1976

Use of polyester resins in photoplasticity

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USE OF POLYESTER RESINS IN
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Use of polyester resins in photoplasticity

by

Loren William Zachary

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
DOCTOR OF PHILOSOPHY

Department: Engineering Science and Mechanics
Major: Engineering Mechanics

Approved:

Signature was redacted for privacy.

In Charge of Major Work

Signature was redacted for privacy.

For the Major Department

Signature was redacted for privacy.

For the Graduate College

Iowa State University
Ames, Iowa

1976
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I. INTRODUCTION

In the experimental stress analysis field, the commonly used experimental techniques can be grouped into two broad areas. The first group consists of point-by-point information gathering devices such as electrical-resistance strain gages and extensometers. Direct information is obtained only at the location of the device. In general, these devices measure the change in length between two points on the surface of a loaded object, and this information is used to calculate an average strain value for the given gage length. Strain gages are inexpensive and are sensitive to small deformations. This has led to their widespread use on both statically and dynamically loaded structures; however, the use of these devices does have limitations. Points of rapid and irregular geometric changes, internal points of three-dimensional models, and requirements for data at a large number of positions are examples of applications that could make the point-by-point method technically or monetarily undesirable.

The second group of experimental procedures makes use of whole field techniques. Moire, holography, grid, and photoelasticity methods are contained in this grouping. These whole field techniques provide information at all points in the model in contrast to the single location information derived from a strain gage.

Photoelasticity, the most advanced of the whole field techniques, requires interpretation of optical patterns in transparent materials subjected to loads and illuminated by polarized light. These optical patterns consist of alternating light and dark bands called fringes and are associated with the temporary double refraction or birefringence that
occurs in most transparent materials while subjected to loads. Two types of fringes are possible; namely, isochromatics and isoclinics. It has been established that the isochromatics are linearly proportional to the in-plane maximum shear stresses (principal stress differences) while the isoclinics indicate the directions of the in-plane principal stresses.

Photoelasticity has been successfully used to establish stress distributions in extremely complicated models. Plane stress, plane strain, and general three-dimensional configurations with static, dynamic, and thermal loads applied have been analyzed. A simple modeling law relating the photoelastic model to the metal prototype has been determined, and the use of "plastic" photoelastic models to model possibly expensive metal prototypes has been well established.

A large selection of polymers are available for use as photoelastic modeling materials. Sheets of photoelastic materials are readily obtainable for use in plane stress studies, and precast blocks of materials are commercially available for use in three-dimensional studies. Several liquid polymers and associated hardeners are available which can be used to cast large blocks or models of predetermined shape. The use of pre-shaped molds is very useful since copies of intricate profiles, such as the irregular shapes of welded joints, can be obtained only in this manner. The availability of appropriate photoelastic materials, the existence of a linear stress-optic law, the establishment of a simple modeling relationship, and the accuracy of photoelasticity has made it a powerful experimental tool.
Photoplasticity, another whole field technique, is the extension of the usual linear elastic photoelasticity technique into the plasticity region. The analysis of plasticity problems by analytical and experimental methods is generally more difficult than those associated with elasticity. The growth and use of the photoplasticity method has been hampered by several factors. Photoplasticity has not had an ample supply of appropriate modeling materials, and the modeling laws are not as simple as those associated with elastic problems.

The number of photoplastic materials available for consideration is restricted because most of the materials used in photoelasticity do not exhibit any large amount of plastic flow or yielding. A second factor limiting the material suitability is the requirement that the photoplastic material must model a particular metal's nondimensional stress-strain characteristics in the elastic, elastic-plastic transition, and plastic regions. Thus, the modeling material must have a very specific stress-strain diagram, and this is a stringent requirement. Materials have been found that to a degree model some metals; however, they are available only in thin sheets and a few preformed shapes. By controlling the test temperature, humidity, and strain rate some modifications in the stress-strain diagrams have been possible, but a universally applicable modeling material has not been found.

The desire to use the benefits of the whole field approach in the area of plasticity has been affected by the lack of materials and by the confusing nature of the materials optical behavior. Some of the materials have an optical response that is a function of strain, while others are
only a function of stress, and still others have a response that is a function of both stress and strain. Further, the strain rate, humidity, and test temperature all have been found to affect some of the materials with respect to their mechanical and optical responses. Photoplasticity, unlike photoelasticity, is without a simple unifying stress and/or strain optic law.

The development of photoplastic modeling materials is an important step in extending the capabilities of photoplasticity. The research presented here characterizes the mechanical and optical properties of a particular polyester system. The polyesters come in liquid form, and the two polyesters used in the studies are commonly referred to as being "rigid" and "flexible." When the two polyesters are mixed together, the shape of the elastic-plastic stress-strain curve can be altered by changing the composition's rigid to flexible resin mixture ratio. The capability of fitting the model's stress-strain characteristics to that of the prototype's, and the capability of using the material for three-dimensional analyses would be an advancement in photoplasticity.
II. REVIEW OF LITERATURE

In 1907, Filon (1) observed that the birefringent properties of glass under pure bending conditions still existed when the elastic limit was reached at the outer fibers of the beam. Rossi (2) and Ambron (3) found in studies of celluloid that a permanent double refraction, residual fringe pattern, was exhibited when the material was strained beyond the elastic limit and then unloaded. Coker and Chakko (4) investigated the stress-strain properties of celluloid in the elastic and inelastic regions. They indicated that the material's optical law was linear with respect to stress when loaded up to twice the elastic limit. Filon and Jessop (5) and Ramspeck (6) also studied celluloid loaded to inelastic strains under creep conditions. Filon and Jessop indicated that the optical and material properties of celluloid were dependent upon the load-time history, and that the birefringence was a function of both stress and strain. Arakawa (7) found that the optical creep properties of the bakelite he studied under compression did not follow the optic law associated with celluloid.

This early work was not extended nor pursued to any great degree until the early 1950's. Since then the majority of research has been involved with two materials, i.e., celluloid and polycarbonate. Several other "plastics" have been tried as possible photoplastic modeling materials. Fried (8) investigated polystyrene, lucite, plexiglass, nylon, cellulose acetate, silver chloride, and celluloid. The first three had low optical sensitivities and were unsuitable for further studies. The nylon was only obtainable in thin sheets and became opaque shortly after the yield point in tension was reached. The fringe patterns in cellulose
acetate became fuzzy and distorted under high stress conditions. Silver chloride, a cubic crystalline material, was ductile, but the grains showed nonuniform birefringence under tensile loads. The crystalline nature of silver chloride may be helpful in studying the slip mechanism in metals.

Hetenyi (9) studied a nylon copolymer available only in thin sheets. The material's mechanical and optical properties were affected by humidity conditions. The tensile tests indicated that the isochromatic fringe orders were linearly related to the longitudinal strain. The tensile specimens subjected to high humidity conditions gave a symmetrical necking phenomenon upon yielding, while low humidity conditions caused unidirectional slip bands to form.

Bayoumi and Frankl (10) used data from tensile creep tests of Catalin 800, CR-39, Marbelette, and other types of bakelite resins to demonstrate the validity of the mathematical relationship they derived between the birefringence and the principal stress difference and principal strain difference. The data indicated that the stress-optic laws for these materials were determined by the state of stress and strain and not by the test temperature or prior loading history. An experimental method was proposed which would separate the principal stress difference term of the optic law from the principal strain difference term. The fringe order obtained under loaded conditions would exhibit the effects of both stress and strain, but the unloaded fringe order would be a function of only the plastic strains. Thus, the loaded and unloaded (residual) fringe orders could be used to separate the stress term from the strain term.
Fried and Shoup (11) used polyethylene to model an aluminum plate with a centrally located hole. They found that the birefringence of polyethylene was linearly related to the principal strain difference. Javornicky (12) indicates that polyethylene in a transparent state might be difficult to obtain. Although very thin specimens may be opaque to visible light, they are transparent to infrared light sources.

Hunter and Schwarz (13) and Hunter (14) used a method utilizing the creep and frozen-stress characteristics of an epoxy resin subjected to a thermal cycle. The maximum temperature in the cycle was well below the critical stress freezing temperature of the material. The resulting frozen, nonlinear, effective stress-strain curve was used to model a mild steel and an aluminum alloy. The three-dimensional elastic-plastic stress distributions along a line of symmetry for a notched-bar subjected to bending and the elastic-plastic stress distributions in plates with centrally located holes and cracks were investigated.

