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Dao Yan Lim
Iowa State University

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Kinematics analysis and elevation control of a bi-directional VSAT antenna

by

Dao Yan Lim

A thesis submitted to the graduate faculty

in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

Major: Mechanical Engineering

Program of Study Committee:
Greg R. Luecke, Major Professor
Atul Kelkar
Ratnesh Kumar

Iowa State University
Ames, Iowa
2015

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ABSTRACT

The VSAT antenna is used for information exchange, which in order to do so it needs to align with the satellite in space. The pre-existing controller lacked the capability to control the antenna elevation motion, which leads to persistent steady state pointing error and occasional high amplitude oscillations. Extended motion testing of the antenna resulted in a few mechanical defects. This thesis explored and demonstrated multiple implementations of the classical controller and resolved the aforementioned problems. Specifically, the implementations are independent PI control, PI control with trajectory generation, PI control with Notch filter and trajectory generation, and PID control with second-order low-pass filter and trajectory generation. The antenna system with the controllers are demonstrated to be closed-loop stable through extensive root-locus analysis. Ultimately, the PI control with trajectory generation demonstrated the most desirable outcome. The systematic approach to analyzing the kinematics of the antenna as well as the system modelling of the dynamics of the antenna are also presented. The kinematics analysis verified that the antenna linkage is kinematically stable within the operating range. The system modelling suggested that the lumped antenna mass may be neglected.
CHAPTER 1. INTRODUCTION

VSAT antennas, also known as Very Small Aperture Terminal, are earth stations with antenna reflector diameter typically no larger than 2 m that allow information exchange between other earth stations or a central hub via satellite in space. These VSAT antennas are typically the primary commercial telecommunication technology used in isolated territories due to the absence of earthbound telecommunication infrastructure. Consequently, there are high demands in rural cities as well as in maritime markets including ocean vessels and oil rigs. According to COMSYS, a consulting firm that specializes in satellite and VSAT systems, there are at least 1,600,000 operating units deployed worldwide presently [1]. Most importantly, the VSAT antennas ensure continuous communication capabilities at any time and location across the globe. When malfunctions occur, for example misalignment with target satellite, temporary downtime disrupts communication, which particularly impacts emergency services. Correspondingly, one of the many sources of antenna pointing error is the servo control system error.

Servo control system error could have several contributors. One is the inaccuracy in the dynamics modeling of the antenna system. For instance, according to Hsia's textbook, Chapter 1, the control of the response of the system is limited depending on the accuracy of the mathematical model in describing its transient response [2]. Complicating the theoretical analysis, most systems include nonlinear elements such as free play in gear drives and friction in joints, which are difficult to characterize from sensor data [3]. The type of control system can also affect antenna positioning steady state error. Theoretically, a type ‘0’ system has finite steady state error when subjected to step input, while a type ‘1’ or a higher type system has zero steady state error with similar input [4]. A set of poorly designed controller gains could potentially render the system
unstable. When a Proportional-Integral-Derivative (PID) controller gains are high, the zeros of the controller are effectively placed to the far left half plane of a pole-zero map in Laplace domain, which results in a fast response. Consequently, when position error is large, the control command will exceed the acceleration limit. This is not ideal, especially in a mechanical system with a large mass as it might excite the oscillatory characteristics of the system. To emphasize such incident, prior to this research, the previously existing controller led the system to persistent vibrations during motion, which caused permanent mechanical defects over time. For example, as illustrated in Figure 1.1, there were scratches and material scraps where the joint at the linear actuator tip and the adjacent wall meet. Besides that, the damper rod that was attached to the bottom of the feedhorn deploying structure was observed to be bent. In order to address the aforementioned issue, many control techniques have been researched and suggested.

Many examples in the literature have demonstrated the implementation of modern controls approach. One recent research focus suggested that a self-tuning fuzzy logic controller [5] outperformed the PID controller in antenna azimuth control. The controller simulation results in this work showed marginally faster response time, zero steady state error, and no overshoot when compared to the PID [5]. In addition, model-based controllers such as Linear Quadratic Gaussian (LQG) and $H_\infty$ controllers have demonstrated pointing accuracy in terms of arcsecond in the presence of wind disturbance. Gawronski, an expert on antenna modelling and control, designed and implemented an LQG and a $H_\infty$ controller to stabilize an antenna system [6]. The author built an observer that estimates the antenna vibrations from the encoder measurements. He reported rms servo errors of 0.39 arcseconds for the LQG controller and 0.18 for the $H_\infty$ controller in elevation, in 10m/s wind gusts. However, the robust controllers might not be able to be implemented in the physical system unless the hardware is upgraded or the acceleration limit is
constrained in software. Although the researches have shown potential improvements when using modern control theory, they were related to larger antennas, specifically deep space terminals with diameter of 34 m - 70 m and majority of the works were on azimuth control only.

Figure 1.1 Close up view of mechanical defects.
The lack of work addressing antenna elevation control motivates this thesis to address the topic. It will begin with the most fundamental, yet important analysis in the mechanical engineering discipline, which is kinematics analysis, covered in CHAPTER 2. The analytical kinematics approach will be used, specifically the vector loop method. Not only will the analysis present analytic solutions to the relationship between linear actuator length and antenna elevation in terms of position and velocity, but it will also determine the location where mechanical singularities are likely to occur. Although acceleration and dynamics analysis are a continuation of kinematics analysis, they are not presented in this thesis because they are more for mechanical design purposes. Next, this thesis will show a continuous time domain approach to system identification in CHAPTER 3. In this chapter, a linear mathematical model that represents a simpler form of dynamical analysis of the system is obtained. Subsequently, CHAPTER 4 presents the controller design and stability analysis as well as hardware results. Lastly, in CHAPTER 5, the implementation of digital filters is demonstrated. Ultimately, this thesis presents a systematic methodology to analyze the antenna linkage, design and implement a classical controller, and couple these with trajectory generation and digital filters in order to stabilize antenna elevation motion.

1.1 VSAT Background

This section will briefly familiarize the reader with the VSAT antenna model that is the subject of this thesis. Essentially, there are three types of VSAT systems: (1) data transmission, also known as uplinking, (2) data receiving, also known as downlinking, and (3) interactive, which is capable of both uplinking and downlinking [7]. The first two types are classified as one-way systems, whereas the last type is a two-way or bi-directional system, and this thesis focuses
on the two-way system. In general, the antenna terminology encompasses the reflector, feedhorn, and the electronic components that perform information exchange. The model developed in this work includes an antenna reflector diameter of 1.2 m and a maximum deployed height of 1.77 m. A full view of the antenna system is shown in Figure 1.2.

The geometric configuration of the antenna model is of an asymmetric paraboloid reflector antenna. In other words, the feedhorn is offset relative to the center of the reflector. One major advantage to such configuration is to maximize radiation efficiency by minimizing beam blockage [8]. The feedhorn acts as a messenger that relays information from the reflector to the receiver or transmitter, and vice versa.

The antenna functions by aligning with target satellite in the geosynchronous orbit. Thus, once deployed, it will remain stationary until operation is terminated. The information exchange of the two-way system is achieved with an Orthogonal Mode Transducer (OMT), which is the waveguide component dedicated to separate signal paths, therefore allowing uplink and downlink connections simultaneously at different rates. The current antenna operates in the Ku-Band range, which is 10 – 18 GHz, and it receives medium beam coverage as well as medium satellite power [9]. It is capable of an uplink rate from 13.75 – 14.50 GHz, and a downlink rate from 10.95 – 12.75 GHz, which is ideal for applications such as video conferencing, audio conferencing, and data sharing [10].

The antenna azimuth and elevation motions are independent. The elevation motion is achieved using a linear actuator, while the azimuth motion uses a DC motor and gearing. A third DC motor installed to rotate feedhorn. This feature allows cross-polarization of the signal.
Figure 1.2 Full body view of the VSAT antenna model SF1200 in its deployed state.
CHAPTER 2. KINEMATICS ANALYSIS

Kinematics is the study of motion without considering static and dynamic forces. Therefore, this chapter will present both position analysis and velocity analysis of the antenna. The velocity analytical solution is the result of the derivative of the position analytical solution. This analysis will include the relationship between the actuation space and Cartesian space of the antenna model.

2.1 Position Analysis

An analytic solution is obtained in this analysis that describes the antenna elevation angle required to point at the satellite in terms of linear actuator length. The antenna elevation angle, also known as look angle, is the angle between the satellite and the horizontal plane of the antenna. The antenna linkage is examined to set up the analysis and simplify the linkage description. The parabolic geometry of the antenna reflector and the offset of the feedhorn define the line of sight. In the final step of position analysis, the vector loop method was used to derive and simulate the analytic solution between the actuator and the pointing angle. The angular parameter is used as the commanded input, while the linear motion at the actuator is used for low-level control.

2.1.1 Linkage Classification

In order to classify the linkage type of the model, the number of joints and links, type of mechanism, and mobility of the linkage were determined. A schematic illustration representing the linkage is shown in Figure 2.1(a). The linkage will be sufficiently described by the solid line,
the dashed line, and the dotted, as shown in Figure 2.1(b). The long dashed line is the line of sight. The shaded shape is of constant lengths and is related by constant angles relative to the solid line and will be regarded as one piece that is part of the linkage.

![Figure 2.1](image)

Figure 2.1  (a) Schematic of linkage superimposed on the side profile of the antenna. (b) Simplified RPRR four-bar linkage.
By definition, a joint is where relative motion occurs between interconnected links. As shown in the simplified linkage illustration, there are three revolute joints, indicated by red dots labeled $O_0$ through $O_2$, that allow rotational motion. Additionally, there is a prismatic joint between $L_R$, linear actuator chassis and $L_S$, linear actuator stroke that permits translational motion. Hence, it is a four-bar slider-crank linkage described as a Revolute-Prismatic-Revolute-Revolute (RPRR) closed chain.

Another interesting note to point out about the linkage is that the links at each joint formed a lower pair. This means that at both revolute and prismatic joints there exists a surface contact as opposed to a point or line contact. One main advantage to such category of kinematic pair is that forces and wears are experienced evenly throughout the surface of the joint. Consequently, this will sustain product life and also reduce maintenance expenses.

To examine the mechanism of the linkage, an inversion of the typical slider crank linkage is demonstrated. A typical slider crank linkage is shown in Figure 2.2, $L_1$ is the fixed link, $L_2$ is the crank, $L_3$ is the coupler, and $L_4$ is the slider block. In this linkage, the slider block is always in translational motion and the path that it travels coincides with the fixed link, as illustrated in Figure 2.2(a) and (b). The linkage of the antenna was an inversion of the typical slider crank linkage, specifically the grounding of the coupler, $L_3$. This effectively changes $L_1$ to a coupler and $L_3$ to a fixed link. As the crank rotates, the coupler experiences both rotational and translational motion and the slider block experiences purely rotational motion, as demonstrated in two different configurations in Figure 2.3(a) and (b). Accordingly, Figure 2.4 shows that the antenna linkage demonstrated the similar mechanism, where $L_0$ is the fixed link, $L_1$ is the crank, and $L_A$ is the coupler. Lastly, the antenna linkage was a non-offset slider crank linkage because the path of the slider block does not always extend at an offset relative to the crank center.
**Figure 2.2** Typical slider crank linkage. (a) Configuration 1, (b) Configuration 2.

**Figure 2.3** An inversion of the slider crank linkage. (a) Configuration 1, (b) Configuration 2.

**Figure 2.4** Antenna linkage as an inversion of the typical slider crank linkage. (a) Configuration 1, (b) Configuration 2.
In terms of the mobility, the antenna as a whole is a spatial linkage due to motion capabilities in both azimuth and elevation components. However when motion in elevation plane only is considered, the mechanism in this work is a planar linkage with one-degree-of-freedom. The linkage can only perform a bi-directional rotation in the elevation plane, rotating the antenna up or down. The number of degrees-of-freedom is verified with the modified Gruebler’s equation as shown in Equation (2.1) [11]:

\[ DF_{(\text{planar})} = 3(n_L - n_j - 1) + \sum_{i=1}^{n_j} f_i \]  

(2.1)

where \( n_L \) is the number of links, \( n_j \) is the number of joints, and \( f_i \) is the number of degrees-of-freedom of each individual joint.

\[ \therefore DF_{(\text{planar})} = 3(4 - 4 - 1) + 4 = 1 \]  

(2.2)

Equation (2.2) verifies that the elevation component is a one-degree-of-freedom planar linkage.

### 2.1.2 Antenna Line of Sight

The parabolic geometry is designed to focus reflected waves at a focal point, under the condition that all incoming waves are parallel to each other. An illustration of a symmetrical parabolic configuration is shown in Figure 2.5(a), where incoming and reflected waves are indicated by solid lines with stealth arrows and dotted lines with normal arrows respectively, the axis of symmetry or center is shown as a dashed line, and the focal point as a dot. The dotted lines represents the surface normal at the spot where wave contact occurs; the line of sight, which is the incoming wave, is parallel to the center axis. The asymmetric paraboloid reflector antenna uses a subsection of the parabola, which is shown in the shaded region and is illustrated in Figure 2.5(b).
Figure 2.5 (a) Reflective property of a parabola. (b) Illustration of the antenna reflector using a subsection of a parabola.
The reason the asymmetric paraboloid reflector uses a subsection of the parabola is that it allows a larger folding range. This is advantageous because the antenna can be deployed on a wide range of terrains and still be able to point at the satellite and can also be folded down when unused.

The line of sight is determined experimentally from the CAD model. First, a line is drawn from the center of the feedhorn, where the focal point is, to the reflector, as shown in Figure 2.6(a). This is equivalent to the reflected wave line in Figure 2.5. Second, a tangent to the reflector surface where the line contacts is drawn and the surface normal is determined accordingly, indicated by a centerline as illustrated in Figure 2.6(b). Third, the angle between the surface normal and the reflected wave line is measured and a line is drawn away from the same surface contact point offset at the same measured angle, as shown in Figure 2.7(a). This is the incoming wave line or the line of sight explained earlier in Figure 2.5. It will be regarded as the center line of sight.

To verify the accuracy of the line of sight, multiple lines of sight are drawn on different spots on the reflector by following the aforementioned steps. They are then compared with the center line of sight. According to Figure 2.7(b), there is an average difference of 0.295° between the center line of sight and both the upper and lower line of sight, which means the lines of sight are fairly accurate. The point angle is determined by superimposing the antenna linkage on the CAD model and then measuring the constant angle offset between the adjacent part and the center line of sight. According to Figure 2.8, this adjacent part is the shaded shape bounded by dashed lines and is a part of the antenna linkage. Ultimately, the angle offset is measured to be 118.88°.
Figure 2.6 (a) Sketching of reflected wave line. (b) Sketching of surface normal.
Figure 2.7 (a) Sketching of line of sight. (b) Sketching of multiple lines of sight.
2.1.3 Position Kinematics

In this section, the analytic solution for the position of the elevation mechanism is derived using vector loop equations. The antenna linkage is illustrated with angle and length parameters as shown in Figure 2.9. Correspondingly, the parameters are described and tabulated in Table 2-1 and Table 2-2. All constant parameters are obtained either directly from CAD or using simple trigonometry. The objective of the analysis is to obtain an expression describing the antenna elevation angle $\theta_\alpha$ in terms of linear actuator stroke length $L_S$ along with the inverse solution.
Figure 2.9 Antenna four-bar linkage and adjacent part with elevation angle parameter, and close-up of elevation mechanism.
### Table 2-1 Antenna linkage length parameters.

<table>
<thead>
<tr>
<th>Description</th>
<th>( L_0 )</th>
<th>( L_1 )</th>
<th>( L_A )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed link</td>
<td>0.4767 m</td>
<td>0.1176 m</td>
<td>0.3926 m + (0:0.2032 m)</td>
</tr>
</tbody>
</table>

### Table 2-2 Antenna linkage angle parameters.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( \theta_0 )</th>
<th>( \theta_A )</th>
<th>( \theta_1 )</th>
<th>( \theta_P )</th>
<th>( \theta_\beta )</th>
<th>( \theta_\alpha )</th>
<th>( \theta_\mu )</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>7.83°</td>
<td>Varies</td>
<td>Varies</td>
<td>129.81°</td>
<td>12.36°</td>
<td>Varies</td>
<td>Varies</td>
<td>Fixed link angle</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Coupler angle</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Crank angle</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>New point angle offset</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Angle between crank and</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>adjacent part</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Elevation angle</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Transmission angle</td>
</tr>
</tbody>
</table>

In the close-up view of the elevation mechanism as shown in Figure 2.9, the elevation angle is expressed in Equation (2.3). This equation is in terms of \( \theta_1 \), which is a function of \( L_S \).

\[
\theta_\alpha = \theta_P - (180° - \theta_1 - \theta_\beta)
\]  

(2.3)

Consequently, an expression describing \( \theta_1 \) in terms of \( L_S \) is derived using the vector loop equations. This is a method that uses both polar form and Cartesian form to solve for the unknown link parameters, as explained in chapter 4 of this textbook [12]. Accordingly, vectors are assigned to the antenna four-bar linkage as shown in Figure 2.10. The varying variable in the linkage denoted by an asterisk is the linear actuator stroke length, \( L_S^* \). The rotating parameters \( \theta_1 \) and \( \theta_\alpha \) are the dependent variables, which are functions of \( L_S^* \).

