Analysis of wind turbine wake characteristics in atmospheric boundary layer winds

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Analysis of wind turbine wake characteristics in atmospheric boundary layer winds

by

Pavithra Premaratne

A dissertation submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

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Program of Study Committee:
Hui Hu, Major Professor
Alric Paul Rothmayer
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Wei Hong

The student author, whose presentation of the scholarship herein was approved by the program of study committee, is solely responsible for the content of this dissertation. The Graduate College will ensure this dissertation is globally accessible and will not permit alterations after a degree is conferred.

Iowa State University
Ames, Iowa
2018

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DEDICATION

I would like to dedicate this dissertation to my beloved parents, my brother and my friends who have helped me tremendously throughout my career.
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ABSTRACT

The wake characteristics behind a Horizontal Axis Wind Turbines (HAWT) sited in Atmospheric Boundary Layer (ABL) winds are investigated by using analytical, experimental and computational techniques. Studies of wake effects are crucial to understand the power generation and wind farm siting. Analytical wake models are investigated and incorporated into an algorithm which accurately predicts the thrust coefficient of wind turbine and flow characteristics in the far wake regions (X/D > 1.0). Instantaneous flow field measurements obtained by using Particle Image Velocimetry (PIV) technique were analyzed using a suit of principal component analysis techniques, such as Proper Orthogonal Decomposition (POD) and Dynamic Mode Decomposition (DMD) to identify the dominant flow structures in the wind turbine wakes. The analysis was focused on the instabilities of the vortex structures, break-up phenomenon, formation of the shear layer and the momentum transport mechanisms. The analysis shows that, in addition to the existence of the well-known tip vortex structures, the vortices generated at the blade mid-span were found to have the twice the circulation of the tip vortex filament. The wake vortices were found to break-up in the downstream region of X/D = 0.6, forming the shear layer. Analysis of momentum transport also shows how the presence of discreet vortex filaments in the near wake hinders the momentum transport by 40%. The effects of this instability involving mid-span and tip vortex filaments are simulated using an unsteady free-wake method, where the filaments are shown to have a mutual attraction which results in a short wave instability. An experimental study was also conducted to compare the aeromechanic performances (i.e., dynamic wind loads and power generation characteristics) of scaled HAWTs in upwind and downwind configurations. A novel approach to measuring torque was introduced to alleviate previous under-predictions that stem from electric power measurements.
CHAPTER 1. INTRODUCTION

Aerodynamic characteristics of wind turbine wake present a substantial challenge in optimizing the output of an individual turbine or in windfarm siting (Bartl, Pierella, & Sætran, 2012). In the present study, we investigate the dominant vortical characteristics, momentum and vorticity transport, and the governing instability mechanisms of the wake emanating from a scaled offshore Horizontal-Axis Wind Turbine (HAWT), operating in turbulent boundary layer flow. We utilize an off-shore boundary layer profile for incoming flow due to the current interests in off-shore HAWT applications, especially to cater the energy needs of the coastal cities in the United States. The near wake (X<1.0D) flow analysis provides crucial performance factors of turbine rotor blades, while the far wake (X>1.0D) analysis determines the turbulence-induced fatigue of the downstream wind turbines as well as the changes in the power output of the downstream turbines (Adaramola & Krogstad, 2011) (Newman, Lebron, Meneveau, & Castillo, 2013).

As depicted in Figure 1.1, once the flow passes through the turbine rotor, a reduction in momentum occurs, causing a region of momentum deficit in the wake. This results in a shear layer, where high velocity components outside the wake regions are mixed with the low velocity components of the deficit region. As the wake progresses axially, it undergoes expansion (Abdelsalam, Boopathi, Gomathinayagam, Hari Krishnan Kumar, & Ramalingam, 2014). The mixing process creates turbulent eddies which result in the wake recovery or wake recharging (Sanderse, 2009). Higher levels of turbulence intensity increase the mixing efficiency, thus decreasing the wake recovery distance. However, such levels lead to blade fatigue in the downstream turbine, thus increasing maintenance costs.
Early wake visualizations conducted using scaled models revealed the presence of a tip vortex and a hub vortex emanating from the rotor assembly. As these vortex filaments propagate downstream, they undergo various instabilities resulting in deformations of the filament and finally breaking up into smaller eddies. PIV studies done at Iowa State University on a scaled commercial wind turbine (NREL 2MW) elucidated the presence of a vortex emanating from the mid-span location of the blade. The measurements were taken for two different incoming ABL winds with 10% and 15% turbulence intensity at the hub height for a power-law type off-shore profile. The presence of higher turbulence intensity in the incoming flow showed the early break-up of vortices compared to case with 10% turbulence intensity (Tian, Ozbay, & Hu, 2014).

Subsequent iterations of this experiment lead to the development of a dual rotor wind turbine, in which a small rotor was placed upstream of the large main rotor. The dual rotor turbine design increased the power output as well the turbulence intensity experienced by the main rotor located downstream (Wang, Tian, Ozbay, Sharma, & Hu, 2016). Stereoscopic and 2D PIV studies done on another scaled turbine design at Monash University featured modal decomposition (POD) and low-order reconstructions of the spanwise planes at a given radial location. The reconstructions revealed the unsteady meander of the vortex cores for phase-locked positions (Nemes, Sherry, Lo Jacono, Blackburn, & Sheridan, 2014). Stereoscopic measurements conducted by Zhang et.al (2012) on a scaled turbine model in turbulent boundary layer flow shows the propagation of the tip vortices to 2-3 diameters downstream, while the hub/root vortices break-up early. Vortex identification criterion such as swirling strength has been employed for this study to identify the regions of circulation (Zhang, Markfort, & Porte-Agel, 2012). Numerous attempts have been made throughout the years to quantify the role of the helical vortical filaments and their governing mechanics.
Hardin (1982) developed a potential flow description for a propagating helical filament using Kapteyn-Krummer series. The derived potential function contained a singularity where the filament intersects with the spanwise planes (Mulinazzi & Zheng, 2014)(Hardin, 1982). This model provided an initial understanding of the induced velocity field, yet it didn’t provide an insight to the unsteady instabilities. New expressions detailing the flow field of a HAWT was pioneered by authors such as Jensen, Larsen and Frandsen(Renkema, 2007), in which momentum conservation and actuator disk theory were used for derivations. The models successfully captured the wake expansion, bi-modal velocity distributions at near wake, Gaussian velocity distribution at far-wake, and the wake –recharging. Recent publications by Porte-Agal and Bastankah provide a three-dimensional analytical flow field for a HAWT based on the Gaussian velocity deficit model(Bastankhah & Porté-Agel, 2014). However, such models still lacked information on the types of instabilities native to helical vortex filaments.

Three primary instability mechanisms have been identified for propagating vortex filaments. Long wave instabilities result at longer wave lengths with no change to core radii. Elliptical instabilities or short wave instabilities result in amplification of short wave perturbations inside the vortex core. In the case of reduced helical pitch, vortex filaments undergo mutual induction.
The effect of an external strain field that perturbs the filaments depends on the tip speed ratio, which in turn determines the spacing between consecutive turns and the presence of other vortical elements in the flow (Sarmast et al., 2014). Widnall used visualization techniques to observe long-wave instability, in which a successive helical turn shows an out of phase displacement. Henningson, Sorenson and Okulov investigated the short wave instability via a simple Kelvin wave approximation. They claimed that the short wave instability is caused by an addition of Kelvin modes to an existing velocity field perturbed by axial and azimuthal Kelvin wave numbers, radial velocity and a frequency (Leweke, Quaranta, Bolnot, Blanco-Rodríguez, & Dizès, 2014) (Hattori & Fukumoto, 2012). In a mutual induction scenario, the amplification of specific waves traveling along the helicoidal wave is deemed responsible for the eventual break-up (Sarmast et al., 2014). The visualization of the three instabilities is shown in Figure 1.2.

![Figure 1.2: Instabilities of helical vortex filaments visualized using a water tunnel (Leweke et al., 2014)](image-url)
PIV measurements on a two bladed HAWT by Lignarolo and Ragni (2014) revealed that the location of instability in wake moves upstream as the tip speed increases (L. E. M. Lignarolo et al., 2014). The mechanism behind the instability observed is a well-known characteristic native to vortex rings and helical filaments called leap-frogging. Leap-frogging causes vortex filaments to expand and contract as they pass through each other (L. E. Lignarolo, Ragni, Simao Ferreira, & van Bussel, 2014) (Wu, Ma, & Zhou, 2006). Variations of the location of instability with varying tip-speed ratios are shown in Figure 1.3.

**Figure 1.3:** Location of instability; \( \lambda = 6.0 \) (top) and \( \lambda = 4.8 \) (bottom) (L. E. M. Lignarolo et al., 2014; L. E. Lignarolo et al., 2014)

Such instabilities, often paired up, lead up to break-up in the vortices which is crucial to the formation of the shear layer. Turbulent momentum and vorticity transport in turbine wake flow provides an insight to the recharging and the dissipation processes which are predominant in determining the distance among turbines in a wind farm. Upon inspecting the vorticity transport equation, a vortex dominated incompressible flow such as turbine wake is predominantly governed by the non-linear terms such as advection and stretching. Vorticity transport is derived from the momentum transport theory via the curl of the velocity (Bernard, 1990). Reynolds
averaging the vorticity transport results in the turbulent advection and stretching terms.

Numerous efforts have been made to relate the Reynolds stress to the said turbulent vorticity terms by Lumley and Tennekes as well as Roshko and Brown (Lumley & Tennekes, 1972) (Brown & Roshko, 2012). The current derivations relates cross-stream derivative of the Reynolds stress to the advection and stretching terms, assuming the stream-wise derivatives to be negligibly small. Recent publications by Hamman, Klewicki and Kirby utilize the lamb vector and its divergence as a means to describe the “unwinding” of the vortices in a channel flow (Hamman, Klewicki, & Kirby, 2008). The unwinding action results in a conversion from angular momentum to linear momentum in the flow field. In this manuscript we focus on investigating both mechanisms to describe the transport processes in the turbine wake flow.

Modeling the behavior of the vortex filaments, namely advection and stretching, presents a challenge due to the modeling constraints, assumptions and the computational power necessary. Current techniques dwell on the traditional grid based approaches coupled with LES or RANS type of turbulence modeling (Trolldborg & Sørensen, 2014). Modeling the rotational flow effects and the shed helical vortices and their dissipation require the explicit modeling of the blades. This increases the computational costs due to the increase in grid points, dynamic grid generation and interpolations between grid domains. Vortex methods based on a Lagrange framework have shown great promise, combined with modern acceleration techniques such as Fast-Multipole-Method (FMM) or Barnes-Hut Tree algorithms (He & Zhao, 2009) (Lectures in Applied Mathematics (Vortex Dynamics and Vortex Methods), 1991). Variations of vortex methods include vortex-particle methods, filament methods, blob based methods, vortex-in-cell methods and lattice-based methods. The presence of N number of particles or filaments in a simulation results in $N^2$ calculations per time step, which in-turn can be reduced to $N \log N$ using the FMM
and tree algorithms. Earliest work on the vortex methods can be traced to Chorin and Bernard (1973), where they simulated the roll-up of a vortex using a time-marching algorithm based on Biot-Savart relationship to describe the advection (Chorin & Bernard, 1973). This research was further expanded by Krasny with simulations of tip vortex roll-up on Trefftz plane and the Kelvin-Helmholtz instabilities using direct integration (Robert Krasny, 1986) (R. Krasny, 1987) (Nitsche, 2001). Vortex methods allow the researchers to focus on the individual vortex characteristics with reduced numerical dissipation (Stock, 2007). Explicit turbulence modeling can be avoided with an adequate number of points and a proper smoothing length among blobs. Viscous effects were introduced via core-spreading and Particle-Strength-Exchange (PSE) in which circulation strength is exchanged between particles, preserving the total circulation in the simulation (Stock, 2007). Application of vortex methods under a free-wake or prescribed-wake framework for rotor flow simulations has been investigated by a handful of authors recently. A helicopter rotor simulation by He and Zhao is shown in Figure 1.4.

Figure 1.4: Helicoper rotor flow simulation by He and Zhao (He & Zhao, 2009)
He and Zhao’s simulation shows a highly detailed cross section of the rotor flow field where tip vortices undergo advection, contraction in the wake (contrary to expansion in wind turbines) and break-up due to leap-frogging related instabilities. Comparison of the downwash velocities between experimental and the simulation shows a satisfactory agreement which couldn’t be attained from a traditional actuator disk or line based approximation. Abedi and Lackner developed filament and Lattice based algorithms simulate the instabilities and calculate the performance of a given rotor with relatively small computational or implementation effort (Abedi, 2013). Circulation strength of the shed vortices along with their initial velocities can be calculated using the lifting line theorem (Leishman, Bhagwat, & Bagai, 2002). Such advances in these methods prompted us to investigate vortex methods as potential candidates to understand the conditions that cause the instabilities observed in our PIV measurements.

This dissertation investigates the analytical wake models for wind turbines in Chapter 2 as well as the development of an iterative method to predict the thrust coefficient of an off-shore wind turbine. Chapter 3 is focused on utilizing modal decomposition techniques to analyze the dominant vortical structures present in the flow along with their advection and dissipation. The spatiotemporal behavior of the dominant is further explored using Dynamic Mode Decomposition and compared with the POD results from Chapter 3. The behavior of the vortical structures identified from Chapter 3 and 4 are simulated using vortex methods to understand the leading causes of instability and break-up. A novel approach to measuring performance of upwind and a downwind turbines based on mechanical torque is discussed in Chapter 6. This is an improvement to the predictions we obtained earlier via measuring the electrical power output.
Chapter 8 sheds light on possible future avenues for wind energy, especially harnessing the power of various environmental flows to increase endurance in airborne platforms.

References


CHAPTER 2. ANALYTICAL WAKE PREDICTIONS

2.1 Abstract

We present a new method to predict the thrust coefficient of a wind turbine along with the span-wise velocity deficit for a far-wake axial station (X). Our model incorporates the Gaussian wake distribution as a baseline with inflow condition pertaining to an offshore environment. The manuscript investigates the wake models that have been used in the past, progressing towards the construction of an iterative prediction algorithm. The wake expansion based on Jensen model along with the maximum velocity deficit occurring at the hub height is also used as input conditions. Based on the validation cases conducted for a prototype model and a scaled model, the iterative algorithm predicted thrust coefficients with less than 6% uncertainty. Velocity profiles were satisfactorily predicted for X>3D for the prototype model, while the scaled model attained increased accuracy for X>6D.

2.2 Introduction

Wake emanating from a Horizontal Axis Wind Turbine (HAWT) has been considered a complex flow phenomenon due to the presence of helical vortex filaments and the external strain field it introduces. A tip vortex and a hub vortex filament propagating downstream breaks up into smaller eddies via instability mechanisms such as leap-frogging or mutual induction. Momentum transport occurs from high velocity regions outside the wake to the velocity deficit region aft of the rotor. Turbulent mixing via the aforementioned eddies enhances the transport process. This phenomenon known as “wake-recovery” determines the output of the downstream turbines in a
wind farm setting. The level of turbulence intensity in the wake determines the fatigue experienced by the downstream turbines which may increase maintenance cost.

Predicting the wake characteristics and the turbine performance via high-fidelity computational methods has yielded detailed solutions with satisfactory performance predictions. However, the computational cost of the modeling unsteady turbulence and resolving blade geometries with finer grids has proven to be immense. Analytical wake and performance prediction methods provide a low-cost alternative to such simulations. Several methods have been suggested in the past to predict wake behind a rotor based on distribution shape of the velocity deficit. The most common profiles include the top hat and Gaussian distributions as shown in Figure 2.1.