Johnson and Goldsmith (15) investigated the characteristics of an amorphous nylon, polyethylene, a type of polyester, cellulose acetate butyrate, natural rubber, and ethylene propylene. The stress-strain properties of the polyester, nylon, and polyethylene plus the birefringent data for polyethylene and nylon are given.

Javornicky (12) has devoted a section of his book to describing the mechanical and optical properties, size and thicknesses available, transparency, etc. of many potential photoplastic materials. His list of references is quite extensive. A major portion of the discussion is with regard to celluloid (cellulose nitrate). Fried (8) found celluloid to be
an acceptable material when loaded under creep conditions. He verified earlier results (4) indicating that the birefringence was linearly related to stress up to twice the elastic limit. The following observations of celluloid were made:

1. The optical and mechanical properties vary with the age of the material.
2. The material exhibits optical and mechanical creep under sustained loads.
3. The properties vary with rate of loading and test temperature.
4. The properties depend on its strain history or previous mechanical treatment.

Ito (16) indicated that, for one- and two-dimensional states of stress, yielding was due to maximum shear stress. Nisida, Hondo, and Hasunuma (17) calibrated celluloid specimens in both tension and compression. They applied the results to beams in elastic-plastic bending, indentations by a wedge, and the compression of a wedge by a flat die.

Frocht and his associates Thomson and Cheng produced a series of papers (18-22) concerned with the use of celluloid in photoplasticity. The general stress-optic law under constant test conditions of temperature, relative humidity, time after loading, and optic path length was established. The birefringence was determined to be a function of the principal stresses and the load path history. The isoclinics were found to be a function of the principal stress directions and not the principal strain directions. When plastic flow occurred, the elastic-plastic principal
strain directions were not necessarily the same as the instantaneous principal stress directions. The principal stress directions were used to determine, by use of the shear difference method, the individual principal stress distributions in tension strips containing fillets and in tension strips having centrally and eccentrically located circular holes.

Monch and Loreck (23) used the anomalous dispersion of birefringence to predict the stress or strain in the plastic region. The fringe order depends on the wavelength of the light used. In the elastic region, the product of the fringe order and the wavelength of the light used, $N\lambda$, was the same for blue mercury light (436 nm) and red light (655 nm). When plastic flow occurred, the dispersion (the difference in the $N\lambda$ products for red and blue light) could be used to determine the stress at the free boundaries and the boundary line between the elastic and plastic regions of a model. This elastic-plastic boundary was determined by noting that the dispersion was zero for the elastic areas and nonzero for the plastic regions.

One problem was indicated by Frocht and Cheng (22) and Monch and Loreck (23). They observed that the celluloid was optically nonhomogeneous when received from the suppliers. A heat treatment was required to eliminate these initial fringe patterns.

Javornicky (24) has reported that celluloid conforms to the Tresca, maximum shear stress, theory of failure. A strip with a centrally located hole was tested and evaluated by using the loaded fringe order, residual fringe order, and birefringent dispersion methods. Javornicky (25) also studied the spreading of the plastic regions in tensile strips containing
central holes and symmetrically located external notches and grooves.

Ito (26) suggested the use of polycarbonate as a photoplastic modeling material. Brill's investigation (27) indicated that from the use of isochromatic patterns, isoclinic inclinations, and strain measurements it could be concluded that the birefringence was a nonlinear function of the principal strain difference. The isoclinics at large strains were aligned with the principal strain directions. The tensile specimens exhibited a necking or extrusion upon yielding, and the isochromatic fringe patterns became inhomogeneous.

Cloud (28) studied the optical creep and dispersion in polycarbonate. The sheets of material had to be annealed because of the anisotropic birefringence in the as-received condition. Dally and Mulc (29) used polycarbonate for the investigation of strain fields associated with unloaded but permanently deformed bodies. Brinson (30) reported the relationship between the plastic thickness changes and plastic isochromatics associated with the formation of slip bands after gross yielding occurred in uniaxial tensile specimens. The rapid increase in fringe density was used by Whitfield and Smith (31) to define the localized yield zone. The fringe density was so high that the fringes became indistinguishable in the yield zone. The yield locus for polycarbonate indicated that the tensile and compressive yield strengths were unequal. Brinson (32) and Theocaris and Gdoutos (33) also used polycarbonate in the study of yield zones.

Noting that celluloid was available only in thin sheets and tubes, that polycarbonate exhibited excessive fringe densities and initial residual stresses, Johnson (34) chose cellulose propionate to perform a
scattered light, three-dimensional, stress and strain analysis. The stress, strain, and optical behavior of the material was studied under slow, stepwise loading in tension and torsion. A grooved shaft subjected to torsional loadings was studied.

Morris and Riley (35) introduced a new material for photoplastic analysis. A rigid polyester resin and a flexible polyester resin were mixed together in varying proportions to control the basic shape of the tensile stress-strain curve. By adjusting this mixture ratio, the nondimensional stress-strain curve of the polyester mixture was made to match that of a certain aluminum. The polyester resins were strain rate dependent, but a limiting strain rate was found below which the stress-strain relationship did not change with varying strain rates. The birefringence in a loaded tensile specimen was found to be a multivalued function of the longitudinal strain; however, upon unloading, the residual fringe orders were a linear function of the longitudinal strain present before unloading occurred. By the use of residual fringe order patterns, the stress and strain concentration factors were obtained for a tensile strip containing a centrally located hole.

Oyinlola (36) used the same polyester resin mixture to model hot-rolled aluminum billets. The residual fringe orders were used to give qualitative information on the internal strain distribution. Longitudinal and lateral slices were taken from rolled polyester billets, and these slices provided the information for three-dimensional fringe order plots.
III. STATEMENT OF PROBLEM

Photoelasticity has proven to be a very valuable experimental tool for determining the stress distributions in two and three dimensional elastic problems. Photoplasticity as applied to elastic-plastic stress distribution problems has much potential and needs to be further developed.

The use of polyester mixture modeling materials, as introduced by Morris and Riley (35), was a new development in photoplasticity that enabled the researcher to modify the basic stress-strain curve of the elastic-plastic modeling material. This modification ability facilitates the matching of model and prototype nondimensional stress-strain curves. To further characterize the modeling properties of the polyester system, the mechanical and optical properties and the methods available for using the optical properties to predict the strain fields in the model must be investigated.

The model-to-prototype transition relationship requires that the model and prototype material behavior under tension, compression, and biaxial states of stress be similar. Since the material characterization tests conducted by Morris and Riley in the feasibility study were limited to uniaxial tension cases, the present study required that the uniaxial compression stress-strain relationship and the biaxial state of stress behavior be obtained. With use of the tension, compression, and biaxial data, a law of yielding for the photoplastic material can be established and compared to the von Mises law of yielding that is frequently associated with ductile metals.
The optical behavior of a photoplastic material can be a function of stress, strain, or a combination of stress and strain. Uniaxial and biaxial loading conditions are required to determine the type of stress- or strain- optical law that is applicable.

The various methods of strain analysis that utilize the optical properties to determine the strain fields must be examined. The methods of analysis explored are

1. Use of unloaded (retained) fringes to predict strain fields in loaded models. Unloaded fringes are obtained from the deformed model with the applied loads removed.

2. Use of fringes in the loaded model to predict the strain fields in loaded models.

3. Use of unloaded (retained) fringes to predict unloaded (retained) strain fields.
IV. MATERIAL PROPERTIES

The model and prototype mechanical properties are used to establish relationships which relate model results to the expected behavior in the prototype. In order for this relationship to be valid, it must be established that the model and prototype materials behave in a similar manner under equivalent conditions. This is accomplished by showing that the general stress-strain relationship is of the same form for both the model and prototype materials.

The next section describes the modeling used in photoplasticity and shows how the general stress-strain relationship of the model and prototype are compared. The remaining sections describe the test apparatus needed to obtain the mechanical and optical data. The results of these tests are presented, and their relationship to the behavior of metal prototype materials is discussed.