The vector loop equations begins by summing all the vectors of the antenna linkage according to their directions. The vector summation is effectively equal to zero since the antenna linkage is a closed kinematic chain. The vector loop of the linkage is shown in Equation (2.4).

\[
\vec{R}_1 + \vec{R}_2 + \vec{R}_3 - \vec{R}_4 = 0
\]  

(2.4)
Next, the vectors are represented in polar forms to reflect both the angle and magnitude of the vector as expressed in Equation (2.5). The magnitude will be represented by the scalar link lengths and the angles will be represented by the angle parameters.

\[ L_R e^{j\theta_A} + L_S^* e^{j\theta_A} + L_1 e^{j\theta_1} - L_0 e^{j\theta_0} = 0 \]  \hspace{1cm} (2.5)

Then Equation (2.5) is transformed into Cartesian forms and subsequently decomposed into \(x\)- and \(y\)-components, which are derived in Equation (2.6) and (2.7) respectively.

\[ L_R (\cos \theta_A + j \sin \theta_A) + L_S^* (\cos \theta_A + j \sin \theta_A) + L_1 (\cos \theta_1 + j \sin \theta_1) - L_0 (\cos \theta_0 + j \sin \theta_0) = 0 \]
Equations (2.6) and (2.7) are rearranged with \( \theta_A \) terms on the left hand side, then squared and summed, as shown in Equation (2.8), where \( C_1 \) and \( C_2 \) are \( L_0 \cos \theta_0 \) and \( L_0 \sin \theta_0 \).

\[
(L_R + L_S^*)^2 = (C_1 - L_1 \cos \theta_1)^2 - (C_2 - L_1 \sin \theta_1)^2
\]

Equation (2.8) is then expanded and rearranged into a transcendental equation as derived in Equation (2.9), where \( A \) is \( C_1 \), \( B \) is \( C_2 \), and \( C \) is \( \frac{C_1^2 + C_2^2 + L_1^2 - (L_R + L_S^*)^2}{2L_1} \).

\[
(C_1 \cos \theta_1 + C_2 \sin \theta_1) = \frac{C_1^2 + C_2^2 + L_1^2 - (L_R + L_S^*)^2}{2L_1}
\]

\[
A \cos \theta_1 + B \sin \theta_1 = C
\]

Subsequently, Equation (2.9) is solved using the tangent of the half angles substitution, as demonstrated in chapter 4 in this text [13]. A quadratic equation with two possible solutions is found, but the feasible solution is expressed in Equation (2.10).

\[
\theta_1 = \tan^{-1} \left( \frac{B}{A} \right) + \tan^{-1} \left( \frac{\sqrt{A^2 + B^2 - C^2}}{C} \right)
\]

\( \theta_A \) is found by following the above steps but with \( \theta_1 \) terms on the left hand side of Equations (2.6) and (2.7). The solution is expressed in Equation (2.11), where is \( C' \) is \( \frac{C_1^2 + C_2^2 + (L_R + L_S^*)^2 - L_1^2}{2(L_R + L_S^*)} \).

\[
\theta_A = \tan^{-1} \left( \frac{B}{A} \right) - \tan^{-1} \left( \frac{\sqrt{A^2 + B^2 - C'^2}}{C'} \right)
\]
By substituting Equation (2.10) into Equation (2.3), the analytic solution of the elevation angle, $\theta_\alpha$, in terms of actuator stroke length $L_S^*$, which is encapsulated in $C$, is obtained and expressed in Equation (2.12). This solution computes the elevation angle output for a given stroke length input, it is also known as the forward solution.

$$\theta_\alpha = \theta_p - \left(180^\circ - \tan^{-1}\left(B/A\right) - \tan^{-1}\left(\frac{\sqrt{A^2 + B^2 - C^2}}{C}\right) - \theta_\beta\right)$$  \hspace{1cm} (2.12)$$

The analytic solution of $L_S^*$ in terms of $\theta_\alpha$ is obtained by manipulating Equation (2.8). This solution calculates the required stroke length input for a given desired elevation angle output, such a solution is called the inverse solution. The derivation is shown in the following.

$$L_S^*{}^2 + 2L_RC_L^* + L_R^2 = C_1^2 + C_2^2 + L_1^2 - 2C_1L_1\cos(\theta_1) - 2C_2L_1\sin(\theta_1)$$

By substituting $\theta_1 = \theta_\alpha + 180^\circ - \theta_p - \theta_\beta$ from Equation (2.3):

$$L_S^*{}^2 + 2L_RL_S^* + L_R^2 - C_1^2 - C_2^2 - L_1^2 + 2C_1L_1\cos(\theta_\alpha + 180^\circ - \theta_p - \theta_\beta)$$

$$+2C_2L_1\sin(\theta_\alpha + 180^\circ - \theta_p - \theta_\beta) = 0$$

A quadratic equation with two possible solutions is found. The feasible solution is shown in Equation (2.13), where $a$ is 1, $b$ is $2L_R$, and $c$ is $[L_R^2 + 2L_1(C_1 \cos(\theta_\alpha + 180^\circ - \theta_p - \theta_\beta) + C_2 \sin(\theta_\alpha + 180^\circ - \theta_p - \theta_\beta)) - C_1^2 - C_2^2 - L_1^2]$.

$$L_S^* = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$  \hspace{1cm} (2.13)$$

Both forward and inverse analytical solutions are simulated and illustrated in Figure 2.11.
The simulation results show how the motion of the elevation angle changes as a function of the actuator length. Additionally, the linkage exhibits a linear motion over most of the operational range of the antenna, specified as 11.6° to 118°. A significant non-linear output 10° can be seen near the upper limit compared with the best-fit linear approximation of motion. In order to ensure pointing accuracy, Equation (2.13) will be used as the position command input to the closed loop control. The transmission angle is examined in Figure 2.12.
The transmission angle of the slider crank mechanism shows that force of the linear actuator is efficiently transmitted to the load, except for when the elevation angle is approaching the upper limit. The resultant force acting on link, $L_1$, which drives the load, is the sine of the transmission angle. This resultant force is tangential to $L_1$, and is equal to the linear actuator force when the transmission angle is at $90^\circ$, which corresponds to a medium elevation angle. The transmission angle decreases to close to $20^\circ$ when approaching high elevation angle. Hence, the magnitude of the resultant force is considerably smaller than the linear actuator force at high elevation angle, while it is at a reasonable magnitude everywhere else within the operating range.

Figure 2.12 Transmission angle of the slider crank at different elevation angle.
Another important note is that there is a kinematic singularity when the actuator is almost fully extended. When the actuator extends so that the total actuator length, \( L_R + L_S \), is at its maximum, it is greater than the combined length of the fixed and the driven link at straight angle by 0.0015 m. Consequently, upon reaching 0.2017 m actuator length, the linkage will succumb to mechanical lock. The velocity analysis is presented next.

### 2.2 Velocity Analysis

In this section, both the angular and linear velocity analytic solutions will be presented. The angular velocity represents the rotational rate of a revolute joint and the linear velocity describes the instantaneous velocity of a point in Cartesian space. The velocity analytic solutions are obtained by differentiating the position vectors from the vector loop equations. This analysis shows the velocity profiles of the joints of the antenna linkage and of the endpoint at the line of sight as a function of the linear actuator velocity. The joint velocity profiles demonstrate the rate of change of the angular displacement of the joints of the antenna linkage. The endpoint linear velocity profile illustrates the motion of the antenna reflector and the feedhorn.

#### 2.2.1 Angular and Linear Velocity Kinematics

The velocity components of the antenna linkage are illustrated in Figure 2.13. At the linkage, \( \dot{\theta}_A \) is the angular velocity of the coupler joint, \( \dot{\theta}_1 \) is the angular velocity of the crank joint, and \( \dot{L}_S^* \) is the linear velocity of the actuator. At the endpoint, \( V_X \) and \( V_Y \) are the horizontal and vertical velocity components, and \( V_R \) is the resultant linear velocity, which has the phase angle of \( \theta_R \). \( V_R \) is also perpendicular to the adjacent part with distance \( R \). The analytic solution for the
angular velocity of the joints of the antenna linkage as a function of the linear actuator velocity will be determined first.

The velocity equation of the antenna linkage is obtained by differentiating Equation (2.5) with respect to time, as expressed in Equation (2.14).

$$\frac{d}{dt} \left( L_R e^{j\theta_A} + L_S e^{j\theta_A} + L_1 e^{j\theta_1} - L_0 e^{j\theta_0} = 0 \right)$$

$$jL_R e^{j\theta_A} \dot{\theta}_A + jL_1 e^{j\theta_1} \dot{\theta}_1 + jL_S e^{j\theta_A} \dot{\theta}_A + jL_1 e^{j\theta_1} \dot{\theta}_1 = 0$$  \hspace{1cm} (2.14)

Since $L_0 e^{j\theta_0}$ is constant with respect to time, it is eliminated after differentiation. Next, by substituting Euler’s identity, the velocity equation is transformed from polar form to Cartesian form, which is shown in Equation (2.15).

$$[L_S^* + j(L_R + L_S^* \dot{\theta}_A)] (\cos \theta_A + j \sin \theta_A) + jL_1 (\cos \theta_1 + j \sin \theta_1) \dot{\theta}_1 = 0$$  \hspace{1cm} (2.15)

Equation (2.15) is decomposed into $x$- and $y$- components shown in Equations (2.16) and (2.17).

$$L_S^* \cos \theta_A - (L_R + L_S^*) \sin \theta_A \dot{\theta}_A - L_1 \sin \theta_1 \dot{\theta}_1 = 0$$  \hspace{1cm} (2.16)

$$L_S^* \sin \theta_A + (L_R + L_S^*) \cos \theta_A \dot{\theta}_A + L_1 \cos \theta_1 \dot{\theta}_1 = 0$$  \hspace{1cm} (2.17)
Figure 2.13 Schematic of joint velocity parameters and Cartesian velocity of endpoint.
The analytic solutions for the angular velocities of the joints are determined by rearranging Equations (2.17) and (2.17) into a matrix form and solved accordingly. The derivation is shown below and the analytic solutions are expressed in Equations (2.19) and (2.19).

\[
\begin{bmatrix}
(L_R + L_S^*) \sin(\theta_A) & L_1 \sin(\theta_1) \\
(L_R + L_S^*) \cos(\theta_A) & L_1 \cos(\theta_1)
\end{bmatrix}
\begin{bmatrix}
\dot{\theta}_A \\
\dot{\theta}_1
\end{bmatrix}
= \begin{bmatrix}
L_S^* \cos(\theta_A) \\
- L_S^* \sin(\theta_A)
\end{bmatrix}
\]

\[
\therefore \begin{bmatrix}
\dot{\theta}_A \\
\dot{\theta}_1
\end{bmatrix}
= \begin{bmatrix}
(L_R + L_S^*) \sin(\theta_A) & L_1 \sin(\theta_1) \\
(L_R + L_S^*) \cos(\theta_A) & L_1 \cos(\theta_1)
\end{bmatrix}^{-1}
\begin{bmatrix}
\dot{L}_S^* \cos(\theta_A) \\
- \dot{L}_S^* \sin(\theta_A)
\end{bmatrix}
\]

\[
\dot{\theta}_A = \frac{-\dot{L}_S^*}{(L_R + L_S^*) \tan(\theta_1 - \theta_A)} \quad (2.19)
\]

\[
\dot{\theta}_1 = \frac{\dot{L}_S^*}{L_1 \sin(\theta_1 - \theta_A)} \quad (2.19)
\]

The endpoint linear velocity is determined by taking the cross product of the angular velocity of the crank, \( \dot{\theta}_1 \), and the distance from the crank to the endpoint, \( R \). This distance is \( R_x \hat{i} + R_y \hat{j} \) in Cartesian space; by referring to Figure 2.9 \( R_x \) is \( R \cos(\theta_1 + \theta_B) \) and \( R_y \) is \( R \sin(\theta_1 + \theta_B) \). The derivation is shown below with the horizontal and vertical velocity components expressed as Equations (2.20) and (2.21). The resultant linear velocity in terms of magnitude and phase angle are shown in Equations (2.22) and (2.23).

\[
\vec{V}_R = \dot{\theta}_1 \times R
\]

\[
\vec{V}_R = \begin{bmatrix} i & j & k \\ 0 & 0 & \dot{\theta}_1 \\ R_x & R_y & 0 \end{bmatrix} = \begin{bmatrix} i & j \\ 0 & 0 \end{bmatrix} = R_x \dot{\theta}_1 j - R_y \dot{\theta}_1 i
\]

\[
V_x = -R_y \dot{\theta}_1 \quad (2.20)
\]

\[
V_y = R_x \dot{\theta}_1 \quad (2.21)
\]
The simulation results of the velocity profiles are illustrated in Figure 2.14 through Figure 2.16 with the linear actuator extending at a constant velocity of 0.01 m/s while staying within operating range.

\[
V_R = \sqrt{V_X^2 + V_Y^2} = |R \dot{\theta}_1| \tag{2.22}
\]

\[
\theta_R = \tan^{-1}\left(\frac{V_Y}{V_X}\right) \tag{2.23}
\]

\[\begin{align*}
\theta_R &= \tan^{-1}\left(\frac{V_Y}{V_X}\right) \\
V_R &= \sqrt{V_X^2 + V_Y^2} = |R \dot{\theta}_1|
\end{align*}\]

Figure 2.14 Velocity profiles of the revolute joints of the antenna linkage.
Figure 2.15  Velocity profile of the magnitude of the endpoint linear velocity.

Figure 2.16  Direction of the endpoint velocity.
At positive actuator velocity, as the antenna is folding up towards the upper limit of the operating range, it is approaching the nonlinear region. All velocities were observed to experience greater rate of change in this particular region. This indicates that the antenna will move relatively faster at higher elevation angle than at lower elevation angle. Note that the linear velocity of the endpoint in Figure 2.15 is always positive and tangent to $R$, so its direction is indicated by the phase angle in Figure 2.16.

Additionally, the angular velocity profiles indicate possible oscillations. In Figure 2.14, there is a change in the direction of the velocity at around 6 seconds, when the linear actuator is extended to $0.08 \, m$, or at the elevation angle of approximately $50^\circ$. During the transition of velocity direction, the linear actuator, or the coupler, $L_A$, is experiencing inertia, thus potentially exciting the oscillatory characteristics. The actual responses, shown in the later chapters, demonstrated otherwise, that there are no significant oscillations except at higher elevation angle of $66^\circ$, when moving down. Though, when stationary, the antenna can be perturbed slightly around the same point by force, which is a behavior observed on the actual antenna. This means that the potential oscillation at zero angular velocity is negated during motion.

This zero angular velocity point is not to be confused with the dead-center position of the linkage. The dead-center position occurs when the instant center of the crank, $L_1$, is coinciding with the pivot of one of the adjacent links and when one of the links is experiencing zero angular velocity. This causes undesirable instability as the direction of the coupler is undecided such that at this position it could rotate in the opposite direction that the mechanism had intended [14]. In this case, the dead-center position of the antenna linkage is when the crank is collinear with the fixed link, $L_0$, which is beyond both the limiting position and the operating range of the linkage, so this poses no concern.
CHAPTER 3. SYSTEM IDENTIFICATION

The objective of system identification is to obtain a mathematical model that describes the dynamical response of the antenna in the elevation axis. The process of identifying the dynamics is divided into two steps. The first step is to identify the linear actuator dynamics, which is the input source. The second step is to include the feedhorn and the antenna mass, which are the driven load. There are many system identification techniques of a dynamical system such as least squares method as explained in chapter 2 in this textbook [15], which uses both input and output measurements. There is also frequency domain decomposition, which is ideal for when only system output measurement is available [3]. The continuous time domain approach will be used for the identification of the linear actuator dynamics. The full equation of motion of the system is obtained using Newton-Euler method. The parameters of the system with load are estimated through trial and error comparisons with hardware measurements. Additionally, this chapter will present a brief specification on the microprocessor of the system.

3.1 System Modeling without Load

In order to obtain the data that exhibit the linear actuator dynamics only, the linear actuator was disassembled from the antenna and powered horizontally. Consequently, this eliminates the gravitational effect and external forces from the load and joint frictions. An open-loop model that represents the linear actuator dynamics is derived in the form of an ordinary differential equation. To characterize the linear actuator parameters, both hardware and simulation results are compared.
3.1.1 Linear Actuator Open-loop Model

The linear actuator dynamics is an electromechanical system that converts the rotational motion of an electric motor to a linear motion. The mechanism is shown in Figure 3.1(a) where the shaft end of the electric motor is represented by a circle, and a schematic of the free-body diagram is shown in Figure 3.1(b). The descriptions of the parameters are tabulated in Table 3-1. The equations of motion of the translational and the rotational systems, based on Figure 3.1(b), are defined in Equation (3.1) and Equation (3.2), which are obtained from force and moment analysis. The counter-clockwise rotation and linear displacement to the right are assumed to be the positive motion. Zero initial conditions are assumed for all the equations in Laplace domain.