![Figure 2.1: Top hat (a) and Gaussian Profile (b)](image)

Such analytical models also take wake expansion and momentum transport into account, thus providing an accurate picture of wake dissipation. However, the models lack an accurate prediction of turbulence induced flow parameters or unsteady flow structures present near the nacelle region. Therefore the models tend to provide better results for far-wake axial stations (X/D > 1.0), where the flow is being re-laminarized\(^3\)\(^4\). The near wake (X/D < 1.0) flow solutions are severely affected by the blade effects, aerodynamic interferences from the hub assembly/tower. The near-wake velocity deficit takes the shape of a bi-modal distribution due to the flow retardation from the hub. Accurate solutions for near-wake flow can be obtained at the expense of high fidelity numerical models with blade modeling and Large-Eddy-Simulations (LES).
2.3 Velocity deficit and performance predictions

Earlier models such as the Jensen model were based on the top hat distribution of velocity deficit. The method assumes there is a constant velocity deficit across the rotor for a given axial station. The Jensen model consists of two equations: a wake diameter calculation accompanied by a deficit prediction given by,

\[ D_w = D \left(1 + 2k_f \left(\frac{x}{a}\right) \right) \]  

\[ \frac{\Delta U}{U_{\infty}} = \frac{1 - \sqrt{1 - C_T}}{(1 + 2k(\frac{x}{a}))^2} \]  

where \( D_w \) denotes wake diameter while \( k_f \) governs wake expansion. Offshore simulations demand a \( k \) value of 0.04 and the on-shore simulations require \( k = 0.075 \). Velocity deficit \( \frac{\Delta U}{U_{\infty}} \) also incorporates the thrust coefficient (\( C_T \)). The Larsen model introduced a Gaussian approximation to the velocity predictions increasing the accuracy. Larsen also developed a second order accurate model that captures the bimodal nature of the near-wake velocity. The first order Larsen model, which also consists of a wake expansion model and a velocity deficit is given by,

\[ R_w(x) = \left(\frac{35}{2\pi}\right)^{\frac{1}{2}} (3c_1^2)^{\frac{1}{2}} (C_T A(x + x_0))^\frac{1}{2} \]  

\[ \frac{\Delta U(x,r)}{U_{\infty}} = -\frac{1}{9} (C_T A(x + x_0)^{-2})^{\frac{1}{2}} \left[ r^2 \left(3c_1^2 C_T A(x + x_0)\right)^{-\frac{1}{2}} - \left(\frac{35}{2\pi}\right)^{\frac{3}{10}} (3c_1^2)^{-\frac{1}{2}} \right] \]  

where the wake radius (\( R_w(x) \)) is given by a relationship between thrust coefficient and the area of the rotor. Parameters such as \( c_1 \) and \( x_0 \) are obtained through secondary calculations as shown in. Radial station is denoted by \( r \). Both models neglect the presence of a nacelle and a tower and generate 2D velocity contours. A sample solution pertaining to the wind tunnel experiment is
shown in Figure 2.2. The sample solution assumes a 10% ambient turbulence for the Larsen model and a 10 m/s incoming flow for both models. A prototype wind turbine (2MW NREL) with a 90 m diameter rotor has been used for this simulation with a thrust coefficient of 0.47. Jensen model uses a wake expansion coefficient of 0.04, assuming off-shore conditions.

The Figure 2.2 depicts the XY plane of the flow solution at the hub height ($Z = Z_H$). Velocities were normalized to $U_{\text{Hub}}$ (10.0 m/s). Upon observation, it is evident that the Jensen and Larsen models have preserved the wake expansion and wake recovery characteristics. Velocity at each axial station increases as the wake propagates thus suggesting wake recovery. The wake diameters along with the minimum velocities (max. deficit) were compared in Figure 2.3.

**Figure 2.2**: Jensen model (Left) and Larsen model (Right)

**Figure 2.3**: Wake diameter (left) and minimum wake velocity (right)
The Larsen model shows the largest wake expansion while the on-shore Jensen model overpredicts the minimum wake velocity at each axial station. Larsen model shows an exponential recovery within the first two diameters converging with Offshore Jensen model. Simplifications done to Jensen model requires the $c_T$ to be less than 1.0.

A new analytical model has been proposed to predict the wake of a HAWT based on a Gaussian velocity deficit distribution. The model assumes a boundary layer flow as an input condition along with $c_T$ in which the 3D velocity distribution can be computed as,

$$\frac{\Delta U}{U_\infty} = \left( 1 - \sqrt{1 - \frac{c_T}{8 \left( k^* \frac{x}{d_0} + 0.2 \sqrt{\beta} \right)^2}} \right) \times \exp \left( -\frac{1}{2 \left( k^* \frac{x}{d_0} + 0.2 \sqrt{\beta} \right)^2} \right) \left( \frac{z - z_h}{d_0} \right)^2 \left( \frac{y}{d_0} \right)^2 \right),$$

where $k^*$ start denotes wake expansion and $\beta$ is a parameter based on the thrust coefficient. Cross sections from the volumetric solution depicting wake velocities are shown in Figure 2.4 for a sample case with a boundary layer inflow.

![Figure 2.4: ZX (at Y/D=0) and ZY (at X/D=2.0) planes of analytical velocities](image)
The solution depicted in Figure 4 clearly depicts the velocity deficit aft of the turbine, wake recharging and expansion, and a bias in the deficit (asymmetric) due to the boundary layer. Span-wise cross section shows the cylindrical nature of the momentum deficit due to the rotor. The circumference of the rotor is highlighted using a dashed line. The boundary layer for this case follows a power-law profile and the wake velocities are normalized to $U_{\text{Hub}}$. However, Eq. (5) does not take ambient turbulence into account and the model is not recommended for predicting near wake data.

Based on the relationship given in Eq. (5) and the Jensen wake expansion model, an iterative algorithm can be constructed to predict $C_T$ the based on the maximum velocity deficit at a given axial station, obtained experimentally or computationally. The algorithm begins with an approximation for the $C_T$ along with the maximum experimental velocity deficit ($C(x)$), normalized axial station, and the blade diameter. A set of primary parameters including $\beta$, normalized standard deviation ($\sigma_d$), and the wake expansion parameter ($k^*$) can be calculated as,

$$\beta = \frac{11 + \sqrt{1 - C_T}}{2 \sqrt{1 - C_T}}$$  \hspace{1cm} (6)

$$\sigma_d = \frac{-C_T}{\sqrt{(8 C(x))^2 - 16 C(x)}}$$  \hspace{1cm} (7)

$$k^* = \frac{\sigma_d - \epsilon}{x/d}$$  \hspace{1cm} (8)

where $\epsilon = 0.2\sqrt{\beta}$. A velocity deficit distribution (2D) for the guessed $C_T$ value can be found using Eq.(5). An estimation for the mass flow rate can be established using the wake expansion equation in the Jensen model \(^6\) and the wake diameter, in which local mass flow rate for each span-wise (Z)
station is calculated. The total wake area has been subdivided into small span-wise sections with a \( \delta \) thickness as shown in Figure 2.5.

![Diagram](image)

**Figure 2.5:** local area \((dA)\) calculation

Mass flow rate \( \dot{m}_i \) local to each \( z \) station can be found using the local area sections and corresponding wake velocities. Integrating local mass flow ratios with corresponding velocity deficit values \((\Delta U_i)\) yields thrust \((T)\) and the \( C_T \) given by,

\[
T = \sum_{i=1}^{n} \dot{m}_i(\Delta U_i)
\]

\[
C_T = \frac{T}{\frac{1}{2} \rho U_{hub}^2 A_d}
\]  

(9)

The new thrust coefficient value is compared with the value obtained in the previous iteration in order to achieve convergence. If the convergence is not achieved, the newly calculated \( C_T \) will be set to the guess value and looped again. The current model is accurate for \( C_T < 1.0 \) and the velocity deficit from the tower and nacelle was neglected, as the resulting drag coefficient is smaller compared to the thrust coefficient of the rotor. A flow chart elucidating the construction of the algorithm is shown in Figure 2.6.
2.4 Results

For the first validation, A Large-Eddy Simulation (LES) conducted for a prototype wind turbine with a blade diameter of 80m, operating in an atmospheric boundary layer with a roughness of 0.00005 m (off-shore) has been used\(^8\). The hub height is 70 m and the span-wise normalized velocity profiles at several axial station has been utilized. Analytical velocity profiles along with the corresponding \(C_T\) predictions are elucidated in Figure 2.7.
Figure 2.7: Velocity profiles (LES Data)

The decrease in velocity deficit as the flow progresses can be observed in Figure 7 along with an expansion in the wake. The prototype turbine rotor has a thrust coefficient of 0.8 and the wake based predictions in the A significant error (~14%) in the $C_T$ predictions has been observed for $X/D = 3.0$. Both analytical and LES velocity predictions shows a significant disagreement in wake diameter and the location where maximum velocity deficit occurs. LES data shows a larger wake expansion occurring at $Z/D = 1.0$, slightly above the hub height. Such discrepancies have
contributed to the error in $c_T$ predictions at 3.0D. The $c_T$ predictions improve as the axial station increases with a 6% or less error for $X = 5D, 7D$ and 10D. The average of the predicted values for the given axial stations yields a value much closer to 0.8 with a 0.31% error.

A set of experimental velocity measurements in the span-wise (Z) direction, obtained through a multi-hole pressure probe (COBRA by TFI Inc.), was used for the second validation case. The experiment featured a scaled wind turbine (2MW NREL turbine scaled to 1:320) performing in atmospheric boundary layer wind. The hub height was set to 225 mm and the rotor diameter was 280 mm in the scaled down model. The rotor assembly contained a nacelle and a hub assembly and the experiment was conducted at the Atmospheric Boundary Layer (ABL) tunnel at Iowa State University. Analytical and experimental wake profiles are shown in Figure 2.8.

Both experimental and the analytical profiles show an expansion and the decrease in the velocity deficit as the flow progresses downstream. A slight downward shift of the maximum deficit from the hub location is observed in experimental profiles in comparison to the analytical profiles. The turbine rotor has a 5-degree angle of attack which drives the wake downward as the wake progress. The deficit values below the hub height show a better agreement with respect to the data points above the hub height. Predicted $c_T$ values were compared to the measured value (from JR3 transducer) of 0.47. Axial stations ($X = 3.0D, 6.0D, 9.0D$) predict the thrust coefficient with a 2% uncertainty which drastically reduces at $X = 12.0D$. The discrepancies between the experiment and analytical deficit profiles result in the over-prediction of $c_T$. 
2.5 Conclusions

An algorithm was constructed to predict thrust coefficient and the associated span-wise velocity
deficit distribution, given the turbine parameters, the maximum velocity deficit at a given axial
station and the incoming flow conditions. The Jensen model was used to determine the wake
diameter at each axial station and a Gaussian velocity deficit model was used to determine the
distribution. A guess for $C_T$ was provided initially, calculating a new value after each iteration until convergence was achieved.

Sets of experimental and computational velocity measurements were used to validate the algorithm. The computational data set stems from a large-eddy simulation conducted for a prototype wind turbine. The average $C_T$ value was predicted to be 0.8 which satisfactorily agrees with the design $C_T$ of the prototype. Near wake results show a 14% error, diminishing significantly in the far wake locations, thus proving the validity of the model in far-wake regimes. The experimental data, acquired for a scaled wind turbine, using a pressure probe also shows satisfactory agreement with analytical predictions. The error remains 2% or less for the comparison of measured and calculated $C_T$ values. Velocity profiles for both cases show a reduction in the momentum deficit due to wake-recharging and an axial expansion which reasonably agrees with the analytical trends observed. Slight discrepancies between the analytical and experimental data can be attributed to the pitch angle of the rotor and aerodynamic characteristics native to the scaled experiment such as the presence of a mid-span vortex filament in the near-wake region. The future work may entail capturing the near wake trends, especially the bi-modal velocity deficit distribution\textsuperscript{10}. Certain limitation in the current model includes the inability to predict performance for rotors with $C_T > 1.0$. A different formulation of the $\beta$ parameter may alleviate this limitation. Future validations could be conducted for different types of atmospheric boundary layer flows such as on-shore scenarios with high ambient turbulence intensity. The current algorithm may be employed by field engineers and experimentalists for faster predictions of $C_T$ and the far wake velocity distributions for off-shore turbines based on a single measurement of max velocity deficit at the hub height.
References


CHAPTER 3. IDENTIFYING COHERENT STRUCTURES

3.1 Experiment setup

The wake flow measurements were conducted at the Atmospheric Boundary Layer (ABL) wind tunnel at Iowa State University. The wind tunnel has a 20 m long test section with a 2.4 m width and a 2.3 m height. An offshore boundary layer was introduced with several rows of a metal chain upwind of the model turbine location. Velocity and turbulence intensity profiles obtained using a COBRA probe (flow measurement devices) are shown in Figure 3.1.

![Figure 3.1: Measured velocity (left) and turbulence intensity (right) profiles](image)

The measured velocity profile was compared with a known power law relationship which describes the boundary layer.

\[
\frac{U}{U_r} = \left( \frac{Z}{Z_r} \right)^\alpha
\]  

(1)
where $\alpha$ is assumed to be 0.11. The reference Z location ($Z_r$) and reference velocity ($U_r$) were taken as the hub height and the velocity at the hub height. The velocity profile was normalized to the velocity at hub height. Upon observation, it is understood that the recorded velocity measurements are in agreement with the power function. The ambient turbulence intensity at the hub height was 10%.

A three bladed horizontal axis wind turbine model (HAWT) scaled to 1:350 ratio to a 2MW industrial prototype by NREL was used for the purpose of this experiment. The rotor and nacelle assembly was constructed using a hard plastic material in a rapid prototyping machine. A metallic rod was used as the tower and the hub height was set to 23 cm from the floor of the test section. The airfoil information across the blade is shown in Figure 3.2. The blade design was based on ERS-100 prototype turbine blades developed by TPI Composites, Inc. The scaled model performs at a tip speed ratio of 5.0, where the optimum $C_p$ occurs. Diameter based Reynolds number ($R_e_D$) is set to 120,000 by adjusting the speed of the wind tunnel Parameters of the model wind turbine are given in Table (3.1).

### Table 3.1: Turbine model parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$R$ (mm)</th>
<th>$H$ (mm)</th>
<th>$d_{pole}$ (mm)</th>
<th>$d_{nacelle}$ (mm)</th>
<th>$\alpha$ (deg.)</th>
<th>$a$ (mm)</th>
<th>$a_1$ (mm)</th>
<th>$A_2$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimension</td>
<td>140</td>
<td>226</td>
<td>18</td>
<td>18</td>
<td>5°</td>
<td>68</td>
<td>20</td>
<td>35</td>
</tr>
</tbody>
</table>
A high resolution PIV system was used to record information of the ZX plane at Y=0 station (symmetrical plane). This plane was illuminated using a ND-YaG laser that emitted two consecutive pulses of 200mJ with a wavelength of 532nm. The air flow was seeded with \( \sim 1\mu m \) diameter oil droplets using a smoke generator. Two 16 bit CCD cameras were used to acquire the necessary images both near field and far field. Both cameras and the laser were synced using a delay generator in order to acquire consecutive image pairs. The user was able to adjust the delay between pulses in order to control the shutters of each camera. Acquired image pairs were stored in a work station. Using this high resolution PIV system, two types of experiments were conducted. A “free run” case demanded the acquisition of 1000 PIV measurements for the purpose of ensemble-averaged flow statistics while the “phase locked” experiment yielded measurements at different phase angles of the blade. Phase locked tests were conducted using an external triggering mechanism and a secondary delay generator. A tachometer measured the
RPM of the rotor and acted as the triggering device. Figure 3.3 depicts a schematic of the experiment setup.

![Experiment Setup](image)

**Figure 3.3**: Experiment Setup

Acquired image pairs were subjected to frame-to-frame cross-correlation in order to obtain the instantaneous velocity distributions. A commercial software package, Insight 3G, was used for this purpose, wherein a 32 x 32 pixel interrogation window was used with a 50% effective overlap between the windows.

### 3.2 Ensemble-averaged results

The post-processed instantaneous velocity fields were statistically averaged and the contour solution is shown in Figure 3.4. A region of velocity deficit due to kinetic energy harvesting can be observed aft of the wind turbine. The vertical velocity gradient depicts the shear layer at a contour level 0.92. Increased contour density near the nacelle and the hub can be attributed to the highly chaotic nature of the fluid. High fluctuations in the flow can be observed closer to the
nacelle, blade tips and the tower. The grey area contains erroneous information due to the shadow of the turbine assembly upon laser illumination.

![Normalized Velocity Magnitude](image)

**Figure 3.4:** Ensemble-averaged velocity distribution

A region of velocity deficit due to kinetic energy harvesting can be observed aft of the wind turbine. The vertical velocity gradient depicts the shear layer at a contour level 0.92. Increased contour density near the nacelle and the hub can be attributed to the highly chaotic nature of the fluid. High fluctuations in the flow can be observed closer to the nacelle, blade tips and the tower. The grey area contains erroneous information due to the shadow of the turbine assembly upon laser illumination.

