Reactor anomaly detection based on noise analysis

George William Hannaman Jr.

Iowa State University

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<td>Axial power distribution shape parameter</td>
<td>( \text{ft}^{-1} )</td>
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<tr>
<td>A</td>
<td>Flow cross sectional area per fuel pin</td>
<td>( \text{ft}^2 )</td>
</tr>
<tr>
<td>( A_c )</td>
<td>Total core flow cross section</td>
<td>( \text{ft}^2 )</td>
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<td>( A_f )</td>
<td>Fuel cross sectional area</td>
<td>( \text{ft}^2 )</td>
</tr>
<tr>
<td>( A_m )</td>
<td>Mechanical equivalent of heat = 778</td>
<td>( \text{ft} \cdot \text{lb/Btu} )</td>
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<td>APF</td>
<td>Axial peaking factor</td>
<td></td>
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<tr>
<td>( A_s(x, t) = A_s^0(x) + a_s(x, t) )</td>
<td>Steam cross sectional area</td>
<td>( \text{ft}^2 )</td>
</tr>
<tr>
<td>( A_w(x, t) = A_w^0(x) + a_w(x, t) )</td>
<td>Water cross sectional area</td>
<td>( \text{ft}^2 )</td>
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<tr>
<td>b</td>
<td>Axial power distribution shape parameter</td>
<td>( \text{radians/ft} )</td>
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<tr>
<td>c</td>
<td>Axial power distribution shape parameter</td>
<td>( \text{radians} )</td>
</tr>
<tr>
<td>( c_p )</td>
<td>Specific heat of water</td>
<td>( \text{Btu/lb} \cdot \text{°F} )</td>
</tr>
<tr>
<td>( c_f )</td>
<td>Specific heat of fuel</td>
<td>( \text{Btu/lb} \cdot \text{°F} )</td>
</tr>
<tr>
<td>( c_c )</td>
<td>Specific heat of cladding</td>
<td>( \text{Btu/lb} \cdot \text{°F} )</td>
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<td>( C_i(t) = C_i^0 + c_i^f(t) )</td>
<td>Precursor concentration of the ( i \text{th} ) delayed neutron group</td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>Small distance</td>
<td>( \text{ft} )</td>
</tr>
<tr>
<td>( D_i )</td>
<td>Neutron diffusion length in the ( i \text{th} ) neutron energy group</td>
<td>( \text{cm} )</td>
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<tr>
<td>( D_e )</td>
<td>Hydraulic diameter</td>
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<td>D(x)</td>
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<td>E</td>
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<td>( \text{Btu} )</td>
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<td>E1, E2</td>
<td>Weighting function</td>
<td></td>
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<tr>
<td>EP1</td>
<td>Weighting function fission power to detector response</td>
<td>( \frac{n}{\text{cm}^2 \cdot \text{s}} ), ( \frac{\text{Btu}}{\text{s}} )</td>
</tr>
</tbody>
</table>
EP2
Weighting function void fluctuation to detector response

\(fp_1, fp_2, fv_2, fv_2\)
Frequencies, Figure 6.1

FABC
Normalization factor for \(D(x)\)

FRAC
Fraction of power delivered to the nonboiling region

\(g\)
Acceleration due to gravity

\(h_{fb}, h_{fnb}\)
Surface heat transfer coefficients, boiling and nonboiling regions

\(h', h''\)
Effective heat transfer coefficients, boiling and nonboiling regions

\(H(x, t) = H^0(x) + h(x, t)\)
Enthalpy of subcooled water

\(H_{fw}\)
Enthalpy of feedwater

\(H_s\)
Enthalpy of saturated steam

\(H_w\)
Enthalpy of saturated water

\(H'_v\)
Latent heat of vaporization

\(H_v\)
Latent heat of vaporization per unit volume of steam

\(J_1, J_2...\)
Constants defined in Equation 4.52

\(k\)
Thermal conductivity of water

\(k_c\)
Thermal conductivity of cladding

\(k_f\)
Thermal conductivity of fuel

\(K_1...K_5\)
Constants defined in Equation 4.84; 4.65

\(\lambda\)
Neutron lifetime

\(L\)
Axial channel length

\(L_b\)
Length of boiling region

\(L_{nb}\)
Length of nonboiling region
\( \Delta L_{nb}(t) \) Small change in the boiling boundary position  ft

\( M_w \) Mass of water in the vessel  lb

\( M_s \) Mass of steam in vessel  lb

\( M_t \) Total mass of coolant in vessel  lb

\( \dot{M}_{fw}, \dot{m}_{fw} \) Steady state, incremental flow rate of feedwater  lb/h

\( \dot{M}_s, \dot{m}_s \) Steady state, incremental flow rate of steam  lb/h

\( \dot{M}_w, \dot{m}_w \) Steady state, incremental flow rate of core water  lb/h

\( N(t) = N^0 + n(t) \) Total fission power  Btu/s

\( N(x, t) = N^0(x) + n(x, t) \) Fission power up to x per unit channel length  Btu/s·ft

\( N'(x, t) = N'^0(x) + n'(x, t) \) Fission power per unit volume of fuel  Btu/s·ft^3

\( P(t) = P + \rho(t) \) Pressure  Psia

\( P_T \) Prandtl number

\( P_f \) Fuel perimeter  ft

\( Q(t) = Q^0 + q(t) \) Total thermal power (coolant)  Btu/s

\( Q(x, t) = Q^0(x) + q(x, t) \) Total thermal power up to x per unit channel length  Btu/s·ft

\( q' \) Heat flux (boiling region average in channel)  Btu/s·ft^2

\( r \) Radial position  ft

\( r(x, t) \) Slip ratio  ft

\( R_c \) Radius core  ft

\( Re \) Reynold's number

\( R_f \) Radius fuel pellet  ft

\( R_p \) Radius of fuel pin  ft
\[ R_T \text{ Thermal resistance} \quad ^\circ \text{F}/\text{Btu}/\text{h}.\text{ft} \]

\[ R' \text{ Effective fuel radius} \quad \text{ft} \]

\[ \text{RPF} \text{ Radial peaking factor} \]

\[ s \text{ Laplace transform variable} \quad \text{s}^{-1} \]

\[ T_f(t) = T_f^0 + 6T_f(t) \]

Temperature of the fuel \quad ^\circ \text{F} \]

\[ T(t) = T^0 + 6T(t) \]

Temperature of the coolant \quad ^\circ \text{F} \]

\[ T_{\text{wall}} \text{ Temperature of cladding} \quad ^\circ \text{F} \]

\[ T_{\text{sat}} \text{ Temperature of saturated water} \quad ^\circ \text{F} \]

\[ T_s \text{ Steam transit time in boiling region} \quad \text{s} \]

\[ T_w \text{ Water transit time in nonboiling region} \quad \text{s} \]

\[ T_R \text{ Recirculation time} \quad \text{s} \]

\[ t \text{ Time variable} \quad \text{s} \]

\[ U(x, t) = U^0(x) + u(x, t) \]

Steam velocity \quad \text{ft/s} \]

\[ U \text{ Average steam velocity} \quad \text{ft/s} \]

\[ U_p \text{ Steam perturbation velocity} \quad \text{ft/s} \]

\[ V(t) = V^0 + u(t) \]

Total steam volume in core \quad \text{ft}^3 \]

\[ W(x, t) = W^0(x) + w(x, t) \]

Water velocity \quad \text{ft/s} \]

\[ W \text{ Average water velocity} \quad \text{ft/s} \]

\[ \text{WET} \text{ Weighting functions for axial flux shape distributions Equation 3.6} \]

\[ WF_b, WF_{nb} \text{ Weight functions} \]

\[ x(t) \text{ General system input} \quad \text{ft} \]

\[ x \text{ Axial position variable} \quad \text{ft} \]
\[ x(t) \] General system input
\[ Y(x, t) \] Ratio of momentum along channel to inlet momentum
\[ y(t) \] General system output

**Transfer functions**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( BA2(x, s) )</td>
<td>Boiling boundary — detector response</td>
<td>void fraction ( \frac{ft}{ft} )</td>
</tr>
<tr>
<td>( BV(s) )</td>
<td>Boiling boundary to void</td>
<td>( \frac{ft^3}{ft} )</td>
</tr>
<tr>
<td>( G(s) )</td>
<td>Zero power reactor transfer function</td>
<td>( \frac{Btu/s}{\Delta k/k} )</td>
</tr>
<tr>
<td>( G_{o1}(s) )</td>
<td>Open loop reactor transfer function</td>
<td>( \frac{Btu/s}{\Delta k/k} )</td>
</tr>
<tr>
<td>( G_{c1}(s) )</td>
<td>Closed loop reactor transfer function</td>
<td>( \frac{Btu/s}{\Delta k/k} )</td>
</tr>
<tr>
<td>( H_{fb}(s) )</td>
<td>Boiling region power to fuel temperature</td>
<td>( \frac{\circ F}{Btu/s} )</td>
</tr>
<tr>
<td>( H_{fnb}(s) )</td>
<td>Nonboiling region power to fuel temperature</td>
<td>( \frac{\circ F}{Btu/s} )</td>
</tr>
<tr>
<td>( H_T(s) )</td>
<td>Total feedback loop transfer function</td>
<td>( \frac{\Delta k/k}{Btu/s} )</td>
</tr>
<tr>
<td>( PA2(x, s) )</td>
<td>Boiling void — detector response</td>
<td>( \frac{Btu/s}{Btu/s} )</td>
</tr>
<tr>
<td>( PB(s) )</td>
<td>Power to boundary</td>
<td>( \frac{ft}{Btu/s} )</td>
</tr>
<tr>
<td>( PH_w(s) )</td>
<td>Power to subcooled water temperature</td>
<td>( \frac{\circ F}{Btu/s} )</td>
</tr>
<tr>
<td>( PTB(s) )</td>
<td>Fission power to coolant power (boiling region)</td>
<td>( \frac{Btu/s}{Btu/s} )</td>
</tr>
<tr>
<td>( PTNB(s) )</td>
<td>Fission power to coolant power (nonboiling region)</td>
<td>( \frac{Btu/s}{Btu/s} )</td>
</tr>
<tr>
<td>( PV(s) )</td>
<td>Power to void</td>
<td>( \frac{ft^3}{Btu/s} )</td>
</tr>
<tr>
<td>( TDHE1 )</td>
<td>Transmission path ( \Delta H ) to detector fission response</td>
<td>( \frac{Btu/s}{Btu/s} )</td>
</tr>
<tr>
<td>( TDHE2 )</td>
<td>Transmission path ( \Delta H ) to detector void response</td>
<td>( \frac{\text{void fraction}}{Btu/s} )</td>
</tr>
<tr>
<td>( TDPE1 )</td>
<td>Transmission path ( \Delta P ) to detector fission response</td>
<td>( \frac{Btu/s}{Btu/s} )</td>
</tr>
<tr>
<td>( TDPE2 )</td>
<td>Transmission path ( \Delta P ) to detector void response</td>
<td>( \frac{\text{void fraction}}{Btu/s} )</td>
</tr>
</tbody>
</table>
Greek symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit/symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_f$, $\alpha_w$, $\alpha_v$</td>
<td>Reactivity feedback coefficients $\Delta k/k/O_F$, $\Delta k/k/O_F$, $\Delta k/k/ft^3$</td>
<td></td>
</tr>
<tr>
<td>$\beta_i$</td>
<td>Delayed neutron fraction $i$th group</td>
<td></td>
</tr>
<tr>
<td>$\gamma(x)$</td>
<td>Steam velocity adjustment coefficient</td>
<td></td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>Ratio of terms</td>
<td></td>
</tr>
<tr>
<td>$\Delta$</td>
<td>&quot;Change in&quot;</td>
<td></td>
</tr>
<tr>
<td>$\Delta H$</td>
<td>Subcooled noise source</td>
<td>Btu/s</td>
</tr>
<tr>
<td>$\Delta P$</td>
<td>Boiling noise source</td>
<td>Btu/s</td>
</tr>
<tr>
<td>$\zeta$, $\zeta_{el}$, $\zeta_{e2}$</td>
<td>System damping ratio</td>
<td></td>
</tr>
<tr>
<td>$\kappa$</td>
<td>$\sqrt{\Sigma/D}$ in Appendix D</td>
<td>cm$^{-1}$</td>
</tr>
<tr>
<td>$\lambda_i$</td>
<td>Delayed neutron decay constant</td>
<td>s$^{-1}$</td>
</tr>
<tr>
<td>$\mu_f$, $\mu_g$</td>
<td>Dynamic viscosity of liquid, gas</td>
<td>lb/h·ft</td>
</tr>
<tr>
<td>$\rho(t)$</td>
<td>Reactivity $\Delta k/k$</td>
<td></td>
</tr>
<tr>
<td>$\rho_c$, $\rho_f$, $\rho_s$, $\rho_w$, $\rho_{fw}$</td>
<td>Density of materials: cladding, fuel, steam, feedwater</td>
<td>lb/ft$^3$</td>
</tr>
<tr>
<td>$\sigma_f$</td>
<td>Effective fission cross section</td>
<td>barns</td>
</tr>
<tr>
<td>$\tau$</td>
<td>General time constant</td>
<td>s</td>
</tr>
<tr>
<td>$\tau_x$</td>
<td>Steam perturbation delay time at $x$</td>
<td>s</td>
</tr>
<tr>
<td>$\tau_{f_{nb}}$</td>
<td>Fuel time constant (nonboiling)</td>
<td>s</td>
</tr>
<tr>
<td>$\tau_{f_{b}}$</td>
<td>Fuel time constant (boiling)</td>
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<tr>
<td>$\phi_s$</td>
<td>Phase angle difference</td>
<td>degree</td>
</tr>
<tr>
<td>$\phi_0$</td>
<td>Initial phase angle</td>
<td>degree</td>
</tr>
<tr>
<td>$\phi^0_1$</td>
<td>Fast neutron flux</td>
<td>n/cm$^2$·s</td>
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<tr>
<td>$\phi^0_2$</td>
<td>Thermal neutron flux</td>
<td>n/cm$^2$·s</td>
</tr>
<tr>
<td>$\psi_{ii}(\tau)$</td>
<td>Auto-correlation function of $i$ function</td>
<td></td>
</tr>
</tbody>
</table>
\( \psi_{ii}(\omega) \)  Power spectral density of \( i \) function
\( \omega \)  Frequency  radians/s

**Notes**

**Superscripts**

\( o \)  Means steady state condition
\( - \)  Means averaged quantity

**Subscripts**

\( b \)  Means boiling region
\( nb \)  Means nonboiling region
\( c \)  Means core
\( cl \)  Means cladding
\( f \)  Means fuel
I. INTRODUCTION
A. Background

In order to realize the economic operation of nuclear power plants it is necessary to minimize reactor downtime caused by system malfunctions. It is possible to reduce unscheduled downtime by continuous surveillance of the reactor components with the aim of detecting incipient malfunctions. Neutron noise analysis has been useful in detecting some types of reactor component malfunctions [1, 2, 3, 4], but will not necessarily detect every reactor anomaly. The most successful method of utilizing neutron noise signals for warning of abnormal changes has been to compare noise signatures produced when an abnormal condition is present with those taken during normal operation.

The term "signature" is used because systems emit "noise" that is characteristic of the system and its physical condition. This signature identifies the system, much as the handwritten signature identifies a person.

The signature can be used in several ways. One is searching the frequency spectrum for characteristic emissions based on calculated models describing the system investigated. A simple example is the calculation and measurement of the vibrational frequencies of a beam. A second way is to examine the signature in the frequency domain, and evaluate it from previous empirical knowledge. A third method is to store the signatures obtained from a system operating with known conditions, and then use this library of signatures as a standard against which to compare signatures obtained at other times or under
other conditions. By use of this comparison technique, trends can be observed, which lead to the prediction of incipient failures.

At the present time the practical application of noise signature analysis to operating power reactors requires a library of signatures based on real or artificially induced difficulties. Comparison of the library signatures with the continuously monitored noise spectra would allow for rapid quantification of incipient abnormalities. Unfortunately, the signature analysis obtained from one reactor does not necessarily apply to another, and signature analysis on each reactor has to be done on an individual basis [5].

Most theoretical anomaly models describe conditions which give rise to reactivity changes which affect the signature of the measured neutron noise signal. The choice of detector size, location, and type has been unspecified in these models. However, the recent development of small self-powered neutron detectors allows for the possibility of monitoring spatial behavior of the neutron variation within the core. Experiments using these small detectors to measure the noise behavior in a Boiling Water Reactor (BWR) core at different locations during steady operation provide basic noise signatures for this reactor type. The same general characteristics would appear in many BWR's. However, the frequencies at which characteristic peaks appear in the frequency domain would be different due to individual nuclear and thermohydraulic design. A model describing the steady state noise behavior is needed to aid in the development of an on-line surveillance and analysis system. A system of this type reduces the chance of a serious malfunction growing from an undetected anomaly. The main
reason for limited application of this technique is inadequate theoretical understanding of the transmission of random processes within a power reactor, particularly when feedback mechanisms must be taken into account. The noise model would also aid in understanding these transmission processes.

B. Purpose of Study

It is the purpose of this work to develop a BWR model for use in studying the stochastic response of an in-core neutron detector which is subjected to boiling induced neutron flux fluctuations. Transfer functions relating detector response to input power variations in the coolant channel are presented. The effects of axial power variations in the coolant channel are studied. The effects of axial power shape, detector location, and steam velocity on the detector response are considered.

The dynamic behavior of BWR cores has been investigated by many authors [6, 7, 8, 9, 10]. The work of Akcasu [11] is of particular significance to this study. Akcasu has developed the transfer functions for a number of the processes in a BWR including the power-void, power-boiling boundary, pressure-void, etc. The transfer functions exist in closed form so that system parameters may be directly related to important time constants. Akcasu's development was based on a sinusoidal axial power distribution. This model was later modified by Wakabayashi [12] to include a sinusoidal shape with
an adjustable phase and period to account for power flattening and variations in the location of peak power.

In the present study, the formulation suggested by Akcasu is utilized to develop transfer functions which describe neutron flux behavior and detector response at various locations along a BWR coolant channel. A linear combination of spatial power shapes of the form $e^{ax} \cdot \sin(bx + c)$ is employed so that very general axial power distributions can be studied. The detector response arising from both reactor power fluctuations and direct void perturbations are considered.

The variations in void content along a boiling channel affects the local neutron flux by changing the neutron absorption and diffusion properties. This "thermalization" effect arising from void variations is viewed as a direct input to the detector response, which adds to variations in the power level to become the total detector response.

The inputs in the model are assumed to be random heat sources originating from the stochastic behavior involved in the heat transfer process, and random amounts of heat appearing in the coolant channel. These inherent heat sources drive the noise model with the reactor operating at a constant average power level. The model response is found to be particularly sensitive to the axial power shape, position of the boiling boundary, coolant flow, and detector location.
II. LITERATURE REVIEW

The development of a neutron noise signature model for BWR's requires a review of experiments and theoretical models developed to explain the dynamic processes occurring in the reactor core. Historically, there has been a large number of noise investigations \[13, 14, 15, 16, 17, 18\] carried out on BWR's due to the fact that they exhibit large neutron flux fluctuations during steady state power operation. The higher noise level observed in BWR's compared to pressurized water reactors (PWR's) has been attributed to their sensitivity to the instantaneous steam volume in the core \[3\]. A generalized noise model driven by reactivity variations was developed by Seifritz \[19\]. This model includes transfer functions relating random processes to reactivity driving functions.

A. Development of Applicable Dynamic Models

Considerable effort has been spent in the study of void behavior in BWR cores. X-ray scanning experiments \[20, 21\] on the SPERT IA reactor were used to measure the void fraction in the axial direction for different constant power levels. The so-called void fraction was actually determined from the mean value of a very noisy signal. Also in this study, the dynamic behavior of steam voidage was investigated by transfer function analysis, i.e. utilizing small oscillation theory. Power oscillations of 10 percent about the mean level were used to drive the void fraction. This response was measured by the variation in X-ray signals. Early BWR dynamic models such as Beckjord's \[7\]
failed to explain the increasing phase lag of the void response at a particular detector location. Frequency response data resulting from rod oscillator experiments on Experimental Boiling Water Reactor (EBWR) were first modeled with empirical transfer functions fitted to the observed break frequencies [6, 22]. In 1960, the model developed by Akcasu [11] from conservation principles was able to adequately describe the at-power EBWR transfer function relating reactivity to power. Akcasu's model employed high and low frequency asymptotes to determine single time constant transfer functions to describe second order behavior in the EBWR model. Thus, some of the fine structure in the "mid-frequencies" is overlooked by application of Akcasu's model.

Jones [23, 24] developed a BWR stability model based on a nodal solution of the basic conservation equations at many positions along the coolant channel. Inherent feedback loops are considered at each node along the channel to give open and closed loop responses. Based on the work of Jones a computer code, FABLE II, was developed and is the standard method for theoretical dynamic analysis of large BWR's [10]. Information about the dynamic behavior of the General Electric BWR's found in the safety analysis reports [25], results in part from dynamic studies performed with FABLE II. The FABLE II frequency response has been compared to experimental data obtained from the General Electric test loop and compared with rod oscillator tests performed at Garigilano Nuclear Reactor. These comparisons indicate that the hydrodynamics incorporated in FABLE II do an adequate job

---

1 Results of this rod oscillator experiment are presented on pages 153-155 [26].
of predicting the channel void response, even though some fine structure is missing near 1.0 Hz.

The main differences between the two BWR dynamic models are:

1. Momentum changes along the channel are assumed proportional to the steam cross section in Akcasu's model, while Jones uses experimental correlations to account for momentum changes along the channel.

2. The heat input to the boiling channel is assumed to be sinusoidal in Akcasu's model, while Jones accounts for axial power shapes by varying the amount of heat delivered to each node along the channel.

3. Reactivity is determined from the contribution of each process modeled in Akcasu's model, while Jones determines a reactivity parameter at each node which is summed to determine the overall reactivity state.

The approach adopted by Akcasu to obtain transfer functions in closed form allows system parameters to be related to important time constants and gains.

B. At Power Noise Models

In 1968 Nomura [15] presented a simple transfer function model of the Japan Power Demonstration Reactor (JPDR) based on the linearized two phase flow equations. This was used to determine the power spectral density (PSD) of the neutron flux fluctuation in a natural circulation BWR. He considered the discrete nature of steam bubble formation during constant heat output as the input noise source. His model neglects feedback due to void generation, but includes temperature and pressure
effects arising from this bubble generation process. The resulting disturbances then introduce reactivity changes which in turn drive the closed loop transfer function of a BWR. This model then distorts a white spectrum heat source to a nonwhite reactivity input which drives the reactor closed loop transfer function. This model gave a fair approximation to the measured PSD (neutron noise signature) which was sufficiently promising to warrant further investigations.

Seifritz [19] derived a generalized expression for the PSD based on fluctuations of the output current from neutron detectors. This expression consists of three noise contributions:

1. the trivial term representing white detection noise,
2. the term accounting for statistical variations in the power caused by the branching process [27], and
3. the dominating power noise term arising from reactivity driving functions.

Additional information about this model is repeated in Appendix C.

Based on Seifritz's formulation, many other authors [28, 29, 30, 31, 32] developed driving functions relating various types of anomalous behavior to a change in reactivity. The types of anomalous behavior considered were:

1. coolant flow fluctuations,
2. vibrating components such as fuel elements or control rods, and
3. inlet temperature fluctuations.
C. Noise Experiments

Albrecht [18] has performed experiments on the University of Washington Nuclear Reactor relating an artificially induced void anomaly to the variations in the neutron flux. Control rod bearing failure has been detected and diagnosed in the High Flux Isotope Reactor (HFIR). This interesting work was reported by Fry [1]. Noise analysis at the Experimental Breeder Reactor II led to the discovery of mechanical component vibrations [2].

Based on Nomura's [15] steady state noise model, Seifritz [33] designed an experiment for the Lingen 540 MW$_{th}$ BWR utilizing self powered in-core detectors [34], which could be moved up and down the channel. The results will serve as a basis for a better understanding of inherent reactivity driving forces in a BWR. Appendix C shows the experimental set up utilized to measure and compute [35] the PSD resulting from the in-core detector signals. The Gaussian shape of the probability density function of the measured noise indicates that linear processes are involved.

Seifritz suggested that the point reactor model is not adequate to describe the change in PSD shapes from a low pass filter characteristic in the lower nonboiling portion of the core to a band pass filter characteristic in the boiling region. He described this behavior as neutron spatial decoupling and suggested that BWR's respond asymmetrically to their inherent reactivity driving forces.

Further testing on the Garigliano reactor demonstrated that the phase shift of the cross power spectral density of two in-core neutron
detectors located along the channel could be used to determine the steam bubble velocity [36]. This important discovery indicates that the in-core neutron detectors are rather directly affected by void fluctuations. Rothman [37] has developed relationships to describe the in-core neutron detector response arising from variations in the local void fraction. The effect of detector size on the rms "bubble" noise was analyzed and the application of this noise source to boiling detection was considered. Previously, Thie [16] referred to this interaction of voids and neutron flux moderation as "thermalization noise" and identified it as a noise source for in-core neutron detectors at the Pathfinder Atomic Power Plant [38].

It is clear from the references cited that noise measurements of the neutron flux fluctuations due to void generation can be used to provide information on system performance without the need to apply external perturbations to the system. Most of the time reactors are operating in the steady power production state, therefore, there is an obvious need for a consistent model which is capable of predicting the observed behavior of neutron noise characteristics and relating them to basic reactor parameters. Such a model would facilitate the interpretation of on-line monitoring systems [4].
III. BWR CHARACTERISTICS

A. Control Philosophy

The core of a BWR consists of an array of fuel bundles cooled by water and steam [39]. The nuclear heat generated by the fission process is controlled by the movement of control rods or recirculation flow. Each independent rod drive enters the core from the bottom, and can be used to shape the power distribution. Steam created in the boiling region is separated from the recirculation water, dried in the top of the primary reactor vessel, and then piped to the high pressure turbine stage. Recirculation water is forced through the core by jet pumps [40] located outside the core barrel, but inside the reactor vessel. The driving force for the jet pumps is provided by two variable speed centrifugal pumps, which draw a fraction of the recirculation water from the vessel and return it with increased pressure. The effect of increasing pressure across the jet pump is to increase the core water velocity above that experienced with natural circulation. A typical BWR core flow system is shown in Figure 3.1.