![Figure 3.1](a) Linear actuator mechanism, (b) free-body diagram of the linear actuator.)
Table 3-1 Parameter description of the linear actuator.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_m$</td>
<td>Angular displacement of motor</td>
<td>Radian</td>
</tr>
<tr>
<td>$\tau_m$</td>
<td>Torque from motor</td>
<td>Nm</td>
</tr>
<tr>
<td>$\tau_l$</td>
<td>Torque from load</td>
<td>Nm</td>
</tr>
<tr>
<td>$J_m$</td>
<td>Motor Inertia</td>
<td>kg·m²</td>
</tr>
<tr>
<td>$b_m$</td>
<td>Viscous damping of the motor</td>
<td>Nm/(rad/s)</td>
</tr>
<tr>
<td>$R$</td>
<td>Gear ratio in terms of radius</td>
<td>m</td>
</tr>
<tr>
<td>$o$</td>
<td>Motor center of rotation</td>
<td>-</td>
</tr>
<tr>
<td>$F$</td>
<td>Output force on the linear actuator</td>
<td>N</td>
</tr>
<tr>
<td>$M_m$</td>
<td>Effective mass of the linear actuator</td>
<td>kg</td>
</tr>
<tr>
<td>$B_m$</td>
<td>Internal friction of the linear actuator</td>
<td>N/(m/s)</td>
</tr>
<tr>
<td>$x_m$</td>
<td>Linear actuator displacement</td>
<td>m</td>
</tr>
</tbody>
</table>

\[
\sum F_{@x_m} \pm = F(t) - B_m \dot{x}_m(t) = M_m \ddot{x}_m(t)
\]

\[
M_m \ddot{x}_m(t) + B_m \dot{x}_m(t) = F(t)
\]

\[
\therefore S(M_m S + B_m)X_m(S) = F(S)
\]

(3.1)

\[
\sum M_o \circ = \tau_m - b_m \dot{\theta}_m(t) - \tau_l = J_m \ddot{\theta}_m(t)
\]

\[
\tau_m = J_m \ddot{\theta}_m(t) + b_m \dot{\theta}_m(t) + \tau_l
\]

\[
\therefore T_m(s) = J_m s^2 \Theta_m(s) + b_m s \Theta_m(s) + T_l(s)
\]

(3.2)

The equation of the combined motion is expressed in Equation (3.3) with the relationship of

\[T_l(s) = R \cdot F(S)\] and \[X_m(S) = R \cdot \Theta_m(s)\] and by substituting (3.3) into (3.3).

\[
T_m(s) = [J_m + M_m R^2] s^2 + (b_m + B_m R^2) s] \Theta_m(s)
\]

(3.3)

The mathematical model of the electric motor is shown in Equation (3.4), which is developed in chapter 6 of this textbook [16]. The voltage input is $V_{in}$, the torque and back e.m.f constants are $K_m$ and $K_b$, and the armature resistance is $R_m$. The inductance is assumed to be negligible.

\[
T_m(s) = [J_m + M_m R^2] s^2 + (b_m + B_m R^2) s] \Theta_m(s)
\]

(3.4)
The displacement output to the voltage input transfer function as defined in Equation (3.5) is obtained by substituting (3.3) into (3.3), then substituting the rotation to translation relationship.

\[
X_m(s) = \frac{RK_m}{R_m} \left[ \frac{1}{(J_m + M_m R^2) s^2 + \left( b_m + B_m R^2 + \frac{K_m K_b}{R_m} \right) s} \right]
\]  

(3.5)

The input signal is in the form of a pulse-width-modulated (PWM) signal and is represented by a signed digital value, which is denoted by \textit{bits}. It is converted to voltage via an amplifier gain, \(K_{\text{amp}}\), then to current by the armature resistance, \(R_m\). The motor torque constant, \(K_m\), converts the current to torque. This constant is unknown, but the thrust constant is known, which is denoted by \(\overline{K_m}\) and has the relationship of \(K_m = R\overline{K_m}\). The linear actuator displacement feedback comes from a potentiometer. It converts the displacement in inches to an unsigned digital value, which is the output denoted by \textit{BITS}, via a potentiometer gain, \(K_{\text{pot}}\). The conversion gain, \(K_{\text{in}}\), converts the English units to the metric system units. The known gain parameters are tabulated in Table 3-2. The actual input-output relationship of the linear actuator is derived and expressed in Equation (3.6). The expanded block diagram of Equation (3.6) is illustrated in Figure 3.2.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>(K_{\text{amp}})</th>
<th>(\overline{K_m})</th>
<th>(R_m)</th>
<th>(K_{\text{pot}})</th>
<th>(K_{\text{in}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>Volts/bit</td>
<td>N/Amp</td>
<td>Ω</td>
<td>BIT/inch</td>
<td>inch/m</td>
</tr>
<tr>
<td>Value</td>
<td>(\frac{24}{2^9} = 0.0468)</td>
<td>556</td>
<td>3.2</td>
<td>(\frac{2^{1^3}}{8} = 1024)</td>
<td>(\frac{100}{2.54} = 39.37)</td>
</tr>
</tbody>
</table>

\(T_m(s) = \frac{K_m}{R_m} (V_{in}(s) - K_b s \Theta_m(s))\)  

(3.4)
With \( V_{in}(s) = \frac{\text{bits}(s)K_{amp}}{R_m} \) and \( X_m(s) = \frac{\text{BITS}(s)}{K_{in}K_{pot}} \)

\[
\text{BITS}(s) = \frac{K_mK_{amp}K_{in}K_{pot}}{R_m} \left[ \frac{1}{s^2 + \left( \frac{b_m + B_mR^2 + K_mK_b}{R^2} \right)s} \right]
\]

\[
\frac{\text{BITS}(s)}{\text{bits}(s)} = \frac{K_mK_{amp}K_{in}K_{pot}}{R_m} \left[ \frac{1}{J_{eq}s^2 + B_{eq}s} \right]
\]

\[
\frac{\text{BITS}(s)}{\text{bits}(s)} = \frac{K_mK_{amp}K_{in}K_{pot}}{R_mB_{eq}} \left[ \frac{B_{eq}}{J_{eq}} \right] \frac{1}{s^2 + \left( \frac{B_{eq}}{J_{eq}} \right)s}
\]

\[
\frac{\text{BITS}(s)}{\text{bits}(s)} = c \left( \frac{a}{s^2 + as} \right)
\] (3.6)

\textbf{Figure 3.2} Block diagram of the open-loop linear actuator model.

Ultimately, the constants to solve for are \( c \) and \( a \) from Equation (3.6). Consequently, the parameters \( J_{eq} \) and \( B_{eq} \) can then be identified. They are the overall effective mass and internal friction of the open-loop model.
The identification of the parameters is done by examining the characteristics of the actual response and comparing with the simulation of the open-loop model. The open-loop transfer function in the Laplace domain rewritten in Equation (3.7), is transformed into the continuous time domain. The input and output notations, *bits* and *BITS*, are replaced with $R(S)$ and $Y(S)$, respectively. The derivation of the continuous time equation is shown below with the assumption of zero initial velocity.

$$G(S) = \frac{Y(S)}{R(S)} = \frac{c \cdot a}{S(S + a)} \quad (3.7)$$

$$Y(S)(S^2 + aS) = R(S)(c \cdot a) \triangleq \dot{y}(t) + \ddot{y}(t)a = (c \cdot a)r(t)$$

$$\therefore (S^2Y(S) - SY(0) - Y(0)) + (SY(S) - Y(0))a = R(S)(c \cdot a)$$

$$Y(S) = \frac{R(S)(c \cdot a)}{(S^2 + aS)} + \frac{Y(0)}{S}$$

Next, the continuous time domain equation is obtained by partial fraction expansion and replacing the step input $R(S)$ with $\frac{Amp}{s}$. Then taking the inverse Laplace transform of $Y(S)$, the final form is expressed in Equation (3.8). The initial position is denoted by $y(0)$.

$$Y(S) = \frac{c \cdot Amp}{a} \left[ -\frac{1}{S} + \frac{1}{S^2} + \frac{1}{S + a} \right] + \frac{Y(0)}{S}$$

$$\therefore y(t) = \mathcal{L}^{-1}\{Y(S)\} = \frac{c \cdot Amp}{a} [-1 + at + e^{-at}] + y(0) \quad (3.8)$$
Equation (3.8) is the combination of an exponential equation, denoted by \( y_1(t) \), and a linear line equation, denoted by \( y_2(t) \), which are shown in Equations (3.9) and (3.10).

\[
y(t) = y_1(t) + y_2(t)
\]

\[
y_1(t) = \frac{c \cdot Amp}{a} e^{-at} \quad (3.9)
\]

\[
y_2(t) = c \cdot Amp \cdot t + y(0) - \frac{c \cdot Amp}{a} \quad (3.10)
\]

Equation (3.9) is a first order system equation that has a time constant of \( \frac{1}{a} \). Hence, this equation will eventually decay and converge to a constant. The open-loop response of the linear actuator dynamics is reflected predominantly by Equation (3.10) once Equation (3.9) becomes a constant. As shown in Figure 3.3, the actual open-loop response resembles a linear line, which is indicated by the circle markers. Then, \( c \) and \( a \) are solved by equating the slope and intercept of Equation (3.10) to the slope and intercept of the linear portion of the open-loop response, which are denoted by \( O.L_m \) and \( O.L_c \) respectively. The solutions are expressed in Equations (3.11) and Equation (3.12).

\[
c \cdot Amp = O.L_m
\]

\[
\therefore c = \frac{O.L_m}{Amp} \quad (3.11)
\]

\[
y(0) - \frac{c \cdot Amp}{a} = O.L_c
\]

\[
\therefore a = \frac{c \cdot Amp}{y(0) - O.L_c} = \frac{O.L_m}{y(0) - O.L_c} \quad (3.12)
\]
3.1.2 Linear Actuator Open-loop Step Response Simulation and Results

The linear actuator is subjected to two different amplitude of step inputs, which are PWM of 150 and 200 bits. Based on the actual response and both Equations (3.11) and (3.12), the average values of $C$ and $a$ and the actuator parameters $J_{eq}$ and $B_{eq}$ are computed and tabulated in Table 3-3. The computed parameters are then substituted into the continuous time domain expression in Equation (3.8) and are simulated and compared with the actual responses, which are illustrated in Figure 3.4 and Figure 3.5.

![Figure 3.3 Linear line characteristic of the open-loop response.](image-url)
Table 3-3 Results of actuator parameters.

<table>
<thead>
<tr>
<th>Parameters\Inputs</th>
<th>$PWM = 150$</th>
<th>$PWM = 200$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O. L_m \left( \frac{BITS}{s} \right)$</td>
<td>218</td>
<td>290</td>
</tr>
<tr>
<td>$O. L_c (BITS)$</td>
<td>2585</td>
<td>3140</td>
</tr>
<tr>
<td>$C \left( \frac{BITS}{bits \cdot s} \right)$</td>
<td>1.4565</td>
<td>1.4521</td>
</tr>
<tr>
<td>$a \left( \frac{1}{s} \right)$</td>
<td>17.47</td>
<td>22.8</td>
</tr>
</tbody>
</table>

Results

$c_{average} = 1.5$ ; $a_{average} = 20$ ; $Beq = \frac{N \cdot s}{m}$ ; $Jeq = 10945 \text{ kg}$

Figure 3.4 Comparisons of actual and simulated actuator response with $PWM$ of 150 bits.
Overall, the mathematical model of the linear actuator is considerably accurate. Both the actual and the simulated responses matched as demonstrated in Figure 3.4 and Figure 3.5. The results of these two experiments showed initially exponential growths with an average time constant of 0.05 seconds. Approximately five time constants later, the rest of the response is linear. Statistically, the average mean squared error of the actual and the simulated responses is $2.895 \text{BITS}^2$. Note that this error is unit dependent and will be reduced significantly if the responses were compared in the metrics system. Ultimately, the effective mass and the internal friction of the linear actuator remains unknown, but the overall effective mass, $J_{eq}$, and internal friction, $B_{eq}$, are estimated. The reason $J_{eq}$ and $B_{eq}$ are so large is that they consist of many other components, which can be recalled from the development of Equation (3.6).
3.2 System Modelling with Load

The estimation of the parameters of the full antenna dynamics is done by comparing the simulation of the linearized model with the actual closed-loop responses. The masses of the antenna reflector and the feedhorn are will be considered. The effect of gravity will also be included, and additional external friction is characterized through trial and error. All hardware limits are identified and included to reflect the model more accurately. The simulation results of the system model with load and without load are compared to examine the differences between the two models. The simulations will be done in both continuous and discrete time domain by using Simulink.

3.2.1 Full Nonlinear Antenna Model

A schematic of the full antenna model is shown in Figure 3.6. The masses of the antenna reflector and the feedhorn are lumped together to simplify the analysis, which is denoted by $M_a$. It is located on the end of the flexible link, which is an extension of link $L_1$ by the lumped mass radius, $R_a$. The flexible link is assumed to be massless. The line of reference used for the lumped antenna mass angle, $\theta_a(t)$, is indicated by the dashed line that is intersecting the crank joint, $b$. A counter-clockwise rotation is assumed to be the positive motion. The antenna damping element, $B_a$, is also considered, which works in the opposite direction to resist the motion. Accordingly, the parameters to be identified for the full antenna model are the lumped mass, mass radius, spring constant, and damping element. The objective is to obtain the full nonlinear equation of motion that describes the dynamics from input force to output linear and angular displacements. Again, they are obtained using force and moment analysis.
Figure 3.6 Full antenna model for dynamics analysis.
The free-body diagram analysis of the model is split into two, as shown in Figure 3.7(a) and Figure 3.7(b), which are the linear actuator and the lumped mass. Both $F_{xa}$ and $F_{ya}$ are the reaction forces. Their directions are chosen arbitrarily and are assigned consistently in equal and opposite directions when dismembered for analysis. The moment analysis of the lumped mass is done first by summing the moment about crank joint, $b$, which the dynamics equation is defined in Equation (3.13).

$$\sum_{M_b} = M_a g R_a \sin(\theta_b(t)) - F_{xa} L_1 \cos(\theta_b(t))$$

$$-F_{ya} L_1 \sin(\theta_b(t)) - B_a \dot{\theta}_b(t) = M_a R_a^2 \ddot{\theta}_b(t)$$ (3.13)

**Figure 3.7** Free-body diagrams of (a) the linear actuator, (b) the flexible link and the lumped mass.
The force analysis of the linear actuator is done by first summing all the forces in the direction of $x_m$, as defined in Equation (3.14).

$$\sum F@x_m\hat{x} = F(t) - B_m\hat{x}_m(t) + M_m g \sin(\theta_a(t))$$

$$+ F_{xa} \cos(\theta_a(t)) - F_{ya} \sin(\theta_a(t)) = M_m \ddot{x}_m(t)$$

(3.14)

To solve for the reaction forces, the forces are summed in the perpendicular direction of linear motion as expressed in Equation (3.15).

$$\sum F@y = F_{ya} \cos(\theta_a(t)) + F_{xa} \sin(\theta_a(t)) - M_m g \cos(\theta_a(t)) = 0$$

(3.15)

Then, Equations (3.14) and (3.15) are used to solve for the reaction forces by using simple matrix algebra. The solved reaction forces are defined in Equations (3.16) and (3.17).

$$F_{xa} = \cos(\theta_a(t)) \left[ M_m \ddot{x}_m(t) + B_m \dot{x}_m(t) - F(t) \right]$$

(3.16)

$$F_{ya} = M_m g + \sin(\theta_a(t)) \left[ F(t) - M_m \ddot{x}_m(t) - B_m \dot{x}_m(t) \right]$$

(3.17)

The full nonlinear equation of motion is obtained by substituting reaction forces into Equation (3.13), which is expressed in Equation (3.18). The nonlinear equation can be simulated to compare with the actual response by using the kinematics solution in CHAPTER 2 and rearranging the equation with $\ddot{x}_m(t)$ on the left-hand side, then integrating $\dot{x}_m(t)$ in Simulink. The known input is $F(t)$ from the actual data, and the known output to compare is displacement in BITS, which can be converted to the same units as $x_m(t)$. But due to the large number of unknown parameters, the simulation of the nonlinear equation is not meaningfully. These unknown parameters are $M_m, B_m, R_a,$ and $B_a$; $M_a$ can be estimated from CAD, which is 36 kg. The linearized model has one less parameter to estimate, which is presented next.
3.2.2 Linearized Antenna Model

The nonlinear equation of motion is linearized in terms of the linear actuator displacement, \( x_m(t) \). The two time-variant angle variables, which are the linear actuator angle, \( \theta_a(t) \), and the lumped antenna mass angle, \( \theta_b(t) \), are converted to displacement. The antenna mass angle is transformed by approximating the displacement with the arc length of the crank loci, as expressed in Equation (3.19). In Figure 3.8, the approximation is fairly accurate until the medium antenna mass angle at around 35°, which corresponds to a medium high elevation angle of 90°. Therefore, this approximation can be used for most of the operation.

\[
F(t) = M_m \ddot{x}_m(t) + B_m \dot{x}_m(t) + \frac{M_a R_a^2 \ddot{\theta}_b(t) + B_a \dot{\theta}_b(t)}{L_1 \cos(\theta_a(t) + \theta_b(t))} \\
+ \frac{(M_m L_1 - M_a R_a) g \sin(\theta_b(t))}{L_1 \cos(\theta_a(t) + \theta_b(t))}
\]  

(3.18)

\[
x_m(t) \cong L_1 \theta_b(t)
\]  

(3.19)
The linear actuator angle is expressed in terms of the antenna mass angle from geometric analysis. The geometric relationship of the two angles is illustrated in Figure 3.9. The solutions are obtained by relating the trigonometry of the right angle triangles formed by the linear actuator and the crank, which are defined in Equations (3.20) and (3.21). The constants $C_1$ and $C_2$ are the fixed link constants, which can be reviewed from the development of Equation (2.8). The sign of Equation (3.20) is flipped to reflect the negative actuator angle. The sign of the sine term of Equation (3.21) is changed to positive to preserve the appropriate resulting sign as the antenna angle changes in both directions. The nonlinear equation is expanded first with the derivatives in terms of $x_m(t)$, which is shown in Equation (3.22). The angle relationships are substituted into the expanded Equation (3.22), as defined in Equation (3.23). The comparisons of nonlinear Equations (3.18) and (3.23) are shown in Figure 3.10.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3_8}
\caption{Comparisons of the approximated and actual antenna mass angle.}
\end{figure}
Figure 3.9 Geometric relationship between actuator angle and antenna mass angle.