A vorticity solution for phase angle of 0 degrees is shown in Figure 3.5. Two distinct vortex cores emanating from the tip and the mid span location break down into smaller eddies at roughly ½ rotor diameters from the tower. The magnitude and the size of the vortex cores decrease as the wake propagates. Root vortices shed at $Z/D = 0.1$ location rotates in the opposite direction to the tip and mid-span vortex patches as anticipated. Root vortices interact with the
flow over the nacelle and decays rapidly downstream. Velocity measurements closer to the
nacelle, turbine blades and the tower have a higher uncertainty due to the reflections.
Governing dynamics of turbine wake can be formulated with respect to vorticity transport as the
flow is dominated by helical vortex filaments. The transport equation for the incompressible and
viscous flow is given by,

\[ \frac{\partial \tilde{\omega}}{\partial t} + (\tilde{u} \cdot \nabla) \tilde{\omega} = (\tilde{\omega} \cdot \nabla) \tilde{u} + \nu \nabla^2 \tilde{\omega} \quad (2) \]

**Figure 3.5**: Normalized vorticity (phase angle = 0 deg.)

The first term on the right hand side stands for the stretching of vortices which can be observed
as and the second term governs viscous diffusion. Vortex stretching cannot be studied in the
current study as volumetric data has not been utilized. Our investigation will predominantly
focus on the advection based momentum transport. Viscous forces can be neglected due to the
high \( Re_D \).
3.3 Proper-Orthogonal-Decomposition

Proper Orthogonal Decomposition (POD) has been an effective method to identify principal structures embedded in turbulent flows by linear decomposition and reconstruction. All of POD modes, which are spatially orthogonal, are ranked by their kinetic energy. Thus, if predominant large-scale structures exist in the turbine wake, they can be extracted by POD and be represented in the first few modes. Singular Value Decomposition (SVD) can be utilized if the number of degrees (rows) is smaller than the number of snapshots (columns). For the purpose of this manuscript, linear decomposition method is discussed³.

A step-by-step construction of the method of snapshots is provided in this section. The subscript denotes the degree of freedom and the superscript defines the image number⁴,⁵.

All the fluctuating velocity components (time averaged components removed) are arranged in a matrix U as:

\[
U = \begin{bmatrix}
u_1 & u_2 & \cdots & u_N \\
\vdots & \vdots & \ddots & \vdots \\
u_M & u_M^2 & \cdots & u_M^N \\
v_1^1 & v_1^2 & \cdots & v_1^N \\
\vdots & \vdots & \ddots & \vdots \\
v_M^1 & v_M^2 & \cdots & v_M^N 
\end{bmatrix}
\] (3)

where M is the number of spatial discrete points and N is the number of the PIV snapshots, which represent the spatial and temporal resolutions of the PIV data respectively. The eigenvalues and eigenvectors of the auto-covariance matrix are calculated as:

\[
\tilde{C} \cdot \mathbf{A}_i = \sigma_i \cdot \mathbf{A}_i \tag{4}
\]

where, \( \tilde{C} = U^T \cdot U \)

and the eigenvalues \( \sigma \) are ranked in a descending order. Each eigenmode is obtained by projecting matrix U onto each eigenvector and then normalized by its norm as:
where \( \phi^i = [\phi^1 \phi^2 \ldots \phi^N] \). The coefficients of each mode can be obtained as

\[
\mathbf{a}^n = \phi^n^T \cdot \mathbf{u}^n
\]  

\[ \text{(6)} \]

Reconstructing the fluctuations of \( n^{th} \) instantaneous solution can be performed by the summation of each mode vector multiplied by the corresponding modal coefficient:

\[
\mathbf{u}^n = \sum_{i=1}^{L} a_i^n \phi^i = \phi \cdot \mathbf{a}^n
\]

\[ \text{(7)} \]

The user has the ability to determine the order of reconstruction (\( L \)) based on the modal energy.

An \( L^{th} \) order POD reconstruction of the \( n^{th} \) instantaneous solution can be obtained by adding the ensemble averaged velocity components:

\[
\mathbf{U}^n = \mathbf{\bar{U}} + \mathbf{u}^n
\]

\[ \text{(8)} \]

where \( \mathbf{\bar{U}} \) is the ensemble-averaged velocity.

POD analysis was performed on a selected region that encompasses the shear layer. Selection of a proper ROI in the wake eliminates noisy and erroneous measurements, thus increasing the accuracy of reconstructions. The selected region is depicted in Figure 3.6.

Figure 3.6: Regions of Interest (ROI)
The velocity magnitude and the vorticity solutions for the selected ROI are presented in Figure 3.7. The velocity magnitude is normalized to velocity at the hub height and the vorticity is normalized to the rotor diameter and the hub velocity.

**Figure 3.7:** Ensemble averaged velocity and vorticity for R1 (Mode 0)

Vorticity has been used as a preliminary parameter in understanding coherent vortical structures buried in a turbulent flow field. However, shortcomings in vorticity include depicting vorticity values where circulating flow does not exist. Swirling strength criterion has been introduced to mitigate this issue and elucidate the coherent structures\(^7\). The direction of the swirling strength of the rotation is determined by the sign of vorticity and based on the formulation of a velocity gradient vector as given by,

\[
\Delta \vec{u} = \begin{bmatrix}
\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\
\frac{\partial v}{\partial x} & \frac{\partial v}{\partial y}
\end{bmatrix}
\]

\[(9)\]
Eigenvalues of the velocity gradient can be calculated as,

$$\lambda = \frac{tr(\Delta \tilde{u}) \pm \sqrt{tr(\Delta \tilde{u})^2 - 4 \ast det(\Delta \tilde{u})}}{2}$$

(10)

Where complex eigenvalues will result if, \(\sqrt{tr(\Delta \tilde{u})^2 - 4 \ast det(\Delta \tilde{u})} < 0.0\). The complex part of the eigenvalue \(\lambda_{ci}\) can be multiplied with \(sgn(\omega_z)\) to obtain swirling strength (SS) as given by,

$$\lambda_{ci} = \sqrt{|tr(\Delta \tilde{u})^2 - 4 \ast det(\Delta \tilde{u})|}$$

(11)

$$SS = sgn(\omega_z) \ast \lambda_{ci}$$

(12)

It can be deduced that the R1 has successfully encompassed the vortex dominant region based on Figure 8, in which a low-order reconstruction can be used to highlight swirling coherent structures. As for the initial results, we observe the cumulative modal energy parameter, calculated from the sorted Eigenvalues. Monitoring the modal energy of the first 2 modes while increasing the number of snapshots provided a convergence criterion to establish the number of snapshot required for the analysis as shown in Figure 3.8.

Figure 3.8: Cumulative modal energy (left) and POD convergence study (right)
The convergence study utilized 996 snapshots and modal energy related mode 1 and 2 show asymptotic behavior after 500 snapshots depicting convergence. First mode contained most of the modal energy (~45%). While the second mode contained around 6% of the modal energy. The cumulative modal energy asymptotes to 1.0 as expected with increasing modal energy. The first few energetic modes correspond to dominant flow features buried within the flow solution. The less dominant modes may correspond to measurement noise and smaller eddies. However, the minimal number of modes required to reconstruct the velocities should be determined. Taking the gradient of the cumulative modal energy curve yields the rate of energy addition to the reconstruction as elucidated in Figure 3.9.

![Gradient of Cumulative Energy vs. Mode Number](image.png)

**Figure 3.9.** Rate of energy addition

Drastic changes in energy addition can be observed up to 5 modes and the rate steadily declines as the mode number increases, eventually approaching zero. Therefore, the low-order reconstructions utilized up to 5 modes. Normalized vorticity along with swirling strength are
parameters of interest, while vorticity serves as a quantifiable representation of dominant structures. A low-order reconstruction \( (a_i \phi_i) \) for an arbitrary snapshot is used derive swirling strength contours which are depicted in Figure 3.10.

\[ (a) \ a_1 \phi_1 + \ldots + a_5 \phi_5 \quad (b) \ a_1 \phi_1 + \ldots + a_5 \phi_5 + \bar{U} \]

(c) Instantaneous Snapshot

**Figure 3.10**: Reconstructions and the Instantaneous

Summation of the fluctuation modes (eg. \( a_1 \phi_1 + a_2 \phi_2 + \ldots + a_n \phi_n \)) results in more pronounced flow features as depicted in Figure 3.10 (a) and (b). The alternating direction of the vortical fluctuations indicates the periodic or coherent nature of the induced flow field by the advecting filament. As the number of cumulative modes increase, the shape and the formation of the coherent structures become apparent. Addition of the time-averaged components to the
fluctuations according to Eq. (8) results in a low-order reconstruction of the original instantaneous solution preserving the buried dominant coherent structures. Summation of all individual fluctuation modes and the time-averaged solution will result in the original instantaneous measurement. A comparison between the low order reconstruction and the instantaneous measurement as illustrated in Figure 3.10 highlights the nature of reconstruction process as a low-pass filter. As the number of modes increases, distinct vortex patches emerge in the reconstructions, never before observed identified in the instantaneous realizations. Reconstructions clearly show vortex patches emanating from the tip and the mid-span (60% of blade radius) locations.

Further confirmations of this claim can be presented using a vorticity based POD (V-POD) method, where out of plane vorticity acts as the variable of interest. This approach reduces the number of degrees of freedom in the snapshot matrix and the magnitude of eigenvalues corresponds to enstrophy of each mode. Low order reconstructions based on V-POD (for X<0.6D) are shown with modal enstrophy content in Figure 3.11. Reconstructions have utilized the first three modes from V-POD based on the enstrophy addition rate. The reconstruction contains 14% of the total enstrophy. The reconstructions clearly depict the alternating vorticity fluctuations in mid-span and tip vortex regimes. The fluctuations also show symmetry until X = 0.4D, in which it transitions to an asymmetric one due to the differences in advecting velocities of each vortex filament. The tip vortex propagates a slightly faster than the mid-span filament and the mid-span filament shows expansion which leads to decrease in proximity to the tip vortex. Further investigation into the beginning of the shear layer (X>0.65D) using V-POD yielded the following low order reconstructions elucidated in Figure 3.12.
\[ a_1 \phi_1 + \ldots + a_3 \phi_3 \]
\[ a_1 \phi_1 + \ldots + a_3 \phi_3 + \bar{U} \]

**Figure 3.11:** Vorticity based POD Results

\[ a_1 \phi_1 + \ldots + a_6 \phi_6 \]
\[ a_1 \phi_1 + \ldots + a_6 \phi_6 + \bar{U} \]

**Figure 3.12:** V-POD Reconstructions for \( X > 0.65D \)
A change in core size can be observed due to the presence of an external strain field. Emergence of vortices acting in the opposite direction can be seen in the low order reconstruction along with a “stretching” of vortex cores. It can be deduced that the decrease in proximity between the filaments observed earlier leads up to lengthening of the vortex cores due to short wave instabilities and counter-rotating vortical perturbations, thus causing the break-up.

Dissipation of circulation of consecutive vortex patches using $\Gamma = \int \omega \, ds$ provides an insight to the strength of the filaments and how they mutually induce velocity perturbations on each other. A phase-averaged data set corresponding to a blade angle of 0° has been utilized where the swirling strength criterion was used to approximate the shape of each patch as shown in Figure 3.13. Circulation strength of each vortex patch is given in Table 3.2.

**Table 3.1**: Turbine model parameters

| Vortex | $|r|$ |
|--------|-----|
| T1     | 0.020 |
| M1     | 0.040 |
| T2     | 0.017 |
| M2     | 0.030 |

**Figure 3.13**: Phase-averaged data (R1+R2)

Initial Mid-span vortex shedding consists of twice the circulation compared to the tip vortex roll-up. Circulation of each vortex patch has decreased due to dissipation effects. This dissipation is a resultant of the turbulent mixing present in the shear layer as the vortices break-up after $X = 0.65D$. 
3.4 Momentum and vorticity transport

In a vortex dominated flow, the momentum transport can be either aided or hindered by the presence of coherent and random fluctuating components. The turbulent momentum transport phenomenon for viscous incompressible flow can be derived using the Reynolds Averaged Navier-Stokes (RANS) equations, where the Reynolds stress (RS) accounts for the momentum transport as given by,

\[
\left[ \frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \bar{u}_j \bar{u}_i}{\partial x_j} \right] = \frac{\partial}{\partial x_j} \left[ \mu \left( \frac{\partial \bar{u}_i}{\partial x_j} \right) - \bar{\rho} \bar{u}_i' \bar{u}_j' \right] - \frac{\partial \bar{p}}{\partial x_i} \tag{13}
\]

The momentum transport is governed via two primary mechanisms as observed in the RANS equation. Microscopic transport happens as a result of viscous stress term while macroscopic transport can be attributed to the Reynolds stress term \( \bar{\rho} \bar{u}_i' \bar{u}_j' \). Vorticity transport given in Eq.14 can also be Reynolds averaged to obtain a vorticity transport term analogous to the RS.

\[
\frac{\partial \Omega_i}{\partial t} + \bar{u}_j \frac{\partial \Omega_i}{\partial x_j} = - \frac{\partial \bar{\omega}_i' \bar{u}_j'}{\partial x_j} + \frac{\partial \bar{\omega}_j' \bar{u}_i'}{\partial x_j} + \Omega_j \frac{\partial u_i}{\partial x_j} + \nu \frac{\partial^2 \Omega_i}{\partial x_i \partial x_j} \tag{14}
\]

where, \( \Omega_i \) depicts the mean vorticity. Vorticity transport due to turbulent fluctuations is given by, \( \frac{\partial \bar{\omega}_i' \bar{u}_j'}{\partial x_j} \) which can be observed by the divergence of lamb vector. Change in vorticity due to fluctuation in strain rates, a three dimensional term analogous to vortex stretching is given by, \( \frac{\partial \bar{\omega}_j' \bar{u}_i'}{\partial x_j} \). Current analysis observes the vorticity and momentum transport terms due to random fluctuations and organized wave components of velocity fluctuations. RS term can be written as a resultant of vorticity transport and the stretching force. The relationship stems from the nonlinear terms that govern advection and the conservation of momentum term.

\[
\left[ \frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} \right] = \frac{\partial}{\partial x_j} \left[ \nu \left( \frac{\partial u_i}{\partial x_j} \right) \right] - \frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} \tag{15}
\]
\[
\frac{\partial u_i u_j}{\partial x_j} = -\varepsilon_{ijk} u_k \omega_k + \frac{\partial}{\partial x_l} \left( \frac{1}{2} u_j u_j \right)
\]

(16)

The advection term can be substituted back into the momentum equation, where a time averaged steady flow equation can be presented as\textsuperscript{12},

\[
-\frac{\partial}{\partial x_l} \left( \frac{\bar{P}}{\rho} + \frac{1}{2} \bar{u}_j \bar{u}_j + \frac{1}{2} \bar{u}'_j \bar{u}'_j \right) + \varepsilon_{ijk} (\bar{u}_j \Omega_k + u'_j \omega'_k) + \frac{\partial}{\partial x_j} \left[ \nu \left( \frac{\partial \bar{u}_i}{\partial x_j} \right) \right] = 0
\]

(17)

For 2D flow, downstream velocity is higher than the cross stream velocity, while the cross stream derivative of mean quantities is higher than the downstream derivative\textsuperscript{13}. As \(\frac{1}{2} \bar{u}_j \bar{u}_j \gg \frac{1}{2} \bar{u}'_j \bar{u}'_j\) and \(\bar{u} u'_j \gg \frac{1}{2} u'_j u'_j\), a simplified expression can be derived as,

\[
\frac{\partial (-u'_1 u'_2)}{\partial x_2} = u'_2 \omega'_3 - u'_3 \omega'_2
\]

(18)

where, the cross stream derivative of Reynolds stress is related to the vorticity transport terms.

The \(u'_2 \omega'_3\) term corresponds to the vorticity transport on cross stream direction(Z or Y), while \(u'_3 \omega'_2\) determines the stretching force associated with varying eddy sizes.

The triple decomposition method is investigated in this manuscript as a means of isolating fluctuating velocity and vorticity components embedded in the shear layer. Both ensemble-averaged and phase-averaged data sets are used for this extraction while Reynolds stress governs momentum transport phenomenon. As per formulation, the instantaneous velocity can be presented as a combination of mean value, contribution of organized wave and random fluctuating components given by

\[
u = \bar{u} + \bar{u} + u' \quad (19)
\]

The method was presented by Hussain and Reynolds in 1986\textsuperscript{14}. The phase averaged velocity can be obtained by averaging velocity measurements for each individual phase\textsuperscript{15}. The Reynolds
stress can be calculated by determining the contributions from coherent flow structures (unsteady) and turbulent flow artefacts. The turbulent velocity components can be calculated by

\[ u' = < u > - u \] (20)

where \(<u>\) denotes phase averaged velocities. Coherent velocity components can be calculated using

\[ \tilde{u} = < u > - \bar{u} \] (21)

Where the time-averaged flow field is denoted by \( \bar{u} \). Based on the definitions developed for the turbulent and coherent velocity components, Reynolds stress can be calculated as

\[ \frac{r_{ij}}{-\rho} = (u' + \tilde{u}) \cdot (v' + \tilde{v}) = u'v' + u'\tilde{v} + v'\tilde{u} + \bar{u} \bar{v} \] (22)

where \( u \) and \( v \) denote velocity components on x and y directions with ensemble averaging. As coherent flow components are uncorrelated to the turbulent components it can be concluded that \( u'\tilde{v} = v'\tilde{u} = 0 \). Therefore the normalized Reynolds stress can be calculated as:

\[ r_{ij} = - \frac{(\bar{u}\tilde{v} + \bar{u'v'})}{U_{hub}^2} \] (23)

The contributions of both coherent and random fluctuations is visualized in Figure 3.14.

