Parton-picture unitary impact-parameter model for multicluster production

Van Chang
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Parton-picture unitary impact-parameter model for multicluster production

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Van Chang

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CHAPTER I. INTRODUCTION

In the parton model (1) hadrons (strongly interacting mesons, e.g., pions and baryons, e.g., nucleons) are depicted as composite objects of constituents, or "partons", which might be quarks. The basic concepts originate from a straightforward, simple interpretation of scaling behavior in deep inelastic scattering, i.e., lepton-nucleon inelastic scattering at energies above the resonance region. In this picture, partons are point-like, spin $\frac{1}{2}$ particles with fixed mass and internal quantum numbers. A few fast partons accompanied by many slower or "wee" partons bind together to form a hadron. When these fast partons and wee partons can be assigned a particular set of internal quantum numbers, they are usually called valence quarks and sea quarks, respectively, in the quark-parton model. In order to avoid the problem of dealing with the complicated strong interactions that bind partons or quarks together, a special Lorentz frame called the "infinite momentum frame" is chosen. In this frame the hadron has momentum $q \rightarrow \infty$. In such a frame the partons or quarks can be approximately treated as free or quasi-free.

It is therefore natural to attempt to apply parton ideas to high energy hadronic interactions. However, in deep inelastic scattering the interactions of the parton with a (virtual) photon is a point interaction whereas in the hadron-hadron case, more of a collective effect must occur because of interactions between the constituent partons.

In high energy hadronic interactions, the most obvious feature is multiparticle production. Experiments with numerous two body initial states at the various high energy laboratories show that around 80% of
the reaction probability goes into such inelastic channels. A basic understanding of the mechanism responsible for multiparticle production certainly has implications for the structure of hadrons, the dynamics of hadronic collisions, the mass spectrum of hadrons and, therefore, a major part of the strong interaction physics.

In studying inelastic channels in which four or more particles are produced, a full description of the angle and momentum distributions for the multiparticle final state is too complicated to allow deduction of underlying dynamics. A more global approach now called "inclusive" analysis therefore became necessary. Here, "inclusive" means that all final states producing a specific particle or particles under study are included together by integration over the kinematic parameters of all the other unspecified particles. The two simplest examples of inclusive measurements are the total cross section and the single particle production cross section with one particle detected. Besides inclusive spectra, two other measurements "multiplicity" and "topological cross section" are important in describing multiparticle production reactions. The charged particle multiplicity is simply the number of charged prongs counted in the experiment and the topological cross section \( \sigma_n \) is the cross section for producing \( n \) charged prongs.

The main general features of multiparticle production are:

1. The transverse momentum \( (q_T) \) distribution for produced particles decreases exponentially. The average value of \( \langle q_T \rangle \), about 0.3 to 0.4 GeV/c, is surprisingly small and is quite independent of
the incident energy and the type of particle or of the multiplicity of produced particles.

2. The average multiplicity grows slowly with the incident energy. Where full phase space can accommodate a growth as rapid as the center of mass c.m. energy $\sqrt{s}$, the observed multiplicity grows only as

$$\langle n_{ch} \rangle \sim \ln s$$

3. At high energies, the invariant momentum distribution is independent of $s$; if written as a function of a properly scaled variable, that is,

$$F(q_L, q_T, s) \sim F(x, q_T)$$

where $x = 2q_L/\sqrt{s}$ is the Feynman scaling variable and $q_L$ is the center of mass longitudinal momentum of the observed particle or secondary produced in the interaction. The range of $x$ is $-1 \leq x \leq 1$.

4. The data behaves differently for small or large values of $|x|$. The region of small $|x|$ ($|x| \lesssim 0.1$) contains a large fraction of pions and makes up the "central pionization" region. These slow pions are not expected to be correlated with either initial state particle, as correlations die out rapidly with the longitudinal momentum difference. At large values of $|x|$, i.e., $|x|$ approaching 1.0, the center-of-mass momenta are large enough that the particles are always associated with one of the
incident particles and such produced particles are said to be in the "fragmentation" region of the target or projectile.

5. There exist strong correlations among secondaries. Secondary particles apparently prefer to come out in the form of "clusters" rather than as single particles.

Models constructed for multiparticle production are mostly generalizations and extensions of models for two body reactions. Among them, multiperipheral models are directly generated from the peripheral model. The basic ingredient of the peripheral model is that peripheral processes, i.e., small momentum transfer to the final state particles, can be described by one particle exchange. The multiperipheral amplitude for multiparticle production is therefore written pictorially as a series of single particle exchanges. However, instead of simple single particle exchange, in a refined picture the amplitude should have such exchanges taken as Regge poles or Regge cuts to give multi-Regge behavior. The multiperipheral model with its simple and transparent mathematical structure provides a specific form for the multiparticle production matrix element. However, as is well-known these multiparticle amplitudes do not satisfy s-channel unitarity. The unitary requirement means that the sum of the probabilities for all the final states must be exactly equal to one.

High energy elastic scattering models constructed in the impact parameter representation, such as the eikonal model (2) and Chou-Yang droplet model (3), have also proved reasonably successful in describing experimental data. The elastic scattering amplitude for two very high
energy protons colliding in the c.m.-system with energy squares \( s \) is

\[
A^e_\ell(\vec{Q},s) = 2is \int d^2\vec{B}(1 - \exp[2i\chi^e_\ell(\vec{B},s)]) \exp(i\vec{Q} \cdot \vec{B}) ,
\]

in the impact parameter (\( \vec{B} \)) representation. The momentum transfer is \( \vec{Q} \), and the quantity \( \chi^e_\ell(\vec{B},s) \) in the S-matrix \( \exp[2i\chi^e_\ell(\vec{B},s)] \) is referred to as the eikonal. In the eikonal model,

\[
\chi^e_\ell(\vec{B},s) = -(2\pi)^{-1} \int_{-\infty}^{\infty} V(\vec{B},z)dz , \tag{1.1}
\]
describing the two point-like protons (interacting through the potential \( V \)) moving with relative velocity \( \vec{B} \) along the \( z \) axis. The integration along the straight line \( z \) axis implies that the protons lose little of their incident momentum. Though the protons are looked upon as point-like, the "potential" \( V(\vec{B},z) \) represents in a sense the structure of the two protons in interaction with one another. Specifically, the real part of this potential contributes to the phase shift and the imaginary part to the absorptive portion of \( \chi^e_\ell(\vec{B},s) \).

A realistic physical interpretation of the high-energy pp interaction has been provided by Chou and Yang (3). In their model, the protons are treated as extended objects and the interaction results from the overlap of the two matter densities, yielding an eikonal function depending on relative impact parameter \( \vec{B} \) and c.m. energy squared \( s \),

\[
\chi^e_\ell(\vec{B},s) = \frac{ic}{2} \int dz \int d^2\vec{B}_A \int d^2\vec{B}_B \rho_A(\vec{B}_A,z)\rho_B(\vec{B}_B,z)\delta(\vec{B}_A - \vec{B}_B - \vec{B}) . \tag{1.2}
\]

Hence the \( \rho_i(\vec{B}_i,z) \) describe the matter densities for nucleon A and B as
a function of transverse and longitudinal coordinates and c is a constant. In the work of Chou and Yang the densities \( \rho(\vec{r},z) \) are simply related to the electromagnetic form factor of the proton.

Extension of this simple geometrical picture to inelastic scattering immediately explains many of the features of multiparticle production reactions. As the collision energy increases, the two incoming particles become thin disks of hadronic matter through the Lorentz contraction. These contracted, rapidly moving matter distribution for the target and projectile determine the distribution of secondaries produced after collision. Due to this collimated motion of the interacting matter disks, the average transverse momentum of those produced particles is small and independent of other kinematical parameters. This then causes the phase space to be effectively damped in \( q_T \), which is also responsible for the slow growth in the multiplicity. At sufficiently high energy when the incoming particles are contracted to thin disk, a further increase in energy will not change the matter distribution significantly. Therefore, the invariant cross section shows scaled behavior.

In this thesis, multiparticle production models constructed directly from those models built up from the impact parameter representation for elastic scattering (4-6) are studied. In such multiparticle production models, generally, the incoming particles are treated as source functions for secondary particles and the multiparticle amplitudes do satisfy s-channel unitarity. However, because of limitations following directly from the original assumptions in the various elastic scattering
models and because of the additional different production mechanisms authors have used in their extensions, each of these multiparticle production models describes certain special features of the data.