To understand the information contained in the neutron detector noise signal, it is important to model mechanisms that affect the neutron density at a detector location. Stability models provide insight into the key parameters affecting the neutron density or reactor fission power. Disturbances can enter the system from either control rod movements or perturbations of a coolant parameter such as pressure, flow, or subcooling. For a change in subcooling, there is a time dependent change in the void content which in turn changes the reactor
Figure 3.1. Reactor vessel internal coolant flow paths [25]
fission power through the negative void-reactivity feedback coefficient. A change in the fission power then appears as a change in the rod surface heat flux after a delay which is referred to as the fuel time constant [10]. This time constant is about 6 to 8 seconds for the 0.5 inch diameter fuel pins used in BWR's.

In addition to the delay introduced by the fuel time constant, another delay is introduced by coolant behavior. After a fission rate perturbation reaches the fuel rod surface as a heat flux perturbation, the channel void content responds by increasing or decreasing in the direction of the heat flux change. This void disturbance is swept through the core in some finite amount of time. The effect of this disturbance on neutron detectors is dependent on mass flow rate, slip velocity, length of boiling region, length of channel, axial power distribution, and detector location. Since the void-reactivity coefficient is negative, it provides an inherent shutdown mechanism and a negative feedback to power.

Another negative feedback mechanism is the Doppler reactivity effect, resulting from the fuel temperature dependence of neutron absorption in $^{238}\text{U}$. This feedback loop reduces the effect of void disturbances of the fission power by attenuating the low frequency response of the zero power reactor transfer function [41].

A third reactivity feedback of importance is the change in subcooled water temperature which affects the water density and thus neutron moderation. This effect is treated as a temperature dependent feedback through a moderator temperature to reactivity coefficient.
In addition to the "natural" feedback characteristics of BWR's, the control methods must be examined. Figure 3.2 shows the basic control systems used in a typical General Electric BWR [25]. Four operating control systems are utilized to maintain the desired reactor to turbo-generator conditions.

First, pressure is held constant by a pressure regulator, unlike conventional plants in which steam flow to the turbine is set by demand. As an example, consider the result of opening the turbine admission valves to increase electrical output. The resulting decrease in pressure causes steam to flash into voids and the negative void coefficient causes the fission power to decrease. This is the opposite of the desired increase in reactor power needed to support increased steam demand. Regulation of pressure and reactor power is achieved by control of a turbine bypass valve which admits reactor steam directly to the condenser. The change in the bypass valve flow counteracts small power demand changes on the turbine, thus achieving constant pressure on the reactor system. Second, the feedwater control system is similar to the conventional fossil plants in that throttling valves control the feedwater flow into the reactor to maintain a constant liquid level in the reactor vessel. Third, neutron absorbing control rods are utilized to change power during start up, maintain criticality during core lifetime, and control the power distribution within the core. Once the reactor is operating at power with the desired power distribution, movement of control rods is expected to be slight. Power distributions within the core are thus static over a period of several hours to days, if the system is above 60 percent power. The fourth and most important
Figure 3.2. Basic BWR control systems [25]
control system for power changes in the range of 60 to 100 percent power is the flow control system. Figure 3.3 is a typical operating map for a BWR [25, 39] which indicates the possible flow-power combinations that can be used during high power operation. Since the fission power is approximately proportional to the recirculation flow, the independent control of recirculation loop flow allows matching of fission power to the demand for electrical grid power. A turbine speed signal related to electrical demand and the pressure regulator valve position signal are compared in the master controller. Should the steam flow available from the reactor be less than that demanded, the master controller acts to increase the recirculation flow until the power demand is satisfied [42].

To relate the physical system to a simple model capable of describing the neutron noise behavior during steady state operation, certain simplifying assumptions about the operating philosophy of the plant are made:

1. Control rod movements are used during normal operation only to maintain or change flux shape, and compensate for fuel burnup.

2. Reactor power level is adjusted by changing the recirculation flow. The relationship between flow and power is determined from the operating map design flow control line shown in Figure 3.3.

3. Constant dome pressure is assumed, since the pressure regulated turbine bypass valve acts rapidly to maintain constant reactor conditions during small changes in the electrical power demand.

4. No significant changes in the liquid operating level occur requiring feedwater flow changes by the control system at steady power
5. The steady state operating conditions for the plant are determined by requiring that the steam exit quality be matched to the design condition exit quality for a given point on the operating map, and a given axial power distribution along the channel.

Figure 3.4 shows the steady state coolant flow, temperature, enthalpy, and pressure of the Duane Arnold Energy Center (DAEC) at
Figure 3.4. Rated operating conditions for model rated thermal conditions [25]. Table 3.1 lists the important physical information needed as input information to the noise model of this reactor.

B. BWR Power Distribution

In operating power reactors, the axial flux distribution is stationary for long periods of time, since the changes in the
Table 3.1. Duane Arnold Energy Center physical modeling data [25]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Computer symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated thermal-hydraulic values</td>
<td>PR</td>
<td>1593.</td>
<td>M \ wth</td>
</tr>
<tr>
<td>Thermal output</td>
<td>MSF</td>
<td>6.84E6</td>
<td>lb/hr</td>
</tr>
<tr>
<td>Steam flow</td>
<td>MCF</td>
<td>50.5E6</td>
<td>lb/hr</td>
</tr>
<tr>
<td>Core flow</td>
<td>MFW</td>
<td>6.82E6</td>
<td>lb/hr</td>
</tr>
<tr>
<td>Feedwater flow</td>
<td>P</td>
<td>1020.</td>
<td>psia</td>
</tr>
<tr>
<td>Nominal dome pressure</td>
<td>FASL</td>
<td>0.76</td>
<td>—</td>
</tr>
<tr>
<td>Core maximum exit void</td>
<td>CHIEI</td>
<td>0.143</td>
<td>—</td>
</tr>
<tr>
<td>Exit quality</td>
<td>ALW</td>
<td>-1.16E-4</td>
<td>Δk/k/°F</td>
</tr>
<tr>
<td>Reactivity coefficients</td>
<td>AFW</td>
<td>-0.98E-5</td>
<td>Δk/k/°F</td>
</tr>
<tr>
<td>Reactor dimensions</td>
<td>ALW</td>
<td>-1.05E-3</td>
<td>Δk/k/% void</td>
</tr>
<tr>
<td>Core length</td>
<td>L</td>
<td>12.0</td>
<td>ft</td>
</tr>
<tr>
<td>Core diameter</td>
<td>DIAC</td>
<td>12.0</td>
<td>ft</td>
</tr>
<tr>
<td>Vessel diameter</td>
<td>DIAV</td>
<td>14.93</td>
<td>ft</td>
</tr>
<tr>
<td>Core flow area</td>
<td>AC</td>
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<td>ft²</td>
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<tr>
<td>Channel flow area</td>
<td>A</td>
<td>0.00234</td>
<td>ft²</td>
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<tr>
<td>Hydraulic diameter</td>
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<td>0.0509</td>
<td>ft</td>
</tr>
<tr>
<td>Fuel pellet radius</td>
<td>RF</td>
<td>0.0199</td>
<td>ft</td>
</tr>
<tr>
<td>Cladding radius</td>
<td>RC</td>
<td>0.0234</td>
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<tr>
<td>Number of fuel assemblies</td>
<td></td>
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Table 3.1. (Continued)

<table>
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<th>Parameter</th>
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<tr>
<td>Fuel rod array</td>
<td></td>
<td>7 x 7</td>
<td></td>
</tr>
<tr>
<td>Clad material — zircaloy-2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fuel material — uranium dioxide</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total weight of fuel</td>
<td></td>
<td>173.500</td>
<td>lbs</td>
</tr>
<tr>
<td>Average enrichment</td>
<td></td>
<td>1.9</td>
<td>w/o %</td>
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### Material properties

#### Thermal conductivity

<table>
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<th>Computer symbol</th>
<th>Value</th>
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<td>Fuel</td>
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<td>3.2</td>
<td>Btu/h·ft·°F</td>
</tr>
<tr>
<td>Cladding</td>
<td>CK</td>
<td>7.0</td>
<td>Btu/h·ft·°F</td>
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<tr>
<td>Water</td>
<td>KW</td>
<td>0.34</td>
<td>Btu/h·ft·°F</td>
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</table>

#### Specific heat

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</thead>
<tbody>
<tr>
<td>Fuel-cladding</td>
<td>CP</td>
<td>0.099</td>
<td>Btu/lb</td>
</tr>
<tr>
<td>Water</td>
<td>CP</td>
<td>1.24</td>
<td>Btu/lb</td>
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</table>

#### Density

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<th>Value</th>
<th>Units</th>
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<tr>
<td>Fuel</td>
<td>RHOF</td>
<td>529.5</td>
<td>lb/ft^3</td>
</tr>
<tr>
<td>Saturated steam</td>
<td>RHOS</td>
<td>2.29</td>
<td>lb/ft^3</td>
</tr>
<tr>
<td>Saturated water</td>
<td>RHOW</td>
<td>46.16</td>
<td>lb/ft^3</td>
</tr>
<tr>
<td>Feedwater</td>
<td>RHOFW</td>
<td>52.80</td>
<td>lb/ft^3</td>
</tr>
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</table>

#### Enthalpy

<table>
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<th>Value</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Saturated steam</td>
<td>HS</td>
<td>1191.6</td>
<td>Btu/lb</td>
</tr>
<tr>
<td>Saturated water</td>
<td>HW</td>
<td>545.4</td>
<td>Btu/lb</td>
</tr>
</tbody>
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Table 3.1. (Continued)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Computer symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feedwater</td>
<td>HFW</td>
<td>397.6</td>
<td>Btu/lb</td>
</tr>
<tr>
<td>Viscosity</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Saturated steam</td>
<td>VISG</td>
<td>1.34E-5</td>
<td>lb/ft·s</td>
</tr>
<tr>
<td>Saturated water</td>
<td>VISL</td>
<td>7.18E-4</td>
<td>lb/ft·s</td>
</tr>
</tbody>
</table>

Distribution are caused by control rod movement, fuel burnup, or xenon oscillations. During noise measurements the change in axial distribution is neglected, however the axial power distribution shifts from the top to the bottom as control rods are withdrawn to compensate for fuel consumption. The effect of changing axial distributions must be considered when comparing noise measurements at different times during the core life. This is accomplished in the model by describing the axial power distribution with a function of the form $e^{ax} \cdot \sin(bx + c)$, where $x$ is the axial core position. The parameters $a$, $b$, and $c$ can be changed to fit the axial distribution at the time of measurement.

The use of flux shaping functions to describe the steady state power distributions requires a normalization factor. The purpose of this factor is to force the integrated value of the distribution function to unity. In this way the total steady state power can be treated as a constant which is independent of spatial integrations. Heat generated in the core is represented as
\[ Q(x, t) = \frac{Q(t)}{L} \cdot D(x), \]

and

\[ q(x, t) = \frac{q(t)}{L} \cdot D(x). \]

\(D(x)\) is normalized to \(L\) so that the change in power level, \(q(t)\), has the same spatial distribution as \(Q(t)\). \(D(x)\) can be determined from

\[
\frac{1}{L} \int_{0}^{L} D(x) dx = 1. \tag{3.2}
\]

In the case of a sinusoidal flux distribution, Equation 3.2 becomes

\[
\frac{1}{L} \int_{0}^{L} F_{ABC} \cdot \sin\left(\frac{\pi x}{L}\right) dx = 1, \tag{3.3}
\]

and

\[
F_{ABC} \times \frac{1}{\pi} \left\{ \cos\left(\frac{\pi x}{L}\right) \right\}_{0}^{L} = 1,
\]

and

\[
F_{ABC} \times \frac{2}{\pi} = 1.
\]

Thus \(F_{ABC}\) equals \(\pi/2\), the normalization factor for \(\sin\left(\frac{\pi x}{L}\right)\), and

\(D(x) = \frac{\pi}{2} \cdot \sin\left(\frac{\pi x}{L}\right)\). For the general shape function considered in the noise model \(D(x) = F_{ABC} \cdot e^{ax} \sin(bx + c)\). \(F_{ABC}\) is then determined from

\[
\frac{1}{L} \int_{0}^{L} F_{ABC} \cdot e^{ax} \cdot \sin(bx + c) dx = 1. \tag{3.4}
\]

Integrating Equation 3.4 and solving for \(F_{ABC}\) gives

\[
F_{ABC} = \frac{L \cdot (a^{2} + b^{2})}{\cos(c) \cdot [a \cdot \sin(bL) - b \cos(bL) + b]}
\]

\[
+ \sin(c) \cdot [e^{aL}(a \cdot \cos(bL) + b \sin(bL)) - a]. \tag{3.5}
\]
To check Equation 3.5, let b = \pi/L, c = 0, and a = 0, the conditions for the sinusoidal case, then FABC degenerates to \pi/2.

By changing the parameters a, b, and c, various "single humped" axial flux shapes can be modeled. Figures 3.5 through 3.10 show the type of flux shapes that can be modeled by this function.

If more complicated shapes must be patterned, weighted sums of D(x) can be used. The total power distribution average is still unity if \sum_{i=1}^{N} WET(i) = 1. Thus the power distribution is described

\[
Q(x) = \sum_{i=1}^{N} WET(i) \cdot D_i(x) \\
= \sum_{i=1}^{N} WET(i) \cdot FABC(i) \cdot e^{aix} \cdot \sin(b_i x + c_i), \quad 3.6
\]

where WET(i) is the weight given each function. Measured axial flux shapes in BWR's appear in some cases to have two "humps" [43], and thus could be modeled with Equation 3.6 by setting i = 2. Figures 3.9 and 3.10 are examples of symmetrical positive and negative damped flux components with equal weighting.

The axial neutron flux shape is assumed to be equal to the heat distribution delivered to the channel. Thus, neutron flux, fission power, and coolant power have the same distribution in the model. For this model it is also assumed that the radial distribution is approximated by a Bessel function \( J_0 \left( \frac{2.405r}{R_c} \right) \)[44]. The power at any position in the core then becomes

\[
Q(x, r) = \bar{Q} \cdot [1.42 \times J_0 \left( \frac{2.405r}{R_c} \right)] \cdot \left[ \sum_{i=1}^{N} WET(i) \cdot FABC(i) \cdot e^{aix} \cdot \sin(b_i x + c_i) \right]. \quad (3.7)
\]
Figure 3.5. Flat flux for \( Q(x) = F_{ABC} \cdot e^{ax} \cdot \sin (bx + c) \)

Figure 3.6. Sinusoidal flux shape for \( Q(x) = F_{ABC} \cdot e^{ax} \cdot \sin (bx + c) \)

Figure 3.7. Negatively damped shape for \( Q(x) = F_{ABC} \cdot e^{ax} \cdot \sin (bx + c) \) characteristic of end of core life
Figure 3.8. Positive damped flux where $Q(x) = FABC \cdot e^{ax} \cdot \sin(bx + c)$ characteristic of beginning of core life

Figure 3.9. Positive and negative damped flux where $Q(x) = \sum_{i=1}^{2} FABC(i) \cdot WET(i) \cdot e^{aix} \cdot \sin(bix + c_i)$

Figure 3.10. Positive and negative damped flux used in base case where $Q(x) = \sum_{i=1}^{2} FABC(i) \cdot WET(i) \cdot e^{aix} \cdot \sin(bix + c_i)$
The radial normalization factor is determined from \( \int_0^R J_0(2.405r/R_c)dr = 1.42 \). A radial position for the detector is then selected by letting RPF equal 1.42 for the center channel, 1.0 for the average radial location, etc. The radial peaking factor as a function of the radial distance becomes,

\[
RPF = 1.42 \ J_0(2.405r/R_c).
\]
IV. BOILING REACTOR NOISE MODEL

The objective of this model is to relate various operating parameters of a forced circulation BWR to information contained in neutron detector fluctuating signal. The approach is to develop transfer functions to describe the dynamic processes of a BWR core at steady operating conditions. The model is driven by small variations of heat into the reactor coolant channel in the boiling and nonboiling regions. The output is taken as the thermal neutron detector response at various locations along the channel arising from fission power oscillations and local thermalization variations.

A. Closed Loop Feedback Model

1. Point reactor

Since the frequency range of interest for this model is 0.1 to 10. Hz, it will be assumed that the point reactor model can be utilized to describe the variation in fission power arising from changes in reactivity. The point model is used assuming that the oscillations are small so that a linear approximation of the kinetic equations is justified. Use of the point kinetics model also assumes that changes in the fission power level appear with the same spatial distribution as the steady state power. The kinetic equations [45] are

\[
\frac{dN(t)}{dt} = \frac{\rho - B}{\lambda} N(t) + \sum_i \lambda_i C_i
\]

and

\[
\frac{dC_i}{dt} = \frac{\beta_i}{\lambda} N - \lambda_i C_i \quad i = 1, 6
\]
where \( N(t) \) = fission power (or neutron density);
\( C_i(t) \) = precursor power of the \( i \)th group;
\( t \) = time;
\( \rho \) = reactivity \((k - 1)/k \approx \Delta k/k\) at critical, the fractional change in neutron reproduction factor;
\( \beta \) = delayed neutron fraction \((\Sigma_i \beta_i)\);
\( \lambda \) = neutron generation time, sec;
\( \lambda_i \) = decay constant for precursor decay, sec\(^{-1}\).

For the purpose of this model it is convenient to choose the one group representation. Let \( N(t) = N^0 + n(t) \), \( C_i(t) = C_i^0 + c_i(t) \), and \( \rho(t) = \rho^0 + \rho(t) \) (\( \rho^0 = 0 \) for a critical reactor). Neglect second order terms, Laplace transform both equations\(^1\), solve for \( C_i(s) \) in Equation 4.2 and substitute back into Equation 4.1 to obtain

\[
sN(s) = \frac{\rho^0 - \beta}{\lambda} N(s) + \frac{1}{\lambda} \left[ \frac{\beta \lambda}{s + \lambda} \right] + \frac{N^0}{\lambda} \rho(s).
\]

Solving for \( N(s) \) then yields

\[
N(s) = \frac{N^0}{\lambda} \cdot \frac{\rho(s)}{s + \beta - \beta^0 - \frac{1}{\lambda} \left[ \frac{\beta \lambda}{s + \lambda} \right]}.
\]

However, for a critical reactor where \( \beta/\lambda \gg \lambda \), and \( \rho^0 = 0 \) the reactor transfer function is simplified to

\[
G(s) = \frac{N(s)}{\rho(s)} = \frac{N^0(s + \lambda)}{\lambda s(s + \beta/\lambda)}.
\]

\(^1\)The Laplace transform of a lower case variable is written as the upper case letter. For example \( \mathcal{L}\{n(t)\} = N(s) \).
Equation 4.5 is often called the one delayed group zero power reactor transfer function [46, 47]. This is because $N^0$ is assumed sufficiently small that no reactivity feedback effects are significant. Typical numbers for the one group approximation of a BWR are

$$\lambda \approx 0.0767 \text{ sec}^{-1}, \quad \ell \approx 5 \times 10^{-5} \text{ sec}$$

and $\beta$ ranges from 0.0072 to 0.005 during the core life$^1$. A computer program, NMOD, described in Chapter V and Appendix G, was written to calculate the transfer function frequency response. It treats $G(s)$ as real and imaginary parts which are functions of the frequency. In the program $G(s)$ becomes

$$TGR(m) = \text{REAL}(w) + \text{IMAG}(w)$$

where

$$\text{REAL}(w) = \frac{\text{REACTOR POWER} \times (\beta - \lambda \ell)}{\ell^2 w^2 + \beta^2}$$  \hspace{1cm} 4.6a$$

$$\text{IMAG}(w) = -\frac{\text{REACTOR POWER} \times (\omega^2 \ell + \lambda \beta)}{(\ell^2 w^2 + \beta^2)w}$$  \hspace{1cm} 4.6b$$

Bode plots of $G(s)$ exhibit the shapes shown in Figure 4.2, page 47.

In power producing BWR's small changes in the power level can affect parameters such as fuel temperature, coolant temperature, and void fraction. Changes in neutron leakage, absorption, and thermalization from such variations in turn modify the reactivity state of the reactor. This reactor behavior is modeled by considering the parameters affected by power changes and describing their effect on the reactivity state with inherent feedback loops.

$^1$ Numerical values are from the Duane Arnold Energy Center training manual, written by the Iowa Electric Light and Power technical staff.
Three important feedbacks considered in this model are fuel temperature, nonboiling coolant temperature, and void fraction. The following sections develop transfer functions to describe these power related feedbacks as they affect reactivity. This approach is similar to Akcasu [11], DeShong and Lipinski [6], Beckjord [7], Thie [8], and many others.

2. Heat transfer

It is the purpose of this section to determine relationships describing the behavior of the fuel temperature resulting from changes in the fission power level. For a small increment of channel length, dx, where heat is transferred only in the radial direction, the following heat balance is utilized,

\[
\text{Heat generated in the fuel = Heat transferred to the coolant + heat stored in the fuel}
\]

Btu/s or watts

\[
N(x, t)dx = Q(x, t)dx + \frac{3}{\delta t} \left\{ \rho f A c f T_f dx \right\}, \quad 4.7
\]

where \( \rho f A c f \) are the lumped fuel-cladding parameters, and \( T_f \) is the volume averaged fuel temperature. Upon linearizing and taking the Laplace transform, Equation 4.7 becomes

\[
N(x, s) = Q(x, s) + \rho f A c f s\delta T_f (x, s). \quad 4.8
\]

Equation 4.8 is utilized in both the boiling and nonboiling regions [48] where the main difference is the method of heat transfer between the cladding and the coolant. It remains then to determine a
relationship between $Q(x, s)$, the coolant power [Btu/s·ft], and the fuel temperature.

a. Boiling region  
In the boiling region the experimental correlation derived by Jens and Lottes [49], $T_{wall} - T_{sat} = 60\left[\frac{q'}{10^6}\right]^{1/4}e^{-p^0/900}$ can be used to estimate the boiling heat transfer coefficient, where $q'$ is heat flux in Btu/h·ft$^2$, $P$ is pressure in psia, and $T$ is the temperature in °F. For the two phase flow at constant pressure the saturation temperature, $T_{sat}$, of the coolant in the boiling region is assumed constant in space and time. Thus, a variation in heat flux results in a change in wall temperature. Because of the poor heat transfer characteristics of oxide fuels, the change in average fuel temperature, $T_f$, can be much greater than the change in $T_{wall}$. Since $T_f$ affects the Doppler feedback mechanism, a relationship between the $T_{wall}$ and $T_f$ is needed to solve Equation 4.8 [47].

In a cylindrical fuel region the temperature behavior is described by the heat balance

$$
\rho_f c_f \frac{dT_f}{dt}(r, t) = k_f \left[ \frac{\partial^2 T_f(r)}{\partial r^2} + \frac{1}{r} \frac{\partial T_f(r)}{\partial r} \right] + N'(r, t) \left[ \frac{\text{Btu}}{\text{h} \cdot \text{ft}^3} \right]
$$

Equation 4.9

assuming the temperature distribution is constant and only the magnitude changes with time. Integrating Equation 4.9 along the radius $r$ yields

$$
\int_0^{R_f} \frac{2\pi}{A_f} \rho_f c_f \frac{dT_f}{dt}(r, t)rdr = \frac{2\pi}{A_f} \int_0^{R_f} k_f (\frac{\partial}{\partial r}) \cdot (r \frac{\partial T_f}{\partial r})dr
$$

$$
+ \frac{2\pi}{A_f} \int_0^{R_f} rN'(r, t)dr,
$$

Equation 4.10
which simplifies to
\[ \rho_{f}c_{f}\frac{\partial T_{f}}{\partial t}(t) = \frac{2\pi}{A_{f}} k_{f}R_{f}\left[\frac{\partial T_{f}}{\partial r}\right]_{\text{at} R_{p}} + \overline{N'}(t), \quad 4.11 \]

where
\[ \overline{T_{f}}(t) = \frac{2\pi}{A_{f}} \int_{0}^{R_{f}} rT_{f}(r, t)dr, \quad 4.12 \]
and
\[ A_{f} = 2\pi \int_{0}^{R_{f}} rdr. \quad 4.13 \]

Let
\[ \left[\frac{\partial T_{f}}{\partial r}\right]_{\text{at} R_{p}} = \frac{T_{\text{wall}} - T_{\text{sat}}}{d} \]

where \( d \) is a small distance. Equation 4.11 becomes
\[ \rho_{f}c_{f}\frac{dT_{f}}{dt} = \frac{2\pi}{A_{f}} k_{f}R_{f}\left[\frac{T_{\text{wall}} - T_{\text{sat}}}{d}\right] + \overline{N'}(t). \quad 4.14 \]

Define \[ \frac{2\pi}{A_{f}} k_{f}R_{f} = h_{b} \]

at the fuel surface in the boiling region which is determined from the Jens-Lottes correlation as
\[ h_{b} = \frac{10^{6}}{15} \times e(P/900) \times \left(\frac{q'}{10^{6}}\right)^{3/4} \left[\frac{\text{Btu}}{h \cdot \text{ft}^2 \cdot \text{F}}\right]. \quad 4.15 \]

Equation 4.14 then becomes
\[ \rho_{f}c_{f}\frac{dT_{f}}{dt} = h_{b}\left[T_{\text{wall}}(t) - T_{\text{sat}}\right] + \overline{N'}(t). \quad 4.16 \]

Solution of Equation 4.16 is simplified by adjusting the heat transfer coefficient to include not only the film drop, but also an allowance for the conduction between the point of average fuel temperature and the wall temperature. Treating the fuel clad material as a
resistance to the average flow of heat in the small length, dx, consider

\[ A_f N' = \bar{N}(s) = -k_f (2\pi r) \frac{dT}{dr}, \quad \text{Btu/s*ft} \quad 4.17 \]

\[ \bar{N}(s) = \frac{T_f - T_{sat}}{\ln(R_f/R_f)} = h_b p_f [T_w - T_{sat}]. \]

Then note that

\[ \bar{N}(s) = \frac{T_f - T_{sat}}{R_{T_b}} \quad \text{where} \quad 4.18 \]

\[ R_{T_b} = \frac{1}{h'_{P_f}} = \frac{\ln(R_f/R_f)}{2\pi k_f} + \frac{\ln(R_f/R_f)}{2\pi k_c} \cdot \frac{\varphi}{\text{Btu/(h*ft)}}. \quad 4.19 \]

Equation 4.19 is then simplified by utilizing the dimensions of the Duane Arnold Energy Center (DAEC) fuel pins [25] and assuming that \( \bar{T}_f \) occurs at 0.707\( R_{T_b} \), it is found that

\[ R_{T_b} = \frac{0.055}{k_f} + \frac{0.0137}{k_c} + \frac{6.8}{h_b} \cdot \frac{\varphi}{\text{Btu/h*ft}}. \quad 4.20 \]