\[
\sin(\theta_a(t)) = \frac{y}{L_R + x_m(t)} = \frac{L_1 \cos(\theta_b(t)) - L_0 \sin(\theta_0)}{L_R + x_m(t)}
\]

\[
\sin(\theta_a(t)) = \frac{C_2 - L_1 \cos(\theta_b(t))}{L_R + x_m(t)} \tag{3.20}
\]

\[
\cos(\theta_a(t)) = \frac{L}{L_R + x_m(t)} = \frac{L_0 \cos(\theta_0) - L_1 \sin(\theta_b(t))}{L_R + x_m(t)}
\]

\[
\cos(\theta_a(t)) = \frac{C_1 + L_1 \sin(\theta_b(t))}{L_R + x_m(t)} \tag{3.21}
\]
\[ F(t) = M_m \ddot{x}_m(t) + B_m \dot{x}_m(t) \]  
\[ + \frac{M_a R_a^2 \ddot{x}_m(t) + B_a \dot{x}_m(t)}{L_1^2 [\cos(\theta_a(t)) \cos(\theta_b(t)) - \sin(\theta_a(t)) \sin(\theta_b(t))] + \frac{(M_m L_1 - M_a R_a) g \sin(\theta_b(t))}{L_1 [\cos(\theta_a(t)) \cos(\theta_b(t)) - \sin(\theta_a(t)) \sin(\theta_b(t))]} \]  
\[ F(t) = \left[ M_m + \frac{M_a R_a^2}{L_1^2 A_1} \right] \ddot{x}_m(t) + \left[ B_m + \frac{B_a}{L_1^2 A_1} \right] \dot{x}_m(t) \]  
\[ + \frac{(M_a R_a - M_m L_1) g \sin(\theta_b(t))}{L_1 A_1} \]  
Where \( A_1 = \frac{\cos(\theta_b(t)) [C_1 + L_1 \sin(\theta_b(t))] - \sin(\theta_b(t)) [C_2 - L_1 \cos(\theta_b(t))]}{L_R + \dot{x}_m(t)} \)

Comparisons of Nonlinear Equations

**Figure 3.10** Verification of the nonlinear equation with one less angle variable.
The nonlinear equations are simulated with arbitrarily set parameters and are subjected to readily available actual control input data, and with \( \ddot{x}_m(t) \) on the left-hand side. Figure 3.10 shows that the simulated results are exactly the same, which verifies the angle relationships.

Equation (3.23) is linearized using Taylor series expansion about the nominal position of \( \theta_{b_0} = 0^\circ \), which is \( x_{m_0} = 0.0826 \) m, and with zero nominal input. This is where the lumped antenna mass is completely vertical and assumed to be at equilibrium. The resulting equation is shown in Equation (3.24); the derivation and the unit dimensions are shown and verified in APPENDIX A. The additional parameters are appended to the block diagram of the linear actuator model as shown in Figure 3.11. They are the inertia and damping element of the lumped mass and the oscillatory element due to the combined masses. For simplicity of analysis, the damping element is assumed to be zero, hence the parameters to be estimated are \( M_m \) and \( R_a \). These parameters are estimated through trial and error comparisons of the simulated and actual responses, which is presented next.

\[
F(t) = (M_m + 72.74M_aR_a^2)\ddot{x}_m(t) + (B_m + 72.74B_a)\dot{x}_m(t)
\]

\[
+(83.95M_m - 713.85M_aR_a)x_m(t)
\]

\[
K_1 = J_{eq} + 72.74M_aR_a^2
\]

\[
K_2 = B_{eq} + 72.74B_a
\]

\[
K_3 = 83.95M_m - 713.85M_aR_a
\]

**Figure 3.11** Block diagram of the open-loop linearized model.
3.2.3 Closed-loop Simulation and Results

In this section, the simulated responses are compared with the actual closed-loop responses. Accordingly, all the hardware limits are included to simulate the response more accurately. The hardware limits are the control input and current saturations, deadband of static friction, and linear actuator velocity limit. Since the linear actuator velocity is based on the current and there is already a saturation on the current, the velocity limit is neglected. More importantly, rounding functions are included in the simulations to reflect the fixed-point arithmetic operation of the microprocessor that controls the linear actuator. The closed-loop simulations are done with a PI controller, the design of the controller is presented in the next chapter. The simulated responses of the linearized model and the linear actuator only model are also compared.

The static friction deadband, in terms of control input, is identified by examining the closed-loop uncompensated response in both directions as shown in Figure 3.12. This is a step response to a displacement of 100 BITS. For upward motion the displacement gradually becomes a constant when the control input is at around 30 bits, for downward motion it is twice less at around 15 bits. The static friction varies in both direction likely due to the effect of gravity. The limits are tabulated in Table 3-4.

<table>
<thead>
<tr>
<th>Limits</th>
<th>Control Input</th>
<th>Current</th>
<th>Static Friction (Up)</th>
<th>Static Friction (Down)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>±255 bits</td>
<td>±10 Amp</td>
<td>30 bits</td>
<td>−15 bits</td>
</tr>
</tbody>
</table>
The block diagrams of the closed-loop compensated system in continuous and discrete time domain are illustrated in Figure 3.13 and Figure 3.14. All the identified limits are included with additional friction blocks to represent the external friction. The block diagrams have second order time-delayed transfer functions, which represent the time delay of the digital system. This time-delayed transfer function has no significant effects on the simulated response as long as the controller gains are low. The effect of time-delayed transfer function and the formulation of the discrete controller are covered in details in CHAPTER 4. The block diagrams of the linear actuator only model in continuous and discrete time are shown in Figure 3.15 and Figure 3.16.

Figure 3.12 Identification of static friction.
Figure 3.13 Block diagram of the closed-loop PI compensated system of the linearized model in continuous time.

Figure 3.14 Block diagram of the closed-loop linearized PI compensated system of the linearized model in discrete time.
Figure 3.15 Block diagram of the closed-loop PI compensated system of the linear actuator only model in continuous time.

Figure 3.16 Block diagram of the closed-loop PI compensated system of the linear actuator only model in discrete time.
The unknown parameters of the linearized model, specifically the effective actuator mass $M_m$ and the lumped antenna mass radius $R_a$, and the external friction are estimated through trial and error. The friction model is governed by the linear actuator velocity, $\dot{L}_S(t)$, viscous friction coefficient, $\mu$, and coulomb friction offset, $F_{offset}$, as defined in Equation (3.25), this is obtained from the Simulink friction block. The viscous friction coefficient has negligible effect on the overall friction because the maximum linear actuator velocity is considerably small, which is less than one in either English or metrics units. Hence, the coulomb friction offset reflects the resulting magnitude of the overall friction. The parameters are tabulated in Table 3-5.

$$Friction = sgn(\dot{L}_S(t)) \cdot (\mu \cdot abs(\dot{L}_S(t)) + F_{offset})$$ (3.25)

The effective actuator mass and the lumped antenna mass radius are guessed as the ratio of the approximate lumped antenna mass and the actual crank length. There are many combinations of ratios that result in fairly accurate simulated responses. For instance, one of the combinations is shown in Table 3-5, which the effective actuator mass and the lumped antenna mass radius are fairly reasonable. However, one of the resulting closed-loop and open-loop roots is unstable. Consequently, the absolute values of the parameters remain inconclusive. The comparisons of the continuous and discrete simulations with friction and the actual response are shown in Figure 3.17. The simulations are done with the proportional and integral gains of 5 and 25

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$M_m$ (kg)</th>
<th>$R_a$ (m)</th>
<th>$\mu$</th>
<th>$F_{offset}$ (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Results</td>
<td>0.25$M_a$ = 9</td>
<td>1.2$L_1$ = 0.14</td>
<td>0.1</td>
<td>115</td>
</tr>
</tbody>
</table>
Both the discrete and continuous responses are fairly identical with the actual response but with slight discrepancies after the first overshoot. This shows that there are other nonlinear mechanical limits that are not accurately represented in the simulations. The only differences between the continuous and the discrete responses are the discretization effect and the marginal difference in the magnitude of overshoot and undershoot. The simulated continuous and discrete responses of the linear actuator only model also show identical results, hence they are not shown again. The continuous responses of the linear actuator only model and the linearized model with and without friction are compared with the actual response, which are illustrated in Figure 3.18(a) and Figure 3.18(b).
Figure 3.18 Displacement comparisons of simulations (a) without friction, (b) with friction.
In Figure 3.18(a), the response of the linearized model has slightly faster rise time and two overshoes just like to actual response. The linear actuator only model matches the actual response at the beginning but shows no second overshoot. When friction is included, as shown in Figure 3.18(b) the response of the linear actuator only model lags behind a little. This is because the effect of gravity has not been included yet. For the linearized model, the gravity term is already in the equation, thus it matches the actual response relatively well.

The block diagrams of the linear actuator only model in Figure 3.15 and Figure 3.16 have modified gravity blocks in the acceleration path, which are labelled as $\bar{g}$. The reason pure gravity constant is not used is that the linear actuator is always at an angle $\theta_a$. Therefore, when resolved in the direction of the linear actuator, it is the multiplication of the gravity constant and the sine component of the actuator angle, which can be reviewed from Figure 3.7(a). The acceleration due to gravity is added to the model by including the kinematics equation in Equation (2.11). The kinematics equation is used because no linear relationship can be formulated.

The simulated response becomes unstable, shown in dashed line in Figure 3.19(b), though the response is not fully shown. This is because the calculated gravity acceleration is higher than the acceleration due to the overall effective mass, $J_{eq}$, as shown in Figure 3.19(a). A correction multiplicative factor of 0.0115 scales the gravity acceleration to match the overall effective mass acceleration, which fixes the simulated response. Since the resultant gravity acceleration shows relatively small changes and has unity magnitude, the corrected gravity acceleration is then a constant of 0.0115. One possible reason the corrected gravity acceleration fixes the response is that the effective linear actuator mass translates and rotates simultaneously. The linear actuator only model only considers the translation motion, so the acceleration due to rotation is not considered. All responses with gravity and friction are compared in Figure 3.20 and Figure 3.21.
Figure 3.19 Inclusion of gravity acceleration to linear actuator only model. (a) Examination of acceleration, (b) Response with gravity acceleration.
Figure 3.20 Comparisons of actual, linearized, and linear actuator only model responses with $K_p$ of 5, $K_i$ of 25. (a) Displacement, (B) Control Input.
Figure 3.21 Comparisons of actual, linearized, and linear actuator only model responses with $K_p$ of 5, $K_i$ of 25. (a) Displacement error, (B) accumulated error.
The comparisons of the simulated results of the linear actuator only model and the linearized model show that they are almost identical with little discrepancies especially near the end of motion. This suggests that the lumped antenna mass has little influence on the dynamics response of the system. Despite the accuracy of the linearized model, the parameters are still inconclusive, which also results in unstable closed-loop roots. Consequently, the linear actuator only model is used to represent the antenna model. An alternative mass-spring-damper analogy is also used to model the antenna for digital filter design as the linear actuator only model does not capture the oscillatory characteristic of the antenna. This is shown later in CHAPTER 5. A brief specification on the microcontroller used to control the antenna system is discussed in the final section of this chapter. The design and implementation of the controller will be presented in the next chapter.

3.3 Microcontroller Specification

The ATmega64 8-bit microcontroller is used to control the antenna system. It has a clock speed of 16 MHz, and it has very limited external memory space. The PWM signal is generated using the available 16-bit timer counter that clears the signal on compare match. The analog feedback is done via interrupt service routine. The control task frequency for the elevation axis is one milliseconds. The microcontroller is capable of performing floating point algorithm in the software, but on the hardware end the results are rounded off to fixed points. Consequently, all computations done in the software are multiplied by a factor of 100 to prevent zero outputs for small decimal results.
CHAPTER 4. CONTROLLER DESIGN

This chapter presents the design of a Proportional-Integral (PI) controller. A trajectory scheme is also introduced to minimize the steady state error overshoot. The controller is designed in the continuous time domain with root locus analysis then transformed into the discrete time domain for implementation on a digital system. A Proportional-Integral-Derivative (PID) controller typically provides a better compensation since it is a band-pass filter, but the resulting control gains are too large to be implemented, see APPENDIX B. Also, the PD controller is not used in this work because it does not eliminate the steady state error of a Type I control system. Therefore a PI controller is used to compensate the system. The hardware results of the compensated system will be presented for comparisons and to verify any improvements.

4.1 Pre-existing Controller

A brief examination of the uncompensated system is presented first. The root locus of the uncompensated system is shown in Figure 4.1. There is a breakaway point at $-10$ on the real-axis, where the closed-loop poles become complex conjugates and the imaginary term goes to infinity with increasing proportional gain. This shows that the proportional controller will cause overshoot and oscillation as the proportional gain increases. The step responses are shown in Figure 4.2 and Figure 4.3.

Figure 4.2 shows the step response to a small displacement of $\pm 100$ BITS, which is an angular displacement of $\pm 1.5^\circ$. The feedhorn is almost parallel to the ground, which is at an elevation angle of $20^\circ$. Consequently, the gravitational effect is relatively large and it reduces the
control effort for downward motion. As shown in Figure 4.2(a), despite the overshoot, the antenna eventually moves to target position when moving down. However, when moving up the antenna overshoots and did not reach the target position, which is illustrated in Figure 4.2(b). This error can intensify if the proportional gain was larger. In Figure 4.3, with the same displacement but the proportional gain is doubled, there is persistent oscillation at the end of the motion. This shows that the proportional controller can make the system go unstable if the gain was too high. Also, it does not always eliminate the pointing error as the compensated system is still a Type I control system.

![Root Locus Plot](image)

**Figure 4.1** Root locus plot of the uncompensated system.
Figure 4.2 Step response of P-control: (a) Moving down. (B) Moving up.
The elevation control of the antenna was previously done using a lead filter. The lead filter is the close equivalent to the PD controller, except that in addition to adding an open-loop zero, it adds an open-loop pole. The equation in the Laplace domain is expressed in Equation (4.1), where the pole of the filter, $p_{LF}$, is larger than the zero, $z_{LF}$. The parameters used are gain $K$ of 300, and the zero and the pole of the filter are located at $-20$ and $-300$. The root locus plot and the step response are illustrated in Figure 4.4 and Figure 4.5.

$$G_{LF}(S) = \frac{K(S + z_{LF})}{S + p_{LF}}$$  \hspace{1cm} (4.1)
Figure 4.4 Root locus of the lead filter compensated system.

Figure 4.5 Step response of lead filter compensated system.
The hardware results show that although the lead filter is a more stable compensator compared to the proportional controller, the steady state error still persists. As shown in Figure 4.5, there is high overshoot and the system becomes idle momentarily and never reaches the target position. This is a response to a very small displacement of 30 BITS, which is an angular displacement of 0.5°. If the displacement was large, the control effort will grow large instantly because of the large magnitude of error.

As demonstrated in the simulation in Figure 4.6, the calculated control input is 300,000 bits, which is at least 1000 times greater than what the hardware can output. As a result, the hardware limits will be maxed out, the linear actuator velocity will move at its peak velocity instantly after a target position is set. This abrupt start can also cause oscillation during motion, which was observed on the antenna.

More importantly, there is a pole-zero cancellation by the lead filter, this is shown in Equation (4.2). It changes the dominant and stable pole of the uncompensated system. In effect, the closed-loop poles are at $-33.81$ and $-266.2$, where the leftmost pole is at least ten times faster than the other pole. Since they are comparably faster than the poles of the uncompensated system, so the overall response is also faster. All in all, the lead filter compensated system is still a Type I control system and the relatively fast response can momentarily destabilize the system.

\[ G_{O.L}(S) = G_{LF}(S) \cdot \frac{Ca}{s(s + a)} \]

\[ \therefore G_{O.L}(S) = \frac{300}{s + 300} \cdot \frac{30}{s(s + 20)} = \frac{9000}{s(s + 300)} \] (4.2)
4.2 PI Control

The PI controller increases the order of the system type. The advantage of a higher order system type is that the steady state error of the compensated system due to a step input can be eliminated. However, the tradeoff is that there will be a slower response time. This tradeoff can be overcome with the addition of trajectory generation, which will be presented in later section.

In the open-loop transfer function, the integral part of the controller adds a pole at the origin of the pole-zero map. The controller also adds a real zero at a desired location to fulfill certain response criteria. The placement of the controller zero and the tuning of the proportional gain dictates the value of the integral gain. The determination of the controller gains is done in the root locus analysis, which will be demonstrated next.
4.2.1 Root Locus Analysis of the PI Controller

The root locus analysis of the PI controller examines the loci of the roots or eigenvalues of the closed-loop compensated system as a result of varying the proportional control gain and the placement of the controller zero. This analysis allows the study of the nature of the closed-loop transient response to a step input. The PI controller equation in the Laplace domain is defined in Equation (4.3), where the controller zero is denoted by \( a_1 \), which is \( \frac{K_I}{K_P} \).

\[
G_c(S) = K_P + \frac{K_I}{S}
\]

\[
\therefore G_c(S) = \frac{K_P}{s} \left( s + \frac{K_I}{K_P} \right) = \frac{K_P}{s} (s + a_1) \quad (4.3)
\]

The feedforward path of the PI compensated system has two poles at the origin and another pole at \(-20\). The root loci corresponding to the pole at \(-20\) will go to \(-\infty\) depending on where the zero of the controller is placed. Whereas for the two poles at the origin, there exists a breakaway point on the imaginary axis, which is the marginal stability line. Accordingly, the zero of the controller, \( a_1 \), must be placed between the pole at \(-20\) and the poles at the origin to pull the root loci away from the marginally stable region towards the stable region, otherwise it will be detrimental to the stability of the system. The effect of the placement of the controller zero is simulated and is illustrated in the root locus and step response plots in Figure 4.8 and Figure 4.8.