These main shortcomings of the unitary impact parameter models are then examined in light of the parton picture for hadron-hadron scattering. In order to keep the language and concepts of point-like particles in the eikonal model, we shall choose the multiperipheral parton picture, with the assumption that the fraction of longitudinal momentum carried by the slow partons damps rapidly along the multiperipheral chain. Our source function is then obtained from this parton picture, giving us a description for particles produced in the central pionization region and in the fragmentation region.

Recent applications of parton-constituent ideas to hadron production in strong interactions through the quark recombination model have been quite successful. This new work was inspired by the foundations laid by Ochs (7) and Das and Hwa (8). Ochs made the empirical observation that the measured \( \pi^+/\pi^- \) ratio in the proton fragmentation region in a proton-proton (p-p) collision is remarkably similar to the \( u/d \) (up/down) quark ratio determined from deep inelastic scattering. This inspired Das and Hwa to seek a fundamental reason for this coincidence. They found that the quark-parton model could be applied to hadron-hadron collisions in the large longitudinal momentum (\( q_L \)), low transverse momentum (\( q_T \)) regime. If a pion is produced at large \( x = 2q_L/\sqrt{s} \), there must have been a large probability for a quark initiating such production to have a large fraction of the momentum. Such valence quark
dominance in specifically leading particle effects had earlier been advocated by Van Hove and Pokorski (9), who suggested leading particles in the final state were due to recombination of the three valence quarks. As emphasized by Das and Hwa, in a fast $\pi^+(\pi^-)$ produced in a proton-proton collision the u(d) quarks of the pion must be one of the original valence u(d) quarks of the incident proton. Therefore, such low $q_T$ hadron events indeed have constituent structure information, and the observation by Ochs receives a natural explanation.

A final comment, separate from properties of partons, on the neglect of spin and isospin in the multiparticle production models may be appropriate. The isospin problems are insignificant because we shall be examining neutral cluster production at high energies. Clusters carrying spin and isospin will generally require quantum number exchange for which the amplitude dies rapidly with energy. The neutral cluster itself is not observed but its decay pions are. Additionally, it has also been shown (10) that the multiplicity distribution of the independent-neutral-emission model can be derived from the unitary impact parameter models discussed above. Since polarization is exceedingly difficult to measure in multiparticle events, we assume spin average calculations will suffice.

In Chapter II, we review unitary impact parameter models and discuss in more detail specific difficulties of each model. In Chapter III, we obtain a more appropriate source function from the multiperipheral parton picture. The portion of this source function appropriate for the pionization region reproduces the main phenomenological features.
suggested by the models in Ref. 4. In Chapter IV, we compare our results with experimental pp reaction data from 70 GeV/c to 400 GeV/c. The main reasons for comparing with pp data are that we have confidence in the parton picture for the proton, pp data are not dominated by nondiffractive features such as charge exchange processes, and we can distinguish the secondary particles from the primary ones. The next three chapters deal with quark recombination model. In Chapter V, we discuss the quark recombination model used by Das and Hwa, Ranft, and Duke and Taylor in quantitative mathematical terms, pointing out limitations. In Chapter VI, a reformulation is introduced to emphasize consistency of the joint momentum probability distribution for hadrons. A sum rule form for the phase space consistency condition is derived. A new scaling type of variable follows from this sum rule for the momentum distribution of the picked-up sea quark. In terms of this new variable, comparison with data are seen in Chapter VII to be very satisfactory. Summary and discussion are reserved for the final chapter.
CHAPTER II. GENERALIZED EIKONAL AND CHOU-YANG MODELS

The models published by Aviv et al. (4) and Auerbach et al. (4) are developed from the eikonal model. The primary particles are treated as point-like objects which lose little of their incident momenta in passing through each other. Source functions associated with the creation or annihilation of the secondary particles are gotten phenomenologically from the multiperipheral model. Therefore only the pionization region is described. There is considerable freedom in choosing source functions from multiperipheral models (as in Ref. 4); therefore, it appears to us that a self-consistent approach based on multiperipheral bootstrap conditions is necessary. This eventually leads to the problem of how to handle a very large number of variables in the determination of the amplitude for $2 \to N$ processes (11).

We now review the unitary impact parameter models of Calucci, Jengo and Rebbi (5) and of Heneyey and Sukhatme (6) which have the source function directly related to the internal structure of primary particles. In the words of Calucci et al. (5), although the original elastic scattering model is also eikonal in nature, source functions are introduced in the Hamiltonian to give an effective inelastic potential additional to the elastic potential. The form of the resulting matrix is

$$S(B,s) = \exp[2i(x^e_1(B,s) + x^{inel}(B,s))]$$

$$x^{inel}(B,s) = \frac{1}{2} \int dK(\xi^+_K(B,s)a^+_K + \xi^-_K(B,s)a^-_K) \quad ,$$  \hspace{1cm} (2.1)

where $x^e_1(B,s)$ is given in Eq. (1.1). Here $\xi^+_K(B,s)$ plays the role of
the source function for the objects produced by the creation operator \( a^+_K \), and \( \xi^+_K \) correspondingly for the annihilation part, with \( \vec{K} \) the cluster momentum. Since we know that multiparticle data can be consistently interpreted using emission of clusters \((12)\) rather than individual pions, we shall treat \( a^+_K \) and \( a^-_K \) as the cluster creation and annihilation operators. The source term \( \xi^+_K(\vec{r},s) \) can be related to a "creation potential" by

\[
\xi^+_K(\vec{r},s) = \frac{e^{i\phi}}{\beta} \int d\vec{r}' dz e^{i\vec{K} \cdot \vec{r}' - i\omega(\vec{r}) z/\beta} V_c(\vec{r}',\vec{r},z),
\]

where

\[
H_c = \int d\vec{r}[A^+\vec{r})V_c(\vec{r},\vec{r}',z) + A(\vec{r})V_c(\vec{r},\vec{r}',z)]
\]

is the term added to the Hamiltonian to describe the production or annihilation of a cluster at the point \( \vec{r} \), and \( \phi \) is a possible phase. Each of these source functions has an extended structure with their centers located at the fast moving primary point-like particles, one of which moves to the left and the other to the right in the center of momentum (c.m.) frame. Particles produced from the overlap of these source functions are therefore also fast moving either to the left or to the right, i.e., only beam and target fragmentation is discussed. In this model, the source functions associated with the two primary point-like particles essentially represent unknown structure of the primary particles which is responsible for multiparticle production. In other words, kinematically the two primary particles are point-like particles but dynamically they possess a structure reflected by the potential. The relation of
the source function to structure inside the primary particle can be seen more clearly in the work of Henyey and Sukhatme (6).

Henyey and Sukhatme have generalized in an analogous way the Chou-Yang model in which the primary particles are extended objects. In such an eikonal model the inelastic creation potential \( V_C(\vec{r}, \vec{B}, z) \) is interpreted as the result of the overlap of the matter clouds of the particles for multiparticle production process,

\[
V_C(\vec{r}, \vec{B}, z) = \int d\vec{B}_A \int d\vec{B}_B \rho_A(\vec{r}, \vec{B}_A, z) \rho_B(\vec{r}, \vec{B}_B, z) \sigma(\vec{B}_A - \vec{B}_B - \vec{B})
\]

Here \( \rho'(\vec{r}, \vec{B}, z) \) represents the density of cluster-producing matter, which may or may not be the same as the matter density for elastic scattering. This model leads to a natural description of the "diffractive dissociation" process. Noting that the result (2.1) is in the form of a coherent state expansion for fixed \( \vec{B} \), one can immediately define observables at fixed \( \vec{B} \). Specifically, the mean multiplicity of produced clusters will be (assuming \( \xi \) as an operator is diagonal)

\[
\bar{n}(\vec{B}, s) = \int d\vec{k} |\xi_K(\vec{B}, s)|^2
\]

and the topological cross sections will be given by the Poisson distribution,

\[
\sigma_n(\vec{B}, s) = \frac{(\bar{n}(\vec{B}, s))^n e^{-\bar{n}(\vec{B}, s)}}{n!}
\]

Strictly speaking, of course, one does not find observables at fixed \( \vec{B} \); instead one should integrate the amplitudes over \( \vec{B} \) and then form the observables. In practice, however, one may use Eqs. (2.4) and (2.5),
integrated over $\hat{B}$, because one is ultimately interested only in parameterization of $E_K(B,s)$.