Given that [46]

\[ k_f = 3.2 \quad \frac{\text{Btu}}{\text{h*ft} \cdot \varphi} \quad k_c = 7.0 \quad \frac{\text{Btu}}{\text{h*ft} \cdot \varphi} \quad h_b \geq 3 \times 10^5 \quad \frac{\text{Btu}}{\text{h*ft}^2 \cdot \varphi} \]

Equation 4.14 becomes

\[ A_f^p c_f \frac{dT_f}{dt} = p_f h'[T_f - T_{sat}] + \bar{N}_b(r, t) \left[ \frac{\text{Btu}}{\text{s*ft}} \right]. \quad 4.21 \]

where

\[ h' = \frac{1}{p_f \times R_{T_b} \times 3600} \left[ \frac{\text{Btu}}{\text{s*ft}^2 \cdot \varphi} \right]. \]
The change in average fuel temperature for a change in the reactor power can be found by solving Equation 4.21.

\[ \delta T_{fb}(x, s) = N_b(x, s) \frac{1}{h'p_f} \frac{1}{1 + \tau_{fb} s} \]  

4.22

Substitution of appropriate numbers gives the following results

\[ \tau_{fb} = \frac{A_f \rho_f c_f}{h'p_f} = \frac{(0.00234 \text{ ft}^2)(529 \text{ lb/ft}^3)(0.079)}{0.0144 \frac{\text{Btu}}{\text{s-ft}}} \approx 6.8 \text{ sec} \]  

4.23

\[ \delta T_{fb}(x, s) = N_b(x, s) \frac{\text{[Btu]}}{s \cdot \text{ft}} \frac{70.\frac{\text{s}}{\text{Btu} \cdot \text{ft} \cdot ^\circ\text{F}}}{1 + 6.8s} \]  

Since the model heat variable is the total reactor heat, \( \bar{N}(s) \) becomes the total heat delivered by all the fuel pins. For example, a change of 1 Btu/s·ft at the average fuel pin becomes a change of 216,384 Btu/s for the whole reactor core. Equation 4.22 is incorporated into the model by utilizing a weighting function, \( WF_b \), which yields

\[ \delta T_{fb}(s) = \frac{WF_b \cdot \bar{N}(s) \cdot \frac{1}{h'p_f}}{1 + 6.8s} \]  

4.24

where

\[ WF_b = \frac{(1 - \text{FRAC}) \times \text{FABC}}{L \times \# \text{ fuel pins}} \]  

4.25

Thus, for an increase in the power level of 1 Btu/s the change in average fuel temperature is approximately 2.5E-4 °F. This increase is felt in the coolant after a time delay related to \( \tau_{fb} \).
b. Nonboiling region  The nonboiling region for this model is defined as the lower portion of the coolant channel where all the heat added increases the coolant temperature to the saturation temperature. It is assumed that subcooled boiling can be neglected. In this region the heat transfer coefficient, $h_{nb}$, at the cladding surface can be estimated from the Dittus-Boelter correlation for forced convection in a circular pipe [19].

$$\text{Nu} = 0.023(\text{Re})^{0.8}(\text{Pr})^{0.4}$$

where the notations are

Nusselt number: $\text{Nu} = \frac{hD_e}{R}$

Reynolds number: $\text{Re} = \frac{D_e \dot{\omega} \rho_e}{\mu_f}$

Prandtl number: $\text{Pr} = \frac{c_p \mu_f}{R}$

Solving Equation 4.26 for the heat transfer coefficient yields

$$h_{nb} = \frac{0.023 R}{D_e} \left( \frac{D_e \dot{\omega} \rho_e}{\mu_f} \right)^{0.8} \left( \frac{c_p \mu_f}{R} \right)^{0.4}$$

Following the same procedure used in the boiling region, the total thermal resistance becomes

$$R_{T_{nb}} = \frac{\ln \left( \frac{R}{R_T} \right)}{2\pi k_f} + \frac{1}{h_{nb} \rho_f} + \frac{\ln \left( \frac{R_p}{R} \right)}{2\pi k_{f1}}$$

Similarly, the equation relating the change in average fuel temperature to a change in the reactor power is determined from

$$A_f \rho_f c_f \frac{dT_{fnb}}{dt} = p_f h''[T_{f_{nb}} - T_w] + N_{nb}$$
which becomes

$$
\delta T_{fb}(x, s) = \frac{WF_{nb} \cdot N_{nb}(x, s) \cdot \frac{1}{h'' p_f}}{1 + \tau_{fb} s},
$$

where

$$
h'' = \frac{1}{p_f \cdot R_{Tnb} \cdot 3600},
$$

$$
\tau_{fb} = \frac{A_f \cdot \rho_f \cdot c_f}{h'' p_f},
$$

and

$$
WF_{nb} = \frac{FRAC \cdot FABC}{L \cdot \# \text{fuel pins}}.
$$

c. Fuel temperature feedback

To utilize this feedback function in the total model two additional factors are considered. First, since the fuel temperature model was derived on a single fuel pin basis, it must be corrected by the number of pins. Second, flux shape weighting should be used to correct for the effectiveness of the temperature feedback at different axial locations, which includes fractions for the boiling and nonboiling regions.

Equations 4.24 and 4.30 are combined and integrated in the axial direction to obtain the reactor feedback loop function.

$$
\delta T_x(s) = \frac{1}{\# \text{pins}} \left\{ \frac{1}{L} \int_0^{L_{nb}} D(x) \cdot \delta T_{nb}(x, s) \, dx + \frac{1}{L} \int_{L_{nb}}^{L} D(x) \cdot \delta T_{b}(x, s) \, dx \right\}.
$$

4.31

To simplify the temperature feedback loop, it is assumed that the nuclear power, coolant power, and weighting function have normalized sinusoidal distributions. Other flux shapes would only modify the
weighting constant slightly, therefore this feedback loop will be assumed independent of flux shape changes. The resulting equation is

$$\delta T_f(s) = \frac{A}{A_c \cdot L} \left\{ \int_0^{L_{ub}} (\frac{\pi}{2})^2 \sin^2 \left( \frac{\pi x}{L} \right) \cdot \frac{\delta N(s)/h''p_f}{1 + s\tau_{nb}} \right.$$  

$$+ \int_{L_{ub}}^{L} (\frac{\pi}{2})^2 \frac{\delta N(s)/h'p_f}{1 + s\tau_{fb}} \sin^2 \left( \frac{\pi x}{L} \right) dx \right\} \left[ ^{\circ}F \cdot ft \right], \quad 4.32$$

where $A_c/A$, the total core flow cross section, divided by the channel flow cross section, equals the number of fuel pins.

This formulation neglects the change in fuel temperature arising from changes in pressure, flow rate, and coolant temperature.

Integrating Equation 4.32 yields

$$\delta T_f(s) = \frac{A}{A_c \cdot L} N(s) \left\{ \frac{E_1/h''p_f}{1 + s\tau_{fb}} + \frac{E_2/h'p_f}{1 + s\tau_{fb}} \right\} \left[ ^{\circ}F \cdot ft \right], \quad 4.33$$

where

$$E_1 = \frac{\pi}{16} \left( \frac{2\pi L_{nb}}{L} - \sin 2 \frac{\pi L_{ub}}{L} \right) \cdot L,$$

and

$$E_2 = \frac{\pi}{16} \left( \frac{2\pi L_{bb}}{L} + \sin 2 \frac{\pi L_{nb}}{L} \right) \cdot L.$$  

Equation 4.33 is then normalized to the total reactor power by dividing $E_1$ and $E_2$ by $L$. The feedback fuel temperature then becomes

$$\delta T_f(s) = \frac{A}{A_c \cdot L} \left\{ \frac{E_1/L \cdot h''p_f}{1 + s\tau_{fb}} + \frac{E_2/L \cdot h'p_f}{1 + s\tau_{fb}} \right\} N(s) \left[ ^{\circ}F \right]. \quad 4.34$$

Use of the fuel feedback is needed in this noise model because it makes the reactor with feedback act as a filter to variations in the fission power stemming from stochastic processes affecting the reactivity. The effect of this "temperature filtering" is to stabilize
the low frequency power response. The model includes two sinusoidally weighted temperature feedback functions

\[
\frac{\delta T_{\text{fb}}(s)}{N(s)} = H_{\text{fb}}(s) = \frac{E_1 \times A}{L \cdot \eta \cdot \rho_f (1 + s\tau_{\text{fb}})^A_c},
\]

4.35

and

\[
\frac{\delta T_{\text{fb}}}{N(s)} = H_{\text{fb}}(s) = \frac{E_2 \times A}{L \cdot \eta \cdot \rho_f (1 + s\tau_{\text{fb}})^A_c}.
\]

4.36

A block diagram of this part of the model is given in Figure 2.1.

d. Heat delivered to coolant  
Heat generated in the fuel elements reaches the coolant channel after a delay time resulting from the thermal characteristics of the fuel and cladding. In the steady state condition, heat generated in the fuel equals heat delivered to the coolant. This is equivalent to letting \( \frac{dT_f}{dt} = 0 \) in Equation 4.21 for the boiling region, or Equation 4.29 in the nonboiling region. Thus the heat delivered to the coolant is

\[
Q_b(x) = p_f h'[T_f - T_{\text{sat}}]
\]

4.37

and

\[
Q_{nb}(x) = p_f h''[T_{\text{fb}} - T_c].
\]

4.38

Substitution of Equations 4.37 and 4.38 into Equations 4.22 and 4.30 respectively, when \( T_{\text{sat}} \) and \( T_c \) are held constant, yields the transfer function for the coolant power arising from fission power changes

\[
\frac{Q_b(s)}{N_b(s)} = \frac{1}{1 + \tau_{\text{fb}} \cdot s},
\]

4.39

and

\[
\frac{Q_{nb}(s)}{N_{nb}(s)} = \frac{1}{1 + \tau_{\text{fb}} \cdot s}.
\]

4.40
Since the steady state fractions of power in the two regions are known, Equations 4.39 and 4.40 can be modified to become

\[ PTB(s) = \frac{1 - \text{FRAC}}{1 + \tau_{fb} \cdot s}, \]

and

\[ PTNB(s) = \frac{\text{FRAC}}{1 + \tau_{fnb} \cdot s}, \]

where \( \text{FRAC} = \frac{\dot{M}_{fw} (H_w - H_{fw}^*)}{Q^0} \). The transfer functions \( PTB(s) \) and \( PTNB(s) \) act as low pass filters in the void and coolant temperature feedback loops to fluctuations in the fission power level. For uranium oxide fuel pins of about 0.5 inch diameter, the fuel time constants range from 6. to 8. seconds. The break frequency associated with \( PTB(s) \) and \( PTNB(s) \) is near 0.023 Hz. However, metallic fuel elements have a much higher break frequency on the order of 0.4 Hz [50, (page 486)]. Thus, the filtering effect on inherent noise sources is much greater in uranium oxide cores than in metal cores.

3. **Power to void**

The moderator-coolant behavior in many light water reactors (LWR) has been modeled by considering the fundamental equations of mass, energy, and momentum. From Appendix A the combined mass-energy equation in linearized form is

\[
\frac{\partial}{\partial x} \left( U^0 \cdot a_s(x, t) + A_s^0 \cdot u(x, t) \right) + \frac{\partial}{\partial t} a_s(x, t) = \]

\[
\left[ \rho_s A_s^0(x) \frac{dH_s}{dP} + \rho_w A_w^0(x) \frac{dH_w}{dP} \right] \frac{dp(t)}{dt}. \]
Figure 4.1. Fuel temperature feedback model

The approximations leading to this equation are:

1. Variations of kinetic and potential energy are small and have been neglected.
2. The pressure change along the channel is neglected.
3. Saturation enthalpies depend only on the pressure.
4. Steam density and latent heat of vaporization per unit steam volumes are constant.
5. Steam and water phases are in equilibrium at any time and position, thus neglecting the possibility of subcooled boiling below the boiling boundary.

6. The power distribution function is constant and power variations are treated as changes in the steady state distribution amplitude.

7. The steady state operating conditions are known at the desired quiescent operating levels.

Equation 4.43 can be solved for $a_s(x, t)$ when $q(x, t)$ and $P(t)$ are specified and if a second relationship between $a_s(x, t)$ and $u(x, t)$ is known. This can be determined by the momentum equation, which depends on small pressure differences along the channel. The change in momentum, however, is due predominantly to the change in steam cross section along the channel for a highly pressurized system. If the change in steam bubble velocity is extremely small, the change in steam cross section dominates the expression $A_s^0 \cdot u(x, t) + a_s(x, t) \cdot U^0$ and is then approximated by $a_s(x, t) \cdot U^0$. An approximation of this type was utilized by Akcasu [11] by increasing the value of $U^0$ in Equation 4.43 to include the effect of increased bubble velocity arising from a change in $A_s$ for a natural circulation reactor. However, in a forced flow high pressure system the value of $U^0$ approaches the actual bubble velocity. This simplification then means that for a void perturbation the steam bubble velocity of the perturbed void is the same as the steady state bubble velocity.

Equation 4.43 then becomes
which is the equation governing the fluid behavior in a coolant channel of a BWR. Equation 4.44 is then simplified in this model by noting that pressure is held constant by the steam by-pass valve system. Therefore, under steady state conditions where \( \frac{dP}{dt} = 0 \) Equation 4.44 becomes

\[
\frac{\partial}{\partial x} a_s(x, t) + \frac{1}{u_0} \frac{\partial}{\partial t} a_s(x, t) = \frac{q(x, t)}{U_0 H V}.
\]

Equation 4.45 is the basic formulation which is used to determine the power to void transfer function \((PV(s))\). In view of assumption 6, \( q(x, t) = \frac{q(t)}{L} \cdot D(x) \), where \( D(x) \) represents the spatial distribution function which is normalized over the core length

\[
\frac{1}{L} \int_0^L D(x) dx = 1.
\]

For a sinusoidal distribution \( D(x) \) becomes \( \pi/2 \cdot \sin\left(\frac{Ux}{L}\right) \), or for more general shapes \( D(x) = FABC \cdot e^{ax} \cdot \sin(bx + c) \). A more detailed discussion of the axial shape function is presented in Chapter 3. Equation 4.45 is then Laplace transformed with respect to \( t \) and becomes

\[
\frac{d}{dx} [a_s(x, s)] + \frac{s}{U_0 H V} a_s(x, s) = \frac{q(s)}{L \cdot H V \cdot U_0} \cdot D(x).
\]

The change in steam volume along the channel may be found by integrating \( a_s(x, s) \) from the boiling boundary to the channel top.
\[ V(s) = \int_{L_{nb}}^{L} a_s(x, s) \, dx. \]  

4.48

The change in steam cross section is determined by solving Equation 4.47, noting that the steam perturbation velocity is equal to the steady state bubble velocity. An integrating factor, \( I(x) = \int_{L_{nb}}^{x} \frac{dx'}{U^0} \), is utilized in the solution of Equation 4.47 which becomes

\[
a(x, s) = e^{-\frac{S}{U} \int_{L_{nb}}^{x} dx'} \cdot \left[ g(s) \int_{L_{nb}}^{x} D(x') \cdot e^{\frac{S}{U} \int_{L_{nb}}^{x'} dx} \right].
\]

4.49

Substitution of Equation 4.49 into 4.48 and solving for the total void change for a change in the heat source amplitude results in

\[
\frac{V(s)}{q(s)} = \frac{1}{L \cdot H_v} \int_{L_{nb}}^{L} \frac{dx}{U^0(x)} \int_{L_{nb}}^{x} D(x') \cdot e^{\frac{S}{U} [x' - x]} \, dx' \cdot \left[ \frac{ft^3}{Btu/s} \right].
\]

4.50

Equation 4.50 is the general form of the power to void transfer function which could be solved numerically for any known \( D(x) \) and \( U^0(x) \) steady state distributions. Care must be exercised in the use of Equation 4.50, since it does not account for the mechanism of subcooled boiling or the delay times resulting from bubble formation, i.e. the time from initial growth to break away from the wall.

A description of bubble formation, presented by El-Wakil [51], indicates that the local surface conditions between the tube and the void determine location, size at detachment from the surface, and local heat transfer coefficient during formation of a particular bubble. This model neglects the bubble formation time assuming that it is much
smaller than the steam transit time. Bubbles are assumed to travel at
the steam velocity which is the slip ratio times the water velocity.
Bubble formation time could be included by utilizing an $e^{-ST}$ factor on
the right side of Equation 4.50, however, as long as $(x - L_{nb})/U > \tau$,
the bubble formation delay can be neglected.

A specific expression for the power to void transfer function can
then be determined for the case of uniform steam velocity and axial
flux distribution of the form $D(x) = FABC \cdot e^{ax} \cdot \sin(bx + c)$. A
summation of several weighted flux shapes would then approximate the
typical distributions found in BWR's [10, 43]. Equation 4.50 then
becomes

$$PV(s) = \frac{FABC}{L\cdot H\cdot V\cdot U} \cdot \int_{L_{nb}}^{L} dx \cdot \int_{L_{nb}}^{x} e^{ax'} \cdot \sin(bx' + c) \cdot \frac{s}{U} \cdot \sin(x' - x) \cdot dx'. \quad 4.51$$

Expanding $\sin(bx + c)$ to $\sin(bx) \cdot \cos(c) + \cos(bx) \cdot \sin(c)$ then
utilizing integration formulas [52] the following equation results.

$$PV(s) = \frac{FABC}{L\cdot H\cdot V\cdot U} \left\{ \frac{\int_{L_{nb}}^{L} J_{1} \cdot \left( \frac{S}{U} \cdot L_{b} - 1 \right) + \left( \frac{S}{U} \cdot L_{b} - 1 \right) \cdot \left( \frac{S}{U} \cdot J_{3} + J_{4} \right)}{S^{2} + \frac{2as}{U} + a^{2} + b^{2}} \right\} \quad 4.52$$

where

$$J_{1} = \frac{\cos(c)}{a^{2} + b^{2}} \left\{ e^{aL} \cdot [a \cdot \sin(bL) - b \cdot \cos(bL)] - e^{aL_{nb}} \cdot [a \cdot \sin(bL_{nb}) - b \cdot \cos(bL_{nb})] \right\} + \frac{\sin(c)}{a^{2} + b^{2}} \left\{ e^{aL} \cdot [a \cdot \cos(bL) + b \cdot \sin(bL)] - e^{aL_{nb}} \cdot [a \cdot \cos(bL_{nb}) + b \cdot \sin(bL_{nb})] \right\}$$
The power to void transfer function approaches a constant at a low frequency and decreases at 20 dB per decade in the high frequency range. This suggests the use of a single time constant to approximate this transfer function. However, in the mid frequency range of 0.1 to 10. Hz, a more exact description of the void behavior is needed to explain the noise measurements observed in in-core detectors. The shape of the power to void transfer function is dependent on the assumed steady state axial flux shape. The magnitude of this transfer function is plotted for several different axial power distributions cases in Figures 4.2 and 4.5. It is observed that for a sin(\(\pi x/L\)) shape, a sharp resonance peak occurs at a frequency of \(\pi/2L\) Hz. This frequency corresponds to the coolant residence time which is equivalent to one-half of a power oscillation (see Appendix F). Behavior of this type is expected in PV(s) since the denominator goes to zero at that
frequency. However, the numerator also goes to zero at $f = \frac{U}{2L}$ ($\omega = \frac{U \pi}{L}$), and the peak in $PV(s)$ reaches a finite value at that frequency.

As the "damping" parameter, $|a|$, is increased, representing a flux distribution skewed up for $|a|$ negative, the resonant peak is broadened in shape and reduced in magnitude. As the flux shape is flattened, by letting $a = 0$, $b \to 0$, and $c = \pi/2$, the peak moves to lower frequencies in the power to void transfer function. The input to $PV(s)$ is heat/unit time, thus to obtain the total void response the variation in the total heat delivered to the boiling region should be considered as input.

The power to void transfer function behavior is important to the detector response in two ways. First, the change in voids acts to change the reactivity state of the reactor and in turn the reactor power level. Second, the change in voids also perturbs the neutron flux by modifying the thermalization properties of the water near the detector.

4. **Power to boundary**

The effect of power variations in the heat source on changes in the boiling boundary is considered in this section. The transfer function associated with heat transfer to the coolant is obviously similar to the boiling region. The difference in the two transfer functions occurs because of the difference in the heat transfer coefficient in the boiling and nonboiling regions.

The heat supplied to the channel in the nonboiling region brings subcooled water to the saturation temperature at the boiling boundary.
A. General point kinetics model

$$G(s) = \frac{N^0(s + \lambda)}{\ell s(s + \beta/\ell)}$$

B. UO\textsubscript{2} lumped fuel model

$$H_f(s) = \frac{\text{Gain}}{(1 + s\tau_f)}$$

C. Power to void

Equation 4.52

Figure 4.2. Magnitude sketches for transfer functions in model
The recirculation water returned to the core inlet mixing plenum by the jet pump system is assumed to be at saturation. This assumption neglects heat lost or gained in the recirculation system. The change in enthalpy along the channel is then viewed as a change in feedwater enthalpy from the inlet up to the boiling boundary. All the heat input into the nonboiling region is added to increase the temperature of the mixed inlet water to the saturation temperature. Since additional heat to the already saturated recirculation water would cause boiling, the recirculation water enthalpy is viewed as a constant until the feedwater enthalpy equals that of the recirculation water at the boiling boundary. Thus the boiling boundary is defined as the channel height where the total heat input equals the amount of heat required to increase the feedwater enthalpy to the saturation condition. A simple boiling channel model is shown in Figure 4.3. Mathematically this can be determined from the heat balance equation for two phase flow which is

$$\frac{\partial}{\partial x} \left( \rho \frac{H_A \rho w}{A_w} \right) + \frac{\partial}{\partial x} \left( \rho \frac{H_A \rho w}{A_w} \right) + \frac{\partial}{\partial t} \left( \rho \frac{A_A H_A + \rho w A_H w}{A_w} \right) = Q_o(x, t). 4.53$$

In the nonboiling region $A_s = 0$ (no steam), $A_w = A_c$, and $\partial/\partial t = 0$ for steady state condition, hence Equation 4.53 becomes

$$\frac{\partial}{\partial x} \left( \rho \frac{H_A w}{A_w} \right) = Q_o(x). \quad [\text{Btu/s} \cdot \text{L}] 4.54$$

Let $M_{fw} = \rho A_w^o$ and boiling begins at $x = L_{nb}$, where the saturation enthalpy at $L_{nb}$ is $H_w$. Hence, the boiling boundary is determined by solving Equation 4.54 as

$$M_{fw} (H_w - H_{fw}) = \int_0^{L_{nb}} Q_o(x) dx \quad \text{Btu/s. 4.55}$$
In the case of a sinusoidal flux distribution Equation 4.55 becomes

\[ L_{nb} = \frac{L}{\pi} \cos^{-1} \left( \frac{Q^0 - \dot{M}_{fw} (H_w - H_{fw})}{Q^0} \right) \] \hspace{1cm} 4.56

For small changes in the power, with feedwater heat input to the channel held constant, changes in enthalpy as a function of position occur. The incremental form of Equation 4.53 becomes

\[ \frac{\partial}{\partial x} h(x, t) + \frac{1}{W^0} \frac{\partial}{\partial t} h(x, t) = \frac{q(x, t)}{M_{fw}}. \] \hspace{1cm} 4.57

By performing a Laplace transformation with respect to time, utilizing \( s/W^0 \times \) an integrating factor \( e^{s/s} \), and noting that \( h(0, t) = 0 \), for constant feedwater inlet enthalpy, Equation 4.57 is solved for \( h(x, s) \) to obtain \( x \)
Small changes in the enthalpy at the boiling boundary give rise to fluctuations in the position of the boiling boundary. In Figure 4.4 channel position is plotted as a function of enthalpy for small changes in the power level. A new rate of enthalpy change is established as a result of power change. Since $H_w$ is constant for a constant pressure, the change in power level is reflected as a small change in the boiling boundary.

$H^0(L_{nb} + \Delta L_{nb})$ is a variable enthalpy about the steady state point $H^0(L_{nb})$. The incremental quantity $h(L_{nb} + \Delta L_{nb})$ is the small change in $H^0(L_{nb})$. Thus

$$h(x, s) = \frac{q(s)}{M_{fw} \cdot L} \int_0^x \frac{-s}{W^o} (x-x') D(x')e \, dx' .$$ 4.58

**Figure 4.4.** Boiling boundary shifts arising from nonboiling region power changes.
\[
H_w = H^0(L_{nb} + \Delta L_{nb}) + h(L_{nb} + \Delta L_{nb})
\]

\[
= H^0(L_{nb} - \Delta L_{nb}) + h(L_{nb} - \Delta L_{nb}).
\]

Then
\[
\frac{dH^0(L_{nb} + \Delta L_{nb})}{dx} = \frac{dH^0(L_{nb} + \Delta L_{nb})}{\Delta L_{nb}} = \frac{Q^0(L_{nb})}{M_{fw}}
\]

and
\[
\Delta L_{nb}(t) = \frac{\dot{M}_{fw}}{Q^0(L_{nb})} (H^0(L_{nb} + \Delta L_{nb}) - H^0(L_{nb}))
\]

\[
\Delta L_{nb}(t) = \frac{\dot{M}_{fw}}{Q^0(L_{nb})} (-h(L_{nb} + \Delta L_{nb})).
\]

Substituting Equation 4.62 into Equation 4.58 at the boiling boundary yields
\[
PB(s) = \frac{\Delta L_{nb}(s)}{q(s)} = -\frac{1}{Q^0(L_{nb})} \int_0^{L_{nb}} D(x') e^{sx'} dx'.
\]

Equation 4.63 is the power to boiling boundary transfer function.