The example parameters used are \( a_1 \) at \(-2\) and \(-80\), and \( K_P \) of 1. The root locus plot shows that, if the controller zero was placed elsewhere from the proposed stable region, the resulting closed-loop response is unstable. For example, for \( a_1 \) at \(-80\), the root at \(-20\) approaches the controller zero, but the root loci at the origin curves away from the imaginary axis to the right side of the pole-zero map, which is the unstable region.
Figure 4.7 Root locus plots of the PI compensated system corresponding to different placement of the zero of the controller.

Figure 4.8 Step response of the PI compensated system with different controller zeros.
The example simulations demonstrated the system response without time delay. The PI controller implemented in the digital system consists of a time delay, $T$, of 0.01 seconds. This time delay is the interval at which the controller function is routinely executed. To reflect the effect of the time delay, a second order Pade approximation is used, which is an order higher than suggested in chapter 4 of this book [17]. A higher order approximation ensures better accuracy and the equation in the Laplace domain is defined in Equation (4.4). Subsequently, it is simulated with the same example parameters above for the stable case and the plots are shown in Figure 4.9 and Figure 4.10.

\[
G_d(s) = e^{-Ts} \cdot G_c(s) \cdot G_p(s)
\]

\[
\because \ G_d(s) = \frac{1 - \frac{T}{2} s + \frac{T^2}{12} s^2}{1 + \frac{T}{2} s + \frac{T^2}{12} s^2} \cdot \frac{K_P}{s} (s + a_1) \cdot \frac{c \cdot a}{s(s + a)}
\]

(4.4)

![Figure 4.9](image)

*Figure 4.9 Effect of time delay on step response of PI compensated system.*
Figure 4.10 Effect of time delay on root locus of PI compensated system and a close-up view of the root locus plot at the origin.

Figure 4.9 shows that there is no significant difference between the step responses of the compensated system with and without time delay. This is only true if the controller zero is placed close enough to the poles at the origin and the proportional gain is kept low. The reason is that the second order Pade approximation introduces two pairs of complex conjugate roots, as depicted in Figure 4.10. These roots are the poles in the left half-plane and the zeros in the right half-plane. If the proportional gain was too large, the root loci between the origin and \(-20\) for the compensated system with time delay will move toward the right half-plane, which is the unstable region. If the controller zero was placed far away from the origin but still within the proposed stable region, the
roots will become more sensitive to the change in proportional gain, therefore the system will become unstable more quickly. This effect is simulated with a $K_P$ of 15 and $a_1$ at $-15$. For the system with time delay, note that only a small portion of the root loci near the origin is in the left half-plane, as shown in Figure 4.11. The step response of the time-delayed system is unstable while the step response of the system without time delay only exhibits high overshoot and decaying oscillations, which is as shown in Figure 4.12.

**Figure 4.11** Effect of time delay on root locus with smaller controller zero and higher proportional gain, and a close-up view at the origin.
Consequently, the PI controller is designed by placing the zero closer to the open-loop poles at the origin at the same time keeping the proportional gain low. The characteristic equation of the closed-loop transfer function is a third order polynomial, as expressed in Equation (4.5). The open-loop zero appears in both the numerator and denominator of the closed-loop system. The closed-loop zero will reduce the rise time at the cost of higher maximum overshoot. The closed-loop poles will have dampening effects on oscillation and overshoot, so this compensates the undesired high overshoot due to the closed-loop zero. The closer the open-loop zero is to the origin, the more dominant the closed-loop complex poles are. This will be examined later in the simulation and results section.

\[
G_{sys}(s) = \frac{G_c(s) \cdot G_p(s)}{1 + G_c(s) \cdot G_p(s)}
\]

\[
\therefore G_{sys}(s) = \frac{caK_p(s + a_1)}{s^3 + as^2 + caK_ps + a_1caK_p}
\]

(4.5)

Ultimately, for a Type II control system, the steady state error is theoretically zero. Correspondingly, it is examined via final value theorem in Equation (4.6), and the zero steady state error shows that the PI controller is indeed an ideal choice of controller for the system. The controller in the discrete time domain will be discussed next.

\[
e_{ss} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)}
\]

\[
\therefore e_{ss} = \lim_{s \to 0} \frac{Amp \cdot s(s + a)}{s^2(s + a) + K_pca(s + a_1)} = 0
\]

(4.6)
Figure 4.12 Effect of time delay on step response with smaller controller zero and higher proportional gain.

4.2.2 Discrete PI Controller

To implement the controller in the software, the error-to-control input path in the system block diagram shown in Figure 3.13 is considered. The controller transfer function is transformed from the Laplace domain into the discrete time domain via z-transform. The Euler integration approximation is used for the z-transform by substituting \( s \) for \( \frac{z-1}{Tz} \). The pulse transfer function of the discrete PI controller is derived and expressed in Equation (4.7).

\[
U(s) = \frac{K_P(s + a_1)}{s} E(s)
\]
Then using the $z$-transform property, the pulse transfer function is converted into a set of difference equations. First, the substitution of $X(z)$ is made to partition the pulse transfer function as shown in Equation (4.8), where $U(z)$ and $X(z)$ consist of the numerator and one over the denominator of Equation (4.8), respectively. This $X(z)$ variable will eventually become a state variable of the integral portion of the controller, as shown in the block diagram in Figure 4.13.

\[
\begin{align*}
\frac{U(z)}{E(z)} &= \frac{U(z)}{X(z)} \cdot \frac{X(z)}{E(z)} = \frac{K_p[(1 + a_1 T)z - 1]}{z - 1} \\
\frac{U(z)}{X(z)} \cdot \frac{X(z)}{E(z)} &= \frac{K_p(z - 1) + K_i T z}{z - 1}
\end{align*}
\]  

(4.8)

Then, the partial pulse transfer functions are transformed into difference equations separately. The derivation of state variable of the integral part is shown first in Equation (4.9), where the initial condition, $x(0)$, is zero. Basically, this difference equation is the accumulated error.

\[(z - 1)X(z) = E(z)\]

\[
\therefore x(k + 1) = e(k) + x(k)
\]  

(4.9)

The difference equation for the control input is defined in Equation (4.10). This is the equation that is being implemented in the software.
Next, just like the continuous time analysis, the steady state error analysis of the discrete PI compensated system is also done by using the final value theorem. The zero steady state error is consistently verified in the discrete system as shown in Equation (4.11).

\[ e_{ss} = \lim_{z \to 1} (z - 1)E(z) = \lim_{z \to 1} (z - 1) \frac{R(z)}{1 + G(z)} \]

\[ e_{ss} = \lim_{z \to 1} (z - 1) \frac{Amp \cdot z}{1 + K_p(z - 1) + K_iTz} \cdot \left\{ \frac{1}{(1 + aT)z^2 - (2 + aT)z + 1} \right\} \]

\[ \therefore e_{ss} = Amp \frac{1}{\infty} = 0 \]
4.2.3 Simulation and Results of the PI Compensated System

The step responses and root locus plots of the systems with different combination of proportional and integral gains are simulated and examined. The controller gains that yield acceptable responses are implemented and tested on the hardware. Subsequently, the results of the compensated system are compared with the response of the uncompensated system to identify the improvement of the controller. The responses with load on the antenna will also be presented. This is to test the disturbance rejection capability of the controller. Additionally, the closed-loop roots of the compensated system will be examined in both the continuous and the discrete time domain to verify the stability.

The controller gains are chosen and tuned experimentally using the aforementioned guides to ensure closed-loop stability. The step responses and the root locus plots for the controller gains are simulated and shown in Figure 4.14 to Figure 4.18. Note that these simulations are done without the nonlinear elements such as the hardware limits and friction, but they will be included later. Since the proportional gains are relatively small and the controller zeros are close enough to the poles at the origin, the effect of time delay on the continuous time step response is insignificant. Correspondingly, the simulations are done without the second order Pade approximation. The step response and the closed-loop transfer function characteristics are tabulated in Table 4-1 and Table 4-2 respectively.

There are performance tradeoffs for all the suggested controller gains. When the controller zero is placed farther away from the integrator, the rise time decreases with increasing maximum overshoot, and vice versa. In Figure 4.14, with $K_p$ of 5 and $K_i$ of 25, the controller zero is at $-5$, there is 45% maximum overshoot, but the system settles the fastest at just over one second with decaying oscillation. This simulated response is denoted by the solid line with circle markers.
Whereas with smaller gains $K_p$ of 1.45 and $K_i$ of 2, the controller zero is at $-1.4$, the magnitude of overshoot and oscillation is reduced by half, which is about 28% overshoot, at the expense of slower response time. This response is illustrated in the dashed line in Figure 4.14, and according to Table 4-1, it has a settling time of approximately four seconds. The cause of the varying responses due to the different gains is examined in the closed-loop eigenvalues, which are obtained from the root locus plots.

Figure 4.14 Step response simulation with different controller gains.
Table 4-1  Step response characteristics of the PI compensated system.

<table>
<thead>
<tr>
<th>Gains</th>
<th>Controller Zero</th>
<th>Percent Overshoot</th>
<th>Settling Time</th>
<th>Rise Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_p$</td>
<td>$K_I$</td>
<td>$a_1$</td>
<td>% PO</td>
<td>$T_s$ (seconds)</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
<td>-5</td>
<td>45</td>
<td>1.1</td>
</tr>
<tr>
<td>1.45</td>
<td>5</td>
<td>-3.45</td>
<td>45</td>
<td>3.75</td>
</tr>
<tr>
<td>1.45</td>
<td>2</td>
<td>-1.4</td>
<td>28</td>
<td>3.77</td>
</tr>
<tr>
<td>2.4</td>
<td>5</td>
<td>-2</td>
<td>28</td>
<td>1.6</td>
</tr>
</tbody>
</table>

The transient response of the closed-loop PI compensated system is reflected predominantly by the complex eigenvalues, which are the dominant complex poles. This is because they have a larger time constant than the remaining real eigenvalues, which is shown in Table 4-2. This can also be verified graphically, for instance the closer the closed-loop pole is to the imaginary axis of the pole-zero map, the more dominant the pole is. In Figure 4.15 to Figure 4.18, the complex eigenvalues are closer to the imaginary axis, therefore they are indeed the dominant poles; all closed-loop eigenvalues are depicted by the square markers in the figures. The reason there is a large overshoot when the controller zero is at $-5$ is that the arctangent angle, $\beta$, due to the complex components of the eigenvalue, is the largest. This arctangent angle dictates the decaying nature of the closed-loop system, such that the dampening effect lessens with increasing angle, which is explained in the pole-zero map illustration in Figure 4.19.

The reason the overshoot increases with increasing proportional gain is that this effectively pulls the real eigenvalue towards the imaginary axis while simultaneously repels the complex eigenvalues away from the imaginary axis. Consequently, the real eigenvalue gradually gains dominance over the complex eigenvalues, and the dampening element becomes less
effective. Figure 4.15 is used to examine this development. At higher proportional gain, the root loci branches of the complex eigenvalue are located to the left of the closed-loop zero, indicated by the circle marker. Hence, the real eigenvalue will ultimately become closer to the imaginary axis as it moves towards the closed-loop zero while the complex eigenvalues diverge away along the branches. This can adversely affect the system response if the proportional gain was high enough that pole-zero cancellation occurs. As a result, the third order system will be reduced to a slow decaying second order oscillatory system.

Figure 4.15 Root locus plot for $K_p$ of 5 and $K_i$ of 25.
Figure 4.16 Root locus plot for $K_p$ of 1.45 and $K_i$ of 5.

Figure 4.17 Root locus plot for $K_p$ of 1.45 and $K_i$ of 2.
Figure 4.18 Root locus plot for $K_p$ of 2.45 and $K_i$ of 5.

Table 4-2 Closed-loop characteristics of PI control.

<table>
<thead>
<tr>
<th>Gains</th>
<th>$K_p$</th>
<th>$K_i$</th>
<th>Eigenvalues</th>
<th>Time Constants (Seconds)</th>
<th>$\beta$ (Degrees)</th>
<th>Damping Ratio $\zeta$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\tau_{\lambda_1}$</td>
<td>$\tau_{\lambda_{2,3}}$</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>25</td>
<td></td>
<td>$\lambda_1 = -12.9, \lambda_{2,3} = -3.56 \pm 6.75$</td>
<td>0.078</td>
<td>0.28</td>
<td>62.2°</td>
</tr>
<tr>
<td>1.45</td>
<td>5</td>
<td></td>
<td>$\lambda_1 = -18, \lambda_{2,3} = -0.975 \pm 2.71$</td>
<td>0.056</td>
<td>1.03</td>
<td>70.2°</td>
</tr>
<tr>
<td>1.45</td>
<td>2</td>
<td></td>
<td>$\lambda_1 = -17.7, \lambda_{2,3} = -1.13 \pm 1.45$</td>
<td>0.056</td>
<td>0.88</td>
<td>52.1°</td>
</tr>
<tr>
<td>2.4</td>
<td>5</td>
<td></td>
<td>$\lambda_1 = -16, \lambda_{2,3} = -2 \pm 2.32$</td>
<td>0.063</td>
<td>0.5</td>
<td>49.2°</td>
</tr>
</tbody>
</table>
Figure 4.19 Effect of arctangent angle of the components of complex eigenvalue on damping.

Since the antenna system is modelled as a linear time-invariant system, the stability of the system can be readily verified by examining the location of the closed-loop eigenvalues.

According to Table 4-2, all the eigenvalues are in the left half-plane of the pole-zero map, which indicates that the system is bounded-input, bounded-output (BIBO) stable. The stability of the discrete system is also examined by looking at the locations of the poles of the closed-loop pulse transfer function as expressed in Equation (4.12). Note that the change in the sample time, $T$, can change the response of the discrete system. For the different controller gains, the corresponding pole locations are tabulated in Table 4-3. The poles are all located in the unit circle of the $z$-plane, hence the stability of the discrete system is consistently verified.

$$G_{cl}(z) = \frac{caK_p T^2 z^2[(1 + a_1 T)z - 1]}{z^3 - \frac{(3 + aT + caK_p T^2)}{a_2} z^2 + \frac{3 + aT}{a_2} z - \frac{1}{a_2}} \quad (4.12)$$

where $a_2 = 1 + aT + caK_p T^2 + caK_p a_1 T^3$
Overall, the simulations show that $K_p$ of 2.4 and $K_I$ of 5 yielded the most desirable response with relatively low overshoot and fast response time. These gains were designed at a later time so the corresponding results will be presented in the trajectory generation section.

Hence, the comparisons of the actual and simulated results of the uncompensated system and compensated system with $K_p$ of 1.45 and $K_I$ of 2 are presented. The step response is illustrated in Figure 4.20. The error plots are also shown in Figure 4.21. The response characteristics are tabulated in Table 4-4. The simulations are done by including all the known hardware limits to reflect the actual system more accurately, as previously shown in Figure 3.15 in CHAPTER 3.

**Table 4-3** Location of poles of the closed-loop pulse transfer function.

<table>
<thead>
<tr>
<th>Gains</th>
<th>Closed-loop poles</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_p$</td>
<td>$K_I$</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>$z_1 = 0.8792, z_{2,3} = 0.9628 \pm j0.0651$</td>
<td></td>
</tr>
<tr>
<td>1.45</td>
<td>5</td>
</tr>
<tr>
<td>$z_1 = 0.8348, z_{2,3} = 0.9899 \pm j0.0269$</td>
<td></td>
</tr>
<tr>
<td>1.45</td>
<td>2</td>
</tr>
<tr>
<td>$z_1 = 0.8375, z_{2,3} = 0.9887 \pm j0.0143$</td>
<td></td>
</tr>
<tr>
<td>2.4</td>
<td>5</td>
</tr>
<tr>
<td>$z_1 = 0.8522, z_{2,3} = 0.9799 \pm j0.0227$</td>
<td></td>
</tr>
</tbody>
</table>

**Table 4-4** Response characteristics of the uncompensated and PI compensated system.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Steady State Error (%)</th>
<th>Overshoot/Undershoot (%)</th>
<th>Rise Time (Seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncompensated</td>
<td>26</td>
<td>26</td>
<td>-</td>
</tr>
<tr>
<td>Actual PI</td>
<td>20</td>
<td>26</td>
<td>0.43</td>
</tr>
<tr>
<td>Simulated PI</td>
<td>17</td>
<td>26</td>
<td>0.43</td>
</tr>
</tbody>
</table>
Figure 4.20 Displacement comparisons with $K_p$ of 1.45 and $K_i$ of 2.

Figure 4.21 Error comparisons with $K_p$ of 1.45 and $K_i$ of 2.
Overall the actual and the simulated results are fairly identical. However, the PI controller demonstrated questionable results as it showed little improvement over the uncompensated system. In Figure 4.20, the uncompensated system never reaches the target and eventually results in high steady state error. The PI compensated system overshoots at an expected magnitude, but never reaches the target, though with a little less steady state error than the uncompensated system, as shown in Figure 4.21. There is a huge discrepancy in the steady state error between the simulations with and without the hardware limits, even though the remaining response characteristics are almost identical.

