAL OF UNCERTAINTY ...........,E15.7,/,  
3' FRACTIONAL REDUCTION OF INTERVAL OF UNCERTAINTY ..........',E15.7,/,  
4' EXTREME ORDINATE DISCOVERED DURING SEARCH.............',E15.7,/,  
5' ABSCISSA OF EXTREME ORDINATE .........................',E15.7,/,  
6' NEW LEFTHAND ABSCISSA OF INTERVAL OF UNCERTAINTY.......',E15.7,/,  
7' NEW RIGHTHAND ABSCISSA OF INTERVAL OF UNCERTAINTY......',E15.7,/,  
8' NUMBER OF FUNCTION EVALUATIONS EXPENDED IN SEARCH ...',115,//*  
XL=XLEFT  
XR=XRIGHT  
112 RETURN

C

***** PROTECTION *****

C

100 IERROR=0  
101 IF(K-1)104,104,102  
102 WRITE(6,103)K  
103 FORMAT(" ****ERROR MESSAGE SUBROUTINE GOLD ****",/,
              '1' I1,'115,' IS NOT 0 OR 1')  
              IERROR=IERROR+1  
104 IF(XR-XL)105,107,107  
105 WRITE(6,106)XL,XR  
106 FORMAT(" ****ERROR MESSAGE SUBROUTINE GOLD ****",/,
              '1' A2,'15.7,' NOT SMALLER THAN A3,'15.7)  
              IERROR=IERROR+1  
607 IF(F)109,109,108  
108 IF(F-1.)113,109,109  
109 WRITE(6,110)F  
110 FORMAT(" ****ERROR MESSAGE SUBROUTINE GOLD ****",/,
              '1' A4,'15.7,' DOES NOT LIE BETWEEN 0. AND 1.')  
              IERROR=IERROR+1  
113 IF(IERROR)111,111,112  
END
Appendix XIX. Computer Program for Linear Regression of $y$ on $x$, $y = a + bx$
LINEAR REGRESSION OF Y ON X, Y = A + B * X

SEE STATISTICAL METHODS BY SNEDECOR (1962)

X IS CAPX, Y IS CAPY, DX IS SMALL X, DY IS SMALL Y

SSX IS SMALL X SQUARE, SSY IS SMALL Y SQUARE, DXY IS SMALL XY

T IS TABULATED VALUE OF t-STATISTIC (AT 3 PROBABILITY LEVELS)

ALPHA = PROBABILITY LEVELS OF 0.1, 0.05, AND 0.01

DIMENSION X(30), Y(30), DX(30), DY(30), SSX(30), SSY(30), DXY(30), $YHAT(30), SYHAT(30), T(3), YLARGE(3, 30), YSMALL(3, 30), ALPHA(3)
READ(5, 20) (ALPHA(K), K=1, 3)
1 READ(5, 10, END=100) N
10 FORMAT(12)
READ(5, 20) (X(I), I=1, N), (Y(J), J=1, N)
READ(5, 20) (T(M), M=1, 3)
20 FCRMAT(8F10.5)
SUMX=0.0
SUMY=0.0
DO 2 I=1, N
SUMX=SUMX+X(I)
2 SUMY=SUMY+Y(I)
XBAR=SUMX/N
YBAR=SUMY/N
SUMDX=0.0
SUMDY=0.0
SUMSSX=0.0
SUMSSY=0.0
SUMDXY=0.0
DO 3 I=1, N
DX(I)=X(I)-XBAR
DY(I)=Y(I)-YBAR
SSX(I)=DX(I)*DX(I)
SSY(I)=DY(I)*DY(I)
DOXY(I)=DX(I)*DY(I)
SUMDX=SUMDX+DX(I)
SUMDY=SUMDY+DY(I)
SUMSSX=SUMSSX+SSX(I)
SUMSSY=SUMSSY+SSY(I)
3 SUMDXY=SUMDXY+DXY(I)
        EN=FLOAT(N)
        B=SUMDXY/SUMSSX
        SYX=SQR((SUMSSY-SUMDXY*SUMDXY/SUMSSX)/(N-2.1))
        A=YBAR-B*XBAR
        SX=SQR(SUMSSX/(EN-1.))
        SY=SQR(SUMSSY/(EN-1.))
        COVX=SX/XBAR
        COVY=SY/YBAR
        SB=SYX/(SQR(SUMSSX))
        SA=SYX/(SQR(1./EN*XBAR*XBAR/SUMSSX))
        TACAL=A/SA
        TBCAL=B/SB
WRITE(6,50) N
50 FORMAT('/',//,10X,'N=',I2,//)WRITE(6,70)
70 FORMAT('/',4X,'XBAR',6X,'YBAR',7X,'SX',8X,'SY',6X,'SYX',6X,'COVX',
$6X,'COVY',8X,'A',8X,'B',8X,'SA',8X,'SB',8X,'TACAL',5X,'TBCAL',/)
WRITE(6,40) XBAR,YBAR,SX,SY,SYX,COVX,COVY,A,B,SA,SB,TACAL,TBCAL
40 FORMAT(32F10.4)
WRITE(6,80)
80 FORMAT('/',3X,'SUMX',5X,'SUMY',5X,'SUMSSX',5X,'SUMSSY',/)
WRITE(6,90) SUMX,SUMY,SUMSSX,SUMSSY
90 FORMAT('/',4F10.4)
WRITE(6,120)
120 FORMAT('/',//,36X,'ALPHA=0.1',11X,'ALPHA=0.05',10X,'ALPHA=0.01',/)
WRITE(6,110)
110 FORMAT(6X,'X',9X,'Y',7X,'YHAT',4X,'YLARGE',4X,'YSMALL',4X,'YLARGE'
$,4X,'YSMALL',4X,'YLARGE',4X,'YSMALL',/)
DO 5 I=1,N
YHAT(I)=A+B*X(I)
SYHAT(I) = SYX*(SQRT(1./EN+DX(I)*DX(I)/SUMSSX))
DO 4 M=1,3
  YLARGE(M,I) = YHAT(I) + T(M) * SYHAT(I)
  YSMALL(M,I) = YHAT(I) - T(M) * SYHAT(I)
4 WRITE(6,30) X(I), Y(I), YHAT(I), YLARGE(M,I), YSMALL(M,I), M=1,3
30 FORMAT(*//,9F10.4)
WRITE(6,50)
GO TO 1
100 STOP
END
Appendix XX. Computer Program for Linear Regression of $y$ on $x$, $y = bx$
LINEAR REGRESSION OF Y ON X, Y = B * X