Henyey and Sukhatme were able to show that the qualitative features which follow from this model are in accord with experiment, but the quantitative results obtained using the Chou-Yang model were far from satisfactory. A simple example of the difficulty can be seen if one assumes a Gaussian matter density, leading to a Gaussian mean multiplicity,

$$\bar{n}(\hat{B},s) = n_0 e^{-B^2/\Delta^2} \quad (2.6)$$

If one now integrates $\bar{n}(\hat{B},s)$ to find the overall topological cross sections,

$$\sigma_n(s) = \int d^2 B \bar{n}(\hat{B},s) \quad (2.7)$$

the results (which can be obtained analytically) bear no resemblance to a Poisson distribution, as can be seen in Fig. 1, regardless of the values of $n_0$ and $\Delta^2$. Since the data somewhat resemble a Poisson function, the Gaussian density is clearly unacceptable; instead it must be a function such that the integral of the Poisson distribution is very close to a Poisson distribution. In addition, Henyey and Sukhatme found various problems with the energy dependence which we shall not discuss in detail here.

In the following chapter, we shall use the multiperipheral parton model of Kogut and Susskind (13) to estimate the $\hat{B}$-dependence of $\bar{n}(\hat{B},s)$ for small $\hat{B}$, and the Chou-Yang model for larger $\hat{B}$. By combining these models, one obtains a function for $\bar{n}(\hat{B},s)$ with exactly the properties
Fig. 1. Probability of finding $n$ charged particles, given a Gaussian source function. The magnitude and slope of the curve depend on the height and width of the Gaussian. The main unacceptable feature is the monotonic decrease in probability as the number of charged particles increases.
required above; we shall make explicit comparisons with the data in Chapter IV.
CHAPTER III. PARTON PICTURE IMPLICATIONS

We now visualize the interactions between two partons colliding at very high energy in terms of the interactions of their constituents. Each proton is pictured, in its infinite momentum frame, as a cloud of partons carrying different fractions $x_i$ of the longitudinal momentum of the entire proton. As shown by Drell and Yan (14), from the energy-momentum conservation relation one may profitably think of three different regimes of parton momentum as responsible for three distinct types of interaction. The rare collisions between "hard" partons, i.e., those carrying a large fraction of the longitudinal momentum, are responsible for high transverse-momentum events, which do not concern us here. Interactions between "wee" partons, those with longitudinal fraction $x_w$ of the order of $1/\sqrt{s}$, are responsible for the pionization region. The "super-wee" partons, with longitudinal fraction $x_{sw} \sim 1/s$, interact to produce fragmentation events and, coherently, also the elastic process. We shall use the multiperipheral parton model of Kogut and Susskind (13) to determine the translation of the latter two effects into impact parameter space in order to obtain a physical model for $\tilde{n}(\beta)$.

For our purposes, the essential features of Kogut and Susskind's model are the following.

1. The longitudinal fraction of momentum is strongly damped along the chain of partons, $x_n \ll x_{n-1}$, as indicated in Fig. 2.
2. Only partons near each other in phase space interact strongly.
3. The transverse motion of the partons is a random walk process.
Fig. 2. Diagram defining the multiperipheral parton picture of the proton, following Kogut and Susskind.
away from the fast leading parton, so that their mean-square transverse distance from it can be calculated, with the results

\[ \langle (B_i - B_n)^2 \rangle \sim \log \left( \frac{1}{x_n^i} \right) , \quad i = A, B \quad (3.1) \]

The first two features suggest that the approximations made in the eikonalization kinematics are still valid; the primary particles can be treated as equivalent to the point-like leading partons which carry all but a small fraction of the momenta. They lose little of their momenta during interaction when the slow partons farther down the chain interact. Therefore we do not need detailed parton wave functions to calculate the amplitude, although required in Ref. (4).

From this third property, we may extract the region of impact parameter space in which the wee and super-wee partons are important. From Eq. (3.1), the mean square impact parameter

\[ \langle B^2 \rangle = \langle [(B_A - B_n) - (B_B - B_n)]^2 \rangle = 2 \log \left( \frac{1}{x_n} \right) , \quad (3.2) \]

where we have neglected the cross term \[2\langle (B_A - B_n) \cdot (B_B - B_n)\rangle\] because \((B_A - B_n)\) and \((B_B - B_n)\) are uncorrelated. Clearly, the super-wee partons, with \(x_{SW} \ll x_{W}\), correspond to larger impact parameters than the wee partons; thus, the fragmentation events are more peripheral than the pionization events. Since these same "super-wee" partons enter into elastic processes, the Drell-Yan-West relation (15) can be used to relate the probability of finding a hard parton accompanied by a multiperipheral chain going down to "super-wee" partons with the high momentum transfer part of the elastic form factor. Regardless of the elastic form factor
used, the essential feature following for the inelastic form factor and hence for $\tilde{n}(B)$ is rapid fall off with $B$ for large $B$. For smaller values of $B$ such that the wee (and not the super-wee) partons dominate, however, we shall extract our result directly from the multiperipheral parton model. As shown by Kogut and Susskind, the probability for finding wee partons in both protons with the same values of $x_n$ is proportional to $s^{\alpha-1}$, i.e., it is constant if $\alpha = 1$. It is thus independent of $x_n$, and therefore by Eq. (3.2) it is independent of the impact parameter as well.

We therefore conclude that the density of partons is independent of $B$ for $B$ in the pionization region, while it falls off rapidly in the fragmentation region. A reasonable model for the density of particle-producing matter will therefore have $\tilde{n}(B) \sim$ constant out to some value $B_0$ and rapidly decreasing thereafter. For the pionization region, we take

$$\tilde{n}_n(B) = n_0, \quad 0 \leq B \leq B_0$$

(3.3)

and for the fragmentation contribution, we add a term

$$n_0(B) = n_0 e^{-(B^2 - B_0^2)/\Delta^2}, \quad B_0 < B < \infty$$

(3.4)

Then, the partial cross section for $n$ cluster production is

$$\sigma_n = \pi \int_0^{B_0^2} dB^2 \frac{n_0 e^{-n_0}}{n!} + \pi \int_{B_0^2}^{\infty} dB^2 \frac{n_0 e^{-n_0}}{n!} e^{-(B^2 - B_0^2)/\Delta^2}$$

$$- n_0 (B^2 - B_0^2)/\Delta^2 - n_0 \exp(-(B^2 - B_0^2)/\Delta^2)$$

$$= \frac{\pi B_0^2 n_0 e^{-n_0}}{n!} + \frac{\pi \Delta^2}{n} \left( 1 - e^{-n_0} \sum_{\ell=0}^{n-1} \frac{n_0^\ell}{\ell!} \right)$$

(3.5)
The total inelastic cross section is

\[ \sigma_{\text{inel}} = \sigma_{0(\text{inel})} + \sum_{n=1}^{\infty} \sigma_n \]

\[ = \sigma_{0(\text{inel})} + \frac{\pi B^2}{2} \left( 1 - e^{-n_0B_0} \right) + \frac{\pi \Delta^2}{2} \sum_{l=1}^{\infty} \frac{(-1)^{l+1} n_0^l}{l \cdot l!} \], \quad (3.6)\]

where \( \sigma_{0(\text{inel})} \) is the zero cluster inelastic cross section corresponding to the two-prong charged particle inelastic cross section in experimental data. From Eqs. (3.5) and (3.6) the average cluster multiplicity is

\[ <n> = \sum_{n=1}^{\infty} \frac{n \sigma_n}{\sigma_{\text{inel}}} = \frac{\pi B^2}{2 \sigma_{\text{inel}}} n_0 + \frac{\pi \Delta^2}{2} n_0 \], \quad (3.7)\]

and the probability for finding \( n \) clusters,

\[ p_n = \frac{\sigma_n}{\sigma_{\text{inel}}} \], \quad (3.8)\]

Besides Eq. (3.3) and Eq. (3.4), other forms of \( B \) dependence for the density of particle-producing matter and also for \( \tilde{n}(B) \) could be

\[ \tilde{n}(B) = n_0 \quad 0 \leq B \leq B_0 \]

\[ = n_0 e^{-B^2/\Delta^2} \quad B_0 < B < \infty \], \quad (3.9)\]

which shows two rather distinct components for the pionization region and the fragmentation region, or

\[ \tilde{n}(B) = n_0 (1 + e^{(B^2 - B_0^2)/\Delta^2}) \]. \quad (3.10)\]
This is a smooth, continuous distribution in which there is no clear line of demarcation in impact parameter between fragmentation and pionization.
CHAPTER IV. EXPERIMENTAL DATA COMPARISON FOR IMPACT PARAMETER PARTON MODEL

As mentioned in Chapter I, the most suitable data on which to test our model result from proton-proton collisions at high energy. Therefore, we compare our results from Chapter III with the experimental pp production data with beam momenta 69 GeV/c (16), 102 GeV/c (17), 205 GeV/c (18), 300 GeV/c (19), and 405 GeV/c (20). Adequate data comparisons result at 50 GeV/c (16) but are not shown in the figures presented.