A. Modeling

The transition from model behavior to predicted prototype behavior can best be considered in terms of similitude and dimensional analysis. The item of interest for both homogeneous, isotropic, linear, elastic materials and nonlinear elastic-plastic materials is the general stress-strain relationship. For linear elastic materials it is well known that the general three-dimensional constitutive relationship between stresses and strains can be written in terms of only two parameters: Young's modulus $E$ and Poisson's ratio $\mu$. The same parameters that apply for a uniaxial state of stress can be used for biaxial and triaxial states of
stress. Linear elastic materials, loaded statically, follow Hooke's law, thus the requirement that the model and prototype material follow the same type of stress-strain law is easily satisfied.

The strain $\varepsilon$ in any direction at a point of an elastic model has a relationship with the principal stresses and material constants that can be presented in a functional form as

$$\varepsilon = f(\sigma_1, \sigma_2, \sigma_3, E, \mu)$$  \hspace{1cm} 1.

By using dimensional analysis, this equation can be written in nondimensional form by using terms called $\pi$-terms. Thus

$$\varepsilon = f \left( \frac{\sigma_2}{\sigma_1}, \frac{\sigma_3}{\sigma_1}, \frac{\mu}{E}, \frac{\varepsilon_1}{\sigma_1} \right)$$  \hspace{1cm} 2.

The strains in a model and prototype are the same if all the $\pi$-terms are equivalent, i.e.,

$$\varepsilon_p = \varepsilon_m$$

$$\text{if} \quad \frac{\sigma_2}{\sigma_1} \bigg|_m = \frac{\sigma_2}{\sigma_1} \bigg|_p$$

$$\frac{\sigma_3}{\sigma_1} \bigg|_m = \frac{\sigma_3}{\sigma_1} \bigg|_p$$

and

$$\frac{\mu}{E} \frac{\varepsilon_1}{\sigma_1} \bigg|_m = \frac{\mu}{E} \frac{\varepsilon_1}{\sigma_1} \bigg|_p$$  \hspace{1cm} 3.

where $m$ represents the model and $p$ represents the prototype.

The stresses in a model and prototype can also be presented in terms of the applied loads $P$, characteristic length of the model $L$, other geometric lengths $\lambda_i$, and the material constants $E$ and $\mu$. Thus
\[ \sigma = g(L, \lambda_1, P, E, \mu) \]

In nondimensional form, Equation 4 becomes

\[ \frac{\sigma L^2}{P} = g \left( \frac{\lambda_1}{L}, \frac{P}{E L^2}, \mu \right) \]

Again the model results can be used to predict prototype behavior if the model and prototype are geometrically similar and loaded proportionally. For example, if

\[ \frac{L}{L} = \left| \frac{\lambda_1}{L} \right|_p \]

\[ \frac{P}{E L^2} = \left| \frac{P}{E L^2} \right|_p \]

and

\[ \mu_m = \mu_p \]

then

\[ \frac{\sigma L^2}{P} = \left| \frac{\sigma L^2}{P} \right|_m \]

With the more complicated problem of nonlinear stress-strain curves, a similar approach of using the uniaxial parameters to help describe the general stress-strain relationship can be used. There are several methods of describing the uniaxial stress-strain curve (37). Ludwik (38) proposed an empirical equation of the form

\[ \sigma = \sigma_0 + m e^n \]

where \( \sigma_0 \) is the yield stress, \( n \) is the strain hardening exponent, and \( m \) is a constant. A popular curve fitting formula introduced by Ramberg and Osgood (39) is
\[ \varepsilon = \frac{\sigma}{E} + k \left( \frac{\sigma}{E} \right)^n \]  \hspace{1cm} 8.

or in a different but equivalent nondimensional form

\[ \frac{\varepsilon E}{S} = \frac{\sigma}{S} + \frac{1-m}{m} \left( \frac{\sigma}{S} \right)^n \]  \hspace{1cm} 9.

where \( E \) is Young's modulus, \( S \) is the secant yield stress, and \( m, k \) and \( n \) are constants. The factor \( m \) which ranges between zero and one is used with \( E \) to define the secant modulus \( mE \). As shown in Fig. 1, the intersection of the line through the origin having a slope \( mE \) with the stress-strain curve gives the secant yield stress. From tests by Ramberg and Osgood, it was found that, on the average, a value of \( m \) equal to 0.7 was reasonable for the sheet aluminum and steel alloys tested. The use of this particular value of \( m \) is not a requirement, and other values for particular materials can be well justified for curve fitting purposes. The value of \( n \) for the aluminum and steel alloys appeared to range from 9 to 19.

The material and modeling relationships for the nonlinear elastic-plastic case can be explored by following the same approach shown for the elastic case. The strain \( \varepsilon \) in any direction is a function of the three principal stresses \( \sigma_1, \sigma_2, \) and \( \sigma_3 \). These three quantities could be lumped into one general stress term \( \sigma \), but this particular sorting out of terms gives a better illustration of the role of the state of stress. Young's modulus, the secant yield stress, and the power coefficient used in the Ramberg-Osgood expression will be assumed to be different in tension and compression for the case being considered. These respective terms will
Fig. 1. Definition of secant yield stress.
be labeled $E_t$, $S_t$, and $n_t$ for tension and $E_c$, $S_c$, and $n_c$ for compression. Poisson's ratio, $\mu$, also is included, but no distinction will be made between its tension and compression behavior. The material properties can also be sensitive to the strain rate, $\dot{\varepsilon}$, which means that time, $t$, must be introduced. Thus, in a functional form, the general strain field can be written as

$$\varepsilon = \phi(\sigma_1, \sigma_2, \sigma_3, E_t, n_t, S_t, E_c, n_c, S_c, \mu, \dot{\varepsilon}, t)$$

For many materials (including the polyester mixtures), an assumption of constant Young's modulus for tension and compression is acceptable. With $E_t = E_c = E$, dimensional analysis gives the following nondimensional form for Equation 10:

$$\frac{\varepsilon E}{S_t} = \phi\left(\frac{\sigma_2}{\sigma_1}, \frac{\sigma_3}{\sigma_1}, \frac{\sigma_1}{S_t}, n_t, n_c, \frac{S_t}{S_c}, \frac{S_t}{E}, \mu, \dot{\varepsilon}, t\right)$$

This form is important since tests of the polyester materials have indicated that the tension and compression properties are different. For those cases where it can be assumed that the tension and compression properties are identical, then

$$\frac{\varepsilon E}{S} = \phi\left(\frac{\sigma_2}{\sigma_1}, \frac{\sigma_3}{\sigma_1}, \frac{\sigma_1}{S}, n, \frac{S}{E}, \mu, \dot{\varepsilon}\right)$$

where $n_t = n_c = n$ and $S_t = S_c = S$.

For transition from model to prototype, the principles of similitude require that all of the individual dimensionless $\pi$-terms indicated above be the same for both the model and prototype. Thus, to use model data to predict prototype strains,
In many problems, the material properties of the prototype are strain rate independent over the strain rates applicable to the problem; thus, the model material must also satisfy this condition since the \( \tau \) term \( \dot{\epsilon}_t \) must be satisfied for both model and prototype. In the research presented here, it will be shown that model material properties can be held constant for practical purposes for all applicable strain rates. Obtaining model properties that are strain rate independent is a major task requiring a considerable amount of experimental control.

The usual material similarity conditions cited in the literature (18,23,31,35,36) are

1. Nondimensional stress-strain curves for model and prototype must be the same.
2. Poisson's ratios must be the same for the model and prototype materials. It has been indicated (18,31,36) that this is required in the plastic range; however, the Poisson's ratio condition must be satisfied at all stages of deformation according to Equations 11 and 12.
3. The law of yielding must be of the same form for the two materials.