SEE STATISTICAL METHODS BY SNEDECOR (1962)

X IS CAPX, Y IS CAPY, XSQ IS XSQUARE, YSQ IS YSQUARE, XY IS CAPXY

T IS TABULATED VALUE OF T-STATISTIC (AT 3 PROBABILITY LEVELS)

ALPHA = PROBABILITY LEVELS OF 0.1, 0.05, AND 0.01

DIMENSION X(15), Y(15), XSQ(15), YSQ(15), XY(15), YHAT(15), SYHAT(15), $T(3), YLARGE(3, 15), YSMALL(3, 15), ALPHA(3), SSX(15), SSY(15), DX(15), DY(215)

READ(5, 20) (ALPHA(K), K = 1, 3)
1 READ(5, 10, END=100) N
10 FORMAT(I2)
READ(5, 20) (X(I), I = 1, N), (Y(J), J = 1, N)
READ(5, 20) (T(M), M = 1, 3)
20 FORMAT(8F10.5)

SUMX=0.0
SUMY=0.0
SUMXSQ=0.0
SUMYSQ=0.0
SUMXY=0.0
DO 2 I=1, N
XSQ(I)=X(I)*X(I)
YSQ(I)=Y(I)*Y(I)
XY(I)=X(I)*Y(I)
SUMX=SUMX+X(I)
SUMY=SUMY+Y(I)
SUMXSQ=SUMXSQ+XSQ(I)
SUMYSQ=SUMYSQ+YSQ(I)
2 SUMXY=SUMXY+XY(I)

EN=FLOAT(N)
XBAR=SUMX/EN
YBAR=SUMY/EN
SUMSSX=0.0
SUMSSY=0.0
DO 3 1=1,N
DX(I)=X(I)-XBAR
DY(I)=Y(I)-YBAR
SSX(I)=DX(I)*DX(I)
SSY(I)=DY(I)*DY(I)
SUMSSX=SUMSSX+SSX(I)
3 SUMSSY=SUMSSY+SSY(I)
B=YBAR/XBAR
SYX=SQR(((SUMSY-SUMXY*SUMXY/SUMXSQ)/(EN-1.)))
SX=SQR((SUMSSX/(EN-1.)))
SY=SQR((SUMSSY/(EN-1.)))
COVX=SX/XBAR
COVY=SY/YBAR
SB=SYX/(SQR(SUMXSQ))
TBCAL=B/SB
WRITE(6,50) N
50 FORMAT(‘/’,//,’10X,’N=’,/)
WRITE(6,70)
70 FORMAT(‘/’,4X,’XBAR’,6X,’YBAR’,7X,’SX’,8X,’SY’,6X,’COVX’,6X,’COVY’
$,8X,’B’,8X,’SB’,7X,’TBCAL’,5X,’SYX’,6X,’SUMSSX’,4X,’SUMSSY’,
$5X,’SUMXSQ’,/)
WRITE(6,40) XBAR,YBAR,SX,SY,COVX,COVY,B,SB,TBCAL,SYX,SUMSSX,SUMSSY
$,SUMXSQ
40 FORMAT(13F10.4)
WRITE(6,120)
120 FORMAT(‘/’,//,’36X,’ALPHA=0.1’,11X,’ALPHA=0.05’,10X,’ALPHA=0.01’,/)
WRITE(6,110)
110 FORMAT(6X,’X’,9X,’Y’,7X,’YHAT’,4X,’YLARGE’,4X,’YSMALL’,4X,’YLARGE’
$,4X,’YSMALL’,4X,’YLARGE’,4X,’YSMALL’,/)
DO 5 I=1,N
YHAT(I)=B*X(I)
SYHAT(I)=(SYX*X(I))/SQR(SUMXSQ)
DO 4 M=1,3
YLARGE(M,I)=YHAT(I)+T(M)*SYHAT(I)
4 YLARGE(M,I)=YHAT(I)-T(M)*SYHAT(I)
5 CONTINUE
4 YSMALL(M,I) = YHAT(I) - T(M) * SYHAT(I)
5 WRITE(6,30) X(I), Y(I), YHAT(I), (YLARGE(M,I), YSMALL(M,I), M=1,3)
30 FORMAT(9F10.4)
   WRITE(6,50)
   GO TO 1
100 STOP
   END