The cause of this discrepancy is that a conditional statement was implemented in the software by the employer during the implementation of the controllers. It automatically ceases motion shortly after the antenna has dawdled for some time to prevent excessive time consumption on the control task. Hence, the control input is zeroed out immediately before the displacement error gradually settles down to zero. In Figure 4.22, towards the end of the actual calculated control input, it suddenly drops to zero despite it is still gradually building up in the negative direction. The reason the antenna dawdles momentarily is that the actual control input is within the deadzone, so no motion is observed until static friction is overcome. If the antenna continues moving with the calculated control input, zero steady state error is expected.

As shown in Figure 4.23, the extended simulation demonstrated that the antenna eventually settles to the target point with zero steady state error, at around 6 seconds. This suggests that the PI compensated system can achieve zero steady state pointing error, while overcoming friction and gravity for a small displacement. However, a 6 seconds response time to make a small displacement of 100 BITS or 1.5° is rather slow. The system with additional load is examined next.
**Figure 4.22** Control input comparisons with Kp of 1.45 and Ki of 2.

**Figure 4.23** Nonlinear simulation of PI compensated system showing zero steady state error.
A 14 kg load is added to the antenna to examine the disturbance rejection capability of the PI compensated system. The setup is shown in Figure 4.24 with the load tied to the back of the antenna. When the antenna is at a medium elevation angle, the center of mass (c.o.m) of the combined load is shifted more to the left of the equilibrium center mass point. Consequently, a pulling force is constantly applied on the linear actuator. In effect, this assists the antenna folding-up motion, which results in a faster rise time. This can be reversed if the center of mass is shifted to the right of the equilibrium center mass point, or by placing the load on the feedhorn as shown in Figure 4.25(b).

*Figure 4.24  Experiment with additional load added to the antenna.*
Figure 4.25 Load placements: (a) behind the antenna reflector, (b) on the feedhorn.
The actual responses of the loaded system are shown in Figure 4.26. The rise time for the upward motion is 15% faster than the downward motion. Similar to the unloaded system, the loaded system shows that the motion is terminated by the program before the control input and the displacement error settle down to zero. Hence, the large steady state error and the abrupt change in the control input are consistently observed in Figure 4.27 and Figure 4.28. If the control task was run uninterrupted, the antenna is expected to move to the target or close to it. This suggests that the loaded system will also have a relatively slow response time. The main contributor of the overshoot is the integral part of the controller. For instance, the accumulated error gets up to about 30 times larger than the largest displacement error, which is shown in both the loaded and unloaded cases in Figure 4.21 and Figure 4.27. A limit must be put on the accumulated error to avoid excessive overshoot, especially for a large displacement.

![Comparisons of Actual Displacements with Load for Up and Down Motions](image)

**Figure 4.26** Actual displacements with load at 50° elevation angle.
**Figure 4.27** Actual errors with load.

**Figure 4.28** Actual calculated control inputs with load.
The disturbance response without the wind load is simulated as shown in Figure 4.29. The displacement error increases marginally and almost instantly. When control input is sufficiently built up to overcome static friction, the error then converges to zero in three seconds. The linear actuator is compressed hence the negative displacement. The magnitude of disturbance error due to gravity is 15 BITS, which is 0.12° error in angular displacement. Thus, the contribution of error due to gravity is relatively small. The disturbance error due to a constant wind load is simulated as shown in Figure 4.30. The simplified schematic that shows the constant wind load on the antenna is shown in Figure 4.31.

For simplicity of analysis, the wind force is assumed to be perpendicular to the crank, shown in Figure 4.31, so the link ratio and the force angle from the reflector to the crank joint are ignored. The reflector is assumed to be a flat plate with a drag coefficient, \( c_d \), of 1.28 [18].

![Disturbance Response Due to Gravity](image)

*Figure 4.29  Disturbance error plot showing effect of gravity.*
Figure 4.30 Disturbance error plot with constant wind load on the reflector.

Figure 4.31 Schematic of wind load on simplified rectangular antenna surface.
Figure 4.32 Resolving the wind force in the direction of the linear actuator.

The antenna reflector is also assumed to be perpendicular to the direction of wind flow for maximum force. The frontal area, $A_F$, that is perpendicular to the wind flow is roughly $1.5 \, m^2$, which is obtained from the CAD drawings. The density of air, $\rho_{air}$, is $1.225 \, \frac{kg}{m^3}$, and the wind speeds for both normal and strong wind are $7 \, \frac{m}{s}$ and $15 \, \frac{m}{s}$, which results in wind forces of $130 \, N$ and $600 \, N$ [19]. To reflect the wind force on the linear actuator, the angle between the wind force and the linear actuator is considered as shown in Figure 4.32. Hence, the resultant wind force when reflected on the linear actuator is only a portion of the full force, which is $F_w \cos(\theta_{prime})$.

As shown in Figure 4.30, it takes longer to compensate for the disturbance error due to the strong wind load than it takes for the normal wind load. Note that the normal wind load reduces the disturbance error due to gravity because in the current wind load configuration, they work against each other. As the wind load gets larger, error due to gravity is overcome, so the error direction of strong wind is the opposite of the normal wind. This means that in the presence of strong wind, the response is predominantly due to the wind load. The simulated response in the presence of
strong wind is shown in Figure 4.33, where it takes 7 seconds to settle to the target with magnified overshoot.

![Step Response with Strong Wind Load](image)

**Figure 4.33** Step response and error plot of PI compensated system with strong wind.

The Pi only control can move the antenna in a stable manner in the presence of external noise. The tradeoff is that it takes a considerably long time to recover from an overshoot, hence many antenna dawdlings have been observed, which is for at least a second. This idle phase triggers a task-terminating condition, which compromises the control task. As a result, the remaining displacement error is left uncompensated, and the steady state pointing error still persists. Even if the condition was removed, the response time is expected to be slow.

Consequently, a trajectory generation scheme is implemented to improve the response time and to reduce the magnitude of overshoot, which is presented next.
4.3 PI Control with Trajectory Generation

The trajectory generation creates a desired path and kinematic profiles for a motion system to track. When considering elevation motion only, the antenna system is a closed kinematic chain, so the path that it travels is fixed. Therefore, the one-dimensional trajectory generation is used to create desired position, velocity, and acceleration profiles to generate a set of desired positions as commanded positions to reach a target. The error to the controller is based on the current position and the current desired position. A new desired position is calculated every cycle until the target is reached. This way both the control input and the error are gradually built up and diminished, which results in a smoother motion. A trajectory generation scheme will be presented next followed with simulation and results.

4.3.1 Trajectory Generation Scheme

The underlying equations that are used to create the kinematic profiles of the antenna are obtained by integrating the acceleration twice. The velocity and position profiles are generated by Equation (4.14) and Equation (4.14). Note that the acceleration term in the desired position equation is approximately zero because of how small the sample time is.

\[
P_{\text{desired}} = \int \int \text{Accel} \, dt \\
P_{\text{desired}} = \int V_{\text{desired}} \, dt \\
\text{where } V_{\text{desired}} = V_{\text{desired, old}} + \text{Accel}_{\text{old}}T \\
P_{\text{desired}} = P_{\text{desired, old}} + V_{\text{desired, old}}T + \frac{1}{2}\text{Accel}_{\text{old}}T^2 \\
P_{\text{desired}} = P_{\text{desired, old}} + V_{\text{desired, old}}T
\]
There are two main conditions that govern the trajectory generation, which are the trajectory position error and the trajectory stop distance. The trajectory position error is the difference between the commanded target position and the current desired position. This condition determines the direction of the trajectory as well as when target is reached. The trajectory stop distance is the distance between the target position and the position to initiate deceleration. It decides when to decelerate the trajectory velocity before approaching the commanded target position. The stop distance can be calculated by solving the general position equation at the time taken to reach the maximum allowable velocity for a given deceleration. The derivation is shown below and the solution is expressed in Equation (4.15), note that the stop distance is a relative distance so the initial position is disregarded.

When the acceleration and the deceleration are identical, this results in a symmetric trajectory, otherwise it is asymmetric. This is illustrated in the example trajectory profiles as shown in Figure 4.34. When the trajectory distance is too small, there might be insufficient distance to start decelerating with the above-defined stop distance, because it is a constant based on the maximum velocity and deceleration. Hence, the solution is to use the middle point of the small trajectory distance as the new stop distance. The calculated desired positions then become the inputs to the PI compensated system. A code snippet is show in Figure 4.35. Ultimately, the advantage of trajectory generation is to provide more control on the overall motion.

\[ P_{\text{stop}} = P_{\text{initial}} + V_{\text{initial}} T_{\text{max}} + \frac{1}{2} Decel T_{\text{max}}^2 \]

where \( V_{\text{initial}} = V_{\text{max}} - Decel T_{\text{max}} \) and \( T_{\text{max}} = \frac{V_{\text{max}}}{Decel} \)

\[ \therefore P_{\text{stop}} = \frac{1}{2} \frac{V_{\text{max}}^2}{Decel} \]  

(4.15)
Figure 4.34 (a) Symmetrical trajectory with identical acceleration and deceleration. (b) Asymmetrical trajectory with different acceleration and deceleration.
Figure 4.35 Code snippet of trajectory generation, written in MATLAB script.
4.3.2 Simulation and Results of the PI Compensated System with Trajectory Generation

The results of the PI controller with $K_P$ of 2.4 $K_I$ of 5 combined with trajectory generation with maximum velocity and acceleration of 350 BITS/s and BITS/s² are presented. A small displacement of 100 BITS is tested first, and the results are shown in Figure 4.36 to Figure 4.38. The position results in Figure 4.36(a) demonstrated significant improvements in terms of reducing both the response time and the steady state error. The system settles to steady state value in 4 seconds, which is 30% faster than the PI only control. The maximum overshoot is 16% which is 4% less than before. Both the actual and simulated position results showed half a second delayed starts. This is because the actual and the desired initial positions are identical, so the error and the control input build up gradually from zero until static friction is overcome, which are observed in Figure 4.37 and Figure 4.38. The same delay is also observed in the velocity in Figure 4.36(b). There is no velocity sensor on the linear actuator so the actual velocity is differentiated from the actual position, which results in high frequency noise. The simulated velocity is less noisy because it is integrated from the acceleration.

There are some discrepancies in both the simulated control input and the simulated accumulated error when compared with the actual results. This is likely due to the inaccurate representation of the full dynamics model. A correction gain of 1.5 is determined through trial and error and is added to the feedforward path of the compensated system. This seems to have resolved the discrepancies slightly when recovering from an overshoot, which is shown in the dashed lines. The steady state discrepancies remain unchanged. This is because the static friction is set to a constant in the simulation; hence this suggests that it varies. Also in Figure 4.37(b), the discretization effect of the actual accumulated error is fairly distinct. This is due to the rounding off to the nearest hundred.
Figure 4.36 Comparisons of desired, simulated, and actual trajectory profiles (a) position, (b) velocity.
Figure 4.37  Comparisons of simulated and actual errors (a) displacement, (b) accumulated.
The results due to a large displacement are shown in Figure 4.39 to Figure 4.41. The displacement is from 1000 BIT to 4000 BIT, which is from 23° to 60° elevation angle. The response showed zero steady state error and no significant oscillations. The antenna settles to the target in just less than 12 seconds with only 0.8% overshoot. Note that the maximum actuator velocity is between 450 BITS/s and 500 BITS/s², so the overall response time can actually be reduced if the maximum trajectory velocity was increased. The dawdling time has also been significantly reduced to less than a second, similar for previous results, so the antenna is almost constantly in motion. The results shown have been of the symmetric trajectory profile. The asymmetric trajectory profile will simply shift the desired position at the beginning or at the end depending on the acceleration and the deceleration, as shown in Figure 4.42.

Figure 4.38 Comparisons of actual and simulated control inputs.
Figure 4.39 Comparisons of desired, simulated, and actual trajectory profiles (a) position, (b) velocity, for a large displacement.
Figure 4.40 Comparisons of simulated and actual errors (a) displacement, (b) accumulated, for a large displacement.
**Figure 4.41** Comparisons of actual and simulated control inputs for a large displacement.

**Figure 4.42** Comparisons of actual results with different trajectory profiles. Kp of 2, Ki of 3, accumulated error limit of 2500, and a 7 kg load at the feedhorn.
The controller for the asymmetric trajectory profile in Figure 4.42 has lower gains, which is not much better than the previous controller. It has a theoretical maximum overshoot of 25% and a settling time of 2.15 seconds. Although with external load added to the antenna, both results with different accelerations outperform the previous controller with only 0.2% overshoot and faster response time for the same large displacement. This reason there is less overshoot is that there is a limit on the accumulated error, so the antenna can settle to steady state faster.

The antenna oscillates briefly when it accelerates slowly from a higher elevation angle, specifically at $66^\circ$. This is where the feedhorn is almost vertical to the ground. The problem exists only for downward motion, which is depicted in Figure 4.43, for controller gains $K_p$ of 2.4 $K_I$ of 5. This can be resolved by increasing the acceleration and/or velocity, which is shown in Figure 4.43(b), where the oscillation is slightly reduced when the acceleration is increased by 200 $BITS/s^2$. Consequently, the maximum acceleration and velocity are used so that the resonant frequency of the feedhorn is not excited.

The results shown in Figure 4.44 has a deceleration of 200 $BITS/s^2$, controller gains $K_p$ of 0.5 $K_I$ of 3, and an accumulated error limit of 700. The control input ramps up to the maximum magnitude in just over a second and the antenna accelerates to the maximum velocity simultaneously. During the motion no oscillation is observed and the antenna settles to the target angle smoothly. The same trajectory profile and controller gains are tested on a slightly different antenna system, which the response is fairly smooth as shown in Figure 4.45. This shows that almost any stabilizing controller can be used to achieve a smooth response as long as an appropriate trajectory profile is created and that the accumulated error is included. Alternative solutions are explored, which is the implementation of the Notch filter, and PID control with low-pass filter, which will be presented in Chapter 5.
Figure 4.43 Oscillation at high angle due to (a) slow acceleration, (b) medium acceleration.
Figure 4.44 Downward motion oscillation eliminated with asymmetric trajectory.

Figure 4.45 Asymmetric trajectory profile with the same trajectory parameters and controller gains as in Figure 4.45, except for deceleration of 20 BITS/s², on a different antenna.
CHAPTER 5. FILTER DESIGN

In this chapter, the design and implementation of the Notch filter and the second order low-pass filter are presented. The Notch filter is implemented to reduce the oscillations of the feedhorn at higher elevation angle. The second order low-pass digital filter smoothens the differentiated error for the exploration of PID control. Ultimately, the PI with Notch filter compensated system and the PID with second order digital filter compensated system, with trajectory generated inputs are examined as alternative controls.

5.1 Notch Filter

The Notch filter introduces complex conjugate zeros to cancel the complex conjugate poles of the plant in order to eliminate the oscillations in the transient response. Exact pole-zero cancellation is unachievable in the physical system because the antenna model is not perfect. In effect, the zeros are placed very close to the actual poles, so the resulting closed-loop eigenvalues due to the complex conjugate poles and zeros of the plant and the filter are constrained within a tight root loci. Additionally, the Notch filter also introduces a new pair of poles in the feedforward path, to maintain the control system type of the model.

5.1.1 Development of the Notch Filter

The placement of the zeros of the Notch Filter depends on the antenna dynamics. Since the linear actuator only model does not have the antenna dynamics, the full equation of motion is used. A mass-spring-damper analogy is used here to obtain the equation of motion. The linearized full equation of motion in CHAPTER 3 is not used because of the uncertainty of the resulting parameters. The free-body diagram and the block diagram of the system is shown in Figure 5.1.
Figure 5.1 Alternate antenna dynamics model (a) free-body diagram of the mass-spring-damper analogy, (b) block diagram simplification of the mass-spring-damper system.
The simplified block diagram in Figure 5.1(b) shows the open-loop transfer function from force input. The simplified form is obtained by dividing the transfer function by $J_{eq}$ and $M_a$. It is expressed as $G_{pa}(s)$ in Equation (5.1) where $\omega_{na}$ is $\frac{K_a}{\sqrt{M_a}}$ and $\zeta_a$ is $\frac{B_a}{2M_a\omega_{na}}$, which they are the damped natural frequency and the damping ratio of the antenna when there is no actuator motion. Note that the spring constant term is neglected because it is divided by the overall effective mass felt by the motor, which is a relatively large number. The antenna natural frequency is obtained by measuring the oscillation of the feedhorn to an impulse input with an accelerometer, which the measured data is shown in Figure 5.2. The measured data has an average damping ratio of 0.0133, which is calculated using the logarithmic decrement method. The Fourier transform of the measured data in Figure 5.3 shows that the antenna has a damped natural frequency of approximate 16 rad/s.

$$G_{pa}(s) = \frac{1}{J_{eq}} \frac{s^2 + 2\zeta_a \omega_{na}s + \omega_{na}^2}{s^3 + (2\zeta_a \omega_{na} + \alpha)s^2 + (2\zeta_a \omega_{na}^2 + \omega_{na}^2)s + \omega_{na}^2 \alpha}$$ \hspace{1cm} (5.1)

The antenna model exhibits a decaying oscillatory response, so the cubic polynomial in the denominator of Equation (5.1) is factorized into one real root and two complex conjugate roots. This is expressed as $G_{paNew}(s)$ in Equation (5.2) where $r_1$ is the open-loop real root and $\zeta$ and $\omega_n$ are the damping ratio and damped natural frequency of the antenna when there is actuator motion. From Figure 5.4, the damped natural frequency is also measured to be approximately 16 rad/s. The damping ratio of the antenna during motion is assumed to be the same since the damped natural frequencies are identical. Figure 5.5 verifies the antenna model.

$$G_{paNew}(s) = \frac{1}{J_{eq}} \frac{s^2 + 2\zeta_a \omega_{na}s + \omega_{na}^2}{s(s + r_1)(s^2 + 2\zeta \omega_n s + \omega_n^2)}$$ \hspace{1cm} (5.2)
Figure 5.2 Normalized raw data of the impulse response of the feedhorn.