When \( D(x) = FABC e^{ax} \cdot \sin(bx + c) \)

\[
PB(s) = \frac{- FABC}{Q^0(L_{nb})} \int_0^{L_{nb}} e^{ax} \cdot \sin(bx + c) e^{-\frac{s}{W}(L_{nb} - x')} dx'.
\]

Utilizing integration formulas [52] the power to boundary transfer function becomes
\[
PB(s) = \frac{- FABC \cdot \left\{ K_2 + sK_1 + (K_4 + sK_5) \cdot \frac{s}{W} \right\}_{L_{nb}}}{Q^0(L_{nb}) \cdot L \left[ \frac{a}{W} \right]^2 + \frac{2as}{W} + a^2 + b^2}
\]

where
\[
K_1 = \frac{e^{aL_{nb}}}{W} \left[ \cos(c) \cdot \sin(bL_{nb}) + \sin c \cdot \cos(bL_{nb}) \right]
\]
\[ K_2 = \cos(c) \cdot e^{aL_{nb}}[a \cdot \sin(bL_{nb}) - b \cdot \cos(bL_{nb})] \]
\[ + \sin(c) - e^{aL_{nb}} \cdot [a \cdot \cos(bL_{nb}) + b \cdot \sin(bL_{nb})] \]
\[ K_3 = \frac{\sin(c)}{w} \]
\[ K_4 = \cos(c) \cdot b + \sin(c) \cdot a. \]

In the case of a sinusoidal distribution, \( K_3 = 0, K_4 = b, K_1 = \sin(bL_{nb})/W, \)
\( K_2 = -b \cdot \cos(b \cdot L_{nb}), \) and \( \text{FABC} = \pi/2. \) However, for the cases tested
this transfer function was found to have behaved approximately as a
simple single time constant transfer function with a single break
frequency. As shown in Figure 4.5 the break frequency for this
transfer function occurs at 1.0 Hz. However, a modification to the
single time constant behavior occurs at about 2.5 Hz. This leveling
in magnitude is attributed to the increase in gain from the delay
function term \( K_2 + K_4 e^{-\left(\frac{s \cdot L_{nb}}{w}\right)} \). An example of this behavior is
given in Appendix F. Power distribution changes due to the "a" parameter had little effect on the frequency dependent shape of
this transfer function.

5. Boundary to void

Fluctuations in the boiling boundary give rise to changes in the
total void fraction. A relationship between the boiling boundary
and the weighted void fraction is needed to complete the void feed-
back loop. As before, let \( \Delta L_{nb}(t - \tau_x) \) be the boundary shift at \( L_{nb} \)
when \( \tau_x \) is zero. In this model \( \tau_x \) then represents the time for the
void perturbation to reach a new position up the channel with the
delay time $\tau_x$. When the wave reaches the channel top steady state is reached. The amplitude of steam cross section change at the boiling boundary, or any position $x$ arising from a power change in the non-boiling region, is proportional to a change in the boundary position.

Equation 4.45 with $\frac{\partial a}{\partial t} = 0$ for the steady state condition before and after a change in the boiling boundary gives

$$U^o \frac{d}{dx} A_s(x) = \frac{Q(x)}{H_v}. \quad 4.65$$

Solution of Equation 4.65 for $A_s(x)$ yields

$$A_s(x) = \frac{1}{H_v U^o} \int_{L_{nb}}^{x} Q(x) dx. \quad 4.66$$

Evaluating Equation 4.66 for $x = L_{nb} - \Delta L_{nb}(t)$ yields

$$a_s(L_{nb}(t)) = -\frac{Q(L_{nb})}{H_v U^o} \cdot \Delta L_{nb}(t) \cdot U(t) \quad 4.67$$

at the boiling boundary. The perturbation front proceeds up the channel with a velocity $U^o$, and the time required to reach a new position $x$, in the interval $L_{nb}$ to $L$ is $\tau_x$. At any position $x$ the change in steam cross section becomes

$$a_s(x(t)) = -\frac{Q(L_{nb})}{H_v U^o} \cdot \Delta L_{nb}(t - \tau_x) \cdot U(t - \tau_x), \quad 4.68$$

where $\tau_x = \int_{L_{nb}}^{x} \frac{dx'}{U^o}$ and $U^o = \frac{dx}{dt}. \quad 4.69$

The void response resulting from changes in $L_{nb}$ is then given as
Use of the Laplace transformation on Equation 4.71 yields

\[ V(x(s)) = \int_0^\infty e^{-st} \cdot \int_{L_{nb}}^{x(t)} \frac{Q(L_{nb})}{H_U U^o} \cdot \Delta L_{nb}(t - \frac{x' - L_{nb}}{U}) \cdot U(t - \frac{x' - L_{nb}}{U}) dx' dt. \]

Rearrangement of Equation 4.72 gives

\[ V(x(s)) = -\frac{Q(L_{nb})}{H_U U^o} \int_{L_{nb}}^{x(s)} dx' \cdot \int_0^\infty \Delta L_{nb}(t - \frac{x' - L_{nb}}{U}) \cdot U(t - \frac{x' - L_{nb}}{U}) e^{-st} dt, \]

\[ V(x(s)) = -\frac{Q(L_{nb})}{H_U U^o} \int_{L_{nb}}^{x(s)} dx' \cdot \left[ \Delta L_{nb}(s) \cdot e^{-s \frac{x' - L_{nb}}{U}} \right], \]

and

\[ \frac{V(x(s))}{\Delta L_{nb}(s)} = -\frac{Q(L_{nb})}{H_U U^o} \cdot \Delta L_{nb}(s) \cdot e^{-s \frac{-x' - L_{nb}}{U}} - \frac{L_{nb}}{U^o} \left| \frac{x(s)}{s} \right|. \]

Thus, the void perturbation up to \( x \) arising from a shift in the boiling boundary is expressed as

\[ \frac{V(x(s))}{\Delta L_{nb}(s)} = -\frac{Q(L_{nb})}{H_U} \cdot \left[ \frac{1 - e^{-s \tau_x}}{s} \right]. \]
In the feedback loop the change in total void volume changes the reactivity state. Since the core residence time is limited to $L_b/U_0 = T_s$, Equation 4.76 is written as

$$\frac{V(s)}{\Delta L_{nb}(s)} = -\frac{Q(L_{nb})}{H_v} \cdot \left[1 - e^{-sT_s}\right]. \quad 4.77$$

The notation used for this transfer function in the block diagrams is

$$BV(s) = -\frac{Q(L_{nb})}{H_v} \cdot \left[1 - e^{-s\frac{L_b}{U}}\right]. \quad 4.78$$

Equation 4.78 represents the total void response in the active core length to a change in the position of the boiling boundary. The feedback effect of the voids is more pronounced in the core center where the flux density is highest rather than near the core boundary. A weighting function could be used in conjunction with $U(t - \tau_x)$ to account for the variation in the void worth along the channel. Another method is to adjust the magnitude of the feedback coefficient itself, to approximate the flux weighting of the void worth.

The behavior of this transfer function is described in Appendix F, and shown in Figure F.1. Delay functions of this type appear in several other transfer functions, namely power to void, power to boundary, and power to water temperature.

6. **Coolant temperature**

Reactor power variations produce local temperature fluctuations in the nonboiling region. Consider a heat balance for a differential
A. Power to void (PV(s))
Equation 4.52

- Positive damped
- --- Base case + - flux

B. Power to boundary (PB(s))
Temperature (PH(s))
Equation 4.65, Equation 4.84

C. Boundary to void (BV(s))
Equation 4.78

Figure 4.5. Additional magnitude sketches for transfer functions in model
volume of fluid at a position $x$ along the nonboiling section of the channel. Heat into the volume equals heat out of the volume plus heat stored in the volume. Heat into the differential volume arises from heat delivered by the fuel element plus heat transported into the volume by the coolant. This heat balance is described by Equation 4.79

$$Q(x, t) + \dot{M}_w c_p T_w(x, t) = \dot{M}_w c_p (T_w(x) + \frac{\partial T_w}{\partial x} dx) + \frac{\partial}{\partial t} (\rho_w A dx_c T_w(x)) \text{ [Btu/s].}$$

Assuming that $c_p = \bar{c}_p$, linearizing the variables $Q, \dot{M}_w, \text{ and } T_w$, and neglecting second order terms Equation 4.79 becomes

$$\frac{q(x, t)}{M_w c_p} = \frac{\dot{M}_w}{M_w} \frac{\partial T_w}{\partial x} + \frac{\partial T_w}{\partial x} + \frac{\rho A}{M_w} \frac{\partial}{\partial t} T_w.$$  

4.80

For constant flow rate $\dot{M}_w = 0$, and noting that $\frac{\dot{M}_w}{\rho A_w} = W$, it follows that

$$\frac{q(x, t)}{\dot{M}_w c_p} = \frac{\partial T_w}{\partial x} + \frac{1}{W} \frac{\partial}{\partial t} T_w.$$  

4.81

Equation 4.81 is solved by performing a Laplace transformation with zero initial conditions on the time variable, then utilizing an integrating factor $e^{\frac{sa}{W} \int dx}$ to solve the resulting first order equation in $x$

$$\frac{d}{dx} T_w(x, s) + \frac{s}{W} T_w(x, s) = \frac{Q(x, s)}{M_w c_p}$$  

4.82

yields

$$T_w(x, s) = \frac{Q(x)}{M_w c_p L} \int_0^x \text{FABC} \cdot e^{ax} \sin(bx+c) \cdot e^{\frac{sa}{W} (x'-x)} \ dx'.$$  

4.83
where $\text{FABC} \cdot e^{ax} \cdot \sin(bx + c)$ is introduced as the spatially dependent weighting function describing the axial flux shape. The change in water temperature arising from variations in the power delivered to the coolant then becomes

$$\frac{\delta T_w(x, s)}{Q(s)} = \frac{\text{FABC} \cdot \frac{s}{M_C L} \left( sK_1 + K_2 + e^{ax} \frac{s}{W} (sK_3 - K_4) \right)}{a^2 + b^2 + 2as + \left( \frac{s}{W} \right)^2}$$

where

$$K_1 = \frac{e^{ax}}{w} \cdot [\cos(c) \cdot \sin(bx) + \sin(c) \cdot \cos(bx)],$$

$$K_2 = e^{ax} \cdot [a \cdot \cos(c) \cdot \sin(bx) + a \cdot \sin(c) \cdot \cos(bx),$$

$$+ b \cdot \sin(c) \cdot \sin(bx) - b \cdot \cos(c) \cdot \cos(bx)],$$

$$K_3 = \frac{\sin(c)}{W},$$

and

$$K_4 = a \cdot \sin(c) + b \cdot \cos(c).$$

$PH_w(s)$, the temperature feedback transfer function, is determined from Equation 4.84 by letting $x = L_{nb}$. This assumes that the non-boiling region power changes cause changes in the coolant temperature, but in the boiling region the coolant temperature is constant and the fluctuations in reactor power level cause void changes. The coolant temperature feedback function then includes flux shape parameters, water velocity, and a time delay function which accounts for the water transit time in the nonboiling section. $PH_w(s)$ has the same form as the power to boundary transfer function, but differs in the magnitude of the gain constant. The previously derived transfer functions are combined to form the BWR feedback model for inherent reactivity driving.
forces. The process transfer functions are summarized in Table 4.1.

A block diagram is presented in Figure 4.6 which shows the combination of these transfer functions into the closed loop reactor transfer function, and the reactivity to power frequency response is displayed in Figure 4.7.

Table 4.1. Summary of transfer functions in the feedback model

<table>
<thead>
<tr>
<th>I. Fuel temperature feedback</th>
<th>Form</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$H_{fnb}(s) = \frac{WF_b/h'p_f}{1 + s\tau_{fnb}}$</td>
<td>$\text{Btu/s} \cdot \text{fuel pin}$</td>
</tr>
<tr>
<td></td>
<td>$H_{fb}(s) = \frac{WF_{nb}/h''p_f}{1 + s\tau_{fb}}$</td>
<td>$\text{Btu/s} \cdot \text{fuel pin}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>II. Water temperature feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>$PH_w(s) = \frac{FABC}{M \cdot c \cdot p} \left[ \frac{sK_1+K_2}{2} + \frac{e}{sW}L_{nb} \right] \left[ \frac{sK_3+K_4}{W^2} \right]$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>III. Void feedback loops</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. $PV(s) = \frac{FABC}{L \cdot H \cdot V} \left[ \frac{S}{U} J_1 + J_2 + \frac{S}{U} L_b - 1 \right] \left[ \frac{S}{U} J_3 + J_4 \right]$</td>
</tr>
</tbody>
</table>

| B. (1) $PB(s) = -\frac{FABC}{Q^o(L_{nb}) \cdot L} \left[ \frac{sK_1s + K_2 + (K_3s + K_4)e}{a^2 + b^2 + \frac{2as}{W} + \left( \frac{s}{W^2} \right)^2} \right] \left[ \frac{s}{W} L_{nb} \right]$ | $\text{Btu/s}$ |

| (2) $BV(s) = -\frac{Q^o(L_{nb})}{H_V} \left[ \frac{-s}{U} \right] \left[ \frac{L_b}{s} \right]$ | $\text{ft}^3/\text{ft}$ |
Figure 4.6. Simplification of the reactor transfer function driven by an external reactivity source.

\[
G_{cl}(s) = \frac{G(s)}{1.0 - G_{op}(s)}
\]

\[
G_{op}(s) = G(s) \cdot \left[ \alpha_v \cdot [PTB(s) \cdot PV(s) + PTNB(s) \cdot PB(s) \cdot BV(s)] + \alpha_w \cdot [PTNB(s) \cdot PH(w(s))] + \alpha_f \cdot [H_{fnb}(s) + H_{fb}(s)] \right]
\]
Figure 4.7. Sketches of combined transfer function behavior
B. Noise Sources

1. Noise sources for other models

The identification of noise sources leading to neutron fluctuations has been the goal of many reactor noise investigations. Cohn [27] was one of the first to identify the fission chain related noise found in zero power reactors. However, as soon as feedback reactivities become important this source is dominated by reactivity related noise and becomes insignificant [3].

An inherent reactivity noise source was postulated in Nomura's [15] model noting that discrete quantities of heat are required to form each bubble during void formation. The randomness of this process then gives rise to reactivity fluctuations through variations in the void fraction, pressure, and temperature. The resulting reactivity change then drives the closed loop BWR transfer function, thus neglecting the feedback void and temperature changes as a driving force.

Seifritz [19] presented a development of a reactivity noise source arising from fluctuations in the coolant flow. Flow variations were assumed to modify the heat transfer coefficient defined by Equation 4.27. A control system approach was used to determine the various transfer functions and feedback paths needed to relate the coolant velocity oscillation to changes in the fuel and coolant temperature. A reactivity driving function then resulted when the appropriate temperature feedback coefficients were utilized.
2. Noise sources for this model

Two sources, similar to bubble generation and coolant flow fluctuation, are considered in this model. Coolant coming into a reactor contains an amount of heat per unit volume. This heat input to the reactor is viewed as a random variable when the reactor is operating at steady state. The randomness of the heat input results from inherent variations of the plant coolant system, such as changes in the condenser flow or temperature, vibrations in the feedwater pumps, nonstatic turbine conditions, and even control system set point changes [42]. The combination of plant fluctuations under steady state conditions then produces heat variation contained in elements of water entering the reactor core. As these water elements enter the heated channel, the heat needed to increase water temperature to saturation conditions varies slightly for each water element. This is viewed as a "white" noise source which is distributed along the channel with the same distribution function as the heat source in the nonboiling region. Specific anomalies in the external reactor system are expected to change the spectral content of this noise source. The source is introduced in the model at a summing junction with the heat supplied from the reactor core as a result of the fission process. This source is similar to Seifritz's except that it appears as a heat source rather than oscillations of the coolant velocity. This white noise heat source is denoted as $\Delta H$.

A second noise source is considered in the boiling region. Bubble formation in a heated channel takes place in an individual way. The amount of wetted surface and the bubble size at detachment are
examples of variable parameters encountered during bubble formation [51]. In the case of saturated water, the amount of heat required to form a unit steam bubble would be a constant. However, the difference in local wetted surface for a unit bubble size gives rise to large variations in the heat transfer coefficient. The large variation in the local heat transfer coefficient varies the local rate at which heat enters a particular bubble. Thus, variation in heat addition rate is viewed as a noise source with a spatial distribution function equivalent to the power distribution in the boiling region. This source is then identical in form to Nomura's heat noise source. However, by introduction of the source at a summing junction its magnitude can be compared with fission power oscillations appearing in the coolant, accounting for the feedback through the reactor for this source. The spectral content of this source is assumed white in the frequency range of interest to simplify the analysis. This bubble formation source is denoted as \( \Delta P \).

Figure 4.9 shows how the noise sources are introduced into the total model. This model, driven by the two sources, results in a detector response which is a function of the position of the detector, the initial operating point, and operating parameters affecting the individual transfer function behavior.

C. Detection Model

Many types of neutron detectors have been used to monitor the neutron flux in power reactors. The choice of neutron detector size, sensitivity, and location are important factors in determining the type
of measurements that can be made. Most of the neutron noise analysis has been done using detectors located outside the vessel. However, the recent development of small self-powered detectors which can be located inside the core opens the possibility of monitoring the spatial neutron noise behavior. The small size allows for movement within the core, and the high sensitivity reduces the need for bulky detectors having a large perturbation on the local neutron density. The time constants are sufficiently short so that they can be used for noise measurements.

The void effect on the thermal neutrons is considered as an important contribution to neutron fluctuations observed by the detector. Thie [16] called this portion of the detector signal "thermalization noise." Rothman [37] modeled the local void effect on detector response by considering the multi-energy group diffusion equations perturbed by changes in the local void fraction around the detector. He concluded that the fluctuation in neutron population is related, by a simple constant, to changes in the void fraction near the detector. He simplified the problem by considering a steady state condition in which the voidage fluctuations followed Poisson statistics, and second order products were neglected. Based on Rothman's thermalization development, it is assumed that the detectors respond to changes in the local void fraction in addition to the variations in fission power.

1. **Power response**

For the space independent model, changes in reactor power are proportional to changes in the neutron density. A cause of fluctuation in the detector signal stems from variations in the power level.
causing changes in the thermal flux at a detector location. The amplitude of the variation in signal current driven by power changes is proportional to the steady state current to reactor power ratio. The movement of the detector to various positions along the channel changes the steady state current in proportion to the thermal flux available at the detector location. The use of an efficiency factor allows the model to account for the spatially dependent power to flux ratios.

\[ E_{PL} = \left( \frac{\Delta \phi}{\Delta N} \right) \times \text{RPF} \times \text{APF}. \]  \hspace{1cm} 4.84

The radial (RPF) and axial (APF) peaking factors are determined by the ratio of the steady state thermal flux at any position in the reactor to the average flux. The average flux is simply determined by [46]

\[ \bar{\phi} = \frac{\text{Power level (watts)}}{3.7E13 \times w(\text{lbs U}_{235}) \times \sigma_f \text{ (effective fission cross section)}}. \]  \hspace{1cm} 4.85

For the DAEC BWR at the beginning of core life Equation 4.85 becomes

\[ \bar{\phi} = \frac{1593E6}{3.7E13 \times 2897 \times 294E-24} = 5.06E13 \text{ n/cm}^2\cdot\text{s}. \]  \hspace{1cm} 4.86

For a small change in power level of 1 Btu/s compared to the operating level of 1.5 E6 Btu/s, the change in average thermal flux would be 3.35E7 n/cm^2\cdot s. The value of the efficiency factor then becomes

\[ E_{PL} = 3.35E7 \times \text{RPF}(r) \times \text{APF}(x) \times \frac{\text{n/cm}^2\cdot\text{s}}{\text{Btu/s}}. \]  \hspace{1cm} 4.87
2. Void thermalization response

The change in neutron density arising from changes in the local void fraction are treated in a slightly different manner. Since local fluctuations in the neutron density are proportional to the variation in the void fraction, consider the axial dependent parameter $a_s(x, s)$ as proportional to the neutron flux variation [37]. Two processes affect variation in $a_s(x, s)$, direct power to void variations and changes in the boiling boundary. From Equation 4.49

$$\frac{a_s(x, s)}{q(s)} = \frac{FABC}{U \cdot L \cdot H_v} \cdot \frac{S}{u} \cdot \frac{x}{e} \cdot \int_{L_{nb}}^{x} \frac{S}{e} \cdot x' \cdot e^{ax'} \cdot \sin(bx' + c) dx'. \quad 4.88$$

After integration and combination of terms, Equation 4.88 becomes

$$\frac{a_s(x, s)}{A_s} = \frac{FABC \cdot e^{ax}}{A_s \cdot L \cdot H_v} \cdot \left[ \frac{K_5(x) + sK_6(x) + e^{-sT(x)}(K_7 + sK_8)}{s^2 + \frac{2a}{U} s + a^2 + b^2} \right] \quad 4.89$$

which is called $PA2(x, s)$ in the model, where

$$K_5(x) = \cos(c) \cdot \left[ a \cdot [\sin(bx) - e^{-a(x-L_{nb})} \cdot \sin(bL_{nb})] 
- b[\cos(bx) - e^{-a(x-L_{nb})} \cdot \cos(bL_{nb})] 
+ \sin(c) \cdot \left[ a \cdot [\cos(bx) - e^{-a(x-L_{nb})} \cdot \cos(bL_{nb})] 
+ b[\sin(bx) - e^{-a(x-L_{nb})} \cdot \sin(bL_{nb})] \right] \right]$$

$$K_6(x) = \frac{1}{U} \cdot \left[ \sin(bx) \cdot \cos(c) + \cos(bx) \cdot \sin(c) \right]$$

$$K_7 = \cos(c)[a \cdot \sin(bL_{nb}) - b \cos(bL_{nb})] 
+ \sin(c) \cdot [a \cdot \cos(bL_{nb}) + b \sin(bL_{nb})]$$
\[ K_8 = \frac{1}{U} \cdot [\sin(bL_{nb}) \cdot \cos(c) - \sin(c) \cdot \cos(bL_{nb})] \]

\[ \tau_x = \frac{x - L_{nb}}{U} \text{ sec} \]

The detector located at \( x \) then responds in part to changes in the local void fraction caused by power fluctuations through the \( PA_2(s) \) transfer function.

Another path leading to changes in the local void fraction at the detector position stems from changes in the boiling boundary. Laplace transforming Equation 4.68 for a step input of \( \Delta L_{nb} \), and considering the delay function response of \( a_s(x, s) \) based on position \( x \) yields

\[ \frac{a_s(x, s)}{\Delta L_{nb}(s)} = \frac{-Q(L_{nb})}{H \cdot U} \cdot e^{-s \cdot \tau_x} , \quad 4.90 \]

where \( \tau_x = (x - L_{nb})/U \), the time for the wave front to reach the position \( x \). The local void fraction response due to changes in the boiling boundary becomes

\[ BA_2(s) = -\frac{Q(L_{nb})}{H \cdot U \cdot A_s} \cdot e^{-s \cdot \tau_x} . \quad 4.91 \]

Since linear processes are considered, the outputs of \( PA_2(s) \) and \( BA_2(s) \) can be summed to give the total change in the local void fraction at \( x \) affecting the detector response.

3. **Combined response**

   The detector response model assumes that the detector current fluctuations are directly proportional to local flux fluctuations near the detector. In this model these variations in flux arise from two sources: (1) power changes, and (2) void fluctuations. The
void fluctuations are space dependent in the axial direction due to the transfer function behavior. The block diagram in Figure 4.8 depicts the detector response model where the efficiencies $E_{P1}$ and $E_{P2}$ relate the magnitude of detector fluctuations due to local void fraction changes and to variations in the fission power level. The examples presented in Appendix D can be used to estimate the values of $E_{P1}(x)$ and $E_{P2}(x)$ for specified locations along the channel. The values used in this model are the result of simple calculations, and neglect the change in the ratio of fast to thermal flux along the voided channel, except for method 3. Assuming the void fluctuations and power response are uncorrelated, the power spectral density of the detector current is proportional to

$$\psi_{dd}(x, \omega) = |E_{P1}(x)|^2 \psi_{nn}(\omega) + |E_{P2}(x)|^2 \psi_{vv}(x, \omega), \quad 4.92$$

where $\psi_{nn}(\omega)$ is the PSD of neutron fluctuation arising from power changes, and $\psi_{vv}(\omega)$ in the PSD of void fluctuations arising from bubble generation. Equation 4.92 compares to Equation E.7 for high power noise models. To simplify the model, the values of $E_{P2}(x)$ and $E_{P1}(x)$ are normalized to the value of $E_{P2}$ at 6.0 feet. The model detector response signature then becomes

$$\psi_{dd}(x, \omega) = \left| \frac{E_{P1}(x)}{E_{P2}(6)} \right|^2 \psi_{nn}(\omega) + \left| \frac{E_{P2}(x)}{E_{P2}(6)} \right|^2 \psi_{vv}(x, \omega), \quad 4.93$$

where the relative units are proportional to the neutron flux at the detector location. The detector response model is given in Figure 4.8. Figure 4.9 shows the total noise model with white noise input sources, reactor feedback and the detector position model.
Boiling boundary fluctuations arising from coolant heat variations

Power fluctuations in boiling region

Local void response at detector position \( x \)

Detector response at \( x \), to thermal flux

Reactor fission power

\[ B_{A1}(x, s) = -\frac{Q(L_{nb})}{H \cdot U \cdot A_c} \cdot e^{-s \cdot \tau(x)} \]

\[ P_{A2}(x, s) = \frac{FABC \cdot e^{ax}}{A_c \cdot L \cdot H \cdot U} \cdot \frac{K_5(x) + sK_6(x) + e^{-s \cdot \tau(x)} \cdot (K_7 + sK_8)}{(\frac{s}{U})^2 + \frac{2a}{U}s + a^2 + b^2} \]

\( E_{P1} \) = change in neutron flux to a change in power level

\( E_{P2} \) = change in thermal flux due to a change in local void content

Figure 4.8. Detector response model
Figure 4.9. Block diagram of BWR noise model driven by $\Delta H$ and $\Delta P$ leading to the detector responses.
V. METHOD OF MODEL ANALYSIS

The purpose of this chapter is to explain the analytical techniques used in analyzing the noise model developed in Chapter IV. A digital computer program, NMOD, was written to utilize key reactor input data to solve for the initial quiescent operating conditions of the reactor [11, 52, 53, 54, 55, 56]. Based on these conditions, the constants in all of the transfer functions are found. Then, the frequency response of individual transfer functions, combination transfer functions relating input to output, and finally the weighted detector responses are calculated in the program. The description of a transfer function in NMOD includes frequency, magnitude, real part, imaginary part, phase angle, and square modulus.

A. Simplification of Block Diagrams

The combined noise input to the detector response is considered as a linear system with multiple inputs and outputs. Using block diagram identities found in linear system analysis texts [57, 58, 59], the model is simplified so that each input to output path is identified with all feedback loops included. Figures 5.1 through 5.4 show the simplified input to output for each source and detector response. At each frequency of interest, individual transfer functions are combined to obtain the total response. To combine the transmission paths, an estimate of the relative weights of the two source strengths is made so that the paths leading to the detector inputs may be
combined, resulting in the total detector response. The procedures are based on the input-output theory presented in Appendix B.

Figure 4.6 represents the reactor model with reactivity as the driving force. The total feedback loop transfer function, $H_T(s)$, results from the products of transfer functions in each feedback loop times the appropriate feedback coefficient. The open loop reactor transfer function then becomes the product of $G_{op}(s) = G(s) \cdot H_T(s)$, and the closed loop reactor transfer function is $G_{cl}(s) = G(s)/1 - G_{op}(s)$.

