2017

Association schemes and designs in symplectic vector spaces over finite fields

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Association schemes and designs in symplectic vector spaces over finite fields

by

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A dissertation submitted to the graduate faculty

in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Major: Mathematics

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Iowa State University

Ames, Iowa

2017

DEDICATION

I would like to dedicate this thesis to my wife Tali. It is not idolization, it is love. You make me feel so loved that I can always pick myself back up and persevere.

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ACKNOWLEDGEMENTS

I would like to thank Professor Sung-Yell Song for being my mentor. He encouraged me to push myself and in turn strengthened my resilience. He advised me to follow my intellectual curiosity as well as my interest in teaching. I am honored to be your student.

I would like to thank my committee and the graduate committee for their support and encouragement. I would like to acknowledge Iowa State for funding my education. To Melanie Erickson and Ellen Olson who helped me tremendously.

I would like to thank my mentor Ulrike Genschel, her support was invaluable.

A special thanks to Mr.Barloon, Dr.Bolles, Dr.Butler and Dr.Johnston. Your dedication to your students and your passion for teaching has inspired me to become a better educator.

To my brother Scott for always being there, my sisters Lizzy and Morissa for demonstrating how to adapt to change and my mother Zita for getting a D on my sixth grade English report.

Lastly, I want to thank my friends Mike, Derek, Chassidy, Diego, Trent and Mishka. You always picked up the phone and made my time at Iowa State enjoyable.

ABSTRACT

In this dissertation, we intended to construct some q-analogue t-designs and association schemes in symplectic vector spaces over finite fields. In this process of searching for designs and association schemes, we found two new families of association schemes, both of which are families of Schurian association schemes. They are obtained from the action of finite symplectic groups or their subgroups

(i) on the sets of totally isotropic projective lines, and

(ii) on subconstituents of the generalized symplectic graphs which are defined on the sets of totally isotropic projective lines as their vertex sets.

The studies of these associations schemes are treated in Chapter 3. We describe these schemes in terms of their character tables and their fusion relations. We also present some tables to list other combinatorial objects that are associated with our association schemes.

CHAPTER 1. INTRODUCTION

In this chapter, we start by introducing the concepts of association schemes, t-designs and q-analogue t-designs. We describe the history and tools required to understand designs and schemes. We then give explicit constructions of known q-analogue t-designs. Following the constructions, we introduce the symplectic space in order to understand the $1\frac{1}{2}$-designs constructed in the symplectic vector space using totally isotropic subspaces in the literature. Some of the constructions are re-explored in Section 2.3 as well as our construction of a $\mathcal{D}$-class association schemes in Chapter 3. Finally, we end with a summary of the thesis organization.

1.1 Preliminaries

Combinatorial design theory was created in the 1920's to assist in the work of the design and analysis of statistical experiments (cf. Stinson, 2003). It is a tool used to figure out if we can arrange elements of a finite set into subsets so that it has certain "balance" properties. Since their discovery, many different types of designs have been introduced. A detailed timeline may be found in (Colbourn and Dinitz, 2006, Chapter 1.2).

Definition 1. A block design is a pair $(P, \mathcal{B})$, where

(i) P is a finite set of elements called points and

(ii) $\mathcal{B}$ is a collection of nonempty subsets of P called blocks.

If $\mathcal{B}$ contains two identical blocks, then it is denoted as a *repeated block* design. A design is *simple* if it is not a repeated block design. A point block pair (p, B) where p is a point in P and B is a block in $\mathcal{B}$ is called a *flag* if $p \in B$; otherwise, it is called an *antiflag*. We can now discuss the simplest balance property that a design can have as described by certain combinatorial and algebraic properties.

Definition 2. A *tactical configuration* also called a 1-design with parameters (v, b, k, r) is a design $(P, \mathcal{B})$, with $|P| = v$ points and $|\mathcal{B}| = b$ blocks such that each block $B \in \mathcal{B}$ contains k points from P and each point in P is contained in r-blocks.

This concept was further generalized by Fischer and Yates in the 1930's and became one of the most studied types of design because of its applications to efficient statistical experiments.

Definition 3. Let v, k and λ be positive integers such that $v > k \geq 2$. A t-design with parameters (v, k, λ) is a tactical configuration $(P, \mathcal{B})$ where $|\{B \in \mathcal{B} | T \subset B\}| = \lambda$ for each t-subset $T \subset P$.

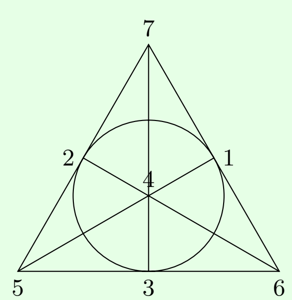
Balanced incomplete block designs also called 2-designs are the most widely used designs in applications. In 1989, Teirlinck proved the existence of ordinary t-designs in Teirlinck (1989) when the known parametric feasibility conditions are satisfied. More recently, Keevash proved the existence of a Steiner triple system $2 - (v, 3, 1)$ for feasible v in Keevash (2015). These results are significant as it justifies searching for new concrete examples of designs.

Example 1. The following is an example of a 2-(7,3,1) design also known as the *Fano plane*, where

$$P = \{1, 2, 3, 4, 5, 6, 7\}$$

$$\mathcal{B} = \{\{1, 2, 3\}, \{2, 4, 6\}, \{1, 4, 5\}, \{1, 6, 7\}, \{2, 4, 6\}, \{2, 5, 7\}, \{3, 4, 7\}, \{3, 5, 6\}\}$$

As seen in the picture below, each line contains three points.



Blocks & Points:

$$\{1, 2, 3\} \supset \{1, 2\}, \{1, 3\}, \{2, 3\}$$
$$\{1, 4, 5\} \supset \{1, 4\}, \{1, 5\}, \{4, 5\}$$
$$\{1, 6, 7\} \supset \{1, 6\}, \{1, 7\}, \{6, 7\}$$
$$\{2, 4, 6\} \supset \{2, 4\}, \{2, 6\}, \{4, 6\}$$
$$\{2, 5, 7\} \supset \{2, 5\}, \{2, 7\}, \{5, 7\}$$
$$\{3, 4, 7\} \supset \{3, 4\}, \{3, 7\}, \{4, 7\}$$
$$\{3, 5, 6\} \supset \{3, 5\}, \{3, 6\}, \{5, 6\}$$

Figure 1.1 Fano plane as 2-design

Definition 4. A design $(P, \mathcal{B})$ is called a *projective plane* if

(i) any two distinct points $p_1, p_2 \in P$ are contained in exactly one block $B \in \mathcal{B}$;

(ii) any two blocks $B_1, B_2 \in \mathcal{B}$ intersect in exactly one point, that is $|B_1 \cap B_2| = 1$;

(iii) there exist four points such that none of the four subsets of three points lie in the same block.

Definition 5. A design $(P, \mathcal{B})$ is called an *affine plane* if

(i) any two distinct points $p_1, p_2 \in P$ are contained in exactly one block $B \in \mathcal{B}$;

(ii) given any antiflag (p, B), there is precisely one flag (p, H) such that $B \cap H = \emptyset$;

(iii) there exists a triangle, that is, three points that are not contained in a common block.

Definition 6. A *divisible design* with parameters $(v, b, k, r; m, \lambda_1, \lambda_2)$ is a triple $(P, \mathcal{B}, G)$ where $(P, \mathcal{B})$ is a tactical configuration and G is a partition of P into m groups such that for any pair of distinct points $p_1, p_2 \in P$

$$|\{B \in \mathcal{B} | p_1, p_2 \in B\}| = \begin{cases} \lambda_1 & \text{if } p_1 \text{ and } p_2 \text{ belong to the same group} \\ \lambda_2 & \text{otherwise} \end{cases}$$

Definition 7. A *partial geometry* with parameters $(v, b, k, r; \tau)$ is a tactical configuration $(P, \mathcal{B})$ such that

(i) any two points are contained together in at most one block, and

(ii) every antiflag (p, B), $p \notin B$, there exists τ blocks containing p that intersect non-trivially with B.

Now, we can construct a design based on flag and antiflag pairs.

Definition 8. A *partial geometric* design with parameters $(v, b, k, r; \alpha, \beta)$ is a tactical configuration $(P, \mathcal{B})$ with parameters (v, b, k, r) such that for every point $p \in P$ and every block $B \in \mathcal{B}$, the number of flags (y, C) such that $y \in B \backslash \{p\}, C \ni p$ and $C \neq B$ is α if $p \notin B$ and β if $p \in B$.

Definition 9. Let $(P, \mathcal{B})$ be a design with $|P| = v$ and $|\mathcal{B}| = b$. The *incidence matrix* of $(P, \mathcal{B})$ is the $v \times b$, $\{0, 1\}$-matrix N whose entries are:

$$N_{i,j} = \begin{cases} 1 & \text{if } p_i \in B_j \\ 0 & \text{otherwise} \end{cases}$$

Definition 10. Supposed that $(P_1, \mathcal{B}_1)$ and $(P_2, \mathcal{B}_2)$ are two designs with $|P_1| = |P_2|$. We say that $(P_1, \mathcal{B}_1)$ and $(P_2, \mathcal{B}_2)$ are *isomorphic* if there exists a bijection $\pi : P_1 \to P_2$ such that

$$[\pi(B) : B \in \mathcal{B}_1] = \mathcal{B}_2.$$

That is, if we map every $p \in P_1$ to P_2 by $\pi(p)$, then the collection of blocks $\mathcal{B}_1$ is transformed into $\mathcal{B}_2$. The *bijection* π is called an *isomorphism*.

Definition 11. Suppose that M and N are both $v \times b$ incidence matrices of designs. Then two designs are isomorphic if and only if there exists a permutation α of $\{1, 2, \cdots, v\}$ and a permutation β of $\{1, 2, \cdots, b\}$ such that

$$M_{i,j} = N_{\alpha(i), \beta(j)}$$

1.2 q-analogue t-designs

Definition 12. A *q-analogue t-design*, denoted by $t - [n, k, \lambda; q]$-design, also referred to as a subspace design, is a collection of k-dimensional subspaces (called blocks) of the vector space $V = \mathbb{F}_q^n$ such that each t-dimensional subspace of V (point) is contained in exactly λ blocks.

If every k-dimensional subspace is selected, then it is denoted as the *trivial design*. Such designs are easily found and give no insight into the algebraic and combinatorial relationship between the blocks and points. If the same k-dimensional subspace is used more than once the design is called a *repeated block design*. When $\lambda = 1$ and $t \geq 2$, we call the designs a *Steiner System*. These designs play a special role as we would have a perfect cover of the subspaces.

Let $\mathcal{P}_t(V)$ denote the collection of all t-dimensional subspaces of the vector space $V = \mathbb{F}_q^n$. It is apparent that we need to be able to count the total number of t-subspaces, $|\mathcal{P}_t(V)|$, as well as how many points are contained in each block.

For this we turn our attention to the q-binomial coefficient also referred to as the Gaussian binomial coefficient which was introduced in Gauss (1808). This definition will be used throughout this dissertation and should be thoroughly reviewed before moving on.

Lemma 1.2.1. *Let n, k be positive integers with $k \leq n$ and let q be a prime power. Then the number of k-dimensional subspaces in the vector space $V = \mathbb{F}_q^n$ is equal to $\binom{n}{k}_q$ where,*

$$\binom{n}{k}_q = \frac{(q^n-1)(q^{n-1}-1)\cdots(q^{n-k+1}-1)}{(q^k-1)(q^{k-1}-1)\cdots(q-1)} = \frac{\prod\limits_{t=n-k+1}^{n}(q^t-1)}{\prod\limits_{t=1}^{k}(q^t-1)}$$

As shown in Rademacher (1977); Carlitz (1970) the Gaussian binomial coefficients act similar to the standard binomial coefficients as the q-binomial coefficients satisfies the recurrence relation

$$\binom{n}{k}_q = x^k\binom{n-1}{k}_q + \binom{n-1}{k-1}_q.$$

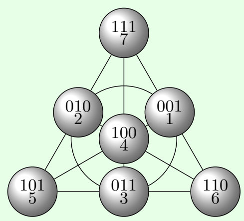
In fact,

$$\lim_{q\to 1}\binom{n}{k}_q = \binom{n}{k}.$$

Example 2. The fano plane from example 1 can also be described as a $1-[3,2,3;2]$-design as seen below. Projective plane of order 2, $PG(2,2) = 1-[3,2,3;2]$

Lastly, we will be discussing the use of subspaces of $V = \mathbb{F}_q^n$ in a graph, we will also do so later in Chapter 2. We wish to mention this here as an interesting natural construction of graphs using subspaces where the Gaussian binomial coefficients in Lemma 1.2.1 appear. The following appears in Godsil and Meagher (2015).

Definition 13. Let $V = \mathbb{F}_q^v$ a vector space of dimension v over the finite field $\mathbb{F}_q$. The *q-Kneser graph* denoted by $qK(v,k)$, where $v \geq 2k$, is the graph whose vertex set is $\mathcal{P}_k(V)$. Two vertices $X, Y \in \mathcal{P}_k(V)$ are adjacent if $\dim(X \cap Y) = 0$.



Blocks & Points:

$$[1,2,3] = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \supset [0,0,1],[0,1,0],[0,1,1]$$

$$[1,4,5] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \supset [0,0,1],[1,0,0],[1,0,1]$$

$$[1,6,7] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \supset [0,0,1],[1,1,0],[1,1,1]$$

$$[2,4,6] = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \supset [0,1,0],[1,0,0],[1,1,0]$$

$$[2,5,7] = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \supset [0,1,0],[1,0,1],[1,1,1]$$

$$[3,4,7] = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \supset [0,1,1],[1,0,0],[1,1,1]$$

$$[3,5,6] = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \supset [0,1,1],[1,0,1],[1,1,0]$$

Figure 1.2 Fano plane as q-analogue 2-design

Calculating the size of the independent set (collection of vertices such that no two are adjacent) requires Lemma 1.2.10 and calculating the eigenvalues also requires a few counting techniques. See Godsil and Meagher (2015) for more details.

The first construction of a nontrivial subspace design with $t \geq 2$ was due to Thomas (1987). This work was soon extended by Suzuki (1990a) and Suzuki (1992). The construction was found to not be extendable past $t = 2$. We will now discuss Thomas' construction followed by Suzuki's construction. For the sage code as well as the complete description of a $2 - [7, 3, 7; 2]$ please see A. Other notable constructions of nontrivial designs can be found in Schram (1989); Miyakawa et al. (1995); Itoh (1989).

1.2.1 Thomas' construction

Let $n \equiv \pm 1 \pmod 6$. Let $\mathbb{F}^* = GF(2^n)^*$, the multiplicative group of $GF(2^n)$, that is the group of all nonzero elements of $GF(2^n)$ under multiplication.

Definition 14. Thomas (1987) Let $n \equiv \pm 1 \pmod 6$ where, $PG(n-1, 2)$ is the projective space whose points are identified with elements of $\mathbb{F}^* = GF(2^n)^*$. Let γ be a primitive element of $\mathbb{F}$, we shall denote γ^b as b.

Under this notation, the set $\{0, a, b\}$ is a line if and only if $\gamma^a + \gamma^b + \gamma^0 = 0$.

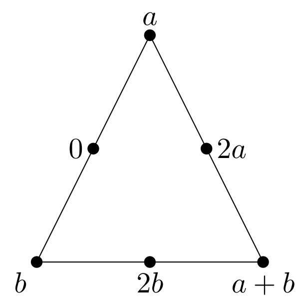


Figure 1.3 Special triangle

Example 3. Let $\{0, a, b\}$ be a line. Then $\gamma^a + \gamma^b + \gamma^0 = 0$. We claim that $\{a, 2a, a+b\}$ and $\{b, a+b, 2b\}$ lie in the same $\mathbb{F}^*$-orbit. Let a and b act on $\{0, a, b\}$ e.g.

$$a\{0, a, b\} = \gamma^a(\gamma^0, \gamma^a, \gamma^b) = (\gamma^a, \gamma^{2a}, \gamma^{a+b}) = \{a, 2a, a+b\},$$

similarly

$$b\{0, a, b\} = \gamma^b(\gamma^0, \gamma^a, \gamma^b) = (\gamma^b, \gamma^{a+b}, \gamma^{b+b}) = \{b, a+b, 2b\},$$

Now, we can construct a special triangle.

Definition 15. A *triangle* is a set $T = \{c_1, c_2, c_3\}$ of three noncollinear points. The lines $< c_i, c_j >$, $1 \leq i < j \leq 3$, are the edges of the triangle. T is said to be a *special triangle* if its edges lie in the same $\mathbb{F}^*$-orbit.

Definition 16. $\mathbb{F}^*$ is said to act *semiregular* on a set X if for any $x, y \in X$ there exists a $f \in \mathbb{F}^*$ such that $fx = y$.

Lemma 1.2.2. *Rao (1969)* $\mathbb{F}$ *acts semiregularly on the sets of lines and planes.*

Lemma 1.2.3. *(Thomas, 1987, Lemma 3) Each plane contains at most one special triangle and each line lies in seven planes.*

Definition 17. A k regular simplex is a k-dimensional regular polytope which is the convex hull of its $k+1$ vertices.

For example, a triangle is a 2-simplex while a tetrahedron is a 3-simplex.

The special triangles will be our 3-dimensional subspaces while the lines are our 2-dimensional subspaces. Under this construction we have the following design.

Theorem 1.2.4. *Thomas (1987) If* $n \equiv \pm 1 \pmod 6$ *and* $n > 5$*, then there exists a nontrivial* $2-[n,3,7;2]$ *design whose blocks are* $\mathcal{B} = \{< T >: \ T \text{ is a special triangle}\}$*, the special triangles defined from Definition 15.*

This was the first nontrivial construction of a family of designs. It is important to note the following about this construction.

Remark 1.2.5.

- When $q \neq 2$, Lemma 1.2.3 fails to hold. We are not guaranteed that each line lies on the same number of planes.
- If we try to identify our points with $k > 1$ the semiregularity as in Lemma 1.2.2 fails.
- Lastly, if we try to look at a k simplex from Definition 17 we can not guarantee a special simplex property as we can in Definition 15.

1.2.2 Suzuki's construction

Let q be a prime power. In this construction we will view the finite field $\mathbb{F}_{q^n}$ as an n-dimensional vector space $V = \mathbb{F}_q^n$ over $\mathbb{F}_q$.

Definition 18. Suzuki (1992) Let U be a 2-dimensional subspace of V. For each $r \in \mathbb{N}$, let

$$L_r(U) =< a_1 \dots a_r | a_i \in U, i = 1, 2, \cdots, r >,$$

the subspace of V generated by all product of r elements of U.

Corollary 1.2.6. *(Suzuki, 1992, Corollary 2.2) Suppose* $n \equiv \pm 1 \pmod{r!}$*, then* $\dim(L_r(U)) = r + 1$.

Theorem 1.2.7. *Suzuki (1992)[Theorem 4.1] If $n \equiv \pm 1 \mod 6$, $n \geq 7$ and q odd, then*

$$B_2 = \left\{ L_2(U) : U \in \binom{V}{2} \right\}$$

forms a nontrivial $2 - [n, 3, q^2 + q + 1; q]$ design.

Lemma 1.2.8. *(Suzuki, 1992, Lemma 4.2) Let $W \subset L_2(U)$, where $U =< x, y >$ Then, W must have one of the following three forms:*

(i) $W =< x^2, y^2 >$

(ii) $W =< x^2, xy >$

(iii) $W =< xy, x^2 + \epsilon y^2 >$, *where ϵ is a fixed nonsquared element in $\mathbb{F}_q$.*

This was a nice generalization of Thomas' constructions. It is important to note the following about this construction.

Remark 1.2.9.

- When $q = 2$ in Theorem 1.2.7 we obtain Theorem 1.2.4.
- When $r \neq 2$, Lemma 1.2.8 fails to hold, that is we are not able to identify the forms of W, so we do not know what elements are in W.

1.2.3 Existence of q-analogue t-designs

Although Thomas was able to construct a q-analogue t-design for $2 - [n, 3, 7; 2]$ in 1987, it was still unknown if they could exist for the other values of n, k and t. The existence of q-analogue t-designs was studied in Ray-Chaudhuri and Singhi (1989). In this paper, they discuss a generalized signed design that is constructed using choice functions f for the blocks and F for the points. Let $V = \mathbb{F}_q^v$, a vector space of dimension v over the finite field $\mathbb{F}_q$.

Definition 19. A *signed* $t - [v, k, \lambda]$ design in V is a function $f : \mathcal{P}_k(V) \to \mathbb{Z}$ such that for all $T \in \mathcal{P}_t(V)$,

$$\sum_{B \in \mathcal{P}_k(V), T \subset B} f(B) = \lambda$$

Such a function is called a $t-[v,k,\lambda]$ design if $f(B) \geq 0$ for all $B \in \mathcal{P}_k(V)$. In a design, we want every point to occur λ times, instead we can generalize this and ask if there is a function F such that each T occurs $F(T)$ times. We can take this a step further and say that each B such that $T \subset B$ contributes a value of $f(B)$.

Definition 20. For a function $F : \mathcal{P}_t(V) \to \mathbb{Z}$, an *integral k-realization* of F is a function $f : \mathcal{P}_k(V) \to \mathbb{Z}$ such that for all $T \in \mathcal{P}_t(V)$,

$$\sum_{B \in \mathcal{P}_k(V), T \subset B} f(B) = F(T)$$

We are not guaranteed the existence of these functions. Before we can state which conditions need to be met in order for these functions to exist and give us a design we need a few more tools to count the interactions of subspaces.

Lemma 1.2.10. *Ray-Chaudhuri and Singhi (1989); Brouwer et al. (2011) Let X be a j-space of $V = \mathbb{F}_q^v (0 < j < v)$. Let*

$$l = \max\{0, k-(v-j)\} \text{ and } m = \min\{k, j\}.$$

For $l \leq h \leq m$, define

$$S_h := \{Y \in \mathcal{P}_k(v) : \dim(Y \cap X) = h\}.$$

Then $\mathcal{P}_k(V)$ is partitioned into the subsets $S_l, S_{l+1}, \cdots, S_m$; that is,

$$\mathcal{P}_k(V) = \cup_{h=l}^{m} S_h, S_{h_1} \cap S_{h_2} = \emptyset \text{ whenever } h_1 \neq h_2, h_1, h_2 \in \{l, l+1, \cdots, m\}.$$

Also we have the following:

(a) The number of k-spaces Y in V such that $\dim(Y \cap X) = h$ is given by

$$|S_h| = q^{(k-h)(j-h)} \binom{v-j}{k-h}_q \binom{j}{h}_q$$

(b)

$$\binom{v}{k}_q = \sum_{h=1}^{m} |S_h| = \sum_{h=l}^{m} q^{(k-h)(j-h)} \binom{v-j}{k-h}_q \binom{j}{h}_q$$

Proof. If $k \leq j$ and $k \leq v-j$, then each k-space Y of V intersect with X trivially or in an h-space for $h = 1, 2, \cdots, k$. It is clear that $S_{h_1} \cap S_{h_2} = \emptyset$ if $h_1 \neq h_2$ for any $h_1, h_2 \in \{0, 1, \cdots, k\}$. So, with $l = 0$ and $m = k$, $S_l, S_{l+1}, \cdots, S_m$ form a partition of the set of all k-spaces of V. We will only be proving this case as the other cases below can be verified in a similar manner

- $k \leq j$ and $k > v-j$
- $k > j$ and $k \leq v-j$
- $k > j$ and $k > v-j$

(a) If $h = 0$, there are q^{kj} k-spaces Y in V disjoint from X corresponding to the q^{kj} canonical matrices of shape $[I_k|A_{k\times j}]$ where I_k is the $k \times k$ identity matrix and $A_{k\times j}$ is a $k \times j$ matrix with q^{kj} unspecified entries.

For h with $l \leq h \leq m$, we show that there are

$$q^{(k-h)(j-h)} \binom{v-j}{k-h}_q \binom{j}{h}_q$$

k-spaces Y in V intersecting the given j-space X in a h-space $Y \cap X$. The reason is that when considering the $(k+j-h)$-space $(Y+X)$ and the h-space $Y \cap X$ as fixed, these k-spaces correspond to a $(k-h)$-space disjoint from the $(j-h)$-space $X \backslash (Y \cap X)$ in the $(k-h+j-h)$-space $(Y+X) \backslash (Y \cap X)$, of which there are $q^{(k-h)(j-h)}$ corresponding to the canonical matrix shape $[I_{k-h}|A_{(k-h)\times(j-h)}]$.

(b) It follows immediately from the above results.

□

This count gives us our first insight into the interaction of t-subspaces and k-subspaces. In fact, $c(k, t, m, l)$ shows the difficulty in counting over a finite field. The deconstruction of the subspace into a block matrix will play an important role in Chapter 2.

Definition 21. Let $d(k,t,m,l)$ be the unique rational number defined by the equations

$$d(k,t,m,l) = 0 \text{ if } l < m$$

$$\sum_{i=m}^{l} d(k,t,m,l) \cdot c(k,t,i,l) = \delta_{m,l},$$

where $\delta_{m,l}$ is the Kronecker delta function which is defined as

$$\delta_{i,j} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise.} \end{cases}$$

Example 4. Let $k = 4, t = 3$ and $q = 2$. Then we can represent the values of $c(4,3,m,l)$ and $d(4,3,m,l)$ for $0 \leq m \leq 3$ and $0 \leq l \leq 3$ in the matrices C and D respectively. Entry $C(i,j)$ will represent $m = i$ and $l = j$.

$$C = \begin{pmatrix} 15 & 1 & 0 & 0 \\ 0 & 14 & 3 & 0 \\ 0 & 0 & 12 & 7 \\ 0 & 0 & 0 & 8 \end{pmatrix}$$

$$D = \begin{pmatrix} \frac{1}{15} & -\frac{1}{210} & \frac{1}{840} & -\frac{1}{960} \\ 0 & \frac{1}{14} & -\frac{1}{56} & \frac{1}{64} \\ 0 & 0 & \frac{1}{12} & -\frac{7}{96} \\ 0 & 0 & 0 & \frac{1}{8} \end{pmatrix}$$

Now, we can move to the main result of the paper, that is the existence of a design.

Theorem 1.2.11. *(Ray-Chaudhuri and Singhi, 1989, Theorem 2.4) Let $k \geq t$ and $v \geq k + t$. Let $V = \mathbb{F}_q^v$. Let $F : \mathcal{P}_t(V) \to \mathbb{Z}$ be a function. Then F has an integral k-realization f if and only if the following holds: For any w-space $W \leq V, 0 \leq w \leq t$ and $(v-t+w)$-space U such that $W \leq U \leq V$,*

$$\sum_{T \in \mathcal{P}_t(V), W \subseteq T} d(k-w, t-w, 0, t - dim(T \cap U))F(T)$$

is an integer.

When $F(T) = \lambda$ for all $T \in \mathcal{P}_t(V)$ we have a standard q-analogue t-design as in Definition 12.

The existence of q-analogue t-designs was taken one step further by Fazeli et al. (2014), which showed that q-analogue t-designs exist for all t. There is still an issue though as constructing the designs is a laborious task. Luckily, as in ordinary designs, we are able to extract other designs from a given q-analogue t-design. This is an important result due to the difficulty of finding a design.

Lemma 1.2.12. *(Suzuki, 1990b, Lemma 4.1(1)) Let D be a $t-[v,k,\lambda;q]$ design. For each $s \in \{0,\dots,t\}$, D is a $s-[v,k,\lambda_s;q]$ designs with*

$$\lambda_s = \lambda \frac{\binom{v-s}{t-s}_q}{\binom{k-s}{t-s}_q} = \lambda \frac{\binom{v-s}{k-s}_q}{\binom{v-t}{k-t}_q}$$

Lemma 1.2.13. *(Suzuki, 1990b, Lemma 4.2) Let D be a $t-[v,k,\lambda;q]$ design. Then the* supplementary design $D^\perp = (V, \{B^\perp | B \in \mathcal{B}\})$ *is a $t-[v, v-k, \lambda\frac{\binom{v-k}{t}_q}{\binom{k}{t}_q}; q]$ design.*

Definition 22. Let $D = (V,\mathcal{B})$ be a $t-[v,k,\lambda;q]$ design. For $U \in \binom{V}{1}_q$, the *derived* design of D in U is defined as

$$Der_U(D) = (V\backslash U, \{B\backslash U : B \in \mathcal{B}, U \subseteq B\}).$$

Definition 23. Let $D = (V,\mathcal{B})$ be a $t-[v,k,\lambda;q]$ design. For $H \in \binom{V}{v-1}_q$, the *residual* design of D in H is defined as

$$Res_H(D) = (H, \{B : B \in \mathcal{B}, B \subseteq H\}).$$

Lemma 1.2.14. *(Kiermaier and Laue, 2015, Lemma 5) Let $D = (V,\mathcal{B})$ be a $t-[v,k,\lambda;q]$ design. Then $Der_U(D)$ is a $(t-1)-[v-1,k-1,\lambda;q]$ design and $Res_H(D)$ is a $(t-1)-[v-1,k,\frac{\lambda_{t-1}-\lambda}{q^{k-t+1}};q]$ design, where λ_{t-1} is defined in Lemma 1.2.12.*

Theorem 1.2.15. *Itoh (1998) Given a 2-$[n,3,q^3(q^n-5-1)/(q-1);q]$ design for an integer $n \equiv 5 \pmod{6(q-1)}$, there exists a $2-[mn, q^3(q^n-5-1)/(q-1);q]$ design for an arbitrary integer $m \geq 3$.*

We are still left with the daunting task of finding a design. To simplify the question of finding a q-analogue t-design, we turn our attention back to Definition 9. Let us rewrite this definition in terms of subspaces.

Definition 24. The incidence matrix A of size $\binom{n}{t}_q \times \binom{n}{k}_q$ is the $\{0,1\}$ matrix whose rows are indexed by the elements in $\mathcal{P}_t(V)$ and whose columns are indexed by the elements of $\mathcal{P}_k(V)$ such that:

$$A_{i,j} = \begin{cases} 1 & \text{if } T_i \subseteq K_j \\ 0 & \text{otherwise} \end{cases}$$

Then, a $t - [n, k, \lambda; q]$ design exists if and only if there exists some vector $x \in \mathbb{F}_2^{\binom{n}{k}_q}$ such that

$$Ax = \lambda \overrightarrow{1}$$

Example 5. We now give an example of an incidence matrix from Definition 9.

Incidence matrix for $1 - [3, 2, 3; 2]$ design from Example 1.

$$A = \begin{array}{cc} & \begin{matrix} \begin{bmatrix}001\\010\end{bmatrix} & \begin{bmatrix}001\\100\end{bmatrix} & \begin{bmatrix}001\\110\end{bmatrix} & \begin{bmatrix}010\\100\end{bmatrix} & \begin{bmatrix}010\\101\end{bmatrix} & \begin{bmatrix}011\\100\end{bmatrix} & \begin{bmatrix}011\\101\end{bmatrix} \end{matrix} \\ \begin{matrix} [100] \\ [010] \\ [001] \\ [110] \\ [101] \\ [011] \\ [111] \end{matrix} & \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix} \end{array}$$

Clearly $A\overrightarrow{1} = 3\overrightarrow{1}$. Because we used every block we have the trivial $1-[3, 2, 3; 2]$ design. To see if another design exists, if $Ax = \lambda x$ for $\lambda = 1, 2$ one would have to do a brute force search over all possible $2^{\binom{3}{2}_2} - 1$ non-zero vectors $x \in \mathbb{F}_2^{\binom{3}{2}_2}$. In fact, we are solving a system of linear Diophantine equations. As n grows, treating this as a system of Diophantine equations become computationally unfeasible to solve. Instead, when $\lambda = 1$, this problem is equivalent to the exact cover problem, which has been studied in Knuth (2011).

The exact cover problem as stated in Knuth (2011) is as follows: *Given a matrix of 0's and 1's, does it have a set of rows containing exactly 1 in each column?*

In Knuth (2011), Knuth details an algorithm to solve this problem. In order to solve for a general λ, one may modify the algorithm as seen in Braun et al. (2016). We are still left with trying to reduce the number of equations in this system. In their seminal work Kramer and Mesner (1976), Kramer and Mesner noticed that they could partition this incidence matrix into smaller classes.

1.2.4 Kramer-Mesner and Singer cycles

The following definitions and results about group actions may be found in (Colbourn and Dinitz, 2006, Chapter 9).

Definition 25. Let X be a finite set and let G be a group acting on X. The *image* of $x \in X$ under $g \in G$ will be denoted by x^g. The *orbit* of x under G is $x^G = \{x^g : g \in G\}$. The orbits under G partition X. The *stabilizer* of x in G is

$$Stab_x(G) = \{g \in G : x^g = x\}.$$

The *normalizer* of A in G, $N_G(A)$ is

$$N_G(A) = \{g \in G : gAg^{-1} = A\}.$$

Proposition 1.2.16. *Let G be a group acting on a finite set X. The stabilizer $Stab_x(G)$ is a subgroup of G and*

$$|G| = |x^G||Stab_x(G)|.$$

Definition 26. A group action is *transitive* if and only if X is a single G-orbit. Thus, a group action, G acting on X, is transitive if and only if for any two elements $x, y \in X$ there exists a group element $g \in G$ such that $x = y^g$.

Definition 27. A group action G on a set S is said to be *doubly transitive* if given any $x_1, x_2, y_1, y_2 \in S$ with $x_1 \neq y_1, x_2 \neq y_2$, there exists a $g \in G$ such that $gx_1 = x_2$ and $gy_1 = y_2$. In other words g maps (x_1, y_1) to (x_2, y_2).

The following is theorem and lemma are known as the Orbit-Stabilizer theorem and Burnside's lemma respectively.

Theorem 1.2.17. *Let G be a group acting on a finite set X.Then*

$$|x^G| = \frac{|G|}{|Stab_x(G)|}.$$

Lemma 1.2.18. *Let G be a group acting on a finite set X. For $g \in G$, let X^g denote the set of $x \in X$ that are fixed by g. Then,*

$$|X/G| = \frac{1}{|G|}\sum_{g \in G}|X^g|,$$

where $|X/G|$ is the number of orbits that G creates in X.

Theorem 1.2.19. *(Kramer and Mesner, 1976, Theorem 2.1) A $t - [v, k, \lambda; q]$-design exists with $G \subseteq Aut(V)$ if and only if there exists a $\{0, 1\}$-solution vector x to the Diophantine system of equations*

$$A_G x = \lambda,$$

where A_G is the incidence matrix indexed by the orbits of the elements under G. If F is the finite field $GF(q)$ of order q, we denote this group by $GL_n(q)$.

Let V be a vector space of size n over a field F. The general linear group $GL_n(F)$ is the group of all automorphisms of V. Alternatively, it is the collection of all $n \times n$ invertible matrices.

Definition 28. A *Singer cycle* is an element of $GL_n(q)$ that generates a cyclic subgroup of order $q^n - 1$.

Definition 29. A polynomial $p(x)$ with coefficients in $GF(q)$ is a *primitive polynomial* if the $\deg(p(x)) = m$ and it has a primitive element γ as a root in $GF(q^m)$.

Definition 30. Let $p(x) = a_0 + a_1x + \cdots + a_{n-1}x^{n-1} + x^n$ be a polynomial over F. The companion matrix of $p(x)$ is defined as the square $n \times n$ matrix over F such that

$$C(p) = \begin{bmatrix} 0 & 0 & \ldots & 0 & -c_0 \\ 1 & 0 & \ldots & 0 & -c_1 \\ 0 & 1 & 0 & \ldots & -c_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & 1 & -c_{n-1} \end{bmatrix}$$

To construct a Singer cycle, one simply finds a primitive polynomial of order $q^n - 1$ and its corresponding companion matrix. A detailed description of the history and further sources on the topics of finite fields may be found in Lidl and Niederreiter (1997).

Example 6. We will construct a Singer cycle in $GL_7(2)$.

- Factor $x^{2^7-1} - 1 = x^{127} - 1$ in $GF(2)$ to find irreducible polynomials
- $x^{127} - 1 = (x+1)(x^7+x+1)(x^7+x^3+1)(x^7+x^3+x^2+x+1)(x^7+x^4+1)(x^7+x^4+x^3+x^2+1)(x^7+x^5+x^2+x+1)(x^7+x^5+x^3+x+1)(x^7+x^5+x^4+x^3+1)(x^7+x^5+x^4+x^3+x^2+x+1)(x^7+x^6+1)(x^7+x^6+x^3+x+1)(x^7+x^6+x^4+x+1)(x^7+x^6+x^4+x^2+1)(x^7+x^6+x^5+x^2+1)(x^7+x^6+x^5+x^3+x^2+x+1)(x^7+x^6+x^5+x^4+1)(x^7+x^6+x^5+x^4+x^2+x+1)(x^7+x^6+x^5+x^4+x^3+x^2+1)$
- Select $x^7 + x + 1$ as our primitive polynomial. Then, its companion matrix is a Singer cycle of order $2^7 - 1 = 127$.

$$S = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

In Itoh (1998) and Miyakawa et al. (1995) the underlying actions on the subspaces to construct the blocks from Thomas (1987) and Suzuki (1992) are shown to rely on Singer cycles. Please see Appendix A for the corresponding sage code of Thomas' construction using Singer cycles and the Kramer-Mesner matrix. In fact, in Itoh (1989), it is shown that there is no nontrivial $2-[6,3,\lambda;2]$ design admitting a Singer cycle for any λ. This brings us to the current research being done in the field.

Theorem 1.2.20. *(Braun et al., 2016, Theorem 3) Let n be an odd prime. Then the normalizer of a Singer subgroup is a maximal subgroup of $GL_n(q)$.*

Lemma 1.2.21. *(Braun et al., 2016, Lemma 4) The normalizer of A_α of a Singer subgroup is self-normalizing in $GL_n(q)$.*

This theorem and lemma tell us that we want to select subgroups of the normalizer of a Singer cycle as the group in the Kramer-Mesner method from Theorem 1.2.19. As noted in Braun et al. (2016), we are still required to perform an intense computer search to find these designs. A more detailed overview of the computational aspects of the problem may be found in Braun (2010); however, these approaches are out of the scope of the author. For further software to construct q-analogue t-designs please see Betten (2013) and Braun (2004).

Below is Thomas' construction using the normalizer of the Singer cycle S from Example 6.

Example 7. Let $n=7, k=3, t=2, q=2$. Let S be the Singer cycle found by taking the companion matrix of the irreducible polynomial x^7+x+1 in $GF(2)$. Let G be the normalizer of the Singer cycle by Theorem 1.2.19 a $2-[7,3,7;2]$-design exists if there is a $\{0,1\}$-solution vector x to the Diophantine system of equations $A_G x^t = 7$. The newly formed incidence matrix partitions the 2-dimensional subspaces into three orbits and the 3-dimensional subspaces in fifteen orbits. The explicit subspaces in the orbits can be found in A.

$$A_G = \begin{pmatrix} 5 & 3 & 2 & 2 & 3 & 1 & 2 & 3 & 2 & 3 & 1 & 1 & 2 & 1 & 0 \\ 1 & 3 & 3 & 2 & 1 & 0 & 3 & 2 & 3 & 2 & 2 & 5 & 2 & 1 & 1 \\ 1 & 1 & 2 & 3 & 3 & 0 & 2 & 2 & 2 & 2 & 4 & 1 & 3 & 5 & 0 \end{pmatrix}$$

The vector $x = [0,0,1,1,0,0,0,1,0,0,0,0,0,0,0]$ gives us $A_G x^t = 7$. Notice that there are other vectors which can lead to this design and we can also find a design with $\lambda = 5$ using $x = [0,0,0,0,0,0,0,0,0,1,0,0,1,0,1]$.

1.3 Connection between association schemes and q-analogue t-designs

In this section we establish the connection between association schemes and q-analogue t-designs. A q-analogue t-design asks about the relationship between t-spaces and k-spaces. By studying these relationships we gain a deeper understanding of the underlying geometry in $\mathbb{F}_q^n$. Specifically, when looking for partial geometric designs as described in Definition 7, where the blocks are k-spaces and the points are t-spaces, we want to know the number of blocks containing two points. The way that these spaces interact will dictate the number of flags and anti-flags. We have seen the relationship of t and k-spaces in Lemma 1.2.10 which allowed us to count the number of subspaces that intersect with a given fixed subspace. Partial geometric designs became of great interest when Brouwer et al. (2012) was able to construct directed strongly regular graphs from a given partial geometric design. Nowak et al. (2016) was able to show that a three-class association scheme contains a partial geometric design. The search for three-class association schemes, partial geometric designs and their derived strongly regular graphs was now formed. Recently, in Chai et al. (2015) and Feng et al. (2016) new graphs were created by studying the symplectic, orthogonal and unitary geometries. In Chai et al. (2015) they create a graph whose vertices are symplectic subspaces satisfying a special bilinear form, furthermore vertices are partitioned into classes based on their distance from a fixed vertex in the graph. Similar relationships have been studied in Rieck (2005), Guo (2010), Gao and He (2013a) and J. Guo and Li (2009). The construction of association schemes in these papers require a strong understanding of the relationships of subspaces and matrix forms. Some simpler association schemes formed from matrices can be found in Wang et al. (2011).

It is from these relationships that we will explore the symplectic geometry and look for new association schemes that may form directly strongly regular graphs in Chapter 3.

1.4 Association schemes

Definition 31. Let X be an n-element set, and let $R_0, R_1, \cdots, R_{\mathcal{D}}$ be subsets of $X \times X := \{(x, y) : x, y \in X\}$ with $R_0 = \{(x, x) : x \in X\}$. The pair $\chi = (X, \{R_i\}_{0 \leq i \leq \mathcal{D}})$ is a *$\mathcal{D}$-class association scheme* if

(i) $R_0 \cup R_1 \cup \cdots \cup R_{\mathcal{D}} = X \times X$

$$R_i \cap R_j = \emptyset \text{ for all } i, j \in \{0, 1, \cdots, \mathcal{D}\}$$

(ii) $R_i^t := \{(y, x) : (x, y) \in R_i\}$ is $R_{i'}$ for some $i' \in \{0, 1, \cdots, \mathcal{D}\}$.

(iii) for any $h, i, j \in \{0, 1, \cdots, \mathcal{D}\}$, there exists a constant p_{ij}^h such that for any $(x, y) \in R_h$,

$$|\{x \in X : (x, z) \in R_i, (z, y) \in R_j\}| = p_{ij}^h$$

An association scheme is $\chi = (X, \{R_i\}_{0 \leq i \leq \mathcal{D}})$ is *commutative* if

$$p_{ij}^h = p_{ji}^h \text{ for all } i, j \in \{0, 1, \cdots, \mathcal{D}\}$$

It is *symmetric* if $R_i^t = R_i$ for all $i \in \{0, 1, \cdots, \mathcal{D}\}$.