**Figure 3.14**: Reynolds stress components: \( \bar{u}\tilde{v} \) (left) and \( \bar{u'}v' \) (right) for R1+R2

The contribution from \( \bar{u}\tilde{v} \) was calculated to be 1.5% of \( \bar{u'}v' \) in (R1+ R2) and concentrated near the turbine blade and the nacelle. The vortex street in the wake region contains Reynolds
stress concentrations (magnitude of 0.004) that dissipate into a broader shear layer starting at $X/D = 0.8$. The RS available aft of the nacelle and tower assembly correspond to the presence of mechanical turbulence. The wake region that encompasses the shear layer has been highlighted in Figure 3.15, where a comparison between decomposed Reynolds stress and swirling strength and strain rate has been drawn.

![Comparison between vorticity(black), strain (blue) and Reynolds stress (color)](image)

**Figure 3.15**: Comparison between vorticity(black), strain (blue) and Reynolds stress (color)

A discrepancy between the locations of high circulation and the locations of stress concentrations was observed suggesting the presence of vortices has hindered the vertical momentum transfer in the near wake region. The majority of RS (<60%) is distributed after the point of instability where turbulent eddies are formed. The strain rate concentrations overlap the regions of high RS or low circulation depicting an inverse relationship between production and vorticity. It can be deduced that the presence of helical vortical filaments tend to hinder the “wake recharging” and should be dissipated immediately. In the absence of volumetric vorticity and velocity data, R1 and R2 regions were analyzed for the vorticity advection term associated related to the cross stream derivative of RS as shown in Figure 3.16.
Random fluctuations show dominance in the vorticity transport phenomenon compared to the organized wave contributions localized on the first tip and mid-span vortex pair. Transport due to random fluctuations is concentrated in the high circulation regions of the vortices. The magnitude of the vorticity transport terms decreases as the vortices advect downstream. This can be further elucidated by plotting the cross-stream derivative of Reynolds stress as shown in Figure 3.17.

The relationship between the dissipation and the cross-stream derivative, as shown in Eq. 18, is further highlighted with highest largest changes occurring at the perimeters of the vortices. This
strain field can be speculated to cause the change in shape of vortex cores, thus resulting in short-wave instability. The Lamb vector \( \vec{l} = \vec{\omega} \times \vec{u} \), which is also referred to as the “vortex force” represents the Coriolis acceleration of a velocity field, which can be used to represent vortex stretching and advection terms such as,

\[
\vec{u} \cdot \nabla \vec{u} = \vec{\omega} \times \vec{u} + \frac{1}{2} \nabla \vec{u}^2 \quad (24)
\]

Irrotational flow results in Beltrami flow which results in Lamb vector. The divergence of Lamb vector provides an insight to the non-homogeneous dynamics of momentum transport with respect to the potential “unwinding” of vortices. The divergence of the lamb vector is a combination of enstrophy and the Flexion product as shown in,

\[
\nabla \cdot \vec{l} = \vec{u} \cdot \nabla \times \vec{\omega} - \vec{\omega} \cdot \vec{\omega} \quad (25)
\]

Positive values of \( \nabla \cdot \vec{l} \) can only occur from positive Flexion products while the negative values are attributed to high enstrophy values. The ensemble averaged solution of \( \nabla \cdot \vec{l} \) for the near-wake region is shown in Figure 3.18.

![Figure 3.18: Divergence of Lamb vector (color) and ensemble-averaged vorticity (solid lines)](image)
The regions where $\nabla \cdot \vec{l} > 0$, flexion acts as a release mechanism for momentum and kinetic energy. Enstrophy stays as a storage mechanism. This can be further elucidated by point out that $\nabla \cdot \vec{l}$ is the source term in the Poisson equation as shown by,

$$\nabla \cdot \vec{l} = -\nabla^2 \phi = -\nabla^2 \left( \frac{p}{\rho} + \frac{u^2}{2} \right) \quad (26)$$

Where $p$ denotes pressure and $\rho$ denotes density. Positive flexions represent straining motions as shown in Figure 19 which are located at the top of each vortex pair. This could be due to the interactions between tip and mid-span vortices as well as tip vortices interacting with high velocity flow outside the influence of the wake. Negative Lamb divergence suggests highly vortical flow (high angular momentum) in which flow energy is minimal.

### 3.5 Conclusions

- High fidelity PIV measurements were acquired for a scaled wind turbine performing in ABL winds with an ambient turbulence of 10% at hub height. The scaled turbine performs at a tip speed ratio of 5.0 where the optimum $C_p$ occurred for the scaled model. Over 1000 instantaneous measurements were acquired along with phase-locked measurements for 8 blade positions. Ensemble averaged results depicted the presence of a tip and a mid-span vortex filaments advecting downstream and breaking up at 0.5D-0.6D into smaller turbulent eddies.

- A low-order modal reconstruction of $X<0.5D$ using Proper-Orthogonal-Decomposition showed the presence of an underlying vortex structure embedded in the wake that carries ~60% of the kinetic energy. Highly circulatory regions were highlighted via swirling
strength criterion. Vorticity based POD (conserving enstrophy) was employed to identify coherent structures at X>0.65D, the region after the break-up of vortices. Majority of the enstrophy (~80%) were distributed among less energetic modes suggesting an absence of coherence in the turbulent wake after vortex break up.

- Momentum transport analysis using RS elucidated the hindrance to wake-recharging caused by both helical vortex filaments. More than 60% of the RS and production was concentrated in the region X>0.65D (critical vortex break-up point). Turbulent vorticity transport depicted vorticity dissipation occurring in the highly circulatory perimeters of the vortex patches, thus showing a correlation with the cross stream derivative of RS as anticipated. Analysis of Lamb vector and its divergence depicted the possible regions of high strain occurring between tip and mid-span vortex cores as well as tip vortex cores and the high velocity flow outside the wake region. This can be speculated as a reason for the rapid deformations in the vortex cores or short-wave instabilities.

References


CHAPTER 4. DYNAMIC MODE DECOMPOSITION

4.1 Introduction

The wind turbine wake is a complex flow phenomenon which includes a multitude of dominant flow structures. Previous computational and experimental studies conducted for an industry-standard three-bladed HAWT, have identified a tip vortex roll-up emanating from the rotor and a hub vortex with three times the circulation of the tip vortex\(^1\). A Gaussian velocity deficit distribution has been identified in the axially expanding wake formations. As the helical discreet tip vortices break-up downstream\(^2\), a shear layer forms where turbulent mixing between high velocity streams outside the wake region mix with the low velocity flow inside the wake. A schematic of these phenomenon are shown in Figure 4.1\(^3\). The turbulent structures present in the near wake region (X/D≤1.0) consist of coherent and random fluctuating components which can be extracted by advanced principal component analysis techniques such as POD (Proper-Orthogonal Decomposition) and Dynamic Mode Decomposition (DMD). While the near wake flow analysis provides the designer with crucial spatial-temporal flow characteristics such as tip and hub vortices. Advanced statistical methods can reveal valuable information about the advection and the breakup of these structures resulting in far-wake turbulence. The intensity of far wake turbulence determines the induced fatigue of the downstream wind turbines as well as the wake recovery aspects\(^4\).
This manuscript presents an analysis of the near-wake region utilizing Exact DMD method. The Standard DMD method was first introduced by Peter Schmid in 2008. While methods such as POD rely on a time-averaged spatial-correlation matrix, DMD relies on a higher degree polynomial to determine temporal behaviors. The POD method produces a set of orthogonal modes that lack any temporal information while the DMD modes alleviate this issue while reducing the truncation error associated with POD based ROM (Reduced-Order-Modeling). A set of high resolution PIV (Particle-Image Velocimetry) measurements of the near wake region were arranged to represent a time series to be subjected to the DMD analysis. The snapshots are assumed to be linearly dependent as they increase in number. The resulting mode shapes and their spatial and temporal behavior (modal growth/decay) were examined. DMD has been used as an analysis tool for both computational and experimental studies. However, using DMD in experimental studies has proven to be a challenge due to measurement noise and the low frequency in data acquisition, especially in measurement techniques such as PIV. Variations of the DMD
formulation have been adopted to mitigate the measurement noise issues such as Exact DMD and Least-Squares DMD\(^9\). However, resolving the issues coupled with low frequency data acquisition has proven to be a challenge. A novel method has been proposed which utilizes time resolved (TR) data from a high frequency intrusive technique such as a probe and non-time-resolved (non-TR) PIV measurements acquired simultaneously. The non-TR data is subjected to a POD analysis, where the modal coefficients along with probe measurements are used to predict a time resolved data set. This requires the knowledge of oscillatory dynamics and integrating complex statistical algorithms ranging from MTD-mLSE and Kalman filters as smoother/filter\(^{10}\). Mitigating the noise in time resolved measurements is crucial to a DMD analysis as the measurement noise introduces a biasing effect to the standard DMD formulation. Total least-squares (tls-DMD) approach assumes a decomposed approach where the initial snapshot matrices can be written as, \(Y = X + X_n\), where \(X_n\) denotes a noise matrix. The tls-DMD algorithm finds the best fit solution by minimizing Frobenius norms of both \(X\) and \(X_n\) matrices, thus producing de-biased eigenvalues\(^{20}\).

### 4.2 Experimental setup

The recent experimental investigations were performed at the Atmospheric Boundary Layer (ABL) wind-tunnel located at the Aerospace Engineering Department at Iowa State University. The wind tunnel has a 20 m long test section with a 2.4 m width and a 2.3 m height. An off shore boundary layer condition was created to obtain the necessary power law velocity profile along the height. The velocity profile was recorded using a COBRA probe (Turbulent Flow Instrumentation) and the data was compared to the theoretical relationship as shown in Figure 4.2. The Figure 4.3 depicts turbulence intensity variations along the z direction.
The power law relationship is given by,

\[
\frac{U}{U_{hub}} = \left[ \frac{Z}{H} \right]^\alpha
\]

Where, \(\alpha\) is assumed to be 0.11\textsuperscript{11}. The velocity profile was normalized using the velocity at hub height 5.45 m/s. Upon observation, it is understood that the recorded velocity measurements are in a good agreement with the power function.

A three bladed horizontal axis wind turbine model (HAWT) scaled to 1:350 ratio to a 2MW industrial wind turbine was used for the purpose of this experiment\textsuperscript{12}. The rotor and nacelle assembly was constructed using a hard plastic material in a rapid prototyping machine. A metallic rod was used as the tower and the hub height was set to 23 cm from the floor of the test section. The airfoil information across the blade is shown in Figure 4.4 along with specific design parameters for the model turbine. The blade design was based on ERS-100 prototype turbine blades developed by TPI Composites, Inc.
Table 4.1: Wind Turbine Geometry Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( R ) (mm)</th>
<th>( H ) (mm)</th>
<th>( d_{\text{pole}} ) (mm)</th>
<th>( d_{\text{nacelle}} ) (mm)</th>
<th>( \alpha ) (deg.)</th>
<th>( a ) (mm)</th>
<th>( a1 ) (mm)</th>
<th>( A2 ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimension</td>
<td>140</td>
<td>226</td>
<td>18</td>
<td>18</td>
<td>5°</td>
<td>68</td>
<td>20</td>
<td>35</td>
</tr>
</tbody>
</table>

The scaled model was set to operate at a tip speed ratio of 5.0, similar to the operating conditions of the prototype. The diameter based Reynolds number was around 120,000 for this experiment and the rotation speed was 2200 RPM for the rotor. A high resolution PIV system was used in the study that recorded information of the ZX plane that passes through the symmetrical plane (\( Y = 0 \)) of the wind turbine. This plane was illuminated using a ND-YaG laser that emitted two consecutive pulses of 200mJ with a wavelength of 532nm. The air flow was seeded with ~ 1μm diameter oil droplets using a smoke generator. Two 16 bit CCD cameras were used to acquire the necessary images both near field and far field. Both cameras and the laser were synced using a delay generator in order to acquire consecutive image pairs. The user was able to adjust the delay between pulses in order to control the shutters of each camera. Acquired images were stored in a
work station. Using this high resolution PIV system, two types of experiments were conducted. A “free run” case demanded the acquisition of 1000 PIV measurements for the purpose of ensemble-averaged flow statistics while the “phase locked” experiment yielded in measurements at different phase angles of the blade. Phase locked tests were conducted using an external triggering mechanism and another delay generator. The extremely high rotational speed of the turbine coupled with the 1Hz data acquisition rate hindered our efforts to obtain a properly time-resolved data set for our analysis. The Figure 4.5 depicts a schematic of the experiment setup.

![Experiment Setup](image)

**Figure 4.5: Experiment Setup**

Phase averaged PIV measurements were utilized for this analysis to create a free run sequence. Snapshots for a given phase were divided into sub-groups, where the average of each group was considered as a snapshot. This approach reduced the measurement noise while preserving phase based the unsteady flow characteristics. The post-processed velocity distributions were properly sequenced to resemble a free-run experiment, thus providing a data set for the DMD analysis.
4.3 Results and discussion

Ensemble-averaged quantities were calculated from the post-processed instantaneous solutions while phase locked solutions were used to depict the phase decomposed coherent structures embedded in the flow. A near wake velocity magnitude solution along with a vorticity solution (Phase Angle = 0 deg.) are shown in Figure 4.6.

![Normalized Velocity Magnitude (U/U_{ref})](image1)

![Normalized Vorticity (\omega D/U_{ref})](image2)

**Figure 4.6:** Ensemble-Averaged velocity magnitude(left) and Normalized vorticity (right)

The velocity magnitude solution clearly shows a momentum deficit region behind the wake. Higher velocity regions are outside of the wake separated by a shear layer where the blade vortices break up to form turbulent eddies. The turbulent eddies aid in the momentum recovery process in the aft of the wake which can be quantified with parameters such as Reynolds stress. Velocity vectors upstream of the turbine have been eliminated due to the errors induced by the shadow effects. The phase-averaged vorticity solution elucidates the presence of a tip vortex, a mid-span vortex and a hub vortex. The tip vortex and the mid-span vortex have been identified as the principal coherent structures in the previous analysis conducted using Proper-Orthogonal Decomposition. The circulation magnitude of the mid-span vortex patch is twice as large as the
circulation magnitude of the tip vortex. Both mid-span and the tip vortices break up at the axial location $X/D = 0.5$ due to the presence of mutually-induced and self-induced strain fields.

### 4.3.1 Dynamic Mode Decomposition

Dynamic Mode Decomposition (DMD) is a novel method to extract both spatial and temporal features embedded in the flow. The dominant coherent structures are assigned to frequencies. This is analogous to the POD method where modes are ranked according to their kinetic energy content. The DMD provides the eigenvalues and eigenvectors of the best-fit linear system relating a snapshot matrix (time = t) and a time-shifted version of the snapshot matrix (time = t + dt) at some later time. The formulation of the DMD method can be done performed using singular value decomposition (SVD) or the QR decomposition. The SVD based DMD is more robust and suitable for experimental data sets while QR based decompositions are better suited for computational data sets. For the purpose of this manuscript, an extension of the DMD method known as the Exact DMD based on SVD is discussed.

A set of time resolved snapshots, containing $m$ number of realizations, can be arranged with $u$ and $v$ velocity components on each column as given in,

$$X = \begin{bmatrix} x_0 & \cdots & x_{m-1} \\ \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots \end{bmatrix}, \quad X' = \begin{bmatrix} x_1 & \cdots & x_m \\ \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots \end{bmatrix}$$  \hspace{1cm} (1)

Where a relationship can be derived using a linear operator,

$$X' = AX$$  \hspace{1cm} (2)

The $X$ matrix can be decomposed using Singular Value Decomposition (SVD) as shown in,
\[ X = U \Sigma V^* \] (3)

where, the conjugate transpose of the non-unitary matrix \( V \) is denoted by \( V^* \). Using the decomposed matrices, a least squares fit \( \tilde{A} \) can be computed by,

\[ \tilde{A} = U'A = U'X'V\Sigma^{-1} \] (4).