From the model developed in Chapter IV the transmission path from each input to its detector output is determined. The boiling source to fission power source (TDPE1) is given in Figure 5.1. This is similar to Nomura's [15] model except that the direct pressure effects and fuel temperature changes arising from the void generation have been neglected. The output unit considered in this model for this transfer function is Btu/s.

Figure 5.2 shows the feedback system representing the boiling source to direct void effects (TDEP2) on the detector, and includes the "feedback" of fission power changes at the noise source input summing junction. The output unit here is a change in void fraction.

The subcooled heat source to reactor power transmission path (TDHE1) is shown in Figure 5.3. This source has two feedback paths leading to power effects, the change in water temperature, and the change in void. This loop is similar to Seifritz's model [19] for fluctuating coolant flow. The output unit for this path is Btu/s.
\[ TDPE_1 = PV(\omega) \cdot \alpha_v \cdot G_{cl}(\omega) \]

Figure 5.1. Transmission path from the boiling noise source to the detector efficiency for fission power changes

The final path considered is the direct effect of the subcooled source on the void fraction (TDHE2). Figure 5.4 shows the block diagram used to calculate this transmission path, including the feedback from the reactor. The output unit is a change in void fraction.

The next step is to determine a weighting factor for each source. The ratio of \( \Delta P/\Delta H \) was picked as 5, which is approximately the fraction of power delivered to the boiling region compared to the fraction in the nonboiling region (4.46 for the base case). The efficiency for the detector response to power oscillations compared to void fluctuations is also needed to determine the total response. Three weighting methods were tried based on the information in Appendix D. The detector response transfer function \( BA_2(x, s) \), \( PA_2(x, s) \), \( EP_2(x) \), and \( EP_1(x) \) are the spatially dependent transfer functions for different detector locations.

The important parameters considered in this study deal with flux shape, core water velocity as determined by an operating map,
$TDPE2 = PA2(x, \omega) (1 + PV(\omega) \cdot \alpha_v \cdot G_{c1}(\omega) \cdot PTB(\omega))$

$+ PV(\omega) \cdot \alpha_v \cdot G_{c1}(\omega) \cdot PTNB(\omega) \cdot PB(\omega) \cdot BA2(x, \omega)$

Figure 5.2. Transmission path from the boiling noise source to the detector efficiency for void variations

magazines of inherent feedback coefficients, relative weight of noise sources, weight of detector noise response to both thermalization noise and power fluctuations, and detector position in the channel. A base case is used to demonstrate the general conditions expected in the detector behavior at three locations in the channel. Parameter variation from the base case demonstrates the effect of this change on the detector response. Analogous changes in measured PSD's from
Figure 5.3. The transmission path for variations in the nonboiling heat source to the detector efficiency for fission power changes

actual reactors may be attributed to changes in the value of that parameter.

This model provides information about the effect of internal parameters on the behavior of PSD's. External reactivity driving functions such as control rod vibration, are not considered in this model, but would appear in the spectrum as a spike at a frequency related to the vibrational mode [2, 60, 61, 62].

B. Description of NMOD

NMOD (Noise Model) is a digital computer program written to describe the in-core neutron detector response to two white noise heat sources. The program uses reactor dimensions, operating conditions,
Figure 5.4. The transmission path for changes in the nonboiling noise source leading to the detector response to void variations and operating coefficients to calculate the steady state constants and gains required for the model transfer functions. The program calculates individual transfer functions as a function of frequency. The output,
based on the complex number at each frequency, includes real and imaginary parts, magnitude, phase, and the square modulus.

Calculated results can then be plotted by subroutine GPlot. Square modulus plots permit qualitative comparisons with PSD measurements from operating reactors. Figure 5.5 explains the sequence of calculations in the program. A listing is also provided in Appendix G.

The program flexibility allows for comparison of input data changes for each transfer function. Study of these transfer functions gives information about the operating conditions which can be observed with noise measurements in both steady state and changes of inherent conditions. Input data are presented in Tables 3.1, 6.1, and 6.2 with a sample input included with the program printout in Appendix G.
Part 1. Initial conditions

(1) Read in steam-water properties at operating conditions
(2) Read in physical reactor data and operating power

Determine flow from operating map function

Calculate slip ratio, recirculation ratio, and steam velocity, based on desired core exit steam quality

Calculate steady state boiling boundary based on sinusoidal flux shape

Calculate lumped parameter transfer function time constants and gains

Output operating conditions and values for lumped transfer functions

Figure 5.5. Flow diagram of NMOD
Part 2. Noise model calculations

Read flux shape parameters, feedback coefficients, detector response factors, and source strength ratios

Calculate normalization factors, output axial flux shape

Determine new boiling boundary due to new flux shape

Calculate transfer function constants based on steady state operating conditions

Calculate the frequency dependent single unit transfer functions — output if needed

Calculate the reactor feedback open and closed loop transfer functions — output if needed

Combine into the path from source to detector — output if needed

Calculate detector response functions based on input weighting functions — output

STOP

Figure 5.5. (Continued)
VI. RESULTS

A series of test cases have been used to analyze the model behavior. Data used in the base case are based on expected normal steady state operating conditions of the DAEC BWR. Changes in various parameters are analyzed with the model, thus generating a library of detector response frequency signatures. The magnitude behavior of these signatures provides information which aids in the interpretation of actual frequency signatures from in-core neutron detectors. Data used in the base case are presented in Table 6.1, and steady state plant parameters are found in Table 3.1. Variations in the base case are listed in Table 6.2.

A. Closed Loop Reactor Transfer Function

The square modulus of the closed loop reactor transfer function for the base case is given in Figure 6.6. This is proportional to the signature of a detector responding to fission power changes resulting from a white noise reactivity driving function with all inherent noise sources zero. The closed loop transfer function, \( G_{cl}(\omega) \), is particularly sensitive to values of the reactivity feedback coefficients. When the coefficients are zero, \( G_{cl}(\omega) \) becomes the zero power reactor transfer function (Figure 4.2). As the fuel temperature feedback loop gain (reactivity coefficient \( \alpha_f \)) is increased, the effect is to reduce the low frequency gain. The published feedback coefficient is used here, since simple sinusoidal flux weighting is included in the derivation of \( H_{fnb}(\omega) \) and \( H_{fb}(\omega) \). In the
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mathematical</th>
<th>Base case value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction of full power</td>
<td>FP</td>
<td>0.9</td>
<td>-</td>
</tr>
<tr>
<td>Desired exit steam quality</td>
<td>CHIE1</td>
<td>0.143</td>
<td>-</td>
</tr>
<tr>
<td>Radial peaking factor</td>
<td>RPF</td>
<td>1.4</td>
<td>-</td>
</tr>
<tr>
<td>Flux shape parameters</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Damping coefficient (1)</td>
<td>a_1*L</td>
<td>ADL(1)</td>
<td>+ 3.0</td>
</tr>
<tr>
<td></td>
<td>a_2*L</td>
<td>ADL(2)</td>
<td>- 3.0</td>
</tr>
<tr>
<td>Damping coefficient (2)</td>
<td>b_1*L</td>
<td>BSL(1)</td>
<td>3.14</td>
</tr>
<tr>
<td></td>
<td>b_2*L</td>
<td>BSL(2)</td>
<td>3.14</td>
</tr>
<tr>
<td>Phase angle (1)</td>
<td>c_1</td>
<td>CB(1)</td>
<td>0</td>
</tr>
<tr>
<td>Phase angle (2)</td>
<td>c_2</td>
<td>CB(2)</td>
<td>0</td>
</tr>
<tr>
<td>Feedback coefficients</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fuel temperature</td>
<td>α_f</td>
<td>ALF</td>
<td>- 0.98E-5</td>
</tr>
<tr>
<td>Water temperature</td>
<td>α_w</td>
<td>ALW</td>
<td>- 5.8E-5</td>
</tr>
<tr>
<td>Void</td>
<td>α_v</td>
<td>ALV</td>
<td>- 1.035E-4</td>
</tr>
<tr>
<td>Ratio of boiling heat source to coolant inlet source</td>
<td>ΔP/ΔH</td>
<td>PDH</td>
<td>5.0</td>
</tr>
<tr>
<td>Mathematical</td>
<td>Computer</td>
<td>Base case value</td>
<td>Units</td>
</tr>
<tr>
<td>--------------</td>
<td>----------</td>
<td>----------------</td>
<td>---------------</td>
</tr>
<tr>
<td>Detector efficiency for power flux</td>
<td>EP1</td>
<td>EP1(3)</td>
<td>1.58E-6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>EP1(6)</td>
<td>2.2E-6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>EP1(9)</td>
<td>1.58E-6</td>
</tr>
<tr>
<td>Detector efficiency for void thermalization flux</td>
<td>EP2</td>
<td>EP2(3)</td>
<td>- 2.75</td>
</tr>
<tr>
<td></td>
<td></td>
<td>EP2(6)</td>
<td>- 1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>EP2(9)</td>
<td>- 0.3</td>
</tr>
<tr>
<td>Detector location</td>
<td>x</td>
<td>XI</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>9</td>
</tr>
</tbody>
</table>

Intermediate calculated parameters of importance

| Water velocity | \( \dot{W}^0 \) | \( W \) | 6.072 | ft/s |
| Steam velocity | \( \dot{U}^0(L) \) | \( U \) | 6.241 | ft/s |
| Boiling boundary | \( L_{nb} \) | \( LNB \) | 2.96 | ft |
| Boiling fuel time constant | \( \tau_{fb} \) | \( TAUFB \) | 6.446 | s |
| Nonboiling fuel time constant | \( \tau_{fnb} \) | \( TAUFNB \) | 7.233 | s |
Table 6.2. Study case variations from base condition

<table>
<thead>
<tr>
<th>Study case</th>
<th>Parameters modified</th>
<th>Base case value</th>
<th>New value</th>
<th>Intermediate values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>PDH</td>
<td>5.0</td>
<td>0.5</td>
<td>No change</td>
</tr>
<tr>
<td>2</td>
<td>FP</td>
<td>0.9</td>
<td>0.75</td>
<td>Lnb = 2.70 ft</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>W0 = 4.37 ft/s</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>U0(L) = 4.745 ft/s</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Tfb = 6.45</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>fnb = 7.48</td>
</tr>
<tr>
<td>3</td>
<td>EP1, EP2 for each</td>
<td>Based on Table D.1 (1)</td>
<td>Based on Table D.1 (2)</td>
<td>No change</td>
</tr>
<tr>
<td></td>
<td>detector position</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>ADL(1) (WET(1))</td>
<td>+ 3.0 (0.5)</td>
<td>+ 4.0 (1.0)</td>
<td>Lnb = 5.94 ft</td>
</tr>
<tr>
<td></td>
<td>ADL(2) (WET(2))</td>
<td>- 3.0 (0.5)</td>
<td>0 (0)</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>ADL(1) (WET(1))</td>
<td>+ 3.0 (0.5)</td>
<td>- 4.0 (1.0)</td>
<td>Lnb = 1.76 ft</td>
</tr>
<tr>
<td></td>
<td>ADL(2) (WET(2))</td>
<td>- 3.0 (0.5)</td>
<td>0 (0)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PDH</td>
<td>5.0</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>ADL(1)</td>
<td>+ 3.0</td>
<td>+ 4.0</td>
<td>Lnb = 2.69 ft</td>
</tr>
<tr>
<td></td>
<td>ADL(2)</td>
<td>- 3.0</td>
<td>- 4.0</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>ADL(1)</td>
<td>+ 3.0</td>
<td>+ 4.0</td>
<td>Lnb = 2.69 ft</td>
</tr>
<tr>
<td></td>
<td>ADL(2)</td>
<td>- 3.0</td>
<td>- 4.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>EP1, EP2</td>
<td>Based on Table D.1 (1)</td>
<td>Based on Table D.1 (3)</td>
<td></td>
</tr>
<tr>
<td>7A</td>
<td>ΔP/ΔH</td>
<td>5.0</td>
<td>50.</td>
<td>Lnb = 2.69</td>
</tr>
<tr>
<td>7B</td>
<td>ΔP/ΔH</td>
<td>5.0</td>
<td>0.01</td>
<td>Lnb = 2.69</td>
</tr>
</tbody>
</table>
frequency range 0.1 to 10 Hz, the temperature feedback effects are small.

The low frequency response of $G_{c1}(\omega)$ up to 2.0 Hz is dominated by the power to void effects. The real part of the power to void transfer function, $(PV)(s)$, changes sign and goes through magnitude oscillations while the imaginary part decreases slowly as frequency is increased. The transport lag function, $1 - 0.5e^{-j\omega \cdot 0.63}$, which causes this oscillatory type of behavior is presented in Figure F.1. When the real part of $(PV)(s)$ becomes a large negative number relative to the imaginary part (approaches a 180° phase shift), reinforcement occurs in the feedback loop. This phenomenon is termed the hydrodynamic phase shift peak [10]. The peak characteristics of $G_{c1}(\omega)$, which have been measured in many BWR's, are then a function of axial power distribution, core flow velocity, and steam bubble residence time. Characteristics of the hydrodynamic peak in $G_{c1}(\omega)$ for the study case are presented in Table 6.3.

The values of $\alpha_v$ and $\alpha_w$ used are taken as 50 percent of the published values to account for flux weighting, since the worth of a void or temperature change is proportional to the flux distribution. Slight modifications to measured reactivity coefficient values can be used to fit the model dynamic behavior to the measured behavior of a particular system. In this model the total feedback is dominated by the power to void loop where $\alpha_v$ is the feedback coefficient.
Table 6.3. Changes in hydrodynamic phase shift peak of the closed loop reactor transfer function \(G_c(\omega)\) for the study cases

<table>
<thead>
<tr>
<th></th>
<th>Peak magnitude (Btu/s)</th>
<th>Peak freq. (Hz)</th>
<th>Frequency bandwidth at 0.9 X peak magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base case, 1 and 2</td>
<td>3.77E8</td>
<td>0.20</td>
<td>0.064</td>
</tr>
<tr>
<td>Case 2 (B) power 0.75</td>
<td>2.50E8</td>
<td>0.20</td>
<td>0.085</td>
</tr>
<tr>
<td>Case 4 (D) (a \cdot L = 4.0)</td>
<td>4.33E8</td>
<td>0.17</td>
<td>0.058</td>
</tr>
<tr>
<td>Case 5 (a \cdot L = -4.0)</td>
<td>3.64E8</td>
<td>0.24</td>
<td>0.062</td>
</tr>
<tr>
<td>Case 6 and 7 (a \cdot L = \pm 4.0)</td>
<td>4.70E8</td>
<td>0.19</td>
<td>0.045</td>
</tr>
</tbody>
</table>

B. Summing Junction Response

Reactivity driving forces arising from the "white" noise heat input sources appear as inputs to \(G_c(\omega)\) with a PSD equivalent to the square modulus of the transmission path transfer function. Since the noise sources are viewed as heat inputs, it is important to consider the heat from reactor power changes returned to the input point. The ratio of heat appearing at the summing junction from the reactor power fluctuations to the noise source heat input is very large around the 0.2 Hz position. The largest magnitude of the ratio is slightly less than 2.0 for all cases. However, the magnitude of the real part is negative and cancels the input vector. Thus for a unit input the resulting loop is stable by itself. This peak occurs at a frequency
of 0.2 Hz in the cases studied, and is viewed as the predominant PSD shaping factor in the low frequency range. The power response at the summing junction is increased by increasing the steady state power, moving the heat distribution from the top to the bottom of the core, and decreasing the power distribution damping factor $|a|$. It is also proportional to the void feedback coefficient. At frequencies greater than .6 Hz, the reactor power response at the summing junction becomes negligible, and the boiling heat input source becomes identical to Nomura's model [15]. This means that the heat feedback from the reactor can be neglected at higher frequencies.

C. Detector Response

Now consider the characteristics of the detector response signatures. Examples of the curves for three detector locations are given in Figures 6.12 through 6.14. To aid in the interpretation of the results, the general characteristics of the detector response are considered. Figure 6.1 shows these characteristics and defines the terms used in the description of them.

The shape of the first peak around 0.2 Hz appearing in the detector response curves results from the second order behavior of the denominators in the system transfer functions. Insight into the system behavior is gained by determining the natural frequency from the characteristic denominator of $PV(s)$ [53]. When $s^2 + 2aUs + u^2(a^2 + b^2) = s^2 + \frac{2\zeta_{n} \omega_n s + \omega_n^2}{\omega_n^2}$, the natural frequency $\omega_n = \sqrt{\frac{u^2(a^2 + b^2)}{\zeta_{n} \omega_n}} = (6.24) \cdot \sqrt{(0.25)^2 + (3.14/12.)^2} = 2.26 \text{ rad/s (0.359 Hz)}$ for the base case.
Also, the damping ratio, \( \zeta = \frac{a}{\sqrt{a^2 + b^2}} = 0.25/\sqrt{(0.25)^2 + (0.26)^2} = 0.69. \)

Because of the sums of axial power distribution functions, feedback loops, and multiple second order poles involved, the exact system damping ratio cannot be calculated. However, an effective damping factor is determined from the ratio of the peak frequency to the natural frequency. This damping factor is used to estimate the degree of hydrodynamic stability. Consider the following definitions:
Figure 6.2. Sketch of the detector response changes due to variations of axial power distribution parameter $|a|$

1. $\omega_{n1} = \sqrt{\frac{c}{b^2}} = \frac{\text{Im}}{L}$, and

$$f_{n1} = \frac{U}{2L}$$

2. $\omega_{n2} = U \sqrt{a^2 + b^2}$, and

$$f_{n2} = \frac{U}{2\pi} \sqrt{a^2 + b^2}$$

3. $\zeta_{e1} = \left[\frac{1 - (f_{pl}/f_{n1})}{2}\right]^{1/2}$

4. $\zeta_{e2} = \left[\frac{1 - (f_{pl}/f_{n2})}{2}\right]^{1/2}$
Utilizing these definitions, the peak value in the detector frequency spectrum, the core steam velocity, and flux shaping parameter used in each case, qualitative conclusions about the system stability can be made. When the values of $\zeta_{e1}$ and $\zeta_{e2}$ approach zero the hydrodynamic stability is reduced. From Table 6.4 it can be seen that when the core power or the flux shape parameter $|a|$ is reduced $\zeta_{e1}$ and $\zeta_{e2}$ are also reduced. These qualitative results agree with the general core design criteria used by General Electric [10].

Information about the system behavior can also be determined from the second peak. This peak results from a delay function of the form described in Appendix F. In real systems the continued oscillations of the delay behavior are damped out due to the combination of slightly different steam transit times from each channel. The model enhances the oscillatory behavior by considering only one time constant for the entire region. Comparison of results indicates that a shift in the delay peak is proportional to a change in the steam bubble velocity.

Comparisons of case 2 with the base case, indicates that the frequency of the second peak is directly proportional to the steam velocity, while the first peak ratio is proportional to the square of the steam velocity. This supports the idea that the first peak results from the second order denominator effect, while the second results from a transport lag function of the form $1 - Ae^{-St}$, where $\tau$ results from a distance divided by the operating steam bubble velocity. Change in the magnitude of $A$, which is a function of power distribution shaping parameters and $L_{nb}$, can shift the peak value of this delay when combined with a second order denominator (illustrated in Figure F.1). In other cases studied
Table 6.4. Detector response information based on test case data

<table>
<thead>
<tr>
<th>Detector position</th>
<th>fp₁</th>
<th>fv₁</th>
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<th>fv₂</th>
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Figure 6.3. Sketch of detector response changes due to changes in core bubble velocity (power).

The simple relationship is not determined for change due to the changed parameters. Comparing the inflection points in the library of detector response signals with lower frequency peaks reveals that the first oscillation may appear elsewhere as an inflection in the detector response. This suggests an interaction between the various transport lags in the system.

Six transport lags appear in the model from the transfer functions, $PV(s)$, $PB(s)$, $BV(s)$, $PH_w(s)$, $BAI(s)$, and $PA2(s)$. However, only three transport lag time constants appear, $T_s = L_b U$ in $PV(s)$ and $BV(s)$, $T_w = L_{nb}/W$ in $PB(s)$ and $PH_w(s)$, and $\tau_x = (x - L_{nb})/U$ in $BAI(s)$ and
PA2(s) where x is the detector position in the channel. Although exact identification of the dominant time lag function causing the peak is difficult, a simple estimate can be made by use of Figure F.1 and the frequency differences of the valley frequencies. The effective time lag is then

\[ \tau_e = 0.63 \cdot \left\lfloor \frac{10}{2\pi(f_{v2} - f_{v1})} \right\rfloor \]

For the cases considered, the dominant time lag constant appears to be the steam transit time \( T_s \), however, evidence of this dominance does not always appear at the same detector position. The position of the delay peak is very sensitive to the values of \( EP1, EP2, \Delta P \) and \( \Delta H \). These are the parameters used to combine the various transmission path transfer functions. For any particular reactor these model parameters are not well known. Test cases were used to determine the detector response sensitivity to values of these parameters. Results indicate that space dependent changes in the detector response occur when \( EP1/EP2 < 1.E-6 \). However, this space dependent sensitivity is also a function of the void feedback coefficient. Decreasing the magnitude of \( \alpha_v \) increases the upper limit on the sensitivity region.

Another interesting phenomenon affecting the delay function valleys occurs at the 6 foot detector position. In several cases a large dip in the magnitude occurs at about 1 Hz. Similar behavior was experimentally observed by Seifritz [33]. Model data show this
to result from a cancellation of the detector void response with the power response at the middle core detector position.

Modeled positions of in-core detectors

\[ \varphi_{9-6} = \varphi_0 - 57.3 \tau \cdot w \]

\[ \varphi_{6-3} = \varphi_0 - 57.3 \tau \cdot w \]

\[ \tau = \frac{\Delta \varphi}{U^0} = \frac{3.0}{6.24} = 0.408 \text{ s} \]

\[ U^0 = 6.24 \text{ ft/s} \]

\[ \varphi(9) \quad - \quad 9 \text{ ft} \]

\[ \varphi(6) \quad - \quad 6 \text{ ft} \]

\[ \varphi(3) \quad - \quad 3 \text{ ft} \]

\[ \text{Active core length} \quad \uparrow \quad 0 \]

\[ 12 \text{ ft} \]

\[ 3 \text{ ft} \]

\[ 6 \text{ ft} \]

\[ 3 \text{ ft} \]

Figure 6.4. Phase shift functional model to check cross power spectral density measurements (see Appendix C)

Utilizing the results presented by Stegemann et al. [36], and repeated in Appendix C, the phase difference of the detector responses at various positions is considered here. The phase difference between detectors at different locations can be measured by cross correlation methods [13, 17]. Figure 6.5 displays behavior of this difference for the study cases. The equation line is based on the known steam velocity and the distance between detectors. For very low frequencies the phase difference follows the equation line in all cases where the detector responded partially to voids. However, the phase difference for the detector void response driven by \( \Delta H \), decreased in agreement with the phase lag equation for all frequencies. The phase difference
agreement for the detector outputs when driven by ΔP was less than 1 Hz, although oscillations in the phase difference occurred at higher frequencies. The approach to a constant phase difference is emblematic of the space independent reactor model. A phase difference of zero would result if detectors responses only to the fission power changes in this model. The case studies indicate that the phase difference behavior is very sensitive to the values of EP1, EP2, P and H, as shown in Figure 6.5. Three detector response curves at each location
from case 7 are included as examples of the combined responses obtained with NMOD when the detector response to both power and void fluctuations are considered. Five other signatures are included (Figures 6.7 through 6.11) which represent the detector response at the upper position for each path leading to the detector model. These signatures result from case 7 with the detector location at 9 feet. The general characteristics in these signatures are similar to the other cases studied, and are presented to show the effect of each noise source on both the detector and fission power responses. The combination of these three inputs leads to the total detector response of Figure 6.14.

Because of the sensitivity of the detector response signatures to the values of EP1 and EP2, the overall response is best viewed as the combination of square modulus transmission transfer functions. The values of the weighting factors, ΔP, ΔH, EP1 and EP2 are functions of various plant operating conditions and are not well known. However, the various transmission functions derived here point to the important experimental parameters to be considered during measurements, and can aid in the interpretation of PSD measurements.

The model results do give insight into the performance of in-core detectors. This knowledge provides a basis for the development of surveillance systems which utilize the information contained in fluctuating noise signals to monitor reactor parameters. Determination of changes in the behavior of each parameter could then lead to the detection of anomalous conditions in the reactor system.
Figure 6.6. Reactor feedback model \( \left( G_{cl}(\omega) \right) \) [\( \frac{\text{Btu/s}}{\Delta k/k} \)]
TDPE1 power response due to boiling source – case 7

Figure 6.7. Transmission path TDPE1
Transmission path TDHE1 power response due to subcooled source - case 7

Figure 6.8. Transmission path TDHE1 [Btu/s]
Figure 6.9. Combined transmission path to power response \( \frac{\text{Btu/s}}{\text{Btu/s}} \)

\[
TDPEL \cdot \frac{\Delta p}{\Delta H} + TDHEL
\]
Figure 6.10. Transmission path TDPE2
Figure 6.11. Transmission path TDHE2

TDHE2 transmission path from subcooled source to detector response at 9 ft – case 7
Detector response at 3 ft — case 7

Figure 6.12. Typical combined detector response at 3 ft
Figure 6.13. Typical combined detector response at 6 ft
Figure 6.14. Typical combined detector response at 9 ft
VII. CONCLUSIONS

There exist a number of fluctuating variables in a BWR such as neutron flux, pressure, flow, void fraction, and temperature. Assuming that the source of these fluctuations arises from stochastic variations in the heat input to the channel leads one to the realization that these fluctuations are not independent, but they are related in some manner to the transmission processes within a BWR core. Certain simplifying assumptions have been adopted in the development of a model of these processes in order to maintain physical meaning of the system parameters. This permits interpretation of the detector response in terms of the physical processes causing the noise. The results of the present model substantiate the experimental results presented by Seifritz [33] and Stegemann et al. [36]. However, the detector response due to voids should be increased relative to the power response in the model. This will increase the frequency range of agreement between the two detectors' phase difference and the steam velocity time lag equation.

The large neutron fluctuations in a BWR can be regarded as a merit rather than a detriment, since in actual practice, the reactor is very stable despite the presence of inherent disturbances which can affect reactivity. The fluctuating detector signal can then provide continuous information about the performance of the system without applying external disturbances.

The important results of this investigation may be qualitatively summarized as follows:
1. It has been shown that space dependent shapes of neutron detector PSD's result partially from direct void effects on the in-core detector response. This behavior is evident in the model even though the space independent reactor feedback model is used to describe reactor power changes.