From Eq. (3.6), the pionization probability is

\[ P_\pi = \pi B_0^2 (1 - e^{-n_0}) / \sigma_{inel} \]  \( (4.1) \)

and the probability of having fragmentation production is

\[ P_D = \frac{1}{\sigma_{inel}} \left( \frac{\pi \Delta^2}{\sigma_{inel}} \sum_{\ell=1}^{\infty} (-1)^{\ell+1} \frac{n_0^{\ell}}{[\ell \cdot \ell!] + \sigma_0(\text{inel})} \right) \]  \( (4.2) \)

where we have included \( \sigma_0(\text{inel}) \) as defined above Eq. (3.7) in the fragmentation region. In terms of \( P_\pi \) and \( P_D \), Eq. (3.7) and Eq. (3.8) can be written as

\[ \langle n \rangle = P_\pi \left( \frac{n_0}{1 - e^{-n_0}} \right) + \left( P_D - \frac{\sigma_0(\text{inel})}{\sigma_{inel}} \right) \left( \frac{n_0}{\sum_{\ell=1}^{\infty} (-1)^{\ell+1} \frac{n_0^{\ell}}{[\ell \cdot \ell!]^{-1}}} \right) \]  \( (4.3) \)

and
Substituting the experimental values for $\langle n \rangle$, $\sigma_0(\text{inel})$, and $\sigma_{\text{inel}}$ and the constraint $P = P_0$ into Eq. (4.3), we find a relation between $P_\pi$ and $n_0$. From this relation and Eq. (4.4), the probability for finding $n$ clusters at each energy is then a function of only one parameter, $n_0$.

To get the probability for finding charged particles, we further make the assumption that the clusters are composed of charged pairs (12), so that

$$\langle n_{\text{ch}} \rangle = 2\langle n \rangle + 2 \quad (4.5)$$

With this assumption, the best results for $P_{\pi}$ vs. $n_{\text{ch}}$ calculated by using $n_0$ (defined by Eqs. (3.3) and (3.4) as a free parameter are shown in Figs. 3-7 as the solid lines for the incident proton momenta, 70 GeV/c, 100 GeV/c, 200 GeV/c, 300 GeV/c and 400 GeV/c. To show the separate explicit contributions of the pionization and fragmentation regions, we plot in Figs. 8-12 these two components respectively for each incident momentum. The "pionization" component which comes from the first term of Eq. (4.4) is a simple Poisson distribution showing the typical distribution of pionization products from multiperipheral models. The actual content of the fragmentation component from the last term of Eq. (4.4) may not necessarily be diffractive; but, comparison with the

\[
P_n = \pi \frac{n^n e^{-n_0}}{n!} \left( P_0 - \frac{\sigma_0(\text{inel})}{\sigma_{\text{inel}}} \right) \left( 1 - e^{-n_0} \sum_{l=0}^\infty \frac{n^l_0}{l!} \right) \sum_{l=1}^\infty \frac{(-1)^{l+1}}{l!} n^l_0 \frac{2(\cdot l!)}{l!}^{-1} .
\]
Fig. 3. Theoretical calculations of the probability for producing \( n \) charged particles in a pp collision at incident momentum 69 GeV/c compared with data from Ref. 16. On the figures \( P_n \) is given by the published \( \sigma_n \) divided by \( \sigma_{\text{inel}} \).
Fig. 4. Theoretical calculation of the probability for producing $n$ charged particles in a $pp$ collision at incident momentum 102 GeV/c compared with data from Ref. 17
Fig. 5. Theoretical calculation of the probability for producing $n$ charged particles in a pp collision at incident momentum 205 GeV/c compared with data from Ref. 18.
Fig. 6. Theoretical calculation of the probability for producing $n$ charged particles in a pp collision at incident momentum 300 GeV/c compared with data from Ref. 19.
Fig. 7. Theoretical calculation of the probability for producing $n$ charged particles in a pp collision at incident momentum 405 GeV/c compared with data from Ref. 20
Fig. 8. Plot showing contributions to the total probability (dashed line) for finding $n_{\text{ch}}$ charged particles at 69 GeV/c incident proton momentum. The dotted curve gives the central pionization contribution and the dot-dashed curve the fragmentation contribution which is compared with the $n_{\text{ch}}^{-2}$ solid curve.
Fig. 9. Plot showing contributions to the total probability (dashed line) for finding $n_{ch}$ charged particles at 102 GeV/c incident proton momentum. The dotted curve gives the central pionization contribution and the dot-dashed curve the fragmentation contribution which is compared with the $n_{ch}^{-2}$ solid curve.
Fig. 10. Plot showing contributions to the total probability (dashed line) for finding $n_{\text{ch}}$ charged particles at 205 GeV/c incident proton momentum. The dotted curve gives the central pionization contribution and the dot-dashed curve the fragmentation contribution which is compared with the $n_{\text{ch}}^{-2}$ solid curve.
Fig. 11. Plot showing contributions to the total probability (dashed line) for finding $n_{ch}$ charged particles at 300 GeV/c incident proton momentum. The dotted curve gives the central pionization contribution and the dot-dashed curve the fragmentation contribution which is compared with the $n_{ch}^{-2}$ solid curve.
Fig. 12. Plot showing contributions to the total probability (dashed line) for finding $n_{ch}$ charged particles at 405 GeV/c incident proton momentum. The dotted curve gives the central pionization contribution and the dot-dashed curve the fragmentation contribution which is compared with the $n_{ch}^{-2}$ solid curve.
$n_{ch}^{-2}$ curves in Figs. 8-12, shows typical fragmentation model behavior exists.

The numerical values of $p_\pi, B_0$ and $\Delta^2$ calculated with the best $n_0$ at each energy are given in Table I. From these values, we note the following results:

1. The probability for pionization production is $\sim 0.75$ for all energies. This feature agrees with Van Hove's and Fialkowski's result from the study of experimental correlation data with the two-component picture (21).

2. As shown in Fig. 13, the values of $n_0$ leading to the best fits essentially have a logarithmic dependence on $s$. The probability for finding pionization products and the nonpionization contribution to $\langle n \rangle$ are calculated to be independent of $s$; from Eq. (4.3), the nearly logarithmic $s$ dependence of $n$ then gives the same logarithmic $s$ dependence for $n_0$.

3. The numerical values of $B_0$ are $\sim 0.89$ fm for all energies. We might think of the constant part of our inelastic structure function as a "black disk" and the rapidly falling tail part as a "gray ring". That the interaction radius of the "black disk" is about the size of a nucleon radius independent of energy is consistent with what one expects from a nonlinear multiperipheral bootstrap approach as suggested in Ref. (4).

4. The same general shape for $\tilde{n}(B)$ as a function of $B$ results regardless of the assumed functional form: Eqs. (3.3) and
Table I. Values of parameters $n_0$, $B_0$, and $\Delta$ defined by Eqs. (3.3) and (3.4) along with the fraction of pionization particles, $P_{\pi}$.

<table>
<thead>
<tr>
<th>Beam Momentum</th>
<th>$P_{\pi}$</th>
<th>$n_0$</th>
<th>$B_0$ (in fm)</th>
<th>$\Delta$ (in fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>70 GeV/c</td>
<td>0.77 $\pm$ 0.07</td>
<td>2.084 $\pm$ 0.035</td>
<td>0.94 $\pm$ 0.04</td>
<td>0.23 $\pm$ 0.04</td>
</tr>
<tr>
<td>100 GeV/c</td>
<td>0.73 $\pm$ 0.05</td>
<td>2.521 $\pm$ 0.045</td>
<td>0.89 $\pm$ 0.03</td>
<td>0.28 $\pm$ 0.03</td>
</tr>
<tr>
<td>200 GeV/c</td>
<td>0.74 $\pm$ 0.08</td>
<td>3.305 $\pm$ 0.057</td>
<td>0.90 $\pm$ 0.06</td>
<td>0.30 $\pm$ 0.05</td>
</tr>
<tr>
<td>300 GeV/c</td>
<td>0.75 $\pm$ 0.05</td>
<td>3.785 $\pm$ 0.042</td>
<td>0.89 $\pm$ 0.04</td>
<td>0.31 $\pm$ 0.03</td>
</tr>
<tr>
<td>400 GeV/c</td>
<td>0.73 $\pm$ 0.06</td>
<td>4.195 $\pm$ 0.070</td>
<td>0.88 $\pm$ 0.05</td>
<td>0.31 $\pm$ 0.04</td>
</tr>
</tbody>
</table>
Fig. 13. The plonization strength $n_0$ defined by Eqs. (3.3) and (3.4) vs. $s$. The two lower points are both from Ref. 16 for incident proton momenta 50 GeV/c and 69 GeV/c
(3.4), Eq. (3.9), or Eq. (3.10). With the same procedure as outlined in the previous chapter, better or equally good $\chi^2$ values to the fits in Figs. 3 to 7 result from Eq. (3.10), while Eq. (3.9) leads to somewhat poorer fits. The comparison of $\tilde{n}(B)$ vs. $B$ for these two best fit cases (Eqs. (3.3) and (3.4) and Eq. (3.10)) are shown in Fig. 14. The curves are quite similar though the continuous one does not allow a clear separation into central and peripheral components. Nevertheless, in all three cases $n^0_0$ characterizes the strength, $B_0$ the pionization range, and $A^2$ determines the slope in the peripheral region.