Definition 32. Let A_i be the $n \times n$ $\{0, 1\}$- matrix representing R_i: i.e.,

$$(A_i)_{xy} = \begin{cases} 1 & \text{if } (x, y) \in R_i \\ 0 & \text{otherwise} \end{cases}$$

The pair $\chi = (X, \{R_i\}_{0 \leq i \leq \mathcal{D}})$ is and association scheme if $A_0, A_1, \cdots, A_{\mathcal{D}}$ satisfies the following requirement:

(i) $A_0 + A_1 + \cdots + A_{\mathcal{D}} = J$, where J is the all-ones matrix and $A_0 = I$ is the identity matrix

(ii) $A_i^t \in \{A_0, A_1, \cdots, A_{\mathcal{D}}\}$ for every $i \in \{0, \cdots, \mathcal{D}\}$

(iii) for any $h, i, j \in \{0, \cdots, \mathcal{D}\}$, there exists a constant p_{ij}^h such that

$$A_i A_j = \sum_{h=0}^{\mathcal{D}} p_{ij}^h A_h$$

Now, χ is *commutative* if $A_iA_j = A_jA_i$ for all $i, j \in \{0, 1, \cdots, \mathcal{D}\}$. χ is *symmetric* if $A_i^t = A_i$ for all $i \in \{0, 1, \cdots, \mathcal{D}\}$.

The matrices $A_0, A_1, \cdots, A_{\mathcal{D}}$ defined above are called the *adjacency matrices* of $\mathcal{X}$ and the graphs $(X, R_1), (X, R_2), \cdots, (X, R_{\mathcal{D}})$ are called the relation graphs of $\mathcal{X}$. The constants p_{ij}^h are called the *intersection numbers* of $\mathcal{X}$.

Let $B_i, i \in \{0, \cdots, \mathcal{D}\}$, be the ith *intersection matrix* defined by

$$(B_i)_{jh} = p_{ij}^h.$$

Then, $B_iB_j = \displaystyle\sum_{h=0}^{\mathcal{D}} p_{ij}^h B_h$.

Let $\mathcal{X} = (X, \{R_i\}_{0 \leq i \leq \mathcal{D}})$ be an association scheme with its adjacency matrices $A_0, \cdots, A_{\mathcal{D}}$ and intersection matrices $B_0, \cdots, B_{\mathcal{D}}$. Then the $\mathbb{C}$-space with basis $\{A_0, \cdots, A_{\mathcal{D}}\}$ is an algebra over the complex numbers called the *Bose-Mesner* algebra of $\mathcal{X}$, denoted by $\mathcal{A}(\mathcal{X})$ or $\langle A_0 \cdots A_{\mathcal{D}} \rangle$. The $\mathbb{C}$-algebra generated by $\{B_0, \cdots, B_{\mathcal{D}}\}$ is called the *intersection algebra* of $\mathcal{X}$. The Bose-Mesner algebra $\mathcal{A}(\mathcal{X})$ and the intersection algebra $\langle B_0, \cdots, B_{\mathcal{D}} \rangle$ are isomorphic $\mathbb{C}$-algebras induced by the correspondence $A_i \to B_i$.

Definition 33. Let $\Gamma = (X, R)$ denote a connected graph with diameter $\mathcal{D}$. Γ is said to be a *distance-regular* graph whenever for all integers i $(i \leq 0 \leq \mathcal{D})$ and for all $x, y \in X$ at distance $\partial(x, y) = i$, the scalars

$$c_i := |\{z \in X : \partial(x, z) = i - 1, \partial(y, z) = 1\}|,$$

$$a_i := |\{z \in X : \partial(x, z) = i, \partial(y, z) = 1\}|,$$

and

$$b_i := |\{z \in X : \partial(x, z) = i + 1, \partial(y, z) = 1\}|$$

are constant and independent of x and y.

Example 8. The Petersen graph is a distance regular graph.

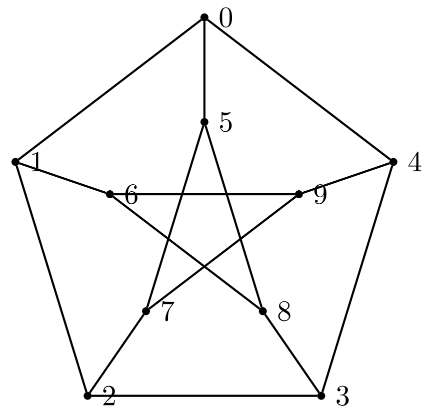


Figure 1.4 Petersen graph

We can now give an example of a known association scheme.

Example 9. Let $\Gamma = (X, R)$ denote a distance-regular graph with diameter $\mathcal{D}$, and define

$$R_i := \{(x, y) : (x, y) \in X, \partial(x, y) = i\}, \ (0 \leq i \leq \mathcal{D}).$$

Then, $Y_\Gamma = (X, \{R_i\}_{0 \leq i \leq \mathcal{D}})$ is a symmetric association scheme.

Using the Petersen graph as Γ, it has diameter $\mathcal{D} = 2$, so we have a 2-class association scheme, $(V(\Gamma), \{R_0, R_1, R_2\})$ with relations

$$R_0 = \{(0,0), (1,1), (2,2), (3,3), (4,4), (5,5), (6,6), (7,7), (8,8), (9,9)\}$$

$$R_1 = \{(0,1), (0,4), (0,5), (1,0), (1,2), (1,6), (2,1), (2,3), (2,7), (3,2), (3,4), (3,8), (4,0), (4,3), (4,9)\}$$

$$\cup\{(5,0), (5,7), (5,8), (6,1), (6,8), (6,9), (7,2), (7,5), (7,9), (8,3), (8,5)\}$$

$$\cup\{(8,6), (9,4), (9,6), (9,7)\}$$

$$R_2 = V(\Gamma) \times V(\Gamma) - R_0 - R_1$$

The intersection matrices of this 2-class association are

$$B_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, B_1 = \begin{bmatrix} 0 & 1 & 0 \\ 3 & 0 & 1 \\ 0 & 2 & 2 \end{bmatrix} \text{ and } B_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 2 \\ 6 & 4 & 3 \end{bmatrix}$$

It is well-known that for a given transitive permutation group G acting on a set Ω, the G-orbits on $\Omega \times \Omega = \{(a,b) : a \in \Omega\}$ constitute an association scheme.

Example 10. Let a finite group G act on a finite set X transitively. Then G acts naturally on $X \times X$ by $(x,y)^g = (x^g, y^g)$. Let $R_0, R_1, \cdots, R_{\mathcal{D}}$ be the orbits of G on $X \times X$, with $R_0 = \{(x,x) : x \in X\}$. Then $\chi = (X, \{R_i\}_{0 \le i \le \mathcal{D}})$ is an association scheme, known as a *group-case* or (*Schurian*) association scheme, and denoted $\chi(G,X)$ in what follows.

Example 11. Let $\chi = (X, \{R_i\}_{0 \le i \le 2})$ be a symmetric association scheme of class 2. Then the first relation graph $\Gamma_1(X, R_1)$ becomes the strongly regular graph with parameters (n, k, λ, ν) given by $n = |X|, k = k_1, \lambda = p^1_{11}, \nu = p^2_{11}$. In this case, we have

$$B_1 = \begin{bmatrix} 0 & 1 & 0 \\ k & \lambda & \nu \\ 0 & k-\lambda-1 & k-\nu \end{bmatrix}, P = \begin{pmatrix} 1 & k & n-k-1 \\ 1 & r & t \\ 1 & s & u \end{pmatrix} \begin{matrix} 1 \\ m_1 \\ m_2 \end{matrix},$$

where $r = -t-1 = \frac{1}{2}(\lambda - \nu + \sqrt{D}), s = -u-1-\frac{1}{2}(\lambda-\nu-\sqrt{D}), m_1 = \frac{1}{2}(n-1-(2k+(n-1)(\lambda-\nu))/\sqrt{D}), m_2 = n - m_1 - 1, D = (\lambda-\nu)^2 + 4(k-\nu)$.

An association scheme on Ω obtained by a transitive action of a group G on Ω is called *Schurian*. For each $x \in \Omega$ and $g \in G$, we will use xg instead of x^g in what follows.

It is a well-known fact that for a transitive permutation group G on Ω, if we let $R_0, R_1, \cdots, R_{\mathcal{D}}$ be the orbits of G on $\Omega \times \Omega$, then the orbits of $Stab_x(G)$ acting on Ω are precisely $R_0(x), R_1(x), \cdots, R_{\mathcal{D}}(x)$ where

$$R_i(x) = \{y \in \Omega : (x,y) \in R_i\}.$$

In this setting, the orbits $R_0(x), R_1(x), \cdots, R_{\mathcal{D}}(x)$ of $Stab_x(G)$ are called the suborbits of G on Ω, and the common number $\mathcal{D}+1$ of suborbits and of G-orbits in $\Omega \times \Omega$ is called the rank of the permutation group G on Ω.

In what follows, the pair G acting on Ω is denoted by (G, Ω). We insist that R_0 is the diagonal $\{(x,x) : x \in \Omega\}$ and so $R_0(x) = \{x\}$, the trivial suborbit. It holds that $R_i(x)g = R_i(xg)$ for all $g \in G,\ 0 \le i \le \mathcal{D}$. The size $k_i := |R_i(x)|$ are called the subdegrees of (G, Ω). Note that the numbers k_i are independent of the choice of x. Since $R_i^t := \{(x, y) : (y, x) \in R_i\}$ is also an orbit of G, there exists some j such that $R_i^t = R_j$. If $R_i^t = R_i$, we say that R_i is self-paired. The transitive permutation group (G, Ω) is called *generously transitive* if all R_i are self-paired; i.e., for each i, and for each $(x, y) \in R_i$, there exists an element $g \in G$ such that $xg = y$ and $yg = x$. With the G-orbits $R_0, R_1, \cdots, R_{\mathcal{D}}$ in $\Omega \times \Omega$, it is shown that $(\Omega, \{R_i\}_{0 \le i \le \mathcal{D}})$ becomes an association scheme. We denote this scheme by $\chi(G, \Omega)$ in what follows. Schurian association schemes are Association schemes obtained from the action of groups on sets. Schurian association schemes have a rich connect to algebra. When a finite group G acts transitively on a finite set Ω, there is a natural one-to-one correspondence between the set Ω and the set $Stab_x(G)\backslash G$ of cosets of a point stabilizer $Stab_x(G)$ for any $x \in G$. Thus the action of G on Ω is identical to that of the action of G on the set of cosets. There is also a one-to-one correspondence between any two of the three sets, the set of 2-orbits (on orbitals) of the permutation group (G, Ω), the set of suborbits of (G, Ω) and the set of double cosets $\{Hg_iH : 0 \le i \le \mathcal{D}\}$ of $H = Stab_x(G)$ in G. This means in terms of algebra, the Bose-Mesner (adjacency) algebra, the centralizer algebra (Hecke algebra) of (G, Ω), and the double coset algebra, the algebra spanned by $T_i := \dfrac{1}{|H|} \displaystyle\sum_{g \in Hg_iH} g$ for $i = 0, 1, \cdots, \mathcal{D}$, are all identical.

Let $\chi = (X, \{R_i\}_{0 \le i \le \mathcal{D}})$ be a symmetric $\mathcal{D}$-class association scheme of order $|X| = n$ with adjacency matrices $A_0, A_1, \cdots, A_{\mathcal{D}}$. Since $A_0, A_1, \cdots, A_{\mathcal{D}}$ pairwise commute, they are simultaneously diagonalizable, thus we have a decomposition $\mathbb{C}^n = V_0 \oplus V_1 \ldots V_{\mathcal{D}}$, where each V_i is a common eigenspace for the matrices A_i. We may suppose that V_0 is the 1-dimensional eigenspace corresponding to the eigenvalue n. Let $E_0 = \frac{1}{n}J, E_1, \cdots, E_{\mathcal{D}}$ denote the primitive idempotents in $\mathcal{A}(\chi)$. The dimension m_j of V_j are called the *multiplicities* of the scheme. Note that

$$m_j = \mathrm{rank}(E_j) = \mathrm{trace}(E_j), \text{ and } E_0 + E_1 + \cdots + E_{\mathcal{D}} = I.$$

Then there are $p_i(j), q_i(j) \in \mathbb{C}$ for all $i, j \in \{0, 1, \cdots, \mathcal{D}\}$ such that

$$A_j = \sum_{i=0}^{\mathcal{D}} p_j(i) E_i \text{ and } E_i = \frac{1}{n} \sum_{j=0}^{\mathcal{D}} q_i(j) A_j.$$

The $(\mathcal{D}+1)\times(\mathcal{D}+1)$ matrices P and Q whose (i,j)-entries are defined by

$$P_{ij} = p_j(i) \text{ and } Q_{ij} = q_j(i)$$

are called the *1st eigenmatrix* and *2nd eigenmatrix* of χ, respectively. The first eigenmatrix of is often called the *character table* of the association scheme. The number $p_j(i)$ is characterized by the relation $A_jE_i = p_j(i)E_i$. Since the A_j are integral matrices, the $p_j(i)$ are algebraic integers. Thus, the $p_j(i)$ are integers if all of the multiplicities $m_j = \text{rank}(E_j)$ are distinct. The numbers q_{ij}^h which satisfy $E_i \circ E_j = (1/|X|)\sum_{h=0}^{\mathcal{D}} q_{ij}^h E_h$ are called the *Krein parameters* of the scheme. Here we use the symbol $\circ$ for Hadamard multiplication, $(A\circ B)_{ij} = (A)_{i,j}(B)_{i,j}$. The Krein parameters are known to be nonnegative real numbers.

1.4.1 Identities of parameters

$p_0(i) = q_0(i) = 1$ holds from $A_0 = I$ and $E_0 = n^{-1}J$. $k_j = p_j(0)$ and $m_j = p_j(0)$ follow from $A_iJ = k_jJ$ and $\text{trace}(E_j) = m_j$. Also $p_i(h)p_j(h) = \sum_{l=0}^{\mathcal{D}} p_{ij}^l p_l(h)$ follows from $A_iA_j = \sum_{l=0}^{\mathcal{D}} p_{ij}^l A_l$. Now we have the following identities between the intersection numbers and the eigenvalues of the scheme. (Here and in what follows, $\overline{a}$ denotes the complex conjugate of a, and $[\mathcal{D}]$ denotes $\{0,1,\cdots,\mathcal{D}\}$.)

- Row Orthogonality

$$\sum_{j=0}^{\mathcal{D}} \frac{1}{k_j} p_j(i_1)\overline{p_j(i_2)} = \delta_{i_1i_2}\frac{n}{m_{i_1}}, \quad i_1, i_2 \in [\mathcal{D}] \tag{1.1}$$

- Column Orthogonality

$$\sum_{i=0}^{\mathcal{D}} m_i p_{j_1}(i)\overline{p_{j_2}(i)} = \delta_{j_1j_2} n k_{j_1}, \quad j_1, j_2 \in [\mathcal{D}] \tag{1.2}$$

The multiplicities, the Krein parameters, and the intersection numbers are calculated from the character table as in the following formulas:

$$m_i = n\left(\sum_{j=0}^{\mathcal{D}} \frac{|p_j(i)|^2}{k_j}\right)^{-1}, \quad i \in [\mathcal{D}] \tag{1.3}$$

$$q_{ij}^h = \frac{m_i m_j}{n} \sum_{v=0}^{\mathcal{D}} p_v(i) p_v(j) \overline{p_v(h)} k_v^{-2} \tag{1.4}$$

$$p_{ij}^h = \frac{1}{nk_h} \sum_{v=0}^{\mathcal{D}} p_i(v) p_j(v) \overline{p_h(v)} m_v, \quad h, i, j \in [\mathcal{D}] \tag{1.5}$$

$$q_j(i) = k_i^{-1} m_j \overline{p_i(j)} \quad \text{for } i, j \in [\mathcal{D}] \tag{1.6}$$

The first two relations are called the *orthogonality relations* of the character table. They are obtained from the matrix identities $PQ = nI$ and $QP = nI$ by writing the identities entry wise in terms of n, k_i, m_j and $p_j(i)$ repeatedly using the last relation (1.6). (1.6) comes from the relation $E_j = \frac{1}{n} \sum_{l=0}^{\mathcal{D}} q_j(l) A_l$. Namely, multiplying both sides by A_i under entry wise product, we have $E_j \circ A_i = n^{-1} q_j(i) A_i$. Then the sum of all entries $E_j \circ A_i$ equals to the trace of $E_j A_{i'}$ and thus $m_j \overline{p_i(j)}$ while the sum of entries on the right becomes $q_j(i) k_i$. Similarly, (1.5) is obtained from the identity $p_{ij}^h A_h = (A_i A_j) \circ A_h$ by comparing their sums of entries, and (1.4) is coming from the equality $\frac{1}{n} q_{ij}^h E_h = (E_i \circ E_j) E_h$ by observing their traces. Now (1.3) follows from (1.1).

Furthermore, $p_i(h) p_j(h) = \sum_{l=0}^{\mathcal{D}} p_{ij}^l p_l(h)$ may be expressed as

$$B_i P' = P' \operatorname{diag}[p_i(0), p_i(1), \cdots, p_i(\mathcal{D})] \tag{1.7}$$

or by

$$P B_i' P^{-1} = \operatorname{diag} \operatorname{diag}[p_i(0), p_i(1), \cdots, p_i(\mathcal{D})].$$

Consequently if we multiply (1.7) by Q' from the right and left we get

$$Q' B_i = \operatorname{diag}[p_i(0), p_i(1), \cdots, p_i(\mathcal{D})] Q'. \tag{1.8}$$

So the l^{th} column vector of P' and the l^{th} row vector of Q' are recognized as a right eigenvector and a left eigenvector of B_i belonging to the eigenvalue $p_i(l)$, respectively.

1.4.2 Fusion schemes

By using fusion and fission techniques to the character tables of given commutative association schemes, we can sometimes construct new commutative association schemes called fusion schemes or fission schemes. It is interesting to observe the nature of many such new schemes, especially, which are coming from the group schemes of finite simple groups. In this section we review some of the facts which we will need in the rest of our dissertation.

Definition 34. Let $\chi = (X, \{R_i\}_{0 \leq i \leq \mathcal{D}})$ and $\tilde{\chi} = (X, \{\tilde{R}_\alpha\}_{0 \leq \alpha \leq e})$ be (commutative) schemes defined on X. If for each $i \in [\mathcal{D}]$, $R_i \subset \tilde{R}_\alpha$ for some $\alpha \in [e]$, then we say that $\tilde{\chi}$ is a *fusion* scheme of χ and χ is a *fission* scheme of $\tilde{\chi}$. Clearly, $e \leq \mathcal{D}$ as we are merging some of the $\mathcal{D}$ classes together from χ.

For the notation, we will denote all the symbols belonging to $\tilde{\chi}$ by a $\tilde{()}$ places over the symbols, such as, $\tilde{p}^{\gamma}_{\alpha\beta}, \tilde{m}_i, \tilde{P}$, etc., whenever we need to distinguish them from those belonging to χ. The following two criteria for a fusion will be used throughout when we construct one.

Lemma 1.4.1. *Bannai and Song (1993) For a given scheme $\chi = (X, \{R_i\}_{0 \leq i \leq \mathcal{D}})$ and a partition $\Lambda = \{\Lambda_0 = \{0\}, \Lambda_1, \Lambda_2, \ldots, \Lambda_e\}$ of $[\mathcal{D}]$, $\tilde{\chi} = (X, \{\tilde{R}_\alpha\}_{0 \leq \alpha \leq e})$ becomes a scheme with relations defined by*

$$\tilde{R}_\alpha = \bigcup_{i \in \Lambda_\alpha} R_i, \text{ for each } \alpha \in [e] \text{ if and only if}$$

(i) $\tilde{R}'_\alpha = \bigcup_{i \in \Lambda_\alpha} R'_i = \bigcup_{j \in \Lambda_{\alpha'}} R_j = \tilde{R}_{\alpha'}$ for some $\alpha' \in [e]$, and

(ii) for any $\alpha, \beta, \gamma \in [e]$, and any $h, l \in \Lambda_\gamma$,

Where the intersection numbers of the fusion scheme for any $\alpha, \beta, \gamma \in [e]$ and any $h, k \in \Lambda_\gamma$ in $\tilde{\mathcal{R}}$ are

$$p^{\tilde{\gamma}}_{\alpha\beta} = \sum_{i \in \Lambda_\alpha} \sum_{j \in \Lambda_\beta} p^h_{ij} = \sum_{i \in \Lambda_\alpha} \sum_{j \in \Lambda_\beta} p^l_{ij}$$

Bannai (1991); Muzichuk (1988) Let $\chi = (X, \{R_i\}_{0 \leq i \leq \mathcal{D}})$ be a scheme, and $\Lambda = \{\Lambda_\alpha\}_{0 \leq \alpha \leq e}$ be a partition of $[\mathcal{D}]$ such that $\Lambda_0 = \{0\}$. Suppose for every $\alpha \in [e]$, $\bigcup_{i \in \Lambda_\alpha} R_{i'} = \bigcup_{j \in \Lambda_{\alpha'}} R_j$ for some

$\alpha' \in [e]$. Then Λ gives rise to a fusion scheme $\tilde{\chi} = (X, \{\tilde{R}_\alpha\}_{0\le\alpha\le e})$ with $\tilde{R}_\alpha = \bigcup_{i\in\Lambda_\alpha} R_i$ if and only if there exists a partition $\Lambda^* = \{\Lambda^*_\alpha\}_{0\le\alpha\le e}$ of $[\mathcal{D}]$ with $\Lambda^*_0 = \{0\}$ such that each $(\Lambda^*_\beta, \Lambda_\alpha)$ block of the character table P of χ has a constant row sum. The constant row sum $\displaystyle\sum_{j\in\Lambda_\alpha} p_j(i)$ for $i \in \Lambda^*_\beta$ of the block $(\Lambda^*_\beta, \Lambda_\alpha)$ is the (β, α)-entry $\tilde{p}_\alpha(\beta)$ of the fusion character table $\tilde{P}$. Consequently, if we know the character table of P of χ and the fusion partition Λ (and Λ^*) of $\tilde{(\chi)}$, we can determine the character table $\tilde{P}$ of $\tilde{\chi}$. Namely, for the given P, first we combine the corresponding columns in each part of the partition Λ, and then from the resulting $\mathcal{D}+1$ by $e+1$ table, delete all the identical rows except one for each part in (Λ^*), then the resulting $e+1$ by $e+1$ table becomes the character table $\tilde{P}$.

By Bannai (1991); Muzichuk (1988), one can always determine the character table of the fusion scheme from that of a given scheme if the fusion partition Λ and Λ^* are known. In some cases, from an investigation of the given character table of a scheme, one can even find a fusion character table without know the fusion pattern in advance. However, the calculation of the character table of a fission scheme for a given scheme is very complicated. Of course, having a character table does not necessarily imply that there always exists a scheme which realizes the table, either. The rest of this section will be dedicated to the discussion of determining the character table of a fission scheme from the given character table of a scheme.

Let $\chi = (X, \{R_i\}_{0\le i\le\mathcal{D}})$ and $\tilde{\chi} = (X, \{\tilde{R}_\alpha\}_{0\le\alpha\le e})$ be (commutative) schemes defined on X. Suppose χ is a fission of $\tilde{\chi}$ with $R_i = \tilde{R}_\alpha$ for $i = 0, 1, \cdots, e-1$, $\displaystyle\bigcup_{i=e}^{\mathcal{D}} R_i = \tilde{R}_e$, and $\displaystyle\sum_{i=e}^{\mathcal{D}} m_i = \tilde{m}_e$. Then, by Bannai (1991); Muzichuk (1988) and the orthogonality relations (1.1) and (1.2), we have the following equations which are useful to determine the fission character table P. For notational simplicity, $\tilde{k}_e, \tilde{m}_e$ and $\tilde{p}_e(e)$ are denoted by k, m and p respectively.

$$p_j(i) = \begin{cases} \tilde{p}_i(i) & \text{if } 0 \le i \le e-1, 0 \le j \le e-1 \\ \tilde{p}_j(e) & \text{if } e \le i \le \mathcal{D}, 0 \le j \le e-1 \\ \frac{k_j}{k}\tilde{p}_e(i) & \text{if } 0 \le i \le e-1, e \le j \le \mathcal{D} \end{cases}$$

For every $e \leq s \leq \mathcal{D}$ and $e \leq u \leq \mathcal{D}$,

$$\sum_{t=e}^{\mathcal{D}} p_t(s) = p$$

$$\sum_{t=e}^{\mathcal{D}} p_t(s)\overline{p_t(s)}k_t^{-1} = \frac{p^2}{k} - \frac{n}{m} + \frac{n}{m_s}$$

$$\sum_{t=e}^{\mathcal{D}} p_t(s)\overline{p_t(u)}k_t^{-1} = \frac{p^2}{k} - \frac{n}{m} \quad \text{if } s \neq e$$

For every $e \leq t \leq \mathcal{D}$ and $e \leq v \leq \mathcal{D}$,

$$\sum_{s=e}^{\mathcal{D}} m_s p_t(s) = \frac{k_t}{k} mp$$

$$\sum_{s=e}^{\mathcal{D}} m_s p_t(s)\overline{p_t(s)} = nk_t + \left(\frac{k_t}{k}\right)^2 (mp^2 - nk)$$

$$\sum_{s=e}^{\mathcal{D}} m_s p_t(s)\overline{p_v(s)} = \frac{k_t k_v}{k^2}(mp^2 - nk), \quad \text{if } t \neq v.$$

These equations are not enough to determine the fission table if $d - e \geq 2$.

Lemma 1.4.2. *(Bannai and Song, 1993) Let $\chi = (X, \{R_i\}_{0\leq i\leq 2})$ be a symmetric association scheme of class 2 with character table*

$$P = \begin{array}{c} \begin{pmatrix} 1 & k_1 & k_2 \\ 1 & r & t \\ 1 & s & u \end{pmatrix} \end{array} \begin{array}{c} 1 \\ m_1 \\ m_2 \end{array}.$$

Suppose $\hat{\chi} = (X, \{\hat{R}_i\}_{0\leq i\leq 3})$ is a non-symmetric fission scheme of χ with three classes such that $\hat{R}_2 = \hat{R}_1{}'$, $\hat{R}_1 \cup \hat{R}_2 = R_1$, $\hat{R}_3 = r_2$, $\hat{E}_1 \cup \hat{E}_2 = E_1$, $\hat{E}_3 = E_2$. Then the character table $\hat{P}$ of $\hat{\chi}$ is given by

$$\hat{P} = \begin{pmatrix} 1 & \frac{1}{2}k_1 & \frac{1}{2}k_1 & k_2 \\ 1 & \rho & \overline{\rho} & t \\ 1 & \overline{\rho} & \rho & t \\ 1 & \frac{1}{2}s & \frac{1}{2}s & u \end{pmatrix} \begin{array}{c} 1 \\ \frac{1}{2}m_1 = \hat{m}_1 \\ \frac{1}{2}m_1 = \hat{m}_2 \\ m_2 = \hat{m}_3 \end{array},$$

where $\rho = \frac{1}{2}(r + \frac{\sqrt{-nk_1}}{\sqrt{m_1}})$

1.5 Overview

This dissertation is organized in the format of my progress to creating a new association scheme. In the general introduction, we discuss the original area of research that I started in, q-analogue t-designs, then state the important background information for the topic.

Chapter 2 contains an outline of the work that inspired the new association scheme. In this chapter, we will introduce the symplectic subspaces as well as results relating to partial geometric designs constructed through regular subgraphs of the generalized symplectic graph.

Chapter 3 contains two new families of $\mathcal{D}$-class association schemes, including constructions of each family.

Chapter 4 contains concluding remarks and recommendations for future research.

CHAPTER 2. SYMPLECTIC GEOMETRY

In this chapter we investigate the known existence and construction results of $1\frac{1}{2}$-designs in finite classical geometries, specifically in Wan (1993), Feng et al. (2016) and Chai et al. (2015). We begin our study of the symplectic vector space and its application to the construction of $1\frac{1}{2}$-designs as proposed by Neumaier (1980). Zhe-xian Wan gave a great foundation for the study of symplectic spaces in Wan (1997) as well as a more detailed construction in Wan (1993). Through his work, we were introduced to the notion of type (m, s) subspaces in the symplectic space as well as totally isotropic subspaces of type $(m, 0)$. Feng et al. (2016) used Wan's construction of the symplectic geometry, Neumaier's generalized $1\frac{1}{2}$-design and Olmez's construction of $1\frac{1}{2}$-designs in Olmez (2014) to create new $1\frac{1}{2}$-designs using type $(m, 0)$, totally isotropic subspaces. To understand their constructions we have to understand what symplectic geometry as well as what a $1\frac{1}{2}$-design is. This section will be dedicated to understanding the symplectic space and the constructions provided in Feng et al. (2016) and Chai et al. (2015) for possible generalizations to other types of totally isotropic subspaces.

2.1 Symplectic vector space and some basic counting

We will now discuss the construction of the symplectic group using congruent matrices and special forms as described in Wan (1993).

Definition 35. Two matrices A and B are said to be *congruent* (cogredient) if there exists an invertible matrix P such that $A = P^t BP$.

Definition 36. A matrix K is called *alternate* if $k_{ij} = -k_{ji}$ for $i \neq j$ and $k_{ii} = 0$ for all $i = 1, \ldots, n$. Similarly, we can say that $xKx^t = 0$ for all $x \in \mathbb{F}_q^n$.

If K is an $n \times n$ alternate matrix of rank 2ν, then ν is called the index of K.

Lemma 2.1.1. *Let K be an alternate matrix with index ν of size $2\nu \times 2\nu$ over $\mathbb{F}_q$. Then for any nonsingular matrix A, AKA^t is an alternate matrix of index ν.*

Proof. By Definition 36, a matrix is alternate if $xKx^t = 0$ for all $x \in \mathbb{F}_q^{2\nu}$. Let A be a nonsingular matrix of size $2\nu \times 2\nu$. Then A may be viewed as a linear transformation. For every $y \in \mathbb{F}_q^{2\nu}$ there exists a unique $x \in \mathbb{F}_q^{2\nu}$ such that $xA = y$. Then, for all $x \in \mathbb{F}_q^{2\nu}$,

$$xAKA^t x^t = yKy^t = 0.$$

Therefore, AKA^t is an alternate matrix. $\square$

Corollary 2.1.2. *(Wan, 1993, Corollary 3.2) Two $n \times n$ alternate matrices over $\mathbb{F}_q$ are congruent, if and only if they have the same rank, and if and only if they have the same index.*

Theorem 2.1.3. *(Wan, 1993, Theorem 3.1) Let K be an alternate $n \times n$ matrix over $\mathbb{F}_q$, then the rank of K is even. Furthermore, if the rank of $K = 2\nu \leq n$, then K is congruent to the matrix*

$$\begin{bmatrix} 0^{(\nu)} & I^{(\nu)} & \\ -I^{(\nu)} & 0^{(\nu)} & \\ & & 0^{(n-2\nu)} \end{bmatrix}.$$

If K is an $n \times n$ alternate matrix of rank 2ν, then the index of K is ν.

The following proof was omitted in Wan (1993) however, a sketch was given and we filled in the gaps below.

Proof. By induction, starting with base case $n = 2$. If $n = 1$, then the index has to be 0, hence $K = \begin{bmatrix} 0 \end{bmatrix}$ and our theorem can be said to be trivially true. When $n = 2$, we obtain the index is $\nu = 1$. Our alternate matrix is of the form

$$K = \begin{bmatrix} 0 & k_{12} \\ -k_{12} & 0 \end{bmatrix}.$$

Let $P = \begin{bmatrix} 1 & 0 \\ 0 & k_{12}^{-1} \end{bmatrix}$. Then,

$$PKP^t = \begin{bmatrix} 1 & 0 \\ 0 & k_{12}^{-1} \end{bmatrix} \begin{bmatrix} 0 & k_{12} \\ -k_{12} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & k_{12}^{-1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Assume that $n > 2$ and that our theorem holds for all $m < n$.

If $K = 0$, we are done as the theorem holds. If $K \neq 0$, there exists an i, j such that $k_{ij} \neq 0$. We can permute the columns and rows to obtain a congruent alternate matrix. We swap $(1, i)$ and $(2, j)$. This gives us a matrix of the form

$$K = \begin{bmatrix} 0 & k_{ij} & \\ -k_{ij} & 0 & \\ & & K' \end{bmatrix},$$

using the matrix $P = \begin{bmatrix} 1 & 0 & \\ 0 & k_{12}^{-1} & \\ & & I \end{bmatrix}$, we have

$$PKP^t = \begin{bmatrix} 0 & 1 & \\ -1 & 0 & \\ & & K' \end{bmatrix}$$

By induction, K' is congruent to

$$\begin{bmatrix} 0 & I & \\ -I & 0 & \\ & & 0 \end{bmatrix}$$

This gives us the matrix

$$\begin{bmatrix} 0 & 1 & & \\ -1 & 0 & & \\ & & 0 & I \\ & & -I & 0 \end{bmatrix}$$

We can swap rows and columns to add another row and column to I as well as $-I$, giving us our congruent matrix. As our matrix is congruent to a matrix of the form above, it clearly has even rank.

□

Corollary 2.1.4. *(Wan, 1993, Corollary 3.3) Let K be an $n \times n$ nonsingular alternate matrix. Then n is necessarily even. Let $n = 2\nu$, then K is congruent to*

$$\begin{bmatrix} 0 & I^{(\nu)} \\ -I^{(\nu)} & 0 \end{bmatrix}.$$

Proof. Apply Theorem 2.1.3 □

Notice that this corollary tells us that it does not matter which form we use to describe our alternating matrix.

Definition 37. Let K be a non-singular $2\nu \times 2\nu$ alternate matrix over $\mathbb{F}_q$. A $2\nu \times 2\nu$ matrix T over $\mathbb{F}_q$ is called a *symplectic matrix* with respect to K if $TKT^t = K$.

As stated in Corollary 2.1.4 we may use any form of our non-singular alternating matrix K. From now on, when we write K, we will use the form

$$K = \begin{bmatrix} 0 & I^{(\nu)} \\ -I^{(\nu)} & 0 \end{bmatrix} \tag{2.1}$$

We will now show that we can form a group by collecting all symplectic matrices.

Proposition 2.1.5. *The collection of all symplectic matrices T with respect to K forms a group under matrix multiplication. The group is called the* symplectic group *of degree 2ν with respect to K over $\mathbb{F}_q$. We denote the symplectic group by $Sp_{2\nu}(q, K)$. If we write $Sp_{2\nu}(q)$, it is implied that K is in the form of equation 2.1.*

Clearly $Sp_{2\nu}(q) \subseteq GL_n(q)$, thus we can act on the vectors $\mathbb{F}_q^{2\nu}$ in a similar manner.

$$\mathbb{F}_q^{2\nu} \times Sp_{2\nu}(q) \to \mathbb{F}_q^{2\nu}$$

$$((x_1, x_2, \ldots, x_{2\nu}), T) \to (x_1, x_2, \ldots, x_{2\nu})T$$

The space $\mathbb{F}_q^{2\nu}$ along with the group action is denoted as the 2ν-dimensional *symplectic space* over $\mathbb{F}_q$, more formally:

Definition 38. Let V be a 2ν-dimensional ($\nu \geq 2$) vector space over $\mathbb{F}_q$ of prime power q, endowed with a non-singular skew-symmetric bilinear form. Then V is called a *symplectic space.*

Definition 39. Let $K = \begin{bmatrix} 0 & I^{(\nu)} \\ -I^{(\nu)} & 0 \end{bmatrix}$ be the representing matrix of the symmetric bilinear form with respect to a suitable basis for V. Then the *symplectic group* $Sp_{2\nu}(q)$ on V is

$$Sp_{2\nu}(q) = \{T \in GL_{2\nu}(q) : TKT^t = K\}.$$

Definition 40. For each 2-dimensional subspace A of V, we use the same symbol A to denote a $2 \times 2\nu$ matrix which represents the 2-dimensional subspace A so that

$$\Omega = \{A \in M_{2\times 2\nu}(\mathbb{F}_q) : \text{rank}(AKA^t) = 0\}.$$

We call Ω the set of *totally isotropic projective lines.*

The vectors inside the symplectic space can be partitioned further into rank classes as $0 \leq \text{rank}(YKY^t) \leq m$ for an m-dimensional subspace Y in V.

Definition 41. A vector $x \in \mathbb{F}_q^{2\nu}$ is called an *isotropic vector* if $xKx^t = 0$.

Proposition 2.1.6. *(Wan, 1993, Page 110) Every $x \in \mathbb{F}_q^{2\nu}$ is isotropic with respect to K.*

Proof. Notice that for any $x \in \mathbb{F}_q^{2\nu}$, we can rewrite $x = (x_1, x_2), x_1, x_2 \in \mathbb{F}_q^{\nu}$ then,

$$xKx^t = x \begin{bmatrix} 0 & I^{(\nu)} \\ -I^{(\nu)} & 0 \end{bmatrix} x^t = \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} x_2 \\ -x_1 \end{bmatrix} = x_1x_2 - x_2x_1 = 0$$

$\square$

Definition 42. Let P be a m-dimensional vector subspace of $\mathbb{F}_q^{2\nu}$. Then the matrix form of P is a $m \times 2\nu$ matrix over $\mathbb{F}_q$. As P is rank m, it is similar to the matrix $P' = \begin{bmatrix} I_m & 0 \end{bmatrix}$. Clearly, $P'KP'^t$ is an alternate matrix. By Theorem 2.1.3, the rank of P' is $2s$. As P is similar to P', P has even rank $2s$. We denote such matrices as type (m, s). We start with a matrix of rank m and project it onto K to form a rank $2s$ matrix.

If $s = 0$, we have matrices of type $(m, 0)$ which form the m-dimensional *totally isotropic subspaces.* If $m = \nu$, then we say a matrix of type $(\nu, 0)$ is a *maximal totally isotropic subspace.* The maximal totally isotropic subspaces are sometimes referred to as the *dual polar space* (Wan and Hua, 1996, Page 181). The collection of all m-dimensional totally isotropic subspaces will be denoted as $D(m, 2\nu; q)$. Matrices of type $(2s, s)$ form the $2s$-dimensional *non-isotropic subspaces.* One can also view this as, $PKP^t = 0$ if P is totally isotropic and $PKP^t = A$, where $A \neq 0$ if P is a non-isotropic subspace.

Clearly as $K = \begin{bmatrix} 0 & I^{(\nu)} \\ -I^{(\nu)} & 0 \end{bmatrix}$, we can form a totally isotropic subspace of dimension $s \leq \nu$ by picking any subspace that only spans s vectors in the first ν columns or last ν columns.

Definition 43. Two vectors $x, y \in \mathbb{F}_q^{2\nu}$ are said to be *orthogonal* with respect to K if $xKy^t = 0$. A vector x that is self orthogonal is exactly an isotropic vector. The *dual subspace* of P denoted by $P^\perp$ is the set of vectors that are orthogonal to every vector of P,

$$P^\perp = \{y \in \mathbb{F}_q^{2\nu} | yKx^t = 0 \text{ for all } x \in P\}$$

Example 12. Recall that for every $x \in \mathbb{F}_q^{2\nu}, xKx^t = 0$. Take $x = e_1$ and $y = e_{v+1}$, then $xKy^t \neq 0$ so they are not orthogonal.

Theorem 2.1.7. *(Wan, 1993, Theorem 3.4)*

1. *A subspace P is totally isotropic if and only if $P \subseteq P^\perp$*

2. *P is non-isotropic if and only if $P \cap P^\perp = (0)$*

Proof.

1. Let P be a totally isotropic subspace. Then, $PKP^t = 0$. By definition $P^\perp = \{y \in \mathbb{F}_q^{2\nu} | yKx^t = 0 \text{ for all } x \in P\}$. As $xKx^t = 0$ for all $x \in P$, $x \in P^\perp$ hence $P \subseteq P^\perp$.

2. ($\Rightarrow$) We will show the contrapositive. Assume that there exists an $x \in P \cap P^\perp$. Then what can we say about $PKP^t = D$? As $x \in P \cap P^\perp$, $x \in P$ and x can be written as a linear combination of the columns of P. There exists some non-zero vector c such that $x = cP$. Then,

$$\begin{aligned} PKP^t &= D \\ cPkP^t &= cD \\ xKP^t &= cD \\ 0 &= cD \end{aligned}$$

Hence, D is singular and P is not non-isotropic. So, by the contrapositive, if P is non-isotropic, then $P \cap P^\perp = (0)$.

($\Leftarrow$) Let $P' = \begin{bmatrix} P \\ P^\perp \end{bmatrix}$. Then, $P'KP'^t = \begin{bmatrix} PKP^t & 0 \\ 0 & P'KP'^t \end{bmatrix}$. As $\dim(P \cup P') = 2\nu$, P' is non-singular and $P'KP'^t$ has full rank thus $\begin{bmatrix} PKP^t & 0 \\ 0 & P'KP'^t \end{bmatrix}$ also has full rank. Therefore, PKP^t has full rank, so it is non-isotropic.

$\square$

Now notice that we want to show that for any subspaces P_1, P_2 of type (m, s) there exists a $T \in Sp_{2\nu}(q)$ such that $P_1 = P_2T$. By Theorem 2.1.3 we know that for any type (m, s) matrix P, PKP^t is similar to

$$\begin{bmatrix} 0^{(s)} & I^{(s)} & \\ -I^{(s)} & 0^{(s)} & \\ & & 0^{(m-2s)} \end{bmatrix}$$

as it is alternate and has rank $2s$. As the matrices are similar there exist Q_1 and Q_2 such that $Q_1P_1KP_1^tQ_1^t = Q_2P_2KP_2^tQ_2^t$. However, Q_1P_1 is not invertible as it is of size $m \times 2\nu$.

The following example will be used to understand the upcoming constructions.

Example 13. Let

$$P = \begin{array}{c} \begin{array}{cccc} m & \nu - m & m & \nu - m \end{array} \\ \begin{pmatrix} I^{(m)} & 0 & 0 & 0 \end{pmatrix} \end{array} \; m$$

First, we shall show that P is totally isotropic.

$$PKP^t = P \begin{array}{cc} m & \\ \begin{pmatrix} 0 \\ I^{(m)} \end{pmatrix} & \begin{array}{c} 2\nu - m \\ m \end{array} \end{array} = 0.$$

So P is of type $(m, 0)$. Next, one can see that the dual space $P^{\perp}$ is of the form

$$\begin{array}{cc} \begin{array}{cccc} m & \nu - m & m & \nu - m \end{array} & \\ \begin{pmatrix} I^{(m)} & 0 & 0 & 0 \\ 0 & I^{(\nu-m)} & 0 & 0 \\ 0 & 0 & 0 & I^{(\nu-m)} \end{pmatrix} & \begin{array}{c} m \\ \nu - m \\ \nu - m \end{array} \end{array}$$

As stated in Theorem 2.1.7, $P \subseteq P^{\perp}$. Clearly, $\operatorname{rank}(P^{\perp}K(P^{\perp})^t) = 2(\nu - m)$ so $P^{\perp}$ is of type $(2\nu - m, \nu - m)$. In the most extreme case, $m = \nu$ we have $P^{\perp}$ is of type $(\nu, 0)$. Since P is totally isotropic, $P \subseteq P^{\perp}$, hence $P = P^{\perp}$.