2. Two important inherent fluctuating heat sources have been identified for the frequency 0.1 to 10. Hz. These sources appear as changes in the heat delivered to the coolant. However, the relative magnitudes of the sources depends on the conditions of the particular reactor system under study.

3. In-core detectors responding to void thermalization effects are good candidates for signal sources in a diagnostic surveillance system, since the auto power spectral densities from their signals contain system operating information. Two types of information can be determined from these signals. First, subtle changes in the normal PSD's give information about the reactor operating parameters which have been described in this work. Second, new frequency peaks appearing in the PSD's indicate the onset of anomalous reactivity conditions such as vibrating control rods, fuel elements, or structural material, which have been described elsewhere [1, 2, 28, 29, 30, 31, 32].
VIII. SUGGESTIONS FOR FUTURE WORK

Considering the objective of developing an on-line surveillance system, the present work provides a basis for several projects which could lead to the design of such a system.

1. Experimental work could be undertaken to verify the thermalization response of in-core detectors. Since this behavior depends on the sensitivity ratio of fast to thermal neutron detectors, various detector materials should be studied.

2. Experimental verification of the internal transfer functions existing within power plants should be carried out. Kerlin [63] has suggested methods of utilizing a multifrequency binary signal on input variables to measure system transfer functions.

3. A multigroup neutron energy system could be utilized to study the void thermalization effect along the voided channel of a BWR. The shift in the neutron energy spectrum along the channel could be considered in detail. This would lead to better values of $\text{EP2}(x)$.

4. Improvements in the model could be made by including random pressure effects.

5. Anomalous reactivity inputs could be derived and the signatures of these inputs on the detector response could be modeled.

6. Based on transfer functions presented in this work, a filtering algorithm [64, 65] could be developed to estimate the time dependent variables from the modeled system. Such an algorithm could be used in a simple on-line monitoring system, by identifying changes in important variables, such as reactivity.
IX. LITERATURE CITED


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40. General Electric Co. 1968. Design and performance of General Electric Boiling water reactor jet pumps. APED-5460. (General Electric Atomic Power Equipment Division, San Jose, Calif.)


X. ACKNOWLEDGMENTS

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XI. APPENDIX A:
DERIVATION OF THE FUNDAMENTAL EQUATION OF MODERATOR DYNAMICS

It is the purpose of this Appendix to show how the coolant dynamics equation

\[ U \frac{\partial}{\partial x} a_s(x, t) + \frac{\partial}{\partial t} a_s(x, t) = \frac{q(x, t)}{H_v} - \frac{1}{H_v} \left[ \rho_s A \frac{dh_s}{dp} \right] \frac{dp}{dt} + \rho_w A_w \frac{dh_w}{dp} \frac{dp}{dt}, \]

A.1

is derived. This is accomplished by applying the basic conservation equations to the reactor coolant channel configuration.

First, consider the conservation of mass in a small volume along the coolant flow path shown in Figure A.1.

Figure A.1. The mass balance of coolant in a small volume of a coolant channel.
The rate coolant mass flows into the control volume equals the rate coolant mass leaves the control volume plus the mass stored in the control volume.

\[ \dot{M}_{\text{in}} + \dot{M}_{\text{out}} = \left( \dot{M}_{\text{in}} + \frac{\partial M}{\partial t} \right) + \left( \dot{M}_{\text{out}} + \frac{\partial M}{\partial t} \right) + \frac{\partial}{\partial t} (dM_w + dM_s) \]

A.2

where

\[ \dot{M}_s = A_s(x, t) \cdot \rho_s \cdot U(x, t), \quad dM_s = A_s \cdot \rho_s \cdot \Delta x \]

and

\[ \dot{M}_w = A_w(x, t) \cdot \rho_w \cdot W(x, t), \quad dM_w = A_w \cdot \rho_w \cdot dx. \]

Substitution of these quantities into Equation A.2 and solving for the time dependent term yields

\[ - \frac{\partial}{\partial t} \left[ (A_s \rho_s \Delta x + A_w \rho_w \Delta x) \right] = \frac{\partial}{\partial x} \left[ A_s \rho_s U \right] \Delta x + \frac{\partial}{\partial x} \left[ A_w \rho_w W \right] \Delta x \]

A.3

Dividing both sides by \( \Delta x \) gives

\[ \frac{\partial}{\partial t} \left[ A_s \rho_s + A_w \rho_w \right] + \frac{\partial}{\partial x} \left[ A_s \rho_s U + A_w \rho_w W \right] = 0. \]

A.4

Equation A.4 is the continuity equation along any coolant channel, or combination of coolant channels when \( A_s \) plus \( A_w \) is the total core cross sectional area.

Next, apply the conservation of energy equation to the differential heated volume of the channel shown in Figure A.2.

The energy of the fluid heated externally along the channel at time \( t \) is the sum of its internal, kinetic, and potential energies. The heat supplied by the fuel to the control volume per unit channel
Figure A.2. Small volume of a coolant channel used to determine the energy in the coolant.

length is $Q(x)$ with units of Btu/s·ft or watt/m. The heat added to the control volume during a time $dt$ is

$$Q(x, t) \cdot dx \cdot dt = \frac{\partial E}{\partial t} \bigg|_{x_o}^{x_o + \Delta x} dx + \frac{\partial E}{\partial x} \bigg|_{t_0}^{t_0 + \Delta t} dt,$$

where $E$ is the total energy of the fluid in the control volume.

For small increments of time and position about $t_o$ and $x_o$ respectively, Equation A.5 becomes

$$Q(x, t) \Delta x \Delta t = \left[ \frac{E(x_o, t + \Delta t) - E(x_o, t)}{\Delta t} \right] \Delta t$$

$$+ \left[ \frac{E(x + \Delta x, t_o) - E(x, t_o)}{\Delta x} \right] \Delta x.$$

The mass of fluid flowing past $x$ in the time interval $\Delta t$ is
\[ \Delta M_t(x, t_o) = [\rho_s A_u + \rho_w A_w](x, t_o) \Delta t, \quad A.7 \]

and the mass flowing past \( x + \Delta x \) in the interval \( \Delta t \) is

\[ \Delta M_t(x + \Delta x, t_o) = [\rho_s A_u + \rho_w A_w](x + \Delta x, t_o) \Delta t. \quad A.8 \]

Similarly the total mass within the control volume at times \( t \) and \( t + \Delta t \) are

\[ \Delta M_t(x, t) = [\rho_s A_u + \rho_w A_w](x, t) \Delta x \quad A.9 \]

and

\[ \Delta M_t(x, t + \Delta t) = [\rho_s A_u + \rho_w A_w](x, t + \Delta t) \Delta x. \quad A.10 \]

The energy of the fluid is the sum of potential, kinetic, and internal energy. However, the kinetic and potential energy contributions are very small compared to the internal energy in a BWR and are neglected in this model. The change in potential energy from bottom to top of core is \( x/A_m = 12 \text{ ft}/778. \text{ ft-lb/Btu} = \frac{0.015 \text{ Btu}}{\text{lb}} \), and the contribution from kinetic energy is \( \frac{W^2}{2} \cdot A_m \cdot g = (5 \text{ f/s})^2/2 \cdot 778 \cdot 32.2 \) \( = 0.0005 \text{ Btu/lb} \). The internal energy is defined as

\[ U_{lw} \equiv H_w - \frac{P}{A_m \rho_w} \quad \text{and} \quad U_{ls} \equiv H_s - \frac{P}{A_m \rho_s}. \quad A.11 \]

The major contribution to the internal energy in BWR's comes from the enthalpy term. At operating conditions

\[ \frac{P}{H_w \rho_w A_m} = 0.0075 \quad \text{and} \quad \frac{P}{H_s \rho_s A_m} = 0.069. \]

By neglecting contributions from the potential, kinetic, and pressure effects on the internal energy, the energy in the time interval \( \Delta t \) at \( x \) and \( x + \Delta x \) becomes
\[ E(x, t_0) = [\rho_w \cdot A_w \cdot \bar{H}_w + \rho_s \cdot A_s \cdot \bar{U}_s \cdot \bar{H}_s](x, t_0) \cdot \Delta t, \quad A.12 \]

and

\[ E(x + \Delta x, t_0) = [\rho_w \cdot A_w \cdot \bar{H}_w + \rho_s \cdot A_s \cdot \bar{U}_s \cdot \bar{H}_s](x + \Delta x, t_0) \cdot \Delta t. \quad A.13 \]

In the distance interval \( \Delta x \) the energy at \( t \) and \( t + \Delta t \) becomes

\[ E(x_o, t) = [\rho_w \cdot A_w \cdot \bar{H}_w + \rho_s \cdot A_s \cdot \bar{H}_s](x_o, t) \cdot \Delta x \quad A.14 \]

and

\[ E(x_o, t + \Delta t) = [\rho_w \cdot A_w \cdot \bar{H}_w + \rho_s \cdot A_s \cdot \bar{H}_s](x_o, t + \Delta t) \cdot \Delta x. \quad A.15 \]

Substituting Equations A.12 to A.14 into Equation A.6, taking the limit as \( x \) approaches \( x_o \), \( t \) approaches \( t_o \), \( \Delta t \) approaches zero, and \( \Delta x \) approaches zero yields

\[ Q(x, t)dx = \frac{\partial}{\partial t} [\rho_w \cdot A_w \cdot \bar{H}_w + \rho_s \cdot A_s \cdot \bar{H}_s]dx + \frac{\partial}{\partial x} [\rho_s \cdot A_s \cdot \bar{U}_s + \rho_w \cdot A_w \cdot \bar{W}_w]dx \quad [\text{Btu/s}]. \quad A.16 \]

The steam enthalpy at saturated conditions equals the latent heat of vaporization per lb plus the saturated water enthalpy which is written as

\[ H_s = H_w + H^i_v. \quad A.17 \]

Assuming that for constant saturation conditions \( \partial H_v^i/\partial t = \partial H_v^i/\partial x = 0 \), Equation A.17 yields

\[ \frac{\partial H_s}{\partial t} = \frac{\partial H_w}{\partial t} \quad \text{and} \quad \frac{\partial H_s}{\partial x} = \frac{\partial H_w}{\partial x}. \quad A.18 \]

Multiplying Equation A.6, the continuity equation, by \( H_w \) yields
where \( \rho_s \) and \( \rho_w \) are constants. Substitution of Equation A.19 into A.16 utilizing the definitions in Equation A.17 and A.18 then yields

\[
Q(x, t) = \rho_s H^I \frac{\partial A_s}{\partial x} + \rho_s H^S \frac{\partial A_s}{\partial t} + \rho_s H^S \frac{\partial A_s}{\partial t} + \rho_s A_s \frac{\partial H_s}{\partial t} + \rho_s A_s \frac{\partial H_s}{\partial t}.
\]

A.20

Let \( H_v = \rho_s H^I \) the latent heat of vaporization per unit volume of steam, and assume that the time dependent changes in enthalpy are functions of pressure changes such that

\[
\frac{\partial H_s}{\partial t} = \frac{dH_s}{dt} \cdot \frac{dP}{dt}.
\]

A.21

Rearranging Equation A.20 then gives

\[
\frac{\partial A_s}{\partial x} + \frac{\partial A_s}{\partial t} = Q(x, t) - \frac{1}{H_v} \left[ \rho_s A_s \frac{dH_s}{dP} + \rho_s A_s \frac{dH_s}{dP} \right] \frac{dP}{dt},
\]

A.22

which is the basic equation describing the behavior of steam and water in a boiling water core.

The following parameters are linearized about their steady state operating conditions

\[
A_s(x, t) = A_s^0(x) + a_s(x, t),
\]

\[
U(x, t) = U^0(x) + u(x, t),
\]

and

\[
Q(x, t) = Q^0(x) + q(x, t).
\]

A.23

Equation A.22 then becomes
when linearized with second order products neglected. To solve
Equation A.24 with a known spatial distribution for \( q(x, t) \) a relationship between \( u(x, t) \) and \( a_s(x, t) \) is needed. Jones [23] solves the
mass, energy, and momentum equations at each node along the channel,
however the nodal method makes simple linear transfer function analysis
more difficult. The individual transfer function identity is main­
tained by using a simple approximation of the momentum equation to
solve for the relationship
\[
A_s^0 u(x, t) + U^0 a_s(x, t) \approx U_p a_s(x, t).
\]

Define the ratio of fluid momentum at the channel inlet to the
momentum at position \( x \) as
\[
Y(x, t) = \frac{\rho_s A W + \rho_s A U}{\rho_s A W^0}.
\]  
A.25

Using average values for the densities and noting that the momentum
in the nonboiling region is nearly constant, the momentum ratio in
the boiling region becomes
\[
Y(x, t) = 1 - \frac{A_s(x, t) \cdot U(x, t)}{A_s^0 \cdot U^0 \cdot r(x, t)} + \frac{\overline{\rho} A_s(x, t) \cdot U(x, t)}{\overline{\rho} A_s^0 \cdot U^0},
\]  
A.26

where \( r(x, t) \) is the slip ratio. The slip ratio in high pressure force
circulation systems is almost one. The spatially dependent change
in momentum is then
\[
\frac{\partial Y(x, t)}{\partial x} \left( \frac{\rho_s A W^0}{\overline{\rho}} \right) = \frac{\partial}{\partial x} \left( A_s(x, t) \cdot U(x, t) \right).
\]  
A.27
\[
\frac{\partial Y}{\partial x} \cdot K = \frac{\partial}{\partial x} (A_s^0(x)u(x, t) + U^0(x)a_s(x, t)).
\]

Since the change in momentum is predominantly due to changes in the steam cross section, and changes in steam velocity are very small, the term \( A_s^0(x)u(x, t) + U^0(x)a_s(x, t) \) is approximated by \( U_p(x) a_s(x, t) \), where \( U_p(x) \) is the steam perturbation velocity and equals a spatially dependent factor, \( Y(x) \), times \( U^0 \)

\[
\frac{\partial Y}{\partial x} K = \frac{\partial}{\partial x} (Y(x) \cdot U^0 \cdot a_s(x, t)).
\]

Equation A.28

In the high pressure forced circulation BWR system used in power plant applications \( Y(x) \approx 1.0 \), thus changes in momentum along the channel are approximately proportional to changes in the steam cross section

\[
\frac{\partial Y}{\partial x} K \approx U^0 \frac{\partial}{\partial x} (a_s(x, t)).
\]

Equation A.29

Assuming that changes in the momentum are proportional to changes in the steam cross section Equation A.24 becomes

\[
U^0 \frac{\partial}{\partial x} a_s(x, t) + \frac{\partial}{\partial t} a_s(x, t) = \frac{q(x, t)}{H_v} - \frac{1}{H_v} \left\{ \rho_s A_s \frac{dH_s}{dP} + \rho_w A_w \frac{dH_w}{dP} \right\} \frac{dP}{dt}.
\]

Equation A.30

Equation A.30 is the moderator dynamics equation used to model the behavior of voids in the coolant channels of a boiling water reactor.
Consider a dynamic system with an input $x(t)$, an impulse response $h(t)$, and an output $y(t)$.

The output of the system is

$$y(t) = \int_{0}^{\infty} s(t - \lambda)h(\lambda)d\lambda$$  \hspace{1cm} (B.1)

as given by the convolution integral with $\lambda$ as the dummy of integration. Introducing the time delay variable $\tau$, and the expectation value of $y(t)$ times $y(t + \tau)$ yields the auto correlation function

$$\phi_{yy}(\tau) = E[y(t) \cdot y(t + \tau)]$$.  \hspace{1cm} (B.2)

Substitution of Equation B.1 into B.2, introducing $\xi$ as the dummy of integration in the second integral, and interchanging the order of integration yields

$$\phi_{yy}(\tau) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(\lambda)h(\xi) \cdot E[x(t - \lambda) \cdot x(t + \tau - \xi)]d\xi d\lambda$$.  \hspace{1cm} (B.3)

By definition of the auto correlation function

$$\phi_{xx}(\tau - \xi + \lambda) = E[x(t - \lambda) \cdot x(t + \tau - \xi)]$$.  \hspace{1cm} (B.4)

Then,
\[
\psi_{yy}(\tau) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} h(\lambda)h(\xi)\psi_{xx}(\tau - \xi + \lambda)d\xi d\lambda. \tag{B.5}
\]

Equation B.5 is quite complicated, but under certain assumptions this relationship is quite useful. When the input is white noise, 
\[
\psi_{xx}(\tau - \xi + \lambda) = K_0 \delta(\tau - \xi + \lambda), \quad \text{and} \quad h(t) = 0 \quad \text{for} \quad t < 0,
\]
Equation B.5 becomes 
\[
\psi_{yy}(\tau) = K_0 \int_{0}^{\infty} h(\lambda)h(\lambda - \tau)d\lambda. \tag{B.6}
\]

The input-output relationship in the frequency domain can be found by use of the Fourier integral transformation of Equation B.5

\[
\hat{\psi}_{yy}(\omega) = \int_{-\infty}^{+\infty} \psi_{yy}(\tau)e^{-j\omega \tau}d\tau \]

\[
= \int_{-\infty}^{+\infty} \left[ \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} h(\lambda)h(\xi)\psi_{xx}(\tau - \xi - \lambda)d\xi d\lambda \right]e^{-j\omega \tau}d\tau. \tag{B.7}
\]

Defining \( \mu = \tau - \xi + \lambda \) and eliminating the \( \tau \) variable gives

\[
\hat{\psi}_{yy}(\omega) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} h(\lambda)h(\xi)\psi_{xx}(\mu)e^{-j\omega(\xi - \lambda - \mu)}d\xi d\lambda d\mu \tag{B.8}
\]

\[
= \int_{-\infty}^{+\infty} h(\lambda)e^{j\omega \lambda}d\lambda \cdot \int_{-\infty}^{+\infty} h(\xi)e^{-j\omega(\xi - \lambda)}d\xi \cdot \int_{-\infty}^{+\infty} \psi_{xx}(\mu)e^{-j\omega \mu}d\mu
\]

\[
= H(+\omega) \cdot H(-\omega) \cdot \psi_{xx}(\omega) = |H(\omega)|^2 \psi_{xx}(\omega) \tag{B.9}
\]

where \( |H(\omega)|^2 \) is the square modulus of \( H(\omega) \), the system transfer function.
Equation B.9 is a very useful relationship, however only the amplitude of the transfer function $H(\omega)$ can be determined and no information with regard to the phase angle can be determined. In the case of white noise Equation B.9 becomes

$$\psi_{yy}(\omega) = K|H(\omega)|^2$$

Equation B.10

where $K$ is a constant over the frequency range of interest. In the case of multiple input systems, Equation B.1 becomes

$$y_i(t) = \int_{-\infty}^{\infty} h_i(\lambda) \cdot x_i(t - \lambda) d\lambda.$$ B.11

Transformed into the frequency domain Equation B.11 becomes

$$Y_i(\omega) = H_i(\omega)x_i(\omega).$$ B.12

In case of a linear system the total output $Y_i(\omega)$ is the sum of contributions from all the inputs.

$$Y(\omega) = \sum_{i=1}^{N} Y_i(\omega) = \sum_{i=1}^{N} H_i(\omega)x_i(\omega).$$ B.13

After following the same procedures outlined for a single input-output system, the power spectral density for the multiple input system becomes

$$\psi_{yy}(\omega) = \sum_{i=1}^{N} \sum_{k=1}^{N} H_i(-\omega) \cdot H_k(\omega)\psi_{ik}(\omega)$$ B.14

where $\psi_{ik}(\omega)$ represents the cross spectral density function between inputs $x_i(t)$ and $x_k(t)$. For the case considered in this work the two input sources are uncorrelated, which means that $i = k$ and Equation B.14 reduces to
\[ \psi_{yy}(\omega) = \sum_{i=1}^{N} |H_{i}(\omega)|^2 \psi_{ii}(\omega). \]

The square modulus, \(|H_{i}(\omega)|^2\), weighted by the relative strength of the \(\psi_{ii}(\omega)\)'s is termed as the square modulus of the detector for this work, and represents the neutron noise signature.
The use of neutron detector signal fluctuations to measure reactor parameters has been used on some reactors where the proper electronic analysis equipment was available. This section includes the details of experiments on actual reactors related to these measurements.

Information from two papers describing the experimental work \cite{33, 36} performed on the 520 MW$_{th}$ Lingen BWR is of particular interest to this work, and is repeated here. Figure C.1 shows the experimental set up and the associated electronic equipment used to perform the in core noise measurements. Probability density function measurements at the core center of the time dependent detector signal fluctuation signify that the wave form is Gaussian and random. This type of wave form indicates that only linear processes are involved in the heat transfer mechanism and in reactivity fluctuations driving the neutron oscillations. Figure C.2 shows a plot of the mean detector current at various axial positions, which represents the axial flux shape along the channel. This general shape could be described by the sum of two weighted shaping functions of the form $e^{ax} \cdot \sin(bx + c)$.

The change of the normalized rms noise amplitude along the channel length indicates that spatial dependent effects of the noise signal must be considered in the description of neutron detector noise. Fourier transforms of the measured autocorrelation functions result in measured power spectral density functions at different
locations along the channel length. Figure C.4 shows the PSD’s at three locations in the heated channel.

Stegemann et al. [36] present additional experimental information from Lingen and Garigliano based on the use of self powered in-core detectors. The significant discovery is that the cross power spectral density phase change from two detectors located at different positions along the channel could be used to determine the velocity of steam bubbles. The slope of the phase change is related to the transport lag of steam bubbles at the two detector positions in the same channel. When the distance between detectors is known the velocity can be calculated by the method shown in Figure C.5. The value determined by the cross power spectral density method was in good agreement with the values from thermohydraulic core design. Also, zero phase shift was observed for detectors at the same channel positions radially separated. Figures C.6 and C.7 show the results of the measurements.
Figure C.1. Experimental equipment set up used to measure the neutron flux at different positions in the coolant channel of the Lingen BWR [33]

Figure C.2. Mean detector current at eleven axial positions, which represents the axial power distribution along the channel [33]
Figure C.3. Change in normalized RMS-noise along the channel positions [33]

Figure C.4. Measured PDS changes with axial detector position [33]
Figure C.5. Method for evaluation of local average steam bubble velocity based on phase shift of the cross power spectral density function of two in-core detectors [36]
Figure C.6. Results of cross power spectral density noise measurements in the BWR core of Garigliano Nuclear Power Plant [36]
Figure C.7. In-core neutron PSD function measured in Lingen BWR core [36]
XIV. APPENDIX D:
VOID THERMALIZATION EFFECT

It is the purpose here to determine constants which relate the local neutron density changes to changes in void fraction flux.

Consider the following simplified method of determining values for EP2: A detector is placed in a boiling channel where the steady state void fraction has a mean value. Based on the mean value a steady state ratio between the fast and thermal flux is established. In a small region, subject to local voidage changes, near the detector the thermal flux is calculated as a function of the initial fast to thermal ratio. Slab geometry is utilized to simplify the calculation.

The two group diffusion equations describing the static flux shape in the slab region shown in Figure D.1 are

\[
\frac{d^2}{dx^2} \phi_1(x) - \frac{\Sigma_1}{D_1} \phi_1(x) = 0 \tag{D.1}
\]

Figure D.1. Slab model of thermal neutron detector in a voided coolant channel subject to given fast to thermal neutron flux at the boundary \( \pm d \)
\[ \frac{d^2}{dx^2} \phi_2(x) - \frac{\Sigma_2}{D_2} \phi_2(x) + \frac{\Sigma_1}{D_2} \phi_1(x) = 0. \quad \text{D.2} \]

A solution of Equation D.2 is

\[ \phi_1 = A \cosh \kappa_1 x \quad \text{where} \quad \kappa_1^2 = \frac{\Sigma_1}{D_1} \quad \text{D.3} \]

and

\[ A = \frac{\phi^0_1}{\cosh \kappa_1 d} \]

Equation D.2 becomes

\[ \frac{d^2}{dx^2} \phi_2 - \kappa_2^2 \phi_2 - \frac{\Sigma_1}{D_2} \cdot \frac{\phi^0_1}{\cosh \kappa_1 d} \cosh \kappa_1 x. \quad \text{D.4} \]

The solution of D.4, rejecting unqualified solutions on physical grounds, \( \phi_2(x) = A' \cosh \kappa_2 x - \gamma A \cosh \kappa_1 x. \) Solving for \( \gamma \) and \( A' \) by substitution into Equation D.6 and use of boundary condition \( \phi_2(\pm d) = \phi^0_2 \) yields

\[ \phi_2(x) = \frac{\phi^0_2 + \phi^0_1 \left( \frac{\Sigma_1/D_2}{\kappa_2^2 - \kappa_1^2} \right)}{\cosh \kappa_2 d} \cdot \cosh \kappa_2 x - \frac{\Sigma_1/D_2}{\kappa_2^2 - \kappa_1^2} \cdot \frac{\phi^0_1}{\cosh \kappa_1 d} \cosh \kappa_1 x. \quad \text{D.5} \]

The following two group constants for H₂O with no voids are utilized in this development [44].

<table>
<thead>
<tr>
<th>Group</th>
<th>( \Sigma ) (Group removal) ( \text{cm}^{-1} )</th>
<th>( D ) (Diffusion length) ( \text{cm} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1</td>
<td>0.0419</td>
<td>1.13</td>
</tr>
<tr>
<td>Group 2</td>
<td>0.0197</td>
<td>0.16</td>
</tr>
</tbody>
</table>
Noting that $\Sigma_v \equiv \Sigma_o \cdot f_w$ and $D \equiv D_o \cdot 1/f_w$ where $f_w$ is the fraction of water in the channel. Equation D.5 at the center of the channel ($x = 0$) becomes

$$\phi_2(0) = \frac{\phi_2^o - 3.04 \phi_1^o}{\cosh \kappa_2 d} + \frac{3.04 \phi_1^o}{\cosh \kappa_1 d}.$$  \hspace{1cm} \text{D.6}

If the ratio of fast to thermal is $\phi_2^o = 1.5 \phi_1^o$, Equation D.6 becomes

$$\phi_2(0) = \phi_2^o \left\{ \frac{-3.56}{\cosh \kappa_2 d} + \frac{4.56}{\cosh \kappa_1 d} \right\}.$$  \hspace{1cm} \text{D.7}

The change of $\phi_2^o/\phi_1^o$ along the channel due to the change in steady state void fraction has been neglected for simplicity. Figure D.2 shows the

![Figure D.2. Ratio of thermal flux at the detector location to thermal flux at the slab channel model boundary for several slab dimensions](image-url)
ratio of the thermal flux at \( x = d \) for various steady state void fractions and boundary conditions. Decreasing the ratio \( \frac{\phi_2^0}{\phi_1^0} \) increases the slope of the curves as shown in Figure D.3. Values of the ratio of thermal flux at the detector position are also calculated for \( \frac{\phi_0^2}{\phi_0^1} \) ratios of 0.1 and 0.2 at \( d = 1 \text{ cm} \). Small local changes in the void fraction would then cause fluctuations of the thermal flux at the detector location. This change can be estimated by taking the slope of a curve in Figure D.2 or Figure D.3. For \( d = 1 \text{ cm} \), and a slow to thermal ratio of 0.67 the slope at 0.35 void is

\[
\text{Slope} = \frac{\phi_0^0 - \phi_1^0}{\phi_0^0} \frac{\phi_0^0}{\phi_0^1} = -0.43 \frac{\phi_0^0}{\phi_0^1}.
\]

At maximum power \( \bar{\phi}_o = 5 \times 10^{13} \text{ n/cm}^2 \cdot \text{ sec} \) and the total void volume is estimated from

\[
\frac{(L - L_{nb}) \cdot A_s^0(x)}{2A_c} \times A_c = \frac{8.623 \times 0.76 \times 44.2}{2.0} = 144.8 \text{ ft}^3.
\]

Then \( \Delta\phi_v = -1.48E11 \Delta V \text{ [ft}^3] \), and recall that \( \Delta\phi = 3.35E7 \cdot \Delta P \text{ [Btu/s]} \).