Each of these items will be discussed in terms of past practices and the relationship to the nondimensional Equations 11 and 12. Using the
general form of Equation 11 (similar arguments hold for Equation 12), it can be shown that uniaxial tension and compression tests establish the \( \pi \) terms \( n_t, n_c, S_t/S_c, S_c/E, \) and \( \mu \). These terms are constant for strain rate independent conditions. The condition that the nondimensional stress-strain curves must be similar is usually checked by using uniaxial data to plot \( \varepsilon E/S_t \) vs. \( \sigma_1/S_t \). This similarity condition is satisfied if the model and prototype curves coincide. Several examples of this procedure exist in reports of past research (18,23,31,34,35,36). The ability of the researcher to change the shape of the stress-strain curve of the model material helps in this task of matching the model and prototype nondimensional data. The uniaxial data must be supplemented with further biaxial and/or triaxial data to assure that the general stress-strain relationship is compatible between the model and prototype. The problems involving plane stress conditions only require that the biaxial conditions be investigated for proper characterization. Triaxial conditions will not be considered in this research.

The condition that Poisson's ratio be the same for model and prototype may not be critical. There are certain types of plane elastic problems in which the stress distribution is not affected by the value of Poisson's ratio (40,41). In three-dimensional photoelasticity, where \( \mu \) is a more important property, the differences in \( \mu \) for the model and prototype have not introduced significant errors. Thus, distorted models, i.e., models and prototypes having unequal \( \pi \) terms, can be used if the pertinent \( \pi \) term variables can be shown to have little influence upon the results. In the elastic-plastic region, the degree of accuracy required
for Poisson's ratio similarity between model and prototype has not been evaluated. Tests designed to investigate this problem would be necessary in order to establish the amount of distortion acceptable for photoplastic modeling.

For large plastic deformations, metals are usually assumed to approach a limiting value of Poisson's ratio equal to 0.5, therefore, this value is commonly used in analytical studies. This condition represents a constant volume process and provides the relationship, applicable in the plastic region only,

\[ \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 0 \]

where \( \varepsilon_1, \varepsilon_2, \) and \( \varepsilon_3 \) are the principal strains. When 2 other independent equations are obtained from photoplasticity or other methods of analysis, this equation can be used as the third equation needed to solve for the three individual principal strain components. Past research has shown that plastics tend to approach this limiting value of Poisson's ratio (35).

At this point it can be seen that after a particular modeling material is selected, several of the \( \pi \) terms become fixed. In particular, the \( \pi \) terms \( n_t, n_c, S_t/S_c, S_t/E, \mu, \varepsilon_t \) are constants or, in the case of the \( \varepsilon_t \) term, do not affect the other parameters. The last of the similarity requirements involves the state of stress existing in the material. The strains are affected not only by the ratio of stresses \( \sigma_2/\sigma_1 \) and \( \sigma_3/\sigma_1 \) but also by the stress amplitude represented by the term \( \sigma_1/S_t \). Since biaxial conditions are being considered in this research \( (\sigma_3 = 0) \), the \( \sigma_2/\sigma_1 \) and \( \sigma_1/S_t \) terms represent the biaxial law of yielding. For the same nondimen-
sional model and prototype strain $\varepsilon E/S_t$, the model and prototype materials must exhibit the same type of law of yielding, sometimes referred to as the theory of failure or yield locus. This implies that for each and every value of nondimensional strain, the law of yielding must be the same for model and prototype. If this condition, along with the condition of similar uniaxial parameters, is satisfied, then the general stress-strain relationship for both model and prototype are proven to be nondimensionally equivalent.

At a given nondimensional strain $\varepsilon E/S_t$, the relationship between the stress ratio and stress amplitude can be explored. For the particular polyesters used in this research, the uniaxial stress-strain curve, as illustrated in Fig. 2, exhibits the nonstrain-hardening characteristic of a stress plateau. The law of yielding characteristics, in the research reported here, will be explored only at nondimensional strains that assure attainment of this plateau. Or in simple terms, yielding will be defined as the condition at which this type of plateau is reached. This condition will be used for both the uniaxial and biaxial stress cases. It will be assumed that the law of yielding obtained in this manner, be it the von Mises, Tresca, internal friction, or other theory of failure, is applicable for all values of nondimensional strain even though the nondimensional strain states in the transition region between elastic and plastic behavior were not fully investigated.

The uniaxial stress cases provide data for instances of zero stress ratio, in fact the yield stresses $T$ in tension and $C$ in compression, Fig. 2, obtained from these tests are key parameters in establishing the law of
Fig. 2. Stress plateau.
yielding. In order for a researcher to make a decision as to what type of
law of yielding is appropriate for the modeling material, stress ratios
other than zero must be utilized. Tests incorporating combinations of
torsion, pressurization, axial tension, and axial compression of thin
walled tubes are common methods of producing biaxial states of stress.
The pressurized thin walled tube was used in this research to provide a
stress ratio of 2. The yield stress obtained under this biaxial stress
condition together with the uniaxial tensile and compressive values were
used in the determination of the law of yielding.

Because of the experimental difficulties of establishing the biaxial
yield criteria, few materials have been evaluated in this manner by past
researchers in the photoplasticity field. The temptation is for the re­
searcher to match the uniaxial tension properties of the model material
and the prototype and then hope that the compression and biaxial proper­
ties are good enough for the analysis envisioned. In reality, the tension,
compression, and biaxial (law of yielding) characteristics of the photo­
plastic material must be established before analyses of a general nature
can be undertaken. Specific cases can be justifiably analyzed with the
limited data if the stress fields are predominantly uniaxial tension, but
it must be realized that some risk is involved when this is done. The
research presented here hopefully begins the further characterization of
the polyester material mixture method reported by Morris and Riley (35).
Data will be presented concerning the tension, compression, and biaxial
mechanical properties of the polyesters along with the optical properties
and associated methods of analysis.
After these material properties are obtained and it is established that the model and prototype materials have similar mechanical properties, the prediction of prototype stress distributions by the use of photoplastic models can be undertaken. As is done in elastic problems, the stress distribution can be expressed in terms of the applied load, geometry, and material properties; i.e.,

$$\sigma = \Psi(L, \lambda_1, P, E, n_t, n_c, S_t, S_c, \mu)$$  \hspace{1cm} (15)$$

where it is assumed that there are no strain rate effects existing. Prescribed boundary displacements can also be included (40), but for many problems the above relationship is adequate. In nondimensional terms, Equation 15 becomes

$$\frac{\sigma L^2}{P} = \Psi \left( \frac{\lambda_1}{L}, \frac{P}{EL^2}, \frac{n_t}{n_c}, \frac{S_t}{S_c}, \frac{S_t}{E}, \mu \right)$$  \hspace{1cm} (16)$$

The process of relating the model stress distributions to the prototype distribution requires that the geometry, load, and material properties be related by the $\pi$ terms shown above or by other equivalent $\pi$ terms (40).
B. Casting Procedure and Model Fabrication

Morris and Riley (35) used a flexible resin (Laminac EPX-126-3) and a rigid resin (Laminac 4116) mixture in their photoplasticity feasibility study. Unfortunately, the flexible resin marketed by the American Cyanamid Co. was an experimental product and is no longer marketed. The rigid resin together with several other flexible resins, however, are still available so that further studies of the mechanical and optical characteristics of the polyester resin system were possible. The substitute flexible resin selected for this research was Laminac 4134. This resin combination yields a model material with somewhat different properties, but the same gross overall behavior as the original mixtures. The new material exhibited material properties more sensitive to test temperature variations than was observed with the original materials. The purpose of this research is to provide all of the tensile properties for the new material (Laminac 4116/4134 mixture) provided in the original feasibility study together with the complete mechanical and optical properties for the new material under uniaxial compression and biaxial stress conditions.

The casting procedure followed for the polyester mixtures uses a volume approach for some of the constituents. For small castings, the small amounts required of the liquid curing agents can be measured accurately with pipettes. Weight measures are used for the viscous resin components. The castings all consisted of the following amounts of curing agents for each 300 grams of the resin mixture:

- 3.0 ml Methyl-Ethyl Ketone Peroxide (MEKP)
- 3.0 ml Laminac Additive 10 (a wax in a styrene monomer)
- 0.7 ml Cobalt Napthanate.
The MEKP and acetalt naphthanate should not be mixed together because of their explosive nature and burning tendencies. Once one is mixed thoroughly with some resin, the other can be safely added to the mixture.

Flat sheets of model material were obtained by casting the resin mixture in molds consisting of two glass plates separated by aluminum bar spacers of the desired thickness. The glass plates were thoroughly cleaned and then sprayed with a mold release agent called Korax 1711 manufactured by Contour Chemical Co. The aluminum bars were lightly coated with an ordinary lubricative grease so that a good seal between the plates and bars was achieved. The plates and bars were held in close contact during the casting process by the application of spring steel binder clips. Morris (42) has published an applications paper dealing with a similar method of casting polyester resins.