Theorem 2.1.8. *(Wan, 1993, Theorem 3.6) Subspaces of type (m, s) exist in the 2ν-dimensional symplectic space if and only if $2s \leq m \leq \nu + s$.*

Theorem 2.1.9. *(Wan, 1993, Theorem 3.7) $Sp_{2\nu}(q)$ acts transitively on subspaces of the same type (m, s).*

Using Theorem 2.1.9 we can take any representative of a type (m, s) matrix and it will tell us how matrices interact in the symplectic space. For $2s \leq m \leq \nu + s$ we can describe a subspace of type (m, s) and it's dual as shown in the following example.

Example 14. The following matrix forms a subspace of type (m, s) using K from equation (2.1).

$$\begin{array}{c}
\begin{array}{cccccc} s & m-2s & \nu+s-m & s & m-2s & \nu+s-m \end{array} \\
\begin{pmatrix}
I^{(s)} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & I^{(s)} & 0 \\
0 & I^{(m-2s)} & 0 & 0 & 0 & 0
\end{pmatrix}
\begin{array}{c} s \\ s \\ m-2s \end{array}
\end{array}$$

It's dual subspace forms a subspace of type $(2\nu - m, \nu + s - m)$ and has the following form:

$$\begin{array}{c}
\begin{array}{cccccc} s & m-2s & \nu+s-m & s & m-2s & \nu+s-m \end{array} \\
\begin{pmatrix}
0 & I^{(m-2s)} & 0 & 0 & 0 & 0 \\
0 & 0 & I^{(\nu+s-m)} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & I^{(\nu+s-m)}
\end{pmatrix}
\begin{array}{c} m-2s \\ \nu+s-m \\ \nu+s-m \end{array}
\end{array}$$

The next general question would be: how many type (m, s) matrices are there? We will be using the notation from Wan (1993). Please be mindful about the number of terms required for each function. Although Lemma 2.1.10 and Theorem 2.1.13 are both labeled N, the function in Lemma 2.1.10 only has two terms while the function in Theorem 2.1.13 has 5 terms.

Lemma 2.1.10 (Lemma 1.2.1)**.** *Let $1 \leq m \leq n$. The number of m-dimensional subspaces of the vector space $\mathbb{F}_q^n$ is*

$$N(m, n) = \frac{\prod_{i=n-m+1}^{n} (q^i - 1)}{\prod_{i=1}^{m} (q^i - 1)}$$

Theorem 2.1.11. *(Wan, 1993, Theorem 3.18) Let $2s \leq m \leq \nu + s$. The number of subspaces of type (m, s) in the 2ν-dimensional symplectic space over $\mathbb{F}_q$ is given by*

$$N'((m, s), 2\nu) = q^{2s(\nu+s-m)} \frac{\prod_{i=\nu+s-m+1}^{\nu} (q^{2i} - 1)}{\prod_{i=1}^{s} (q^{2i} - 1) \prod_{i=1}^{m-2s} (q^i - 1)}$$

Corollary 2.1.12. *(Wan, 1993, Corollary 3.19) Let $1 \leq m \leq \nu$. The number of m-dimensional totally isotropic subspaces (type $(m, 0)$) in $\mathbb{F}_q^{2\nu}$ is*

$$N'(m, 2\nu) = \frac{\prod_{i=\nu-m+1}^{\nu} (q^{2i} - 1)}{\prod_{i=1}^{m} (q^i - 1)}$$

Theorem 2.1.13. *(Wan, 1993, Theorem 3.27) Let $2s \leq m \leq \nu + s$ and $\max\{0, m_1 - s - s_1\} \leq \min\{m - 2s, m_1 - 2s_1\}$. Let P be a fixed subspace of type (m, s). Then the number of subspaces of type (m_1, s_1) contained in P is given by the following formula:*

$$N(m_1, s_1; m, s; 2\nu) = \sum_{k=\max\{0, m_1-s-s_1\}}^{\min\{m-2s, m_1-2s_1\}} q^{2s_1(s+s_1-m_1+k)+(m_1-k)(m-2s-k)} F^*(k),$$

where

$$F^*(k) = \frac{\prod_{i=s+s_1-m_1+k+1}^{s} (q^{2i} - 1) \prod_{i=m-2s-k+1}^{m-2s} (q^i - 1)}{\prod_{i=1}^{s_1} (q^{2i} - 1) \prod_{i=1}^{m_1-2s_1-k} (q^i - 1) \prod_{i=1}^{k} (q^i - 1)}$$

Proof. See (Wan, 1993, Page 135). □

Theorem 2.1.14. *(Wan, 1993, Theorem 3.38) The number of type (m, s) subspaces containing a type (m_1, s_1) subspace,*

$$N'(m_1, s_1; m, s; 2v) = N(2\nu - m, \nu + s - m; 2\nu - m_1, \nu + s_1 - m_1; 2\nu)$$

Proof. Let P_1 be a subspace of type (m_1, s_1) contained in P, a subspace of type (m, s), $P_1 \subseteq P$. Then, $P^\perp \subseteq P_1^\perp$. Thus, we can instead count the number of $P^\perp$'s in $P_1^\perp$. Recall that $P^\perp$ will be of type $(2\nu - m, \nu + s - m)$ while $P_1^\perp$ will be of type $(2\nu - m_1, \nu + s_1 - m_1)$. Giving us our desired result of counting the number of type $(2\nu - m, \nu + s - m)$ subspaces in a type $(2\nu - m_1, \nu + s_1 - m_1)$ subspace. It is evident that $m_1 \leq m$ and $s_1 \leq m$ so that $2\nu - m_1 \geq 2\nu - m$ and $\nu + s_1 - m_1 \geq \nu + s - m$. This gives us our desired result of

$$N(2\nu - m, \nu + s - m; 2\nu - m_1, \nu + s_1 - m_1; 2\nu),$$

where $N(m_1, s_1; m, s; 2\nu)$ is defined in Theorem 2.1.13. □

2.2 $1\frac{1}{2}$-designs and their parameters

The partial geometric designs introduced in Section 1.1 are also known as $1\frac{1}{2}$-designs. Now that we have a general idea of the symplectic group, we can turn our attention to the basics of $1\frac{1}{2}$-designs.

Definition 44. A $t\frac{1}{2}$-design is a t-design $\mathcal{B}$ over P for which there are integers $\alpha_0, \ldots, \alpha_t$ such that $\alpha(T, B) = \alpha_i$ for every block $B \in \mathcal{B}$ and every t-subset T of P with $|T \cap B| = i$ $(i = 0, \ldots, t)$, where $\alpha(T, B)$ is the number of flags compatible with (T, B), that is the number of (x, A) where $T \subseteq A$ and $x \in B$ but $x \notin T$.

So for t=1, we have α_0 and α_1 only two parameters. In general, what we are doing is characterizing the global property of flags by locally identifying the intersection size of $|T \cap B|$.

$1\frac{1}{2}$-designs have been of great interest since their connection to strongly regular graphs as well as association schemes was described in Brouwer et al. (2012) and Nowak et al. (2016). Many new constructions have appeared such as in Feng et al. (2016) and Feng and Zeng (2016). The motivation of Chai et al. (2015) was a result of trying to create a new strongly regular graph from a $1\frac{1}{2}$-design. Much of the work was already described in Feng and Zeng (2016).

A *strongly regular graph* is a (simple, undirected) graph G such that every vertex is adjacent with the same number of other vertices. The number of vertices adjacent with two distinct vertices a, b depends only on whether a and b are adjacent or not.

Theorem 2.2.1. *(Neumaier, 1980, Theorem 3.11) An incidence structure $\mathcal{B}$ is a $1\frac{1}{2}$-design with parameters $(v, k, b, r; \alpha, \beta)$ if and only if its incident matrix A satisfies the equations*

$$AJ = rJ, JA = kJ, AA^tA = nA + \alpha J$$

where

$$n = r + k + \beta - \alpha - 1$$
$$v = \frac{k(kr - n)}{\alpha}$$
$$b = \frac{r(kr - n)}{\alpha}$$

$$\beta = \alpha + n + 1 - r - k$$

$$k + r \leq n + \alpha + 1 \leq kr$$

Lemma 2.2.2. *(Neumaier, 1980, Lemma 3.12) If A is the incidence matrix of a proper $1\frac{1}{2}$-design, then $N = AA^t$ satisfies*

$$NJ = krJ, N^2 = nN + \alpha rJ$$

2.3 Construction of $1\frac{1}{2}$-designs with symplectic subspaces

For the following construction we need to rewrite the definition of a $1\frac{1}{2}$-design. Let $s(x, B)$ be the number of flags (y, C) such that $y \in B \backslash \{x\}, x \in C$ and $C \neq B$. For two different points $p, q \in \mathcal{P}$, denote λ_{pq} as the number of blocks containing both p and q. Then the property for a $1\frac{1}{2}$-design is equivalent to saying that for every point $x \in \mathcal{P}$ and every block $B \in \mathcal{B}$,

$$s(x, B) = \begin{cases} \sum_{y \in B} \lambda_{xy} = \alpha, & \text{if } x \notin B, \\ \sum_{y \in B \backslash \{x\}} (\lambda_{xy} - 1) = \beta, & \text{otherwise} \end{cases}$$

Corollary 2.3.1. *(Chai et al., 2015, Corollary 2.3)*

Let $1 \leq k < m \leq \nu$. Then the number of m-dimensional totally isotropic subspaces in the symplectic space $\mathbb{F}_q^{2\nu}$ containing a given k-dimensional totally isotropic subspace is

$$N(k, m; 2\nu) = \frac{N'(m, 2\nu)N(k, m)}{N'(k, 2\nu)}$$

Proof. Each totally isotropic subspace of dimension k is contained in the same number of m-dimensional totally isotropic subspaces. Similarly if we take a m-dimensional totally isotropic subspace and take all of its k-dimensional subspaces, each will be totally isotropic as well. $\square$

It should be noted that this formula and proof will not hold when counting the number of subspaces containing a non-isotropic subspace. Notice that a type $(m, s), s > 0$ space always contains the 1-dimensional totally isotropic subspaces. In fact, it may contain totally isotropic subspaces of size $1, \cdots, m - 1$. Therefore, we do not have the case described in the proof above.

Please see (Wan, 1993, Theorem 3.38) for the formula used to count the number of subspaces of type (m, s) containing a given subspace of type (m_1, s_1) where $m_1 \leq m$ and $s_1 \leq s$.

2.3.1 First construction

Our natural instinct is to fix ν and take the totally isotropic subspaces of dimension k and m, $k < m$ and try to create a design. We would let the points be all isotropic subspaces of dimension k and the blocks be the isotropic subspaces of dimension m. Then we want to find a collection of m-dimensionally totally isotropic subspaces that contain all of the points. If we take all m-totally isotropic spaces then we know that each k-dimensional subspace will appear exactly $\frac{N'(m,\nu)N(k,m)}{N'(k,\nu)}$ times. So we could have a type of design, however, it is trivial as we take all isotropic subspaces.

We will let $D(k, 2\nu; q)$ represent the total number of totally isotropic subspaces of dimension k in the vector space $\mathbb{F}_q^{2\nu}$ using the form K from equation 2.1.

Corollary 2.3.2. *Let $\mathcal{P} = D(k, 2\nu; q), \mathcal{B} = D(m, 2\nu; q)$ and $1 \leq k < m \leq \nu$. For $x \in \mathcal{P}$ and $B \in \mathcal{B}, x \in B$ if and only if $x \subset B$ as subspaces of $\mathbb{F}_q^{2\nu}$. Then the incidence structure $\mathcal{T}_1 = (\mathcal{P}, \mathcal{B}, \epsilon)$ is a 1-design.*

Proof. By the corollary proof above, we have $vr = bk$. Under the construction above we have:

- v, the number of points is $|D(k, 2\nu; q)| = N'(k, 2\nu)$.
- b , the number of blocks is $|D(m, 2\nu; q)| = N'(m, 2\nu)$.
- k, the number of points in each block, $N(k, m)$ as a totally isotropic space has all totally isotropic subspaces
- r, the number of blocks containing a point, $N(k, m; 2\nu)$.

□

Theorem 2.3.3. *(Chai et al., 2015, Theorem 2.4) Let $\mathcal{P} = D(k, 2\nu; q), \mathcal{B} = D(m, 2\nu; q)$ and $1 \leq k < m \leq \nu$. For $x \in \mathcal{P}$ and $B \in \mathcal{B}, x \in B$ if and only if $x \subset B$ as subspaces of $\mathbb{F}_q^{2\nu}$. Then the* *incidence structure* $\mathcal{T}_1 = (\mathcal{P}, \mathcal{B}, \epsilon)$ *is a* $1\frac{1}{2}$*-design if an only if* $k = 1$ *and* $m = \nu$*. Furthermore, when* $k = 1$ *and* $m = \nu$*, the parameters of* $\mathcal{T}_1$ *is*

$$((q+1)(q^2+1), (q+1)(q^2+1), q+1, q+1, 1, 0)$$

if $\nu = 2$*, and*

$$v = \frac{q^{2\nu}-1}{q-1}, b = \prod_{i=1}^{\nu}(q^i+1), k = \frac{q^{\nu}-1}{q-1}, r = \prod_{i=1}^{\nu-1}(q^i+1)$$

$$\alpha = \frac{q^{\nu-1}-1}{q-1}\prod_{i=1}^{\nu-2}(q^i+1), \beta = \frac{q(q^{\nu-1}-1)}{q-1}(\prod_{i=1}^{\nu-2}(q^i+1)-1)$$

We will now clarify the proof given in Chai et al. (2015) by adding a few more details that were left to the reader.

Proof. By the above corollary, $\mathcal{T}_1$ is a 1-design for all k and m. The rest of our claim will be shown as follows: First, when $k = 1$ and $m = \nu$, we will show that $\mathcal{T}_1$ is a $1\frac{1}{2}$-design by calculating α and β. For all other k and m we will construct multiple blocks to show that the value α for antiflags is not fixed as required.

1. **Let $k = 1$ and $m = \nu$.** We know that type (m, s) matrices are transitive under the symplectic space. Thus, any representative of points preserves the geometric properties of the space. For the sake of simplicity, we can let $x = e_1 \in \mathbb{F}_q^{2\nu}$. Pick a block $B \in \mathcal{B}$, we then have two cases:

   - If $x \in B$, then x is orthogonal to any $y \in B$. There are $N(1, m) - 1$ 1-dimensional subspaces in B that can form a totally isotropic 2-dimensional subspace with x. Each 2-dimensional space is contained in exactly $N(2, m; 2\nu) - 1$ m-totally isotropic subspaces (blocks) different from B. Thus,

   $$\beta = \sum_{y \in B \setminus \{x\}} (\lambda_{xy} - 1) = (N(1, m) - 1)(N(2, m; 2\nu) - 1)$$

   - If $x \notin B$, then there exists a non-orthogonal point $y \in B$. This has to be true as $m = \nu$, making B a maximally totally isotropic subspace. If x was orthogonal to every point in B then the space spanned by $x + B$ would be a totally isotropic subspace of dimension

$\nu+1$ which contradicts the maximality of $m=\nu$. Using our representative of the points $x=e_1$, we can see that B must contain a point y such that $y_{\nu+1}=1$ so that $xKy^t \neq 0$. We can row reduce B by zeroing out all entries in the $\nu+1$ column to see that the other $\nu-1$ rows are still orthogonal to x. Excluding this point y there are $N(1,m-1)$ 1-dimensional subspaces in B that can form a totally isotropic 2-dimensional subspace with x. Each 2-dimensional space is contained in exactly $N(2,m;2\nu)$ m-totally isotropic subspaces (blocks). Thus,

$$\alpha=\sum_{y\in B}(\lambda_{xy})=(N(1,m-1))(N(2,m;2\nu))$$

2. **Let $k=1$ and $m<\nu$.** One may check that the β is still fixed for flags. However when $x\notin B$ and m is not maximal, x can be orthogonal to all $y\in B$ or non-orthogonal to some $y\in B$. These blocks can easily be constructed.

   - Assume $x\notin B$ and there exists a $y\in B$ such that $xKy^t=0$. Then as above we can exclude this point y and there are $N(1,m-1)$ 1-dimensional subspaces in B that can form a totally isotropic 2-dimensional subspace with x. Each 2-dimensional space is contained in exactly $N(2,m;2\nu)$ m-totally isotropic subspaces (blocks). Thus

$$\alpha=\sum_{y\in B}(\lambda_{xy})=(N(1,m-1))(N(2,m;2\nu))$$

   - Assume $x\notin B$ and $xKy^t=0$ for all $y\in B$. There there are $N(1,m)$ 1-dimensional subspaces in B that can form a totally isotropic 2-dimensional subspace with x. Each 2-dimensional space is contained in exactly $N(2,m;2\nu)$ m-totally isotropic subspaces (blocks). Thus

$$\alpha=\sum_{y\in B}(\lambda_{xy})=(N(1,m))(N(2,m;2\nu))$$

   As $1=k<m<\nu$, $N(1,m)>N(1,m-1)$. Hence, α is not fixed so we can not have a $1\frac{1}{2}$-design.

3. **$k>1$.** Again, when $k>1$, one may check that the β is still fixed for flags; however, when $x\notin B$ and $k>1$ we run into a new problem. Even if $x\notin B$, $0\leq \dim(x\cap B)<k$, that is

we can have a non-trivial intersection of the block and the point which will greatly affect our value of α. Let $\dim(x \cap B) = t$ so $0 \leq t < k$. Intuitively, we know that α will not be fixed as $B \subseteq B^{\perp}$, so we can find other totally isotropic subspaces using the vectors in $B^{\perp}$. Similarly, notice that there has to exist $\nu - m$ vectors in $B^{\perp}$ that are not in B which can extend B to a maximally totally isotropic subspace. To see this, recall that every type (m, s) subspace is transitive under the symplectic group. As there exists maximally totally isotropic subspaces and a maximal totally isotropic space contains a totally isotropic subspace of dimension m, then we can view the maximal subspace as an extension of the m space. Hence, since m spaces are transitive, all m spaces are extendable to totally isotropic subspaces. For the rest of this section we will assume that x and B_t for $0 \leq t < k$ represents the following point and block:

$$x = \begin{array}{cccccc} & k & \nu-k & k & \nu-k & \\ & \left(I^{(k)} \right. & 0 & 0 & \left. 0 \right) & k \end{array}$$

Clearly, $xKx^t = 0$ so $x \in \mathcal{P}$.

$$B_t = \begin{array}{c} \begin{array}{cccccccc} t & k-t & m-k & \nu-m & t & k-t & m-k & \nu-m \end{array} \\ \begin{pmatrix} I^{(t)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I^{(k-t)} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I^{(m-k)} & 0 \end{pmatrix} \end{array} \begin{array}{c} t \\ k-t \\ m-k \end{array}$$

Similarly, $B_t K B_t^t = 0$ for all $0 \leq t < k$ so $B_t \in \mathcal{B}$.

(i) Assume $m < 2k - t$. Then, it is impossible to form a block such that $x, y \in B$ as $\dim(x + y) = 2k - t > m$. Thus,

$$\alpha = \sum_{y \in B} \lambda_{xy} = 0.$$

(ii) Assume $m \geq 2k - t$. Now pick a $y \in B_t$. Notice that when $m \geq 2k - t$, we can fit two points in a block so $\dim(B_t) \geq \dim(x + y) = \dim(x) + \dim(y) - \dim(x \cap y) = m \geq$

$2k - \dim(x \cap y)$. Thus, $\dim(x \cap y) \geq 2k - m$ and $\dim(x \cap y) \geq 0$ thus, $\max\{0, 2k - m\} \leq \dim(x \cap y) \leq \dim(x \cap B_t) = t$. We will count the number of points z in B_t that are orthogonal to x and $\dim(x \cap z) = w$. We will first notice that $\dim(x \cap B_t) = t$ is fixed as x is fixed and B is fixed.

We can restate our problem as follows:

Given a vector space V, an m-dimensional subspace B_t and a t-dimensional subspace T in B_t, we look to see how many k-dimensional subspaces K of B exist such that $\dim(B \cap K) = w$ and are orthogonal to x. This problem has been dealt with many times, most notably in q-analogues of t-designs by Ray-Chaudhuri and Singhi. Using their notation: For any k-subspace K of V and a $(k-l)$-subspace K_1 of K, $c(k, t, m, l)$ is the number of t-subspaces T of K satisfying the condition that the dimension of $T \cap K_1$ is $t - m$. The only nuance is that we have to pick our vectorspaces such that they are orthogonal to x. By our fixed form of B_t, we are not dealing with all of B, we can only use $m - k$ vectors that are orthogonal, that is normally we would have $m - t$ vectors we can't pick however, due to orthogonality we can only pick $m - k$. Using a similar notation, the number of points z in B such that $\dim(x \cap z) = w$ is

$$C_w = c'(m, k, k - w, m - t) = q^{(t-w)(k-w)} \binom{m-k}{k-w}_q \binom{t}{w}_q$$

Now, each of these points can form a subspace with x, $\dim(x + z) = 2k - w$. The number of totally isotropic subspaces of dimension m that contain a given $2k - w$ subspace is $N(2k - w, m; 2\nu)$. So for a given B_t, that is a block B with $\dim(B \cap x) = t$, we have

$$\alpha = s(x, B_t) = \sum_{w=\max\{0,2k-m\}}^{t} C_w N(2k - w, m; 2\nu)$$

- When $m < 2k, t$ can be in case (i) and (ii). We have $\alpha = 0$ in case (i). Now, for case (ii), notice that $N(2k - w, m; 2\nu) \neq 0$ when $m \geq 2k - w$, which gives us $w \geq 2k - m$. Note that $N(2k - w, m; 2\nu)$ is not always 0. Given a totally isotropic subspace of dimension r we can extend it to a totally isotropic subspace of dimension $r + 1, r + 2, \cdots, \nu$. So, given a fixed x, we can take a dimension $k-1$ subspace and extend it to a block of dimension m such that $x \notin B$ and $t = \dim(x \cap B) = k-1$. We will always be able to have $2k-(k-1) = k+1 \leq m$, giving us $N(2k-w, m; 2\nu) = 1$. When this occurs, $C_w > 0$ as $t > w$ by assumption and if $m-k < k-w$, then $w < 2k-m$ which contradicts $w \geq 2k-m$. Therefore, we will have non-zero values. So there are some $\alpha > 0$ giving us

$$|\{s(x, B_t)|0 \leq t < k\}| > 1$$

which contradicts only having a single value for α.

- When $m \geq 2k$ we only have case (ii). We have already shown what occurs when $k = 1$ and $m < \nu$ so now we can assume $k \geq 2$. In the instance when $k \geq 2$ we can always construct blocks such that $t = 0$ or 1. Notice that $\max\{0, 2k-m\} = 0$ as $m \geq 2k$.

$$s(x, B_0) = \sum_0^0 C_w N(2k-w, m; 2\nu) = C_0 N(2k, m; 2\nu)$$

$$s(x, B_1) = \sum_0^1 C_w N(2k-w, m; 2\nu) = C_0 N(2k, m; 2\nu) + C_1 N(2k-1, m; 2\nu)$$

As discussed above, $N(2k-1, m; 2\nu) > 0, C_1 > 0$. Thus, $s(x, B_0) < s(x, B_1)$. So there are multiple values for α

□

We have shown that for this construction there is only one possible way to obtain a $1\frac{1}{2}$-design. This occurs if $k = 1$ and $m = \nu$. The graph we constructed is also known as the *dual polar graph* [Page 212]Wan and Hua (1996).

Note that this construction fails as C_w's vary. By placing restrictions on our blocks we can reduce the number of blocks containing a point. In order to do this, we need to further understand the lattice structure of the totally isotropic subspaces in the symplectic space. A similar problem was studied in Rieck (2005) in which the author constructs association schemes on the set of totally isotropic subspaces under specific intersection sizes of the blocks and points. An explicit formula for the intersection numbers of the general association scheme is still an open problem. In Chapter 3, we get closer to understanding the general relationship of a similar association scheme.

2.3.2 Second construction

The second construction is a bit more complicated and follows the work of Brouwer et al. (2012). To begin, we will define a new form

$$M = \begin{pmatrix} 0 & I^{(m)} & & \\ -I^{(m)} & 0 & & \\ & & 0 & I^{(v-m)} \\ & & -I^{(v-m)} & 0 \end{pmatrix}, \tag{2.2}$$

where $1 \leq m \leq \nu$. Recall that all symplectic forms are equivalent. The ones we establish will simplify our counting arguments.

Definition 45. The *generalized symplectic graph* Γ over $\mathbb{F}_q$ is the undirected graph whose vertices are with all m-dimensional subspaces X of $\mathbb{F}_q^{2\nu}$ such that $XMX^t = 0$. The two vertices X and Y are adjacent, denoted $X \sim Y$ if and only if $\dim(X \cap Y) = m - 1$ and $\operatorname{rank}(XMY^t) = 1$.

Again, for simplicity, in our counting we will fix an m-dimensional subspace P,

$$P = \begin{array}{c} \begin{array}{cccc} m & m & \nu - m & \nu - m \end{array} \\ \begin{pmatrix} I^{(m)} & 0 & 0 & 0 \end{pmatrix} \end{array} m$$

Notice that $P \in V(\Gamma)$ as $PMP^t = 0$.

Definition 46. Let $0 \leq d \leq m-1$ and $\max\{0, 2m - \nu - d\} \leq r \leq m - d$. Then

$$S_P(r,d) = \{N \in V(\Gamma) | \operatorname{rank}(PMN^t) = r, \dim(P \cap N) = d\}.$$

Now, we have partitioned the vertices in Γ using P and $S(r,d)$. We would like to know when vertices in two different parts are joined by an edge.

Lemma 2.3.4. *(Zeng et al., 2013, Lemma 2.2) For any vertex $Q_1 \in S_P(r_1, d_1)$, there exists a vertex $Q_2 \in S_P(r_2, d_2)$ such that $Q_1 \sim Q_2$ if and only if r_1, r_2, d_1 and d_2 satisfy one of the following conditions:*

1. $d_1 = d_2$ *and* $|r_1 - r_2| = 1$

2. $d_1 - d_2 = r_2 - r_1 = 1, -1$ *or* 0

Notice that for our fixed P, $\partial(P, Q) = 1$ is only satisfied for the set $S_P(1, m-1)$.

Definition 47. Let $\Gamma^{(e)}(P) = \{X \in V(\Gamma) | \partial(P, X) = e\}$.

Using this definition, the following observation was made in Chai et al. (2015). $\Gamma^{(1)}(X)$ is the set of neighborhoods of X in Γ and will be labeled as $\Gamma(X)$. $\Gamma(P) = S(1, m-1)$ and

$$\Gamma^{(2)}(P) = \begin{cases} S_P(0, m-1), & \text{if } m = 1 < \nu, \\ S_P(2, m-2) & \text{if } 1 < m = \nu, \\ S_P(0, m-1) \cup S_P(2, m-2), & \text{otherwise.} \end{cases}$$

Lemma 2.3.5. *(Zeng et al., 2013, Lemma 3.1) The graph induced by $\Gamma(P)$ has $\frac{q^m - 1}{q-1}$ connected components. Every component is a regular graph with $q^{2(\nu-m)+1}$ vertices and with valency $(q-1)q^{2(\nu-m)}$, and the corresponding vertex set $\Gamma(i, \gamma)$ is composed of the following points: $\Gamma(i, \gamma, a, \alpha, \beta) =$*

$$\begin{array}{ccccccccc} i-1 & 1 & m-i & i-1 & 1 & m-i & \nu-m & \nu-m & \\ \multicolumn{8}{c}{\left(\begin{array}{cccccccc} 0 & a & 0 & 0 & 1 & \gamma & \alpha & \beta \\ I^{(i-1)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\gamma^t & I^{(m-i)} & 0 & 0 & 0 & 0 & 0 \end{array}\right)} & \begin{array}{c} 1 \\ i-1 \\ m-i \end{array} \end{array}$$

where the integer $0 < i \leq m$ and $\gamma \in \mathbb{F}_q^{(m-i)}$.

We once again follow the proof given in Zeng et al. (2013), filling in the details left to the reader.

Proof. Assume we have an $X \in \Gamma(P)$. Since $X \in S_P(1, m-1)$ as $\partial(P, X) = 1$, we know that $\dim(P \cap X) = m - 1$ and $\operatorname{rank}(PMP^t) = 1$. Any representative of X can be represented in its reduced row echelon form.

$$P = \begin{array}{cc} \begin{array}{cccc} m & m & \nu - m & \nu - m \end{array} & \\ \begin{pmatrix} I^{(m)} & 0 & 0 & 0 \end{pmatrix} & m \end{array}$$

Therefore, X must have $I^{(m-1)}$ in its reduced row echelon form in the first m rows and, it must have a non-zero entry in the second m rows so that $\operatorname{rank}(PMX^t) = 1$. This tells us that X must have a submatrix of the form $\begin{pmatrix} A & 0 & 0 & 0 \end{pmatrix}$ where $\operatorname{rank}(A) = m - 1$ and A is of size $m \times (m-1)$.

$$X = \begin{array}{cc} \begin{array}{cccc} m & m & \nu - m & \nu - m \end{array} & \\ \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ A & 0 & 0 & 0 \end{pmatrix} & \begin{array}{c} 1 \\ m-1 \end{array} \end{array}$$

As X has to be rank m, $\operatorname{rank}([x_2 x_3 x_4]) = 1$. We can assume $\operatorname{rank}(x_2) = 1$. Since $X \in \Gamma$ and $X \in \Gamma(P)$ we have to satisfy the following two conditions:

- $XMX^t = 0$. Notice that $x_1 x_2^t = x_2^t x_1$ and $x_3 x_4^t = x_4^t x_3$.

$$X^t = \begin{array}{cc} \begin{array}{cc} 1 & m-1 \end{array} & \\ \begin{pmatrix} x_1^t & A^t \\ x_2^t & 0 \\ x_3^t & 0 \\ x_4^t & 0 \end{pmatrix} & \begin{array}{c} m \\ m \\ \nu - m \\ \nu - m \end{array} \end{array}$$

$$XMX^t = \begin{array}{c} \begin{array}{cccc} m & m & \nu-m & \nu-m \end{array} \\ \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ A & 0 & 0 & 0 \end{pmatrix} \end{array} \begin{array}{c} \begin{array}{cc} 1 & m-1 \end{array} \\ \begin{pmatrix} x_2^t & 0 \\ -x_1^t & A^t \\ x_4^t & 0 \\ -x_3^t & 0 \end{pmatrix} \end{array} = \begin{pmatrix} 0 & -x_2A^t \\ Ax_2^t & 0 \end{pmatrix}$$

Thus, we must have $Ax_2^t = 0$.

- $\operatorname{rank}(PMX^t) = 1$.

$$PMX^t = \begin{array}{c} \begin{array}{cccc} m & m & \nu-m & \nu-m \end{array} \\ \begin{pmatrix} I^{(m)} & 0 & 0 & 0 \end{pmatrix} \end{array} \begin{array}{c} \begin{array}{cc} 1 & m-1 \end{array} \\ \begin{pmatrix} x_2^t & 0 \\ -x_1^t & A^t \\ x_4^t & 0 \\ -x_3^t & 0 \end{pmatrix} \end{array} = \begin{pmatrix} x_2^t & 0 \\ 0 & 0 \end{pmatrix}$$

Hence, $\operatorname{rank}(x_2^t) = 1$. Since $Ax_2^t = 0$, there exists a $T \in GL_{m-1}(\mathbb{F}_q)$ such that

$$TA = \begin{array}{cc} \begin{array}{ccc} i-1 & 1 & m-1 \end{array} & \\ \begin{pmatrix} I^{(i-1)} & 0 & 0 \\ 0 & -\gamma^t & I^{(m-i)} \end{pmatrix} & \begin{array}{c} i-1 \\ m-i \end{array} \end{array}, \gamma \in \mathbb{F}_q^{(m-i)}$$

If we let $T' = \begin{bmatrix} 1 & 0 \\ 0 & T \end{bmatrix}$ then,

$$X = \begin{array}{cc} \begin{array}{cccccccc} i-1 & 1 & m-i & i-1 & 1 & m-i & \nu-m & \nu-m \end{array} & \\ \begin{pmatrix} 0 & a & 0 & x_{2,i-1} & x_{2,1} & x_{2,m-i} & \alpha & \beta \\ I^{(i-1)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\gamma^t & I^{(m-i)} & 0 & 0 & 0 & 0 & 0 \end{pmatrix} & \begin{array}{c} 1 \\ i-1 \\ m-i \end{array} \end{array}$$

Now, since $XKX^t = 0$, we must have $x_{2,i-1} = 0$ and $x_{2,1}\gamma = x_{2,m-i}$. Notice that $x_{2,2} \neq 0$, otherwise $\mathrm{rank}(x_{2,2}) = 0$ which contradicts the fact that it has rank 1. We can fix $x_{2,1} = 1$ so that $x_{2,m-i} = \gamma$. This gives us our desired form: $X = \Gamma(i, \gamma, a, \alpha, \beta) =$

$$\begin{array}{cc} \begin{array}{cccccccc} i-1 & 1 & m-i & i-1 & 1 & m-i & \nu-m & \nu-m \end{array} & \\ \begin{pmatrix} 0 & a & 0 & 0 & 1 & \gamma & \alpha & \beta \\ I^{(i-1)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\gamma^t & I^{(m-i)} & 0 & 0 & 0 & 0 & 0 \end{pmatrix} & \begin{array}{c} 1 \\ i-1 \\ m-i \end{array} \end{array}$$

X represents every possible type of vertex in $\Gamma(P)$. Each X can be further partitioned by the reduction of A. As $P = \begin{bmatrix} I^{(m)} & 0 & 0 & 0 \end{bmatrix}$, we can partition the X based on which $m-1$ vectors we select from $I^{(m)}$. This means that there are $\binom{m}{m-1}_q = \binom{m}{1}_q$ ways to select a subspace of dimension $m-1$ from $I^{(m)}$. So there are $\binom{m}{m-1}_q$ different partitions of the vertices in $\Gamma(X)$. Each of these partitions of X form a connected subgraph. We will call the collection of all vertices in Γ of a fixed type X a *cluster*. Thus, there are $\binom{m}{m-1}_q$ clusters. In each cluster, as $\mathrm{rank}(x_2^t) = 1$, and x_3, x_4 are free variables, we have $(q-1)q^{(\nu-m)}q^{(\nu-m)} = (q-1)q^{2(\nu-m)}$ neighbors for each vertex. If $x_2^t = 0$, we would not have a neighbor. We would still have a vertex in the cluster hence we would have $q^{2(\nu-m)+1}$ total vertices. Now, assume that there is a vertex X in cluster 1 and a vertex Y in cluster 2 that are not connected. Since each cluster has $(q-1)q^{2(\nu-m)}$ neighbors, there must be a total of $2(q-1)q^{2(\nu-m)} \geq q^{2(\nu-m)+1} > q^{2(\nu-m)+1} - 2$ vertices in the neighborhood of these two clusters. So, we have more vertices in the neighborhood than vertices in a cluster. The cluster must overlap and must be connected.

Now, we will count the number of connected components in $\Gamma(P)$. Each connected component depends on $\gamma \in \mathbb{F}_q^{(m-i)}$ for $0 < i \leq m$, so we just have to count the total number of possible γ's for varying i. The number of connected components is

$$\sum_{i=1}^{m} q^{m-i} = \frac{q^m - 1}{q - 1}.$$

$\square$

Lemma 2.3.6. *(Zeng et al., 2013, Lemma 3.2) The graph induced by* $S_P(0, m-1)$ *has* $\frac{q^m-1}{q-1}$ *connected components. Every component is a regular graph with* $\frac{q(q^{2(\nu-m)}-1)}{q-1}$ *vertices and with valency* $q^{2(\nu-m)}$*. The corresponding vertex set of a connected component will be denoted* $\overline{\Gamma(i,\gamma)}$

Before we move on to the main result, let us understand what we have constructed and why it is important. As stated in Section 1.3 and Brouwer et al. (2012), if we find a $1\frac{1}{2}$-design, we have a directed strongly regular graph. By Lemmas 2.3.5 and 2.3.6, we have have found regular subgraphs existing inside the distance classes. Specifically, distance classes one and two from P contain regular graphs. Our goal now is to create a strongly regular graph. By looking at the relationship of a cluster $C_1 \in \Gamma(P)$ of distance one and a cluster $C_2 \in S_P(0, m-1)$ of distance two we can actually form the strongly regular graph.

Using the notation in Chai et al. (2015), $\Gamma(i,\gamma)$ is a component of $\Gamma(P)$, a connected regular subgraph whose vertices are in $S_P(1, m-1)$. Similarly, $\overline{\Gamma(i,\gamma)}$ is a component of $S_P(0, m-1)$, a connected regular subgraph whose vertices are in $S_P(0, m-1)$.

Theorem 2.3.7. *(Chai et al., 2015, Theorem 3.4) Let the integers* m *and* i *satisfy* $1 \leq m \leq \nu$ *and* $0 < i \leq m$*. Suppose* $\gamma \in \mathbb{F}_q^{(m-i)}, \mathcal{P} = \Gamma(i,\gamma)$ *and* $\mathcal{B} = \{\Gamma(X) \cap S(1, m-1) | X \in \overline{\Gamma(i,\gamma)}\}$*. For* $x \in \mathcal{P}$ *and* $B \in \mathcal{B}$*,* $x \in B$ *if and only if* x *belongs to* B*. Then, the incidence structure* $\mathcal{T}_2 = (\mathcal{P}, \mathcal{B}, \epsilon)$ *is a* $1\frac{1}{2}$*-design with parameters*

$$v = q^{2(v-m)+1}, b = \frac{q(q^{2(\nu-m)}-1)}{q-1}, k = q^{2(\nu-m)}(q-1), r = q^{2(\nu-m)}-1,$$

$$\alpha = q^{4(\nu-m)+1} - 2q^{4(\nu-m)} + q^{4(\nu-m)-1} - q^{2(\nu-m)+1} + q^{2(\nu-m)}$$

and

$$\beta = q^{4(\nu-m)+1} - 2q^{4(\nu-m)} + q^{4(\nu-m)-1} - 2q^{2(\nu-m)+1} + 2q^{2(\nu-m)} + 2.$$

We will now provide an example that illustrates Lemma 2.3.5, Lemma 2.3.6 and Theorem 2.3.7.

Example 15. Let $\nu = 3, m = 2$ and $q = 2$. First we will show the components described in Lemmas 2.3.5 and 2.3.6.

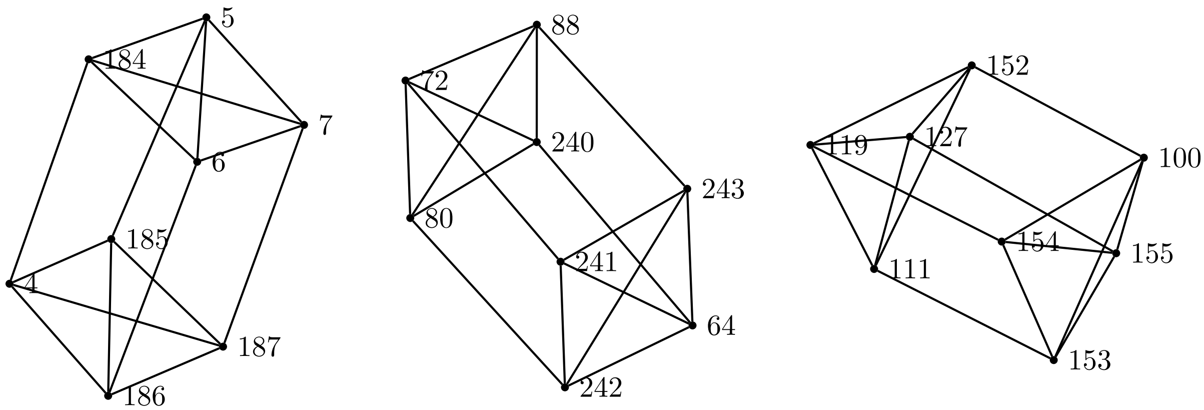


Figure 2.1 Neighbor graph, $\Gamma(P)$

The following is $\Gamma(P) = S_P(0, m-1) = S_P(0,1)$. Each vertex number corresponds to the index in the list of all vertices. For example, $P_4 =$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}.$$

For a full list of the corresponding matrices to each vertex please see Appendix B.

Each of these subgraphs are what we call a cluster in the proof of Lemma 2.3.5. The authors denote each of these cluster as components. The vertices in each of the components can be described by a matrix $X = \Gamma(i, \gamma)$. We will see the explicit blocks and points at the end of this example.

Similarly, we can construct the component of $\overline{\Gamma(i,\gamma)} = S_P(0,1)$.

Now, we take a component say $\{1, 2, 3, 216, 217, 232\}$. Now, we find the neighborhood of each vertex in this component. Due to the transitivity of subspaces in the symplectic space, the neighborhood of a vertex in our graph should behave exactly like $\Gamma(P)$. All of these neighborhood graphs will be regular graphs as described in Lemma 2.3.5. For example $P_1 =$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix},$$

gives us the following neighborhood.

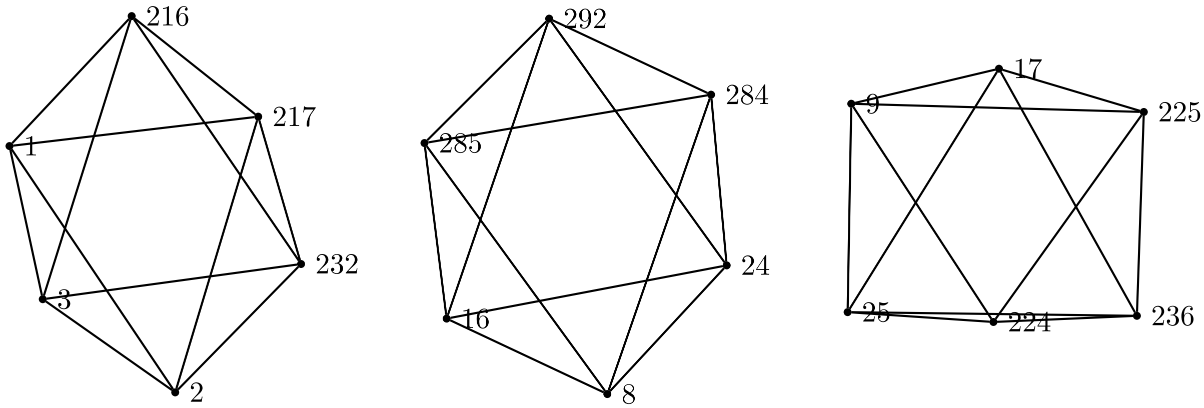


Figure 2.2 Distance 2 components, $S_P(0,1)$

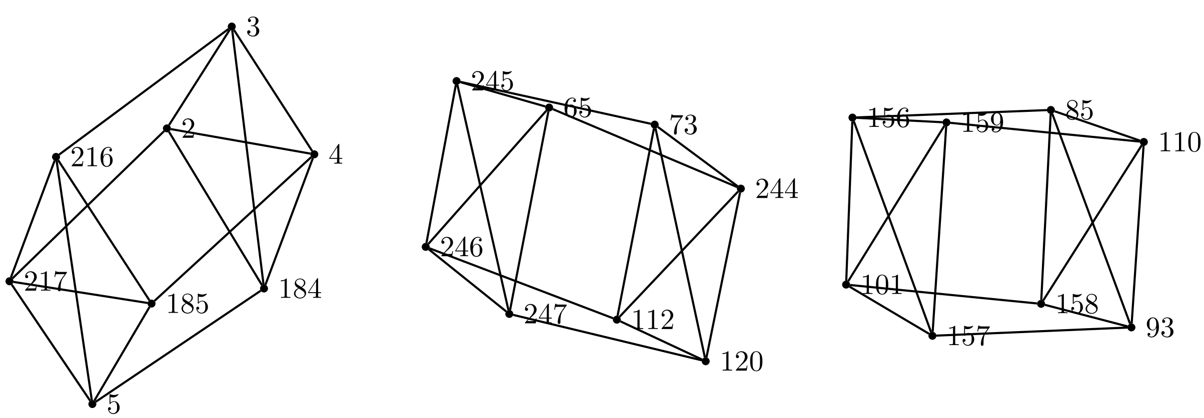


Figure 2.3 Neighbor graph of vertex in $S_P(0,1)$

We then take each of these neighborhoods and intersect them with $\Gamma(P)$. Each of the six vertices neighborhoods intersected with $\Gamma(P)$ will then give rise to a block and the points will all lie in one cluster.