5. We note that the $\chi^2$ per point is unity or less for all energies except the highest where three or four data points clearly seen in Fig. 7 fluctuate excessively about the smooth curve. Since $\tilde{n}(B,s)$ determines the average charge multiplicity which rises as $\ln s$ (22), the normalization $n^0_0$ could be taken as $N_0\ln s$ for any energy to formally bring $\chi^2$/degree of freedom to the statistically significant level despite the data fit of Fig. 7. The same statement holds for the continuous (Woods-Saxon) shape of Eq. (3.10).
Fig. 14. Distributions for $\bar{n}(B)$ vs. $B$ for the discontinuous case given by Eqs. (3.3) and (3.4), solid line, and Eq. (3.10), dot-dash line
CHAPTER V. NAIVE QUARK RECOMBINATION PHYSICS

As discussed in Chapter III, particles produced with finite mass and limited transverse momentum in high energy hadron-hadron interactions are due to either "wee" parton or "super-wee" parton exchange. Those particles produced from exchange of "super-wee" parton have a large longitudinal momentum fraction of the initial particle, $1 > x > 1/\sqrt{s}$, thus forming the fragmentation region particles. Producing these fragmentation events requires much longer interaction times than do those coming from pionization events due to "wee" parton exchange. A simple estimate of the ratio would be

$$\frac{\tau_{SW}}{\tau_{W}} \sim \frac{z_{SW}}{z_{W}} \frac{\sqrt{s}}{\sqrt{s}} \sim \sqrt{s}.$$ 

During this "long" interval, it is highly probable that one of the valence quarks (the leading partons) of the initial hadron recombines with a slow sea antiquark to make a meson in the fragmentation region. Based on this idea Das and Hwa (8) formulated the quark recombination model, and their approach has been recently adopted by other authors (23). It is useful to discuss the basic formulae, as improvements in the next chapter will be easier to introduce. For the recombination of, e.g., a $\pi^+$ meson from a $u$ valence quark and a $\bar{d}$ sea quark, these authors write the inclusive $\pi^+$ production cross section in the fragmentation region as
\[
\frac{d\sigma}{dx} = \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} F(x_1, x_2) R(x_1, x_2, x) \quad (5.1)
\]

The variables \(x_1\) and \(x_2\) give \(x\) values for the \(q\) and sea \(\bar{q}\) constituents, \(F(x_1, x_2)\) is the scale-invariant, two quark momentum distribution function for having \(q(\bar{q})\) with longitudinal momentum fraction \(x_1(x_2)\), and \(R(x_1, x_2, x)\) is the \(q-\bar{q}\) two body recombination function

\[
R(x_1, x_2, x) = \alpha_M \frac{x_1 x_2}{x^2} \delta \left( 1 - \frac{x_1}{x} - \frac{x_2}{x} \right) \quad (5.2)
\]

argued for by Das and Hwa (8) on the basis of counting rules. It was noted that a sum rule requirement on \(R(x_1, x_2, x)\) exists

\[
\int_0^1 d\xi_1 \int_0^1 d\xi_2 \ R(\xi_1, \xi_2) = 1 \quad , \quad \xi_i = \frac{x_i}{x} \quad (5.3)
\]

implying \(\alpha_M = 6\). An uncorrelated two quark distribution for \(F\) was assumed

\[
F(x_1, x_2) = F_q(x_1) F_{\bar{q}}(x_2) \rho(x_1, x_2) \quad (5.4)
\]

where, e.g., for \(q = u\), \(F_u(x) = xu(x)\) for the up quark momentum distribution, \(F_{\bar{q}}\) is the corresponding distribution for the sea antiquark which has a small argument typically, and \(\rho(x_1, x_2)\) is the phase space factor justified as assumed (8)

\[
\rho(x_1, x_2) = \beta_M (1 - x_1 - x_2) \quad (5.5)
\]

where \(\beta_M\) is a constant which can depend on the meson species. Substitution of Eqs. (5.2), (5.4) and (5.5) into Eq. (5.1) leads to (23)
As emphasized by Miettinen (23), this approach can be only approximately correct, as the joint quark distribution Eq. (5.4) does not integrate to the single quark distribution. Duke and Taylor therefore examine ratios of various inclusive particle distributions so that the sensitivity to the approximations will be minimized by cancellation. (They also take the $\beta_M$ all equal.)

The straightforward generalization of Eqs. (5.1)-(5.7) with the same approximations to processes proceeding by recombination of three quarks was done by Ranft (23), e.g., the recombination function has the obvious form

$$R_3(x_1, x_2, x_3, x) = \alpha^B \xi_1 \xi_2 \xi_3 \delta(\xi_1 + \xi_2 + \xi_3 - 1)$$ (5.7)

However, the three quark joint momentum probability distribution generalized from Eq. (5.5) is found to be insufficient and Ranft introduces another parameter $\gamma$ to allow data to be fitted. For inclusive proton production, for example

$$F_3(x_1, x_2, x_3) = \beta^p F_u(x_1) F_u(x_2) F_d(x_3)(1 - x_1 - x_2 - x_3)^\gamma$$ (5.8)

Data to which this approach is applied include high energy leading protons, neutrons, and $\Lambda$ hyperons with low $q_T$ produced in $p$-$p$ collisions.
VI. KINEMATICAL CONSIDERATIONS ON QUARK RECOMBINATION

We follow the basic reasoning of the quark recombination model discussed in Chapter V to try to improve and study limitations of these ideas. It might appear that Eq. (5.4) could be regarded from the purely mathematical point of view as completely general when the function \( p \) is left unspecified. Clearly, the form in Eq. (5.5) is quite specific. However, Das and Hwa (8) emphasize that they want to try a factorized form with the valence quark and sea quark distributions as explicit factors as an ansatz to be used in the asymmetric situation \( x_q \gg x_q^- \). The consequences and limitations of such an assumption have not been pursued in the literature.

We start by showing that one can assume Eq. (5.4) is rigorously true everywhere and arrive at a contradictory appearing result. It is required that the quark-antiquark joint momentum distribution \( F(x_q, x_q^-) \) should be integrable on either variable to yield within a constant the single quark or antiquark momentum distribution in the other variable, as emphasized by Miettinen (23). If the single quark and antiquark momentum distributions are factorized explicitly as in Eq. (5.4), this requirement can then be phrased in terms of momentum sum rules (24):

\[
\langle x_q \rangle F_q(x_q) = \int_0^{1-x_q} dx_q^- F(x_q, x_q^-) \quad (6.1)
\]

and

\[
\langle x_q^- \rangle F_q^-(x_q^-) = \int_0^{1-x_q} dx_q F(x_q, x_q^-) \quad (6.2)
\]
where \( \langle x^- \rangle = \int_0^1 dx^- \cdot F_q(x^-) \) and \( \langle x^- \rangle = \int_0^1 dx \cdot F_q(x) \). One can then cancel out \( F_q(x^-) \cdot (F_q(x^-)) \) on both sides of Eq. (6.1) (Eq. (6.2)) and solve for \( \rho(x_q, x^-) \), as we shall show. The two solutions are in general not equal, even though the factorized form Eq. (5.4) suggests that they should be. This result indicates that the apparent symmetry between \( q \) and \( \bar{q} \) in Eq. (5.4) is not real.