The linear operator is eigen-decomposed to obtain the eigenvalues as,

\[ \tilde{A}W = \Lambda \] (5)

Where \( W \) denotes Ritz eigenvectors while \( \Lambda \) stands for Ritz eigenvalues which are approximations of the initial values. A set of unscaled DMD modes can be constructed using the previous information,

\[ \phi = X'V\Sigma^{-1}W \] (5).

Amplitude based scaling for the DMD modes can be computed as,

\[ K = \frac{\phi}{X(:,1)} \] (6)

Where the final scaled modes is attained by taking the cross product between the modal matrix and the diagonal of the amplitudes.

\[ \tilde{\phi} = \phi \times diag(K) \]

The second norm of each mode will yield the corresponding modal energy\(^{17}\). The temporal behavior of each mode can be assessed by observing the growth rate and its respective frequency\(^{18}\).

\[ \sigma = \frac{\log |\Lambda|}{dt} \]

\[ \omega = \frac{\text{arg}(\Lambda)}{dt} \]
A DMD analysis was performed on selected regions in the shear layer based on the spatial-temporal evolution of the tip and mid-span vortices. Selection of a proper ROI in the wake eliminates noisy and erroneous measurements, thus increasing the accuracy of mode shapes while decreasing the computational time. The three regions of interest are shown in Figure 4.7.

![Fig 4.7: Region of Interest (ROI)](image)

Upon observing the phase locked data, it can be deduced that R1 encapsulates the area where discreet vortices are present, while R2 contains aftermath of vortex break-up. An overlap window, R3 was also specified to capture the temporal dynamics of the vortex break-up process. The analysis was focused on several parameters pertaining to the spatial-temporal behavior of DMD modes such as modal energy, growth/decay rate, and the eigenvalues. Results obtained for R1 region are shown in Figure 4.8 with several modes highlighted. For the purpose of this initial analysis, five modes have been selected ranging from the highest to the lowest modal energy. The first mode corresponds to the ensemble averaged solution for the given time series.\textsuperscript{17}
Figure 4.8: Results – R1 and Normalized Vorticity of Mode 0 \( \frac{\omega D}{U_{hub}} \)

The frequency of 118 Hz (mode 1) contains the second highest amount of modal energy while mode 2 contains roughly the same amount of energy with a temporal growth rate. The negative frequencies correspond to the complex conjugate resulting from the decomposition process. Observing the real and imaginary components of the eigenvalues determines the stability and energy of each mode. Eigenvalues with positive real components contain the highest energies with less damping or positive growth and are worthy of analysis. Mode 4
(lowest energy) and mode 3 are high frequency and low energy modes with severe temporal decaying. Vorticity plots corresponding to each mode are elucidated in Figure 4.9.

![Normalized Vorticity plots](image)

(a) Mode 1 ($\phi_1$)  
(b) Mode 2 ($\phi_2$)  
(c) Mode 3 ($\phi_3$)  
(d) Mode 4 ($\phi_4$)

**Figure 4.9:** Normalized Vorticity plots ($\frac{\omega_D}{U_{hub}}$)

Mode 0 depicted in Figure 4.8 depicts a vorticity solution which shares an identical trend with the ensemble-averaged solution. The effects of mid-span and tip vortex patches can be observed,
where the mid-span vortex contains twice the circulation of the tip vortex\textsuperscript{19}. The 1\textsuperscript{st} mode, which contains the second highest energy, elucidates an axisymmetric mode propagating downstream, while undergoing a temporal decay or a negative growth rate. A comparison can be made to a POD analysis conducted with 900+ instantaneous snapshots obtained from the same experiment. A low order reconstruction of velocity fluctuations is shown in Figure 4.10 which depicts the dominant axisymmetric mode\textsuperscript{19}. The POD mode also shows a spatial decay in vorticity and changes to the vortex cores in downstream. The 2\textsuperscript{nd} mode depicts an out-of-phase relationship with the 1\textsuperscript{st} mode along with severe deformation in the alternating vortex cores. Temporal growth of such a mode demonstrates elliptical/short-wave instability, thus contributing to the break-up of the vortical structures. A distinct pattern of three spatially periodic vortex patches in the span-wise direction were observed in the 3\textsuperscript{rd} DMD mode, which has a higher temporal decay rate, the highest frequency and significantly lower modal energy than the 1\textsuperscript{st} and the 2\textsuperscript{nd} mode. The 4\textsuperscript{th} mode which contains the lowest energy, shows vortex patches corresponding to tip and mid-span filaments with high circulation at the center of the field. These patches spatially decay after 0.25D.

![Normalized Vorticity](image)

**Figure 4.10:** Dominant axisymmetric POD mode (velocity based)
The subsequent DMD analysis of R2 and R3 windows focuses on the most energetic and temporally dominant modes that do not correspond to the ensemble-averaged solution. Modal energies along with the temporal behavior (growth or decay) are presented here and the ensemble averaged correlation (mode 0) has been highlighted. Results from analyzing the R2 window are shown in Figure 4.11. The R2 region encompasses majority of the shear layer which occurs after vortices breakup downstream. It was investigated for spatial-temporal behavior due to its importance in recharging the flow via turbulent mixing.

**Figure 4.11:** Results for R2 window including modal vorticity plots
A vorticity plot reveals artifacts of spatially decaying mode 1 to have the highest energy and a temporal growth. The 2\textsuperscript{nd} mode with similar modal energy and a slight temporal dampening again elucidates severe deformations in the vortex cores. The disappearance of the coherent vortical structures can be attributed to the increasing levels of turbulent kinetic energy and the momentum transport. The analysis suggests a transition in the flow field from an axially symmetric mode to an asymmetric mode during the initial break-up process. This warrants a DMD analysis on the region of instability identified from 0.4D to 0.8D. Results for the R3 window are shown in Figure 4.12.

**Figure 4.12:** Results for R3
The highest energetic mode in R3 analysis has the highest growth rate of all the modes. The axisymmetric mode observed in R1 changes its nature to asymmetric due to the difference in propagation velocities of each helical filament, where the tip vortex filament has a higher axial velocity than the mid-span filament. Changes in the modal shapes can be attributed to short-wave instability and mutually induced perturbations by consecutive helical turns which manifest into break-up of the filaments. Finally, we present a full field analysis (R1+R2) that encompasses the whole shear layer ranging from the discreet vortices to the turbulent eddies after instability in Figure 4.13. The full field analysis captures a dominant mode (1st mode) which extracts the alternating vortex patches observed before along with its spatial decay. The mode also shows temporal decay and clearly shows the axial expansion of the wake. The mode encompasses the axisymmetric and asymmetric nature of the vortex fluctuations before and after X/D = 0.5 and clearly depicts the reducing proximity between the filaments, which may lead up to elliptical-like instabilities changing vortex core radii and in turn causing break-up.

**Figure 4.13:** Results for R1 + R2
4.4 Conclusion

Flow characteristics of the wake induced by a three-bladed HAWT were measured using a high-resolution PIV system. Instantaneous realizations obtained were subjected to a fundamental statistical analysis where ensemble averaged and phase-averaged solutions were analyzed. Ensemble averaged solutions revealed the presence of a velocity deficit area behind the rotor and a high velocity flow above the rotor. Presence of a shear layer was also observed between the velocity deficit region and the high velocity flow region outside the wake. Phase-averaged solutions elucidated the presence of discrete vortex patches, resulting from multiple helical filaments intersecting normal to the field of view. A tip vortex, a vortex filament emanating from the mid-span location and the strong hub vortex that contours the nacelle of the wind turbine were identified. The mid-span vortex has twice the circulation of a tip vortex and both filaments reach instability via break-up at a critical axial station of $X/D = 0.5$.

Dynamic Mode Decomposition, an advanced statistical tool designed for extracting spatial and temporal dominant structures in turbulent flow was utilized for in-depth analysis of the phase-averaged data sets. Multiple regions of interests were declared encompassing the shear layer due to its importance in wake recovery. Dominant modes were selected based on their energy content and the temporal growth or reduced decay rates. The R1 window ($0 < X/D < 0.5$) contained discreet
tip and mid-span patches before the critical axial station of instability. The analysis yields two axisymmetric, spatially decaying high energy modes with a small visible offset in the axial direction. Mode 1 shows a slight temporal decay, while the 2nd mode shows a growth and an out-of-phase relationship with the 1st one. The offset in advection is caused by the difference of velocities of the tip and mid-span filaments. As the vortices break-up into smaller eddies in the R2 window (X/D>0.5), the axisymmetric modes manifest into asymmetric ones. Despite their spatially decaying nature, the modes show positive growth rate for the 1st mode and slight decay for the 2nd mode. This prompted a further investigation involved declaring an overlapping region (R3) between R1 and R2 where the instability occurs. The high energy mode in R3 also shows bias towards an asymmetric configuration with temporal growth contrary to the temporal decay observed in R1. It can be deduced that the axisymmetric nature of the underlying turbulent patterns diminish as the time progress as the asymmetric nature shows growth after the transition, causing an instability. A full field analysis (R1+R2) elucidates both asymmetric and axisymmetric modes and shows an overall temporal decay in the mode.

This analysis should be improved with a data set that contains more phase angles, while trying to preserve the high-frequency data that gets filtered in the phase averaging process. It can be further validated using an extended DMD method such as tls-DMD, while reducing the bias induced by measurement noise. A different formulation of the DMD based on the temporal axis and a single spatial axis (x-t or y-t) can yield spatial growth/decay rates and the temporally dominant mode shapes. A new energetic criterion weighted by the growth/decay rate should take temporal dominance into account when presenting modal energy. The high rotation frequency
(36 Hz) coupled with the extremely low data acquisition rates still hinders our ability to a time-resolved data set and this can only be addressed through a computational data set.

References


CHAPTER 5: MODELING TURBINE NEAR WAKE CHARACTERISTICS VIA VORTEX METHODS

5.1 Introduction

Coherent structures in the wind turbine wake and their roles in the stability of the wake have been analyzed through many high fidelity grid based computational techniques. The wind turbine flow in the near wake region (X/D<1.0) contains very complex flow characteristics and the traditional grid based methods require substantial rotational turbulence modeling and upstream turbulence modeling. With the recent advances in computer technology, Lagrangian vortex methods have gained popularity in simulating a multitude of microscopic and macroscopic flow phenomenon. Vortex methods also present the user with advantages such as absence of numerical viscosity, easier formulation as the vorticity transport equation gets rid of pressure term, and the absence of complex grid generation. In this manuscript, we propose blob and filament based vortex-methods (2D and 3D) to model the near-wake vortex structures and the conditions leading up to the vortex “break-up” phenomenon due to mutually and internally induced strain fields.

This study was inspired by a wind turbine wake analysis study conducted at the Atmospheric Boundary Layer (ABL) wind tunnel at Iowa State University, using a scaled mode (1:350) of a 2 MW commercial wind turbine by TPI composites\(^1\). Velocity fields in the near wake and far wake locations were measured by a high resolution digital PIV system. A phase-averaged measurement depicted in Figure 5.1, elucidated the presence of a secondary vortex filament emanating from 0.3D of the blade. The circulation strength in the mid-span vortex patch is twice as large as the tip vortex patches. The vortex patches break up around X/D = 0.5 axial station into turbulent eddies, which has not been observed in previous wind turbine wake studies which showed a dominating tip vortex propagating into far wake regions (X/D>1.0) before break-up. We suggest modeling this
problem with a corrected rotor model to introduce tip and mid-span and hub vortices and observe the induced instabilities as the wake propagates downstream.

![Normalized Vorticity Plot](image)

**Figure 5.1**: Normalized Vorticity Plots based on PIV

A specific region of interest is highlighted in Figure 1(a) which clearly shows four discreet vortex patches present before the critical break-up point. An ensemble-averaged solution shows both filaments axially expanding and converging after X/D = 0.5 as highlighted in Figure 5.1(b) where interactions between filaments cause the formation of turbulent eddies in the shear layer. Dynamics of the helical vortex filaments are governed by advection, viscous diffusion and the stretching of the helical vortex filaments. A plethora of studies done previously managed to simulate the rotor wakes of helicopters and wind turbines using blob-based and filament based vortex methods. Filament based methods have utilized free-wake and prescribed wake models, while employing simplified rotor models such as lifting line, actuator line and actuator disk models. C. He and J. Zhao used a viscous vortex blob based method to simulate 3D flow structures shed by a helicopter rotor using the lifting line theorem, where the blade circulation
was calculated using Kutta-Joukowski theorem. Near-wake downwash velocities from the solutions were compared to experimental data set with satisfactory agreement. Leishman studied the ground effect on helicopter wake using a filament based method, where a mirrored wake solution was used to simulate the ground plane. Leishman solves the free-wake method in an iterative and a time marching method and concludes that best results are obtained from the time-marching algorithm. H. Abedi used free-wake and prescribed wake models to simulate the propagation of a tip and hub vortex filaments which showed instabilities propagating to far wake. Abedi utilized the Vortex-Lattice-Method (VLM) based on the lifting line theorem, in which the blade surface is replaced by vortex panels. Vortex methods also allow the users to model flow instabilities, such as mutual induction, long-wave instability and short-wave instability which play dominant roles in rotor wake problems.

5.2 Methodology

Early work done in developing potential functions for velocity fields induced by helical filaments was investigated as a preliminary analysis. A potential function was developed for the stream function solution induced by a propagating filament using modified Bessel functions in Kapteyn-Kummer series. The stream function inside the helix is given by,

\[
\Psi = \frac{\Gamma \rho^2}{4\pi k^2} - \frac{\Gamma a \rho}{\pi k^2} \sum_{m=1}^{N} K'_m \left( \frac{am}{k} \right) I'_m \left( \frac{\rho m}{k} \right) \cos(m\theta) \quad \rho < a
\]  

(1)

While the stream function outside the filament is given by,

\[
\Psi = -\frac{\Gamma}{2\pi} \ln \rho - \frac{\Gamma a \rho}{\pi k^2} \sum_{m=1}^{N} K'_m \left( \frac{\rho m}{k} \right) I'_m \left( \frac{am}{k} \right) \cos(m\theta) \quad \rho > a
\]  

(2)
This causes a singularity at the location of the filament \((\rho = a)\) and should be omitted during the calculations. The super-position principle for stream functions can be used to derive an expression for multiple filaments where, \(a_1 < \rho < a_2\) region between the helixes will reveal the dynamics present. However, this approach doesn’t take the unsteady effects into account which prompted an investigation into the formulation of unsteady Lagrange methods.

Lagrange particle methods has been explored initially by Alexander Chorin, where a vortex roll-up has been simulated using a de-singularized Biot-Savart relationships\(^6\). In the recent work done, criterions such as particle strength exchange via viscous splitting techniques have been employed for an accurate modeling of viscous effects. Lagrange particle methods also present problem of non-physical velocity gradients induced by nodal concentrations which can be alleviated using “remeshing” techniques. Recent developments in Tree codes and Fast Multipole Methods (FMM) have reduced the computational costs due to direct integration from \(N^2\) to \(N \log (N)\)\(^7\).

Governing dynamics of vortex methods in viscous, incompressible flow stem from the vorticity transport equation as given by,

\[
\frac{\partial \vec{\omega}}{\partial t} + (\vec{u} \cdot \nabla)\vec{\omega} = (\vec{\omega} \cdot \nabla)\vec{u} + \nu \nabla^2 \vec{\omega} \quad (3)
\]

The first term on the right hand side stands for the stretching of vortices which is a three dimensional phenomenon and the second term governs viscous diffusion. The advection term \(((\vec{u} \cdot \nabla)\vec{\omega})\) can be solved using a kernel function of varying order and the Biot-Savart law. For the purpose of this abstract we present the formulation of a two dimensional algorithm which will later be extended to an unsteady 3D vortex filament algorithm. The induced velocity components can be calculated using the Biot-Savart Kernels\(^2\),
\[ \vec{u} = \sum_{j=1}^{N} K(r, \delta)(\vec{x} - \vec{x}_j) \times \Gamma_j \quad (4) \]

Where \( \delta \) represents the width of the blob and \( r = |\vec{x} - \vec{x}_j| \). The order of the kernel function can be changed based on the simulation but for the purpose of this manuscript we have used a second order kernel (a Gaussian distribution) as given by,

\[ K(r, \delta)(\vec{x} - \vec{x}_j) = \frac{(\vec{x} - \vec{x}_j)}{2\pi r^2} \left(1 - e^{-\frac{r^2}{\delta^2}}\right) \quad (5) \]

The formulation of the kernel functions are subjected to change depending on the simulation run.