The ratio of detector efficiencies is then

\[
\frac{EP_1}{EP_2} = \frac{3.35E7 \times AFF \times RPF}{1.48E11 \times \text{slope}} \cdot \left[1 \frac{\text{Btu/s}}{1 \text{/ft}^3}\right].
\]

\[
= \frac{(9.65E-5)(PF)}{\text{Slope}}.
\]

Thus, for a void fraction of 0.35 (about 6.0 feet up the channel) the ratio \( EP_1/EP_2 = -2.2E-4 \cdot \frac{\text{Btu/s}}{1 \text{/ft}^3} \). The ratios at other positions can be determined by considering the slope at a void fraction corresponding to the new channel height. At 3.0 feet the void fraction is very small, while at 9.0 feet it is slightly less than the exit
Figure D.3. Ratio of thermal flux at the detector location to thermal flux at the slab channel model boundary for several thermal to fast flux ratios.

As the ratio EP1/EP2 decreases, a larger contribution to the detector response arises from void effects. Due to the uncertainties in the values of EP2 at various locations the weighted combinations of
Table D.1. Methods of weighting void thermalization to fission power fluctuations

<table>
<thead>
<tr>
<th>Position</th>
<th>$\phi_2/\phi_1$</th>
<th>Flux weighting</th>
<th>Slope of Figure</th>
<th>Slope normalization to 6 ft</th>
<th>EP1/EP2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base case</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Method 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.67</td>
<td>0.707</td>
<td>- 1.18</td>
<td>2.75</td>
<td>- 5.7E-5</td>
</tr>
<tr>
<td>6</td>
<td>0.67</td>
<td>1.0</td>
<td>- 0.43</td>
<td>1.0</td>
<td>- 2.2E-4</td>
</tr>
<tr>
<td>9</td>
<td>0.67</td>
<td>0.707</td>
<td>- 0.13</td>
<td>0.3</td>
<td>- 5.3E-4</td>
</tr>
<tr>
<td>Case 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Method 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.1</td>
<td>0.707</td>
<td>- 1.87</td>
<td>1.0</td>
<td>- 3.6E-5</td>
</tr>
<tr>
<td>6</td>
<td>0.1</td>
<td>1.0</td>
<td>- 1.87</td>
<td>1.0</td>
<td>- 5.1E-5</td>
</tr>
<tr>
<td>9</td>
<td>0.1</td>
<td>0.707</td>
<td>- 0.74</td>
<td>0.395</td>
<td>- 9.2E-5</td>
</tr>
<tr>
<td>Case 7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Method 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.67</td>
<td>0.707</td>
<td>- 1.18</td>
<td>1.38</td>
<td>- 5.4E-5</td>
</tr>
<tr>
<td>6</td>
<td>0.2</td>
<td>1.0</td>
<td>- 0.85</td>
<td>1.0</td>
<td>- 1.13E-4</td>
</tr>
<tr>
<td>9</td>
<td>0.1</td>
<td>0.707</td>
<td>- 0.81</td>
<td>0.95</td>
<td>- 8.4E-5</td>
</tr>
</tbody>
</table>

Transfer functions calculated by NMOD yield qualitative information about the measured PSD's rather than exact magnitudes.

As voids increase in the core less water is available for moderation of the neutron spectrum. Thus a change in the energy spectrum at different elevations in a BWR is expected to arise from the increase in void fraction. In the simple two group slab channel model this effect is modeled by changing the ratio of $\phi_2/\phi_1$ at the boundary for
different channel positions. An example of this is presented as the last three entries in Table D.1 based on information from Figure D.3.

Since the output of the detector position transfer functions is the change in void fraction, the values of EP1 and EP2 are normalized to the value of EP2 at 6 feet. This normalization makes the detector response proportional to changes in the neutron flux by a factor which relates a change in unit voids at the middle detector position to changes in the neutron flux. Because the detector efficiencies are not considered in this model, the term relative units is used to describe outputs proportional to the detector current fluctuations at a particular location.
Seifritz's theoretical noise model, presented here, has been utilized by several authors [28, 29, 30, 31, 32] to develop noise signature models based on various stationary stochastic reactivity driving forces with zero mathematical expectations. The development of this general noise model assumes that a linear delayed critical-reactor system responds to a reactivity input vector whose components are made up of a series of \( I \) reactivity driving forces, \( \rho_1, \rho_2, \ldots, \rho_I \). It is further assumed that these components are correlated among one another, i.e., \( \rho_j \rho_k \neq 0, j, k = 1 \ldots I, j \neq k \). This accounts for situations such as coupled component vibrations in various core regions.

A source term introduced by Cohn [27] is included to account for neutron fluctuation due to the branching processes. Furthermore, the space-independent or volume averaged behavior of the neutron field is assumed i.e. the point kinetic model. The desired quantity to be evaluated is the power spectral density (PSD, \( \psi_{nn}(\omega) \)) of the fluctuation output current of a neutron detector. The general form, consisting of three additive terms is

\[
\psi_{nn}(\omega) = 2W_d \delta_c^2 + 2W_d ^2 F_q ^2 D |G_{cl}(\omega)|^2 + 2W_d ^2 F_q ^2 |G_{cl}(\omega)|^2 \cdot |\rho_{dx}(\omega)|^2
\]

where
- \( W_d \) is the detector efficiency \( \text{counts/fission} \)
- \( F \) is the total fission rate \( \text{fissions/s} \)
- \( \bar{q}_c \) is the average charge per neutron detected in a detector \( \text{coulomb/detection} \)
$D_v$ is relation width of the normalized probability distribution for the number of prompt neutrons per fission. This factor is called the Diven parameter and equals $(\bar{\nu}^2 - \bar{\nu})/\bar{\nu}^2$ where $\bar{\nu}$ is the mean number of prompt neutrons released per fission [17, pp. 52-54].

The frequency dependent terms are the reactor transfer function with inherent feedback loops closed and the reactivity driving functions. The at-power reactor transfer function is

$$G_{cl}(\omega) = \frac{G(\omega)}{1 - G(\omega) \times \left[ \sum_{m=1}^{N^O} \alpha_m \cdot H_m(\omega) \right]}$$

where $N^O$ is the steady state power level ($N^O$ in Btu/s = F/3.29E13 or $N^O$ in watts = F/3.1E10)

$\alpha_m$ is the power reactivity coefficient for the mth feedback path ($\delta k/k$ Btu or $\delta k/k$/watt)

$H_m(\omega)$ is the mth feedback path transfer function

$G(\omega)$ is the zero power reactor transfer function.

The general expression for the reactivity driving forces is the sum of the auto power spectral density and the cross power spectral density terms arising from reactivity input components, and is given as

$$|\rho_{dr}(\omega)|^2 = \sum_{i=1}^{T} |\rho_{i}(\omega)|^2 + \sum_{j=1}^{T} \sum_{k=1}^{T} \rho_j(\omega)\rho_k^*(\omega).$$

When the reactivity input components are independent of one another Equation E.3 reduces to
\[ |\rho_{dr}(\omega)|^2 = \sum_{n=1}^{N} |\rho_n(\omega)|^2 . \quad \text{E.4} \]

If only one perturbation \(\rho(t)\) drives the system, Equation E.4 yields

\[ |\rho_{dr}(\omega)|^2 = |\rho(\omega)|^2 . \quad \text{E.5} \]

The fact that the third term dominates in a power reactor can easily be shown by considering the ratio, \(\Gamma\) of the third term in Equation E.1 to the second term.

\[ \Gamma = \frac{\text{Noise contribution due to reactivity driving forces}}{\text{Noise contribution due to internal branching processes}} = F \frac{|\rho_{dr}(\omega)|^2}{D_v} \approx K N^0 \beta^2 |\zeta_{dr}(\omega)|^2 , \quad \text{E.6} \]

\((K = 3.9E6 \text{ for } N \text{ in watts or } 4.01E10 \text{ for Btu/s, } |\zeta_{dr}(\omega)|^2 \text{ in } (\text{cent})^2 \text{ per cycle/second units, } D_v = 0.8)\) which is called the "power signal to background ratio". Assuming a realistic reactivity excitation spectral density of \(10^{-2} \text{ cent}^2/\text{cycles} \cdot \text{s}\) and \(\beta = 0.0064\), shows that the power signal term dominates when the power level is greater than 0.6 watt.

When first and second terms in Equation E.1 are neglected for high power noise models, the PSD becomes

\[ \psi_{dd}(\omega) = 2W_dq^2F^2 |G_{cl}(\omega)|^2 \cdot |\rho_{dr}(\omega)|^2 . \quad \text{E.7} \]

\((A^2 \cdot \text{sec})\)

The mean detector current resulting from a constant fission rate is given by

\[ \bar{i} = q_c W_d F \quad \text{(in A)} . \quad \text{E.8} \]
In Seifritz's model the fluctuating detector current is assumed proportional to changes in the fission rate by defining the normalized root mean square of the detector current as

$$NRMS = \left[ \int_0^\infty \frac{|\psi_{nn}(\omega)|^2 \, d\omega}{2\pi \tau^2} \right]^{1/2} = \text{const.} \quad \text{E.9}$$

This formulation does not allow for PSD changes along the channel when the space-independent reactor model is used. In the present work, however, the detector current is assumed proportional to the incident thermal neutron flux, which is driven by both void fluctuations and fission rate.
THE RESONANCE EFFECT

Functions of the form \( 1 - e^{-sT_s} \) give rise to a resonance effect in the frequency response plots. The term \( e^{-j\omega T_s} \) is a unit vector in the real and imaginary plane which rotates with a continually decreasing phase angle. The cancellation and reinforcement of the constant 1.0 then appears as an increase or decrease in the magnitude as a function of frequency.

The physical interpretation of this behavior in reactor cores, which is similar to heat exchangers [66], arises because the reactor is forced in a distributed manner; i.e., the void fraction is changed along the entire length of the core and not at just one end. The peaks and valleys in the frequency response correspond to an integral number of half cycles along the boiling length, when the distribution function \( D(x) \) is constant.

If the void residence time corresponds to 1.5 cycles of the power oscillation, some of the elements of coolant are exposed to higher than normal cladding temperatures for two-thirds of the time and to lower than normal one-third of the time. For other elements, the proportions are reversed, and the difference in exposure tends to increase the amplitude ratio. When the void resonance time is equal to exactly one or more full cycles, all the elements are exposed to a higher than normal cladding temperature for half the time and the amplitude ratio is thus relatively low.
If \( D(x) \) is not constant, heat added during different half cycles will vary with position. However, coolant elements continuously enter the heating regions and the delay function frequency response is similar to the constant heat case. Examination of \( PV(s) \) in Equation 4.52 indicates that the ratio of the delayed portion to the general response can change for different heat distributions.

A simple example of the magnitude behavior of a delay function is shown in Figure F.1. The transfer functions \( G_1(s) \), \( G_2(s) \), and \( G_3(s) \) are similar to those derived from the coolant dynamics equation. The shape of \( G_3(s) \) can be modified greatly by changing the relative position of the denominator time constants (8. seconds and 0.5 seconds) with respect to the transport lag time constant (0.63 seconds).
Figure F.1. Magnitude plots for an example transfer function of the form found in the solution of coolant dynamics equations.
XVII. APPENDIX G:

COMPUTER PROGRAM LISTING
C* NMOD A PROGRAM TO ESTIMATE THE DETECTOR FREQUENCY RESPONSE *
C* ARISING FROM WHITE Noise HEAT INPUTS *
C* SUBROUTINES INCLUDE THE ISU SIMPLOTTER PACKAGE FOR DATA PLOTS **

COMMON NPTS,OL,DA1,DA2,DA3,XLAB,YLAB,DATLAB
REAL*4 MFWF,MSF,MP,LNB,LBL,LNBK,LBM,K3,K2,P,D1,DI,DR1,DI2
REAL*4 L,LC,LEV,LS,MU,KW,LJP,LL,MM,LM,LCM,LEV4,MCF,MCFM,KS
DIMENSION DA1(100),DA2(100),DA3(100),OL(100),OM(100)
DIMENSION XLAB(5),YLAB(5),DATLAB(5)
DIMENSION A0L(5),BSL(5),CBL(5),AD(5),BS(5),FABC(5),P8(5),COC(5),
COMPLEX TPV(100),TPVS(100),TPB(100),TPBS(100),U1,U2,U3,DN1,D1,
1 D2,BN1,BN2,BN3,CMPY,L,CEXP,CONJG
COMPLEX THWS(100),TGR(100),TFB(100),TFNB(100),TVB(100),TPCNB(100)
COMPLEX TPCB(100),DFB,DNB
COMPLEX TDEP1(100),TDEP2(100),TDEP3(100),TDHE1(100),TDHE2(100),
1TDHE3(100),B5,B6,B7,B8,B9,B10
COMPLEX DN2,U4,U5,U6,TVR(100),TVRS(100),TAB(100),BB1
1001 FORMAT('1',15X,' REACTOR MODEL INPUT PARAMETERS/*
1002 FORMAT('0',5X,' INPUT STEADY STATE CONDITIONS*/
1003 FORMAT('0',10X,' POWER LEVEL = ',F9.5,' TIMES RATEDOP*,F10.5,
1' MWTH,'F15.3,'BTU/HR'/*
1004 FORMAT('0',10X,' DOME PRESSURE = ',F10.4,' PSIA OR',F20.4,' KN/M
1 **2'/)
1005 FORMAT('0',1DX,' DESIRED STEAM EXIT QUALITY = ',F9.5'/
1006 FORMAT('0',1DX,' FEEDWATER ENTHALPY = ',F10.4,' BTU/LBM OR = ',
1F10.4,'KJ/KG'/*,11X,' FEEDWATER DENSITY = ',F10.4,'LBM/FT**3 OR = '
2,F10.4,'KG/M**3'/
1007 FORMAT('0',5X,' THERMODYNAMIC PROPERTIES OF WATER-STEAM AT OPERATI
1NG CONDITIONS*/
1008 FORMAT('0',10X,' SATURATED STEAM ENTHALPY = ',F10.4,' BTU/LBM OR = '
1F10.4,'KJ/KG'/*,10X,' SATURATED WATER ENTHALPY = ',F10.4,
2'BTU/LBM OR = ',F10.4,'KJ/KG'/*,11X,' SPECIFIC HEAT OF WATER = '

//STPPl EXEC WAVFIV,REGION.GO=176K,TIME.GO=(,25)
//GO,SYSTIN DD *
$JOB 'HANNAMAN',TIME=25,PAGES=40
C**************************************************************************************
C* NMOD A PROGRAM TO ESTIMATE THE DETECTOR FREQUENCY RESPONSE *
C* ARISING FROM WHITE Noise HEAT INPUTS *
C* SUBROUTINES INCLUDE THE ISU SIMPLOTTER PACKAGE FOR DATA PLOTS *
C**************************************************************************************
COMMON NPTS,OL,DA1,DA2,DA3,XLAB,YLAB,DATLAB
REAL*4 MFWF,MSF,MP,LNB,LBL,LNBK,LBM,K3,K2,P,D1,DI,DR1,DI2
REAL*4 L,LC,LEV,LS,MU,KW,LJP,LL,MM,LM,LCM,LEV4,MCF,MCFM,KS
DIMENSION DA1(100),DA2(100),DA3(100),OL(100),OM(100)
DIMENSION XLAB(5),YLAB(5),DATLAB(5)
DIMENSION A0L(5),BSL(5),CBL(5),AD(5),BS(5),FABC(5),P8(5),COC(5),
COMPLEX TPV(100),TPVS(100),TPB(100),TPBS(100),U1,U2,U3,DN1,D1,
1 D2,BN1,BN2,BN3,CMPY,L,CEXP,CONJG
COMPLEX THWS(100),TGR(100),TFB(100),TFNB(100),TVB(100),TPCNB(100)
COMPLEX TPCB(100),DFB,DNB
COMPLEX TDEP1(100),TDEP2(100),TDEP3(100),TDHE1(100),TDHE2(100),
1TDHE3(100),B5,B6,B7,B8,B9,B10
COMPLEX DN2,U4,U5,U6,TVR(100),TVRS(100),TAB(100),BB1
1001 FORMAT('1',15X,' REACTOR MODEL INPUT PARAMETERS/*
1002 FORMAT('0',5X,' INPUT STEADY STATE CONDITIONS*/
1003 FORMAT('0',10X,' POWER LEVEL = ',F9.5,' TIMES RATEDOP*,F10.5,
1' MWTH,'F15.3,'BTU/HR'/*
1004 FORMAT('0',10X,' DOME PRESSURE = ',F10.4,' PSIA OR',F20.4,' KN/M
1 **2'/)
1005 FORMAT('0',1DX,' DESIRED STEAM EXIT QUALITY = ',F9.5'/
1006 FORMAT('0',1DX,' FEEDWATER ENTHALPY = ',F10.4,' BTU/LBM OR = ',
1F10.4,'KJ/KG'/*,11X,' FEEDWATER DENSITY = ',F10.4,'LBM/FT**3 OR = '
2,F10.4,'KG/M**3'/
1007 FORMAT('0',5X,' THERMODYNAMIC PROPERTIES OF WATER-STEAM AT OPERATI
1NG CONDITIONS*/
1008 FORMAT('0',10X,' SATURATED STEAM ENTHALPY = ',F10.4,' BTU/LBM OR = '
1F10.4,'KJ/KG'/*,10X,' SATURATED WATER ENTHALPY = ',F10.4,
2'BTU/LBM OR = ',F10.4,'KJ/KG'/*,11X,' SPECIFIC HEAT OF WATER = '
4, F10.4, 'LBM/FT**3 OR = ', F10.4, 'KG/M**3' */, 11X, 'DENSITY OF STEAM = '
5, F10.4, 'LBM/FT**3 OR = ', F10.4, 'KG/M**3')

1009 FORMAT('O', '31X, 'HS/P = ', F10.6, ' BTU/LBM-PSI OR = ', F10.6, ' KJ/KG-K/M**2'
1/M**2', '31X, 'HW/P = ', F10.6, ' BTU/LBM-PSI OR = ', F10.6, ' KJ/KG-K/M
2*M**2', '31X, 'DT/DP = ', F10.6, ' F/PSI OR = ', F10.6, ' K/KN/M**2')

1010 FORMAT('O', '5X, 'BASIC PHYSICAL DIMENSIONS AND OPERATING COND' '/)
1011 FORMAT('O', '31X, 'CORE LENGTH = ', F5.2, ' FT OR = ', F5.2, ' M'
1 25X, 'CORE DIAMETER = ', F5.2, ' FT OR = ', F5.2, ' M'
1 25X, 'VESSEL DIAMETER = ', F5.2, ' FT OR = ', F5.2, ' M'
1 25X, 'VESSEL WATER LEVEL = ', F5.2, ' FT OR = ', F5.2, ' M'
1 25X, 'FEEDWATER INLET LEVEL = ', F5.2, ' FT OR = ', F5.2, ' M'
1 25X, 'CHIMNEY HEIGHT = ', F5.2, ' FT OR = ', F5.2, ' M'

1013 FORMAT('O', '10X, 'MASS FLOW RATE IN CORE = ', E14.6, ' LB/HP OR = '
1, E14.6, ' KG/S'/)

1014 FORMAT('O', '10X, 'EXIT QUALITY = ', F9.5, ' VOID FRACTION = ', F9.5,
1 'SLIP RATIO = ', F9.5, ' RECIRC. RATIO = ', F6.2')

1, F6.3, 'M'/)

1, F6.3, 'M/S'/)

1, F6.3, 'M/S'/)

1018 FORMAT('O', '10X, 'LATENT HEAT PER UNIT VCL = ', F9.2, ' BTU/FT3 OR = '
1, F9.2, ' KJ/M**3'/)

1019 FORMAT('O', '20X, 'EFFECTIVE MASSES OF WATER FOR MODEL'/)

1020 FORMAT('O', '10X, 'CORE MASS = ', F10.4, ' LBM OR = ', F10.4, ' KG'/, 11X, '
1 DOWNCOMER MASS = ', F10.4, ' LBM OR = ', F10.4, ' KG, ' 2X, 'RECYC. SYSTEM = '
2', F10.4, ' LBM OR = ', F10.4, ' KG'/)

1021 FORMAT('O', '10X, 'HEAT TRANSFER AREA = ', F10.2, ' FT2 OR = ', F10.2,
1 'M2, 2X, 'AVE. CHANNEL HEAT FLUX = ', F10.2, ' BTU/HR FT2 OR = ', F10.2
2, 'W/M2'/)

1022 FORMAT('O', '10X, 'REYNOLDS = ', F10.2, ' PRANDTL = ', F10.6, '/)

1023 FORMAT('O', '10X, 'NONBOILING HEAT TRANS COEF. = ', F9.2, ' BTU/HR-FT2F'
1, F10.4, ' W/M2-K'/)

1024 FORMAT('O', '10X, 'BOILING HEAT TRANSFER COEF. = ', F9.2, ' BTU/HR-FT2F'
1, F14.4, ' W/M2-K'/)
1025 FORMAT(*0*,10X,'AVE HEAT FLUX IN CHANNEL * = *,F9.2,*BTU/HP-FT2 *
1,*,F10.2,*W/M2*)

1026 FORMAT(*1*,20X,'GAIN AND TIME CONSTANTS FOR SIMPLIFIED MODEL*')

C******************************
C \# OF FLUX SHAPES OF THE FORM EXP(AX)*SIN(BX+C) ARE NEEDED TO DESCRIBE
C THE AXIAL FLUX DISTRIBUTION
C NPTS = \# OF FREQUENCY POINTS CONSIDERED

READ(5,2)N,NPTS
2 FORMAT(16I5)
3 FORMAT(4F20.5)
NT= 1. +NPTS

C READ PHYSICAL DIMENSIONS IN FT OR FT2
C L= CORE LENGTH, LC = WATER LEVEL ABOVE THE CORE, LEV = WATER LEVEL ABOVE
C CORE BOTTOM, FWL = FEEDWATER FLOW PATH LENGTH, D=FUEL PIN OD, DE =HYDRAULIC
C DIAMETER, DIAC= CORE DIA, DIAV = VESSEL DIA

READ(5,1)L,LC,LEV,FWL,D,DE,DIAC,DIAV

C A= CHANNEL FLOW AREA, AC= TOTAL CORE FLOW AREA, LS=SHAPING LENGTH,
C RPF = RADIAL PEAKING FACTOR, FDC = FPAC. OF METAL IN DOWN COMER

READ(5,1) A, AC,LS,RPF,FD

C FK = THERMAL CONDUCTIVITY OF THE FUEL BTU/HR FT F CK SAME FOR CLADDING
C RF = RADIUS OF FUEL LB/FT3, RC THE SAME FOR CLADDING

READ(5,1) FK,CK,RF,RC

C AVSP = PUMPING AREA OF CONTROL PUMPS *LJP LENGTH OF JET PUMP DRIVE LOOP

READ(5,1) AVSP,LJP

C READ THERMODYNAMIC PROPERTIES OF WATER-STEAM AT OPERATING CONDITIONS
C UNITS IN ENGLISH BTU/LBM LBM/FT3 *BTU/PSI, BTU/LM-F
C HS = ENTHALPY OF STEAM,WATER(SAT), AND FEEDWATER BTU/LBM
C CP = SPECIFIC HEAT OF WATER BTU/LBM F, CPF = SAME FOR FUEL
C DTDP, DHSP DHWP = THE CHANGE IN SATURATION CONDITIONS DUE TO PRESSURE

READ(5,1) HS, HN, HFW, CP,CPF,DTDP,DHSP, DHWP

C DENSITIES OF WATER, STEAM, FEEDWATER, AND FUEL LB/FT3

C******************************
PPAD(5,1) PHOW, RHOS, RHOFW, PHOF
C MU = VISCOSITY OF AVE NONBOILING H2O LBM/FT HR, KW = THERMAL COND OF
C SAME IN BTU/HR FT F
C*****************************************************************************
READ(5,1) MU, KW
C DESIRED EXIT STEAM QUALITY, DOME PRESSURE, AND THE FRACTION OF RATE POWER
C FP= FRACTION OF OPERATING POWER, P= PRESSURE PSIA, CHIE1= THE DESIRED
C EXIT QUALITY OPERATING CONDITIONS
C*****************************************************************************
READ (5,1) FP, P, CHIE1
C VISG, VISL = DYNAMIC VISCOSITY OF THE SATURATED STEAM AND WATER CONDITIONS
C THE UNITS ARE LBM/FT-SEC
C*****************************************************************************
READ (5,1) VISG, VISL
C FORMAT(8F10.5)
C CONVERT ENGLISH UNITS TO INTERNATIONAL UNITS (SI)
C CONVERSION FACTORS
C
CC = 4.1868
HH = 2.326
HT = 5.6783
LL = .3048
MM = .45359237
PI = 3.1415927
PP = 6.8948
QQ = .29307
RP = 16.019
TT = 5./9.
AA = LL*LL
C ENTHALPY IN BTU/LBM TOKJ/KG
HFWM = HH*HFW
HSM = HH*HS
HWM = HH*HW
C PRESSURE IN PSI TO KN/M**2
PM = P*PP
C SPECIFIC HEAT IN BTU/LBM-F TO KJ/KG-K
CPM = CC*CP
CPFM = CC*CPF
C DENSITY IN LBM/FT**3 TO KG/M**3