We can now construct the design described in Theorem 2.3.7.

The points are $\{4, 5, 6, 7, 184, 185, 186, 187\}$

While the blocks are

$B_0 =$

$$\{184, 185, 4, 5\}$$

$B_1 =$

$$\{184, 186, 4, 6\}$$

$B_2 =$

$$\{184, 187, 4, 7\}$$

$B_3 =$

$$\{185, 187, 5, 7\}$$

$B_4 =$

$$\{185, 186, 5, 6\}$$

$B_5 =$

$$\{186, 187, 6, 7\}$$

The points are the exact points seen in the first cluster of $\Gamma(P)$.

We know that the vertices can be partitioned into distance classes $\Gamma^{(1)}(P), \Gamma^{(2)}(P), \cdots, \Gamma^{(d)}(P)$. However, as we try to use other distance classes, we run into an issue of finding a regular subgraph. Even in the case of distance 2 vertices, the set $S_P(2, m-2)$ is not explicitly connected regular components as we can see using the above example:

We need to further understand how the vertices inside each $\Gamma^{(e)}(P)$ are connected and attempt to characterize them. Furthermore, we need to study the relationship between the distance classes.

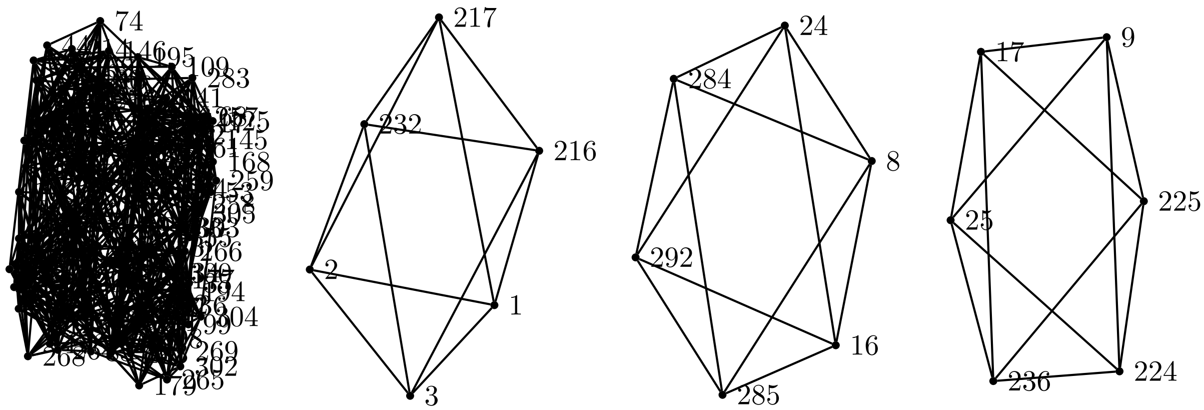


Figure 2.4 Distance 2 component, $S_P(2,0)$

Although we are only focusing on the symplectic geometry, similar arguments for the orthogonal space can be found in Feng et al. (2016). In general, many of the results are given in terms of the symplectic, orthogonal and unitary groups as well as their singular forms.

Definition 48. (Wan, 1993, Chapter 3.3) Let

$$K_1 = \begin{bmatrix} K & 0 \\ 0 & 0^{(l)} \end{bmatrix},$$

where K is the $2\nu \times 2\nu$ alternate matrix as defined in Theorem 2.1.3. The set of all $(2\nu + l) \times (2\nu + l)$ nonsingular matrices T over $\mathbb{F}_q$ satisfying

$$TK_lT^t = K_l$$

forms a group called the *singular symplectic group* of degree $2\nu + l$ and index ν over $\mathbb{F}_q$ and denoted by $Sp_{2\nu+l,\nu}(q)$.

We will only focus on the finite symplectic case however, our results can be realized in the orthogonal and unitary groups as well as their singular counterparts, as in Definition 48, with a bit of work. From Lemma 2.3.4 we can identify which distance classes are incident. The dual polar case, $m = \nu$, for the symplectic, unitary and orthogonal groups has been thoroughly studied in the following works Ma et al. (2011); Rieck (2005); Wan (2009); Wang et al. (2011). Currently, work is being done to understand the non-dual polar and non-isotropic cases. The first step is understanding how these distance classes intersect by studying the orbits of type (m, s)-subspaces under finite singular classical groups which was done in Guo (2010) and Guo and Wang (2009). These classes give rise to association schemes which can be found in Rieck (2005); Gao and He (2013b); Guo and Wang (2009); Gao and He (2013b) and Wang and Gao (2015).

As the distance classes are formed by $S_P(r, d)$ from Definition 46, we need to understand what vertices make up specific $S_P(r, d)$'s. By identifying these classes and the relationship between an $X \in S_P(r_1, d_2)$ and $Y \in S_P(r_2, d_2)$, we will be able to understand the distance classes. It became clear that the $S_P(r, d)$'s give rise to a previously unknown association scheme. All of the current research creates association schemes whose classes are formed by partitioning the Cartesian pairs of the totally isotropic subspaces (using maximally totally isotropic subspaces). Instead, we had the idea that follows in the next section.

We will now focus our attention to the association schemes constructed in Rieck (2005) and Ma et al. (2011). Note that Ma et al. (2011) was motivated by the work of Rieck (2005) and many of the theorems are viewed over the singular classic groups. The authors of Ma et al. (2011) were able to extend the results in Rieck (2005) by calculating the intersection numbers for their association scheme. Thus, if we let $l = 0$ in Ma et al. (2011), we will obtain the results described in Rieck (2005). In the next section, we will begin by introducing the association schemes that these authors studied.

2.4 Rieck and Gao's association schemes motivated by Grassmann graph

We will now describe the association scheme found in Rieck (2005). In this paper, Rieck's construction is motivated by the distance in the Grassmann graph.

Definition 49. Let $n \geq 2d$ and $V = \mathbb{F}_q^n$. The *Grassmann graph* $J_q(n, d)$ is the graph with the set of d-dimensional subspaces of V as its vertex set $\binom{n}{d}_q$ and two vertices X, Y are adjacent whenever $\dim(X \cap Y) = d - 1$.

Let $V = \mathbb{F}_q^{2\nu}$, where q is an odd prime. Equip the space with a non-degenerate symmetric bilinear form with Witt index ν, in other words we will use K from equation 37. Then, for any integer $1 \leq m \leq \nu$ the points of our association scheme are the m-dimensional totally isotropic subspaces of the 2ν-dimensional orthogonal vector space over $\mathbb{F}_q$. We denote this set also by $D(m, 2\nu; q)$.

Definition 50. Rieck (2005) Let U and W be m-dimensional totally isotropic subspaces of V. If $\dim(U \cap W) = m - k$ and $\dim(U^{\perp} \cap W) = m - \gamma$, then we say that U and W are (k, γ)-*associates*, also denoted k_γ, where U^γ is the dual subspace as in Definition 43.

Let $G = J_q(n, m)$, then two m-dimensional totally isotropic subspaces U, W that are k_γ associates are distance k in the Grassmann graph $J_q(n, d)$.

Theorem 2.4.1. *Rieck (2005) Let $\mathcal{N} = D(m, 2\nu; q)$, the collection of all m-dimensional totally isotropic subspaces of the orthogonal vector space $\mathbb{F}_q^{2\nu}$. Let $\mathcal{R}_{k_\gamma}$ be the collection of all ordered pairs of m-dimensional totally isotropic subspaces that are k_γ-associates. Then with $D(m, 2\nu; q)$ as the set of points, the relations $\mathcal{R}_{k_\gamma}$, ($0 \leq \gamma \leq k \leq m$) forms an association scheme.*

Now, the association scheme studied in Ma et al. (2011) is as follows:

Theorem 2.4.2 (Theorem 1.1)**.** *Ma et al. (2011) Let X be the set of all maximally totally isotropic subspaces of the orthogonal vector space $\mathbb{F}_q^{2\nu}$. For any two elements of X define*

$$R_i = \{(P, Q) |\, \dim(P \cap Q) = \nu - i\},$$

where $0 \leq i \leq \nu$. Then we obtain a family of symmetric association schemes.

Recall, Theorem 2.1.7 that is if U is totally isotropic subspace then $U \subseteq U^{\perp}$. If U is maximal totally isotropic then $U = U^{\perp}$ and Definition 50 forces $k = \gamma$ so that our ordered pairs $\mathcal{R}_{k_k}$ in Theorem 2.4.1 is the collection of all pairs of maximal totally isotropic subspaces satisfying $\dim(U \cap W) = \nu - k$.

Now, the goal is to calculate the intersection numbers of the association schemes.

For further clarity, Rieck (2005) is studying the association scheme formed over the finite classical groups for $1 \leq m \leq \nu$. While Ma et al. (2011) is studying the association scheme formed over finite singular classical groups for $m = \nu$. The two association schemes are exactly the same when $l = 0$ and $m = \nu$. Thus, the associated intersection numbers will only overlap in this case, so we will be looking at the intersection numbers as derived in Ma et al. (2011) with $l = 0$.

Theorem 2.4.3. *(van Lint and Wilson, 2001, Theorem 25.2) The number of surjective linear transformations from an n-dimensional vector space to an m-dimensional vector space V over $\mathbb{F}_q$ is*

$$\sum_{k=0}^{m} (-1)^{m-k} \binom{m}{k}_q q^{nk+\binom{m-k}{2}}$$

Proof. See page 338 of van Lint and Wilson (2001). □

Corollary 2.4.4. *(Wan, 1993, Theorem 1.10) The number of $m \times n$ matrices of rank i over $\mathbb{F}_q$ is given by the formula:*

$$N(i, m \times n, q) = q^{\frac{i(i-1)}{2}} \binom{m}{i}_q \prod_{t=n-i+1}^{n} (q^t - 1)$$

Proof. Follows from Theorem 2.4.3.

□

Lemma 2.4.5. *(Ma et al., 2011, Lemma 2.1) Let $\mathcal{S}(t, m)$ be the set of all $m \times m$ symmetric matrices of rank t over $\mathbb{F}_q$. Then,*

$$|\mathcal{S}(t, m)| = q^{\lfloor \frac{t}{2} \rfloor (\lfloor \frac{t}{2} \rfloor + 1)} \frac{\prod_{l=\lfloor \frac{t}{2} \rfloor + 1}^{m} (q^l - 1)}{(\prod_{l=1}^{\lfloor \frac{t}{2} \rfloor} (q^l + 1))(\prod_{l=1}^{m-t} (q^l - 1))}$$

Lemma 2.4.6. *(Ma et al., 2011, Lemma 2.3) Let $1 \leq m \leq n$. Then the number of $m \times n$ matrices (AB) of rank r over $\mathbb{F}_q$, with A symmetric is given by*

$$N^*(r; m \times n)$$

$$= \sum_{t=r-\min\{n-m,r\}}^{r} q^{t(n-m)} N(r-t, (m-t) \times (n-m), q) \left| \mathcal{S}(t, m) \right|,$$

where $\mathcal{S}(t, m)$ is defined above in Lemma 2.4.5.

Theorem 2.4.7. *(Ma et al., 2011, Theorem 2.5) Let* $1 \leq m \leq \nu$. *Fix two maximal totally isotropic subspaces* P *and* Q, *such that* $\dim(P \cap Q) = i$. *Where* $p^i_{jk}(\nu, \nu; m)$ *is the set of all maximal totally isotropic subspaces,* $R \in D(\nu, 2\nu; q)$, *such that* $\dim(P \cap R) = j$ *and* $\dim(R \cap Q) = k$. *Then*

$$p^i_{jk}(\nu, \nu; m) =$$

$$\sum_{\substack{\alpha+\gamma \leq \nu-i, \beta+\rho \leq i, \beta+\gamma=k \\ \alpha+\beta+\gamma+\rho=m, \beta \leq j \leq \alpha+\beta \\ \alpha+\beta-j-\min\{\alpha+\beta-j, v-i-\gamma-\alpha\} \leq t \leq \alpha+\beta-j}} q^{(\alpha+\gamma)(i-\beta-\rho)+\rho(2\nu-i-m)+\frac{\rho(\rho+1)}{2}} F^*(i, \alpha, \beta, \gamma, \rho) N^*_\alpha(\alpha+\beta-j; \alpha \times (\nu-i-\gamma)),$$

where

$$F^*(i, \alpha, \beta, \gamma, \rho) = \binom{\nu - i}{\alpha}_q \binom{i}{\beta}_q \binom{\nu - i - \alpha}{\gamma}_q \binom{i - \beta}{\rho}_q$$

Corollary 2.4.8. *(Ma et al., 2011, Corollary 2.6) Let* $1 \leq m \leq \nu$. *Then,*

$$p^\nu_{jj}(\nu, \nu; m) = q^{(m-j)(\nu-m)+\frac{(m-j)(m-j+1)}{2}} \binom{\nu}{j}_q \binom{\nu - j}{m - j}_q$$

Notice that their association scheme uses the fact that totally isotropic subspaces are transitive sets, so they only need to fix two representatives. As we can see the intersection numbers are difficult to calculate and give little understanding to the structure of the $S(r, d) = \{x, y \in D(m, 2\nu; q), \operatorname{rank}(xKy^t) = r, \dim(x \cap y) = d\}$ classes used to construct a $1\frac{1}{2}$-design in Theorem 2.3.7. Our goal is to determine whether there exists a 3-class association scheme that gives rise to new $1\frac{1}{2}$-designs or strongly regular graphs.

Before we move on to the next chapter, we will make a few remarks. The dual polar graphs (in the extreme case of $m = \nu$) form a family of well-known distance-regular graphs (which are P- and Q-polynomial association schemes, see Brouwer (1989)). Applying the enumeration results of Wan (1993), many researchers in Wans school calculated the parameters of dual polar graphs. As a generalization of dual polar graphs, mathematicians Zeng et al. (2013); Tang and Wan (2006); Liu et al. (2012); Ma et al. (2011) have been studying various subspaces of a given dimension in classical (symplectic, unitary and orthogonal) spaces. The focus of studies in this vain has been in the set of the totally isotropic subspaces with the same dimension in a classical space forms an orbit under the action of the corresponding classical group. Researchers try to determine the orbitals and the rank of the permutation action and calculate the length of each suborbit (or the size of each orbital).

In the next chapter, we have explored the general cases with $1 < m < \nu$. We have taken the partition of the totally isotropic 2-dimensional subspaces into $S(C_i)$s as association relations and we calculated their character tables. However, existence of such association schemes has been already discussed in the work of Derr (1980); Wei and Wang (1996).

CHAPTER 3. ASSOCIATION SCHEMES AND THEIR INTERSECTION NUMBERS

3.1 Association schemes defined on $D(m, 2\nu; q)$

In this section, we construct an association scheme on the set $\Omega = D(m, 2\nu; q)$ of all m-dimensional totally isotropic subspaces of symplectic vector space of degree 2ν over $\mathbb{F}_q$. Let x and y be two elements of Ω. Being m-dimensional totally isotropic subspaces in the 2ν-dimensional symplectic vector space over $\mathbb{F}_q$, we know that $\mathrm{rank}(xKx^t) = \mathrm{rank}(yKy^t) = 0$ where $K = \begin{bmatrix} 0 & I^{(v)} \\ -I^{(v)} & 0 \end{bmatrix}$ as before. Furthermore, there exist nonnegative integers r, $(0 \le r \le m)$ and d, $(0 \le d \le m)$ such that $\mathrm{rank}(xKy^t) = r$ and $\dim(x \cap y) = d$. With an appropriate set of pairs $\{(r_i, d_i) : 0 \le i \le \mathcal{D}\}$, every pair $(x, y) \in \Omega \times \Omega$ holds $\mathrm{rank}(xKy^t) = r_i$ and $\dim(x \cap y) = d_i$ for some i, $0 \le i \le \mathcal{D}$. Without loss of generality, let $C_0 = (0, m), C_1 = (r_1, d_1), \cdots, C_{\mathcal{D}}(r_{\mathcal{D}}, d_{\mathcal{D}})$ be the collection of all feasible (rank,dimension)-pairs for given m and ν.

Then, $\Omega \times \Omega$ is partitioned by the subsets $S(C_0), S(C_1), \cdots, S(C_{\mathcal{D}})$ where

$$S(C_i) = \{(x, y) \in \Omega \times \Omega : (\mathrm{rk}(xKy^t), \dim(x \cap y)) = C_i\}, \quad 0 \le i \le \mathcal{D}.$$

Theorem 3.1.1. *Let $\Omega = D(m, 2\nu; q)$ and $S(C_i)$ for $i = 0, 1, \ldots, \mathcal{D}$ as the above. If we define the relations $R_i \subseteq \Omega \times \Omega$ by $R_i = S(C_i)$ for $i = 0, 1, \ldots, \mathcal{D}$, then $\mathcal{X} = (\Omega, \{R_i\}_{0 \le i \le \mathcal{D}})$ is a $\mathcal{D}$-class symmetric association scheme.*

Proof.

(1) $R_0 = \{(x, x) : x \in \Omega\}$.

(2) Every pair (x, y) of elements $x, y \in \Omega$ belongs to C_i for some $0 \le i \le \mathcal{D}$ according to the rank of xKy^t and the dimension of $x \cap y$. The i is uniquely determined, and so, it follows that $R_0, R_1, \ldots, R_{\mathcal{D}}$ form a partition of $\Omega \times \Omega$.

(3) For each i, $0 \leq i \leq \mathcal{D}$, $R_i^t = R_i$ holds.

(4) For every $h, i, j \in \{0, 1, \ldots, \mathcal{D}\}$, and for any $(x, y) \in R_h$, the number of $z \in S_x(C_i) \cap S_y(C_j)$ is a constant p_{ij}^h. The intersection number is independent from the choice of (x, y) in $S(C_h)$ as

$$|S_x(C_i) \cap S_y(C_j)| = |S_z(C_i) \cap S_w(C_j)|$$

as long as $(z, w) \in S(C_h)$. This can be proved by using the facts that $G = Sp_{2\nu}(q)$ acts transitively on Ω and doubly transitively on each $S(C_i)$, together with the fact that $S_x(C_0), S_x(C_1), \ldots, S_x(C_{\mathcal{D}})$ are the orbits of the point stabilizer $Stab_x(G)$ acting on $\Omega - \{x\}$. See Lemma 3.2.2 below for further details.

We note that the existence of this association scheme has been noticed by Wei and Wang (1996). □

3.2 Counting $S_x(r, d)$ the (r, d)-associates of a point x

Fix $0 < m \leq \nu$ and let q be a power of a prime p. Let, $\Omega = D(m, 2\nu; q)$, the collection of all totally isotropic subspaces of type $(m, 0)$ in the symplectic vector space of degree 2ν over $\mathbb{F}_q$. Now extend Definition 46 to any point in Ω, so that we can partition Ω around any one of its points and include the point itself by allowing $d = m$. Recall that the set of totally isotropic subspaces is a transitive set under the action of G by Theorem 2.1.9 thus, the sizes of our partitions will be the same for any point $x \in \Omega$.

Before we can construct our association scheme, whose underlying set will be Ω and relations $R_0, \cdots, R_{\mathcal{D}}$ will be the subsets of $\Omega \times \Omega$, we need to calculate $\mathcal{D}$, the number of classes.

Lemma 3.2.1. *Let $m = \nu - a$, for $0 \leq a < \nu$ and let $\Omega = D(m, 2\nu; q)$, the collection of all totally isotropic subspaces of dimension m. Then $\overline{\mathcal{D}}$ is the number of tuples (r, d), $0 \leq r, d \leq m$ such that $S(r, d) > 0$, is equal to:*

$$\overline{\mathcal{D}} = \begin{cases} \frac{(m+1)(m+2)}{2} & \text{if } \nu \leq 2a \\ (\nu - m + 1)(m - a + 1) + \frac{a(a+1)}{2} & o.w. \end{cases}$$

We then let $\mathcal{D}$ be the number of tuples (r, d) excluding the class $S(0, m)$. Hence,

$$\mathcal{D} = \overline{\mathcal{D}} - 1.$$

Proof. The proof will be shown in two cases based on when the max is 0 or $2m - \nu - d$, where $m = \nu - a$. Notice that $2m - \nu - d \leq 0$ when $2(\nu - a) - \nu - d = \nu - 2a - d \leq 0$. The largest value occurs when $d = 0$, giving us $\nu - 2a \leq 0$. When $\nu \leq 2a$ we have that $2m - \nu - d \leq 0$ for all $0 \leq d \leq m$.

- **Case 1:** As stated above, when $\nu \leq 2a$, $2m - \nu - d \leq 0$ for all $0 \leq d \leq m$. When $d = i$, $0 \leq r \leq m - i$. Thus there will be $(m - i + 1)$ values of r for each $0 \leq d \leq m - 1$.

$$\sum_{i=0}^{m} (m - i + 1) = \left(\sum_{i=1}^{m+1} i \right) = \frac{(m+2)(m+1)}{2}.$$

- **Case 2:** When $\nu > 2a$, we are no longer guaranteed that the lower bound of r is 0.

  (i) if $0 \leq d \leq m - a$, we have that $2m - \nu - d \geq 2m - \nu - (m - a) = m - \nu + a = 0$ as $m = \nu - a$. Thus, $\max\{0, 2m - \nu - d\} = 2m - \nu - d$ and $2m - \nu - d \leq r \leq m - d$ has exactly $m - d - (2m - \nu - d) + 1 = \nu - m + 1$ values giving us:

$$\mathcal{D} = \sum_{d=0}^{m-a} \nu - m + 1 = (m - a + 1)(\nu - m + 1)$$

  classes.

  (ii) if $m - a < d \leq m$, then $\max\{0, 2m - \nu - d\} = 0$ and $0 \leq r \leq m - d$. Once again this gives us the geometric sum:

$$\sum_{d=m-(a+1)}^{m} (m - d + 1) = \sum_{i=1}^{a} i = \frac{a(a+1)}{2}$$

  Combining the two options we have

$$(m - a + 1)(\nu - m + 1) + \frac{a(a+1)}{2}$$

$\square$

Definition 51. Let $0 \leq d \leq m$ and $\max\{0, 2m-\nu-d\} \leq r \leq m-d$. For any $z \in \Omega$, let the (r,d)-associates of z be the collection,

$$S_z(r,d) = \{y \in \Omega |\, \mathrm{rank}(zKy^t) = r, \dim(z \cap y) = d\}$$

Proposition 3.2.2. *Let $z \in D(m,s,2\nu;q)$ and r,d be values such that $S_z(r,d)$ exists. Then, $H = Stab_z(Sp_{2\nu}(q))$ sends $x \in D(m,s,2\nu;q)$ to $y \in D(m,s,2\nu;q)$ and if $x \in S_z(r,d)$, $y \in S_z(r,d)$.*

Proof. Let $x \in D(m,s,2\nu;q)$. Then $\mathrm{rank}(xKx^t) = 2s$. As $H \subset Sp_{2\nu}(q)$, for any $T \in H$ we have $TKT^t = K$. Thus, $(xT)K(xT)^t = xTkT^tx^t = xKx^t$. So $\mathrm{rank}(xKx^t) = \mathrm{rank}(xTK(xT)^t) = 2s$.

Let $x \in S_z(r,d)$ and $H = Stab_z(Sp_{2\nu}(q))$. We will show that if $x \in S_z(r,d)$ then $xT \in S_z(r,d)$, for all $T \in H$. Recall that $TKT^t = K$.

As $x \in S_z(r,d)$, $\mathrm{rank}(zKx^t) = r$ and $\dim(z \cap x) = d$. For any $T \in H$ we have the following:

$$\mathrm{rank}(zK(xT)^t) = \mathrm{rank}(zKT^tx^t) = \mathrm{rank}(zTKT^tx^t) = \mathrm{rank}(zKx^t) = r$$

$$\dim(z \cap xT) = \dim(zT \cap xT) = \dim(z \cap x) = d,$$

where $\dim(zT \cap xT) = \dim(z \cap x)$ as T can be viewed as a linear transformation which maps the subspace $z \cap x$ to the subspace $(z \cap x)T$.

Lastly, we will need to show that $xT \in D(m, 2\nu; q)$. As $x \in D(m, 2\nu; q)$,

$$xTK(xT)^t = xTKT^tx^t = xKx^t = 0.$$

Hence, $xT \in D(m, 2\nu; q)$.

□

Furthermore,

If $z = yT_1$ and $y = zT_2$. Then, $z = zT_2T_1$, hence $T_2T_1 \in Stab_z(Sp_{2\nu}(q))$. If $y = xT'$, then $yT_1 = xT'T_1$ and $z = xT'T_1$.

So if $z = yT_1$ where $T_1 \in H$. Then if $y = xT_2$, we have $yT_1 = xT_2T_1$, $z = xT_2T_1$

3.3 Basic enumerations

We would now like to calculate the size of $|R_i| = |\Omega||S_z(C_i)| = |\Omega|k_i$, where k_i is the subdegree, for $i = 0, \cdots, \mathcal{D}$. Recall that $|S_z(C_i)| = |\{y : (z, y) \in S(C_i)\}|$. As H is transitive on $S_z(C_i)$, we can fix z as

$$z = \begin{array}{c} \begin{array}{cccc} m & m & \nu - m & \nu - m \end{array} \\ \begin{pmatrix} I^{(m)} & 0 & 0 & 0 \end{pmatrix} \end{array} \; m \, .$$

The rest of the counting arguments are counting $|\{y : (z, y) \in S(C_i)\}|$. To obtain R_i from Theorem 3.1.1, we can multiply $|\{y : (z, y) \in S(C_i)\}|$ by $|\Omega|$.

Lemma 3.3.1. *Let x be of type $(m, 0) \in Sp_{2\nu}(q)$. Let*

$$\chi_d = |\{y \in D(m, 2\nu; q) : \dim(x \cap y) = d\}|,$$

with $\chi_m = 1$ as x is the only matrix that will intersect itself. Then,

$$\chi_i = \binom{m}{i}_q \left(N(i, m; 2\nu) - \sum_{j=i+1}^{m} \chi_j \right)$$

Proof. Starting with $\chi_m = 1$, we have

$$\chi_{m-1} = \binom{m}{m-1}_q (N(m-1, m; 2\nu) - 1)$$

as there are $\binom{m}{m-1}_q$ totally isotropic $(m-1)$-dimensional subspaces of an m-dimensional totally isotropic space and each one is contained in $N(m-1, m; 2\nu)$ totally isotropic subspaces of dimension m including the original space x, thus we subtract 1 to remove it.

Assume that $\chi_i = \binom{m}{i}_q \left(N(i, m; 2\nu) - \sum_{j=i+1}^{m} \chi_j \right)$ for $i \geq 1$.

Then let's find the size of χ_{i-1}. There are $\binom{m}{i-1}_q$ choices for an $i-1$ totally isotropic subspace in an m-dimensional totally isotropic subspace each of which is contained in exactly $N(i-1, m; 2\nu)$ totally isotropic subspaces of dimension m. Notice that the subspaces also contain all totally isotropic subspaces that contain an $i, i+1, \cdots, m$. Thus we obtain our desired formula. $\square$

Proposition 3.3.2. *The subspace*

$$y = \begin{array}{c} \\ \\ \\ \end{array}\begin{array}{cccccc} d & m-d & m-r & r & 2(\nu-m)-m-d-r & m-d-r \end{array}$$

$$y = \begin{pmatrix} I^{(d)} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I^{(r)} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I^{(m-d-r)} \end{pmatrix} \begin{array}{c} d \\ r \\ m-d-r \end{array}$$

is in $S_z(r,d)$ *that is* $yKy^t = 0$ *and* $\operatorname{rank}(zKy^t) = r$ *while* $\dim(z \cap y) = d$.

Proof. Let

$$\begin{array}{cccc} m & m & \nu-m & \nu-m \end{array}$$

$$y = \begin{pmatrix} A & 0 & 0 & 0 \\ 0 & 0 & 0 & C \\ 0 & B & 0 & 0 \end{pmatrix} \begin{array}{c} d \\ m-d-r \\ r \end{array},$$

where $\operatorname{rank}(A) = d, \operatorname{rank}(B) = r$ and $\operatorname{rank}(C) = m-d-r$, so that $\operatorname{rank}(y) = m$. Then

$$\begin{array}{cccccc} d & m-d & m-r & r & \nu-m & \nu-m \end{array}$$

$$yKy^t = \begin{pmatrix} 0 & 0 & 0 & AB^t & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -BA^t & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{array}{c} d \\ m-d-r \\ r \end{array}$$

If $-BA^t = 0$ and $AB^t = 0$ then $y \in D(m, 0; 2\nu)$. Let $A = \begin{bmatrix} I^{(d)} & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & I^{(r)} \end{bmatrix}$. Then $AB^t = 0$ and $-BA^t = 0$. Finally, when $\operatorname{rank}(C) = m-d-r$, y is an m-dimensional totally isotropic subspace.

As $zK = \begin{array}{c} \begin{array}{cccc} m & \nu-m & m & \nu-m \end{array} \\ \begin{pmatrix} 0 & I^{(m)} & 0 & 0 \end{pmatrix} \end{array} m$ then $zKy^t = I^{(m)}B^t = B^t$. As $\operatorname{rank}(B) = r$ we have $\operatorname{rank}(B^t) = r$. Lastly, $z \cap y = A$ as z spans the first m rows and columns. Therefore, $\dim(z \cap y) = \operatorname{rank}(A) = d$. Thus, $y \in S_z(r,d)$. $\square$

Proposition 3.3.3.

$$|S_z(0,m-i)| = \binom{m}{m-i}_q q^{i^2} N'(i,2(\nu-m)) \textit{ for } 0 \le i \le m$$

and has the form

$$\begin{array}{cc} & \begin{array}{ccccc} m-i & i & m & \nu-m & \nu-m \end{array} \\ S_z(0,m-i) = & \begin{pmatrix} I^{(m-i)} & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & \beta & \gamma \end{pmatrix} \begin{array}{c} m-i \\ i \end{array} \end{array}$$

Proof. Let $y = \begin{bmatrix} A & B & C & D \end{bmatrix}$ then,

$\dim(z \cap y) = m-i$ and $\text{rank}(zKy^t) = 0$. Notice that $zKy^t = B^t$, hence $\text{rank}(B) = 0$ so $B = 0$. As $\dim(z \cap y) = m-i$, we have $\text{rank}(A) \ge m-i$. We can rewrite $A = \begin{bmatrix} I^{(m-i)} & 0 \\ 0 & \alpha \end{bmatrix}$ as we can clear the rows and columns using row reduction and for any $\alpha \in \mathbb{F}_q^{i^2}$. Lastly, we notice that we can clear the first $m-i$ rows of the matrix, giving us the form

$$\begin{array}{cc} & \begin{array}{ccccc} m-i & i & m & \nu-m & \nu-m \end{array} \\ S_z(0,m-i) = & \begin{pmatrix} I^{(m-i)} & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & \beta & \gamma \end{pmatrix} \begin{array}{c} m-i \\ i \end{array} \end{array}$$

Lastly, as y must be totally isotropic of dimension m we must have $yKy^t = 0$, multiplying this out we obtain :

$yKy^t = \begin{bmatrix} \beta & \gamma \end{bmatrix} K_{2(\nu-m)} \begin{bmatrix} \beta^t \\ \gamma^t \end{bmatrix}$ has to be rank i. Notice that this is exactly, the number of totally isotropic subspaces of type $(i,0)$ in $2(\nu-m)$. Which would be $N'(i,2(\nu-m))$. There are $\binom{m}{m-i}_q$ choices for $I^{(m-i)}$ and q^{i^2} choices for α. Combining all of these results we obtain our desired formula:

$$|S_z(0,m-i)| = \binom{m}{m-i}_q q^{i^2} N'(i,2(\nu-m)) \text{ for } 0 \le i \le m$$

□

Proposition 3.3.4. *Let A be an $n \times n$ matrix over $\mathbb{F}_q$. Then,*

$$\operatorname{rank}(A - A^t) = 2k, k \in \mathbb{N}$$

Proposition 3.3.5. *The number of $m \times m$ matrices over $\mathbb{F}_q$, A such that $\operatorname{rank}(A - A^t) = 2s$, where $0 \leq s \leq \lfloor \frac{m}{2} \rfloor$ is $q^{\frac{m(m+1)}{2}}$*

Proof. Now, notice that $A - A^t$ is skew symmetric. So $A - A^t = S = \begin{bmatrix} 0 & S' \\ -S' & 0 \end{bmatrix}$, a skew-symmetric matrix, where $S^t = -S$. Clearly, $|\{A \in M_n(\mathbb{F}_q) : \operatorname{rank}(A - A^t) = 0\}| = q^{\frac{m(m+1)}{2}}$. Notice that $\operatorname{rank}(A - A^t) = 0$ is the 0 matrix. Then, for each symmetric matrix S of rank i, we can let $A' = A + \begin{bmatrix} 0 & S' \\ 0 & 0 \end{bmatrix}$, so that $A - A^t = S$. That is, the number of type S matrices is the same as rank 0. Thus, each class has the same number of elements and we have exactly $q^{\frac{m(m+1)}{2}}$, $m \times m$ matrices such that $\operatorname{rank}(A - A^t) = 2s$ for $0 \leq s \leq \lfloor \frac{m}{2} \rfloor$.

$\square$

Proposition 3.3.6. $|S_z(r, d)| = \binom{m}{d}_q \binom{m-d}{r}_q (q^{\frac{r(r+1)}{2}}) \left(\sum_{p=0}^{\min\{r, 2(\nu - m)\}} N(p, r \times (2(\nu - m)), q) \right)$ *where $r + d = m$.*

Proof. Let $y = \begin{bmatrix} A & B & C & D \end{bmatrix}$ Notice that $\operatorname{rank}(zKy^t) = \operatorname{rank}(B^t) = r$. Similarly, $\operatorname{rank}(z \cap y) = d$ where $r + d = m$ hence we can always reduce any matrix y to the following form:

$$\begin{array}{cccccccc} & d & m-d=r & m-r=d & r & 2(\nu-m) & \\ y = & I^{(d)} & 0 & 0 & 0 & 0 & d \\ & 0 & \alpha_2 & 0 & I^{(r)} & \beta & r \end{array}$$

Then

$$yKy^t = -\alpha_2^t + \alpha_2 + \begin{bmatrix} 0 \\ \beta \end{bmatrix} \begin{bmatrix} 0 & I^{(\nu-m)} \\ -I^{(\nu-m)} & 0 \end{bmatrix} \begin{bmatrix} 0 \\ \beta \end{bmatrix}^t = 0,$$

So there are $\binom{m}{d}_q$ choices for $I^{(d)}$. As we can see this is exactly the same issue as in the case above, just using smaller matrices. We have $q^{r(r+1)/2}$ values for each α and we want to find the β of size

$r \times 2(\nu - m)$ of rank i. Lastly, we have $\binom{m}{d}_q$ choices for $I^{(d)}$ and $\binom{m-d}{r}_q$ choices for $I^{(r)}$, giving us our desired formula:

$\binom{m}{d}_q \binom{m-d}{r}_q (q^{\frac{r(r+1)}{2}}) \left(\sum_{p=0}^{\min\{r,2(\nu-m)} N(p, r \times (2(\nu - m)), q)\right)$

□

Proposition 3.3.7.

$$|S_z(r,d)| = \binom{m}{d}_q \binom{m-d}{r}_q q^{\frac{r(r+1)}{2}} \left(\sum_{p=0}^{\min\{r,2(\nu-m)\}} N(p, r \times (2(v-m)), q) \right) N'(m-d-r, 2(\nu-m)) q^{(m-d-r)^2}$$

where $r + d < m, r > 0$ and $d > 0$.

Proof. Let

$$x = \begin{array}{cc} \begin{array}{ccc} m & m & 2(\nu-m) \end{array} & \\ \begin{pmatrix} A & B & C \end{pmatrix} & m \end{array}$$

and

$$y = \begin{array}{cc} \begin{array}{ccc} m & m & 2(\nu-m) \end{array} & \\ \begin{pmatrix} D & E & F \end{pmatrix} & m \end{array}$$

Then

$$xKy^t = -BD^t + AE^t + C \begin{bmatrix} 0^{(\nu-m)} & I^{(\nu-m)} \\ -I^{(\nu-m)} & 0^{(\nu-m)} \end{bmatrix} F^t$$

Let $a = m - r - d$. We claim that

$$y = \begin{array}{cc} \begin{array}{cccccccccc} d & a & r & d & a & r & \nu-(m+a) & a & \nu-(m+a) & a \end{array} & \\ \begin{pmatrix} I^{(d)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_1 & 0 & 0 & 0 & 0 & \beta_1 & \beta_2 & \beta_3 & I^{(a)} \\ 0 & 0 & \alpha_4 & 0 & 0 & I^{(r)} & \beta_4 & \beta_5 & \beta_6 & \beta_7 \end{pmatrix} & \begin{array}{c} d \\ a \\ r \end{array} \end{array}$$

Originally we have

$$\begin{array}{cc} & \begin{array}{cccccccccc} d & a & r & d & a & r & \nu-(m+a) & a & \nu-(m+a) & a \end{array} \\ y = & \left(\begin{array}{cccccccccc} I^{(d)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_1 & \alpha_2 & 0 & 0 & 0 & \beta_1 & \beta_2 & \beta_3 & I^{(a)} \\ 0 & \alpha_3 & \alpha_4 & 0 & 0 & I^{(r)} & \beta_4 & \beta_5 & \beta_6 & \beta_7 \end{array}\right) \begin{array}{c} d \\ a \\ r \end{array} \end{array}$$

which gives us

$$yKy^t = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -\alpha_2^t & -\alpha_4^t \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \alpha_2 \\ 0 & 0 & \alpha_4 \end{bmatrix} + C \begin{bmatrix} 0^{(\nu-m)} & I^{(\nu-m)} \\ -I^{(\nu-m)} & 0^{(\nu-m)} \end{bmatrix} C^t$$

Now if C is totally isotropic, then $\alpha_2 = 0$ otherwise

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -\alpha_2^t & -\alpha_4^t \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \alpha_2 \\ 0 & 0 & \alpha_4 \end{bmatrix} \neq 0.$$

By Proposition 3.3.5 the number of matrices A that satisfy the totally isotropic case must be the same number as the non-isotropic case. Thus, α_2 must always be 0, otherwise we would be able to construct more cases for a non-isotropic class.

Similarly, as $I^{(r)}$ does not have to be in the exact position given, we can have it in the following form:

$$\begin{array}{cc} & \begin{array}{cccccccccc} d & a & r & d & a & r & \nu-(m+a) & a & \nu-(m+a) & a \end{array} \\ y = & \left(\begin{array}{cccccccccc} I^{(d)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_1 & \alpha_2 & 0 & I^{(r)} & 0 & \beta_1 & \beta_2 & \beta_3 & \beta_4 \\ 0 & \alpha_3 & \alpha_4 & 0 & 0 & 0 & \beta_5 & \beta_6 & \beta_7 & I^{(a)} \end{array}\right) \begin{array}{c} d \\ r \\ a \end{array} \end{array}$$

which gives us

$$yKy^t = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\alpha_1^t & -\alpha_3^t \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & \alpha_1 & 0 \\ 0 & \alpha_3 & 0 \end{bmatrix} + C \begin{bmatrix} 0^{(\nu-m)} & I^{(\nu-m)} \\ -I^{(\nu-m)} & 0^{(\nu-m)} \end{bmatrix} C^t$$

Now if C is totally isotropic, then $\alpha_3 = 0$ otherwise

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -\alpha_1^t & -\alpha_3^t \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & \alpha_1 & 0 \\ 0 & \alpha_3 & 0 \end{bmatrix} \neq 0.$$

By Proposition 3.3.5 the number of matrices A that satisfy the totally isotropic case must be the same number as the non-isotropic case. Thus, α_3 must always be 0, otherwise we would be able to construct more cases for a non-isotropic class.