Figure 5.3 Fourier transform of the data to find the damped natural frequency of the feedhorn.
Figure 5.4 Close-up view of the oscillations in Figure 4.43(b).

Figure 5.5 Verification of antenna model using the mass-spring-damper analogy. Simulated with $K_p$ of 5, $K_i$ of 25, and with friction and gravity included.
Ultimately, the complex zeros of the Notch filter are placed at the same location as the complex conjugate poles of $G_{pa_{new}}(s)$ to reduce the oscillatory response. The poles of the filter are determined experimentally by varying their locations while examining the root locus, which is presented in the simulation and results section. Real poles are added to avoid introducing unwanted oscillations. Consequently, the Notch filter is the Laplace domain is expressed in Equation (5.3).

$$G_{NF}(s) = \frac{s^2 + 2\zeta \omega_n s + \omega_n^2}{(s + n_1)(s + n_2)}$$  \hspace{1cm} (5.3)

In order to implement the Notch filter, Equation (5.3) is transformed into the discrete domain. The $z$-transform is done using Euler integration approximation and the Notch filter in the $z$-domain is expressed as $G_{NF}(z)$ in Equation (5.4), where $Y_{NF}(z)$ is the filtered output and $R_{NF}(z)$ is the control input.

$$G_{NF}(z) = \frac{Y_{NF}(z)}{R_{NF}(z)} = \frac{(1 + 2\zeta \omega_n T + \omega_n^2 T^2)z^2 - 2(1 + \zeta \omega_n T)z + 1}{(1 + (n_1 + n_2)T + n_1 n_2 T^2)z^2 - (2 + (n_1 + n_2)T)z + 1}$$  \hspace{1cm} (5.4)

The difference equation is shown in Equation (5.5), where the inputs and filtered outputs with the _old subscripts are old values, which are initialized to zero.

$$y_{NF}(k) = \frac{(1 + 2\zeta \omega_n T + \omega_n^2 T^2)r_{NF}(k) - 2(1 + \zeta \omega_n T)r_{NF}(k - 1)}{(1 + (n_1 + n_2)T + n_1 n_2 T^2)}$$

$$+ \frac{r_{NF}(k - 2) + (2 + (n_1 + n_2)T)y_{NF}(k - 1) - y_{NF}(k - 2)}{(1 + (n_1 + n_2)T + n_1 n_2 T^2)} \hspace{1cm} (5.5)$$

$$y_{NF}(k) = \frac{(1 + 2\zeta \omega_n T + \omega_n^2 T^2)r_{NF}(k) - 2(1 + \zeta \omega_n T)r_{NF,OLD}}{(1 + (n_1 + n_2)T + n_1 n_2 T^2)}$$

$$+ \frac{r_{NF,OLD,OLD} + (2 + (n_1 + n_2)T)y_{NF,OLD} - y_{NF,OLD,OLD}}{(1 + (n_1 + n_2)T + n_1 n_2 T^2)}$$
5.1.2 Simulations and Results of the PI-Notch Compensated System

The results and the root locus analysis of the PI-Notch compensated system are presented in this section. The feedforward transfer function is solved numerically and is expressed in Equation (5.6). It relates the control input to the digital output, so all the conversion gains are absorbed in the transfer function. Note that it is still a Type I control system after the estimated dynamics have been included. The closed-loop transfer function of the uncompensated system with the factorized roots is defined in Equation (5.7). The equation shows that there is already pole-zero cancellation with the lightly damped complex poles, which suggests that the Notch filter is not required. The simulated root locus plot of the transfer function is shown in Figure 5.6.

\[
G_{OLNew}(s) = \frac{30s^2 + 12.77s + 7680}{s(s^3 + 20s^2 + 264.3s + 5011)} \quad (5.6)
\]

\[
G_{CLNew}(s) = \frac{(s + 0.2 + j16)(s + 0.2 - j16)}{(s + 1.6)(s + 18.4)(s + 0.2 + j16)(s + 0.2 - j16)} \quad (5.7)
\]

Actual pole-zero cancellation is not realizable in the physical system. Consequently, to reflect the actual system the damping ratio of the antenna, when there is actuator motion, is assumed to be slightly higher, such as \(\zeta\) of 0.05. The new closed-looped transfer function, \(G_{CL2New}(s)\), is defined in Equation (5.8). The simulated root locus that reflects the actual system slightly more accurately is shown in Figure 5.7. The resulting eigenvalues that lie along the tight root locus between the complex poles and zeros are the dominant closed-loop poles. Accordingly, these very lightly damped complex poles cause oscillations

\[
G_{CL2New}(s) = \frac{(s + 0.2 + j16)(s + 0.2 - j16)}{(s + 1.6)(s + 18.4)(s + 0.83 + j16)(s + 0.83 - j16)} \quad (5.8)
\]
Figure 5.6 Root locus plot of the open-loop transfer function of the mass-spring-damper system.

Figure 5.7 Root locus plot with assumed slightly different complex poles and zero locations.
For the purpose of demonstration, pole-zero cancellation is assumed to occur when the Notch filter is added. Therefore the lightly damped closed-loop poles disappear. The filter also adds two open-loop poles, which affects the closed-loop response depending on where the poles of the Notch filter are placed. The closed-loop system is potentially unstable if the filter poles are placed between the poles of the open-loop system. For example, in Figure 5.8, the resulting root locus branch near the origin crosses the imaginary axis to the unstable region when the poles of the filter are placed at -10. Although the complex poles near the origin are highly damped, which reduces the overshoot and oscillation of the response, the closed-loop system can become unstable if the controller gains were too high.

Figure 5.8 Root locus plot of the Notch filter compensated system with filter poles at -10.
The closed-loop Notch filter only compensated system has a closed-loop stable response, if the filter poles are placed far away from the poles of the open-loop system. For instance, in Figure 5.9, when the filter poles are at -30, the entire root locus occupies the stable region. The response characteristics of stable closed-loop depends of the proportional gain of the filter only compensated system. Figure 5.9 shows a unity proportional gain simulation, which the resulting dominant closed-loop poles are real, therefore the response is overdamped. If the gain was high, the dominant closed-loop poles will become complex, and hence the response will be oscillatory. The response is also slightly slower because one of the real closed-loop poles is considerably closed to the origin.

The effect of the Notch filter on the response of the PI compensated system is also examined. The root locus of the compensated system is simulated with $K_p$ of 2.5 and $K_I$ of 4, as shown in Figure 5.10 and Figure 5.11, with the filter poles at -20 and -30, respectively. The simulated step response plots and characteristics are demonstrated in Figure 5.12 and Table 5-1.

![Root Locus Editor for Open Loop 1(OL1)](image)

**Figure 5.9** Root locus plot of the Notch filter compensated system with filter poles at -30.
Figure 5.10  Root locus of the PI-Notch compensated system with filter roots at -20.

Figure 5.11  Root locus of the PI-Notch compensated system with filter roots at -30.
Figure 5.12  Step response of the compensated system with different pole placements of the filter.

<table>
<thead>
<tr>
<th>Roots</th>
<th>Rise Time (s)</th>
<th>Settling Time (s)</th>
<th>% Overshoot</th>
<th>Damping Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>-20</td>
<td>0.399</td>
<td>3.24</td>
<td>44.3</td>
<td>0.5</td>
</tr>
<tr>
<td>-30</td>
<td>0.675</td>
<td>7.67</td>
<td>46</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Table 5-1  Step response characteristics of the closed-loop PI Notch compensated system.

The closed-loop response of the PI-Notch compensated system is almost similar to the Notch filter only compensated system. The closer the filter poles are to the PI compensated system, the faster response time, and vice versa. In Figure 5.11, the dominant complex eigenvalues of the filter poles at -20 are further away from the origin than that of the filter poles at -30, hence the system settles faster. Also, the damping ratio is higher, so there is less oscillation. The changes to the overshoot is almost insignificant because the location of the closed-loop zeros is unaffected by the filter, as shown in Figure 5.11 and Figure 5.12. The closed-loop response becomes unstable more easily if the filter poles are placed between the two open–loop poles of

}````
the system. In Figure 5.13, with the filter poles at -10, way below the feedhorn resonant frequency, majority of the root locus at the origin shifts to the unstable region. With the same controller gains, the closed-loop system is unstable as there are two complex eigenvalues on the right-half plane.

The filter is implemented on the control input shown in Figure 4.43(b) and the simulated results are shown in Figure 5.14. The magnitude of the filtered control input is maintained when the filter poles are at -16, while it is reduced when they are at -20. To ensure closed-loop stability, the filters poles are designed to be equal to or greater than the magnitude of the resonant frequency, but less than twice the magnitude so that the control input can overcome static friction. Ultimately, the final Notch filter form that is implemented in the digital system is expressed as $y_{NF\_new}(k)$ in Equation (5.9), which uses the $\zeta$ of 0.05, $\omega_n$ of 16 rad/s, and the real poles at -20.

$$y_{NF\_new}(k) = 0.7r_{NF}(k) - 1.4r_{NF\_OLD} + 0.7r_{NF\_OLD\_OLD} + 1.7y_{NF\_OLD} - 0.7y_{NF\_OLD\_OLD}$$

\[ (5.9) \]

**Figure 5.13** Root locus of the PI-Notch compensated system with filter roots at -10.
Figure 5.14 Filtered control input of the oscillatory data in Figure 4.43(b).

The actual and simulated displacement results of the PI-Notch compensated system are shown in Figure 5.15. The oscillations during the starting motion from high elevation angle have been attenuated. Both the actual and the simulated results are almost identical, and they appear to be lagging behind the desired trajectory from the start. This outcome is expected; the simulations earlier showed increased rise time when the filter poles are placed at -20. The slopes of the desired trajectory and the actual and simulated displacements at maximum velocity are different. This is because the actual maximum linear actuator velocity is less than 500 BITS/s. Despite the differences with the desired trajectory, the response characteristics are maintained with the addition of the filter. For instance, there is only 1.8% maximum overshoot and 0.13% steady state error. The displacement results show an overall stable and smooth response. The actual and simulated control inputs are shown in Figure 5.16.
Figure 5.15  Results using $K_p$ of 2.5, $K_i$ of 4, acceleration and velocity of 400 BITS/$s^2$ and 500 BITS/$s$, deceleration of 200 BITS/$s^2$, and accumulated error limit of 1200.

Figure 5.16  Comparisons of calculated and filtered control inputs.
The results in Figure 5.16 agree with the simulations earlier, which is when the Notch filter poles are at -20 the magnitude of the filtered control input is reduced and has less oscillations. The actual control input shows two distinct and brief cycles of oscillations, which are attenuated in the actual filtered control input. The simulated unfiltered control input shows a fairly close match with the actual data, but the simulated filtered control input shows otherwise. This suggests that the simulated Notch filter might be offset by a dc gain. Finally, the PI-Notch compensated system is indeed closed-loop stable as the eigenvalues are all in the left-half plane of the pole-zero map, as shown in Table 5-2.

<table>
<thead>
<tr>
<th>Poles of Notch Filter</th>
<th>Eigenvalues</th>
</tr>
</thead>
<tbody>
<tr>
<td>-20</td>
<td>$\lambda_{1,2} = -25.7 \pm 12.2$, $\lambda_{3,4} = -0.8 \pm 16$, $\lambda_{5,6} = -1.2 \pm 2.1$, $\lambda_7 = -6.2$</td>
</tr>
<tr>
<td>-16</td>
<td>$\lambda_{1,2} = -33.1 \pm 12.3$, $\lambda_{3,4} = -0.8 \pm 16$, $\lambda_{5,6} = -0.5 \pm 1.3$, $\lambda_7 = -12.8$</td>
</tr>
</tbody>
</table>

5.2 Second-order Digital Low-pass Filter

The design of a low-pass filter is examined in this section. The derivative part of the PID controller uses the differentiated error, which is contaminated with noise. This is due to the differentiation of the slightly noisy error measurements at a fairly fast rate. In order to attenuate the noise in the differentiated error, the second-order, digital low-pass filter is used. The development of the PID controller is presented in APPENDIX B.
5.2.1 Development of the Second-order Digital Low-pass Filter

The filter is designed by choosing the appropriate cutoff frequency and the filter damping ratio. Measurements with frequency components higher than the cutoff frequency are considered as noise and are attenuated. The filter damping ratio dictates the oscillatory characteristic of the filtered measurement. The second-order filter is the preferred filter because it attenuates noise twice as fast as the first-order filter. The filter in the Laplace domain is defined as $G_{LP}(s)$ in Equation (5.10), where $\omega_c$ is the cutoff frequency and $\zeta_{LP}$ is the filter damping ratio.

$$G_{LP}(s) = \frac{\omega_c^2}{s^2 + 2\zeta_{LP}\omega_c s + \omega_c^2}$$  \hspace{1cm} (5.10)

The actual differentiated error is illustrated in Figure 5.17, which contains high frequency noise. The cutoff frequency is determined by examining the Fourier transform of the differentiated error as shown in Figure 5.18. Accordingly, the cutoff frequency can be chosen between the two high frequency components, 13 Hz and 24 Hz. The former is chosen as the cutoff frequency, to ensure that most of the high frequency noise is attenuated. A standard damping ratio of 0.7 is chosen in order to maintain the form of the low frequency signal once it is reconstructed. The numerical expressions of the filter in the Laplace and discrete domains are shown in Equations (5.11) and (5.12). The filter in $z$-transform is shown in Equation (5.13).

$$G_{LP,new}(s) = \frac{6724}{s^2 + 114.8s + 6724}$$  \hspace{1cm} (5.11)

$$G_{LP}(z) = \frac{Y_{LP}(z)}{R_{LP}(z)} = \frac{\omega_c^2T^2z^2}{(1 + 2\zeta_{LP}\omega_cT + \omega_c^2T^2)z^2 - 2(1 + \zeta_{LP}\omega_cT)z + 1}$$  \hspace{1cm} (5.13)

$$\therefore y_{LP}(k) = \frac{r_{LP}(k)\omega_c^2T^2 + 2(1 + \zeta_{LP}\omega_cT)y_{LP}(k - 1) - y_{LP}(k - 2)}{(1 + 2\zeta_{LP}\omega_cT + \omega_c^2T^2)}$$

$$y_{LP}(k) = 0.24r_{LP}(k) + 1.1y_{LP,old} - 0.35y_{LP,old,old}$$  \hspace{1cm} (5.12)
**Figure 5.17** Actual differentiated error.

**Figure 5.18** Fourier transform of the differentiated error.
5.2.2 Simulations and Results of the Second-Order Low-pass Filter on PID

In this section, the actual and simulated results of the filter and the root locus analysis of the PID with the filter are presented. When the second-order low-pass filter is implemented on the differentiated error as previously shown in Figure 5.17, the filtered differentiated error is significantly less noisy, which is illustrated in Figure 5.19. There are instances where the amplitude of the filtered differentiated error is high, but not as comparably high as the unfiltered differentiated error.

![Second-order Low-pass Filter on Differentiated Error](image)

*Figure 5.19* Comparisons of actual and filtered differentiated error.

The filter adds a pair of complex poles and a real zero to the open-loop transfer function of the PID controller, which is expressed as $G_{PID,LP}(s)$ in Equation (5.14). The implementation of the filter in the continuous time domain is illustrated in the block diagram in Figure 5.20.
Figure 5.20 Block diagram of PID controller with the second-order low-pass filter.

The low-pass filter does not affect the closed-loop stability of the PID compensated system. Due to the high cutoff frequency of the filter, the root locus plots of the PID compensated system with and without the filter are almost identical. The simulations are done with $K_P$ of 2, $K_I$ of 3, and $K_D$ of 0.25. The complex eigenvalues due to the cutoff frequency are so large that they are far away from the root locus at the origin, as shown in Figure 5.21. Consequently, the root locus at the origin of the system with the filter is identical to that of the system without filter, as shown in Figure 5.22 and the close-up view in Figure 5.21. The only other difference is the root locus branches due to the complex poles diverge away and cross over to the unstable region.

Therefore, the system can become unstable if the proportional gain was extremely high.

Additionally, the root locus of the system with the filter, on the real axis, goes to the real zero that

\[
G_{PID,LP}(s) = K_P + \frac{K_D \omega_c^2 s}{s^2 + 2\zeta_{LP}\omega_c s + \omega_c^2} + \frac{K_I}{s}
\]

\[
G_{PID,LP}(s) = \frac{K_P \left[s^3 + \left(2\zeta_{LP} \omega_c + \frac{K_D}{K_P} \omega_c^2 + \frac{K_I}{K_P} \omega_c^2 \right)s^2 + \left(2\zeta_{LP} \omega_c \frac{K_I}{K_P} + \omega_c^2 \right)s + \frac{K_I}{K_P} \omega_c^2 \right]}{s(s^2 + 2\zeta_{LP} \omega_c s + \omega_c^2)}
\]

\[
\therefore G_{PID,LP}(s) = \frac{K_P \left[(s + a_{c1})(s + a_{c2})(s + a_{c3}) \right]}{s(s^2 + 2\zeta_{LP} \omega_c s + \omega_c^2)}
\]

(5.14)
is introduced by the filter. Whereas for the system without filter the root locus goes to infinity.