PHOSM = RHOS*RP
RHOWM = RHOW*RR
PHOFWM = PHOFW*FP

C CHANGE IN SAT H FOR A CHANGE IN PRESSURE BTJ/LBM/PSI TO KJ/KG/KN/M**2

DHSPM = DHSP*HH/PP
DHWP = DHWP*HH/PP

C CHANGE IN TEMPERATURE FOR A CHANGE IN PRESSURE F/PSI TO K/KN/M**2

DTDP = DTD*TT/PP

C LENGTHS IN FT TO METERS

LM = L*LL
LCM = LC*LL
LEVM = LEV*LL
FWLM = FWL*LL
DIACM = DIAC*LL
DIAVM = DIAV*LL
DELM = DE*LL
OM = D*LL

C AREA IN SQUARE FT TO M**2

AM = A*AA
PF = PI*D
AF = PI*D*D/4.
AFM = AF*AA
ACM = AC*AA
PFM = PF*AA

C CALCULATION OF FLOW IN CORE FROM THE OPERATING MAP

C THIS FUNCTION SHOULD BE MODIFIED FOR DIFFERENT REACTORS

PR = FP*(5437.07E6)
PRM = PR*(2.9307E-7)
MCF = PR*0.0146 - 28.86E6
MCFM = MCF*126.
PRL = PPF*PR/L
PM = P*6.8948

C CALCULATION OF THE EXIT VOID, SLIP, RECIRCULATION RATE

C START WITH ESTIMATED VOID FRACTION

FASL = 0.52 + 0.22*CHIE1
ETA = 1. - RHOS/RHOW
C
C BANKOFF CORRECTION MODIFIED BY JONES
C AND EXTENDED BY HUGHMAK PAGE 288 IN SIFTO VOL #2
C FROM REFERENCES 123,53,55
C
R = 3.53125 - .1875E-3*P + .58594E-6*P*P
GA = MCF/(AC*3600.)
GV = 32.2
C2 = GA*GA/(PHOW*RHOW*GV*DE)
C1 = GA*DE/(VISL*(1.-FASL)+VISG*FASL)
XV = FASL*RHOS/(PHOS*FASL +RHOW*(1.-FASL))
C3 = (1.+XV)*RHOW/((1.-XV)*PHOS)
SI = (C1**0.16667)*(C2**.125)*C3
IF(SI-50) 31,30,30
30 KS = .874*EXP(.0008727*SI)
GO TO 32
31 KS = .783*EXP(.00295*SI)
32 CONTINUE
PRINT,SI,KS
IF(KS.GE.1.0) KS = .999
IF(SI-15.) 5,5,8
5 KS = .71 + .9045E-4*P
KS = KS+(1.-KS)*FASL**R
CHIE2 = ((1.0-ETA)*FASL)/(KS - ETA*FASL +(1.-KS)*FASL**R)
DCHI = CHIFI - CHIE2
IF(ABS(DCHI).LT.0.0008) GO TO 15
10 FASL = FASL +.333*CHI
GO TO 25
15 SLIP = (1.-FASL)/(KS - FASL-(1.-KS)*FASL**R)
C CALCULATED THE RECIRCULATION RATIO
RP = (1.-CHIE2)/CHIE2
MSF = MCF/(1.+RR)
MFWF = MSF
C CALCULATE THE BOILING BOUNDARY (LNB)
C ASSUMING AXIAL POWER DISTRIBUTION IS SIN(BX)
ARG = (PR-2.*MFWF*(HW-HFW))/PR
LNB = L* ARCOS(ARG)/PI
FP.AC = MFWF*(HW-HFW)/PR
LNB Mayer = LN8*LL
LB = L-LNB
LBM = LB*LL
C INTERMEDIATE CALCULATIONS OF INTEREST
SLA = (1.+SLIP)/2.
C AVERAGE STEAM VELOCITY
WC = MCF/(RHOW*AC*3600.)
WCM = WC*LL
MR = MSF*RR
U = WC*SLA
UM = U*LL
C PARAMETERS NEEDED FOR AKCASU'S LUMPED TIME CONSTANT MODEL
BE = PI*LNB/L
AL = PI*LB/L
BES = PI*LNB/LS
ALS = PI*LB/LS
WS = PI*U/L
WW = PI*WC/L
TW = LNB/WC
TS = LB/U
COB = COS(BES)
SIB = SIN(BES)
COB2 = (1.-COB)*(1.-COB)
SI2B = SIN(2.*BES)
SI2B = SIN(2.*BES)
CO2B = COS(2.*BES)
COA = COS(ALS)
CO2A = COS(2.*ALS)
SIA = SIN(ALS)
SI2A = SIN(2.*ALS)
HV = RHOS*(HS-HW)
HVM = RHOS*(HSM-HWM)
C AREA OF DOWNCOMER WITH FRACTION OF METAL =FDC
ADC = PI*(DIAC+DIAV)*(DIAV-DIAC)*.25*FDC
RHODC = (RHDFW+RR*RHOW)/(1.+RR)
EMC = AC*RHOW*(LEV-ETA*FASL*(LC+LB/2.))
EMDC = ADC * RHODC * LEV
EMJP = 2. * RHODC * LJP * AVSP

C MASS IN LBM TO KG
EMCM = EMC * MM
EMDCM = EMDC * MM
EMJPM = EMJP * MM
RE = 3600. * DE * WC * RHOW / MU
PRN = CP * MU / KW

C NONBOILING HEAT TRANSFER COEF
HNB = 0.023 * (1.0 / DE) * (DE ** 0.8) * PRN ** 0.4
BHNB = HT * HNB
HTA = PF * L * AC / A

C RPF = POWER IN CHANNEL / POWER IN AN AVE CHANNEL
Q = PR * RPF / HTA
CHPO = Q * AF
QM = Q * 3.1546

C BOILING HEAT TRANSFER COEF
HB = (1. * 1E6 * EXP (P / 900.) * (0 / 1. * 1E6) ** 0.75) / 15.
HBM = HT * HB
HTAM = HTA * AA
WRITE (6, 1001)
WRITE (6, 1002)
WRITE (6, 1003) FP, PRM, PR
WRITE (6, 1004) P, PM
WRITE (6, 1005) CHF1
WRITE (6, 1006) HFW, HF4M, RHOFW, RHOFWM
WRITE (6, 1007)
WRITE (6, 1008) HS, HSM, IH1, HWM, CP, CPM, RHOW, RHOWM, RHOS, RHOSM
WRITE (6, 1009) DHSP, DHSPM, DHWP, DHWPM, DTDP, DTDM
WRITE (6, 1010)
WRITE (6, 1011) L, LM, DAC, DACM, DIAV, DIAVM, LEV, LEVM, FWL, FWLM, LC, LCM
WRITE (6, 1041) AL, BE, TW, TS

C FUEL TEMPERATURE FEEDBACKS
RNBO = 0.055 / FK + ALOG(RC / RF) / (PI * CK) + 1.0 / (HNB * PF)
RBO = 0.055 / FK + ALOG(RC / RF) / (PI * CK) + 1.0 / (HB * PF)
E1 = PI * (2. ** BE - SI2B) / (16.)
E2 = PI * (2. ** AL + SI2B) / (16.)
WRITE(6,1026)
WRITE(6,1039)
WRITE(6,1027) GTNB,GTNB,M,TABN
1027 FORMAT('latitude',10X,'power (NB) to fuel temp. = ',F9.6,'F/3TU/HR')
1,F10.8,'K/W',F9.3/)
WRITE(6,1028) GTB,GTBM,TABM
1028 FORMAT('latitude',10X,'power (B) to fuel temp. = ',F9.6,'F/3TU/HR')
1,F10.8,'K/W',F9.3/)
WRITE(6,1032) GQV,GQVM,TAVQ
1032 FORMAT('latitude',10X,'power to void = ',F9.6,'F/3BTU/S')
1,F10.4,'M**3/W',F9.3/)
WRITE(6,1033) GBV,GBVM,TAVB
1033 FORMAT('latitude',10X,'boiling boundary to void = ',F9.2,'F/3BTU/S')
1,F10.4,'M**3/M',F9.3/)
TAURB = TW*(1.-COB)/(BE*SB)
GRB =-(1.-COB)/(2.*PRB)
GRBM = GRB*LL/(QQ/3600.)
WRITE(6,1035) GRB,GRBM,TAURB
1035 FORMAT('latitude',10X,'power to boundary = ',F9.6,'F/3TU/HR')
1,F14.9,'M/W',F9.5/)
1038 FORMAT('latitude',15X,'intermediate calculations of interest')
1039 FORMAT('latitude',15X,'interim time constants (sec)')
1041 FORMAT('latitude',5X,'alpha = ',F6.3,'beta = ',F6.3,'tw = ',F6.3,'ts = ',F6.3)
WRITE(6,1043) GPWT,GPWTM,TAUWT
1043 FORMAT('latitude',10X,'power to water temp. = ',F14.9,'F/3TU/HR')
1,F14.9,'K/W',F9.7/)
C
C NOISE MODEL CALCULATIONS
1062 FORMAT('latitude',4X,'flux input factors',3X,ADL,6X,BSL,6X,CB,6X,WT)
1,F4X,'fabc'/)
WRITE(6,1062)
DO 45 I=1,N
C*************************************************
READ(5,1)ADL(I),BSL(I),CB(I),WT(I)
C
C CALCULATE THE WEIGHTING FUNCTION FABC FOR EACH FLUX SHAPE
C
C
AD(I) = ADL(I)/L
BS(I) = BSL(I)/L
SIC(I) = SIN(CB(I))
COC(I) = COS(CB(I))
CALL UPPER(AD(I),BS(I),CB(I),L,P1(I),P2(I))
P8(I)= AD(I)*AD(I) + BS(I)*BS(I)
FABC(I) = L*P8(I)/(COC(I)*P1(I)+BS(I))*SIC(I)*(P2(I)-AD(I))
WRITE(6,1) AD(I),BS(I),CB(I),WET(I),FABC(I)

45 CONTINUE
C CALCULATE THE ASSUMED FLUX PROFILE IN THE AXIAL L DIRECTION
WRITE(6,1050)
1050 FORMAT('0',20X,'ASSUMED FLUX PROFILE ',/4X,'X/LCR',4X,'FLUX')
1051 FORMAT(' ',F10.3,F15.5)
C FLUX PROFILE
   X = 1.0
   DO 80 I=1,21
      FLUX = 0
   DO 81 K=1,N
      FLUXI = FABC(K)*WET(K)*EXP(ADL(K)*X)*SIN(BSL(K)*X*CB(K))
      FLUX = FLUX +FLUXI
   81 CONTINUE
   WRITE(6,1051)X,FLUX
   X = X-.05
80 CONTINUE
C
C GENERATE THE LIST OF OMEGA'S TO BE USED IN CALL THE TRANS FUNCTIONS
C OMEGA'S FIRST AND LAST
C
C*************************************************************************************************************************
READ(5,1)OFST,OLST
DOM = ALOG(OLST/OFST)/NPTS
DO535 I=1,NPTS
   OM(I) = OFST*EXP(DOM* (I-1))
   OL(I) = ALOG10(OM(I))/6.28
535 CONTINUE
C FEEDBACK LOOP COEF FOR FUEL TEMP, H2O TEMP, AND VOID
C*************************************************************************************************************************
READ(5,3) ALF, ALW, ALV
4 FORMAT(20A4)
C ***********************************************************************
READ(5,4) XLAB, YLAB, DATLAB
PRBS = FRAC*PL/3600.
GWH = 3600./(MCF*CP*L)
C DETERMINE THE NEW BOILING BOUNDARY DUE TO THE OVERALL VIEW SHAPE
C USING THE FRACTION OF POWER IN THE NONBOILING REGION AS THE BALANCE
C PARAMETER
C
Y = LNB
51 CONTINUE
FUNF = 0.0
DO 50 I = 1, N
UL1 = COS(CB(I))*AD(I)*SIN(BS(I)*Y) - BS(I)*COS(BS(I)*Y)
UL2 = SIN(CB(I))*AD(I)*COS(BS(I)*Y) + BS(I)*SIN(BS(I)*Y)
ZL = BS(I)*COS(CB(I)) - AD(I)*SIN(CB(I))
FUNF = FUNF + FABC(I) * (EXP(AD(I)*Y) * (UL1 + UL2) + ZL) * WET(I) / (P8(I)*L)
PRINT 1, UL1, UL2, ZL, FUNF
50 CONTINUE
D = FRAC - FUNF
PRINT 1, D, FUNF, FRAC, Y
IF(ABS(D) < .001) 52, 53, 53
53 Y = Y + D
GO TO 51
52 LNB = Y
LB = L - LNB
W = WC
TSM = LB/U
TWM = LNB/W
WRITE (6,1) W, LNB, LB, TSM, TWS
C
DO 82 J = 1, NPTS
TPBS(J) = (0.0, 0.0)
TPVS(J) = (0.0, 0.0)
82 CONTINUE
C
THE I LOOPS SUM THE RESPONSES FROM THE VARIOUS FLUX SHAPES CONSIDERED
THE J LOOP CALCULATES THE TRANSFER FUNCTION FREQUENCY RESPONSE ON OMEGA

DO 29 I=1,N
CINB = COS(BS(I)*LNB)
SNB  = SIN(BS(I)*LNB)
EXLNBI = EXP(AD(I)*LNB)
P3  = EXLNBI*(AD(I)*SNB-BS(I)*CINB)
P4  = EXLNBI*(AD(I)*CBN+BS(I)*SNB)
C1  = (COC(I)*(P1(I)-P3)+SIC(I)*(P2(I)-P4))/(P8(I)*U)
C7  = AD(I)*SIC(I)*COC(I)*BS(I)
C2  = [C7*(P1(I)-P3)+(AD(I)*SIC(I)-BS(I)*COC(I))*(P2(I)-P4)]/P8(I)
C3  = LB*EXLNBI*(COC(I)*SNB-SIC(I)*CINB)/U
C4  = LB*EXLNBI*(AD(I)*SIC(I)*SNB-SIC(I)*CINB)
C8  = SIC(I)/W
C5  = COC(I)*P3 + SIC(I)*P4
C6  = EXLNBI*(COC(I)*SNB+SIC(I)*CINB)/W
GPB = -WET(I)*FABC(I)*3600./{PRL*FRAC}
GPV = FABC(I)*FET(I)/(HV*L*U)
DO 60 J=1,NPTS
RD  = P8(I)*OM(J)*OM(J)/(U*U)
DI  = 2.*OM(J)*AD(I)/U
DR2 I = P8(I) - (OM(J)*OM(J))/(U*U)
DI2 I = 2.*AD(I)*OM(J)/W
U1  = CMPLX(C2,C1*OM(J))
U2  = CMPLX(C4,C3*OM(J))
U3  = CEXP(CMPLX(0,0,-TSM*OM(J)))
D1  = CMPLX(RD,DI)
D2  = CMPLX(DR2,DI2)
DN1 I = CMPLX(0,0,TSM*OM(J))
BN1 I = CMPLX(C5,C6*OM(J))
BN2 I = CMPLX(C7,C8*OM(J))
BN3 I = CEXP(CMPLX(0,0,-OM(J)*TWM))
TPV(I) = GPV*(U1 + U2*(U3-1.0)/DN1)/D1
TPVS(J) = TPV(I)+TPVS(J)
TPB(I) = GPB*(BN1+BN2*BN3)/D2
TPBS(J) = TPB(I) + TPBS(J)
TVB(J) = PRBS*(1.-U3)/(OM(J)*HV)
60 CONTINUE
29 CONTINUE
DO 61 J=1,NPTS
C REACTOR ONE GROUP TRANS FUNCTION WITH B/L BRFAK BETA= .00646L= 5E-5
C LAMBDA = .0767 SEC-1
GDUM = (25.*E-10*OM(J)*OM(J) + 4.*095E-5)*3600.
GRE = PR*0.006395/GDUM
GIM = PR*4.9E-5*OM(J)*OM(J)*5.E-5/(OM(J)*GDUM)
TGR(J) = CMPLX(GRE,-GIM)
DFB = CMPLX(1.,AUFB*OM(J))
DFNB = CMPLX(1.,AUFB*OM(J))
TFNB(J) = 3600.*GTNB/DFNB
TFB(J) = 3600.*GTB/DFB
TPCNB(J) = FRAC/DFNB
TPCB(J) = (1.-FRAC)/DFB
THWS(J) = GW*TPBS(J)/GPB
B5 = ALF*(TFB(J) + TFB(J)) + ALW*TPCNB(J) + THWS(J)
B6 = ALV*(TPCB(J) + TPVS(J))
B7 = ALV*(TPCNB(J) + TPBS(J) + TVB(J))
TVR(J) = TGR(J)*(B5+B6+B7)
B8 = TGR(J)/(1.0-TVJ(J))
TPB(J) = B8
TPV(J) = ALV*B8*TPCB(J)*TPVS(J)
C CALCULATE THE IMPORTANT FEEDBACK TRANSFER FUNCTIONS
TDPE1(J) = TPVS(J)*ALV*B8
TDH=1(J) = B8*(ALW*THWS(J) + ALV*TPBS(J)*TVB(J))
61 CONTINUE
C
C WRITE OUT SEQUENCE
C
WRITE (6,1060)
1060 FORMAT('1',15X,' THIS IS THE FREQ. RESPONSE POWER TO VOID', /
1,5X,'OMEGA',6X,' FREQUENCY',9X,'dB',11X,'REAL',10X,'IMAG',8X,'/
2 ABSVALUE',8X,' PHASE',5X,' SQUARE MOD'/)
DO 55 I=1,NPTS
CALL TWRT(TPVS(I),OM(I),DA1(I),DA2(I),DA3(I))
55 CONTINUE
CALL GPlot
1075 FORMAT(*1',5X,'OPEN LOOP REACTOR FEEDBACK MODEL'/)
WRITE(6,1075)
WRITE(6,1065)
DO 99 J=1,NPTS
CALL TWRT(TVR(J),OM(J),DA1(J),DA2(J),DA3(J))
99 CONTINUE
CALL GPlot
1069 FORMAT(*1',5X,'PATIO OF OUTPUT TO DP SOURCE AT SUMMING POINT'/)
WRITE(6,1069)
WRITE(6,1065)
DO 96 J=1,NPTS
CALL TWRT(TPV(J),OM(J),DA1(J),DA2(J),DA3(J))
96 CONTINUE
CALL GPlot
1068 FORMAT(*1',5X,'TOTAL REACTOR FBK TRANSFER FUNCTION WITH ALV = ',E14.4
1,'DK/FT3, ALW = ',E14.4,'DK/FT3, ALF = ',E14.4,'DK/FT3'/)
WRITE(6,1068)ALV,ALW,ALF
WRITE(6,1065)
DO 97 J=1,NPTS
CALL TWRT(TPB(J),OM(J),DA1(J),DA2(J),DA3(J))
97 CONTINUE
CALL GPlot
1066 FORMAT(*1',15X,'REACTOR POWER RESPONSE TO DP SOURCE WITH FEEDBACK'/)
WRITE (6,1066)
WRITE(6,1065)
DO 90 J=1,NPTS
CALL TWRT(TDE1(J),OM(J),DA1(J),DA2(J),DA3(J))
90 CONTINUE
CALL GPlot
1070 FORMAT(*1',15X,'REACTOR POWER RESPONSE TO DH WITH FEEDBACK'/)
WRITE (6,1070)
WRITE(6,1065)
DO 93 J=1,NPTS
CALL TWRT(TDHE1(J),OM(J),DA1(J),DA2(J),DA3(J))
CONTINUE
CALL GPLTO
DO 62 K=1,3
READ(5,1) PDH, E, PI, EP2, X1
XD = (X1-LNB)/LB
TSD = (X1-LNB)/U
DO 63 I=1,N
CNB = COS(BS(I)*LNR)
SNB = SIN(BS(I)*LNR)
EXLNB = EXP(AD(I)*LNB)
P4 = EXLNB*(AD(I)*CNB + BS(I)*SNB)
P3 = EXLNB*(AD(I)*SNB - BS(I)*CNB)
C7 = AD(I)*SIC(I)*COC(I)*BS(I)
C3 = LB*EXLNB*(COC(I)*SNB - SIC(I)*CNB)/U
C41 = LB*EXLNB*(AD(I)*(COC(I)*SNB - SIC(I)*CNB))
C4 = -LB*EXLNB*BS(I)*(COC(I)*CNB - SIC(I)*SNB) + C41
CALL UPPER(AD(I), BS(I), COC(I), CB(I), XI, P1(I), P2(I))
G1 = (COC(I)*(P1(I)-P3) + SIC(I)*(P2(I)-P4))/P8(I)*U
G2 = (C7*(P1(I)-P3) + (AD(I)*SIC(I)-BS(I)*COC(I))*P2(I)-P4))/P8(I)
G3 = XD*C3
G4 = XD*C4
GPV = FABCCI*HET(I)/(HV*L*U)
DO 63 J=1,NPTS
RD = P8(I)-OM(J)*OM(J)/(U*U)
DI = 2.0*OM(J)*AD(I)/U
D1 = CMPLX(RD, DI)
TVRS(J) = CMPLX(RD, DI)
DN2 = CMPLX(0.0, TSD*OM(J))
U4 = CMPLX(G2, G1*OM(J))
U5 = CMPLX(G4, G3*OM(J))
U6 = CEXP(CMPLX(0.0, -TSD*OM(J)))
TVF(I) = (GPV/AC) *(U4 + U5*(U6-1.0)/DN2)/D1
TVRS(J) = TVF(I) + TVRS(J)
TAB(J) = -PPB*U6/(AC*HV*U)
63 CONTINUE
DO 65 J=1,NPTS
C INCLUDING THE CORE AVERAGED FEEDBACK REACTIVITY COEF.
08 = TPB(J)
B81 = (TPCB(J) * TVRS(J) + TPDB(J) * TPBS(J) * TAB(J))
B82 = TPVS(J) * ALV * B81 + B82 * (TPBS(J) * ALV * TVRS(J) + THWS(J) * ALW) * BB1
TDHE2(J) = TPB(S(J) * TAB(J) + BB1 * (TPBS(J) * ALV * TVRS(J) + THWS(J) * ALW) * BB1
TDHE3(J) = TDPE2(J) * PDH + TDHE2(J)
TDPE3(J) = TDPE2(J) * PDH + TDHE3(J)
TFB(J) = TDPE3(J) * EP1 + TDHE3(J) * EP2

65 CONTINUE
C WRITE OUT SEQUENCE FOR DETECTOR DEPENDENT LOCATIONS
WRITE(6, 1072) EP1, EP2
WRITE(6, 1065)
DO 95 J = 1, NPTS
CALL TWRT(TDPE3(J), OM(J), DA1(J), DA2(J), DA3(J))
95 CONTINUE
WRITE(6, 1074) EP1, EP2, PDH, XI
WRITE(6, 1065)
DO 98 J = 1, NPTS
CALL TWRT(TFB(J), OM(J), DA1(J), DA2(J), DA3(J))
98 CONTINUE
CALL GPLOT
62 CONTINUE
1072 FORMAT('1', 5X, 'BOILING NOISE TO DET. VOID RESPONSE GIVEN THAT EP1 = 1', E14.4, 'EP2 = ', E14.4, ')
1073 FORMAT('1', 5X, 'NOISE SOURCES TO POWER RESPONSE GIVEN THAT EP1 = 1', E14.4, 'EP2 = ', E14.4, ')
WRITE(6, 1073) EP1, EP2
WRITE(6, 1065)
DO 92 J = 1, NPTS
CALL TWRT(TDPE3(J), OM(J), DA1(J), DA2(J), DA3(J))
92 CONTINUE
CALL GPLOT
1067 FORMAT('1', 15X, 'VOID RESPONSE TO XI FROM BOILING SOURCE WITH FB')
WRITE (6,1067)
WRITE(6,1065)
DO 91 J=1,NPTS
   CALL TWRT(TDPE2(J),OM(J),DA1(J),DA2(J),DA3(J))
91 CONTINUE
CALL GPLOT
1071 FORMAT('1',15X,'VOID RESPONSE AT XI FROM DH WITH FEEDBACK')/
WRITE (6,1071)
WRITE(6,1065)
DO 94 J=1,NPTS
   CALL TWRT(TDHE2(J),OM(J),DA1(J),DA2(J),DA3(J))
94 CONTINUE
CALL GPLOT
STOP
END

C
C
SUBROUTINE UPPER(A,B,CB,Y,D,F)
   X=EXP(A*Y)
   C = CCS(B*Y)
   S = _SIN(B*Y)
   D = X*(A*S-B*C)
   F =X*(A*C + B*S)
RETURN
END

C
C
SUBROUTINE TWRT(Y,OM,SML,DBL,PHAS)
   COMPLEX Y,CONJG
   F = OM/6.2832
   ABSV = CABS(Y)
   PHAS = 57.29578*ATAN2(AIMAG(Y),REAL(Y))
   DBL = 20. * ALOG10(ABSV)
   SM = Y*CONJG(Y)
   SML = ALOG10(SM)
   WRITE (6,10)OM,F,DBL,ABSV,PHAS,SM
10 FORMAT(8E14.4)
RETURN
END

C
C
C
SUBROUTINE GPLOT
COMMON NPTS,OM(100),DA1(100),DA2(100),DA3(100),XLAB(5),YLAB(5),
1 DATLAB(5)
DIMENSION GLAB(5)
C **********************************************
READ(5,3) GLAB
3 FORMAT(20A4)
CALL OPIGIN(7.5,0.0,1)
CALL GRAPH(100,OM,DA1,3,5,-5.5,-7.5,2.,-1.0,0.0,0.0,0.0,
1XLAB,YLAB,GLAB,DATLAB)
RETURN
END

C
C
C
INPUT DATA

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<td>-4.0</td>
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<td>FREQUENCY (HZ)</td>
<td>SQ MODULUS REL UNITS+4, P.9</td>
<td>CASE 7</td>
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<tr>
<td>POWER TO VOID NOFB</td>
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<td>OPEN LOOP REACTOR TF</td>
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<td>CLOS LOOP REACTOR TF</td>
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<tr>
<td>DP S TO PWR RES WFB</td>
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<tr>
<td>DH S TO PWR RES WFB</td>
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<tr>
<td>DP/DH EP1 EP2 DETECTOR POSITION</td>
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<td>DP-VOID ES AT X1</td>
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<tr>
<td>DH-VOID ::S AT X1</td>
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<tr>
<td>//GO*FTL4F001 DD DSNAMES=&amp;SM,UNIT=SCRATCH,DISP=(NEW,PASS),</td>
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<td>// SPACE=(800,120,15),DCB=(RECFM=VS,LRECL=796,BKSIZE=800)</td>
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<tr>
<td>//SMPLTTR EXEC PLOT, PLOTTER=INCRMNMTL, FORM=W</td>
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