After the mixture had been slowly combined to minimize air bubbles, the mixture was poured into the prepared molds. The polymerization occurs at room temperature. Because of difficulties encountered in keeping the polyester mixture from adhering to the plates and subsequently causing a self-induced tearing and fracturing due to curing shrinkage, the mixture was removed from the mold before complete polymerization was completed. This method gave sheets of material suitable for tension, compression, and pure bending test specimens.

The mixture was room cured for 24 hours and then post cured for 16 hours at 90°C. The materials exhibit very little time-edge stress effect, caused by moisture absorption at the free edges, and the machining characteristics are excellent. The castings have a slight reddish-brown tint
due to the presence of the cobalt napthanate, but this has little effect on the light transmission. For the sodium light source (589.3 nm) used the light transmission was good, and it appears that the material can be used in thick castings needed in three-dimensional studies.

The thin walled cylinder specimens used for the biaxial tests had an outside diameter of 100 mm, a wall thickness of 4 mm, and a length of 400 mm. The specimens were formed by spin casting the viscous resin mixture inside a 100 mm diameter polished aluminum tube. The wall thickness was controlled by varying the amount of mixture introduced into the aluminum tube. The curing cycle used for the cylinders was the same as that used for the flat sheets.

The ends of the cylinders were sealed by using aluminum attachment rings and end plates. The rings were approximately 40 mm long and were machined to fit over the outside surface of the cylinder with sufficient clearance to facilitate bonding. The adhesive used to bond the rings to the cylinders consisted of 100 pbw of Ciba 502 epoxy resin, 10 pbw of 951 hardener, and 0.7 pbw of aluminum powder. A complete model showing the cylinder, attachment rings, end plates bolted to the rings, and quick attach hydraulic coupler for the pressure line is shown in Fig. 3.

The same end rings were used for all of the cylinders tested in this research. To remove the aluminum rings from the polyester cylinder, the cylinder/ring combination was placed in an environmental chamber capable of achieving a temperature of -50°C. When the chilled cylinder was lightly tapped, the rings slipped off of the cylinder.
Fig. 3. Thin-walled cylinder test specimen.
Aluminum templates were taped onto the model material and tension, compression, and beam test specimens were machined with the use of a high speed router. The tension specimen test sections were 150 mm long and 13 mm wide. Tension specimens were made from 6 mm and 13 mm thick polyester material. Compression specimens had test sections of 60 mm length and 13 mm width and depth. Rectangular beam specimens of 13 mm thickness and 38 mm depth had test section lengths of 75 mm.

Moiré grids were placed on the beam specimens. The surface of the beam was prepared for the application of a photographic emulsion by a single tool fly-cutting operation. This process removed the smooth outer surface and provided a good surface for the application of the Kodak KPR photoresist solution. The photoresist was wiped onto the surface with tissues saturated in the photoresist. This provided a thin emulsion that was thoroughly dried by a heat lamp. Thin-film 1000 lines/inch (39370 lines/m) grid negatives were placed on the dried surface, and the specimen and negative were exposed to ultraviolet light created by a carbon-arc plate maker. For this particular material and method, the exposure time was 3 minutes, and the emulsion was developed with continuous agitation in standard Kodak photoresist developer for 3 minutes. The line contrast was enhanced by the use of a Kodak black photoresist dye. Glass plate master grids were placed on the beam specimens and the resulting moiré patterns of the deformed beam provided the information necessary to determine the strains.
C. Experimental Apparatus

Complete mechanical and optical properties for the 4116/4134 polyester mixture under uniaxial tensile, compressive, and biaxial loadings were determined. Major test conditions of applied strain rates and temperatures had to be considered. The mixtures exhibited material properties that were highly dependent upon these two conditions. Thus, strain rates of 1 (μm/m)/sec were desired and temperatures controllable to within 1/4°C were required.

1. Uniaxial tests

The tensile-testing machine used for all of the uniaxial determinations had a crosshead whose speed could be accurately controlled. Different strain rates, as measured at the gage length, were achieved by varying the crosshead speeds. This capability was required since the stress-strain curve of a flexible-rigid resin mixture is strain rate dependent. It was observed early in the program that the material properties were also sensitive to temperature variations. In order to maintain control of the temperature, a circulating mineral oil bath, capable of maintaining the oil temperature within 1/4°C of a desired temperature, was designed and used. The thermostatically controlled heater of the type commonly used in viscometers contained a pump and was used to circulate the heated oil. The oil bath tank consisted of plexiglass walls and was mounted directly on the tensile-testing machine.

The compression specimens were placed in a sliding compression cage that allowed the use of a tensile-testing machine to apply a compressive load. The beams were placed in four point loading which produced a
constant moment in the test section. These loading conditions are schematically illustrated in Fig. 4.

Uniaxial tension test data were plotted automatically on an x-y recorder by using the output from a load cell indicator and a signal from a clip-on transducer capable of measuring strains up to 30%. This method was used for the mechanical property test to obtain the stress-strain curves. The clip-on transducer caused some local distortions in the fringe patterns; therefore, when the optical data were recorded, two lines were scribed on the model with an ink pen, and a traveling microscope was used to measure displacements in the specimen. Twenty-five mm gage lengths were used on the tension specimens. A micrometer was used to measure the specimen thickness so that Poisson's ratio could be calculated.

The compression test specimens also used inked lines to give 13 mm gage lengths. Poisson's ratio was obtained in a manner similar to that used for the tension cases.

The clip-on transducer was fabricated from cold-rolled 7030 brass stock of 0.5 mm thickness. The general shape of the transducer is shown in Fig. 5. Two strain gages were placed on the inner and outer surfaces of the formed brass at the points indicated and were attached to a commercial strain indicator and a 100:1 voltage amplifier before being used as an input to the x-y recorder. Two small holes were drilled in the brass and straight pins were soldered into place. The transducer was attached to the tensile specimens by sticking these straight pins into the model.

As the model stretched, the original gage length $L_g$ increased by an amount $\delta$. This caused the original radius of curvature $r_1$ of the
Fig. 4. Loading methods: a) compression  
   b) pure bending.
Fig. 5. Longitudinal strain transducer.
transducer to change to $r_2$. The resulting strains recorded by the strain
gages gave a signal proportional to the longitudinal strain in the model.
Calibration of the transducer indicated that strains up to 30% could be
linearly related to the signal produced by the strain gages.

The original radius of the transducer used was approximately 40 mm,
the thickness was 0.5 mm, the depth was 7.5 mm, and the gage length was
25 mm. This arrangement kept the maximum strains at the strain gage loca­
tion within the capabilities of the strain gages. The strain gage suppli­
ers estimate that the particular strain gages employed were usable up to
strains of 3 - 5%.

2. Pressurized cylinder tests

The thin-walled cylinder specimens were loaded with a pneumatic-
hydraulic system shown schematically in Fig. 6. Maximum strain rates were
maintained below the limiting value established for strain rate effect
independence in the uniaxial tension and compression tests. The pressure
applied to the cylinder was controlled with two microneedle valves. The
first valve regulated the flow of high-pressure nitrogen gas into the top
part of the accumulator. The accumulator acts as an interface between the
gas and hydraulic fluid. The second microneedle valve controlled the flow
of the hydraulic fluid from the bottom portion of the accumulator into the
test cylinder. Each of these valves had 0.80 mm orifices. The needles
were mounted on 20 turn threaded stems which had vernier scales attached.
The needles could be accurately set at 200 locations between the fully
closed and the fully open positions. By adjusting the two needle posi­
tions, the strain rate in the specimen could be adequately controlled.
Fig. 6. Schematic diagram of the system used to test the thin-walled cylinder specimens.
The second valve also restricted the flow rate into the specimen to a level which eliminated any serious danger due to sudden fracture of the cylinder. The pressures in the specimen were monitored with Bourdon tube pressure gages.

The temperature was controlled by the same method used in the uniaxial tests. Thus, all of the thin-walled cylinder specimens were tested in a mineral oil bath which could be maintained at the desired temperature ± 1/4°C.