In the quark recombination picture, there is lack of symmetry between the roles of quark and antiquark. The physical picture we started with has an undisturbed hard quark dominating the dynamics. It seems more logical to factor only the momentum distribution of the hard (i.e., high momentum) valence quark since its properties appear relatively independent of the probe. The sea distributions appear to be dependent on the model of study, on the other hand, and it may as well be lumped with the unknown correlation, phase-space factor, and modifications of their distributions due to the disturbance by the interaction as a new function. The necessity of the enhanced sea (8.23) might be an indication of this disturbance. The requirement that integration over the quark variable to give a function proportional to the antiquark momentum distribution in Eq. (6.2) suggests that the sea antiquark also goes through the interaction region unaffected. In terms of the \( x \) distribution \( \tilde{q}(x^-) \) itself (i.e., \( F_q(x^-) = x^- \tilde{q}(x^-) \)), \( \tilde{q}(x^-) \) has an infrared singularity so it is not surprising that inconsistencies result. Ranft (23), for example, writes his constraint (which is not strictly correct) on \( \rho \) for three quark recombinations only in the order consistent with
Eq. (6.1). Thus, Eq. (5.4) will not be used here, through all recent work in the literature (8,23) does make this assumption.

We therefore postulate the joint momentum distribution as,

\[ F(x_q,x_q^-) = F_q(x_q^-) \eta(x_q^-) \]  \hspace{1cm} (6.3)

The function \( \eta(x_q^-) \) lumps together our ignorance of the sea quarks, their correlations with valence quarks, and modifications of their distributions. From Eq. (6.3) and discussions above, only the momentum sum rule Eq. (6.1) is consistent with the original picture of the quark recombination model. The upper limit on the right hand side integral explicitly points out a kinematical restriction on the \( x_q^- \) value. Substitution of Eq. (6.3) into Eq. (6.1) gives

\[
\int_0^{1-x_q^-} dx_q^- F_q(x_q^-) = \int_0^{1-x_q} dx_q^- \eta(x_q^-) , \hspace{0.5cm} x_q \neq 1 . \hspace{1cm} (6.4)
\]

For mathematical consistency of Eq. (6.4) we note that \( F_q(1) = 0 \), i.e., all the momentum can not be carried by a single parton. This is in agreement with the fact that the probability of finding a wee parton at \( x \) is equal to \( \frac{dx}{x} \) and therefore each parton must have some momentum in the bremsstrahlung analysis of the original parton picture (1). It is also a consistency condition on Eq. (6.4).

Now we make a variable change on the right side of Eq. (6.4), \( x_q^- + y = x_q^-/(1-x_q^-) \) simply rescaling \( x_q^- \), so the integrands can be compared to get

\[
F(x_q^-,x_q^-) = F_q(x_q^-) F_q(x_q^-/(1-x_q^-))/(1-x_q^-) , \hspace{0.5cm} x_q \neq 1 . \hspace{1cm} (6.5)
\]
This is a specific solution to the integral Eq. (6.4). As such it is not unique, but is of interest because it incorporates the kinematical requirements of the momentum sum rule (6.1) and it can be integrated on the antiquark argument to yield the single quark distribution. Since Eq. (6.4) follows from Eq. (6.1), the form postulated in Eq. (6.3) is justified and \( \eta(x_q, x^-) = F_q(x^-/1-x_q)/(1-x_q) \) does not factorize into the form of Das and Hwa (8) in Eq. (5.4).

If one also assumes that \( F(x_q, x^-) \) contains a factor \( F_q(x^-) \), as in Das and Hwa's form (5.4), one can follow an analogous procedure using (6.2) rather than (6.1) to obtain

\[
F(x_q, x^-) = F_q(x^-) F_q(x_q/1-x^-)/(1-x^-) \quad (6.6)
\]

Clearly the expression for \( F(x_q, x^-) \) given by (6.5) and (6.6) are not necessarily identical for general \( F_q(x_q) \) and \( F_q(x^-) \). Indeed, if one evaluates

\[
F_q(x_q) F_q\left(\frac{x_q}{1-x_q}\right)(1-x_q)^{-1} = F_q(x_q) F_q\left(\frac{x^-}{1-x^-}\right)(1-x^-)^{-1}
\]

at \( x_q = 0 \), it follows that

\[
F_q(0) F_q(x_q) = F_q(x_q) F_q(0)(1-x_q)^{-1}
\]

i.e.,

\[
1 - x_q = 1
\]

Thus the two forms are mutually inconsistent (25), and Das and Hwa's assumption (5.4) is untenable. This is our reason for making the less restrictive assumption (6.3).
To close this chapter, we use this new form for the joint momentum distribution to calculate the $x$ distribution for meson quark recombination and the modified sea quark distributions. Inserting Eq. (6.5) into Eq. (5.1) we find for meson recombination

$$\frac{d\sigma_M}{dx} = \frac{\alpha_M}{x} \int_0^x \frac{dx_q}{1-x_q} F_q(x_q) F_q\left(\frac{x-x_q}{1-x_q}\right), \quad x_q \neq 1. \quad (6.7)$$

We note that in ratios of meson $x$ distributions, the factors $\alpha_M$ cancel out. Using Eq. (6.5) and the momentum sum rule Eq. (6.2), we have the modified sea quark distribution

$$F_q'(x_q) = \frac{1-x_q}{\int_0^1 dx_q F_q(x_q) F_q'(x_q/(1-x_q))/(1-x_q)} \quad \int_0^1 dx_q F_q(x_q). \quad (6.8)$$

In general, these modified sea quark distributions are different from the original ones inside the proton. They are also dependent on the flavor of the valence quark with which they combine. We have evaluated $F_q'(x_q)$ using the valence quark distributions of Field and Feynman (26) for $F_q(x_q)$ and our sea quark distributions which we shall determine later in Chapter VII for $F_q'(x_q)$. The results are shown in Fig. 15. The comparison between $F_q'(x_q)$, which indicates the degree of self-consistency in the assumption (6.3), is quite satisfactory.

It is interesting to mention that the specific solution Eq. (6.6), although it is not physically reasonable, after being substituted into Eq. (5.1) with the specific form of $R$ as in Eq. (5.2) gives
Fig. 15. Comparison of some of the modified sea antiquark distributions $F_q^i(x_q)$ with the original sea antiquark distributions $F_q(x_q)$. (a) $F_q^i(x_q)$ of $\bar{q}$ antiquark when combined with $u$ quark (solid line) and $F_q(x_q)$ (dot-dash line). (b) $F_q^i(x_q)$ of $u$ antiquark when combined with $d$ quark (solid line) and $F_q(x_q)$ (dot-dash line).
\[ \frac{d\sigma_M}{dx} = \frac{r M}{x} \int_0^x \frac{dx_q}{(1-x+sx_q)} \ F_q \left( \frac{x_q}{1-x+sx_q} \right) F_q(x-x_q) . \]

If we change \( x_q \to x_q' = x_q/(1-x+sx_q) \), this yields the same expression as Eq. (6.7).
CHAPTER VII. EXPERIMENTAL DATA COMPARISON FOR THE QUARK RECOMBINATION MODEL

A. Meson Production Ratios

The \( \pi^+, \pi^-, K^+, \) and \( K^- \) mesons are the easiest mesons to detect in a reaction. Experimental inclusive measurements on these data exist and ratios of particles produced as a function of Feynman \( x \) can be constructed. From the four particle production ratios \( \pi^+/\pi^- \), \( K^+/K^- \), \( \pi^+/K^+ \), and \( \pi^-/K^- \) as functions of \( x \), Duke and Taylor (23) deduced sea quark distributions by use of the naive quark recombination model described in Chapter V. They used the valence quark distributions of Field and Feynman (26) with the accepted forms for sea quark distributions

\[
F_u(x) = x u_{\text{sea}}(x) = x \bar{u}(x) = u_0(1 - x)^n_u
\]  
\[
F_d(x) = x d_{\text{sea}}(x) = x \bar{d}(x) = d_0(1 - x)^n_d
\]  
\[
F_s(x) = x s_{\text{sea}}(x) = x \bar{s}(x) = s_0(1 - x)^n_s
\]

for \( q = \text{up, down and sideways sea quarks} \). From Eq. (5.6) a representative particle production ratio in the recombination model (23) is

\[
\frac{A}{B} = \frac{\beta_A \int_0^x dx_1 F_{Aq}(x_1) F_{Aq}^-(x - x_1)}{\beta_B \int_0^x dx_1 F_{Bq}(x_1) F_{Bq}^-(x - x_1)}
\]

where \( A, B \) represent \( \pi^+, K^+, \ldots \) etc. and \( \beta_A = \beta_B = \beta_M \) are taken to be species independent. Our consistency sum rule Eq. (6.6) shows that this approximation can be improved upon.
Our approach given in Chapter VI leads to Eq. (6.6) replacing Eq. (5.5). Meson production ratios as functions of \( x \) in the consistent quark recombination model are

\[
\frac{A}{B} = \frac{\int_0^x \frac{dx_1}{1-x_1} F_A(x_1) F_A\left(\frac{x-x_1}{1-x_1}\right)}{\int_0^x \frac{dx_1}{1-x_1} F_B(x_1) F_B\left(\frac{x-x_1}{1-x_1}\right)},
\]

(7.3)

where the phase space constant \( \lambda \) has indeed been explicitly shown to be species independent. Using Eqs. (7.1a-c) for the sea distributions and also the Field-Feynman valence quark distributions, sea functions are found as

\[
x \bar{u}(x) = 0.60(1-x)^5
\]

(7.4a)

\[
x \bar{d}(x) = 1.15(1-x)^9
\]

(7.4b)

\[
x \bar{s}(x) = 0.115(1-x)^6
\]

(7.4c)

by using Eq. (7.3) to fit particle ratios as functions of \( x \). To obtain these distributions Eq. (7.4a-c), the powers \( n_u, n_d, \) and \( n_s \) were restricted in the search procedure to integers and the symmetry requirements used in the second paper of Ref. 23 was dropped. Results for four different particle ratios are shown in Figs. 16 through 19.