This gives us our desired form:

$$y = \begin{array}{c} \begin{array}{cccccccccc} d & a & r & d & a & r & \nu-(m+a) & a & \nu-(m+a) & a \end{array} \\ \begin{pmatrix} I^{(d)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_1 & 0 & 0 & 0 & 0 & \beta_1 & \beta_2 & \beta_3 & I^{(a)} \\ 0 & 0 & \alpha_4 & 0 & 0 & I^{(r)} & \beta_4 & \beta_5 & \beta_6 & \beta_7 \end{pmatrix} \end{array} \begin{array}{c} \\ d \\ a \\ r \end{array}$$

Now, we have that

$$yKy^t = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\alpha_4^t \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \alpha_4 \end{bmatrix} + C \begin{bmatrix} 0^{(\nu-m)} & I^{(\nu-m)} \\ -I^{(\nu-m)} & 0^{(\nu-m)} \end{bmatrix} C^t$$

Therefore, we can choose any value for α_1 giving us q^{a^2} choices. Let $CKC^t = 0$. Then, $\alpha_4 = \alpha_4^t$. So there are $q^{\frac{r(r+1)}{2}}$ choices for α_4. Now, $\begin{bmatrix} \beta_1 & \beta_2 & \beta_3 & I^{(a)} \end{bmatrix}$ must be of rank a as well as totally isotropic as in Proposition 3.3.3. There are $N'(a, 2(\nu - m))$ such subspaces. Lastly, $\begin{bmatrix} \beta_4 & \beta_5 & \beta_6 & \beta_7 \end{bmatrix}$ can be of rank $0, 1, \cdots, \min\{r, 2(\nu - m)\}$. As in Proposition 3.3.6 we can use Theorem 2.4.4 to count the number of rank i matrices of size $m \times n$ over $\mathbb{F}_q$, giving us $\sum_{p=0}^{\min\{r,2(\nu-m)\}} N(p, r \times (2(v - m)), q)$ choices. In total, we have $\binom{m}{d}_q$ choices for $I^{(d)}$ from z, we have $\binom{m-d}{r}_q$ choices for $I^{(r)}$, $q^{\frac{r(r+1)}{2}}$ choices for α_4, q^{a^2} choices for α_1, $N'(a, 2(\nu - m))$ choices for the $a \times 2(\nu - m)$ rank a row and $\sum_{p=0}^{\min\{r,2(\nu-m)\}} N(p, r, 2(v - m), q)$ choices for the last $r \times 2(\nu - m)$ row of y. This gives us our desired formula of:

$$|S_z(r,d)| = \binom{m}{d}_q \binom{m-d}{r}_q q^{\frac{r(r+1)}{2}} \left(\sum_{p=0}^{\min\{r,2(\nu-m)\}} N(p, r \times (2(v-m)), q) \right) N'(m-d-r, 2(\nu-m)) q^{(m-d-r)^2}$$

□

3.4 List of constructed association schemes

We will use the following order, where $(x,y) \in R_i = (r_i, d_i)$ implies that $\operatorname{rank}(xMy^t) = r_i$ and $\dim(x \cap y) = d_i$ where

$$M = \begin{pmatrix} 0 & I^{(m)} & & \\ -I^{(m)} & 0 & & \\ & & 0 & I^{(v-m)} \\ & & -I^{(v-m)} & 0 \end{pmatrix},$$

$$R_0 = (0,2)$$

$$R_1 = (0,1)$$

$$R_2 = (1,1)$$

$$R_3 = (1,0)$$

$$R_4 = (2,0)$$

When $\nu > 3$,

$$R_5 = (0,0)$$

We have also fixed our representatives to count x and y via the form

$$y = \begin{array}{c} \begin{array}{ccccccc} d & m-d & m-r & r & \nu-m & \nu+d+r & m-d-r \end{array} \\ \begin{pmatrix} I^{(d)} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I^{(r)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I^{(m-d-r)} \end{pmatrix} \end{array} \begin{array}{c} d \\ r \\ m-d-r \end{array}$$

Theorem 3.4.1. *For $m = 2$, $\nu = 3$ and $\Omega = D(2, 6; q)$ with the relations*

$$R_0 = S(0,2), R_1 = S(0,1), R_2 = S(1,1),$$

$$R_3 = S(1,0), R_4 = S(2,0)$$

we have a 4-class association scheme with the following character table:

$$\begin{array}{c} \begin{bmatrix} 1 & q(q+1)^2 & q^3(q+1) & q^4(q+1)^2 & q^7 \\ 1 & (2q-1)(q+1) & q(q^2-1) & q^2(q-2)(q+1) & -q^4 \\ 1 & -q-1 & q(q^2+1) & -q^3(q+1) & q^4 \\ 1 & -q-1 & 0 & q(q+1) & -q^2 \\ 1 & q^2-1 & -q(q+1) & -q(q^2-1) & q^3 \end{bmatrix} \end{array} \begin{array}{l} 1 \\ \frac{1}{2}q(q^2+1)(q^2+q+1) \\ \frac{1}{2}q(q+1)(q^3+1) \\ q^3(q^4+q^2+1) \\ q^2(q^4+q^2+1) \end{array}$$

Theorem 3.4.2. *For $m = 2$, $\nu > 3$ and $\Omega = D(2, 2\nu; q)$ with the relations defined by*

$$R_0 = S(0,2), R_1 = S(0,1), R_2 = S(1,1),$$

$$R_3 = S(1,0), R_4 = S(2,0), R_5 = S(0,0),$$

we have 5-class association schemes with the following character table:

$$\begin{array}{cc}
\begin{bmatrix}
1 & q(q+1)\frac{q^{2(l-2)}-1}{q-1} & q^{2(l-2)+1}(q+1) & q^{2(l-2)+2}(q+1)\frac{q^{2(l-2)}-1}{q-1} & q^{4(l-2)+3} & q^4\frac{(q^{2(l-3)}-1)(q^{2(l-2)}-1)}{(q^2-1)(q-1)} \\
1 & p_1(1) & q^{l-2}(q^{l-1}-1) & p_3(1) & -q^{3(l-2)+1} & q^2(q^{2(l-3)}-1)\frac{q^{l-2}-1}{q-1} \\
1 & p_1(2) & q^{l-2}(q^{l-1}+1) & p_3(2) & q^{3(l-2)+1} & -q^2(q^{2(l-3)}-1)\frac{q^{l-2}+1}{q-1} \\
1 & -q-1 & 0 & q^{2l-5}(q+1) & -q^{2(l-2)} & -q(q^{2(l-3)}-1) \\
1 & -(q^{l-2}+1)(q+1) & q^{l-2}(q+1) & -q^{l-2}(q+1)(q^{l-2}+1) & q^{2(l-2)+1} & q(q^{l-3}+1)(q^{l-2}+1) \\
1 & (q^{l-2}-1)(q+1) & -q^{l-2}(q+1) & -q^{l-2}(q^{l-2}-1)(q+1) & q^{2(l-2)+1} & q(q^{l-3}-1)(q^{l-2}-1)
\end{bmatrix}
&
\begin{array}{c} 1 \\ m_1 \\ m_2 \\ m_3 \\ m_4 \\ m_5 \end{array}
\end{array}$$

where

$$p_1(1) = (q^{l-1}+q^2+q-1)\frac{q^{l-2}-1}{q-1}$$

$$p_3(1) = q^{l-1}(q^{l-1}-q^{l-2}-q^{l-3}-1)\frac{q^{l-2}-1}{q-1}$$

$$= 1-q^{3l-5}+q^{2l-3}-q^{l-2}+(q^{2l-4}+q^{l-1}+q-1)\frac{q^{l-2}-1}{q-1}.$$

$$p_1(2) = \frac{(q^l+q^2)(q^l-q^3-q^2+q)}{q^3(q-1)}$$

$$p_3(2) = -\frac{\left(q^3+q^{l+2}-q^{l+1}-q^l\right)\left(q^2+q^l\right)q^{l-6}}{q-1}$$

$$m_1 = \frac{(q-1)q^l+q^{2\,l}-q}{2\,(q-1)},\ m_2 = -\frac{(q-1)q^l+q-q^{2\,l}}{2\,(q-1)},\ m_3 = \frac{q^4-\left(q^4+1\right)q^{2\,l}+q^{4\,l}}{q^5-2\,q^3+q},$$

$$m_4 = \frac{q^4-\left(q^3-q\right)q^{3\,l}-\left(q^4+1\right)q^{2\,l}+\left(q^3-q\right)q^l+q^{4\,l}}{2\,(q^6-2\,q^4+q^2)},$$

$$m_5 = \frac{q^4+\left(q^3-q\right)q^{3\,l}-\left(q^4+1\right)q^{2\,l}-\left(q^3-q\right)q^l+q^{4\,l}}{2\,(q^6-2\,q^4+q^2)}$$

3.5 Action of stabilizer

Note that the action of the stabilizer of a point on the set $\Omega = D(m, 2\nu; q)$ will yield different orbit sizes based on type of subspace selected. That is, it depends solely on s for $z \in D(m, s, 2\nu; q)$.

Let $H = Stab_z(Sp_{2\nu}(q))$ where $z \in \Omega$, then $|H| = \frac{|Sp_{2\nu}(q)|}{|\Omega|}$ and H acts on Ω. Then, the action yields orbits $\mathcal{O}_0, \mathcal{O}_1, \cdots, \mathcal{O}_{\mathcal{D}}$. There is a bijection between the orbits obtained by the action of H on Ω and the cosets of $L \backslash H$, where $L = Stab_{x_i}(H)$ for $x_i \in \mathcal{O}_i$.

Similarly, if we take a $z' \in D(m, s, 2\nu)$, we can let $H' = Stab_{z'}(Sp_{2\nu}(q))$, then $|H'| = \frac{|Sp_{2\nu}(q)|}{N(m,s,2\nu)}$. There will also be a bijection between the orbits obtained by the action of H' on Ω and the cosets of $L' \backslash H'$, where $L = Stab_{x_i}(H)$ for $x_i \in \mathcal{O}'_i$. In fact, $|L'| = \frac{|H'|}{k_i}$.

Recall that $0 \leq s \leq \lfloor \frac{\nu}{2} \rfloor$. In the special case when $\nu = 3$ we only have one non-zero value for s, $s = 1$. If we take a $z' \in D(m, 1, 6)$, we can let $H' = Stab_{z'}(Sp_6(q))$, then $|H'| = \frac{|Sp_{2\nu}(q)|}{N(m,1,6;q)}$. There will also be a bijection between the orbits obtained by the action of H' on Ω and the cosets of $L' \backslash H$, where $L' = Stab_{x_i}(H')$ for $x_i \in \mathcal{O}_i$.

The action of the stabilizer of an isotropic subspace on the symplectic space can be viewed as an action of the permutation group of a subgroup of the automorphism group of the generalized symplectic graph on the same space. Similarly, the action of the stabilizer of a non-isotropic subspace on the symplectic space can be viewed as an action of the permutation group of a subgroup of the automorphism group of the generalized non-isotropic graph, whose vertices are subspaces of type (m, s_1), where $s_1 \neq 0$.

3.5.1 The case $\nu = 3$, $m = 2$, $q = 2$

Let $\Omega = D(2, 6; 2)$. Using our formula from Proposition 3.3.7 we have the following values for k_i:

$$k_0 = 1, k_1 = 18, k_2 = 24, k_3 = 144, k_4 = 128$$

$A_{315}[2] =$

$$\begin{array}{c}\begin{pmatrix} 1 & 18 & 24 & 144 & 128 \\ 1 & -3 & 10 & -24 & 16 \\ 1 & 3 & -6 & -6 & 8 \\ 1 & 9 & 6 & 0 & -16 \\ 1 & -3 & 0 & 6 & -4 \end{pmatrix}\end{array}\begin{array}{c} 1 \\ 27 \\ 84 \\ 35 \\ 168 \end{array}$$

As we have seen $G = Sp_6(2)$ acts transitively on each $S_x(C_i)$ for $i = 1, 2, \cdots, \mathcal{D}$. Thus we have a Schurian association scheme $\chi(G, S_x(C_i))$ for each i. Let z be an element of $D(2, 0, 6; 2)$.

Let $\Delta = S_z(0, 1)$, where $|\Delta| = 18$. Let $H = Stab_z(Sp_6(2))$. Then, $|H| = 4608$. H is transitive on $S_z(0, 1)$ and H acts on $\Delta \times \Delta$. We will obtain a 4-class association scheme. We will have two orbits of size 72, one orbit of size 144 and two orbits of size 18.

Furthermore, we can write the character table for this scheme due to the classification of association schemes with small vertices by Miyamoto and Hanaki (1998)

The above association scheme is identified in Miyamoto and Hanaki's list of order 18 schemes, specifically table # 19.

$$A_{18}[19] = \begin{pmatrix} 1 & 1 & 4 & 4 & 8 \\ 1 & 1 & 4 & -2 & -4 \\ 1 & 1 & -2 & 4 & -4 \\ 1 & 1 & -2 & -2 & 2 \\ 1 & -1 & 0 & 0 & 0 \end{pmatrix}\begin{array}{c} 1 \\ 2 \\ 2 \\ 4 \\ 9 \end{array}$$

Let $\Delta = S_z(1, 1)$, where $|\Delta| = 24$. Let $H = Stab_z(Sp_6(2))$. Then, $|H| = 4608$. H is transitive on $S_z(1, 1)$ and H acts on $\Delta \times \Delta$. We will obtain a 4-class association scheme. We will have two orbits of size 72, one orbit of size 384 and one orbit 24.

Furthermore, we can write the character table for this scheme due to the classification of association schemes with small vertices by Miyamoto and Hanaki (1998). The above association scheme is identified in Miyamoto and Hanaki (1998) list of order 24 schemes, specifically table # 36.

$$\begin{array}{c} \begin{pmatrix} 1 & 1 & 3 & 3 & 16 \\ 1 & 1 & 3 & 3 & -8 \\ 1 & -1 & 3 & -3 & 0 \\ 1 & 1 & -1 & -1 & 0 \\ 1 & -1 & -1 & 1 & 0 \end{pmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 9 \\ 9 \end{array}$$

Then we have two possible stabilizers. Specifically if we pick an $z_1 \in D(2,0,6;2)$, we will have $|Stab_{z_1}(G)| = \frac{|G|}{N'(2,6;2)} = 4608$. If $z_2 \in D(2,1,6;2)$ then $|Stab_{z_2}(G)| = \frac{|G|}{N'((2,1),6;2)} = 4320$.

Let $H = Stab_{z_1}(G)$. Then H acts on $N(2,6)$. We will obtain 9 orbits of the following orders 128,144,24,48,128,144,16,18,1.

$\mathcal{O}_0 = z_1$

$\mathcal{O}_1 = \{y \in D(2,0,6;2) : (z_1, y) \in R_1\}, |\mathcal{O}_1| = 18$

$\mathcal{O}_2 = \{y \in D(2,0,6;2) : (z_1, y) \in R_2\}, |\mathcal{O}_2| = 24$

$\mathcal{O}_3 = \{y \in D(2,0,6;2) : (z_1, y) \in R_3\}, |\mathcal{O}_3| = 144$

$\mathcal{O}_4 = \{y \in D(2,0,6;2) : (z_1, y) \in R_4\}, |\mathcal{O}_4| = 128$

$\mathcal{O}_5 = \{y \in D(2,1,6;2) : (z_1, y) \in (0,0)\}, |\mathcal{O}_5| = 16$

$\mathcal{O}_6 = \{y \in D(2,1,6;2) : (z_1, y) \in (1,1)\}, |\mathcal{O}_6| = 48$

$\mathcal{O}_7 = \{y \in D(2,1,6;2) : (z_1, y) \in (1,0)\}, |\mathcal{O}_7| = 144$

$\mathcal{O}_8 = \{y \in D(2,1,6;2) : (z_1, y) \in (2,0)\}, |\mathcal{O}_8| = 128$

Let's look at the action of H on $\mathcal{O}_1$. As seen above, we obtain $A_{18}[19]$. Furthermore, $A_{18}[1]$ and $A_{18}[4]$ are found as fusion schemes of $A_{18}[19]$.

$A_{18}[1] =$

$$\begin{pmatrix} 1 & 17 \\ 1 & -1 \end{pmatrix} \begin{array}{c} 1 \\ 17 \end{array}, \quad \tilde{R}_0 = R_0, \tilde{R}_1 = R_1 \cup R_2 \cup R_3 \cup R_4$$

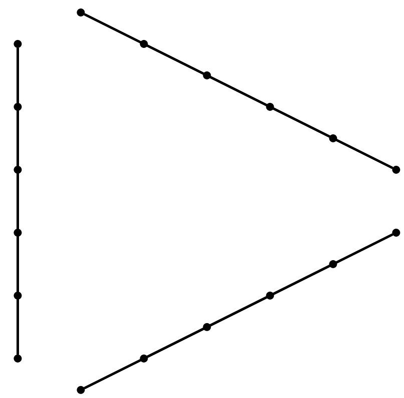


Figure 3.1 $K_{6,6,6}$ from fusion scheme

$A_{18}[4] =$

$$\begin{array}{c}\begin{pmatrix} 1 & 5 & 12 \\ 1 & 5 & -6 \\ 1 & -1 & 0 \end{pmatrix}\end{array}\begin{array}{c} 1 \\ 2 \\ 15 \end{array} \quad , \quad \tilde{R}_0 = R_0, \tilde{R}_1 = R_1 \cup R_3, \tilde{R}_2 = R_2 \cup R_4$$

Furthermore, the 1^{st} relation graph is a complete SRG(18,12,6,12) which is the complete tripartite graph $K_{6,6,6}$ whose complement is the disjoint union of three copies of the complete graphs on 6 vertices. and (18,5,4,0).

Let's look at the action of H on $\mathcal{O}_5$.

These association schemes can be identified in Hanaki's table. Specifically tables # 1 and #5.

Character Table: $A_{16}[1]$=

$$\begin{pmatrix} 1 & 15 \\ 1 & -1 \end{pmatrix}\begin{array}{c} 1 \\ 15 \end{array}$$

$A_{16}[5]$=

$$\begin{pmatrix} 1 & 6 & 9 \\ 1 & 2 & -3 \\ 1 & -2 & 1 \end{pmatrix}\begin{array}{c} 1 \\ 6 \\ 9 \end{array}$$

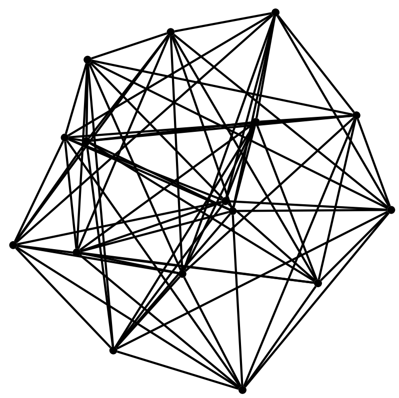


Figure 3.2 $SRG(16,9,4,6)$ from action on $\mathcal{O}_5$

Furthermore, we can identify the strongly regular graph with parameters $(16,9,4,6)$ in Brouwer (2008).

3.5.2 $1\frac{1}{2}$-design as fusion scheme

Let's look at the action of H on $\mathcal{O}_6$. We obtain the following 9-class association scheme.

$$B_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 4 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6 & 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 \end{pmatrix} B_2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$B_3 = \begin{pmatrix} 0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&2&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0 \\ 2&0&2&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&2&0&0 \\ 0&2&0&0&0&0&0&0&0&0 \\ 0&0&0&0&2&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&2&2 \\ 0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0 \end{pmatrix} \quad B_4 = \begin{pmatrix} 0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&6&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&2&0&0&0 \\ 8&8&8&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&6&0&0&0 \\ 0&0&0&0&0&0&0&0&8&8 \\ 0&0&0&8&0&8&0&0&0&0 \\ 0&0&0&0&0&0&0&4&0&0 \\ 0&0&0&0&0&0&0&4&0&0 \end{pmatrix}$$

$$B_5 = \begin{pmatrix} 0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&6&0&4&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0 \\ 0&2&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&6&0&0 \\ 6&4&6&0&0&0&0&0&0&0 \\ 0&0&0&0&6&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&6&6 \\ 0&0&0&0&0&0&3&0&0&0 \\ 0&0&0&0&0&0&3&0&0&0 \end{pmatrix} \quad B_6 = \begin{pmatrix} 0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&6&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&2&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&8&8 \\ 0&0&0&0&6&0&0&0&0&0 \\ 8&8&8&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&8&0&0 \\ 0&0&0&4&0&4&0&0&0&0 \\ 0&0&0&4&0&4&0&0&0&0 \end{pmatrix}$$

$$B_7 = \begin{pmatrix} 0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&6&0&0 \\ 0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&2&2 \\ 0&0&0&8&0&8&0&0&0&0 \\ 0&0&0&0&0&0&0&0&6&6 \\ 0&0&0&0&0&0&8&0&0&0 \\ 8&8&8&0&0&0&0&0&0&0 \\ 0&0&0&0&4&0&0&0&0&0 \\ 0&0&0&0&4&0&0&0&0&0 \end{pmatrix} \quad B_8 = \begin{pmatrix} 0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&3&3 \\ 0&0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&4&0&0&0 \\ 0&0&0&0&0&0&0&3&0&0 \\ 0&0&0&4&0&4&0&0&0&0 \\ 0&0&0&0&4&0&0&0&0&0 \\ 4&2&0&0&0&0&0&0&0&0 \\ 0&2&4&0&0&0&0&0&0&0 \end{pmatrix}$$

$$B_9 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Furthermore, we can construct 3-class fusion schemes. Specifically, when $\tilde{R}_0 = R_0, \tilde{R}_1 = \bigcup_{i\in[9]-\{2,3\}} R_i, \tilde{R}_2 = R_2, \tilde{R}_3 = R_3$ we obtain the $1\frac{1}{2}$-design with parameters (48, 48, 44, 44; 1760, 1776).

Let $H = Stab_{z_2}(G)$. Then H acts on $N(2,6)$. We will obtain 9 orbits of the following orders , 120,135,45,45,180,90,20,15,1.

$\mathcal{O}_0 = z_2$

$\mathcal{O}_1 = \{y \in D(2,0,6;2) : (z_1, y) \in (0,0)\}, |\mathcal{O}_1| = 15$

$\mathcal{O}_2 = \{y \in D(2,0,6;2) : (z_1, y) \in (2,0)\}, |\mathcal{O}_2| = 120$

$\mathcal{O}_3 = \{y \in D(2,0,6;2) : (z_1, y) \in (1,0)\}, |\mathcal{O}_3| = 135$

$\mathcal{O}_4 = \{y \in D(2,0,6;2) : (z_1, y) \in (1,1)\}, |\mathcal{O}_4| = 45$

$\mathcal{O}_5 = \{y \in D(2,1,6;2) : (z_1, y) \in (0,0)\}, |\mathcal{O}_5| = 20$

$\mathcal{O}_6 = \{y \in D(2,1,6;2) : (z_1, y) \in (2,1)\}, |\mathcal{O}_6| = 45$

$\mathcal{O}_7 = \{y \in D(2,1,6;2) : (z_1, y) \in (1,0)\}, |\mathcal{O}_7| = 180$

$\mathcal{O}_8 = \{y \in D(2,1,6;2) : (z_1, y) \in (2,0)\}, |\mathcal{O}_8| = 90$

Let's look at the action of H on $\mathcal{O}_1$.

These association schemes can be identified in Hanaki's table. Specifically tables # 1 and #4.

Character Table: $A_{15}[1] =$

$$\begin{array}{cc} \begin{pmatrix} 1 & 14 \\ 1 & -1 \end{pmatrix} & \begin{array}{c} 1 \\ 14 \end{array} \end{array}$$

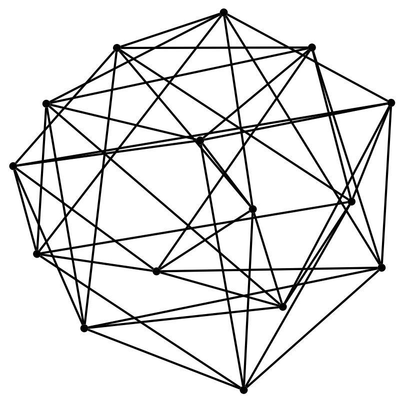


Figure 3.3 $SRG(15,6,1,3)$ from action on $\mathcal{O}_1$

$A_{15}[4] =$

$$\begin{array}{cc} \begin{pmatrix} 1 & 6 & 8 \\ 1 & -3 & 2 \\ 1 & 1 & -2 \end{pmatrix} & \begin{array}{c} 1 \\ 5 \\ 9 \end{array} \end{array}$$

Furthermore, we can identify the strongly regular graph with parameters $(15,6,1,3)$ in Brouwer (2008).

Let's look at the action of H on $\mathcal{O}_5$.

These association schemes can be identified in Hanaki's table. Specifically tables # 1 and #10. Character Table: $A_{20}[1] =$

$$\begin{array}{cc} \begin{pmatrix} 1 & 19 \\ 1 & -1 \end{pmatrix} & \begin{array}{c} 1 \\ 19 \end{array} \end{array}$$

$A_{20}[10] =$

$$\begin{array}{cc} \begin{pmatrix} 1 & 1 & 9 & 9 \\ 1 & -1 & 3 & -3 \\ 1 & -1 & -3 & 3 \\ 1 & 1 & -1 & -1 \end{pmatrix} & \begin{array}{c} 1 \\ 5 \\ 5 \\ 9 \end{array} \end{array}$$

3.5.3 The case $\nu = 3$, $m = 2$, $q = 3$

Let $\Omega = D(2, 6; 3)$. Using our formula from Proposition 3.3.7 we have the following values for k_i:

$$k_0 = 1, k_1 = 48, k_2 = 108, k_3 = 1296, k_4 = 2187$$

$A_{3640}[2] =$

$$\begin{pmatrix} 1 & 48 & 108 & 1296 & 2187 \\ 1 & 20 & 24 & 36 & -81 \\ 1 & -4 & 30 & -108 & 81 \\ 1 & -4 & 0 & 12 & -9 \\ 1 & 8 & -12 & -24 & 27 \end{pmatrix} \begin{matrix} 1 \\ 195 \\ 168 \\ 2457 \\ 819 \end{matrix}$$

Let $\nu = 3$, $m = 2$,$q = 3$.

Let $\Delta = S_z(0, 1)$, where $|\Delta| = 48$. Let $H = Stab_z(Sp_6(3))$. Then, $|H| = 2519424$. H is transitive on $S_z(0, 1)$ and H acts on $\Delta \times \Delta$. We will obtain a 4-class association scheme. We will have two orbits of size 432, one orbit of size 1296 and one orbit of size 96.

Character Table:

$$\begin{pmatrix} 1 & 9 & 9 & 27 & 2 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 9 & -3 & -9 & 2 \\ 1 & -3 & 9 & -9 & 2 \\ 1 & -3 & -3 & 3 & 2 \end{pmatrix} \begin{matrix} 1 \\ 32 \\ 3 \\ 3 \\ 9 \end{matrix}$$

Then we have two possible stabilizers. Specifically if we pick an $z_1 \in D(2, 0, 6; 3)$, we will have $|Stab_{z_1}(G)| = \frac{|G|}{N'(2,6;3)} = 2519424$. If $z_2 \in D(2, 1, 6; 3)$ then $|Stab_{z_2}(G)| = \frac{|G|}{N'((2,1),6;3)} = 1244160$.

Let $H = Stab_{z_1}(G)$. Then H acts on $N(2, 6)$. We will obtain 9 orbits of the following orders , $2187, 1296, 108, 324, 4374, 2592, 81, 48, 1$.

$\mathcal{O}_0 = z_1$

$\mathcal{O}_1 = \{y \in D(2, 0, 6; 2) : (z_1, y) \in R_1\}, |\mathcal{O}_1| = 48$

$\mathcal{O}_2 = \{y \in D(2,0,6;2) : (z_1, y) \in R_2\}, |\mathcal{O}_2| = 108$

$\mathcal{O}_3 = \{y \in D(2,0,6;2) : (z_1, y) \in R_3\}, |\mathcal{O}_3| = 1296$

$\mathcal{O}_4 = \{y \in D(2,0,6;2) : (z_1, y) \in R_4\}, |\mathcal{O}_4| = 2187$

$\mathcal{O}_5 = \{y \in D(2,1,6;2) : (z_1, y) \in (0,0)\}, |\mathcal{O}_5| = 81$

$\mathcal{O}_6 = \{y \in D(2,1,6;2) : (z_1, y) \in (1,1)\}, |\mathcal{O}_6| = 324$

$\mathcal{O}_7 = \{y \in D(2,1,6;2) : (z_1, y) \in (1,0)\}, |\mathcal{O}_7| = 2592$

$\mathcal{O}_8 = \{y \in D(2,1,6;2) : (z_1, y) \in (2,0)\}, |\mathcal{O}_8| = 4374$

Let's look at the action of H on $\mathcal{O}_5$.

Character Table:

$A_{81}[1] =$

$$\begin{array}{cc} \begin{pmatrix} 1 & 80 \\ 1 & -1 \end{pmatrix} & \begin{matrix} 1 \\ 80 \end{matrix} \end{array}$$

$A_{81}[2] =$

$$\begin{array}{cc} \begin{pmatrix} 1 & 32 & 48 \\ 1 & 5 & -6 \\ 1 & -4 & 3 \end{pmatrix} & \begin{matrix} 1 \\ 32 \\ 48 \end{matrix} \end{array}$$

We can identify the strongly regular graph (81,32,13,12) in Brouwer (2008).

Let $H = Stab_{z_2}(G)$. Then H acts on $N(2,6)$. We will obtain 9 orbits of the following orders , $2160, 1280, 160, 320, 2880, 1920, 90, 40, 1$.

$\mathcal{O}_0 = z_2$

$\mathcal{O}_1 = \{y \in D(2,0,6;2) : (z_1, y) \in (0,0)\}, |\mathcal{O}_1| = 40$

$\mathcal{O}_2 = \{y \in D(2,0,6;2) : (z_1, y) \in (2,0)\}, |\mathcal{O}_2| = 2160$

$\mathcal{O}_3 = \{y \in D(2,0,6;2) : (z_1, y) \in (1,0)\}, |\mathcal{O}_3| = 1280$

$\mathcal{O}_4 = \{y \in D(2,0,6;2) : (z_1, y) \in (1,1)\}, |\mathcal{O}_4| = 160$

$\mathcal{O}_5 = \{y \in D(2,1,6;2) : (z_1, y) \in (0,0)\}, |\mathcal{O}_5| = 90$

$\mathcal{O}_6 = \{y \in D(2,1,6;2) : (z_1, y) \in (2,1)\}, |\mathcal{O}_6| = 320$

$\mathcal{O}_7 = \{y \in D(2,1,6;2) : (z_1, y) \in (1,0)\}, |\mathcal{O}_7| = 2880$

$\mathcal{O}_8 = \{y \in D(2,1,6;2) : (z_1, y) \in (2,0)\}, |\mathcal{O}_8| = 4080$

Let's look at the action of H on $\mathcal{O}_1$.

Character Table:

$A_{40}[1] =$

$$\begin{array}{cc} \begin{pmatrix} 1 & 39 \\ 1 & -1 \end{pmatrix} & \begin{array}{c} 1 \\ 39 \end{array} \end{array}$$

$A_{40}[2] =$

$$\begin{array}{cc} \begin{pmatrix} 1 & 12 & 27 \\ 1 & 2 & -3 \\ 1 & -4 & 3 \end{pmatrix} & \begin{array}{c} 1 \\ 25 \\ 15 \end{array} \end{array}$$

We can identify the strongly regular graph (40,12,2,4) in Brouwer (2008).

3.6 The case $\nu = 4$, $m = 3$, $q = 2$

Let $\Omega = D(3,8;2)$. Let

$$R_0 = (0,3), R_1 = (0,2), R_2 = (3,0), R_3 = (1,1), R_4 = (1,2), R_5 = (2,0), R_6 = (2,1)$$

Using our formula from Proposition 3.3.7 we have the following values for k_i:

$$k_0 = 1, k_1 = 42, k_2 = 4096, k_3 = 1008, k_4 = 56, k_5 = 5376, k_6 = 896$$

$B_0 =$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$B_1 =$

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 42 & 13 & 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 21 & 0 & 0 & 16 & 0 \\ 0 & 24 & 0 & 15 & 36 & 3 & 9 \\ 0 & 4 & 0 & 2 & 3 & 0 & 0 \\ 0 & 0 & 21 & 16 & 0 & 19 & 24 \\ 0 & 0 & 0 & 8 & 0 & 4 & 9 \end{pmatrix}$$

$B_2 =$

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 21 & 0 & 0 & 16 & 0 \\ 4096 & 2048 & 1456 & 1536 & 2048 & 1408 & 1664 \\ 0 & 0 & 378 & 256 & 0 & 384 & 288 \\ 0 & 0 & 28 & 0 & 0 & 16 & 32 \\ 0 & 2048 & 1848 & 2048 & 1536 & 1984 & 1728 \\ 0 & 0 & 364 & 256 & 512 & 288 & 384 \end{pmatrix}$$

$B_3 =$

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 24 & 0 & 15 & 36 & 3 & 9 \\ 0 & 0 & 378 & 256 & 0 & 384 & 288 \\ 1008 & 360 & 63 & 162 & 252 & 81 & 144 \\ 0 & 48 & 0 & 14 & 0 & 6 & 9 \\ 0 & 384 & 504 & 432 & 576 & 450 & 504 \\ 0 & 192 & 63 & 128 & 144 & 84 & 54 \end{pmatrix}$$

$B_4 =$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 4 & 0 & 2 & 3 & 0 & 0 \\ 0 & 0 & 28 & 0 & 0 & 16 & 32 \\ 0 & 48 & 0 & 14 & 0 & 6 & 9 \\ 56 & 4 & 0 & 0 & 4 & 0 & 3 \\ 0 & 0 & 21 & 32 & 0 & 34 & 0 \\ 0 & 0 & 7 & 8 & 48 & 0 & 12 \end{pmatrix}$$

$B_5 =$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 21 & 16 & 0 & 19 & 24 \\ 0 & 2048 & 1848 & 2048 & 1536 & 1984 & 1728 \\ 0 & 384 & 504 & 432 & 576 & 450 & 504 \\ 0 & 0 & 21 & 32 & 0 & 34 & 0 \\ 5376 & 2432 & 2604 & 2400 & 3264 & 2416 & 2832 \\ 0 & 512 & 378 & 448 & 0 & 472 & 288 \end{pmatrix}$$

$B_6 =$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 8 & 0 & 4 & 9 \\ 0 & 0 & 364 & 256 & 512 & 288 & 384 \\ 0 & 192 & 63 & 128 & 144 & 84 & 54 \\ 0 & 0 & 7 & 8 & 48 & 0 & 12 \\ 0 & 512 & 378 & 448 & 0 & 472 & 288 \\ 896 & 192 & 84 & 48 & 192 & 48 & 148 \end{pmatrix}$$

Proposition 3.6.1. *When $m = \nu$ and $l = 0$, the association scheme constructed in Lemma 3.1.1 is a fission scheme of Theorem 2.4.1.*

Proof. Merge the orbits of the same intersection size in Lemma 3.1.1. □

3.7 Remarks

Remark 3.7.1. We have constructed the following strongly regular graphs, $1\frac{1}{2}$-designs and association schemes. We have highlighted certain parameters of Table 3.1 which corresponds to the current color code found in Brouwer (2008). We have also highlighted unknown parameters in orange.

Table 3.1 Parameters of strongly regular graph from fusion schemes

srg	q	ν	m	(r, d, s)	H	**Fusion Classes**
(18,12,6,12)	2	3	2	(0,1,0)	$Stab_{z_1}(G)$	[[1, 3], [2, 4]]
(15,6,1,3)	2	3	2	(0,0,0)	$Stab_{z_2}(G)$	[[1], [2]]
(25,8,3,2)	2^2	2	2		$Stab_{z_2}(G)$	[[1,3], [2]]
(48,36,24,36)	3	3	2	(0,1,0)	$Stab_{z_1}(G)$	[[1, 3], [2, 4]]
(40,12,2,4)	3	3	2	(0,0)	$Stab_{z_2}(G)$	[[1], [2]]
(16,9,4,6)	2	3	2	(0,0,1)	$Stab_{z_1}(G)$	[[1], [2]]
(81,32,13,12)	3	3	2	(0,0,1)	$Stab_{z_1}(G)$	[[1], [2]]
(48,32,13,12)	2	3	2	(1,1,1)	$Stab_{z_1}(G)$	[[1, 3, 4, 5, 6, 7, 8, 9], [2]]
(48,44,40,44)	2	3	2	(1,1,1)	$Stab_{z_1}(G)$	[[1, 4, 5, 6, 7, 8, 9], [2, 3]]
(45,30,15,30)	2	3	2	(1,1,0)	$Stab_{z_2}(G)$	[[1, 3, 4], [2, 5]]
(120,117,114,117)	2	3	2	(2,0,0)	$Stab_{z_2}(G)$	[[1, 3, 4, 5, 6, 7, 8, 9], [2]]
(120,111,102,111)	2	3	2	(2,0,0)	$Stab_{z_2}(G)$	[[1, 3, 4, 5, 6, 7, 9], [8, 2]]
(120,114,108,114)	2	3	2	(2,0,0)	$Stab_{z_2}(G)$	[[1, 3, 4, 5, 6, 7, 8], [9, 2]]
(120,21,6,3)	2	3	2	(2,0,0)	$Stab_{z_2}(G)$	[[1, 3, 9], [2, 4, 5, 6, 7, 8]]
(120,63,30,36)	2	3	2	(2,0,0)	$Stab_{z_2}(G)$	[[1, 3, 4, 7, 8, 9], [2, 5, 6]]
(120,108,96,108)	2	3	2	(2,0,0)	$Stab_{z_2}(G)$	[[1, 3, 4, 5, 6, 7], [8, 9, 2]]
(135,132,129,132)	2	3	2	(1,0,0)	$Stab_{z_2}(G)$	[[1], [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]]
(135,108,81,108)	2	3	2	(1,0,0)	$Stab_{z_2}(G)$	[[1, 3, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15], [2, 4, 6]]
(135,120,105,120)	2	3	2	(1,0,0)	$Stab_{z_2}(G)$	[[1, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14, 15], [9, 2, 13]]
(135,126,117,126)	2	3	2	(1,0,0)	$Stab_{z_2}(G)$	[[1, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14], [2, 11, 15]]
(135,96,57,96)	2	3	2	(1,0,0)	$Stab_{z_2}(G)$	[[1, 4, 5, 12, 13], [2, 3, 6, 7, 8, 9, 10, 11, 14, 15]]
(135,102,69,102)	2	3	2	(1,0,0)	$Stab_{z_2}(G)$	[[1, 4, 5, 14, 15], [2, 3, 6, 7, 8, 9, 10, 11, 12, 13]]
(135,114,93,114)	2	3	2	(1,0,0)	$Stab_{z_2}(G)$	[[1, 12, 13, 14, 15], [2, 3, 4, 5, 6, 7, 8, 9, 10, 11]]
(135,90,45,90)	2	3	2	(1,0,0)	$Stab_{z_2}(G)$	[[1, 4, 5, 12, 13, 14, 15], [2, 3, 6, 7, 8, 9, 10, 11]]
(108,81,54,81)	3	3	2	(1,1,0)	$Stab_{z_1}(G)$	[[1], [2, 3, 4, 5, 6]]
(160,120,80,120)	3	3	2	(1,1,0)	$Stab_{z_2}(G)$	[[1, 3], [2, 4, 5]]
(144,142,140,142)	2	3	2	(1,0,0)	$Stab_{z_1}(G)$	[[1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15], [2]]
(144,140,136,140)	2	3	2	(1,0,0)	$Stab_{z_1}(G)$	[[1, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15], [2, 3]]
(144,126,108,126)	2	3	2	(1,0,0)	$Stab_{z_1}(G)$	[[1, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15], [2, 7]]
(144,124,104,124)	2	3	2	(1,0,0)	$Stab_{z_1}(G)$	[[1, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15], [2, 3, 7]]
(144,52,16,20)	2	3	2	(1,0,0)	$Stab_{z_1}(G)$	[[1, 10, 3, 7], [2, 4, 5, 6, 8, 9, 11, 12, 13, 14, 15]]
(144,66,30,30)	2	3	2	(1,0,0)	$Stab_{z_1}(G)$	[[1, 3, 5, 6, 7, 10, 13], [2, 4, 8, 9, 11, 12, 14, 15]]
(144,78,42,42)	2	3	2	(1,0,0)	$Stab_{z_1}(G)$	[[[1, 5, 7, 9, 10, 11, 13], [2, 3, 4, 6, 8, 12, 14, 15]]
(144,138,132,138)	2	3	2	(1,0,1)	$Stab_{z_1}(G)$	[[1, 11], [2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16]]
(144,122,100,122)	2	3	2	(1,0,1)	$Stab_{z_1}(G)$	[[1, 10, 7], [2, 3, 4, 5, 6, 8, 9, 11, 12, 13, 14, 15, 16]]
(144,120,96,120)	2	3	2	(1,0,1)	$Stab_{z_1}(G)$	[[1, 10, 13, 7], [2, 3, 4, 5, 6, 8, 9, 11, 12, 14, 15, 16]]
(160,120,80,120)	2	3	2	(1,0,1)	$Stab_{z_1}(G)$	[[1, 3], [2, 4, 5]]
(9,4,1,2)	2	3	3		$Stab_{z_2}(G)$	[[1], [2, 3]]
(40,12,2,4)	3	3	2	(0,0,0)	$Stab_{z_2}(G)$	[[1], [2]]]
(36,20,10,12)	3	2	2		$Stab_{z_1}(G)$	[[8, 1, 4, 5], [2, 3, 6, 7]]]

Table 3.2 $1\frac{1}{2}$-designs obtained from some fusion schemes

$\mathbf{1\frac{1}{2}}$**-designs**	q	ν	m	(r,d,s)	H	**Fusion Classes**
(12, 12, 9, 9; 54, 63)	2	2	2		$Stab_{z_1}(G)$	[[1],[2],[3]]
(12,12,8,8;32,48)	2	2	2		$[Stab_{z_1}(G):H_1]=2$	[[1], [2, 4, 5, 6, 7], [3]]
(48, 48, 44, 44; 1760, 1776)	2	3	2	(1,1,1)	$Stab_{z_1}(G)$	[[1, 4, 5, 6, 7, 8, 9], [2], [3]]
(192,192,184,184;34592,34608)	2	4	2		$Stab_{z_1}(G)$	[[1, 4, 5, 6, 7, 8, 9], [2], [3]]
(144,144,140,140;19040,19056)	2	3	2	(1,0,0)	$Stab_{z_1}(G)$	$[[1],[2],[3,4,\cdots,15]]$

Table 3.3 Association scheme character tables in Miyamoto and Hanaki (1998)

Table Number	q	ν	m	Notes	H	Fusion Classes
$A_6[1]$	2	2	2		$Stab_{z_1}(G)$	[[1, 2]]
$A_6[2]$	2	2	2		$Stab_{z_1}(G)$	[[1], [2]]
$A_6[3]$	2	2	2		$Stab_{z_2}(G)$	[[1], [2]]
$A_8[1]$	2	2	2		$Stab_{z_1}(G)$	[[1, 2, 3]]
$A_8[3]$	2	2	2		$Stab_{z_1}(G)$	[[1, 3], [2]]
$A_8[5]$	2	2	2		$Stab_{z_1}(G)$	[[1],[2],[3]]
$A_9[1]$	2	2	2		$Stab_{z_2}(G)$	[[1,2,3]]
$A_9[2]$	2	2	2		$Stab_{z_2}(G)$	[[1], [2,3]]
$A_9[3]$	2	2	2		$Stab_{z_2}(G)$	[[1, 3], [2]]
$A_9[5]$	2	2	2		$Stab_{z_2}(G)$	[[1], [2],[3]]
$A_{12}[1]$	2	2	2		$Stab_{z_1}(G)$	[[1,2,3,4,5,6,7]]
$A_{12}[2]$	2	2	2		$Stab_{z_1}(G)$	[[1], [2, 3, 4, 5, 6, 7]]
$A_{12}[4]$	2	2	2		$Stab_{z_1}(G)$	[[1, 6, 7], [2, 3, 4, 5]]
$A_{12}[5]$	2	2	2		$Stab_{z_1}(G)$	[[1, 3, 4, 6], [2, 5, 7]]
$A_{12}[9]$	2	2	2		$Stab_{z_1}(G)$	[[1], [2, 5, 7], [3, 4, 6]]
$A_{12}[10]$	2	2	2		$Stab_{z_1}(G)$	[[1], [2, 5, 6], [3, 4, 7]]
$A_{12}[3]$	3	2	2		$Stab_{z_1}(G)$	[[1],[2,3]]
$A_{12}[6]$	3	2	2		$Stab_{z_1}(G)$	[[1],[2],[3]]
$A_{15}[1]$	2	3	2	(0,0,0)	$Stab_{z_2}(G)$	[[1,2]]
$A_{15}[4]$	2	3	2	(0,0,0)	$Stab_{z_2}(G)$	[[1],[2]]
$A_{16}[1]$	2	3	2	(0,0,1)	$Stab_{z_1}(G)$	[[1, 2]]
$A_{16}[5]$	2	3	2	(0,0,1)	$Stab_{z_1}(G)$	[[1],[2]]
$A_{16}[3]$	3	2	2		$Stab_{z_2}(G)$	[[1],[2,3]]
$A_{16}[6]$	3	2	2		$Stab_{z_2}(G)$	[[1,3],[2]]
$A_{16}[12]$	3	2	2		$Stab_{z_2}(G)$	[[1],[2],[3]]
$A_{18}[1]$	2	3	2	(0,1,0)	$Stab_{z_1}(G)$	[[1, 2, 3, 4]]
$A_{18}[4]$	2	3	2	(0,1,0)	$Stab_{z_1}(G)$	[[1, 3],]2, 4]]
$A_{18}[19]$	2	3	2	(0,1,0)	$Stab_{z_1}(G)$	[[1], [2],[3],[4]]
$A_{20}[1]$	2	3	2	(0,0,1)	$Stab_{z_2}(G)$	[[1,2,3]]
$A_{20}[10]$	2	3	2	(0,0,1)	$Stab_{z_2}(G)$	[[1],[2],[3]]
$A_{20}[3]$	2^2	2	2		$Stab_{z_1}(G)$	[[1],[2]]
$A_{24}[1]$	3	2	2		$Stab_{z_2}(G)$	[[1,2,3,4,5,6]]
$A_{24}[6]$	3	2	2		$Stab_{z_2}(G)$	[[1, 5], [2, 3, 4, 6]]
$A_{24}[1]$	3	2	2		$Stab_{z_2}(G)$	[[12,3,4,5,6]]
$A_{24}[1]$	3	2	2		$Stab_{z_2}(G)$	[[12,3,4,5,6]]
$A_{25}[1]$	2^2	2	2		$Stab_{z_2}(G)$	[[1,2,3]]
$A_{25}[2]$	2^2	2	2		$Stab_{z_2}(G)$	[[1],[2,3]]
$A_{25}[3]$	2^2	2	2		$Stab_{z_2}(G)$	[[1,3],[2]]
$A_{25}[13]$	2^2	2	2		$Stab_{z_2}(G)$	[[1],[2],[3]]
$A_{30}[1]$	5	2	2		$Stab_{z_1}(G)$	[[1,2,3]]
$A_{30}[4]$	5	2	2		$Stab_{z_1}(G)$	[[1],[2,3]]
$A_{30}[14]$	5	2	2		$Stab_{z_1}(G)$	[[1],[2],[3]]

CHAPTER 4. SUMMARY AND DISCUSSION

4.1 General discussion

The connection between q-analogue t-designs, $1\frac{1}{2}$-designs and association schemes was considered in Section 1.3. It was shown that certain families of 3-class association schemes yield $1\frac{1}{2}$-designs. The construction of these schemes require the use of the groups and counting techniques prevalent in q-analogue t-designs. In this dissertation, we set out to answer the following question: Can we find a new 3-class association scheme which gives rise to a partial geometric design using symplectic geometry? Although I was unable to answer this question in general, I was able to produce a few examples of 3-class association schemes that are $1\frac{1}{2}$-designs as found in Table 3.7. In Chapter 1, I discussed q-analogue t-designs and the tools used to find them. I give the explicit SAGE code to construct the blocks and points of the first non-trivial q-analogue t-design in Appendix A. I then show the current methods to find new q-analogue t-designs using the Kramer-Mesner method as well as a search algorithm. In Chapter 2, I introduced symplectic geometry and $1\frac{1}{2}$-designs. I showed two known constructions of $1\frac{1}{2}$-designs and describe the blocks and points for a specific example. I then discussed the issues in generalizing the construction and identified the relationships we need to study more. Finally, in Chapter 3, I rediscovered a known family of association schemes and I discovered a new family of Schurian schemes. Specifically, I found the general character table for $m = 2$ and have constructed several concrete examples. Furthermore, I look at fusion schemes from our Schurian scheme, and find previously unknown constructions of strongly regular graphs, association schemes and $1\frac{1}{2}$-designs.