The longitudinal and hoop strains in the cylinder were determined with a traveling microscope and grid lines inked on the outside surface of the cylindrical specimen. The gage lengths used for the longitudinal and hoop directions were 32 and 25 mm, respectively.

D. Uniaxial Mechanical and Optical Properties

The tensile mechanical and optical properties of the polyester system had to be reexamined because the flexible resin used by Morris and Riley in the feasibility study was no longer manufactured. The tension tests were used to establish that the new material was qualitatively similar to the original mixture. No particular prototype material is being quantitatively modeled with respect to the Ramberg-Osgood parameters as was done in the original work; however, this research is aimed at providing the general characteristics required to evaluate the mixture as a potential modeling material.

This section presents the new tensile data plus the compression test results. The uniaxial stress-strain relationships and the optical properties are further explored on tests of beams in pure bending. With the
use of beam specimens, possible stress analysis procedures are illustrated and the validity of the mechanical properties is assessed with the use of the equilibrium checks.

1. Tension and compression

Fig. 7 shows engineering stress-strain curves for seven mixtures of flexible and rigid polyester resins. The tests were conducted at room temperature at a constant strain rate of 90 (µm/m)/sec. As can be seen from the figure, the mixture ratio can be used to significantly alter the stress-strain curve. These tests indicate that the replacement flexible resin (Laminex 4134) yielded a more flexible mixture when compared with the Morris and Riley mixture. It was observed early in the program that the material properties were also sensitive to temperature variation, thus the remaining tests were conducted with controlled test temperatures.

Fig. 8 shows results for one particular mixture ratio, namely 70 percent rigid and 30 percent flexible, at several different strain rates. This ratio was selected on the basis of the curves shown in Fig. 7. The initial test was conducted at 35°C and at an average strain rate of 1.1 (µm/m)/sec. When the strain had reached 6.7 percent, it was obvious that a desired maximum stress level of approximately 2.5 ksi (17.5 MN/m²) would not be achieved, so the test was terminated. Tests were then conducted at several different temperatures, and 28°C was selected as the test temperature for the rate dependency work. The rate dependency of the 70 percent rigid/30 percent flexible mixture at 28°C is shown for both tensile and compressive uniaxial loadings in Fig. 8. It can be seen that the yield stress decreases only about 10 percent for a 350 percent reduction in
Fig. 7. Stress-strain curves for several mixtures of rigid and flexible polyester resins at a constant strain rate.
Fig. 8. Tensile and compressive stress-strain curves for a 70/30 mixture of rigid and flexible polyester resins at a number of strain rates.
strain rate [from 3.9 (μm/m)/sec to 1.1 (μm/m)/sec] in the range of strain from 4 to 12 percent. Thus it seems reasonable to assume that a stress-strain curve independent of strain rate is achieved if the maximum strain rate in a model is kept below 1 (μm/m)/sec. At higher levels of compressive strain (>4 percent), this assumption is obviously not valid. This result indicates that the compressive properties are more strain rate dependent than the tensile properties. The data in Fig. 8 also indicate that the compressive yield stress for the 70/30 mixture is approximately twice as large as the tensile yield stress.

The uniaxial results as shown in Fig. 8 are consistent with the general stress-strain form given by Equation 11. The Young's modulus can be considered equivalent in tension and compression, but the other Ramberg-Osgood terms, power coefficient and yield stress, are not equal in tension and compression. Thus, from this information it can be concluded that the photoplastic material can be used to model prototypes having unequal tension and compression properties, but prototypes having equal properties can not be modeled exactly. The material could be used as a distorted model for modeling primarily uniaxial tensile or compressive stress fields. This would mean that the π terms for model and prototype would be unequal, and the researcher applying this method would have to assess the errors introduced when such a modeling process was incorporated. Such an application was conducted in the Morris and Riley study. Plates with centrally located holes were used and stress concentrations were successfully obtained.
Another area of interest investigated was the effect of thickness of the casting on the stress-strain curve. The concern here was based on a fear that a thicker casting might cure differently than a thinner one. At the slowest strain rate, 6 mm and 13 mm thick tensile specimens were tested. These were the two thicknesses used in subsequent tests. No appreciable difference was observed for these two specimens.

Lateral as well as longitudinal strains were obtained from tests of tension and compression specimens. The plot shown in Fig. 9 depicts the relationship between the longitudinal and lateral strains. The slope of this curve represents the Poisson's ratio, and a 0.47 value for Poisson's ratio appears to be appropriate for this material. The equivalent result reported for the Morris and Riley mixture was 0.45. Both of these results can be considered adequate for photoplastic modeling in the plastic region when compared with the constant volume assumption of 0.5 for plastic flow in metals.

Tests were also conducted to obtain the optical properties of the material. A diffused light polariscope with a sodium light source (589.3 nm) was used for all of these tests. Fig. 10 shows the fringe order per unit thickness as a function of uniaxial tensile and compressive strain. It is interesting to note that the birefringence, in a specimen under either tensile or compressive loadings, changes sign at a stress level near the yield strength of the material. This behavior produces a fringe order distribution which is a multivalued function of strain. Unload or retained birefringence, on the other hand, yields a fringe order distribution which is a single-valued linear function of strain that is independent
Fig. 9. Poisson's ratio.
Fig. 10. Loaded and retained optical response as a function of tensile and compressive strain for a 70/30 mixture of rigid and flexible polyester resins at test temperatures of 28 and 35 °C.
of the test temperature. This behavior is similar to the behavior noted by Morris and Riley for the original polyester material under tensile loadings. Fig. 10 shows that for a constant test temperature the compression curve has the same shape as the tension curve; however, the peaks are of different magnitudes, and the 28°C compression zero crossing occurs at 6% strain compared to 3.5% strain for tension. The unloaded compression data points lie on the same straight line as the tension points. For convenience in future analyses, positive birefringence has been associated with the unloaded tensile curve.

From this study it is evident that the loaded birefringence curve would be difficult to use to establish strain distributions in a general strain field. The unloaded fringe order-strain curve offers a more traditional relationship for establishing the strain field in an elastoplastic problem. As will be seen later, however, other factors must be considered when this unload method is used. Fortunately, the optical properties of the material were observed to be independent of both temperature and strain rate.

2. **Beam in pure bending**

The rectangular beam specimen loaded in four point bending was selected for the second series of tests. The specimen used contained a large region under pure bending with areas of both tension and compression occurring simultaneously. The strain rates in the beam also range from zero at the neutral surface to maximum values at the outer fibers. The beam specimen thus provides data for evaluating the assumption of strain rate independence below a certain strain rate if the outer fiber rate is
limited to this value. Equilibrium checks provide a means for evaluating
the validity of the overall test procedure.

Moiré grids were placed on the beams, when the beam tests were con-
ducted, to provide a means for making an independent determination of the
strains. One half of the beam from top to bottom contained a 1000 lines/inch (39370 lines/m) grid oriented vertically; the other half contained
lines oriented horizontally. The moiré data thus provided principal
strain information over the full depth of the beam.

The test procedure followed in the beam tests was the same as that
developed for the uniaxial tests; namely, the load was applied slowly and
then removed suddenly at a number of load levels in different specimens.
Optical patterns were recorded photographically. Fig. 11 and 12 show
typical light and dark field fringe patterns and the fringe order distribu-
tion in a beam immediately before and after removal of the load, respec-
tively. The presence of three zero-order fringes in the patterns of Fig.
11 verifies the gross overall optical behavior of the material previously
indicated in Fig. 10.