The theory curve in these figures 16, 17, 18, and 19 follow from Eq. (7.3) with \( A/B = \pi^+ / \pi^- = \left( \frac{d\sigma}{dx} \right)_{pp+\pi^+} / \left( \frac{d\sigma}{dx} \right)_{pp+\pi^-} \), with \( A/B = K^+ / K^- \)

\[
= \left( \frac{d\sigma}{dx} \right)_{pp+K^+} / \left( \frac{d\sigma}{dx} \right)_{pp+K^-}, \text{ with } A/B = \pi^- / K^- = \left( \frac{d\sigma}{dx} \right)_{pp+\pi^-} / \left( \frac{d\sigma}{dx} \right)_{pp+K^-}.
\]
Fig. 16. The $\pi^+/\pi^-$ production ratio in p-p collisions as a function of $x$. The solid curve is calculated from our sea quark distributions Eq. (7.4) with the Field-Feynman valence quark distributions in the consistent quark recombination model. Experimental data are from Fermilab and ISR, Refs. 27 and 28, respectively.
Fig. 17. The $K^+/K^-$ production ratio in p-p collisions as a function of $x$. The solid curve is calculated from our sea quark distributions Eq. (7.4) with the Field-Feynman valence quark distributions in the consistent quark recombination model. Experimental data are from Fermilab and ISR, Refs. 27 and 28, respectively.
Fig. 18. The $\pi^+/K^+$ production ratio in p-p collisions as a function of $x$. The solid curve is calculated from our sea quark distributions Eq. (7.4) with the Field-Feynman valence quark distributions in the consistent quark recombination model. Experimental data are from Fermilab and ISR, Refs. 27 and 28, respectively.
Fig. 19. The $\pi^-/K^-$ production ratio in p-p collisions as a function of $x$. The solid curve is calculated from our sea quark distributions Eq. (7.4) with the Field-Feynman valence quark distributions in the consistent quark recombination model. Experimental data are from Fermilab and ISR, Refs. 27 and 28, respectively.
and with \( A/B = \pi^+ / K^+ = \left( \frac{d\sigma_{pp+\pi^+}}{dx} \right) / \left( \frac{d\sigma_{pp+K^+}}{dx} \right) \), respectively, plotted versus \( x \). The data shown are from experiments at Fermi-
lab (27) and ISR (28). These data are functions of \( x \); they give values in Eq. (7.4). We note that \( K^- \) meson is formed from re-
combination of two sea quarks and therefore its production is strongly suppressed. These sea quark distributions are shown in Fig. 20. The curve labeled \( x_u(x) \) (\( x_d(x) \)) is the sum of the Field and Feynman valence and our new Eq. (7.1) up (down) quark dis-
tributions.

An interesting test of these functions is supplied by recently measured \( \eta \) inclusive production given as a ratio to \( \pi^0 \) production in a Fermilab experiment (29). In the notation of Eq. (7.3), the \( \pi^0/\eta \) ratio as a function of \( x \) is

\[
\frac{\pi^0}{\eta} = \frac{\int_0^x \frac{dx_1}{(1-x_1)} \frac{1}{2} \left[ F_u(x_1)F_u(\tilde{x}_1) + F_d(x_1)F_d(\tilde{x}_1) \right]}{\int_0^x \frac{dx_1}{(1-x_1)} \frac{1}{6} \left[ F_u(x_1)F_u(\tilde{x}_1) + F_d(x_1)F_d(\tilde{x}_1) + 4F_s(x_1)F_s(\tilde{x}_1) \right]}
\]  

(7.5)

where \( \tilde{x}_1 = (x - x_1)/(1 - x_1) \). In the range \( x = 0.05 \) to \( x = 0.95 \) this ratio Eq. (7.5) varies from 2.92 to 3.00. This small variation is due to the term \( \frac{2}{3} (1 - x_1)F_s(x_1)F_s(\frac{x - x_1}{1 - x_1}) \) in the denominator which is insig-
ificant because both \( F_s \)'s are sea distributions with \( s_0 = 0.115 \) in Eq. (7.4c). Any form for the \( F_s^- \) distribution leads to a constant ratio.

Nevertheless, the agreement of this ratio with experiment, both in magnitude and flatness, is unusual confirmation of quark-parton
Fig. 20. Quark momentum distributions within the proton plotted as functions of $x$. The sea quark functions $x_u(x)$, $x_d(x)$, and $x_s(x)$ for up, down, and sideways sea contributions, respectively, are plotted from Eq. (7.4). The curve labeled $x_u(x)$ ($x_d(x)$) is the sum of the Field-Feynman valence quark distribution and our sea quark distribution to give the total momentum fraction carried by all up (down) quarks in a proton undergoing fragmentation.
recombination ideas. (The possibility of $q_T$ dependence in this ratio is hinted in the limited data, and this point is being explored.)

B. Inclusive Distributions

Of considerable interest are possible tests of this new scaling variables appearing in Eq. (6.17). To this end, we first examine the observation made by Ochs (7) on the ratio of $\pi^+ / \pi^-$ inclusive distributions. The sea quark distribution is a function very peaked at small values of its argument $x$, so it might be approximated by a $\delta$-function

$$F_{\text{sea}}(x) = F_{\text{sea}}(0) \delta(x)$$

in either Eq. (7.2) or (7.3). The $\pi^+ / \pi^-$ ratio with the consistency scaling variable is given in Eq. (7.3) and in Eq. (7.2) without requiring consistency. Both forms give

$$\frac{\pi^+}{\pi^-}(x) = \frac{u(x)}{d(x)} ,$$

the ratio of up to down quark distributions. However, the specific single particle distributions according to Eq. (6.6) with the scaling variable then has the form

$$\frac{d\sigma}{dx}\bigg|_\pi \propto \frac{F_{\pi q}(x)}{x(1-x)} ,$$

which is quite different from the approximation following from Eq. (5.6)

$$\frac{d\sigma}{dx}\bigg|_\pi \propto \frac{F_{\pi q}(x)(1-x)}{x} .$$
Therefore it seems logical to examine the pion inclusive $\frac{d\sigma}{dx}$ distributions directly in the fragmentation region without taking ratios. Our consistent quark recombination model expression Eq. (6.6), written explicitly for $\pi^+ (\pi^-)$ production, is

$$\frac{d\sigma}{dx} \bigg|_{(\pi^+)} = \frac{\alpha \lambda}{x} \int_0^x dx_1 \, F_{u_1}(x_1) \, F_{d_1}(\frac{x - x_1}{1 - x_1})(1 - x_1), \quad (7.6)$$

whereas the elementary quark recombination model gives

$$\frac{d\sigma}{dx} \bigg|_{(\pi^+)} = \frac{\alpha \beta (1 - x)}{x} \int_0^x dx_1 \, F_{u_1}(x_1) \, F_{d_1}(x - x_1). \quad (7.7)$$

Comparison of these two expressions with each other and with experimental data (30) is shown in Fig. 21. The curvature of the data, particularly the $\pi^+$, in this $\frac{d\sigma}{dx}$ vs. $x$ plot is reproduced very nicely with Eq. (7.6) and our new variable. We note that Pokorski and Van Hove (31) suggest that the explanation of the difference in slope between the low and high $x$ fragmentation regions is resonance creation and subsequent decay into a final $\pi$ meson. Our calculation suggests resonance creation is not a dominant effect; other authors (23) also argue that for $x > 0.5$ resonance production is not an important source of pions. A very recent investigation (32) places by explicit calculation an upper limit of 25% on resonance production contributions.
Fig. 21. Calculated values of $d\sigma/dx$ as a function of $x$ for the inclusive reactions $pp \rightarrow \pi^\pm +$ anything. The solid line follows from the consistent quark recombination model of Chapter VI and the dot-dashed line from the simpler elementary quark recombination model of Chapter V. The data shown are taken from Figs. 3 and 4 of Ref. 31. Both theoretical curves and data are unnormalized
CHAPTER VIII. DISCUSSION AND CONCLUSIONS

In this thesis, we examine two cases where the parton picture is applied to multiparticle production in high energy proton-proton collisions. Basically, the different pieces of information deduced from these two applications overlap little with each other. From the models with the impact parameter representation we obtained the transverse structure of partons in the proton while from the quark recombination model we learned the longitudinal structure.