4.2 Future research

I was unable to answer our original question in general and the problem remains open. Moving forward, I want to identify when the family of Schurian association schemes yields $1\frac{1}{2}$-designs and strongly regular graphs. The latter have been studied by many researchers. See Brouwer (2008); Spence (2006). In Chapter 2, using the generalized symplectic graph, I created an example where the distance 2 neighborhood is not regular. There were three regular components which raises the following questions:

1. Can we characterize the distance classes of sizes larger than 2?

2. Can I classify all neighbors of a vertex in distance class i for $0 \leq i \leq \mathcal{D}$?

Similarly, I would like to understand the automorphism group of the generalized non-isotropic graph as studied in Chai et al. (2015) with respect to the generalized symplectic graph. In Chapter 3, I was able to find the intersection matrices of size $m = 2$ and would like to find the general formula for the intersection numbers. Lastly, I will extend this study to the orthogonal and unitary geometries.

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APPENDIX A. ADDITIONAL MATERIAL

Suzuki sage code

```
from collections import Counter
m=1
q=2
K.<b> = GF(q^m)
n=7
V = VectorSpace(K,n)
NV.<a> = GF(q^(m*n))
##convert to value then back to field, (0,b,1) should be q^0 +bq^1,
\\which would be 1+2*4

%time
zer = [0]*((n))
VtoF={}
##Dictionary that contains the conversion from Vectorspace to underlying field
VtoF[tuple(zer)]=0
FtoV={}
FtoV[0] = tuple(zer)
##Dictionary that contains the conversion from the Field to the Vectorspace
FtoV[0]= zer
tempv=[0]*(n)
for i in range((n)):
    tempv[i] = a^i
tempv.reverse()
tempv = matrix(tempv)
A = list(V)
A.sort()
T = list(NV)
T.sort()
for i in range(q^(m*n)):
    VtoF[tuple(A[i])] = T[i]
    FtoV[T[i]] = A[i]

ylist = list(NV)
ylist.remove(0)
xlist= list(NV)
for i in K:
    if i in xlist:
        xlist.remove(i)

%time
AllSpaces={}
SpaceGens ={}
for y in ylist:
```

```
    for x in xlist:
        temptriple = [y,y*x,y*x^2] #original
        tempm=[]
        for l in temptriple:
            tempm+=[FtoV[l]]
        tempmat = matrix(tempm).rref()
        if tempmat in AllSpaces:
            AllSpaces[tempmat] +=1
            SpaceGens[tempmat].append((y,x))
        else:
            AllSpaces[tempmat]= 1
            SpaceGens[tempmat]= [(y,x)]

keep=[]
for i in AllSpaces.iterkeys():
    keep+=[rank(i)]

Counter(keep)
}
```

Kramer-Mesner singer cycle

```
n=7
k=3
t=2
q=2
G = GL(n,q)
V = VectorSpace(GF(q),n)
S=list(V.subspaces(t))
K = list(V.subspaces(k))
F.<x> = GF(q)[]
factor(x^(q^(2*n)-1)+1) ##Pick irreducible polynomial to
\\construct your singer cycle
r =companion_matrix(x^7+x^4+1)
GL7 = gap.GL(n,q)
Nr = GL7.Normalizer(r)
SubG = gap.ConjugacyClassesSubgroups(Nr)

##We construct our singer cycle as well as the Normalizer
of the singer cycle. We then find all subgroups of the SingerCycle

import time
start = time.time()

matrixlist=[]

for j in gap.Elements(Nr):
    matrixlist+=[Matrix(j,GF(q))]


end = time.time()
print(end - start)

##We go through each of the elements in gap and convert them to sage

start = time.time()

supermatrixlist=[]
for i in SubG:
    submatrixlist=[]
    for j in gap.Elements(gap.Representative(i)):
        submatrixlist+=[Matrix(j,GF(q))]

    supermatrixlist+=[submatrixlist]

end = time.time()
print(end - start)
##We go through each of the elements in gap and convert them to sage


start = time.time()
orbitK=[]
used = list(V.subspaces(k))
```

```
while len(used)>0:
    temp=[]
    i=used.pop()
    for j in matrixlist:
        b=span((j*i.basis_matrix().transpose()).transpose())
        if b not in temp:
            temp +=[b]
            if b in used:
                used.remove(b)
    orbitK.append(temp)
end = time.time()
print(end - start)
##We go through the list and find the orbits of the blocks under each group
defined above

start = time.time()
orbitT=[]
used = list(V.subspaces(t))
while len(used)>0:
    temp=[]
    i=used.pop()
    for j in matrixlist:
        b=span((j*i.basis_matrix().transpose()).transpose())
        if b not in temp:
            temp +=[b]
            if b in used:
                used.remove(b)
    orbitT.append(temp)
end = time.time()
print(end - start)
##We go through the list and find the orbits of the points under each group
defined above



def kspacecounter(j,V,k):
    import time
    matrixlist=j
    start = time.time()
    orbitK=[]
    used = list(V.subspaces(k))
    while len(used)>0:
        temp=[]
        i=used.pop()
        for j in matrixlist:
            b=span((j*i.basis_matrix().transpose()).transpose())
            if b not in temp:
                temp +=[b]
                if b in used:
                    used.remove(b)
        orbitK.append(temp)
    end = time.time()
    print(end - start)
```

```
    return(orbitK)
##creates the k-spaces representative for kramer mesner


def tspacecounter(j,V,t):
    import time
    matrixlist=j
    start = time.time()
    orbitT=[]
    used = list(V.subspaces(t))
    while len(used)>0:
        temp=[]
        i=used.pop()
        for j in matrixlist:
            b=span((j*i.basis_matrix().transpose()).transpose())
            if b not in temp:
                temp +=[ b]
                if b in used:
                    used.remove(b)
        orbitT.append(temp)
    end = time.time()
    print(end - start)
    return(orbitT)
import time
##creates the t-spaces representative for kramer mesner




def KramerCounter(trep,Kspace):
    if len(trep)==0:
        return 0
    if len(Kspace)==0:
        return 0
    trep1 =  trep[0]
    count=0
    for i in Kspace:
        if trep1.is_subspace(i):
            count+=1
    return count

allthematrices=[]
start=time.time()
for i in supermatrixlist:
    orbitT=tspacecounter(i,V,t)
    orbitK = kspacecounter(i,V,k)
    SS= matrix(ZZ,len(orbitT),len(orbitK),lambda x, y: KramerCounter(orbitT[x],orbitK[y]))
    allthematrices+=[SS]
    print SS

## Gives the Kramer Mesner Matrix for each Group used at the start.
```

Table A.1 Blocks for Thomas' $2-[7,3,7;2]$ design.

Orbits for Kramer-Mesner matrix and Thomas' construction, 2-[7,3,7;2]

The following values are the blocks that will form the $2-[7,3,7;2]$ design.