The optical and moiré data from the beam specimens provided the
opportunity to evaluate three possible methods of analysis, namely:

a) Use of unloaded (retained) fringes to predict loaded strain
fields.

b) Use of loaded fringes to predict loaded strain fields.

c) Use of unloaded (retained) fringes to predict unloaded (retained)
strain fields.
Fig. 11. Light and dark field isochromatic fringe patterns and fringe order distribution in a beam specimen under load.
Fig. 12. Light and dark field isochromatic fringe patterns and fringe order distribution in a beam specimen immediately after unloading.
3. **Analysis procedures**

Proceeding with the concept that the linear unloaded fringe order–loaded strain relationship could be used in the traditional manner of method (a) above, a series of beam tests was conducted. As indicated in Fig. 13, three beams were tested to a maximum principal strain difference of 10 percent on the tension side and 9 percent on the compression side. The moiré data indicated that the longitudinal strain was a linear function of distance from the neutral surface at all levels of load. This is in agreement with the fundamental beam assumption that transverse planes remain plane in a beam being subjected to a pure bending moment. Fig. 12, however, indicates that the unloaded fringe order is not a linear function of distance from the neutral surface of the beam. Thus, the unloaded fringe order cannot be a linear function of the loaded longitudinal strain. Fig. 13 shows a plot of unloaded fringe order per unit thickness as a function of the loaded principal strain difference indicated by the moiré data. The relationship is slightly nonlinear and shows a zero offset tendency with respect to the elastic neutral surface of the beam. The beam in pure bending experiences a uniaxial state of stress until the curvature becomes sufficient for significant radial stresses to develop. Unlike the tension and compression specimens, however, a residual stress distribution develops in the beam specimen when it is unloaded. If the material of the beam has identical tensile and compressive stress–strain properties, the magnitudes of the residual stresses are symmetric with respect to the elastic neutral surface (the signs are opposite in the two halves). If the tensile and compressive stress–strain properties of the
Fig. 13. Retained optical response as a function of loaded principal strain difference in three different beam specimens.
beam material are significantly different (the compressive yield stress is approximately double the tensile yield stress in the polyester material), the residual stress distribution which develops upon unloading will not exhibit symmetry with respect to the elastic neutral surface of the beam. The higher compressive yield strength causes the zero stress position to shift away from the elastic neutral surface towards the compressive side of the beam. The nonlinearity and the zero offset shown in Fig. 13 can be attributed to the residual stresses which remain in the beam after the loads are removed. The presence of the residual stresses in complicated models present serious analysis difficulties. If the unloaded fringe orders in a complicated model are used to predict the loaded strains, errors will be introduced if a linear strain-optic law is used. Since the residual stresses and their associated strains affect the resulting isochromatic fringe pattern, and since the residual stress distribution is a function of the particular model geometry, no unique calibration can be devised to account for the presence of the residual stresses. Thus, "exact" answers using this method cannot be obtained for complicated geometries where residual stresses will always exist. Approximate answers may be obtained, however, and their usefulness will depend upon the researcher's ability to ascertain the effects of the residual stress state. The data presented in Fig. 13, together with the previous work by Morris and Riley (35), seem to indicate that a class of stress concentration problems may fit into the category where useful results can be obtained.

In instances where the residual stress magnitudes are significant or where more accurate solutions are needed, the optical patterns obtained
under load must be used to obtain principal strain information; i.e., the
loaded fringes are used to predict loaded strain fields (method b). The
multivalued relationship between fringe order and strain complicates the
analysis, but the residual stress problem is not present. The unloaded
optical patterns can be used to advantage as an aid in the interpretation
of the loaded patterns. The beam studies together with pressurized thin-
walled cylinder studies indicate that the loaded fringe order is a function
of the principal strain difference. These results are presented in the
Biaxial Mechanical and Optical Properties section under the subheading
Strain-optic relationship (pp. 58-59).

Forming operations such as hot rolling, cold rolling, drawing, and
extruding are widely used manufacturing processes. To date, relatively
little information is available regarding strain distributions associated
with these operations except for the limited data obtained by using coarse
grid networks on the surface or on planes of symmetry in the visoplasticity
method. The use of suitable photoelastic models where retained fringes
may be used to predict retained strain distributions could provide much
useful information (43,44).

The beam tests provided a means for initiating studies in this impor-
tant area. Both unload optical data and principal strain differences from
moiré data were available from several beams at a number of times after
unloading. The results from two beams are presented in Fig. 14. In this
figure, the retained optical response is plotted as a function of the re-
tained principal strain difference. Data was recorded in one beam 450
sec. after the load was removed. In the second beam, data was recorded
Fig. 14. Retained fringe order vs. retained strain.
120 and 440 sec. after unloading. Fig. 14 shows a linear relationship between optical response and strain difference. In a series of tests by other researchers (43), in which the same mixture ratio of polyester resins was used, similar results were obtained. The test conditions for their study were radically different; i.e., the applied compressive strain rate was approximately 3000 times the maximum rate experienced by the beam, the temperature was 17°C higher, and the relaxation times ranged from 30 minutes to 98 hours as compared to a maximum of 8 minutes in the beam tests. In spite of these differences, the retained optical-strain relationship was the same. Their data is plotted as the two special points on Fig. 14. The implication that can be drawn from these results is that the retained optical response is independent of both strain rate and temperature. The mechanical properties will be different for the two test conditions, but the optical properties are a function only of the instantaneous state of strain.

Fig. 15 shows a typical plot of the retained fringe order per unit thickness in a beam at different times after the load was removed. Several different locations in the beam were monitored and values for both the tension and compression sides are given. In this particular test, the fringe orders changed very little beyond 800 sec. after the load was removed. This data indicates that fringe patterns should be recorded within 20 sec. of removal of the load if they are to be used to predict loaded strains.
Fig. 15. Retained fringe order as a function of time after removal of the load.
4. **Equilibrium checks**

One further purpose of the beam tests was to provide additional data concerning the strain rate independence of the mechanical properties below a certain threshold value. This was accomplished by using the longitudinal stress distribution in the beam for a series of axial load and bending moment equilibrium checks. Moiré data provided the longitudinal and transverse strains for use with the stress-strain curve presented in Fig. 8. Strain and stress distributions across a typical beam are shown in Fig. 16. Such distributions yielded equilibrium checks within ±10 percent for all of the distributions studied. This indicates that the assumed strain rate independence of the material below a certain rate is reasonable.

E. **Biaxial Mechanical and Optical Properties**

The research outlined in this section was conducted to determine the strain-optical and yield characteristics, under a biaxial state of stress, of a polyester material suitable for photomechanics studies in the inelastic region of material response. In the previous section, complete mechanical and optical properties were presented for the material under uniaxial tensile and compressive loadings. The uniaxial tests indicated that the mechanical properties of the material could be changed significantly by modifying the ratio of flexible to rigid polyester resin, the loading rate, or the test temperature.

An important aspect of model-to-prototype scaling for the inelastic region of material response concerns the yield characteristics of the model and prototype materials under multiaxial states of stress.
Fig. 16. Stress and strain distributions in a typical beam under load.
Experimental results for ductile metals are usually in good agreement with the von Mises criterion of yielding. The data from the previous uniaxial tensile and compressive tests, together with the biaxial data from the thin-walled cylinder tests, indicate that the polyester model material may follow a pressure-modified von Mises yield criterion (45,46) which accounts for differences in tensile and compressive yield strengths.

1. Strain-optic relationship

The research of Morris and Riley (35) and the present research, using optical data from simple tension and compression tests, have indicated that the fringe order at a point in a specimen under load is a multivalued function of the strain at the point while the retained fringe order upon removal of the load is a single-valued linear function of the retained strain. In all of this previous work, the state of stress was uniaxial; therefore, the two principal strains $\varepsilon_1$ and $\varepsilon_2$ were related to each other and it was impossible to determine whether the optical response was related to the principal strain difference $(\varepsilon_1 - \varepsilon_2)$, the principal strain sum $(\varepsilon_1 + \varepsilon_2)$, or some other combination of the individual principal strains.

With the pressurized thin-walled cylinder specimens, the state of stress is biaxial, so additional data is provided to help establish the functional relationship between optical response and strain. It was observed that as the pressure was slowly increased in the cylinder, the longitudinal strain initially increased but quickly stabilized at a relative low value of approximately 0.003 $\mu$m/m. The hoop strain increased throughout the loading sequence. Thus, both the stress and strain fields
in the thin-walled cylinder are much different than those encountered in the uniaxial tests.

The data in Fig. 17, from three different types of tests, clearly indicate that the fringe order at a point in a loaded specimen is a multi-valued function of the instantaneous principal strain difference at the point. Upon removal of the load, the retained fringe order is a single-valued linear function of the retained principle strain difference as indicated in Fig. 14.