In the generalized eikonal or Chou-Yang model, we study what the matter, or parton, "overlap" distribution responsible for particle (cluster) production is when two protons collide at high energies. The parton description of Kogut and Susskind suggests to us that there is little change in this overlap distribution-production potential when the impact parameter is small; and, the superwee partons are responsible for the rapid fall-off as the proton-proton impact parameter gets large. The quark-gluon cluster production picture (12) of Pokorski and Van Hove can be consistent with such a picture; the gluon cloud is easily stripped in peripheral collisions and spilled liberally in central collisions to make for no distinction between individual clusters produced by the two different mechanisms. The distribution deduced is quite different from that predicted by electromagnetic form factors in the conventional extension of the Chou-Yang model.

In the third Chou-Yang paper in Ref. 3 the original point-like interaction $\delta(\vec{b}_A - \vec{b}_B - \vec{b})$ function is generalized to $F(\vec{b}_A - \vec{b}_B - \vec{b})$
and the original c-number matter density function for elastic scattering is replaced by a corresponding q-number one to incorporate diffractive excitation processes. Our model is essentially equivalent in principle to this type of extension of the Chou-Yang model. The replacement of the \( \delta(\vec{\delta}_A - \vec{\delta}_B - \vec{\delta}) \) function for point interactions by \( F(\vec{\delta}_A - \vec{\delta}_B - \vec{\delta}) \) produces a smearing of the interaction in impact parameter space. In the pionization region, it is not surprising therefore that \( \bar{n} \) is not given by the electric or magnetic distributions. Rather, independent interactions of slow partons in the two protons can be depicted by polyperipheral diagrams (33). The analogy of our production mechanism to a polyperipheral diagram with only one cluster produced from each chain can be seen from the s-matrix given in Eq. (2.1). On the other hand, the diffractive fragmentation region is described by a matter density somewhat similar to the original elastic density in the inelastic channels of the q-number version of the Chou-Yang model.

We now comment on the relation of this portion of our work with that of Leader et al. (34) and with that of Snider and Wyld (35). Leader et al. showed quantitatively that any form in which the opacity \( \Omega(\vec{\delta}, s) \) from Chou-Yang model factorizes into \( K(s)f(\vec{\delta}) \) can not be fitted by the elastic pp data. Our form for \( \bar{n} \) obtained from inelastic data might well be expected to look like \( \Omega(\vec{\delta}, s) \). The growth of the tail region (large \( B \)) with energy appears similar; and the lack of factorization shows up in Table I in the energy dependence of \( A \). (The growth in the tail region corresponds to only one of the solutions of Leader et al., which now seems preferred.) There are clear similarities between
our diffractive contribution expressions for topological cross sections $\sigma_n$ for $n$ clusters and the total inelastic cross section in Eq. (3.5) and Eq. (3.6) and those of Snider and Wyld. In their "naive eikonal model" a Gaussian distribution in impact parameter for the amplitude of a single conventional Regge Pomeron exchange is assumed and then iterated as the basic "phase" in eikonalization. Fig. 1 of the text illustrates the basic monotonic decrease of $\sigma_n$, which results with Gaussians. Their less naive model, which is an "energy conserving eikonal model" with ladders of multiparticle chain production sharing the available energy, appears to incorporate pionization more adequately into the multi-Regge model. Finally, for further reference, a substantial version of this portion containing the parton-picture impact-parameter model in this thesis has appeared in published form (36).

In the quark recombination model, an outgoing large $x$, small $q_T$ meson produced in a $p$-$p$ collision results when one of the fast valence quarks in an initial proton combines with a sea antiquark. For recombination producing a meson, the cross section is given by Eqs. (5.1)-(5.6) in the naive, straightforward approach. In these equations, the quark-antiquark joint momentum distribution function $F(x_1, x_2)$ can not be integrated on the antiquark variable to yield the single quark momentum distribution. Our approach treated in detail in Chapter VI overcomes this difficulty. The condition can be written in the form of a sum rule, Eq. (6.4), from which the meson production cross section $\frac{d\sigma}{dx}$ in Eq. (6.6) depends on the sea quark distribution with a scaled variable. In meson production ratios, the effect of this new variable is apparent from the
final quark distributions given in Eq. (7.4). These are written in the form of Field and Feynman, for example, but parameters come out differently. Both SU(3) and SU(2) symmetry breaking is apparent from the differences in the powers nq and coefficients u_o, d_o, and s_o. Other authors have also found it necessary to break these symmetries (8,23,26). Field and Feynman (26) suggest that the presence of two u quarks and only one d quark suppresses uu pairs over dd pairs. Das and Hwa (8) and Ranft (23) base their conclusions on the relative sizes of the plateau in π⁺ and π⁻ production spectra. In the present calculations with the scaled argument it appeared impossible to fit all four particle ratios simultaneously without breaking these symmetries.

It might be noted that controversy exists about exactly what is the most appropriate form for the function F(x₁,x₂). Undoubtedly, Eq. (5.4) has the limitations noted in Ref. (23) and that noted in Chapter VI. Nevertheless, Hwa (38) particularly has emphasized that modifications being made on Eq. (5.4) by the authors in Ref. (23) do not improve the credibility of the model (33).

Our sea quark scaled variable can be argued for and a similar dependence deduced from an improved Kuti-Weisskopf model calculation (38). The general probability distribution can be written

\[ dp_n(x'_i, i=1, 2, 3; x_j, j=1,...n) = \prod_{i=1}^{3} Gv_i(x'_i) \ dx'_i \ \frac{1}{n!} \ \prod_{j=1}^{n} g_{sea_j}(x_j) \ dx_j \]

\[ \times \delta(1 - \sum_{i=1}^{3} x'_i - \sum_{j=1}^{n} x_j) \]
where the valence and sea quark functions, \( G_{v_i}(x) \) and \( g_{\text{sea}_j}(x) \), might behave as powers of \( x \) with \( G_{v_i}(x) \sim x^{-\frac{1}{2}} \) and \( g_{\text{sea}_j}(x) \sim x^{-1} \) according to Regge theory (38). By following the procedure given in Eqs. (B.3) to (B.8) of the Kuti-Weisskopf paper and integrating over the variables for unseen quarks, one finds a phase space factor \((1 - x_1 - \ldots - x_m)^\mu\) in the momentum distribution for \( m \) quarks. The power \( \mu \) depends on parameters (and powers) in \( G_{v_i}(x) \) and \( g_{\text{sea}_j}(x) \). The \( m \)-quark momentum distribution function following from the specific solution of the quark recombination model in Chapter VI would be

\[
F(x_1, \ldots, x_m) \propto \frac{F_1(x_1)F_2\left(\frac{x_2}{1-x_1}\right) \ldots \ldots F_m\left(\frac{x_m}{1-x_1 - \ldots - x_{m-1}}\right)}{(1-x_1)(1-x_1-x_2)\ldots \ldots (1-x_1 - \ldots - x_{m-1})}
\]

Then, with functions of the type given in Eq. (7.1), this same Kuti-Weisskopf phase space factor \((1 - x_1 - \ldots - x_m)^\mu\) results. Certainly, insofar as data comparison or fitting of data is concerned the approach in Chapter VI leads to a much less cumbersome equation to work with.


22. Various power laws are also possible; see, e.g., E. Suhonen et al., Phys. Rev. Lett. 31, 1567 (1973). However, for a range 50 - 400 GeV/c such as is being worked with here, there is no way to distinguish $\ln s$ from $s^y$.


24. H. I. Miettinen, phone communication to K. E. Lassila.

25. This proof was pointed out by Dr. S. J. Wilson.


30. ISR data obtained by the group working with Prof. H. Sens, quoted in Refs. 8 and 31.


37. R. C. Hwa, Lecture at the Thirteenth Moriond Conference in 1978 (to be published).

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