$$\begin{pmatrix} 1&0&1&0&0&0&0\\ 0&1&1&0&0&1&1\\ 0&0&0&1&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&0&1&0&1\\ 0&0&1&1&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&0\\ 0&0&1&0&0&1&0\\ 0&0&0&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&1&0&0&0&0&0\\ 0&0&0&0&1&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&1&1\\ 0&1&0&0&0&1&1\\ 0&0&0&1&0&1&0 \end{pmatrix} \begin{pmatrix} 1&1&0&0&1&0&0\\ 0&0&1&1&0&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&1&0&0&1&0\\ 0&0&0&1&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&0&1\\ 0&1&1&0&0&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&0&1&0&1&0&0\\ 0&0&0&1&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&0&1&0&0\\ 0&0&0&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&1&0&1&0&0&0\\ 0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&1&0&1&0&0\\ 0&0&0&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&0&1&1&0\\ 0&0&1&0&0&0&0\\ 0&0&0&1&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&1&1\\ 0&1&0&0&0&0&0\\ 0&0&1&0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&1&0&1&1&0\\ 0&1&1&0&0&1&0\\ 0&0&0&1&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&1&0&1&1\\ 0&0&1&1&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&0&0&0&0\\ 0&0&1&0&1&0&1\\ 0&0&0&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&0&0\\ 0&1&1&0&0&1&0\\ 0&0&0&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&1&0&0&0&1&0\\ 0&0&1&0&1&1&0\\ 0&0&0&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&1&0&0\\ 0&1&1&0&0&1&1\\ 0&0&0&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&1&0&1&1\\ 0&0&1&1&0&1&1\\ 0&0&0&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&1&0\\ 0&1&1&0&0&1&0\\ 0&0&0&1&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&1&0&0&1\\ 0&0&1&1&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&1&0&0&1&1\\ 0&0&0&1&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&0&1&0&0&1&0\\ 0&0&0&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&1&0\\ 0&0&1&1&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&1&0&0&1&0&0\\ 0&0&0&1&1&0&1\\ 0&0&0&0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&1&1&0&1&0&0\\ 0&0&0&1&0&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&0&1&0&1\\ 0&0&1&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&0&1&0&0\\ 0&0&1&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&0&0&0&0\\ 0&0&1&1&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&1&1&0&1&0\\ 0&1&1&1&0&0&0\\ 0&0&0&0&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&1&0&1&1\\ 0&0&1&1&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&0&0&1&1\\ 0&0&1&0&1&0&0 \end{pmatrix} \begin{pmatrix} 1&1&0&1&0&1&1\\ 0&0&1&0&0&0&0\\ 0&0&0&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&1&0&0&0\\ 0&0&1&1&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&0&0&1&1\\ 0&0&0&1&0&0&1 \end{pmatrix} \begin{pmatrix} 1&1&0&1&1&0&0\\ 0&0&1&1&1&0&1\\ 0&0&0&0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&1\\ 0&0&1&0&0&0&1\\ 0&0&0&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&0&1&0&1\\ 0&0&1&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&1&1&1\\ 0&1&1&0&1&1&0\\ 0&0&0&1&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&1&1&0&1&1\\ 0&0&0&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&1&0\\ 0&1&0&0&1&1&0\\ 0&0&0&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&0&0&0&1\\ 0&0&1&1&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&0&1&0&1\\ 0&0&1&0&1&0&1\\ 0&0&0&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&1&1&0&0\\ 0&0&1&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&1&0&1&0&0\\ 0&1&1&0&1&1&1\\ 0&0&0&1&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&1&0&0&1&1\\ 0&0&0&1&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&0&1&0&0&1&1\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&0&0\\ 0&1&1&0&1&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&0&0\\ 0&1&0&0&1&0&0\\ 0&0&0&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&0&0\\ 0&0&1&0&0&0&0\\ 0&0&0&0&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&0&0&0&0\\ 0&0&1&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&1&0&0\\ 0&0&1&0&1&0&0\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&0&0\\ 0&1&1&1&0&1&1\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&1&0&1&0\\ 0&0&1&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&1&1\\ 0&1&1&1&0&0&1\\ 0&0&0&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&1&0&1&0\\ 0&0&1&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&0&0&1&1\\ 0&0&1&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&0&1\\ 0&0&1&1&0&0&1\\ 0&0&0&0&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&0&1&0&1\\ 0&0&1&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&1&0&1&1\\ 0&0&1&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&1&0&0&0\\ 0&0&1&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&1&1\\ 0&1&1&0&0&0&1\\ 0&0&0&1&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&0&1&1&0\\ 0&0&1&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&1&0\\ 0&0&1&0&1&1&1\\ 0&0&0&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&1&1&0&1&0\\ 0&1&1&1&0&1&1\\ 0&0&0&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&1&0\\ 0&0&1&0&0&1&1\\ 0&0&0&1&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&0&1&0&1\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&0&0&0&1\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&1&1&0&0&0\\ 0&1&1&0&0&0&1\\ 0&0&0&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&0&0&1&1\\ 0&0&1&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&1&0&1&1\\ 0&0&1&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&0&1&0&1\\ 0&0&1&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&1&0&0&1&1&1\\ 0&0&1&0&1&0&1\\ 0&0&0&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&0&1&0&1&1&0\\ 0&0&0&1&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&1&1&0&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&0&1&0&1\\ 0&0&1&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&0&1&0&0&0&0\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&1&0\\ 0&1&1&0&0&0&0\\ 0&0&0&1&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&1&1&0&0&1\\ 0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&0&1&0&0\\ 0&0&1&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&0&0&1&1\\ 0&0&1&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&0&0&0&0\\ 0&0&1&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&1&0&0&1\\ 0&0&1&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&0&1&0&1&1&0\\ 0&0&0&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&1&0&1&1\\ 0&0&1&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&1&0&1&1&1\\ 0&0&0&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&1&1&0&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&1\\ 0&1&0&1&0&1&1\\ 0&0&1&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&0&1&0&1&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 0&1&0&0&1&0&0\\ 0&0&1&0&1&1&0\\ 0&0&0&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&1&0&1&0&0\\ 0&0&0&1&1&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 0&0&1&0&1&0&0\\ 0&0&0&1&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 0&1&0&0&0&0&1\\ 0&0&1&0&0&1&0\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 0&1&0&0&0&0&0\\ 0&0&1&1&0&0&0\\ 0&0&0&0&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&0&0&1&1\\ 0&0&1&0&0&1&1\\ 0&0&0&1&0&0&0 \end{pmatrix}\begin{pmatrix} 0&1&0&1&0&0&1\\ 0&0&1&0&0&0&0\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&0&1&1&1\\ 0&0&1&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&1&1&1&1\\ 0&0&1&0&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&0&1&0&0\\ 0&0&1&0&0&0&0\\ 0&0&0&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&1&1&1&0\\ 0&0&1&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&0&1\\ 0&1&1&0&0&0&1\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&0&0\\ 0&1&0&0&0&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&0&1&0&1&0&0\\ 0&0&0&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&1&0&1&1&0\\ 0&0&0&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&1&1&0&1&1\\ 0&1&1&1&0&1&0\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&0&1&1&0&1&1\\ 0&0&0&0&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&1&0&1\\ 0&1&0&0&1&0&1\\ 0&0&0&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&1&0&1&0\\ 0&0&1&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&1&0&1&0&1&1\\ 0&0&1&1&0&0&0\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&0&0&1&1\\ 0&0&1&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&0&1&0&0&0&1\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&1&0&0&0\\ 0&0&1&0&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&0&0&0&0\\ 0&0&1&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&1&0&1&0\\ 0&0&1&1&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&1&1\\ 0&1&0&0&1&0&0\\ 0&0&1&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&1&0&1&1\\ 0&0&1&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&1&0&0&0&0\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&0&1&1&0\\ 0&0&1&1&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&0&0&0&1\\ 0&0&0&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&0\\ 0&1&0&0&0&1&1\\ 0&0&0&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&1&1&0&0\\ 0&0&1&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&0&0&1&0\\ 0&0&1&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&0&1&0&1\\ 0&0&0&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&1&1&1&1\\ 0&0&1&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&1&1&1&0\\ 0&0&1&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&1&1&0&1\\ 0&0&1&1&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&0&1&0&1\\ 0&0&1&0&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&0&0&0&0\\ 0&0&1&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&1&1&0&1&0\\ 0&1&0&0&0&1&1\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&1&1&1&1\\ 0&0&1&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&1&1\\ 0&1&0&0&0&1&0\\ 0&0&0&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&1&0&0&1&1&1\\ 0&0&1&0&0&1&0\\ 0&0&0&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&1\\ 0&1&0&1&0&0&1\\ 0&0&1&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&0&0&0&1\\ 0&0&1&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&0&1&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&0&1\\ 0&0&0&1&0&0&0\\ 0&0&0&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&0&0&1&1\\ 0&0&0&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&0\\ 0&1&1&1&1&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&0&0&1&0\\ 0&0&1&0&0&0&1\\ 0&0&0&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&0&1&0&1\\ 0&0&1&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&1&0&1&1&0&1\\ 0&0&1&1&1&0&1\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&1&1&0&1&1\\ 0&1&1&1&0&1&1\\ 0&0&0&0&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&1&0&0&0\\ 0&0&1&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&0&0\\ 0&1&1&0&1&1&0\\ 0&0&0&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&0&0&1&1\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 0&1&0&0&0&1&0\\ 0&0&1&0&1&1&1\\ 0&0&0&1&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&1&1&0&1\\ 0&0&1&1&1&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 0&1&0&1&1&0&1\\ 0&0&1&1&0&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 0&1&0&0&0&1&0\\ 0&0&1&0&1&1&1\\ 0&0&0&1&1&0&0 \end{pmatrix}\begin{pmatrix} 0&1&0&0&1&1&1\\ 0&0&1&0&1&1&0\\ 0&0&0&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&0&0&1&0\\ 0&0&0&1&0&0&1\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 0&1&0&0&1&1&1\\ 0&0&1&0&1&0&1\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&0&0\\ 0&1&0&1&1&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&1&0\\ 0&1&0&0&1&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&1&0&1\\ 0&1&0&0&0&0&1\\ 0&0&0&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&1&1&0&1\\ 0&0&1&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&0&0&1&0\\ 0&0&0&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&0&1&0&0&1&0\\ 0&0&0&1&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&1&1&0&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&0&0\\ 0&1&0&1&1&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&0&1&1&1\\ 0&0&0&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&0&1&1&1\\ 0&0&1&0&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&0&0&1&0\\ 0&0&0&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&1&0&0&1\\ 0&0&1&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&0&0&1&0\\ 0&0&1&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&1&1&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&1&1\\ 0&1&0&0&1&1&0\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&1&1&0\\ 0&0&1&0&1&1&1\\ 0&0&0&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&0&1&1&1\\ 0&0&1&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&1&1&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&0&0&1&0\\ 0&0&1&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&1&1\\ 0&1&1&0&1&0&1\\ 0&0&0&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&0&0&0&1\\ 0&0&0&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&0&0&0&0\\ 0&0&1&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&1&1&0&1\\ 0&1&1&0&1&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&1&1&0&1&1\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&1\\ 0&1&0&1&0&1&0\\ 0&0&1&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&0\\ 0&1&0&0&1&0&0\\ 0&0&0&1&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&0&1&0&0&0&0\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&1&1\\ 0&0&1&0&1&0&0\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&0&1&1\\ 0&0&0&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&0&1&0&1&1&0\\ 0&0&0&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&1&1&0&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&1&0&1\\ 0&0&1&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&1&0&0&1\\ 0&0&1&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&1&1&0&0&0\\ 0&0&0&0&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&0&1&1&1&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&1&0&0&1&1\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&0&0&1&0&1&0\\ 0&0&0&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&1&0&0&0&1\\ 0&0&0&1&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&0&1&1&0&1&0\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&1&0&1\\ 0&0&1&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&0&1&1&0&0&1\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&0&1&1\\ 0&0&1&1&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&0&1&0&1\\ 0&0&1&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&0&1&0&1\\ 0&0&1&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&0&0&1&1\\ 0&0&1&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&0&1&1&0&0&1\\ 0&0&0&0&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&0&0&1&1\\ 0&0&1&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&1&1&0&1&0\\ 0&0&0&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&0\\ 0&1&0&0&0&1&1\\ 0&0&0&0&1&1&0 \end{pmatrix}\begin{pmatrix} 0&1&0&1&1&0&1\\ 0&0&1&0&0&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&0&0&1&0\\ 0&0&0&1&0&1&1\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 0&1&0&0&1&1&0\\ 0&0&1&0&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 0&1&0&0&0&0&1\\ 0&0&1&0&1&0&0\\ 0&0&0&1&0&0&0 \end{pmatrix}\begin{pmatrix} 0&1&0&0&0&0&0\\ 0&0&0&1&1&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&0&1&0&0\\ 0&0&0&1&0&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 0&1&0&0&0&1&0\\ 0&0&1&0&0&1&0\\ 0&0&0&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&0&1\\ 0&1&0&0&0&0&0\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&0&1&0&0\\ 0&0&1&0&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&0&0&0&0\\ 0&0&1&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&1&1&0&1&0\\ 0&1&1&0&0&0&0\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&0&1&0&0\\ 0&0&1&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&0&0&0&0\\ 0&0&1&0&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&1&0&1&1\\ 0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0 \end{pmatrix} \begin{pmatrix} 0&1&0&0&1&0&1\\ 0&0&1&0&1&1&0\\ 0&0&0&1&0&1&0 \end{pmatrix} \begin{pmatrix} 0&1&0&0&0&1&0\\ 0&0&1&1&0&1&1\\ 0&0&0&0&1&0&1 \end{pmatrix} \begin{pmatrix} 0&1&0&1&0&1&1\\ 0&0&1&1&0&0&1\\ 0&0&0&0&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&0&0&1&0\\ 0&0&1&0&1&0&1\\ 0&0&0&1&1&0&1 \end{pmatrix} \begin{pmatrix} 0&1&0&1&0&1&1\\ 0&0&1&1&0&0&0\\ 0&0&0&0&1&1&0 \end{pmatrix} \begin{pmatrix} 0&1&0&0&1&1&1\\ 0&0&1&0&1&0&1\\ 0&0&0&1&0&1&0 \end{pmatrix} \begin{pmatrix} 0&1&0&0&0&1&1\\ 0&0&1&0&1&1&0\\ 0&0&0&1&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&1&0&0&1\\ 0&1&0&0&0&1&0\\ 0&0&0&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&1&1&0&1\\ 0&0&1&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&1&1&1&1\\ 0&0&1&1&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&0&0&1&1\\ 0&0&1&1&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&1&0&1&0\\ 0&0&1&1&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&0&1&1&1\\ 0&0&1&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&1&1&0&0&1\\ 0&1&0&1&0&0&0\\ 0&0&0&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&1&1&1\\ 0&0&0&1&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&1&1&1&1\\ 0&0&1&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&1&0&1&0&1\\ 0&0&0&1&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&0&1&0\\ 0&0&1&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&1&0&0&0\\ 0&0&0&0&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&1&1&1&0\\ 0&0&1&1&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&0&1&1&0&1&1\\ 0&0&0&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&0\\ 0&0&1&0&1&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&1&0&0&0&0&1\\ 0&0&1&0&0&1&0\\ 0&0&0&0&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&0&0&0&0\\ 0&0&1&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&1&0&1&1&0\\ 0&1&0&0&1&0&0\\ 0&0&0&1&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&1&1\\ 0&1&1&0&0&0&0\\ 0&0&0&1&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&1&0&0&1\\ 0&0&1&0&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&1&0&1&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&1&0&1&1&0\\ 0&0&0&1&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&0&1&0&0&1&1\\ 0&0&0&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&0&0&1\\ 0&0&0&1&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&0&0&1\\ 0&0&0&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&1&1&0\\ 0&0&0&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&1&1&0&0\\ 0&0&1&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&1&1&0&0\\ 0&0&1&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&0&1&0&0\\ 0&0&1&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&1&1&0&0\\ 0&0&1&0&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&1\\ 0&0&1&0&0&0&0\\ 0&0&0&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&1&1&0&0&1\\ 0&0&0&0&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&0&0&0&1\\ 0&0&1&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&0&1&1&0\\ 0&0&1&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&1&1\\ 0&1&0&0&0&0&0\\ 0&0&0&1&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&1&1&0&1\\ 0&0&1&1&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&1&1&1&1\\ 0&0&1&1&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&1&1&1&0&0\\ 0&1&0&1&1&0&1\\ 0&0&0&0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&1&0&1&1&1\\ 0&1&0&0&0&1&0\\ 0&0&0&1&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&0&1&1&1\\ 0&0&1&1&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&0&0&1&0\\ 0&0&1&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&0&1&1&1\\ 0&0&0&1&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&1&0&1&1&1\\ 0&0&0&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&0&1&1&0\\ 0&0&1&1&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&1&0&1&1\\ 0&0&1&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&0&0&0&1\\ 0&0&1&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&1\\ 0&0&1&0&1&0&1\\ 0&0&0&1&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&1&0&1&1\\ 0&0&1&0&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&1&1&0&0\\ 0&0&1&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&1&0&0&0&1\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&1\\ 0&1&1&0&1&1&0\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&0&1\\ 0&0&1&1&0&1&0\\ 0&0&0&0&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&1&0&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&1&0&0&0\\ 0&0&1&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&0&1&0&0&0&1\\ 0&0&0&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&1&0&1&0\\ 0&0&1&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&0&0&1&0\\ 0&0&0&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&0\\ 0&1&0&0&1&1&1\\ 0&0&0&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&1&1&1&1\\ 0&0&1&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&1&0\\ 0&0&1&1&1&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&1&0&0&1\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&1&1&0&0\\ 0&1&1&0&1&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&1&1&0&1\\ 0&0&1&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&1&0&1&1\\ 0&0&1&0&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&1&0&1&1\\ 0&0&1&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&1&0\\ 0&1&1&1&0&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&1&0&0&1\\ 0&0&1&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&0&1\\ 0&1&1&0&1&1&1\\ 0&0&0&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&1&1&0&1\\ 0&0&1&0&1&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&1&1&1\\ 0&0&1&0&1&1&0\\ 0&0&0&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&0&0&0&0\\ 0&0&1&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&1&1&0&0&0&0\\ 0&0&0&1&0&1&0\\ 0&0&0&0&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&1&1&0&0\\ 0&0&1&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&0&0&0&1\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&0&1&1&0\\ 0&0&1&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&0&0\\ 0&1&1&0&0&0&0\\ 0&0&0&1&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&0&1&0&0&0&0\\ 0&0&0&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&1&1&0&1\\ 0&0&1&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&1&1&0\\ 0&0&1&0&0&0&1\\ 0&0&0&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&0&1\\ 0&1&0&0&0&1&0\\ 0&0&0&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&1&1&1&0\\ 0&0&1&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&1\\ 0&1&0&1&0&1&0\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&0&1&1&1\\ 0&0&1&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&1&1&0&0&1\\ 0&1&1&0&0&1&0\\ 0&0&0&0&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&1&1&0&0\\ 0&0&1&0&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&0&1&0&0\\ 0&0&1&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&0&0&1&1\\ 0&0&1&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&1&0&1\\ 0&0&1&0&1&0&0\\ 0&0&0&1&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&0&0&0&1\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&1&1\\ 0&1&1&0&1&0&0\\ 0&0&0&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&1&1&0&1&0\\ 0&1&0&0&0&0&0\\ 0&0&0&0&1&1&0 \end{pmatrix}\begin{pmatrix} 0&1&0&0&0&0&1\\ 0&0&0&1&0&0&1\\ 0&0&0&0&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&0&0&1&0\\ 0&0&1&0&0&1&1\\ 0&0&0&1&1&1&0 \end{pmatrix}\begin{pmatrix} 0&1&0&0&0&0&1\\ 0&0&0&1&0&1&1\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 0&1&0&0&0&0&1\\ 0&0&1&0&1&1&1\\ 0&0&0&1&0&1&1 \end{pmatrix}\begin{pmatrix} 0&1&0&0&1&1&0\\ 0&0&1&0&0&1&1\\ 0&0&0&1&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&0&1&1&1\\ 0&0&1&0&0&0&1\\ 0&0&0&1&0&1&1 \end{pmatrix}\begin{pmatrix} 0&1&0&1&1&0&0\\ 0&0&1&0&0&0&1\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&0&1&0&1&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&0&1&0&0&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&1&0\\ 0&1&0&0&0&0&0\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&0&0&1&0&0&0\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&1&0&1&0&0\\ 0&0&0&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&0&1&0&0\\ 0&0&1&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&0&0&0&0\\ 0&0&1&0&1&0&1 \end{pmatrix}\begin{pmatrix} 0&1&0&0&0&0&0\\ 0&0&1&0&1&1&1\\ 0&0&0&1&0&0&0 \end{pmatrix}\begin{pmatrix} 0&0&1&1&0&0&1\\ 0&0&0&0&1&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 0&1&0&0&1&0&1\\ 0&0&1&0&0&0&1\\ 0&0&0&1&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&1&0&0&0\\ 0&0&1&0&1&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 0&1&0&0&0&1&1\\ 0&0&1&0&0&1&0\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 0&1&0&1&0&1&0\\ 0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 0&1&1&0&0&0&0\\ 0&0&0&1&0&0&1\\ 0&0&0&0&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&1&0&0&0\\ 0&0&1&1&0&1&1\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&1&0&0&0\\ 0&1&0&1&0&0&0\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&1&1&1&1\\ 0&0&1&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&1&1&1&0\\ 0&0&1&1&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&0&1&1&1\\ 0&0&0&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&0&0&1&0\\ 0&0&1&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&0&1\\ 0&1&1&0&1&0&1\\ 0&0&0&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&1&1&0&1\\ 0&0&1&0&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&0&0&1&0\\ 0&0&1&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&1&1&1&0\\ 0&0&1&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&1&1&0&1&1\\ 0&1&0&1&0&1&1\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&0&1&1&1\\ 0&0&1&0&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&0&1&0&0\\ 0&0&1&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&0&0&0&0\\ 0&0&1&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&1&1&0&0\\ 0&0&1&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&1&1&1&1\\ 0&0&1&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&0&1&1&0\\ 0&0&1&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&0&1\\ 0&1&0&0&1&0&1\\ 0&0&0&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&0&0&1&0\\ 0&0&1&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&0&0&1&1\\ 0&0&0&1&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&0&1\\ 0&1&0&0&0&0&1\\ 0&0&0&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&1&0&1&1&0\\ 0&0&0&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&0&0&1&1\\ 0&0&1&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&1&1&0&1&0\\ 0&0&0&0&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&0&1&1&0&1&1\\ 0&0&0&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&0&1&1&0&0&0\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&1&1\\ 0&1&0&0&1&0&1\\ 0&0&0&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&1&0&1&0\\ 0&0&1&1&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&0&1&0&1\\ 0&0&0&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&0&1&1&0&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&0\\ 0&1&1&1&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&0&0&1&0&0&1\\ 0&0&0&0&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&1&1\\ 0&1&0&0&0&1&1\\ 0&0&1&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&0&1\\ 0&0&1&0&1&1&1\\ 0&0&0&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&1&1&1&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&1&1&0&1\\ 0&0&1&0&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&0&0&1&0\\ 0&0&1&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&1&1&1&0\\ 0&0&1&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&1&0&1&0&1\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&0&0&1&1\\ 0&0&0&1&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&1&0&0&1\\ 0&1&0&0&1&0&1\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&0&1&1&0&1&0\\ 0&0&0&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&1&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&1&0&0&1\\ 0&0&0&0&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&1&1&1&0\\ 0&0&1&0&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&1&0&1&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&0&1&0&1&0&0\\ 0&0&0&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&0&1&0\\ 0&0&1&0&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&0&0&0\\ 0&0&0&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&0&1&0&1\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&0&1&0&0&1&1\\ 0&0&0&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&1&0&0&0&1\\ 0&0&0&1&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&0&1&1&0\\ 0&0&1&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&1&0&0&1&0&0\\ 0&0&1&0&1&1&1\\ 0&0&0&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&0&1\\ 0&1&0&0&0&0&1\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&1&1&0&1&1\\ 0&1&1&1&0&1&1\\ 0&0&0&0&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&0&1&0&1\\ 0&0&1&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&1&0&1&1\\ 0&0&1&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&1\\ 0&1&0&0&0&1&1\\ 0&0&1&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&0&1&0&0&0&0\\ 0&0&0&0&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&0&1&0&0\\ 0&0&1&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&1&1&0&0&0\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&0&0&0&0\\ 0&0&1&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&1&0&0&0&1\\ 0&0&0&1&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&0&1&0&0&1&1\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&1&1&0&0\\ 0&0&1&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&1&1&0&0&0&0\\ 0&0&0&1&0&0&1\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&0&0&0&1\\ 0&0&1&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&0&1&1&0\\ 0&0&1&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&0&0\\ 0&0&0&1&1&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&1&1&1&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&1&0&1&1&0\\ 0&0&0&1&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&1&0&1&0\\ 0&1&0&1&0&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&0&1&0&0&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&0&1&0&0&1&0\\ 0&0&0&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&0&0&1&0&0&0\\ 0&0&0&0&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&1&0&0&0\\ 0&1&0&1&0&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&1&1&0&0\\ 0&0&1&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&0&0&0&1\\ 0&0&1&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&1&0&1&0&1&0\\ 0&0&1&0&0&1&1\\ 0&0&0&0&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&1&1&0\\ 0&1&1&0&1&1&1\\ 0&0&0&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&1\\ 0&1&0&0&1&0&1\\ 0&0&0&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&0&0&1&1\\ 0&0&1&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&0&1&1&0\\ 0&0&1&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&1&0&0&0&1\\ 0&0&0&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&0&1&1&0\\ 0&0&1&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&0&0&0&1\\ 0&0&1&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&0&0&1&1&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&0&0\\ 0&0&0&1&0&0&1\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&0&0\\ 0&0&1&0&0&1&1\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&1\\ 0&1&0&1&1&0&0\\ 0&0&1&0&1&1&1 \end{pmatrix}\begin{pmatrix} 0&1&0&1&0&1&0\\ 0&0&0&0&1&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&1&0&0&0\\ 0&0&1&0&0&1&0\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 0&1&0&0&0&1&1\\ 0&0&1&0&1&1&0\\ 0&0&0&1&0&1&1 \end{pmatrix}\begin{pmatrix} 0&1&0&0&1&0&1\\ 0&0&1&0&0&1&1\\ 0&0&0&1&1&1&1 \end{pmatrix}\begin{pmatrix} 0&1&0&0&0&0&1\\ 0&0&1&1&1&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&0&1&1&1\\ 0&0&1&0&0&0&1\\ 0&0&0&1&1&1&0 \end{pmatrix}\begin{pmatrix} 0&1&0&0&1&1&0\\ 0&0&1&0&0&0&1\\ 0&0&0&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&1&0&0&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&0&0&0&0\\ 0&0&1&0&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&0&1&0&0\\ 0&0&0&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&1&1&0\\ 0&0&1&0&1&0&0\\ 0&0&0&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&0\\ 0&1&1&0&1&0&0\\ 0&0&0&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&0&1&0&0&0&0\\ 0&0&0&1&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&1&0&0\\ 0&1&1&0&1&1&1\\ 0&0&0&1&0&1&1 \end{pmatrix}\begin{pmatrix} 0&1&0&0&0&0&1\\ 0&0&1&0&0&0&1\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 0&1&0&1&0&0&0\\ 0&0&1&1&0&0&0\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 0&1&0&0&0&1&0\\ 0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&0&0&0&0\\ 0&0&1&0&0&0&0\\ 0&0&0&1&1&0&1 \end{pmatrix}\begin{pmatrix} 0&1&0&0&0&0&0\\ 0&0&0&1&0&1&0\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 0&1&0&0&0&0&0\\ 0&0&1&0&0&1&1\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 0&1&0&0&1&0&0\\ 0&0&1&0&0&0&0\\ 0&0&0&1&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&0&0&1&0\\ 0&0&0&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&1\\ 0&1&0&0&1&1&1\\ 0&0&1&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&1&0&0&1&0\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&1&1&0&1\\ 0&0&1&1&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&0&1&0&0&0&1\\ 0&0&0&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&1&1&1&0\\ 0&0&1&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&0&1\\ 0&1&0&1&0&1&0\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&1&0&0&1&1\\ 0&0&0&0&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&1&0\\ 0&1&0&0&0&0&0\\ 0&0&0&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&0&1&0&0\\ 0&0&1&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&0&1&0&0&0&1\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&1&0&0&1&0\\ 0&0&0&0&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&1&0&0&0\\ 0&0&1&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&0&1&0&0&0&0\\ 0&0&0&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&0&0&1&0\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&1\\ 0&1&0&0&1&1&1\\ 0&0&0&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&0&0&1&0\\ 0&0&0&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&1&1&1&1\\ 0&0&1&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&1&1&1&0\\ 0&0&1&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&1\\ 0&1&0&1&1&1&0\\ 0&0&1&1&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&1&1&0&1\\ 0&0&1&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&0&0&1&0\\ 0&0&1&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&0&1&1&0&0&1\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&1&0&0&1&1\\ 0&0&0&0&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&0&1&0&1\\ 0&0&0&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&0&1&1&1\\ 0&0&1&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&0&0\\ 0&1&0&0&0&1&1\\ 0&0&0&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&1&1&0&1\\ 0&0&1&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&1&1&1&1\\ 0&0&1&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&1&1&0&1\\ 0&0&1&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&1&0&1&1\\ 0&0&1&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&1&0&1&1\\ 0&0&1&1&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&1&1&1&0\\ 0&0&1&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&0&0&1&0\\ 0&0&1&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&0&1&1&1\\ 0&0&1&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&0\\ 0&1&0&1&0&0&1\\ 0&0&0&0&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&1&0&0&1&0\\ 0&0&0&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&0&1&0&0&1&1\\ 0&0&0&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&0&0&1&1&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&0&0&0&1\\ 0&0&1&0&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&1&1\\ 0&1&0&0&1&1&0\\ 0&0&1&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&0&0&1&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&1&0&0&0\\ 0&0&1&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&0&1&0&0&0&1\\ 0&0&0&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&1&0&0&1&0\\ 0&0&0&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&1&0&1&0&0&1\\ 0&0&1&1&0&0&0\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&0\\ 0&1&1&0&0&1&1\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&1&0&1&0\\ 0&0&1&1&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&0&0&1&0&1&0\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&0&0\\ 0&1&1&0&0&0&0\\ 0&0&0&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&0&0\\ 0&0&1&0&0&1&0\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&0&0\\ 0&1&0&0&1&0&0\\ 0&0&0&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&0&1&0&1&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&1&0&0&1\\ 0&0&1&0&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&0&0&0&0\\ 0&0&1&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&1&0&1&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&0&1&0&0&1&1\\ 0&0&0&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&0&1\\ 0&1&1&0&0&1&1\\ 0&0&0&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&0&1&1&0\\ 0&0&0&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&0&1&1&0\\ 0&0&1&1&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&1&0&0&1&1\\ 0&0&0&1&0&1&1\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&0&0&0&1\\ 0&0&1&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&1\\ 0&1&0&0&0&0&1\\ 0&0&0&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&0&1\\ 0&1&1&0&0&1&0\\ 0&0&0&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&1&0&1&1\\ 0&0&1&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&1&0&0&1&0\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&0&1&1&0&1&1\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&1&0&0&1&1\\ 0&0&0&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&0&1&0&1\\ 0&0&1&0&1&1&0\\ 0&0&0&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&0&1&0&1&0&1\\ 0&0&0&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&1&1&0&0\\ 0&0&1&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&0&1&0&0&1&1\\ 0&0&0&0&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&0&1&1&1\\ 0&0&1&0&1&0&1\\ 0&0&0&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&0\\ 0&1&1&0&1&1&1\\ 0&0&0&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&1&0&0&1&1\\ 0&0&0&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&0&1&1&0\\ 0&0&0&1&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&0&0&0&1\\ 0&0&1&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&0&0\\ 0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&1&0&1&0\\ 0&0&1&0&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&0&1&0&0\\ 0&0&1&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&1&0&0&1\\ 0&0&1&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&0&0\\ 0&0&0&0&1&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&1&0&0&0&0\\ 0&0&0&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&0&1&1&0\\ 0&0&1&0&0&0&0 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&1&1\\ 0&0&1&0&0&1&1\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&0&0&0&0\\ 0&0&1&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&0&0\\ 0&1&0&0&1&0&0\\ 0&0&0&1&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&0&0\\ 0&1&0&1&0&0&0\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&0&0&0&1\\ 0&0&1&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&0&0\\ 0&1&1&0&1&0&0\\ 0&0&0&1&0&1&1 \end{pmatrix}\begin{pmatrix} 0&1&0&0&0&0&0\\ 0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&0&1&1&1\\ 0&0&1&0&1&1&1\\ 0&0&0&1&1&1&0 \end{pmatrix}\begin{pmatrix} 0&1&1&0&0&0&1\\ 0&0&0&1&0&1&1\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 0&1&0&0&1&0&0\\ 0&0&0&1&0&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 0&1&0&0&1&1&0\\ 0&0&1&0&0&1&0\\ 0&0&0&1&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&1&0&0&1\\ 0&0&1&0&0&1&0\\ 0&0&0&0&1&1&0 \end{pmatrix}\begin{pmatrix} 0&0&1&0&0&0&0\\ 0&0&0&1&1&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&1&0&1\\ 0&0&1&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0\\ 0&0&0&1&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&0&0&0\\ 0&0&1&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&0&1&1\\ 0&0&1&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&1&0&0\\ 0&0&1&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&0&0&1&0 \\ 0&0&1&1&0&0&0 \\ 0&0&0&0&1&1&0 \end{pmatrix} \begin{pmatrix} 0&1&0&0&0&1&1 \\ 0&0&1&1&0&1&1 \\ 0&0&0&0&1&0&0 \end{pmatrix} \begin{pmatrix} 0&1&0&0&0&1&1 \\ 0&0&1&0&0&0&0 \\ 0&0&0&0&1&1&0 \end{pmatrix} \begin{pmatrix} 0&1&0&0&1&0&1 \\ 0&0&1&0&1&0&1 \\ 0&0&0&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&0&1&0&1 \\ 0&0&1&0&1&1&0 \\ 0&0&0&1&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&1 \\ 0&1&1&0&0&0&1 \\ 0&0&0&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&1 \\ 0&1&0&0&0&1&1 \\ 0&0&1&1&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&1&0 \\ 0&1&0&1&0&1&1 \\ 0&0&1&0&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&1&0&0&1 \\ 0&1&1&1&0&0&0 \\ 0&0&0&0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&0 \\ 0&0&1&0&1&0&1 \\ 0&0&0&1&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&0 \\ 0&1&0&0&1&0&1 \\ 0&0&1&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&1 \\ 0&1&0&1&1&0&1 \\ 0&0&1&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&0&1 \\ 0&1&0&1&0&0&0 \\ 0&0&0&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&1 \\ 0&1&0&0&0&1&1 \\ 0&0&1&1&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&1 \\ 0&1&0&1&1&1&1 \\ 0&0&1&1&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&1&1&0&1&0 \\ 0&1&0&0&0&1&0 \\ 0&0&0&0&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&0&0 \\ 0&1&0&0&1&1&1 \\ 0&0&1&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&0 \\ 0&1&0&1&0&1&0 \\ 0&0&1&1&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0 \\ 0&0&1&1&0&1&1 \\ 0&0&0&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0 \\ 0&1&1&0&0&1&0 \\ 0&0&0&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&0 \\ 0&0&1&0&1&0&1 \\ 0&0&0&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0 \\ 0&1&0&1&0&1&1 \\ 0&0&1&1&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0 \\ 0&1&1&0&0&1&1 \\ 0&0&0&1&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0 \\ 0&0&1&0&1&1&0 \\ 0&0&0&1&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&0 \\ 0&1&1&0&0&1&0 \\ 0&0&0&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&1&1&0 \\ 0&1&0&0&0&0&1 \\ 0&0&0&1&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&1 \\ 0&1&0&0&1&1&0 \\ 0&0&1&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&1&1 \\ 0&1&1&1&0&0&0 \\ 0&0&0&0&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&0&1&1&0 \\ 0&0&1&0&0&1&1 \\ 0&0&0&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&1&0&1 \\ 0&1&1&0&0&1&0 \\ 0&0&0&1&0&1&1 \end{pmatrix} \begin{pmatrix} 1&1&0&0&0&0&1 \\ 0&0&1&0&1&1&0 \\ 0&0&0&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&1&0 \\ 0&1&0&1&1&0&0 \\ 0&0&1&1&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&1 \\ 0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&1&1&0&0&0 \\ 0&1&1&0&0&0&0 \\ 0&0&0&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&1 \\ 0&1&0&0&1&0&0 \\ 0&0&1&1&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&1 \\ 0&1&1&0&1&0&0 \\ 0&0&0&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&1&0&0&1&0 \\ 0&0&0&1&0&0&0 \\ 0&0&0&0&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&0&1 \\ 0&0&1&0&0&0&0 \\ 0&0&0&0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&1 \\ 0&0&1&0&1&0&0 \\ 0&0&0&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&1 \\ 0&1&0&1&0&1&1 \\ 0&0&1&1&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&0&0 \\ 0&1&0&0&1&1&1 \\ 0&0&1&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&1&1 \\ 0&1&0&0&0&1&0 \\ 0&0&1&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&1 \\ 0&1&0&1&1&1&0 \\ 0&0&1&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&1 \\ 0&1&0&1&1&0&1 \\ 0&0&1&0&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&0 \\ 0&1&0&1&1&1&1 \\ 0&0&1&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&0 \\ 0&1&0&1&0&1&1 \\ 0&0&1&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&1 \\ 0&1&0&1&0&0&0 \\ 0&0&1&1&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&1 \\ 0&1&0&1&1&1&1 \\ 0&0&1&0&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&1&0&1&0 \\ 0&1&0&0&0&0&0 \\ 0&0&0&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&0 \\ 0&1&0&1&0&1&0 \\ 0&0&0&0&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&0&0 \\ 0&1&0&1&1&0&1 \\ 0&0&1&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&1&0 \\ 0&1&0&0&1&0&0 \\ 0&0&1&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&1 \\ 0&1&0&0&0&1&0 \\ 0&0&1&0&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&0 \\ 0&1&0&0&0&1&0 \\ 0&0&0&1&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&1 \\ 0&1&0&1&0&1&0 \\ 0&0&0&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&1 \\ 0&1&0&1&1&0&1 \\ 0&0&1&1&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&0&0&0&1 \\ 0&0&1&0&0&0&1 \\ 0&0&0&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&1&0 \\ 0&1&0&1&1&1&0 \\ 0&0&1&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&1 \\ 0&1&0&0&1&1&1 \\ 0&0&1&1&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&1&1 \\ 0&1&1&0&0&1&0 \\ 0&0&0&0&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&0 \\ 0&0&1&0&0&1&0 \\ 0&0&0&1&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0 \\ 0&1&0&1&0&0&1 \\ 0&0&1&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0 \\ 0&1&0&1&0&0&0 \\ 0&0&1&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0 \\ 0&0&1&1&1&0&1 \\ 0&0&0&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&0 \\ 0&1&1&0&1&1&0 \\ 0&0&0&1&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0 \\ 0&1&0&1&0&1&0 \\ 0&0&0&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0 \\ 0&1&1&0&1&1&1 \\ 0&0&0&1&0&0&0 \end{pmatrix} \begin{pmatrix} 0&1&0&0&1&1&0 \\ 0&0&1&0&0&1&1 \\ 0&0&0&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&0&0&0&1 \\ 0&0&0&1&0&0&0 \\ 0&0&0&0&1&1&0 \end{pmatrix} \begin{pmatrix} 0&1&0&0&0&1&0 \\ 0&0&0&1&1&1&0 \\ 0&0&0&0&0&0&1 \end{pmatrix} \begin{pmatrix} 0&1&0&0&1&1&1 \\ 0&0&1&0&0&1&0 \\ 0&0&0&1&0&1&1 \end{pmatrix} \begin{pmatrix} 0&1&0&0&0&0&1 \\ 0&0&1&1&1&0&0 \\ 0&0&0&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&0&1&0&1 \\ 0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&1 \end{pmatrix} \begin{pmatrix} 0&1&0&1&1&1&0 \\ 0&0&1&0&0&1&0 \\ 0&0&0&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&1 \\ 0&1&1&1&0&0&1 \\ 0&0&0&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&1 \\ 0&0&1&0&1&1&0 \\ 0&0&0&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&0&0 \\ 0&1&0&0&1&0&1 \\ 0&0&1&0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&0 \\ 0&1&0&1&0&0&1 \\ 0&0&1&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&1&0 \\ 0&1&0&0&0&1&1 \\ 0&0&0&1&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&0 \\ 0&0&1&1&1&1&0 \\ 0&0&0&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&1 \\ 0&1&1&1&0&0&0 \\ 0&0&0&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&0 \\ 0&1&0&1&1&1&1 \\ 0&0&1&0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&1&0 \\ 0&1&0&0&1&1&1 \\ 0&0&0&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&1&0&1&1&1 \\ 0&1&1&0&1&0&1 \\ 0&0&0&1&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&0&0 \\ 0&1&0&0&0&0&1 \\ 0&0&0&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&1 \\ 0&1&0&0&1&1&0 \\ 0&0&0&1&0&0&0 \end{pmatrix} \begin{pmatrix} 1&1&0&0&0&1&0 \\ 0&0&1&0&0&1&1 \\ 0&0&0&1&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&0 \\ 0&1&0&1&1&0&0 \\ 0&0&0&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&1&1 \\ 0&1&0&1&0&1&1 \\ 0&0&1&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&0 \\ 0&1&0&0&0&1&1 \\ 0&0&1&1&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&0 \\ 0&1&1&1&0&0&0 \\ 0&0&0&0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&1 \\ 0&1&0&0&1&0&1 \\ 0&0&1&1&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&1 \\ 0&1&1&0&0&0&1 \\ 0&0&0&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&1 \\ 0&0&1&0&1&0&1 \\ 0&0&0&1&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&0 \\ 0&0&1&0&1&1&0 \\ 0&0&0&1&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&1 \\ 0&1&0&0&0&1&0 \\ 0&0&1&0&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&1&0&1 \\ 0&1&0&1&1&0&1 \\ 0&0&0&0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&0&0 \\ 0&1&0&1&1&1&1 \\ 0&0&1&1&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&1&0&1&0&1 \\ 0&1&0&0&0&1&0 \\ 0&0&0&1&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&0 \\ 0&1&0&1&1&0&1 \\ 0&0&1&1&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&1&1&0 \\ 0&1&0&0&1&1&1 \\ 0&0&0&1&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&0 \\ 0&1&0&0&1&1&1 \\ 0&0&1&1&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&1&0&1&0&0 \\ 0&1&1&0&0&0&1 \\ 0&0&0&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&1&1&0&0&0&0 \\ 0&0&0&1&1&0&0 \\ 0&0&0&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&0&0&1&1 \\ 0&0&1&0&1&1&1 \\ 0&0&0&1&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&1&1&0&1&0 \\ 0&1&1&1&0&1&1 \\ 0&0&0&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&1 \\ 0&1&0&1&0&0&1 \\ 0&0&1&1&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&0 \\ 0&0&1&0&0&1&0 \\ 0&0&0&1&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&0 \\ 0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0 \end{pmatrix} \begin{pmatrix} 0&1&1&0&0&0&1 \\ 0&0&0&1&0&0&0 \\ 0&0&0&0&1&1&1 \end{pmatrix} \begin{pmatrix} 0&1&0&1&0&0&0 \\ 0&0&1&0&1&1&0 \\ 0&0&0&0&0&0&1 \end{pmatrix} \begin{pmatrix} 0&1&1&0&1&1&0 \\ 0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&1&0&0&0\\ 0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 0&0&1&0&0&0&1\\ 0&0&0&1&1&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 0&0&1&0&0&1&0\\ 0&0&0&1&0&0&0\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 0&1&0&1&0&0&1\\ 0&0&1&0&0&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&1&0&1\\ 0&1&1&0&1&1&1\\ 0&0&0&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&1&1&0&0\\ 0&0&1&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&0&1\\ 0&1&0&0&1&1&0\\ 0&0&0&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&1&0\\ 0&1&1&0&0&1&1\\ 0&0&0&1&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&1&0&1&0\\ 0&0&1&0&0&1&1\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&0&0&0&1\\ 0&0&1&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&1&1\\ 0&0&1&0&1&0&1\\ 0&0&0&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&1&0&0&1\\ 0&0&1&1&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&1&0&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&1&0&0&1&1\\ 0&0&0&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&0&1&1&0&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&1&1\\ 0&0&1&0&0&1&0\\ 0&0&0&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&1&0\\ 0&0&0&1&0&0&0\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&0&0&1&0&1&0\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&1&1&1&0&1\\ 0&1&1&1&1&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&1&1&0\\ 0&0&1&0&0&0&1\\ 0&0&0&1&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&0&0&1&1\\ 0&0&1&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&1&0&1&0&1&0\\ 0&0&0&0&1&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&1&0&0&0\\ 0&0&1&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&1\\ 0&1&0&0&1&0&1\\ 0&0&0&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&1&0&0&1\\ 0&0&1&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&1\\ 0&1&0&0&0&0&0\\ 0&0&0&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&0&1&0&0\\ 0&0&0&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&0&1&1&0&0&0\\ 0&0&0&0&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&0&1&1&1\\ 0&0&1&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&1&1&1&1\\ 0&0&1&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&1&0&0&0&0\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&0&0&1&0\\ 0&0&1&0&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&1&0&0&0\\ 0&0&1&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&1&0&0&0\\ 0&0&1&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&1&0&1&1\\ 0&0&1&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&1&0&1&0\\ 0&0&1&1&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&0&0&0&0\\ 0&0&1&0&0&0&1\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&0&0\\ 0&1&1&1&0&0&0\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&1&0&1&0\\ 0&0&1&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&0&0&0&1\\ 0&0&1&0&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&0&0&0&0\\ 0&0&0&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&1&0\\ 0&0&1&0&0&0&0\\ 0&0&0&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&1&1&0&0&0\\ 0&1&1&1&0&0&1\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&0&1&1&0\\ 0&0&1&0&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&1&1&0&0\\ 0&0&1&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&0&1&0&0\\ 0&0&1&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&1&0&0&1\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&0\\ 0&1&1&0&1&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&0&0&0&0\\ 0&0&0&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&0&1\\ 0&0&1&0&1&0&0\\ 0&0&0&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&1&1&1&0\\ 0&0&1&0&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&0&0&1&0\\ 0&0&1&0&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&0&1&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&1&1&1&0&0&0\\ 0&0&0&0&1&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&1&0&0&1&0\\ 0&0&0&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&0&1&0&0&1&0\\ 0&0&0&1&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&1&0&1\\ 0&1&1&0&0&1&1\\ 0&0&0&1&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&0&0&1&1\\ 0&0&0&1&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&0&1&0&1\\ 0&0&1&1&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&1&0\\ 0&0&1&1&0&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&1&0&1&0&1\\ 0&0&0&1&0&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&1&0&1&0&0&0\\ 0&0&1&0&0&1&0\\ 0&0&0&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&0\\ 0&0&1&1&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&1&0&0&1&0&1\\ 0&0&0&1&1&0&1\\ 0&0&0&0&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&1&0&0&1\\ 0&0&1&1&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&0&1\\ 0&1&1&0&0&1&1\\ 0&0&0&1&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&0&1\\ 0&1&1&0&0&1&0\\ 0&0&0&1&1&0&0 \end{pmatrix} \begin{pmatrix} 0&1&0&0&0&0&0\\ 0&0&1&0&0&0&1\\ 0&0&0&1&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&0&0&0&1\\ 0&0&1&0&1&0&1\\ 0&0&0&0&0&1&0 \end{pmatrix} \begin{pmatrix} 0&1&1&1&0&0&1\\ 0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix} \begin{pmatrix} 0&1&0&0&1&1&0\\ 0&0&1&0&1&1&0\\ 0&0&0&1&1&0&0 \end{pmatrix} \begin{pmatrix} 0&1&0&0&1&0&0\\ 0&0&1&0&0&1&1\\ 0&0&0&1&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 0&0&1&0&0&0&0\\ 0&0&0&1&0&0&1\\ 0&0&0&0&1&1&1 \end{pmatrix} \begin{pmatrix} 0&1&0&0&1&0&1\\ 0&0&1&0&1&0&0\\ 0&0&0&1&0&0&1 \end{pmatrix} \begin{pmatrix} 1&1&0&0&0&1&1\\ 0&0&0&1&0&0&0\\ 0&0&0&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&0\\ 0&0&1&0&0&1&0\\ 0&0&0&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&0&0\\ 0&1&0&1&0&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&1&0&0&1&0&1\\ 0&0&1&0&0&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&1&0&0\\ 0&1&0&1&0&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&1&0&1&1&0\\ 0&0&0&1&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&1&1&1&0\\ 0&1&1&1&1&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&1&1&0\\ 0&0&0&1&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&0&0&1\\ 0&0&0&1&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&0&1&0\\ 0&0&1&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&1&1&1\\ 0&0&1&1&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&0&1&0&1&1&1\\ 0&0&0&1&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&1&1&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&1&0&1&0&1\\ 0&0&0&1&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&0&1&0&1&1&1\\ 0&0&0&1&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&1&0&1&0&1\\ 0&1&1&1&1&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&1&1&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&0&0\\ 0&0&1&1&0&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&0&0&1&1\\ 0&0&1&0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&1&1&0&0&0&1\\ 0&0&0&1&0&0&1\\ 0&0&0&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&1&0\\ 0&1&0&0&1&0&1\\ 0&0&0&1&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&0&1\\ 0&1&0&1&1&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&0&0&1&0\\ 0&0&0&1&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&1&1\\ 0&1&0&0&1&1&1\\ 0&0&1&0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&1&0&0&1\\ 0&0&1&1&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&1&1&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&0&1\\ 0&1&0&0&1&1&1\\ 0&0&0&1&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&0&0&1&0\\ 0&0&1&1&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&0&1\\ 0&1&1&0&0&1&0\\ 0&0&0&0&1&0&0 \end{pmatrix} \begin{pmatrix} 1&1&0&0&1&0&1\\ 0&0&1&0&0&0&0\\ 0&0&0&1&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&1&1&0\\ 0&1&0&0&0&0&0\\ 0&0&0&1&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&1&1&0&0&1\\ 0&1&1&0&0&1&1\\ 0&0&0&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&0&1&0&0\\ 0&0&1&1&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&1&0&0&0\\ 0&0&1&1&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&1&0&1&1\\ 0&0&1&0&0&0&1\\ 0&0&0&0&1&0&1 \end{pmatrix} \begin{pmatrix} 0&1&0&1&0&0&0\\ 0&0&1&1&0&1&1\\ 0&0&0&0&1&1&1 \end{pmatrix} \begin{pmatrix} 0&1&0&0&0&1&0\\ 0&0&1&0&0&0&1\\ 0&0&0&0&1&0&1 \end{pmatrix} \begin{pmatrix} 0&1&0&1&0&1&0\\ 0&0&1&1&0&0&0\\ 0&0&0&0&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&0&0&1&0\\ 0&0&1&0&0&0&0\\ 0&0&0&1&1&0&1 \end{pmatrix}\begin{pmatrix} 0&1&0&1&0&0&0\\ 0&0&1&1&0&0&0\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 0&1&0&0&0&0&0\\ 0&0&0&1&0&1&0\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 0&1&0&0&1&0&0\\ 0&0&1&0&0&0&1\\ 0&0&0&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&0&0\\ 0&1&0&0&0&1&0\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&1&1&0&1\\ 0&0&1&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&1&1&1&1\\ 0&0&1&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&0&1&1&1&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&1&0&1&0&1\\ 0&0&0&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&0&1&1&1\\ 0&0&1&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&1&1&1&0\\ 0&0&1&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&1&1&1\\ 0&0&1&0&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&1&0&1&1\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&0&0&0\\ 0&0&1&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&0&1&0\\ 0&0&1&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&1&1&0&1\\ 0&0&1&0&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&1&1&1&0\\ 0&0&1&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&1&0&0\\ 0&0&1&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&0&1&0&1&0&0\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&1&1&0&1&1\\ 0&0&0&0&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&0&0&0&0\\ 0&0&1&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&1&0\\ 0&1&0&0&1&0&0\\ 0&0&0&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&1&0&1&0\\ 0&0&1&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&0&1&0&0&0&0\\ 0&0&0&0&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&1&0&1&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&0\\ 0&1&1&0&1&1&0\\ 0&0&0&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&1&1&1\\ 0&0&1&0&0&1&1\\ 0&0&0&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&0&0\\ 0&1&0&0&0&0&1\\ 0&0&0&1&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&0&0&0&1\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&0&1&1&0\\ 0&0&0&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&1\\ 0&1&0&1&1&0&0\\ 0&0&1&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&1&1&0&0\\ 0&0&1&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&0&1&1&0\\ 0&0&1&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&0&0\\ 0&1&1&0&1&1&0\\ 0&0&0&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&0&0&0&1\\ 0&0&1&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&1&0&1&0\\ 0&0&1&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&1&0&0&0\\ 0&0&1&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&0&0\\ 0&0&1&0&1&1&0\\ 0&0&0&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&1\\ 0&1&0&1&0&1&1\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&1&1&1\\ 0&0&1&0&1&0&0\\ 0&0&0&1&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&0&0&0&0\\ 0&0&0&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&1&0&0&0\\ 0&1&0&1&0&0&1\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&0&0&1&0\\ 0&0&1&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&0&1&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&1&0&1&0\\ 0&1&1&0&1&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&1&1&1&0\\ 0&0&1&0&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&0&1&0&1\\ 0&0&1&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&0&1&1&1&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&1&0&0&0\\ 0&1&1&1&0&0&0\\ 0&0&0&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&1&0&0&1&0&0\\ 0&0&1&0&1&1&0\\ 0&0&0&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&1&0&0&1\\ 0&0&1&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&1\\ 0&1&0&0&0&1&1\\ 0&0&0&1&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&1&1&0&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&1&0&0&0&1\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&1\\ 0&1&0&0&1&0&0\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&1\\ 0&1&0&1&1&1&0\\ 0&0&1&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&1 \\ 0&1&0&0&0&0&0 \\ 0&0&0&0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&1 \\ 0&1&0&0&1&1&1 \\ 0&0&1&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&0 \\ 0&0&1&0&0&0&0 \\ 0&0&0&1&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&1 \\ 0&1&0&1&1&1&1 \\ 0&0&1&0&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&1 \\ 0&1&0&0&1&0&0 \\ 0&0&0&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&1&0 \\ 0&1&0&1&0&0&0 \\ 0&0&0&0&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&0 \\ 0&1&0&0&0&0&1 \\ 0&0&1&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&1 \\ 0&1&0&0&0&0&0 \\ 0&0&1&1&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&1 \\ 0&0&1&0&0&1&1 \\ 0&0&0&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&1&1 \\ 0&1&0&0&1&1&0 \\ 0&0&1&0&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&1 \\ 0&1&1&0&1&0&0 \\ 0&0&0&1&0&1&1 \end{pmatrix} \begin{pmatrix} 0&1&0&1&0&1&1 \\ 0&0&1&0&0&0&1 \\ 0&0&0&0&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&1&0&1&1 \\ 0&0&1&0&0&1&0 \\ 0&0&0&0&1&1&0 \end{pmatrix} \begin{pmatrix} 0&1&0&0&1&0&0 \\ 0&0&1&1&1&0&0 \\ 0&0&0&0&0&0&1 \end{pmatrix} \begin{pmatrix} 0&1&0&1&0&0&1 \\ 0&0&1&1&0&0&1 \\ 0&0&0&0&1&1&1 \end{pmatrix} \begin{pmatrix} 0&1&0&0&1&1&1 \\ 0&0&1&0&0&0&0 \\ 0&0&0&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&0&0&0&0 \\ 0&0&1&0&1&0&1 \\ 0&0&0&0&0&1&1 \end{pmatrix} \begin{pmatrix} 0&1&0&0&0&1&0 \\ 0&0&1&0&1&1&0 \\ 0&0&0&1&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&1 \\ 0&1&0&0&0&1&0 \\ 0&0&1&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&1 \\ 0&1&1&0&1&0&1 \\ 0&0&0&1&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&0&0 \\ 0&1&0&1&1&0&0 \\ 0&0&0&0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&0 \\ 0&1&0&0&0&0&1 \\ 0&0&0&1&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&0 \\ 0&1&0&0&1&1&0 \\ 0&0&0&1&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&1 \\ 0&1&0&0&1&1&1 \\ 0&0&1&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&1 \\ 0&0&1&0&1&1&1 \\ 0&0&0&1&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&1&0&1 \\ 0&1&1&0&1&0&1 \\ 0&0&0&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&0 \\ 0&1&0&1&1&1&1 \\ 0&0&1&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&0&0 \\ 0&1&0&0&1&1&1 \\ 0&0&0&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&0&1&0&0 \\ 0&0&1&1&1&0&0 \\ 0&0&0&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&1&1&0 \\ 0&1&0&0&0&1&0 \\ 0&0&0&1&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&0 \\ 0&1&0&1&1&0&1 \\ 0&0&1&0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&1&1&0&0&1 \\ 0&1&0&1&0&0&1 \\ 0&0&0&0&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&1 \\ 0&1&0&1&0&1&0 \\ 0&0&1&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&1&0&0&0&1&0 \\ 0&0&1&1&0&0&1 \\ 0&0&0&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&0&0 \\ 0&1&0&0&0&1&1 \\ 0&0&1&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&0 \\ 0&1&0&1&0&0&0 \\ 0&0&1&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&0 \\ 0&1&0&0&1&0&1 \\ 0&0&1&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&0 \\ 0&1&0&1&0&1&1 \\ 0&0&1&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&1&1&0&1&0 \\ 0&1&1&1&0&0&1 \\ 0&0&0&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&1 \\ 0&1&0&0&0&0&1 \\ 0&0&0&1&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&0 \\ 0&1&1&0&0&0&1 \\ 0&0&0&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&0&0 \\ 0&0&0&1&0&1&1 \\ 0&0&0&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&0&1 \\ 0&1&0&1&0&1&0 \\ 0&0&1&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&1 \\ 0&1&0&0&1&1&0 \\ 0&0&1&1&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&0 \\ 0&1&0&1&1&0&0 \\ 0&0&1&1&0&1&0 \end{pmatrix} \begin{pmatrix} 1&1&0&0&0&0&0 \\ 0&0&1&0&0&0&1 \\ 0&0&0&0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&1&1 \\ 0&1&0&0&1&0&1 \\ 0&0&1&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&0 \\ 0&0&1&1&0&0&1 \\ 0&0&0&0&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&1&0 \\ 0&1&1&0&0&1&1 \\ 0&0&0&1&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&1 \\ 0&0&0&1&0&1&0 \\ 0&0&0&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&1 \\ 0&1&0&0&0&1&1 \\ 0&0&1&1&0&0&0 \end{pmatrix} \begin{pmatrix} 1&1&0&1&0&0&1 \\ 0&0&1&1&0&1&0 \\ 0&0&0&0&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&1 \\ 0&1&1&0&0&0&1 \\ 0&0&0&1&1&0&1 \end{pmatrix} \begin{pmatrix} 0&1&1&0&0&0&0 \\ 0&0&0&1&1&0&1 \\ 0&0&0&0&0&1&1 \end{pmatrix} \begin{pmatrix} 0&1&0&0&1&1&0 \\ 0&0&1&0&1&0&0 \\ 0&0&0&1&1&1&1 \end{pmatrix} \begin{pmatrix} 0&1&0&0&0&0&1 \\ 0&0&0&1&0&0&0 \\ 0&0&0&0&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 0&0&0&1&0&1&0\\ 0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 0&1&0&0&1&1&0\\ 0&0&1&0&0&1&0\\ 0&0&0&1&0&1&0 \end{pmatrix}\begin{pmatrix} 0&0&1&0&0&0&0\\ 0&0&0&1&0&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 0&1&0&0&0&0&0\\ 