2. **Material properties**

Uniaxial tensile and compressive stress-strain curves were obtained at a test temperature of 28°C and a strain rate of 1.5 m/m per second for the batch of material used in fabricating the cylinders. The tension specimen, which was removed in the longitudinal direction from an unused cylinder, gave the stress-strain curve shown in Fig. 18. This curve indicates that the spin-casting fabrication procedure for the cylinders had little or no effect on the mechanical behavior of the material. This stress-strain curve, when compared with curves from previous uniaxial tests, is well within the limits of variation normally encountered between different batches of plastic material. The 2% offset yield strength obtained from this curve is approximately 2.75 ksi (19 MN/m²). The compression specimen gave a 2% offset yield strength of 3.7 ksi (25.5 MN/m²). The hoop stress in the thin-walled cylinder reached a stress plateau value of approximately 2.83 ksi (19.5 MN/m²).
Fig. 17. Fringe order versus principal strain difference for the 70/30 polyester mixture.
Fig. 18. Stress-strain curve from a test coupon removed from a cylinder.

\[ \dot{\varepsilon} = 1.5 \text{ (um/m)/sec} \]

\[ \text{TEMP} = 28^\circ\text{C} \]
3. Yield criterion

An important consideration in model-to-prototype scaling in the inelastic region of material response concerns the yield characteristics of the two materials under multiaxial states of stress. In classical plasticity theory for ductile materials it is normally assumed that: (1) the material is isotropic and homogeneous, (2) yielding is not influenced by the hydrostatic component of the stress state, (3) the tensile and compressive yield strengths are equal, and (4) constant volume is maintained as deformation occurs. Experiments with ductile metals indicate that either a von Mises or a Tresca yield criterion gives reasonable results.

Recent research with crystalline and glassy amorphous polymers indicates that conditions (2) and (3) above are not usually satisfied. In the present study it was observed that the ratio of compressive to tensile yield strengths was approximately 1.35. This ratio is approximately the same as that observed by other investigators (45,46) for polycarbonate, polystyrene, PMMA, high density polyethylene and polyvinylchloride. No direct evidence is available for the polyester mixtures concerning the effect of the hydrostatic component of the stress state on yielding.

In two recent papers by Raghava and Caddell (45,46), a pressure-modified von Mises criterion, which accommodates differences in tensile and compressive yield strengths, was proposed for polymers. The form of the relationship for \( \sigma_3 = 0 \) (plane stress) is:

\[
s_1^2 + s_2^2 - s_1s_2 + (C - T)(s_1 + s_2) = CT
\]
where $C$ is the compressive yield strength in a uniaxial specimen
$T$ is the tensile yield strength in a uniaxial specimen
$\sigma_1$ and $\sigma_2$ are the principal stresses.

This modified von Mises yield criterion showed excellent agreement with experimental data for all of the polymers previously mentioned. If the modified von Mises criterion is normalized with respect to the tensile yield strength $T$ and plotted for the case of $C/T = 1.35$, the curve shown in Fig. 19 is obtained. The points representing the tensile and compressive yield strengths automatically fall on the curve, and data from the cylinder tests show excellent agreement. This indicates that the modified von Mises yield criterion may represent the yield behavior of the polyester mixtures under multiaxial states of stress. Further tests with other stress ratios are needed to fully confirm this behavior.

Since the modified von Mises yield criterion reduces to the von Mises criterion for materials with equal tensile and compressive yield strengths, further study seems justified to see if the $C/T$ ratio can be reduced by proper selection of mixture ratio or test temperature.

4. Fracture behavior

All of the cylinders used in the study fractured longitudinally at strain levels far below the ultimate strain achieved in the uniaxial tensile tests. A typical cylinder, showing the fully developed fracture pattern, is shown in Fig. 20. The fracture may have resulted from a significant decrease in ductility due to the tensile biaxial state of stress, or it may have been caused by imperfections in the tubes due to the
Fig. 19. Yield locus data for the 70/30 polyester mixture.
Fig. 20. Fractured thin-walled cylinder.
manufacturing process. Similar studies using small cylinders machined from solid blocks of material may provide the data needed to resolve the fracture problem.
V. CONCLUSIONS AND RECOMMENDATIONS

The complete mechanical and optical properties for the polyester material are presented under both tensile and compressive uniaxial loads. These results indicate that the stress-strain curves for the material can be modified significantly by changing the mixture ratio or the test temperature. The curves are also strain rate dependent, but a limiting value of strain rate exists below which, for practical purposes, the material can be considered strain rate independent. Optical data from the simple tension and compression tests indicates that the fringe order at a point in a specimen under load is a multivalued function of the instantaneous strain at the point. The retained fringe order at the point in the specimen upon removal of the load, however, is a single-valued linear function of the retained strain at the point.

The optical data from the pressurized thin-walled cylinder specimens, together with the tensile and compressive data, indicate that the fringe order at a point in a specimen is a function of the instantaneous principal strain difference at the point. This dependence on principal strain difference can not be determined from uniaxial tests since one principal strain is simply a fraction of the other. This relationship holds for most materials in the elastic range but had not been established previously for this material in either the elastic or the inelastic ranges. Under load, the function is nonlinear and multivalued. Upon removal of the load, however, the retained fringe order at a point is a single-valued linear function of the retained principal strain difference at the point.
Data from the uniaxial tension and compression tests, together with limited data from the pressurized thin-walled cylinder tests, indicates that the polyester material follows a modified von Mises yield criterion which accounts for differences in the tensile and compressive yield strengths. This type of yield behavior has been noted by other investigators in polymers such as polycarbonate, polystyrene, and polyvinylchloride.

The results of this program indicate that the polyester mixture is a suitable model material for many studies in the inelastic range of material response. With the developing wide use of plastic materials, the similarity in yield behavior between these materials and the model material should permit a wide variety of studies in the inelastic range with no scaling difficulties. The difference in yield behavior between the model material and ductile metals may produce serious modeling problems unless the tensile and compressive yield strengths of the model material can be brought closer together by suitable selection of mixture ratio or test temperature. Further work in this area should be undertaken.

There are several research areas that need to be explored. These areas include the following:

a. The fracture behavior of the material is disturbing and tests on thin-walled cylinders machined from cast blocks of the polyester material should be conducted. This will help determine whether the fracture behavior was due to specimen irregularities or basic material susceptibility to biaxial tension fields.
b. Once the fracture behavior is clarified, then a program to determine what test conditions, material mixtures, additives, etc. will alter the tension-compression yield strength ratio can be instituted. If the yield strength ratio approaches 1, then the pressure-modified von Mises yield criterion will become the normal von Mises yield criterion.

c. Further biaxial tests are needed to establish the validity of the yield criterion. Biaxial states of stress other than the 2:1 ratio of pressurized cylinders used in this research would be required.

d. It is possible that other rigid and flexible polyester resins exist that will give improved optical and mechanical properties. Only a very limited number of polyesters have been investigated in this and past research; thus, the search for and classification of alternate polyesters might be considered.

e. The ability to alter the shape of the polyester mixture stress-strain curve could be used to study the effect of the stress-strain relationship on the stress and strain concentration factors of a typical model such as a plate with centrally located hole or a plate with edge notches. The effects of the stress-strain curve shape could be obtained by using several mixture ratios and/or test temperatures. Results could be compared to theoretical work such as presented by Budiansky and Vidensek (47) on stress and strain concentration factors of a circular hole in a plate subjected to uniaxial tension.
Other areas of research most certainly exist, but it seems prudent to methodically establish the basic material characteristics before highly complicated stress and strain analysis problems are studied.
VI. BIBLIOGRAPHY


VII. ACKNOWLEDGMENTS

I wish to thank Professor W. F. Riley for his guidance during this project. He has continuously demonstrated unique but simultaneously fundamental approaches to solving stress analysis problems. I consider my association with Professor Riley to be one of the most important aspects of my graduate education. I also wish to thank Professors C. P. Burger and F. M. Graham for providing many hours of advice and consultation that brought forth many fruitful results.

My wife Sharon and children Kelly and Laura have provided a home life that made it possible for me to spend those odd hours required to conduct the experiments. Thank you for the sacrifices that were required to reach "our" goal.

This research work was supported by the Engineering Research Institute at Iowa State University through a grant from the U.S. Army Research Office.