Their resulting closed-loop responses are the same, because the real zero is considerably large that it does not affect the root locus near the origin. The comparisons of the step responses of both the systems, as shown in Figure 5.23, verify that they are indeed the identical. In the actual responses, shown later, the signal will be contaminated with noise due to actual differentiation of the error. Ultimately, the closed-loop system is stable as shown in the response characteristics Table 5-3.

Figure 5.21 Root locus of PID compensated system with the second-order low-pass filter.
Figure 5.22 Root locus of PID compensated system without the low-pass filter.

Figure 5.23 Verification of effect of low-pass filter on the PID compensated system.
Table 5-3  Closed-loop response characteristics of PID compensated system with and without low-pass filter

<table>
<thead>
<tr>
<th>Systems</th>
<th>Rise Time (s)</th>
<th>Settling Time (s)</th>
<th>Maximum Overshoot (%)</th>
<th>Eigenvalues</th>
</tr>
</thead>
<tbody>
<tr>
<td>With Low-pass Filter</td>
<td>0.447</td>
<td>2.45</td>
<td>19.5</td>
<td>( \lambda_{1,2} = -51.5 \pm j54.5, \lambda_{3,4} = -1.1 \pm j1.5, \lambda_5 = -30.4 )</td>
</tr>
<tr>
<td>Without Low-pass Filter</td>
<td>0.472</td>
<td>2.46</td>
<td>20</td>
<td>( \lambda_{1,2} = -1.1 \pm j1.52, \lambda_5 = -25.2 )</td>
</tr>
</tbody>
</table>

The actual and simulated control inputs of the PID compensated system with and without the low-pass filter are demonstrated in Figure 5.24 and Figure 5.25, respectively. The same controller gains are used, and the symmetric trajectory profile is used with acceleration and velocity of 200 \( BITS/s^2 \) and 500 \( BITS/s \). The second-order low-pass filter prevents unwanted oscillations from being introduced to the system, specifically for PID control. The simulation with the unfiltered differentiated error shows that the resulting control input is contaminated with noise. Conversely, the simulated control input is comparably smoother when the differentiated error is filtered, which matches the actual control input. Consequently, the second-order low-pass filter ensures that the PID controller compensates the system in a stable manner.

The actual and simulated displacement results of the PID compensated system with the second-order low-pass filter are illustrated in Figure 5.26. The results are also compared with the PI control. The PID with low-pass filter showed no significant improvement over the PI control. For instance, both compensated systems have almost similar displacement responses, except during the starting and overshoot phases. Also, the PID with low-pass filter shows a slightly smoother start and has 0.5% more maximum overshoot than the PI control.
Most importantly, the oscillations at high elevation angle have been eliminated, which is observed in both systems. This is because the maximum actuator velocity is used to generate the desired trajectory. At the same time, the actual maximum actuator velocity is lower than the desired maximum trajectory velocity, therefore the actual and simulated responses are showing different trajectory slopes. For example, the desired maximum velocity is roughly 50 \textit{BITS}/s higher than the actual maximum actuator velocity, as shown in Figure 5.27. Note that the actual velocity is slightly noisy, because it is differentiated from the actual displacement, whereas the simulated velocity is integrated from the simulated acceleration.

Ultimately, the PID compensated system with the second-order low-pass filter, and trajectory generation, is demonstrated to be a reasonable alternative control, just like the PI-Notch control. The closed-loop response is stable and the overall motion is smooth with negligible overshoot and steady state error.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig524.png}
\caption{Comparisons of control inputs with filtered and unfiltered differentiated error.}
\end{figure}
**Figure 5.25** Comparisons of actual and simulated control inputs with filtered differentiated errors.

**Figure 5.26** Comparisons of actual and simulated displacement results of PID with low-pass filter and PI only, with trajectory generation.
Figure 5.27  Velocity comparisons of the actual and simulated systems with filtered differentiated errors.
CHAPTER 6. DISCUSSION AND CONCLUSION

The methods of analyzing and subsequently stabilizing the elevation motion of the antenna have been presented. First, the kinematics analysis of the slider crank mechanism of the antenna linkage have been demonstrated. The inverse and forward analytic kinematics solutions, in terms position and velocity, were obtained to relate the antenna elevation angle to the linear actuator stroke length, and vice versa. The position kinematics solution can replace the low-level command to the controller with a meaningful dimension quantity. Additionally, the mechanical limits and the linearity of the elevation motion have been. The entire operating range of the antenna in the elevation axis was within the mechanical limits, and was approximately linear. Correspondingly, the antenna linkage was kinematically stable.

Multiple versions of the equations of motion that describe the elevation motion of the antenna have been obtained and compared. The results showed that for most of the antenna operation, the lumped antenna mass can be neglected. This was due to the large value of the overall effective mass and the internal damping element of the combination of the linear actuator and the motor. Consequently, the simulated responses of the full antenna dynamics, the mass-spring-damper system, and the linear actuator only model were fairly identical. Although not entirely identical with the actual response, all system models were able to capture most of the response characteristics of the actual closed-loop response. The only minor discrepancies were the magnitude of the subsequent overshoots and the brief underdamped oscillations at higher elevation angle. This was likely due to the inaccurate representation of the full antenna dynamic, such as the masses of the flexible feedhorn link and the stiff reflector, as well as other
unaccounted nonlinear characteristics. Therefore the linear actuator only model was used with more confident in this work as the aforementioned antenna parameters were inconclusive.

The PI controller with trajectory generation was more advantageous than the independent PI controller. The maximum overshoot for the controller with trajectory generation was significantly reduced. For a small displacement, the maximum overshoot of the PI compensated system with trajectory generation was 4% less than the independent PI compensated system. For a large displacement, the average maximum overshoot of the PI compensated system with trajectory generation was less than 1%, which there was essentially no oscillations. Also, there was more control over the response time of the closed-loop system. This is because the response time of the compensated system with trajectory generation was governed by the trajectory profile, whereas the response time of the independent PI compensated system was governed by the control gains. For instance, the different trajectory profiles, as shown in Figure 4.42 and Figure 4.44, demonstrated that the system with the control gains $K_P$ of 2 and $K_I$ of 3, and the system with the control gains $K_P$ of 0.5 and $K_I$ of 3 have similar response times. As a result, the Pi control with trajectory generation indeed outperformed the independent PI control.

The PI compensated system with trajectory generation was the most preferred controller. This is because the only requirements to ensure a smooth and stable closed-loop response were an appropriate trajectory profile, an accumulated error, and any combination of stabilizing controller gains. The stability was also maintained in the presence of external disturbances. The PID compensated system with trajectory generation and a second-order low-pass filter on the differentiated error was the secondary controller. The reason is that the bigger the derivative gain, the bigger the rest of the controller gains, hence more unwanted overshoots for both small and large displacements are expected. So the derivative gain was kept relatively small, and the actual
closed-loop response of the PID compensated system was effectively similar to that of the PI compensated system. Lastly, the accuracy of the antenna model impede the understanding of the effect of the Notch filter on the antenna model. For instance, the roughly estimated full antenna model demonstrated that there was already a pair of open-loop complex zeros near the lightly damped open-loop complex poles, which suggested that the Notch filter complex zeros will have little to no effect on the open-loop complex poles. However, the actual results showed the opposite, that the oscillations at high elevation angle were attenuated by the Notch filter. Also, another tradeoff was that latency was introduced to the response due to Notch filter poles. All in all, the presented classical controllers have been extensively examined in both the root locus and step response analysis and the compensated systems were all closed-loop stable. In the end, the final controller implemented in the actual system was the PI control with trajectory generation. This same control scheme was also implemented and worked on other antenna models with slightly different dynamics.

For future works, the antenna azimuth control should be considered. The lumped antenna mass should be included because the required azimuth torque is dependent on the angle between the lumped antenna mass and the ground. When the mass is at a right angle, the azimuth motor feels almost no effective antenna mass. When the mass is normal to the ground, the maximum effective antenna mass is felt by the azimuth motor.

The kinematics chain should be extended to a two-degree-of-freedom full antenna linkage. Therefore, multiple inverse solutions will be available and the viable or the most optimal solution will be used. The kinematics solutions will provide the azimuth joint and linear actuator parameters as reference input to the controller for simultaneous or independent motion. A different trajectory scheme will be explored, which describes the desired trajectory of the
effective antenna mass directly. The corresponding desired trajectories for the linear actuator and the azimuth motor are then solved using the inverse kinematics solutions.

Multiple control approaches should be explored. Two classical controllers with trajectory generation can be used to independently control each axis. Another approach is to use full-state feedback control. Both the controllability and the observability of the system will be examined first to ensure viable use of modern control. If the system was observable, then the joint rates can be estimated. Since most modern controllers are extremely fast, the control effort can accumulate to the maximum instantly, so when designing the gains, the desired closed-loop poles should be placed such that they will have reasonable response characteristics. Besides that, a slower trajectory profile can also be used.
REFERENCES


This appendix shows the linearization of Equation (3.23), which is shown again below.

\[
F(t) = \left[ M_m + \frac{M_a R_a}{L_1^2 A_1} \right] \ddot{x}_m(t) + \left[ B_m + \frac{B_a}{L_1^2 A_1} \right] \dot{x}_m(t) + \frac{(M_a R_a - M_m L_1) g \sin(\theta_b(t))}{L_1 A_1} \dot{x}_m(t)
\]

Where \( A_1 = \frac{\cos(\theta_b(t)) \left[ C_1 + L_1 \sin(\theta_b(t)) \right] - \sin(\theta_b(t)) \left[ C_2 - L_1 \cos(\theta_b(t)) \right]}{L_R + x_m(t)} \)

This equation is simplified and becomes Equation (A1.1). The term to be linearized is \( f_3(t) \), because linearization of time derivative terms remain unchanged. The operating conditions can be determined by change of coordinates and the setting the derivatives of the changed coordinates to zero and solving for the appropriate parameters. Due to the number of unknowns in the equation, it becomes indeterminate. Therefore, the linearization is done about the equilibrium points \( \theta_{b0} \) of \( 0^\circ \) and \( x_{m0} \) of 0.0826 m with the assumption that the lumped antenna mass is at equilibrium when it is completely vertical.

\[
F(t) = f_1(t) + f_2(t) + f_3(t)
\]  
(A1.1)

The linearization of the \( f_3(t) \) term is shown using Taylor’s series expansion, which is expanded in the form as expressed in Equation (A1.2).
The solution of $f_3(t)$ is expressed in Equation (A1.3). Note that the resulting unit is in Newton.

$$f_3(x_{m0}, \theta_{b0}) = 0$$

$$\frac{\partial f_3}{\partial x_m}(x_{m0}, \theta_{b0}) (\Delta x_m) = 0$$

$$\frac{\partial f_3}{\partial \theta_b}(x_{m0}, \theta_{b0}) (\Delta \theta_b) = \left[ \frac{g(L_R + x_m)(M_mL_1 - M_aR_a)}{C_1L_1} \right] \theta_b(t)$$

$$f_3(t) = (83.95s^{-2})(M_mL_1 - M_aR_a)\theta_b(t)$$

(A1.3)

By substituting Equations (A1.3) back into Equation (A1.1), the final form of the linearized equation is defined in Equation (A1.4), which is the same as Equation (3.24). The result $(70.74m^{-2})$ is one over the product of $A_1$ and $L_1^2$.

$$F(t) = (M_m + (70.74m^{-2})M_aR_a^2)\ddot{x}_m(t) + (B_m + (70.74m^{-2})B_a)\dot{x}_m(t) + (83.95s^{-2})(M_mL_1 - M_aR_a)\theta_b(t)$$

$$+ (83.95s^{-2})M_m - \left( 713.85 \frac{1}{m^2} \right) M_aR_a \right) x_m(t)$$

(A1.4)
APPENDIX B

PID CONTROLLER

This appendix shows the development of the PID controller. The controller in the Laplace domain is defined as $G_{PID}(s)$ in Equation (A2.1), where $a_{c1}$ and $a_{c2}$ are the controller zeros.

$$G_{PID}(s) = K_P + K_D s + \frac{K_I}{s}$$

$$G_{PID}(s) = K_D \left( \frac{s^2 + \frac{K_P}{K_D} s + \frac{K_I}{K_D}}{s} \right)$$

$$G_{PID}(s) = K_D \left( \frac{s^2 + (a_{c1} + a_{c2}) s + a_{c1} a_{c2}}{s} \right) \quad (A2.1)$$

Where $K_P = K_D (a_{c1} + a_{c2})$ \& $K_I = K_D a_{c1} a_{c2}$

The controller equation in the discrete domain is shown below, it is not expanded to preserve and demonstrate the error structures that the controller gains are multiplied with. The unexpanded pulse transfer function of the controller is shown in Equation (A2.2), which is can also be expressed as a parallel combination of three pulse transfer functions in Equation (A2.3).

$$G_{PID}(z) = \frac{U_c(z)}{E_c(z)} = K_P + K_D \frac{z - 1}{Tz} + K_I \frac{Tz}{z - 1} \quad (A2.2)$$

$$\frac{U_c(z)}{E_c(z)} = \frac{U_p(z)}{E_p(z)} + \frac{U_d(z)}{E_d(z)} + \frac{U_i(z)}{E_i(z)} \quad (A2.3)$$

Where $U_c(z) = U_p(z) + U_d(z) + U_i(z)$ \& $E_c(z) = E_p(z) = E_d(z) = E_i(z)$

The proportional part of the controller is expressed in Equation (A2.4).
\[ \frac{U_P(z)}{E_p(z)} = K_P \]

\[ \therefore u_P(k) = K_P e_P(k) \quad (A2.4) \]

The derivative part of the controller is expressed in Equation (A2.5), where \( \frac{e_D(k)}{T} - x_D(k) \) is the differentiated error.

\[ \frac{U_D(z)}{E_D(z)} = \frac{U_D(z) X_D(z)}{X_D(z) E_D(z)} = K_D \frac{z - 1}{T z} \]

Where \( \frac{U_D(z)}{X_D(z)} = K_D (z - 1) \) & \( \frac{X_D(z)}{E_D(z)} = \frac{1}{T z} \)

\[ u_D(k) = K_D (x_D(k + 1) - x_D(k)) \]

\[ x_D(k + 1) = \frac{e_D(k)}{T} \]

\[ \therefore u_D(k) = K_D \left( \frac{e_D(k)}{T} - x_D(k) \right) \quad (A2.5) \]

The integral part of the controller is expressed in Equation (A2.6).

\[ \frac{U_I(z)}{E_I(z)} = \frac{U_I(z) X_I(z)}{X_I(z) E_I(z)} = K_I \frac{T z}{z - 1} \]

Where \( \frac{U_I(z)}{X_I(z)} = K_I T z \) & \( \frac{X_I(z)}{E_I(z)} = \frac{1}{z - 1} \)

\[ u_I(k) = K_I T x_I(k + 1) \]

\[ x_I(k + 1) = e_I(k) + x_I(k) \]

\[ \therefore u_I(k) = K_I T (e_I(k) + x_I(k)) \quad (A2.6) \]
By adding equations (A2.4), (A2.5), and (A2.6), the difference equation of the PID controller is expressed in Equation (A2.7), which is equivalent to the form in Equation (A2.8).

\[
 u(k) = K_P e_P(k) + K_D \left( \frac{e_D(k)}{T} - x_D(k) \right) + K_I T (e_I(k) + x_I(k)) \quad \text{(A2.7)}
\]

\[
 control_input = K_P \cdot error + K_D \cdot diff\_error + K_I \cdot T \cdot accum\_error \quad \text{(A2.8)}
\]

To demonstrate that the PID control is too robust for this work, three cases with different controller zero placements with the same unity \( K_D \) gain are shown. The first case shows one of the controller zeros at -5, between the open-loop poles of the system in Equation (3.6), and another zero at -25, left of the open-loop poles of the system. The root locus and step response plots are shown in Figure A2.1 and Figure A2.2. The step response show that the rise time is less than 0.1 seconds and the settling time is 0.6 second. The resulting gains are \( K_P \) of 30 and \( K_I \) of 125. Although the root locus indicate closed-loop stable, the gains are exceedingly high and the response is too fast that it is practically unachievable.

![Step Response](image)

Figure A2.1 Step response of PID compensated system for case 1.
For the second case, both controller zeros are placed to the left of the open-loop poles of the system, which are at -25 and -35. The root locus and step response plots are shown in Figure A2. 3 and Figure A2. 4. The resulting gains are $K_p$ of 60 and $K_i$ of 875. The response time is even faster and the gains are irrationally high. For the last case, the controller zeros are placed between the open-loop poles of the system, which are at -2 and -3. The root locus and step response plots are shown in Figure A2. 5 and Figure A2. 6. The resulting gains are $K_p$ of 5 and $K_i$ of 6. Both the response time and the gains seem more reasonable for this case. Ultimately, the PID controller is too robust that it could damage the system over time, unless the controller zeros are placed very close to the origin, and the derivative gains are kept relatively low that it has almost no effect on the control input. Subsequently, the PID compensated system response will be similar to the response of the PI compensated system.
Figure A2. 3  Step response of PID compensated system for case 2.

Figure A2. 4  Root locus of the PID compensated system for case 2.
Figure A2. 5 Step response of PID compensated system for case 3.

Figure A2. 6 Root locus of PID compensated system for case 3.