A variety of methods have been suggested to model viscous diffusion from core-spreading technique to Particle Strength Exchange (PSE) method\(^8\). PSE method redistributes circulation strength among particles as advanced in time while conserving the total circulation in all the vortex blobs. The time derivative of \( \Gamma \) can be presented as,

\[ \frac{d\Gamma_p}{dt} = \nu \epsilon^{-2} \sum_q (\Gamma_q - \Gamma_p) \eta \left(\frac{x_p - x_q}{\epsilon}\right) \quad (6) \]

Where \( p \) denotes the particle of interest and the \( q \) denotes the rest of the vortex blobs. The function \( \eta \) can be determined as,

\[ \eta(\vec{x}) = \frac{C}{1 + |\vec{x}|^2} \quad (7) \]

for \( |\vec{x}| \leq 2 \). If the \( |\vec{x}| > 2 \), the value of the function goes to zero. The constant \( C \) has a value of 0.835. A simplistic approach based on core-spreading technique has been used in the present study as where the time dependent circulation can be calculated as,

\[ \Gamma = \Gamma_0(1 - e^{4\nu t}) \quad (8) \]
where, the initial circulation of a blob is given as $\Gamma_0$. However this approach contains numerical inconsistencies which can be alleviated via reducing the size of the vortex blobs\(^7\).

Both advection and PSE equations are advanced in time using a 4\(^\text{th}\) order Runge-Kutta scheme.

In a three dimensional formulation presented in the final manuscript vortex stretching effects will also be taken into account. The final velocity for each particle at the end of a time step can be calculated as,

$$\bar{\vec{U}} = \bar{\vec{U}}_\infty + \bar{\vec{U}}_{\text{induced}} \quad (9),$$

where, $\bar{\vec{U}}_\infty$ represents the free-stream. The vorticity for each 2D vortex blob can be derived as\(^2\),

$$\bar{\omega}_j = \sum_{j=1}^{N_j} \Gamma_j \frac{e^{-r^2/\delta^2}}{\pi \delta^2} \quad (10)$$

The point vortices can be interpolated to a Cartesian grid in order to plot the vorticity contours in a given field. The final manuscript will also cover re-meshing techniques to overcome nonphysical particle concentrations in high velocity gradients\(^9\).

Expanding the 2D particle algorithm to a 3D framework for rotor simulations increases the number of particles, thus increasing the number of calculations per time step. In order to alleviate this issue, a vortex ring based approach has been proposed. The consecutive helical turns can be represented as vortex rings with axisymmetric expansion and contraction. This allows each vortex to be represented by a point in a cylindrical coordinate system\(^{21}\). The advection of the vortex ring can be described using elliptical integrals of first and second kind ($K, E$) as,

$$U_z = \frac{\Gamma_i}{2\pi \sqrt{(z-z_i)^2 + (r+R_i)^2}} \left[ K(m) + \frac{R_i^2 - r^2 - (z-z_i)^2}{(z-z_i)^2 + (R_i - r)^2} E(m) \right] \quad (11)$$
\[ U_r = \frac{\Gamma_i (z - z_i)}{2\pi r \sqrt{(z - z_i)^2 + (r + R_i)^2}} \left[ K(m) - \frac{R_i^2 + r^2 + (z - z_i)^2}{(z - z_i)^2 + (R_i - r)^2} E(m) \right] \]  

(12)

where, \( m = 4r R_i / ((z - z_i)^2 + (r + R_i)^2) \). The circulation strength can be deduced from an actuator disk approximation as given by,

\[ \Gamma_i = \frac{1}{2} U_\infty C_T \Delta t \]  

(13)

The time interval \( \Delta t \) depends on the rotational frequency of the wind turbine, thus the helical pitch. The illustration in Figure 5.2 shows the shedding and the propagation of a single vortex ring.

![Fig 5.2: Coordinate system for vortex ring approach](image)

The 2D algorithm was extended to a three-dimensional framework using a free-wake, filament based method. The free-wake method takes boundary layer flow into account while the filaments are free to deform, thus having the ability to capture the expansion of the wake and the conditions leading up to an instability\(^3,10\). Stretching effects of the vortices can be observed along with the
viscous diffusion. The vector representation for the velocity induced by a unit filament (AB) at a
given point (C) is elucidated in Figure 5.3.

![Diagram of velocity induced at point C](image)

**Fig 5.3:** Velocity Induced at point C

Velocity induced at the given point from the unit filament is given by,

\[
V_{\text{ind}} = \frac{\Gamma}{4\pi r_1 r_2} \left( \frac{\vec{r}_1 \times \vec{r}_2}{r_1 r_2 + \vec{r}_1 \cdot \vec{r}_2 + \delta^2} \right)
\]

(14)

where, \( \delta \) represents a smoothing factor or a cut-off radius. This expression can be further
expanded to add the viscous effects to the vortex core using a simple diffusion model as
\(^{10,11}\),

\[
V_{\text{ind}} = K_v \frac{\Gamma}{4\pi} \left( \frac{r_1 + r_2}{r_1 r_2 + \vec{r}_1 \cdot \vec{r}_2} \right)
\]

(15)

where,

\[
K_v = \frac{h^n}{(r_c^{2n} + h^{2n})^{1/n}}
\]

(16)

The \( h \) stands for the perpendicular distance from the filament to the point of interest given by,

\[
h = \frac{|\vec{r}_1 \times \vec{r}_2|}{|\vec{L}|}
\]

(17)
The factor \( n = 2 \) is recommended for the tip vortices of a rotor and the core radius can be found using a growth model \( r_c = \sqrt{4\alpha \nu t} \) where \( t \) represents the current time and \( \alpha = 1.25643 \). Kinematic viscosity is denoted as \( \nu \). A flow chart has been presented in Figure 5.4 depicting the flow of information and the time integration loop. For the purpose of this manuscript, we will be using an unbounded filament with known circulation.

![Flow chart](image)

**Figure 5.4**: Construction of a simple algorithm

### 5.3 Validation and results

Preliminary results for these simulations were obtained from the stream function representations discussed previously using modified Bessel functions. An iso-surface tangential to the propagating filament (+Z direction) is shown in Figure 5.5, along with its stream-function. In the stream function representation, the filament is intersecting normal to the XY plane at (1,0)
location. At the point of intersection, a high concentration of streamlines is present suggesting the presence of a sharp velocity gradient in which complex flow features are present.

Figure 5.5: Velocity iso-surface (left) and stream-function (right)

A stream-function representation for a tip vortex filament alongside a mid-span vortex filament was developed using the superposition principle. Theoretical velocities were compared to the experimental solutions obtained from ensemble-averaged PIV measurements as shown in Figure 5.6.

Figure 5.6: stream-function (left) and velocity comparison (right)
The complexities of the flow is revealed at $X = 0.6$ and $X = 1.0$ where the filaments intersect orthogonally with the plane of interest. The stream function solution is asymmetric in the radial direction with streamlines converging and diverging near singularities suggesting significant gradients in velocity fluctuations. The velocity distributions between filament intersections show a rapid decrease in velocity magnitude and an immediate increase as the observer gets closer to the tip vortex. The analytical solution manages to capture the decreasing trend in velocity magnitude aft of the mid-span filament but doesn’t capture the gradual increase towards the tip vortex. The current model doesn’t evaluate velocity induced by the consecutive turns in $z$ direction or the diameter of the filament (core size). The effects of the ambient turbulence also do not factor in to the model.

### 5.3.1 Kelvin Helmholtz Instability (Validation 1)

Kelvin-Helmholtz instability occurs due to a velocity shear in a single continuous fluid. This is a short-wave instability which can be simulated with a de-singularized kernel functions given as$^{20}$,

$$
\frac{dx_i}{dt} = -\frac{1}{2N} \sum_{j=1,i\neq j}^{N} \frac{\sinh(2\pi(y_i - y_j))}{\cosh(2\pi(y_i - y_j)) - \cos(2\pi(x_i - x_j)) + \delta^2} \tag{18}
$$

$$
\frac{dy_i}{dt} = -\frac{1}{2N} \sum_{j=1,i\neq j}^{N} \frac{\sinh(2\pi(x_i - x_j))}{\cosh(2\pi(y_i - y_j)) - \cos(2\pi(x_j - x_k)) + \delta^2} \tag{19}
$$

where the kernels are de-singularized using by declaring a blob width of $\delta^{12}$. The initial Lagrange grid was generated using with the following circulation strengths and spatial perturbations.

$$
x_i = \Gamma_i + 0.01 \sin(2\pi\Gamma_i) \quad y_i = -0.01 \sin(2\pi\Gamma_i)
$$
The circulation strengths are distributed among the points as $\Gamma_i = (i - 1)\Delta\Gamma$. The evolution of this instability in the time is shown in Figure 5.7. The simulation was run using 400 blobs and a blob width of 0.5. As the time progresses, the vortex filament starts to deform with a counter-clockwise moment. However, the simulation doesn’t take surface tension of the material into account. This reproduction of KH instability phenomenon paved the way to the development of a generalized solver based on Biot-Savart law and a better understanding of vortex blob methods.

**Figure 5.7:** Evolution of the KH Instability
5.3.2 Elliptic Wing Tip Vortices Simulation (Validation 2)

Based on the work by Krasney\textsuperscript{13}, We also reproduced the case of tip vortices shed by an elliptically loaded wing. A vortex filament with equally spaced blobs was declared as an initial grid. The circulation strength changed from positive to negative along the filament with the particles in the middle having the lowest circulations. Minima and maxima occurred at the end of the filaments. The simulation was run for 4 seconds with 1000 vortex blobs and the final results are shown in Figure 5.8.

![Figure 5.8: Tip Vortex roll-up at different times](image)

The Lagrange grid and the circulation strengths were interpolated on to a regular grid in order to construct a vorticity plot as shown in Figure 5.9 using an interpolation kernel.
The solution was symmetric and produced tip vorticity patches of opposite signs as intended. A qualitative observation with the literature confirmed the validity and the accuracy of the algorithm. This initial simulation with an inviscid flow assumption was configured to include viscous effects via core-spreading (CS) and comparison is shown in Figure 5.10.

**Figure 5.9:** Tip vortices in a regular grid

**Figure 5.10:** Viscous diffusion model
The solutions obtained from core-spreading viscous diffusion scheme shows variations in vortex roll-up and the expansion of the filament compared to the inviscid case. Introduction of viscosity has adversely affected the rate of roll-up on the tip vortices as well as the stretching of the filament. A high velocity gradient can be seen at the core of each vortex which may lead to numerical instabilities if neglected.

5.3.3 Formation of self-similar vortex sheets (Validation)

The formation of the self-similar vortex sheets can also be studied using the de-singularized Biot-Savart kernel\textsuperscript{14}. Position of the Lagrange particles is advanced in time via fourth order RK4 and the initial grid location and the corresponding circulation is given by,

\[ x(\alpha, 0) = \cos(\alpha), \quad y(\alpha, 0) = \sin(\alpha), \quad \text{and} \quad \Gamma(\alpha) = (1 - \cos(\alpha))/2.0 \]

where, \(0 \leq \alpha \leq \pi\). The vortex ring has an initial velocity and the initial and the final results are shown in Figure 5.11.

![Figure 5.11: Formation of vortex rings (at t = 0 and t = 0.075s)](image)
The final solution depicts a roll-up in identical vortex rings with an extending tail in the axis of symmetry. Further simulations require a smoothing scheme where Lagrange points can be added between rapidly expanding filaments to prevent non-physical artifacts.

5.3.4 Wind Turbine Rotor Simulation

A simplified rotor model based on the actuator disk model was implemented to model the tip vortices emanating from a rotor\textsuperscript{15}. A vortex blob is generated at each tip location, at every time step, with opposite circulation strengths and an initial unit velocity. The circulation strength magnitude for each tip vortex is given by,

\[
\Gamma = \frac{\Delta P}{\rho} \Delta t \quad (20)
\]

where \(\Delta P = \frac{1}{2} \rho U^2 C_T\). A thrust coefficient \(C_T\), was determined assuming the optimum axial induction (a = 1/3). The case was run for 5.0 seconds and the results are shown in Figure 5.12.

![Figure 5.12: Particle locations (Right) and Interpolated vorticity solution (Right)](image_url)

The solution shows symmetry along the axis \(Y = 0.0\) as the wake propagates downstream. Vorticity was calculated for individual points and interpolated on to a regular grid. The
positive and negative circulation strengths had resulted in two tip vortex filaments with equal and opposite vortex strengths. The wake has undergone expansion as the vortex blobs propagate downstream.

This 2D method can be further extended to simulate the interactions between a mid-span and tip vortex, where a secondary stream of vortices is shed from the R = 0.3 location analogous to what has been seen in the PIV measurements. The circulation of the mid-span vortex has been set to a value twice as high as that of the tip vortex filament. The aggregate behavior of the discreet vortices in XY plane for a normalized hub velocity of 1.0 is provided in Figure 5.13. The mid-span vortex blobs show roll up in the near-wake regions, while the tip vortices roll-up in the far-wake. The wake shows an expansion in the near-wake region and a decrease in proximity between the mid-span and the tip vortex blobs at X = 0.5D, as highlighted in red, paving the way for a short wave instability. Short wave instability will lead to merging of vortical cores thus causing break-up and diffusion into the far wake.

![Vortex interactions in 2D](image)

**Figure 5.13:** Vortex interactions in 2D

The 2D representation can be further improved using the axisymmetric vortex ring approximation discussed earlier. The leap frogging of the vortex due to contractions and
expansions were captured by this method along with the effects induced by the external strain fields represented by the Kernel functions. Results shown in Figure 5.14 depict two simulations of rotor flow ($C_t = 0.47$),

(a) Isolated tip vortex propagation

(b) Tip Vortex influenced by a mid-span vortex analogous to our experiment

Figure 5.14: Vortex interactions in axisymmetric solver
Presence of a mid-span vortex has caused the primary tip vortex filament to become unstable in the near wake region of the rotor. In the presence of an isolated tip vortex, leap-frogging among consecutive helical turns can be observed in the far wake regions. However, we can only observe the conditions leading up to the break-up along with the aggregate behavior of shed vortex ring throughout a given time period, due to the modeling constrains embedded in this method.

Short wave instability can also be simulated using the vortex blobs. Sample solutions for two vortices with similar and different circulation distributions are shown in Figure 5.15. Lagrange grid based solution was interpolated into a 2D Cartesian grid where the potential function was obtained.

![Diagram](image.png)

(a) $\Gamma_1 = \Gamma_2$

(b) $\Gamma_1 > \Gamma_2$

**Figure 5.15**: Stages of a core instability potential function

Short-wave instabilities or elliptical instabilities which take place between vortices with similar circulations yield a symmetric elliptical vortex after they merge\(^{16}\). An asymmetric solution is obtained for the merging between two vortices with different circulations, similar to the situation observed between mid-span and tip vortices.
The three-dimensional filament based algorithm was utilized to run inviscid, incompressible simulations of the vortex behavior. A Kelvin wave assumption was used to determine the initial geometry of an un-bounded vortex filament. Normalized input including wave radius and the wave length has been used for the initial conditions. No boundary conditions or a specific rotor model has been used. The Kelvin wave equations are given by\textsuperscript{17},

\begin{align*}
y &= a \cos(kx - \omega t) \\
z &= a \sin(kx - \omega t)
\end{align*}

where \( k \) stands for wave number and \( a \) stands for the amplitude of the wave. Results obtained from the filament based algorithm are shown in Figure 5.16. As the time progresses both filaments showed expansion. The axial propagation was neglected for the moment to focus on the span-wise effects.

Vortex stretching term doesn’t have to be explicitly solved as each point is part of a continuous filament. As shown in Figure 5.16, the mid-span vortex stretches surpassing the tip vortex filament. The mid-span vortex gets closer to the tip vortex, which could lead up to break-up instability. Some of the instances where both filaments have converged on to each other are highlighted in red boxes as potential candidates for break-up phenomenon. Such qualitative trends obtained from the filament based algorithm agreed with trends observed in the 2D vortex-blob algorithm as shown in Figure 5.14.
5.4 Conclusion

Vortex blob methods (2D) and filament based methods (3D) were employed to understand the behavior of flow structures observed in the near wake regions of a HAWT. PIV wake measurements of a scaled (1:350) turbine revealed the presence of a tip vortex, a mid-span vortex and a hub vortex that dissipates aft of the nacelle. The tip and the mid-span vortex advect downstream until $X/D = 0.5$, where they start to break-up due to the presence of an external
strain field. The vortex blobs break-up into turbulent eddies creating a shear layer where momentum is transferred from high velocity fluid outside wake to the low velocity fluid elements. This is also known as wake recharging which plays a crucial role in determining the distance between each turbine in a wind-farm setting. Absence of volumetric measurements presented a challenge in identifying a primary mechanism behind vortex break-up experimentally. Unsteady vortex methods allowed us to perform a qualitative investigation into the near-wake region, with a relatively lower computational cost and a smaller implementation time line.

Implementation of the vortex blob based method showed its ability to perform a variety of validations, including vortex roll-up in an elliptically loaded wing, KH instability and the deformation of a self-similar vortex ring. The vortex blob method coupled with an actuator disk model was used to simulate the aggregate behavior of tip vortices and mid-span vortices. The mid-span vortex, with twice the circulation of tip vortex blobs, showed roll-up behavior in the near-wake region. A convergence between tip and mid-span vortices was observed prior to X/D = 0.5. This was further verified by the axisymmetric simulations, where the introduction of mid-span vortices with twice the circulation strength of tip vortices leads to instability in near-wake regions due to reduced helical pitch and converging behavior. Such convergence leads into short-wave instabilities in the vortex cores, which could result in the final-break up into smaller eddies as the vortices merge to create asymmetric vortex structures. Simulation of unbounded tip and mid-span vortex filaments via a three-dimensional vortex filament algorithm also confirmed the converging behavior previously observed as a precursor to the short-wave instability.

As for the future work, a rotor model will be introduced to the filament based solution with a criterion to handle instability. Lifting line based rotor models are currently being investigated for
this purpose. This will place proper boundary and initial conditions to ensure the accuracy of the filament based simulations. Using a tree code or a Fast Multipole Method (FMM) may improve the computational costs that arise due to $N^2$ calculations that has to be performed at every given time step. Simulations resulting in severe vortex expansion or stretching may lead to non-physical numerical artifacts that can be mitigated via a smoothing criterion which introduces grid points into expanding filaments. A vortex-in-cell approach may also alleviate the non-physical numerical effects due to grid points congregating at regions where high velocity gradients are present. However, this approach also introduces numerical diffusion and interpolation errors which tend to grow as the time progresses.

References


CHAPTER 6: MEASURING PERFORMANCE OF UPWIND AND DOWNWIND TURBINES

6.1 Introduction

Off-shore wind energy applications related to Horizontal-Axis-Wind-Turbines have been gaining momentum in the last few years. This is predominantly due to the space requirements imposed in wind farm siting. The distance between two turbines required for the maximum wake recharging increases the competition for space on land. Moving the conventional turbine prototypes to off-shore farms brings a brand new set of challenges. Turbines need to adapt to the extreme weather conditions present in the ocean, challenges in maintenance and tidal conditions. As a result of these requirements, downwind turbines have been gaining popularity, especially the designs by Hitachi Heavy Industries [6]. Downwind turbines have the rotor assembly located downstream from the nacelle, while the rotor of an upwind design is located in front of the nacelle as shown in Figure 6.1.

Figure 6.1: Upwind and Downwind Turbines
Downwind turbines reduce tower strikes in which blade deflection due to gust loads result in contact with the tower. Upwind turbines mitigate this issue by manufacturing rigid blades, thus increasing manufacturing costs. The ability to withstand gusts and flutter loads without damage makes downwind turbine an excellent candidate for Off-shore applications. Downwind turbines have proven to generate more power for upward flow inclination angles compared to their upwind counterparts[1]. However, the presence of a tower and a nacelle upstream creates a blockage (shadow effect) which decelerates the incoming flow, thus reducing the power output. Even though the manufacturing costs are relatively low for downwind turbines, maintenance costs can be higher due to continuous flutter and gust loads experienced in the off-shore environment.

The current research investigates the differences in performance of upwind and downwind turbines[1][2]. We measure the mechanical torque, wake profile behind each configuration and the thrust coefficient of the rotor, for optimal power settings. A scaled turbine was utilized for this experiment, along with highly accurate torque and force transducers. Velocity measurements were acquired by a COBRA probe (TFI Inc.). All the experiments were conducted at the Atmospheric-Boundary-Layer (ABL) wind tunnel at Iowa State University.

Our previous work utilized a DC motor, housed inside the nacelle, directly coupled with the scaled rotor assembly. The motor was connected to a variable load resistor to achieve different tip-speed ratios for a given incoming hub-velocity. The voltage and the current output from the motor for each TSR were measured in order to determine the TSR that produces maximum power coefficient. This approach resulted in a power coefficient severely under-predicted compared with the anticipated value (Cp = 0.44) for the prototype[3].
Measuring the direct mechanical torque alleviates the inefficiencies that stem from converting mechanical power to electrical power. DC motors operating as generators tend to be highly inefficient. Using a high quality DC motor with less friction in the brushes, improved our answer by 500% but it still yielded a 90% under-prediction. Our current design alleviates this issue by directly measuring the mechanical torque in a shaft driven by the wind turbine rotor.

6.2 Governing Dynamics

Power coefficient of a rotor operating in constant angular velocity ($\omega$) is given by,

$$C_p = \frac{\tau \omega}{\frac{1}{2} \rho U_{hub}^3 A} \quad (1)$$

where, $\tau$ represents torque and the rotor disk area is denoted by $A$. Power coefficients were acquired for different tip speed ratios ($\lambda$) as given by,

$$\lambda = \frac{\omega R}{U_{hub}} \quad (2)$$

A variety of torque measurement techniques have been employed to measure the direct mechanical torque induced by the rotor assembly. A porous disk type of representation was considered by Meneveau[4][5], where a set of strain gauges were utilized to build an in-house torque transducer. For our experiment, we use a rotational torque transducer that operates as a torsion spring or a mechanical oscillator. The general equation of motion is given by,

$$I \frac{d^2 \theta}{dt^2} + C \frac{d\theta}{dt} + k\theta = \tau(t) \quad (3)$$
where, the moment of inertia is given by \( I \), angular damping is given by \( C \), torsion spring constant is given by \( k \) and the driving torque is denoted by \( \tau(t) \). A solution can be derived for a case with no driving torque \( (\tau(t) = 0) \) in which,

\[
\tau = -k\theta \quad (4)
\]

and the natural frequency is given by,

\[
f_n = \frac{1}{2\pi} \sqrt{\frac{C}{T}} \quad (5)
\]

The thrust coefficient for a wind turbine rotor can be derived based on mass and momentum conservation and it is given by,

\[
C_T = \frac{T}{\frac{1}{2} \rho A U_{hub}^2} \quad (6)
\]

Forces acting on the rotor can be measured using a strain gauge or a load cell.

**6.3 Experiment setup and procedures**

Our design was based on a commercial prototype, NREL 2 MW by TPI composites. The turbine was scaled down 1:175 from its original design. Previous power and thrust measurement experiments were carried out using 1:350 scaled turbines. Scaling up the design allows the blades to operate at a higher Reynolds number than its predecessor, reducing the performance under-predictions and outputting “measurable” performance with reduced uncertainty. The scalability between the prototype and the scaled version is ensured by operating the design at a prescribed range of \( \lambda = 4.5 – 6.0 \). The geometric parameters for the scaled version are given in Table 6.1 along with a depiction in Figure 6.2 [3].
**Table 6.1: Scaled model parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$R$ (mm)</th>
<th>$H$ (mm)</th>
<th>$d_{pole}$ (mm)</th>
<th>$d_{nacelle}$ (mm)</th>
<th>$\alpha$ (deg.)</th>
<th>$a$ (mm)</th>
<th>$a1$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dimension</strong></td>
<td>280</td>
<td>452</td>
<td>36</td>
<td>50</td>
<td>0°</td>
<td>228</td>
<td>20</td>
</tr>
</tbody>
</table>

**Figure 6.2: Turbine geometry and airfoil information**

Compared to the previous designs, the new design featured a larger nacelle which housed the DC motor, torque transducer, and a coupler to connect both shafts. The aft of the nacelle was extended with a nose cone design to reduce turbulence intensity of the incoming flow, when operating in downwind configuration. The nose cone, the rotor blades and the hub were manufactured via rapid prototyping using Veorwhite hard plastic material. The nacelle was constructed using Plexiglas and the tower was constructed using an aluminum tube.

A HBM T20WN torque transducer was coupled with the rotor (driver) and a DC motor(driven). The transducer has a measuring range of 2 Nm with an uncertainty of 0.02%. The transducer is capable of measuring rotational and stationary torque along with the RPM.
The DC motor acts as a variable load once coupled with varying resistances. Voltages corresponding to torque and angular velocity are acquired by MX400B channel box, a 4-channel box dedicated for analog input. A schematic given in Figure 6.3 elucidates the configuration inside the nacelle.

**Figure 6.3**: Power measurement setup (left), torque transducer (lower-right) and the DC motor (top-right)

The acquired analog signals from the transducer for torque and voltage were further processed using CatmanEasy™ software. The software provided by HBM allows easy integration between the transducer and the computer, allowing users to obtain measurements in real-time, derive secondary parameters if necessary, and store the time series data sets. A sample screen is shown in Figure 6.4.
Experiments were conducted at the Atmospheric Boundary Layer (ABL) wind tunnel at Iowa State University. The wind tunnel has a 20 m long test section with a 2.4 m width and a 2.3 m height[3]. The experiments were focused on wake profiles pertaining to offshore wind turbine applications. Therefore, an offshore boundary layer was introduced with several rows of a metal chain upwind of the model turbine location. Prior to installing the turbine, velocity and turbulence intensity profiles were obtained using a COBRA probe (Turbulent Flow Instrument, Inc.). Transverse boundary velocity profile across the tunnel was acquired using dual-axis traverse system with the probe mounted on it. The same methodology was used again. The profiles are shown in Figure 6.5.

The measured velocity profile was compared with a known power law relationship which describes the boundary layer.

\[
\frac{U}{U_r} = \left( \frac{Z}{Z_r} \right)^\alpha 
\]  

(7)
where $\alpha$ is assumed to be 0.11[3]. The reference $Z$ location ($Z_r$) and reference velocity ($U_r$) were taken as the hub height and the velocity at the hub height. The velocity profile was normalized to the velocity at hub height.

Upon observation, it is understood that the recorded velocity measurements are in agreement with the power function. The ambient turbulence intensity at the hub height was 10%. The diameter based Reynolds number was 220,000, while the avg chord based Reynolds number was calculated to be 10,000 based on $U_{\text{Hub}} = 5.5$ m/s. The rotor configuration was designed to switch between upwind and downwind and the final design is presented in Figure 6.6

Torque measurements were carried were out for variable driven loads resulting from 1 to 1000 Ohm resistors. The RPM changed with every load, resulting in highest loads at lower resistances. Torque measurements for each of these tip-speed ratios were used to derive $C_p$ values and the $\lambda$ for maximum $C_p$ was identified. Thrust measurements using the JR3 load cell, along with spanwise-wake measurements for each axial station using COBRA probe was carried out for the $\lambda$ related to $C_{p_{\text{max}}}$.
Figure 6.6: (a)/(b) turbine inside the wind tunnel and (c) inside the nacelle
6.4 Results

The torque measurements for a range of $\lambda$ were used to calculate $C_p$ for both upwind and downwind turbine configurations. Results are elucidated in Figure 6.7.

Figure 6.7: $C_p$ vs. TSR

The average torque necessary was obtained by averaging the torque data signal. The highest $C_p$ occurs at $\lambda = 4.5$, within the operational range of the prototype. The newly calculated $C_p$ was closer to 0.1 and both upwind and downwind configurations yield curves extremely close to each other. Uncertainties were much higher at the high $\lambda$ as well as low $\lambda$. The spanwise wake profiles acquired using COBRA probe is given by Figure 6.8. The downstream axial locations were measured from the location of the rotor. A clear discrepancy in velocity deficit ($0.2 \frac{U}{U_{Hub}}$) between upwind and downwind configurations
was observed in the near wake station (X = 0.75D). The upwind turbine has the highest velocity deficit, roughly 60% compared to the hub velocity. However, the bi-modal shape, a characteristic pertaining to near-wake region is not observed for the upwind configuration.

![Figure 6.8: Span-wise wake profiles at downstream axial stations](image)

This is due to the presence of a larger nacelle, a nose cone and the proximity of the probe to the aft of the nacelle. A larger nacelle was required to house the torque transducer and the motor assembly. The downwind turbine configuration produced the bimodal velocity distribution for the near wake axial location. Both velocity profiles recover at X = 2.0D, resulting in the anticipated Gaussian profile. The discrepancy between the velocity deficits
has further reduced to 0.1 \( \frac{U}{U_{\text{Hub}}} \) as the wake recovers. As the wake progresses downstream the discrepancy further reduces, eventually converging at \( X = 4.0D \). The turbulence intensity at hub height (\( I_{uu} \)) remained around 20%. It can be speculated that the discrepancy in \( C_{p_{\text{max}}} \) at \( = 4.5 \), in which upwind turbine has a higher \( C_p \) value, can be attributed to the higher velocity deficit observed for upwind turbine. However, the discrepancy in \( C_p \) shown in Figure 6.7 falls within the measurement uncertainty. The presence of a large nacelle (large blunt body) has contributed to the increased velocity deficit. Wake recovery for both turbines along the axial direction for both turbine configurations is shown in Figure 6.9.

Upon comparing the maximum deficits at \( X = 0.75D \) and \( X = 4.0D \) for upwind and downwind turbines, it can be deduced that the upwind turbine shows a faster recovery compared to its counterpart. The faster recovery of upwind turbine is attributed to the presence of a nose cone along with higher turbulent intensity present in the wake. The initial turbulent intensity aft of the rotor is amplified due to the presence of the large nacelle downstream. The faster recovery suggests that an upwind configuration is suitable for a wind
farm setting. Higher blade vibrations were observed in the downwind turbine, which is due to the increased turbulence intensity (due to the nacelle upstream) of the incoming flow.

6.5 Conclusions

A set of experiments to measure the performance of an upwind and a downwind HAWT was carried out at the ABL wind tunnel at Iowa State University. A mechanical torque transducer coupled with a DC motor was driven by a rotor, in which a power coefficient was derived based on the mechanical torque for a range of tip-speed ratios. This method is a significant departure from our previous approach of measuring mechanical power output by the DC motor. The inefficiencies in mechanical to electrical power conversions resulted in a severe under-prediction (C\text{p} = 0.01) compared to the anticipated power coefficient(C\text{p}=0.44). A scaled model of a commercial prototype was constructed using rapid prototyping and standard manufacturing techniques, which was then placed in an off-shore boundary layer flow. Once the maximum power setting was identified, thrust and velocity measurements were conducted.

A HBM (T20WN) series torque transducer was used to obtain a high-frequency data series for RPM and mechanical torque. For a range of \( \lambda \), averaged mechanical torque values were used to derive C\text{p} for upwind and downwind turbine configurations. The maximum C\text{p} of 0.1 occurs at \( \lambda = 4.5 \), which is a significant improvement from earlier prediction of 0.01. However, this is still 25% of the C\text{p} reported for the prototype. A further investigation into the blade design unveiled a significant loss in performance based on the airfoil and Reynolds number it performs at. The turbine blades were constructed using S18, S19 and S20 airfoils with angles at different axial stations. A XFOIL simulation of airfoils for Reynolds numbers
experienced by scaled model and prototype depicts a 60% reduction in the lift coefficient for the scaled turbine model. This contributes towards the under-prediction present in the current \( C_{p_{\text{max}}} \).

Span-wise Velocity measurements in the turbine wake at pre-determined axial locations were obtained for the maximum power setting. The velocity deficits of upwind and downwind turbines show a distinct discrepancy at near wake and converge at far wake locations. As per wake recovery, upwind turbine wake elucidates a faster recovery and a turbulent intensity of 20% or more throughout the wake profile. Presence of a larger nacelle to house the torque transducer, results in the change of bi-modal shape native to near wake profiles, increased downstream turbulence in upwind wake as well as high amplitude blade vibrations present in downwind turbine configurations.

References


CHAPTER 7: GENERAL CONCLUSIONS

Flow characteristics of a wind turbine were studied experimentally in a wind tunnel, focusing on coherent structures, wake instabilities, and transport of momentum and vorticity. Near wake flow is highly dynamic, requiring advanced modeling of rotational turbulence. Current rotor models such as Actuator Disk (AD) and Actuator Line (ACL) reduce computational cost but fail to capture blade related characteristics. Wind tunnel measurements obtained via Particle Imaging Velocimetry (PIV) provided reliable flow measurements with a 2% uncertainty and a 5% uncertainty on derived quantities such as vorticity. Our measurements revealed important wake characteristics such as a secondary helical vortex shed from the mid-span location causing both tip and mid-span vortex filaments to reach instability in the near wake region. Velocity and vorticity distributions through ensemble averaging elucidated the wake reaching instability in the near wake region, contrary to the trends obtained from CFD simulations where the helical wake propagates to far wake regions before breaking up to turbulent eddies. The secondary helical vortex filament emanating from 0.6R blade span was also observed and investigated for its potential to cause significant velocity perturbations.

Based on the initial results, we localized on a region of interest within the flow field for a principal component analysis. This region encompassed the vortex patches immediately aft of the rotor, and the region extended up to the axial station (X/D = 0.6) where the vortex breakup occurs. A POD and a DMD analysis was conducted to identify the dominant coherent structures in the highly turbulent near wake region. Convergence of the POD study determined the number of snapshots required as well as the number of modes
sufficient for a high energy and low-order reconstructions. The analysis isolated principal
flow components that account for 68% of the modal energy spread among the first 7 Eigen-
modes. A modal reconstruction of fluctuating components of an instantaneous measurement
clearly showed vortex patches with alternating directions as they propagate downstream.
Addition of the time averaged mean clearly showed the extracted vortex patches which were
initially observed in phase-averaged solutions. However, there were no observable modal
dependencies among the modal coefficients. The DMD analysis showed spatially decaying
energetic modes, similar to POD fluctuation reconstructions with varying temporal growth or
decay depending on the ROI. The pre and post vortex break-up events contained highly
energetic modes that are opposite in phase and in temporal growth. The temporal growth and
deformations of vortical cores suggest the presence of short-wave instability.

The development of an expression was required to understand the propagation and
the instability of the tip and mid-span vortex patches. In the absence of three-dimensional
data for accurate modeling, a Kelvin wave assumption was used to represent the filaments.
The critical point of instability for the filaments was assumed to be $X/D = 0.6$ due to the
vortex breakup observed in the measurements. A qualitative observation elucidated decrease
in spacing between consecutive spiraling blade tip vortices (helical pitch) due to geometric
scaling of the model as well as the rotor RPM. An initial approach of a stream function
derivation showed regions prone to interactions between the mid-span and the tip vortices
and the induced velocities. The unsteady nature of such perturbations, represented by point
based and filament based Biot-Savart kernels demonstrated the effects of short-wave and
mutual induction driven instabilities. An axisymmetric vortex ring solver and a filament
based solver simulated mutual and self-induction present within the helices while a blob-
based algorithm elucidated the asymmetric elliptical instability between vortex patches. However, the absence of lateral velocity data resulted in unsolved instability criterions in y direction. Presence of velocity perturbations that lead to vortex breakups, prompted us to investigate the vertical momentum and vorticity transports in the near wake shear layer between the tip and mid-span vortices.

As shown in Figure 3.1, the shear layer that lies between the wake region and high velocity flow outside is directly responsible for the wake recovery. The turbulence present in this region governs the energy output and the blade fatigue of the downstream turbines, thus making the transport process worthy of investigation. Transport equations were derived for both momentum and vorticity where each process was influenced by a time-averaged mean, a coherent component and a random fluctuation. Stretching effects could not be extracted due to the lack of three-dimensional data. Phase locked measurements were subjected to triple decomposition which produced phase averaged Reynolds stress (RS) distributions. The presence of positive stress values signified vertical momentum transport which aids wake recovery. The RS contribution due to coherent flow structures was around 1.5% of the contribution from the turbulent fluctuating quantities. Prior to the vortex breakup, RS was predominantly concentrated between the vortices shed by the turbine. A plot of the production term, RS and vorticity as shown in Figure 3.15 elucidated the absence of production or RS in the cores of vorticity patches. As the vortex patches break up to smaller eddies after X/D>5.5 or 6.0, the beginning of a shear layer with high RS becomes apparent. Most of the RS (>60%) in the phase averaged near wake solution was concentrated aft of the critical station of instability. Such trends were observed in all the phase averaged data, suggesting that the presence of coherent vortex patches is hindering the wake recovery.
Turbulence at the nacelle and tower assembly also leads to high RS which was not investigated in this study.

In order to understand the role of vortex patches in the wake, we analyzed the terms pertaining to the vorticity transport. The advection terms of vorticity showed enstrophy dissipation in the perimeters of the vortices, where high circulation was present. Comparison between cross-stream derivative of RS and the vorticity advection term ($< \omega' v' >$) yielded a satisfactory agreement, thus reflecting on our initial hypothesis between momentum and vorticity transport. Divergence of the Lamb vector highlights the strain field that exists between tip and mid-span vortices due to their proximity as well as between the tip vortices and the high velocity regions. This “unwinding” or positive flexion can also be considered a contributing factor in wake-recharging and the formation of the shear layer.

This study has provided a detailed insight to the wake recovery process in wind turbine flows and we managed to successfully identify the governing dynamics behind this complex process. Novel power measurements conducted using a mechanical torque transducer improved the previous measurement by 900%, removing the error due to mechanical-to-electrical conversion efficiency. Future work should be focused on understanding the generation and properties of random fluctuations during comprehensive momentum recovery studies. Effects of blade boundary layer should be thoroughly examined to understand the generation and evolution of the secondary helix (mid-span) downstream. Chapter 8 will discuss the development of an airborne wind energy system that can be used increase the endurance of an unmanned aerial vehicle.
CHAPTER 8: FUTURE WORK – AIRBORNE WIND ENERGY

8.1 Introduction

Unmanned Aerial Vehicles (UAVs) have been at the forefront of many different disciplines ranging from military surveillance to social media in the recent times. These vehicles are currently designed to accommodate a wide variety of engineering requirements such as increased loitering times, increased range and improved maneuverability. Increasing the endurance of a UAV, especially used in long duration reconnaissance, requires meticulous energy management throughout the missions. Utilizing non-renewable energy sources readily available in the environment such as solar energy, to replenish the batteries of electrical UAVs has been extensively investigated. Recent advances in kite based mechanisms to harness high-altitude wind energy will significantly reduce the space requirements set forth for contemporary wind farm while exceeding the output\(^1\). In our current study we propose the conceptual design of a UAV, which replenishes its batteries using environmental flows such as thermal plumes. Recent studies on such concept vehicles have been carried out by NASA (Cloud Swift project) and by Dr. Etan Fisher of Shamoon College of engineering in Israel\(^2,3\). Thermal plumes are air currents that rise from the grounds due to heating during the day time. Glider aircraft as well as animals such as turkey vultures utilize these vertical air currents to gain altitude\(^4\). Once the maximum altitude is reached, gliders can leave the plume in a descent flight converting potential energy to kinetic energy.

We consider a glider-like UAV, designed to gain altitude via thermal soaring. An iterative algorithm was designed to find the optimal flight conditions for the glider in both ascent and descent phases of the flight. The glider is equipped with a two blade rotor which can be used to harvest energy via wind-milling. It can also be used as a propeller providing additional
thrust when required. The design process utilized a commercial glider UAV (DDG-1000) as a baseline model. The baseline design is equipped with an 11” x 7” folding propeller powered by a 2200 mAh LiPo battery. An energy cycle was established where the glider uses a wind-milling propeller to generate power during the descent phase of the flight\(^5\), aided by the gravity while the rotor acts as a propeller during the ascent phase, compensating for any additional drag. Governing equations for steady flight conditions were used in tandem with XFOIL to obtain the optimal configuration for power generation and consumption.

### 8.2 Methodology

The flight of the UAV is decomposed to two phases; descent and the ascent phase. Steady flight conditions were assumed for both phases. During the descent phase, the glider leaves the thermal plume and loses altitude due to gravity, converting potential energy to kinetic energy. The schematic in Figure 8.1 shows the attitude of the flight and the forces acting on it\(^6\).

![Figure 8.1: Descent phase of the flight](image)
The drag \( D_{\text{total}} \) acting on the body can be decomposed to the drag due to the propeller \( D_p \) and the drag acting on the body and the wing \( D_{\text{body}} \). Relationships can be derived for steady flight conditions as given by,

\[
L \cos(\theta_{\text{glide}}) + D_{\text{total}} \sin(\theta_{\text{glide}}) = W \quad (1)
\]

\[
L \sin(\theta_{\text{glide}}) - D_{\text{total}} \cos(\theta_{\text{glide}}) = 0 \quad (2)
\]

Equations (1) and (2) based on force equilibrium can be further manipulated to find the glide velocity,

\[
V_{\text{descent}} = \sqrt{\frac{2W}{\rho \cos(\theta_{\text{glide}}) S(C_L + C_D \tan(\theta_{\text{glide}})) + 4A_{\text{Disk}} a'(1 - a') \tan(\theta_{\text{glide}})}}, \quad (3)
\]

where,

\[
\theta_{\text{glide}} = \tan^{-1}\left(\frac{SC_D + 4A_{\text{Disk}} a'(1 - a')}{SC_D}\right) \quad (4)
\]

and the area of the wing is denoted by \( S \), \( a' \) for axial induction and \( A_{\text{Disk}} \) represents the rotor area\(^7\). The drag coefficient \( C_D \) can be calculated by estimating the induced, skin friction and form drag as shown in,

\[
C_D = C_{D0} + KC_L^2 \quad (5)
\]

where, \( K = \frac{1}{\pi AR e_{\text{oswald}}} \) and \( C_L \) represents 3D lift coefficient. \( C_{D0} \) can be found using the wetted area\( (S_{\text{wet}}) \), skin friction\( (C_f) \) and form factors\( (\text{FF}) \). The flow is assumed to be turbulent and an expression can be derived for \( C_{D0} \) as\(^6\)

\[
C_{D0} = \frac{C_f F F S_{\text{wet}}}{S_{\text{ref}}} \quad (6)
\]

Power of the rotor during the descent phase can be calculated as,

\[
P = 2A_d a'(1 - a')V_{\text{descent}}^3 \quad (7)
\]
During the ascent phase of the flight, the glider uses a helical ascent pattern to reach the highest altitude within the thermal. The glider has a banked attitude towards the center of the thermal, creating a centrifugal force component derived from both lift and drag. The Figure 8.2 depicts the flight attitude and the forces acting on the glider.

![Figure 8.2: Ascent phase of the flight](image)

For each bank angle ($\phi$), ascent velocity ($V_\infty$) can be calculated using,

$$V_{ascent} = \frac{\sqrt{g R \tan \theta}}{\cos \theta_{ascent}} \quad (8)$$

Force acting on the rotor can be written as an expression of lift and drag coefficients,

$$F_{rotor} = \frac{1}{2} \rho V^2 \cos^2 \theta_{ascent} - \frac{C_L \sin \theta_{ascent}}{2} \quad (9)$$

where $C_L$ and $C_D$ were evaluated for a range of angle of attacks. To maintain steady turning flight, centrifugal force is present along with a component of $F_{xz}$ acting to support the weight:

$$F_{xz} = L \cos \theta_{ascent} + D \sin \theta_{ascent} \quad (10)$$

$$F_{xz} \sin \phi = \frac{m V^2}{R} \quad (11)$$

$$F_{xz} \cos \phi = W \quad (12)$$
where the mass of the glider is given by \( m \) and \( R \) denotes the radius of the thermal. The radius of the thermal can be calculated by\(^2\),

\[
R = \frac{1}{2} \left( 0.203 \right) \left( \frac{Z}{z_l} \right)^{\frac{1}{3}} \left( 1 - 0.25 \frac{Z}{z_l} \right) z_l
\]  

(13)

where, \( z_l \) denotes the mixing layer thickness. Equations pertaining to both descent and ascent depend on an external XFOIL module to find the flight parameters. In the descent phase, velocity that satisfies steady state flight conditions was found using an iterative numerical scheme for a series of angle of attacks. In the ascent phase, \( F_{\text{rotor}} \) was calculated for a series of bank angles and angles of attack. If \( F_{\text{rotor}} > 0 \), additional thrust is required for a steady ascent while \( F_{\text{rotor}} < 0 \) allows the rotor to be used as a turbine. The amount of drag can be controlled using variable pitch. The flow of information in the algorithm is given in Figure 8.3.
8.3 Results

For validation purposes, a benchmark case based on a commercial thermal riding UAV was introduced. The UAV is a scale model of the DDG-1000 glider design and features a folding propeller for additional thrust. Parameters for the glider is given are given in Figure 8.4.

![Diagram of UAV](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flight Mass</td>
<td>1.7 kg</td>
</tr>
<tr>
<td>Wing span (b)</td>
<td>2.63 m</td>
</tr>
<tr>
<td>Wing Area</td>
<td>0.285 m² (AR = 24)</td>
</tr>
<tr>
<td>Fuselage Length(L)</td>
<td>1.130 m</td>
</tr>
<tr>
<td>Airfoil</td>
<td>HQ3012</td>
</tr>
<tr>
<td>Propeller Diam. (D)</td>
<td>0.28 m</td>
</tr>
<tr>
<td>Battery</td>
<td>2200 mAh LiPo (11.1V)</td>
</tr>
</tbody>
</table>

**Figure 8.4:** Parameters of the benchmark case

The rotor was assumed to be an efficient wind turbine (effective at wind milling) with an axial induction of 1/3. The glider uses a high aspect ratio wing with a rectangular planform with no sweep. The original design features a small forward sweep that was neglected to simplify the design process. The original design also features a 2200 mAh LiPo battery that should be charged using a charging circuit. Efficiencies and mechanics associated with a charging circuit are omitted for the purpose of this study. Due to the presence of a slender body, profile drag created from the body can be considered negligible. The airfoil HQ3012 is a cambered geometry where the maximum thickness occurs at the 35% of the chord, while the max. camber occurs at 50% of the chord. The geometry of the airfoil along with the
$C_l/C_d$ vs. $\alpha$ curve are elucidated in Figure 8.5. The $C_l/C_d$ increases with the increase in Reynolds number, resulting in a maximum value between 6°-8° angles of attack.

A thermal with a 2 km altitude was used in this calculation, with thermal velocities ranging from 2-5 m/s. The change in radius values with each z station is shown in Figure 8.6. The thermal radius increases with the altitude, finally converging on 150 m at the top. An exponential growth of the radius can be seen after $z = 30$ m. Therefore, flight conditions for ascent should be investigated for a range of radii. Results for the descent phase are given in Figure 8.7. We observe the converged glide velocity for each angle of attack ($\alpha$) along with the normalized energy extracted.

**Figure 8.5:** Airfoil geometry (left) and performance (right)
**Figure 8.6:** Altitude vs. Thermal radius

**Figure 8.7:** Descent performance
As the angle of attack increases, the glide angle decreases along with the glide velocity. The increasing $\alpha$ results in the increase of lift and the drag ($\sim KC_L^2$), which in turn slows down the descent of the glider. The vertical component of the descent velocity is used to calculate the descent time of the glider, in which it loses 90% of the altitude (2 km). The energy extracted throughout the whole descent was normalized to total energy stored at the battery, 2200 mAh. According to the normalized energy plot, the rotor extracts more than 20% of the energy stored at the LiPo cells.

Ascent performance was evaluated for three thermal radii for the same series of $\alpha$ used in the descent performance. The tangential velocity component corresponding to each thermal radii was found initially for a series of bank angles ranging from $5^0$ to $80^0$. The ascent angle was found using the tangential and the upward thermal velocity (normal component). For each flight velocity, an angle of attack that satisfies the steady flight conditions given in Eq (11) and Eq (12) is calculated, along with the corresponding rotor force ($F_{\text{rotor}}$). Results for the ascent phase are shown in Figure 8.8 for multiple radii.

![Figure 8.8: Ascent performance](image)
As the radius of the thermal increases, the thrust required increases with decreasing bank angle ($\phi$). For a thermal radius of 50 m, the required force $F_{rotor} < 0$ in which a drag force is acting on the rotor. The appropriate drag force can be provided by wind-milling a rotor. The required force goes to zero for $R = 100$ m and $R = 150$ m for the $\phi$ angles of $60^0$ and $40^0$.

Operating in bank angles above the critical values for a given thermal radius will require the rotor to operate as a propeller providing thrust, which will use a portion of the energy stored in the LiPo cells. The $F_{rotor}$ parameter also depends on the angle of attack of the wing in which higher $\alpha$ values result in $F_{rotor} < 0$. For a thermal radius of 50 m, the glider is able to perform in the range of $\alpha$ between $-1^0$ to $14^0$. As the thermal radius increases from 100 m to 150 m, the $F_{rotor}$ curves shift towards the left, resulting $F_{rotor} > 0$ for $\alpha < -2^0$. The decreasing $\alpha$ results in a decrease in $C_L$, thus resulting in a requirement for a thrust as depicted in equation 11. The range of $\alpha$ suitable for the successful ascent of the glider can be established as $-2^0$ to $14^0$. The glider should also maintain $\phi < 40^0$ throughout the ascent with varying thermal radii. Upon observing the $\alpha$ vs. $\phi$ plot, the $\phi$ increases with decreasing...
Increasing $\phi$, results in an increase in the tangential velocity, thus causing the lift to increase. The parameter $\alpha$ reduces in turn to satisfy the steady state flight conditions.

References