0&0&1&0&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&0&1\\ 0&1&1&1&1&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&1&0\\ 0&0&1&0&1&1&1\\ 0&0&0&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&0&1&0&1\\ 0&0&0&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&1&1&1&0&0\\ 0&1&1&1&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&1&1&0&0\\ 0&0&1&1&0&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&0&1\\ 0&0&0&1&0&0&1\\ 0&0&0&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&0&0&1&1\\ 0&0&1&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&0&1&0&0\\ 0&0&1&0&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&1&0&0&1\\ 0&1&1&0&0&0&1\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&1&0&1\\ 0&0&1&0&0&0&0\\ 0&0&0&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&1&1&1&0&1\\ 0&1&1&0&0&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&0&0&0&0\\ 0&0&1&1&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&0&0&1&1\\ 0&0&0&1&0&0&1\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&1&0\\ 0&0&1&0&1&0&0\\ 0&0&0&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&1&0&1&0&0&0\\ 0&0&1&0&0&0&1\\ 0&0&0&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&0&1&0&1\\ 0&0&1&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&1&0&0&0\\ 0&0&1&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&0&1\\ 0&1&1&0&1&1&0\\ 0&0&0&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&0&1\\ 0&1&1&1&0&1&1\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&1&0&1&0&0&0\\ 0&0&1&1&1&0&1\\ 0&0&0&0&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&1&0&1\\ 0&1&0&0&0&1&1\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&0&0\\ 0&1&1&0&1&0&1\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&0&1&0&1\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&0&0\\ 0&0&1&1&0&1&0\\ 0&0&0&0&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&0&0&1&0\\ 0&0&1&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&1&1&0&1\\ 0&0&1&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&1&1&1&0\\ 0&0&1&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&1\\ 0&1&0&0&0&1&1\\ 0&0&0&1&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&0&1&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&0&1\\ 0&0&1&0&0&1&0\\ 0&0&0&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&1&0&0&1\\ 0&0&1&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&1\\ 0&0&0&1&0&0&0\\ 0&0&0&0&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&0&1\\ 0&1&0&0&0&1&1\\ 0&0&0&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&1&1&1&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&0&0&0&1\\ 0&0&1&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&1&0&0&1\\ 0&1&1&1&0&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&0&1&0&0\\ 0&0&1&1&1&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&1&0&0&1\\ 0&0&1&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&0&1&0&1&1&0\\ 0&0&0&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&0&1&0&1\\ 0&0&1&0&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&1&1&1\\ 0&1&0&0&0&1&1\\ 0&0&0&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&1&1&0&0&0\\ 0&0&0&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&1&1&1&1\\ 0&0&1&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&1&1\\ 0&1&1&0&1&0&1\\ 0&0&0&1&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&1&1&0&1\\ 0&0&1&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&0&1&1&1\\ 0&0&1&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&1&0&1&1&0&1\\ 0&0&1&1&1&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&1&1&1&0\\ 0&0&1&0&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&0&0&1&0\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&1&0&1&0&0\\ 0&0&0&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&0&0&0&0\\ 0&0&1&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&0&1&0&1&0&0\\ 0&0&0&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&1&0 \\ 0&1&0&0&1&0&0 \\ 0&0&1&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&1&1&0&1&1 \\ 0&1&1&0&0&1&1 \\ 0&0&0&0&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&0 \\ 0&1&0&1&0&1&1 \\ 0&0&1&1&0&1&1 \end{pmatrix} \begin{pmatrix} 1&1&0&0&0&1&1 \\ 0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&1 \\ 0&1&0&1&0&1&0 \\ 0&0&1&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&0&1 \\ 0&0&1&0&0&0&1 \\ 0&0&0&0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&1&1&0&0&1&0 \\ 0&0&0&1&0&1&1 \\ 0&0&0&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&1 \\ 0&1&0&0&0&0&1 \\ 0&0&0&1&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&1&1&0 \\ 0&1&1&0&0&0&1 \\ 0&0&0&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&0 \\ 0&1&0&0&1&1&0 \\ 0&0&1&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&1 \\ 0&1&0&1&1&0&0 \\ 0&0&1&1&0&1&0 \end{pmatrix} \begin{pmatrix} 0&1&0&0&0&1&0 \\ 0&0&0&1&0&1&1 \\ 0&0&0&0&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&0&1&1&1 \\ 0&0&1&0&0&1&1 \\ 0&0&0&1&0&1&0 \end{pmatrix} \begin{pmatrix} 0&1&0&0&1&0&1 \\ 0&0&1&0&0&0&0 \\ 0&0&0&1&0&0&0 \end{pmatrix} \begin{pmatrix} 0&1&0&0&0&1&0 \\ 0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1 \end{pmatrix} \begin{pmatrix} 0&1&0&0&0&0&1 \\ 0&0&1&0&1&0&0 \\ 0&0&0&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&0&0&0&0 \\ 0&0&1&0&0&1&0 \\ 0&0&0&1&1&1&0 \end{pmatrix} \begin{pmatrix} 0&1&0&1&1&0&0 \\ 0&0&1&1&1&0&0 \\ 0&0&0&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&1 \\ 0&1&0&1&1&0&1 \\ 0&0&1&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&1&1 \\ 0&1&0&0&1&1&1 \\ 0&0&1&1&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&1 \\ 0&1&0&1&1&1&1 \\ 0&0&1&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&1 \\ 0&1&0&1&0&1&1 \\ 0&0&1&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&1 \\ 0&1&0&1&0&1&1 \\ 0&0&1&1&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&0 \\ 0&1&0&0&0&1&0 \\ 0&0&1&0&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&1 \\ 0&1&0&1&1&1&0 \\ 0&0&1&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&0 \\ 0&0&1&0&1&1&0 \\ 0&0&0&1&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&0&0 \\ 0&1&1&1&0&0&1 \\ 0&0&0&0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&1&1 \\ 0&1&0&1&0&1&1 \\ 0&0&1&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&1 \\ 0&0&1&0&1&0&1 \\ 0&0&0&0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&1 \\ 0&1&1&0&0&0&1 \\ 0&0&0&1&0&0&1 \end{pmatrix} \begin{pmatrix} 1&1&0&0&0&0&0 \\ 0&0&1&0&1&1&1 \\ 0&0&0&1&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&1 \\ 0&1&1&0&1&1&1 \\ 0&0&0&1&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&1&0&1 \\ 0&1&1&0&0&0&0 \\ 0&0&0&1&0&1&0 \end{pmatrix} \begin{pmatrix} 1&1&0&0&0&1&1 \\ 0&0&1&0&1&0&0 \\ 0&0&0&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&1&1&0&0&1 \\ 0&1&1&0&0&1&0 \\ 0&0&0&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&1&1&0&0&1 \\ 0&1&0&0&0&0&0 \\ 0&0&0&0&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&0 \\ 0&1&0&0&1&0&0 \\ 0&0&1&1&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&0 \\ 0&1&0&0&0&1&1 \\ 0&0&1&0&0&0&0 \end{pmatrix} \begin{pmatrix} 1&1&0&0&0&1&1 \\ 0&0&1&1&0&1&1 \\ 0&0&0&0&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&0&1 \\ 0&1&0&0&0&0&1 \\ 0&0&0&0&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&0&0 \\ 0&1&1&0&1&0&0 \\ 0&0&0&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&0&0 \\ 0&1&0&0&0&0&0 \\ 0&0&0&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&0 \\ 0&1&0&0&1&0&0 \\ 0&0&1&1&0&0&1 \end{pmatrix} \begin{pmatrix} 1&1&0&0&0&0&0 \\ 0&0&1&0&1&0&0 \\ 0&0&0&1&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&1 \\ 0&1&0&0&0&1&1 \\ 0&0&1&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&0 \\ 0&1&0&1&1&0&0 \\ 0&0&1&0&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&1 \\ 0&0&0&0&1&0&0 \\ 0&0&0&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&0&0 \\ 0&1&0&1&0&1&0 \\ 0&0&1&0&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&1 \\ 0&1&0&1&0&0&1 \\ 0&0&1&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&0 \\ 0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&0 \\ 0&1&1&0&0&0&0 \\ 0&0&0&1&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&1 \\ 0&1&0&0&1&0&0 \\ 0&0&1&0&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&1&1 \\ 0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0 \end{pmatrix} \begin{pmatrix} 0&1&0&0&0&1&0 \\ 0&0&1&0&1&1&1 \\ 0&0&0&1&0&1&0 \end{pmatrix} \begin{pmatrix} 0&1&0&1&0&0&0 \\ 0&0&1&1&0&1&0 \\ 0&0&0&0&1&1&1 \end{pmatrix} \begin{pmatrix} 0&1&0&0&0&1&0 \\ 0&0&1&0&0&0&1 \\ 0&0&0&1&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&1&0&0&0 \\ 0&0&0&0&1&0&1 \\ 0&0&0&0&0&1&0 \end{pmatrix} \begin{pmatrix} 0&1&0&1&0&1&0 \\ 0&0&1&1&0&1&1 \\ 0&0&0&0&1&1&1 \end{pmatrix} \begin{pmatrix} 0&1&0&0&1&1&1 \\ 0&0&1&0&0&0&1 \\ 0&0&0&1&1&0&1 \end{pmatrix} \begin{pmatrix} 0&1&0&0&1&1&1 \\ 0&0&1&0&0&0&1 \\ 0&0&0&1&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&0&1 \\ 0&1&0&0&0&1&0 \\ 0&0&1&1&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&1&1&0&1&1 \\ 0&1&0&1&0&1&1 \\ 0&0&0&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&1 \\ 0&1&0&0&1&1&1 \\ 0&0&1&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&0 \\ 0&1&0&1&1&1&1 \\ 0&0&1&1&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&1&0&1 \\ 0&1&1&0&1&0&1 \\ 0&0&0&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&0 \\ 0&1&0&1&1&0&1 \\ 0&0&1&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&1&0&0&1&0&0 \\ 0&0&1&0&1&1&0 \\ 0&0&0&1&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&0 \\ 0&1&0&1&0&0&0 \\ 0&0&1&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&1 \\ 0&0&1&0&0&0&1 \\ 0&0&0&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&0&0 \\ 0&1&0&1&0&0&0 \\ 0&0&1&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&0 \\ 0&1&0&1&0&1&1 \\ 0&0&1&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&1 \\ 0&1&0&1&0&1&0 \\ 0&0&1&1&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&1 \\ 0&1&1&1&0&0&0 \\ 0&0&0&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&1&1 \\ 0&1&0&1&0&1&0 \\ 0&0&1&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&1 \\ 0&1&0&0&0&1&1 \\ 0&0&1&1&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&1 \\ 0&1&1&1&0&1&0 \\ 0&0&0&0&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&0&0 \\ 0&1&0&0&1&0&1 \\ 0&0&1&1&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&1 \\ 0&1&0&0&1&0&1 \\ 0&0&1&1&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&0 \\ 0&1&0&0&0&1&1 \\ 0&0&1&1&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&1 \\ 0&1&0&0&0&1&1 \\ 0&0&0&0&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&0 \\ 0&0&1&1&0&0&1 \\ 0&0&0&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&0 \\ 0&1&1&0&1&1&0 \\ 0&0&0&1&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&0 \\ 0&1&0&0&0&1&1 \\ 0&0&1&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&1&1&0&1&0 \\ 0&1&1&1&0&1&0 \\ 0&0&0&0&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&0 \\ 0&0&1&1&0&1&1 \\ 0&0&0&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&1&0&0&0&1&1 \\ 0&0&1&1&0&0&0 \\ 0&0&0&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&1&0&0 \\ 0&1&0&0&1&0&1 \\ 0&0&0&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&1 \\ 0&1&0&1&0&1&0 \\ 0&0&1&1&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&0&1&0&1 \\ 0&0&0&1&1&0&0 \\ 0&0&0&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&1 \\ 0&0&1&0&0&1&0 \\ 0&0&0&1&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&1 \\ 0&1&0&1&0&0&1 \\ 0&0&1&0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&0 \\ 0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&1&0 \\ 0&1&1&0&0&0&1 \\ 0&0&0&1&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&0 \\ 0&1&0&1&0&0&1 \\ 0&0&1&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&1&1&1&1&0&0 \\ 0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&1 \\ 0&1&0&1&1&0&1 \\ 0&0&1&1&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&1 \\ 0&1&0&0&0&1&0 \\ 0&0&1&1&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&0 \\ 0&1&0&1&0&1&1 \\ 0&0&0&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&0 \\ 0&1&1&0&1&0&1 \\ 0&0&0&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&0 \\ 0&1&0&1&1&1&1 \\ 0&0&1&1&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&1&0 \\ 0&1&0&0&1&1&1 \\ 0&0&1&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&1 \\ 0&0&1&0&1&1&0 \\ 0&0&0&1&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&1 \\ 0&1&0&0&0&1&1 \\ 0&0&0&0&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&0 \\ 0&1&0&1&1&0&1 \\ 0&0&1&1&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&0 \\ 0&1&0&1&1&1&0 \\ 0&0&1&0&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&1 \\ 0&1&0&0&1&0&1 \\ 0&0&1&0&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&0 \\ 0&1&0&0&0&0&0 \\ 0&0&1&1&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&1&0 \\ 0&1&0&0&0&1&0 \\ 0&0&0&0&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&1&1 \\ 0&1&0&1&1&1&1 \\ 0&0&1&1&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&1 \\ 0&1&0&0&0&1&0 \\ 0&0&0&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&1 \\ 0&1&0&0&1&1&0 \\ 0&0&1&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&1 \\ 0&1&0&0&1&1&1 \\ 0&0&0&1&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&0&1&1&0\\ 0&0&0&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&0\\ 0&1&0&0&0&0&1\\ 0&0&0&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&1\\ 0&1&0&0&0&0&1\\ 0&0&1&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&1&1&1&0\\ 0&0&1&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&1&1&0&0\\ 0&0&1&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&0&1&1&1\\ 0&0&1&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&0&0&0&1\\ 0&0&1&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&1&1&0&1\\ 0&0&1&0&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&1&1\\ 0&1&0&0&1&0&0\\ 0&0&0&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&1&0&1&1\\ 0&1&0&0&0&1&0\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&0&0&0&0\\ 0&0&0&1&1&1&0 \end{pmatrix}\begin{pmatrix} 0&1&0&0&0&1&1\\ 0&0&1&0&0&1&0\\ 0&0&0&1&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 0&0&1&0&0&1&1\\ 0&0&0&1&0&0&1\\ 0&0&0&0&1&1&0 \end{pmatrix}\begin{pmatrix} 0&1&0&1&1&0&0\\ 0&0&1&0&1&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 0&1&1&0&1&0&1\\ 0&0&0&1&0&0&1\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 0&1&0&0&0&0&1\\ 0&0&1&1&0&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&0&1&1&0\\ 0&0&1&0&1&1&1\\ 0&0&0&1&0&0&0 \end{pmatrix}\begin{pmatrix} 0&1&0&0&1&0&1\\ 0&0&0&1&1&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&0&1&1&1\\ 0&0&1&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&0&1&0&1\\ 0&0&0&1&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&0&0\\ 0&1&0&0&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&0&1&1&1&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&1&1&1&0\\ 0&0&1&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&1&0&1&0&1\\ 0&0&0&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&1&0&1&1\\ 0&1&0&1&0&0&1\\ 0&0&0&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&1&1&0&0\\ 0&0&1&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&0&0&0&1\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&0&0&0&0\\ 0&0&1&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&0&0&0&1\\ 0&0&1&0&0&0&0\\ 0&0&0&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&1\\ 0&0&0&1&0&1&0\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&1&0&0&0&0\\ 0&0&0&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&0&1&1&0\\ 0&0&1&0&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&0&0&0&1\\ 0&0&1&0&0&0&0 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&0&1\\ 0&0&1&0&0&1&1\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&0&1&0&1&0&0\\ 0&0&0&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&0&0&0&0\\ 0&0&0&1&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&1&0&0&1\\ 0&1&0&0&1&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&1&0&1&0&0\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&1\\ 0&1&1&0&0&0&0\\ 0&0&0&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&1&1&0&0&1&0\\ 0&0&0&0&1&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&1&0&0&0\\ 0&0&1&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&0&0&1&1\\ 0&0&1&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&0&1&0&1\\ 0&0&0&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&1&0&0&0\\ 0&1&1&1&1&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&1&0&0&1\\ 0&0&1&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&0&1&0&0&0&1\\ 0&0&0&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&0&1&0&0\\ 0&0&1&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&1&0&0&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&0&0&1&0&0&1\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&0&0&0&0\\ 0&0&1&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&0&1&0&0&0&0\\ 0&0&0&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&0&1&0&1&0&0\\ 0&0&0&1&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&1&0&0&0&1\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 0&1&1&0&1&0&0\\ 0&0&0&1&1&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 0&1&0&0&1&0&0\\ 0&0&1&0&1&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 0&0&1&0&1&0&0\\ 0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&0&0&1&0\\ 0&0&1&0&0&1&0\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 0&1&0&1&0&0&1\\ 0&0&1&1&0&1&0\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 0&1&0&1&0&1&1\\ 0&0&1&0&0&0&0\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 0&1&0&0&0&0&0\\ 0&0&1&0&1&1&1\\ 0&0&0&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&0&1\\ 0&0&0&1&0&1&0\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&1&0&1&0\\ 0&0&1&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&1&0&0&1&0\\ 0&0&0&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&1&0\\ 0&0&1&0&0&0&1\\ 0&0&0&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&1&0&0&0\\ 0&1&1&0&0&1&1\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&1&0&1&0&1&0\\ 0&0&1&1&0&0&0\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&1&0&0&0\\ 0&0&1&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&1&0&1&0&0&0\\ 0&0&1&1&0&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&1&0&0&1&0\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&1&0&0\\ 0&0&1&0&1&1&1\\ 0&0&0&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&0&1\\ 0&0&0&1&0&0&1\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&1&0&0&0&1\\ 0&0&0&0&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&0&1&0&1&1&1\\ 0&0&0&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&0&1\\ 0&1&1&0&0&0&1\\ 0&0&0&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&1\\ 0&1&0&1&1&0&0\\ 0&0&1&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&0&1&1&0\\ 0&0&0&1&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&1&0&0&0\\ 0&1&0&0&0&0&1\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&0&1&0&0&1&1\\ 0&0&0&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&1&1&0&0\\ 0&0&1&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&0\\ 0&1&0&0&0&0&1\\ 0&0&0&1&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&1&0&1&1&0\\ 0&0&0&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&0&1&0&1\\ 0&0&1&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&0&1&1&1&0&1\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&1&1&0&1&1\\ 0&0&0&0&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&0&1\\ 0&1&0&0&0&1&1\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&1&0&0&0\\ 0&0&1&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&0&1&0&0&0&1\\ 0&0&0&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&1&0&1&1&0\\ 0&0&0&1&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&0&1&0&0\\ 0&0&1&0&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&0&0&0&0\\ 0&0&1&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&0&1&0&0\\ 0&0&1&0&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&1&0&0&0&0\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&0&0&0&0\\ 0&0&1&0&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&1&1&0&1&1\\ 0&1&0&0&0&0&0\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&0&0&0&0\\ 0&0&0&0&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&1&0&1&0\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&1\\ 0&1&0&1&1&0&1\\ 0&0&1&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&0&1&0&0\\ 0&0&1&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&0&0&1&0\\ 0&0&1&0&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&1&1&1&1\\ 0&0&1&0&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&1&0&0&0\\ 0&0&1&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&1&0\\ 0&0&0&1&1&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&0&1&0&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&1&1&0&0\\ 0&0&1&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&0&0&0&1\\ 0&0&0&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&0&1\\ 0&1&0&0&1&1&0\\ 0&0&0&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&1&1&1&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&1&0&0&0&1\\ 0&0&0&1&0&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&1&0&1&1\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&1&1&0\\ 0&0&1&0&1&1&0\\ 0&0&0&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&0&0\\ 0&1&1&0&1&1&0\\ 0&0&0&1&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&1&0&0&1\\ 0&0&1&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&1&0&1&0\\ 0&0&1&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&1&1&0&0&0&0\\ 0&0&0&0&1&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&1&1&0\\ 0&1&0&0&1&0&1\\ 0&0&0&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&0&0&1&0\\ 0&0&1&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&1&1&0&0\\ 0&0&1&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&0&1&1&0\\ 0&0&1&1&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&1&0&1\\ 0&1&0&0&0&1&1\\ 0&0&0&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&1&1&1&1\\ 0&0&1&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&1&0&0&0\\ 0&1&0&0&0&0&1\\ 0&0&0&0&1&1&0 \end{pmatrix}\begin{pmatrix} 0&1&0&0&0&0&1\\ 0&0&1&0&0&1&1\\ 0&0&0&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&0&1&0&1\\ 0&0&1&0&1&1&0\\ 0&0&0&1&1&1&0 \end{pmatrix}\begin{pmatrix} 0&1&1&0&0&1&0\\ 0&0&0&1&0&1&1\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 0&1&0&0&0&1&1\\ 0&0&1&1&0&1&1\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 0&1&0&0&1&1&0\\ 0&0&1&0&1&0&1\\ 0&0&0&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&0&0&0&1\\ 0&0&1&0&1&1&0\\ 0&0&0&1&1&0&1 \end{pmatrix}\begin{pmatrix} 0&0&1&0&0&1&1\\ 0&0&0&1&0&1&1\\ 0&0&0&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&0&1&0&0\\ 0&0&1&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&0&1&0&0&0&0\\ 0&0&0&0&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&0&1&0&1\\ 0&0&1&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&0&0&1&1\\ 0&0&1&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&1&1&0&0&0\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&1&0&1&1\\ 0&0&1&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&0&0&0&0\\ 0&0&1&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&0&0&1\\ 0&0&1&0&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&0&0&0\\ 0&0&0&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&0&1&0&1&0&0\\ 0&0&0&1&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&1&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&1&0&1&0&0\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&0&1&0&0&1&1\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&1&0&0&0&0\\ 0&0&0&1&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&0&0&0&0\\ 0&0&1&0&0&0&0\\ 0&0&0&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&0&1&1&0\\ 0&0&1&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&0&0&0&0\\ 0&0&1&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&0&0\\ 0&1&0&0&0&0&1\\ 0&0&0&0&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&0&0\\ 0&0&0&1&0&1&0\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&1&0&0&0&0\\ 0&0&0&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&1&1&0&0\\ 0&0&1&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&0&0&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&1&0&0&1\\ 0&0&1&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&1&1&0&1&0\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&0&1&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&0\\ 0&1&1&0&1&0&0\\ 0&0&0&1&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&1&0&0&1\\ 0&0&1&0&0&1&0\\ 0&0&0&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&0&1&0&0&0&0\\ 0&0&0&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&1&0&1\\ 0&0&1&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&0&1&1&0&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&1&0&0&1&0\\ 0&0&0&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&0&0&0&1&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&0&1&1\\ 0&0&0&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&0&1&0&0&1&0\\ 0&0&0&1&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&1&0&0&1&1\\ 0&0&0&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&1&1\\ 0&1&1&0&0&1&1\\ 0&0&0&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&1&0&1&0\\ 0&0&1&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&1&0&1&1&0\\ 0&0&0&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&0&1\\ 0&0&0&0&1&0&1\\ 0&0&0&0&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&0&1&0&1&1&1\\ 0&0&0&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&1&0&0&1&0&1\\ 0&0&1&0&0&0&1\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&1&0&0&0\\ 0&0&1&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&0&0\\ 0&1&1&0&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&0&1&1&0&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&0&0\\ 0&1&1&0&0&1&1\\ 0&0&0&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&0&0\\ 0&0&0&1&0&0&0\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&1&1&0&0&0&0\\ 0&0&0&1&0&1&0\\ 0&0&0&0&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&0&1&0&0\\ 0&0&1&0&0&1&0\\ 0&0&0&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&1&0&0&1\\ 0&0&1&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&0&0\\ 0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&0&0\\ 0&0&1&0&1&1&1\\ 0&0&0&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&1&0&1&0&0\\ 0&0&0&1&1&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&0&1\\ 0&1&1&0&0&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&1&0\\ 0&0&1&0&0&1&0\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&1&0&0&1\\ 0&0&1&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&0&0\\ 0&1&1&1&0&1&1\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&0\\ 0&1&1&0&1&1&0\\ 0&0&0&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&1&0&0&0\\ 0&0&1&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&1&1&0&1&1\\ 0&0&0&0&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&0&1&0&0&0&1\\ 0&0&0&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&1\\ 0&1&0&0&1&0&1\\ 0&0&1&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&1&0&1&0&0&1\\ 0&0&1&1&1&0&1\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&0&0\\ 0&1&0&0&0&1&1\\ 0&0&0&1&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&0&1&0&1\\ 0&0&1&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&0&1&0&0\\ 0&0&1&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&0&0&0&0\\ 0&0&1&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&1\\ 0&1&1&1&0&0&0\\ 0&0&0&0&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&1&0&1&1\\ 0&0&1&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&0&0&1&1\\ 0&0&1&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&0&1\\ 0&0&1&0&0&0&0\\ 0&0&0&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&1&0&1&1\\ 0&0&1&1&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&0&0&1&1\\ 0&0&1&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&1&1&0\\ 0&0&1&0&1&0&1\\ 0&0&0&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&0&1&1&0&0&0\\ 0&0&0&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&0&1&0&1\\ 0&0&1&0&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&1&1&0\\ 0&1&1&0&1&1&1\\ 0&0&0&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&1&1&0&0&1\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&0&1&1&0\\ 0&0&1&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&1&1&0&0&1\\ 0&1&0&0&0&0&1\\ 0&0&0&0&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&0&0&1&1\\ 0&0&0&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&1\\ 0&1&0&1&1&0&0\\ 0&0&1&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&0&0&1&0\\ 0&0&1&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&0\\ 0&1&0&0&1&0&1\\ 0&0&0&1&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&1&1&1&1\\ 0&0&1&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&0\\ 0&1&1&0&0&0&0\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&0&1&0&0\\ 0&0&1&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&0&1&0&0\\ 0&0&1&0&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&0&0&0&0\\ 0&0&1&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&1&0&1&0&0&1\\ 0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&0&0&0&0\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&0&0&0&0\\ 0&0&1&0&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&1&0&0&1\\ 0&0&0&0&1&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&1&0\\ 0&1&1&0&0&1&1\\ 0&0&0&1&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&0&0&1&1\\ 0&0&0&1&0&1&0 \end{pmatrix} \begin{pmatrix} 1&1&0&0&1&1&0\\ 0&0&1&1&0&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&0&1&0&1\\ 0&0&1&1&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&1&0&0&1&0\\ 0&0&0&1&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&1\\ 0&0&1&0&0&1&0\\ 0&0&0&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&1&0&1\\ 0&1&1&0&0&0&1\\ 0&0&0&1&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&1&0&1&1\\ 0&0&1&1&0&1&0 \end{pmatrix} \begin{pmatrix} 1&1&0&0&0&0&0\\ 0&0&1&0&1&0&1\\ 0&0&0&0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&0&0\\ 0&1&1&0&1&1&1\\ 0&0&0&1&1&0&0 \end{pmatrix} \begin{pmatrix} 1&1&0&0&0&1&0\\ 0&0&1&0&1&1&0\\ 0&0&0&1&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&0&1&1&1\\ 0&0&1&0&1&1&1\\ 0&0&0&1&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&1&0&0\\ 0&1&1&1&0&0&1\\ 0&0&0&0&0&1&0 \end{pmatrix} \begin{pmatrix} 0&1&1&0&0&0&1\\ 0&0&0&1&0&1&1\\ 0&0&0&0&1&1&0 \end{pmatrix} \begin{pmatrix} 0&1&0&0&0&1&1\\ 0&0&1&1&0&1&0\\ 0&0&0&0&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&0&1&0&1\\ 0&0&1&0&1&1&1\\ 0&0&0&1&1&1&0 \end{pmatrix} \begin{pmatrix} 0&1&0&0&0&1&1\\ 0&0&1&1&0&0&0\\ 0&0&0&0&1&1&0 \end{pmatrix} \begin{pmatrix} 0&0&1&0&0&1&1\\ 0&0&0&1&0&1&0\\ 0&0&0&0&1&1&0 \end{pmatrix} \begin{pmatrix} 0&1&0&0&1&0&1\\ 0&0&1&0&1&1&0\\ 0&0&0&1&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&0&0&0&1\\ 0&0&1&1&0&0&1\\ 0&0&0&0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&0&1&0&1\\ 0&0&1&1&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&0&0&1&1\\ 0&0&1&1&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&1&1&0&1&0\\ 0&1&1&1&0&1&0\\ 0&0&0&0&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&0&0&1&0\\ 0&0&1&1&0&0&1\\ 0&0&0&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&0&0&1&1\\ 0&0&1&1&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&0&0&1&1\\ 0&0&0&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&0&1&0&1\\ 0&0&1&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&1&0&0&1\\ 0&0&1&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&1\\ 0&0&1&0&1&0&1\\ 0&0&0&1&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&1&0&1&0\\ 0&0&1&0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&1\\ 0&0&1&0&0&0&1\\ 0&0&0&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&1&0&1&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&1&1&0&0&1\\ 0&0&0&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&1&0&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&1&0&1&1\\ 0&0&1&0&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&0&1\\ 0&1&1&0&1&1&1\\ 0&0&0&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&1&0&0&0&1&1\\ 0&0&1&0&1&0&1\\ 0&0&0&1&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&0&0&0&1\\ 0&0&1&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&0&1&1&0\\ 0&0&1&1&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&1&0&1&1\\ 0&0&1&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&1&1&0&0\\ 0&0&1&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&0&1&1&1\\ 0&0&1&1&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&0&1&0&1\\ 0&0&0&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&0&0&0&1\\ 0&0&1&1&0&0&1\\ 0&0&0&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&1&1\\ 0&1&0&0&0&1&1\\ 0&0&0&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&1&1\\ 0&1&1&0&0&1&1\\ 0&0&0&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&0&0&1&0\\ 0&0&1&1&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&1&1&0&1\\ 0&0&1&1&1&1&1 \end{pmatrix} \begin{pmatrix} 0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0\\ 0&0&0&0&1&0&1 \end{pmatrix} \begin{pmatrix} 0&1&0&0&1&1&1\\ 0&0&1&0&1&0&0\\ 0&0&0&1&1&0&1 \end{pmatrix} \begin{pmatrix} 0&1&0&1&0&0&1\\ 0&0&1&1&0&1&1\\ 0&0&0&0&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&0&1&0&0\\ 0&0&1&0&0&1&0\\ 0&0&0&1&0&1&0 \end{pmatrix} \begin{pmatrix} 0&0&1&1&0&0&0\\ 0&0&0&0&1&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix} \begin{pmatrix} 0&1&0&0&0&1&0\\ 0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix} \begin{pmatrix} 0&1&1&0&0&0&0\\ 0&0&0&1&0&1&0\\ 0&0&0&0&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&0&1&0&1&1&0\\ 0&0&0&1&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&1&0&0&0&1\\ 0&0&0&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&0&0&1&1\\ 0&0&1&1&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&1&0&1&1\\ 0&0&1&0&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&1&1&0&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&1\\ 0&0&1&0&1&0&1\\ 0&0&0&1&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&0&1&0&1\\ 0&0&1&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&1&0&0&0\\ 0&0&1&0&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&1&0&0&0&0\\ 0&0&0&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&1&0\\ 0&1&0&0&0&1&1\\ 0&0&0&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&1\\ 0&0&1&1&0&0&1\\ 0&0&0&0&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&0&1&0&1\\ 0&0&1&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&1&0&1&0\\ 0&0&1&0&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&0&0&0&0\\ 0&0&1&1&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&1&0&0&0&1\\ 0&0&0&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0\\ 0&0&0&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&0&0&1&0&0&1\\ 0&0&0&0&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&0&1&0&1&0&0\\ 0&0&0&1&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&1&0&0\\ 0&0&1&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&0&0&0\\ 0&0&1&1&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&1&0&0&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&1&0&1&1&0\\ 0&1&0&0&1&1&1\\ 0&0&0&1&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&1&1&1&1\\ 0&0&1&1&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&0&0&1&0\\ 0&0&0&0&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&1&1&1\\ 0&1&0&0&0&1&0\\ 0&0&0&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&1&1&1&0\\ 0&0&1&1&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&1&1&0&1\\ 0&0&1&1&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&1&1&1&0\\ 0&0&1&1&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&1&0&0&1\\ 0&0&0&0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&1&0\\ 0&0&1&0&1&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&1&0\\ 0&1&1&0&1&1&1\\ 0&0&0&1&0&0&1 \end{pmatrix} \begin{pmatrix} 1&1&0&0&1&1&0\\ 0&0&1&0&1&0&1\\ 0&0&0&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&1&0&0&0&1\\ 0&0&0&1&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&1\\ 0&0&1&0&0&1&0\\ 0&0&0&1&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&1&1&0&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&0&1&1&0\\ 0&0&1&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&0&0&0&1\\ 0&0&1&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&1&0&0&0\\ 0&0&1&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&1&0&0&1&0&0\\ 0&0&1&0&1&1&0\\ 0&0&0&1&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&1&1&0\\ 0&1&1&0&1&1&0\\ 0&0&0&1&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&1&0&1&0\\ 0&0&1&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&1&1&0&1&1\\ 0&1&0&1&0&1&1\\ 0&0&0&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&1&1&1\\ 0&1&0&0&0&0&1\\ 0&0&0&1&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&1&0&0&0&1\\ 0&0&0&1&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&0&1\\ 0&0&0&1&0&1&1\\ 0&0&0&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&1&0&1&0\\ 0&0&1&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&0&1&1&0\\ 0&0&1&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&1&0&1&0&0&0\\ 0&0&1&0&0&0&1\\ 0&0&0&0&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&1&1&0&0\\ 0&0&1&1&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&1&0&0&0\\ 0&0&1&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&1&0&1&0\\ 0&0&1&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&1&0&1&1\\ 0&0&1&1&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&1&0&0&0\\ 0&0&1&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&1&0&1&0\\ 0&0&1&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&1&1&0&0&0\\ 0&0&0&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&0\\ 0&0&1&0&0&0&1\\ 0&0&0&0&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&0&1\\ 0&0&0&0&1&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&1&0&1&0&0&0\\ 0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&1&0\\ 0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&1&0&0&1\\ 0&0&1&1&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&1&1\\ 0&1&0&0&1&0&0\\ 0&0&1&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&1&0&1&0\\ 0&0&1&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&1&0&0&0&0\\ 0&0&0&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&0&1&1&0\\ 0&0&0&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&0&0&0&1\\ 0&0&0&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&0\\ 0&0&0&1&1&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&1&1&0&0\\ 0&0&1&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&1&1&1&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&0&0&0&1\\ 0&0&0&1&0&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&1&0&1&1&0&0\\ 0&0&1&0&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 0&1&0&1&1&0&1\\ 0&0&1&0&1&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 0&1&0&0&0&1&0\\ 0&0&1&0&1&1&0\\ 0&0&0&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&0&1&0&1\\ 0&0&1&0&0&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 0&1&0&0&0&0&0\\ 0&0&1&0&1&0&1\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 0&1&0&0&0&1&1\\ 0&0&0&1&0&0&0\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 0&1&0&1&1&1&0\\ 0&0&1&1&1&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&0&0&1&0\\ 0&0&1&1&0&1&0\\ 0&0&0&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&1&0&1&1&0\\ 0&0&0&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&0&1&0&0&1&1\\ 0&0&0&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&0&0&0&1\\ 0&0&0&1&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&0&0&0&1\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&0&1&1&0\\ 0&0&0&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&1&1&0&0\\ 0&0&1&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&1&1&0&0\\ 0&0&1&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&1&1&0&1\\ 0&0&1&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&0&0&1&1\\ 0&0&1&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&0&1&1&1\\ 0&0&1&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&0&0&1&0\\ 0&0&0&0&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&1&0&1&0\\ 0&1&0&1&0&0&0\\ 0&0&0&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&1&1&1&1\\ 0&0&1&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&1&0&1&0\\ 0&0&1&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&1&1&1&1\\ 0&0&1&0&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&0&1&1&1&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&1&1&0&1\\ 0&0&1&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&1&0&1&0&1\\ 0&0&0&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&0&1&1&1\\ 0&0&1&0&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&1&1\\ 0&1&0&0&0&1&0\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&1&1&1&0\\ 0&0&1&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&1&0&1\\ 0&0&1&1&1&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&1&0\\ 0&1&0&1&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&1&0&1\\ 0&1&1&0&1&1&0\\ 0&0&0&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&1&0&0&0\\ 0&0&1&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&1&0&0&1\\ 0&0&1&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&0&1&0&0&1&0\\ 0&0&0&0&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&1&0&1&1&1\\ 0&0&0&1&0&0&0 \end{pmatrix}\begin{pmatrix} 0&1&1&0&0&0&1\\ 0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 0&0&1&0&1&1&0\\ 0&0&0&1&1&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 0&1&1&0&0&1&0\\ 0&0&0&1&0&0&1\\ 0&0&0&0&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&1&0&0&1\\ 0&0&1&1&0&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 0&0&1&0&0&1&0\\ 0&0&0&1&0&0&1\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 0&0&1&0&1&0&1\\ 0&0&0&1&1&0&1\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 0&1&1&0&0&0&1\\ 0&0&0&1&0&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&1&1&1\\ 0&1&1&0&1&1&1\\ 0&0&0&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&1&1&0&0\\ 0&0&1&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&0&0\\ 0&1&0&0&1&1&0\\ 0&0&0&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&1&0&0&1&1\\ 0&0&0&1&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&0&1&0&0&1&1\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&0&0&0&1\\ 0&0&1&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&1&0&0&1&0&0\\ 0&0&1&0&1&0&1\\ 0&0&0&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&0&0&0&0\\ 0&0&1&0&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&1&0&0&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&0&1\\ 0&1&0&0&1&0&0\\ 0&0&0&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&0&1&0&1&0&0\\ 0&0&0&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&1&1\\ 0&0&1&0&0&0&0\\ 0&0&0&1&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&1&0&1&0\\ 0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&1&0&1&0&0\\ 0&0&0&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&0&1&0&1&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&1&1&0&0&1&1\\ 0&0&0&1&0&0&0\\ 0&0&0&0&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&0&1&0&0\\ 0&0&1&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&1&1&0&1&0\\ 0&1&1&0&0&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&0&1&0&0&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&1&0&1&0&0\\ 0&0&0&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&0&1\\ 0&1&0&0&0&0&0\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&1\\ 0&1&0&1&1&0&1\\ 0&0&1&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&0&0&1&0\\ 0&0&0&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&0&1&1&1\\ 0&0&1&1&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&0&1&1&1\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&1&1&0&1\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&1&1&1&1\\ 0&0&1&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&0&0&1&0\\ 0&0&1&0&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&1&1\\ 0&1&0&0&1&1&1\\ 0&0&0&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&1&1&1&1\\ 0&0&1&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&0&0\\ 0&1&1&0&1&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&1&0&1&1&0&0\\ 0&0&1&1&1&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&1&0&0&1\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&1&1&0&1\\ 0&0&1&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&0&0&1&0\\ 0&0&0&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&0&1&0&0\\ 0&0&1&0&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&0&0&0&0\\ 0&0&1&0&0&0&0 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&1&0\\ 0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&1&1&0&0&0\\ 0&1&1&0&0&0&0\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&0&1&0&0\\ 0&0&1&0&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&0&0&0&0\\ 0&0&1&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&0&0&0&0\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&1&1&0&1\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&0&0&1&0\\ 0&0&1&0&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&1&1&1&1\\ 0&0&1&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&1&1&0&1\\ 0&0&1&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&0&1&1&1\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&0&0\\ 0&1&0&0&0&1&0\\ 0&0&0&1&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&0&1&1&1\\ 0&0&1&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&1\\ 0&1&0&1&0&0&0\\ 0&0&0&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&0&1&1&1\\ 0&0&1&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&1&0&1&0\\ 0&0&1&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&1&1&0&1\\ 0&0&1&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&0&0&1&1\\ 0&0&1&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&0&0&1&0\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&1&1&1&1\\ 0&0&1&1&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&1&1&1\\ 0&1&0&0&0&1&0\\ 0&0&0&1&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&1&1&0&1\\ 0&0&1&1&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&1&1&1&1\\ 0&0&1&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&1&0&0&1&0&1\\ 0&0&1&1&1&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&1&1&0\\ 0&1&1&0&1&0&1\\ 0&0&0&1&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&0&1&1&1\\ 0&0&1&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&1&1&1&0\\ 0&0&1&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&1&0&1&1&1\\ 0&0&0&1&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&1&0&1&1\\ 0&0&1&1&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&0&1&0&1&0&1\\ 0&0&0&1&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&1&0&1\\ 0&0&1&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&1&1&0&0&1\\ 0&0&0&0&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&0&1&1&0&0&0\\ 0&0&0&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&0&1&1\\ 0&0&1&1&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&0\\ 0&0&0&1&0&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&1&0&0&1\\ 0&0&1&0&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&1&1&1\\ 0&1&1&0&0&0&1\\ 0&0&0&1&1&1&1 \end{pmatrix} \begin{pmatrix} 1&1&1&0&0&0&1\\ 0&0&0&1&1&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&1\\ 0&0&1&0&0&1&0\\ 0&0&0&1&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&1&0&0&1\\ 0&0&1&0&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&1&0&0\\ 0&0&0&1&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&1&1&1\\ 0&0&1&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&0&0&1\\ 0&0&1&0&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&0&1&0\\ 0&0&0&0&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&0&0&0\\ 0&0&0&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&1&1&0&0\\ 0&0&1&1&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&1&1&0&1\\ 0&0&1&0&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&0&1&0&0\\ 0&0&1&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&1&1&0&0\\ 0&0&1&0&1&0&0 \end{pmatrix} \begin{pmatrix} 1&1&0&0&1&0&0\\ 0&0&1&0&0&0&0\\ 0&0&0&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&1&1&0&0&1\\ 0&1&1&1&0&0&1\\ 0&0&0&0&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&0&0&0&1\\ 0&0&1&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&0&1&1&0\\ 0&0&1&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&1&0&1&0&0\\ 0&1&0&0&0&0&0\\ 0&0&0&1&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&0&1&0&1\\ 0&0&1&0&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&0&0&0&0\\ 0&0&1&1&0&1&0 \end{pmatrix} \begin{pmatrix} 1&1&1&0&0&0&1\\ 0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&1&0&1&1&1\\ 0&1&1&0&0&0&0\\ 0&0&0&1&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&0&1&0&0\\ 0&0&1&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&0&0&1&1\\ 0&0&1&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&1\\ 0&0&1&0&0&0&0\\ 0&0&0&1&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&1&0&1&1\\ 0&0&1&0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&1&0&1&0&0\\ 0&1&1&1&0&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&1&0&1&1\\ 0&0&1&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&1&1\\ 0&1&1&0&1&1&1\\ 0&0&0&1&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&1&0&0&1\\ 0&0&1&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&1&0&0&1&1&0\\ 0&0&1&0&1&1&0\\ 0&0&0&1&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&0&0&0&1\\ 0&0&1&0&1&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&1&0&0&1&0\\ 0&0&0&0&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&0&1&0&0\\ 0&0&1&1&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&1&0&0&1&1\\ 0&0&0&0&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&0&0&0&0\\ 0&0&0&1&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&1&0&0&0\\ 0&0&1&1&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&0\\ 0&0&1&0&0&0&0\\ 0&0&0&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&1\\ 0&0&1&0&0&0&1\\ 0&0&0&0&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&1&1&0&0\\ 0&1&1&1&1&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&1&0&0&1&1&1\\ 0&0&1&0&1&1&1\\ 0&0&0&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&0&0\\ 0&1&0&0&1&0&1\\ 0&0&0&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&0&0\\ 0&1&1&1&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&1&1&0&1\\ 0&0&1&1&0&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&1&0\\ 0&0&0&1&0&0&1\\ 0&0&0&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&0&0&1&1\\ 0&0&1&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&1&0&0&1\\ 0&0&1&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&1&1\\ 0&1&1&1&0&1&1\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&1&1&0&1&0&1\\ 0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&1&1&1&0&1\\ 0&1&1&0&0&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&0&1&0&0&1&0\\ 0&0&0&1&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&0&1&1&0\\ 0&0&0&1&1&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&0&1\\ 0&0&1&0&1&1&1\\ 0&0&0&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&1&1\\ 0&0&1&0&0&0&1\\ 0&0&0&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&0&1&1&1\\ 0&0&1&1&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&0&0&1&0\\ 0&0&0&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&1&1&0&1\\ 0&0&1&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&1&0&0&0\\ 0&1&1&0&0&1&0\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&1&1&1&0\\ 0&0&1&1&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&1&0&0&1\\ 0&1&0&1&0&1&0\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&0&0\\ 0&1&1&0&0&0&0\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&0&1&0&1&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&0&0\\ 0&0&1&1&0&1&0\\ 0&0&0&0&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&1&1\\ 0&1&0&1&0&1&1\\ 0&0&1&0&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&0&0&0&0\\ 0&0&1&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&0&1&0&0\\ 0&0&1&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&1&0&1&0&0\\ 0&0&0&1&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&1&1&1&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&0&0\\ 0&0&1&0&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&0&0&0&1\\ 0&0&0&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&0&0\\ 0&0&0&1&0&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&1&0&0\\ 0&1&0&0&1&1&0\\ 0&0&0&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&0&0&1&1&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&1&1&0&0\\ 0&0&1&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&1&1&1&1\\ 0&0&1&0&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&0&1&1&1\\ 0&0&1&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&0&0&1&0\\ 0&0&1&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&1&1&1&0\\ 0&0&1&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&0&1&0&1\\ 0&0&1&1&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&0&0&1&1\\ 0&0&1&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&0&1\\ 0&1&0&1&0&1&1\\ 0&0&0&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&1&1&0&0\\ 0&0&1&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&1\\ 0&1&1&0&0&0&1\\ 0&0&0&1&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&0&1&1&0\\ 0&0&1&0&0&1&1\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&0&1&1&0\\ 0&0&1&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&0&1\\ 0&0&0&1&1&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&0&0&0&1\\ 0&0&1&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&0&1&0&1\\ 0&0&0&1&0&0&1\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&0&0&1&1\\ 0&0&1&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&0&0&0&0\\ 0&0&1&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&0&1&0&1\\ 0&0&1&0&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&0&1&0&0\\ 0&0&1&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&1&0&0&1\\ 0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&1&0&0&0&0\\ 0&0&0&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&1&1\\ 0&0&1&0&0&0&0\\ 0&0&0&1&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&1&0&1&0\\ 0&0&1&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&0&1&0&0&0&1\\ 0&0&0&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&1&1&0&0&1\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&1&0&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&1&0&0&1\\ 0&0&1&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&0&1&0&1&0&1\\ 0&0&0&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&1&0&1&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&0&0&1&0\\ 0&0&0&0&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&1&1&0&1\\ 0&0&1&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&0&1&0&1\\ 0&0&1&0&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&1&1&0&0&0\\ 0&1&0&0&0&1&1\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&1&1&1&0\\ 0&0&1&0&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&0&0&0&0\\ 0&0&1&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&1&1&1&1\\ 0&0&1&1&1&1&1 \end{pmatrix}\begin{pmatrix} 0&1&0&1&0&0&0\\ 0&0&1&0&0&0&0\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 0&1&0&0&1&1&1\\ 0&0&1&0&0&1&1\\ 0&0&0&1&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&0&1&0&0\\ 0&0&1&0&1&1&0\\ 0&0&0&1&0&1&1 \end{pmatrix}\begin{pmatrix} 0&1&0&0&0&0&1\\ 0&0&1&0&1&0&0\\ 0&0&0&1&0&1&0 \end{pmatrix}\begin{pmatrix} 0&1&0&1&0&1&1\\ 0&0&1&1&0&1&1\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 0&1&0&0&0&1&0\\ 0&0&1&0&0&0&1\\ 0&0&0&0&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&0&0&0&0\\ 0&0&1&0&1&1&0\\ 0&0&0&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&0&1&0&0&0&1\\ 0&0&0&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&0&0\\ 0&1&0&1&0&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&1&1\\ 0&0&1&1&0&1&0\\ 0&0&0&0&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&1&0&0&0\\ 0&0&1&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&1&0&0&0&1\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&1&0\\ 0&1&1&0&1&1&0\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&1\\ 0&1&0&1&0&1&0\\ 0&0&1&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&1&0&1\\ 0&1&1&0&0&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&1&1\\ 0&0&1&0&1&0&0\\ 0&0&0&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&0&1\\ 0&1&1&0&1&0&0\\ 0&0&0&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&0&0&0&0\\ 0&0&1&0&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&1&1\\ 0&1&0&0&1&0&0\\ 0&0&0&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&0&1&0&0&0&0\\ 0&0&0&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&1&1&1&0&0&0\\ 0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&0&0&0&1\\ 0&0&1&1&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&0&0\\ 0&1&1&0&0&1&0\\ 0&0&0&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&1\\ 0&1&0&0&0&1&1\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&0&1&0&1\\ 0&0&1&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&0&0\\ 0&0&0&1&0&1&1\\ 0&0&0&0&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&0&1&0&0\\ 0&0&1&0&0&1&1\\ 0&0&0&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&1&1&0&0\\ 0&0&1&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&1&0&0&0\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&0&1&0&0&1&1\\ 0&0&0&0&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&1&0&1&0&0\\ 0&0&0&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&0&1&1&0\\ 0&0&1&0&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&0&0&0&1\\ 0&0&1&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&1&1\\ 0&1&0&0&1&0&0\\ 0&0&0&1&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&0&0&0&0\\ 0&0&1&1&1&0&1 \end{pmatrix}\begin{pmatrix} 0&1&0&1&0&0&0\\ 0&0&1&1&1&0&1\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 0&1&0&1&0&0&0\\ 0&0&1&1&0&0&1\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 0&1&0&0&0&1&0\\ 0&0&1&1&0&1&0\\ 0&0&0&0&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&1&0&1&0\\ 0&0&1&0&0&0&1\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 0&1&0&0&0&1&1\\ 0&0&1&0&1&0&1\\ 0&0&0&1&0&0&1 \end{pmatrix}\begin{pmatrix} 0&1&0&0&1&1&1\\ 0&0&1&0&0&0&1\\ 0&0&0&1&1&0&0 \end{pmatrix}\begin{pmatrix} 0&1&0&1&0&0&0\\ 0&0&1&0&0&0&1\\ 0&0&0&0&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&0&0&1&0\\ 0&0&1&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&1&0&0&0\\ 0&1&0&1&0&1&1\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&0&1&1&1\\ 0&0&1&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&1&1&1&1\\ 0&0&1&1&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&1&0&1&0&1\\ 0&0&0&1&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&1&1&0&1\\ 0&0&1&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&0&1&0&1&1&0\\ 0&0&0&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&0&0\\ 0&0&1&0&0&0&1\\ 0&0&0&0&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&0&0\\ 0&1&1&0&0&1&1\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&1&0&0&0\\ 0&0&1&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&1&0&0&1&0\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&0&1&0&0&0&0\\ 0&0&0&1&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&0&1&0&0\\ 0&0&1&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&0&0\\ 0&1&0&0&0&0&0\\ 0&0&0&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&0&0&1&1\\ 0&0&1&0&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&1&1&1&1\\ 0&0&1&0&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&1&1\\ 0&1&0&0&1&0&1\\ 0&0&1&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&1&1&1&0\\ 0&0&1&1&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&0&0&1&0\\ 0&0&1&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&1&0&1&1\\ 0&0&0&0&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&0&1&1&1\\ 0&0&1&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&1&0&1&1&0\\ 0&0&0&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&1&1&0&0\\ 0&0&1&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&1&1&0&0\\ 0&0&1&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&0&1&0&0&1&1\\ 0&0&0&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&1&1&0&1&0\\ 0&1&0&0&0&0&1\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&1&1\\ 0&1&0&0&1&1&0\\ 0&0&0&1&1&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&0&0&0&1\\ 0&0&0&1&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&1&0&1\\ 0&0&0&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&0&1&1&0&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&1&1&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&0&0&1&0&0&1\\ 0&0&0&0&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&0&1&1\\ 0&0&1&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&0&1&0&1&1&1\\ 0&0&0&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&1&1&1&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&0&1&0&0\\ 0&0&1&0&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&0&0&0&0\\ 0&0&1&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&1&0\\ 0&1&1&0&0&0&1\\ 0&0&0&0&1&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&0\\ 0&0&0&1&0&0&1\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&1&0&0&1&0&1\\ 0&0&1&0&1&0&0\\ 0&0&0&1&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&1&0&0&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&1&1\\ 0&0&1&0&0&0&0\\ 0&0&0&1&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&0&0&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&1&0&0&1\\ 0&0&1&1&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&0&0\\ 0&1&1&1&0&1&0\\ 0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&0&0\\ 0&1&0&0&1&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&1&0&0\\ 0&1&1&0&1&0&0\\ 0&0&0&1&0&0&0 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&1&0\\ 0&0&1&0&0&1&0\\ 0&0&0&0&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&0&0&0&0\\ 0&0&1&0&0&0&0\\ 0&0&0&1&1&1&1 \end{pmatrix}\begin{pmatrix} 1&1&0&0&0&0&1\\ 0&0&0&1&0&0&1\\ 0&0&0&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&0&1&1&0\\ 0&0&0&1&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&1&1&0&0\\ 0&0&1&1&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&0&1&1&1\\ 0&0&1&0&0&0&1\\ 0&0&0&1&0&1&1 \end{pmatrix}\begin{pmatrix} 1&0&0&1&0&1&1\\ 0&1&0&0&0&0&1\\ 0&0&1&0&1&1&1 \end{pmatrix}\begin{pmatrix} 1&0&1&0&0&0&1\\ 0&1&1&1&1&0&1\\ 0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&1&1&1&0\\ 0&1&0&1&0&0&0\\ 0&0&1&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&0&0&1&0\\ 0&0&0&1&0&1&0\\ 0&0&0&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&1&0&1&0\\ 0&0&1&1&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&0&1\\ 0&1&1&0&0&1&0\\ 0&0&0&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&1&0&0&1&1&1\\ 0&0&1&0&0&0&1\\ 0&0&0&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&1&0&0&1\\ 0&1&1&0&0&1&1\\ 0&0&0&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&1&0&0&0&1&0\\ 0&0&1&1&0&0&0\\ 0&0&0&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&1&0&0&0\\ 0&0&1&1&0&1&1 \end{pmatrix} \begin{pmatrix} 1&1&0&1&0&0&0\\ 0&0&1&0&0&0&0\\ 0&0&0&0&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&1&1&0\\ 0&1&0&0&0&0&0\\ 0&0&0&1&0&0&1 \end{pmatrix} \begin{pmatrix} 1&1&0&1&0&0&0\\ 0&0&1&0&1&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&0&1\\ 0&0&0&1&0&0&0\\ 0&0&0&0&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&1&0&0&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&0&1&0&0\\ 0&0&1&1&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&0&1\\ 0&1&1&0&1&0&0\\ 0&0&0&1&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&1&1&0&0\\ 0&0&1&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&0&1&1&0\\ 0&0&1&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&0&0&0&1\\ 0&0&1&1&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&1&0&1\\ 0&1&0&0&1&0&1\\ 0&0&0&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&1\\ 0&0&1&0&0&1&1\\ 0&0&0&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&1&0&1&1&1\\ 0&0&0&1&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&0&0&1&1\\ 0&0&1&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&0&1&0&1\\ 0&0&0&1&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&0&0&1&0\\ 0&0&1&1&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&1&1&1&1\\ 0&0&1&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&0&1\\ 0&1&0&0&0&1&1\\ 0&0&0&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&1&1&0&0\\ 0&0&1&1&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&1&1&0&1&1\\ 0&1&0&0&0&0&1\\ 0&0&0&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&0&1&1&0\\ 0&0&1&1&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&1&0&0&1&0\\ 0&0&0&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&1\\ 0&0&1&0&1&0&0\\ 0&0&0&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&0&0&1&1\\ 0&0&1&0&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&1&0\\ 0&1&1&0&0&0&0\\ 0&0&0&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&1&0&0&1\\ 0&0&1&1&0&1&1\\ 0&0&0&0&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&0&0&0&0\\ 0&0&0&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&0&1&0&0\\ 0&0&1&1&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&0&1&1&1\\ 0&0&0&1&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&1&1&1&0\\ 0&0&1&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&1&1&0&1\\ 0&0&1&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&1&0&1&0&1\\ 0&0&0&0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&0&0&1&0\\ 0&0&1&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&1&1\\ 0&1&0&1&1&1&1\\ 0&0&1&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&1\\ 0&0&1&0&1&0&1\\ 0&0&0&1&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&0&0\\ 0&0&0&0&1&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&0&1&0&0&0\\ 0&0&1&0&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&0&0&1&1\\ 0&0&1&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&1&0&0&1\\ 0&0&1&1&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&1&0&1\\ 0&1&0&0&1&0&1\\ 0&0&0&1&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&1&1&1&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&0&0&0&0&0\\ 0&0&1&0&0&0&1\\ 0&0&0&1&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&1&0&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&1&1\\ 0&1&0&1&0&0&1\\ 0&0&1&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&1&0&1&0\\ 0&0&1&0&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&1&0\\ 0&1&1&0&1&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&1\\ 0&0&1&0&0&0&1\\ 0&0&0&1&1&1&1 \end{pmatrix} \begin{pmatrix} 1&1&0&0&0&0&1\\ 0&0&1&0&1&0&1\\ 0&0&0&1&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&1&1&0&0&1\\ 0&0&0&0&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&0 \\ 0&1&0&0&0&1&0 \\ 0&0&0&1&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0 \\ 0&1&0&0&1&1&1 \\ 0&0&0&1&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0 \\ 0&1&1&0&1&0&1 \\ 0&0&0&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0 \\ 0&1&0&1&1&0&1 \\ 0&0&1&0&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&0 \\ 0&0&1&1&1&0&0 \\ 0&0&0&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0 \\ 0&1&0&1&0&0&1 \\ 0&0&0&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0 \\ 0&1&0&1&1&1&1 \\ 0&0&1&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&0 \\ 0&0&1&0&0&0&1 \\ 0&0&0&0&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&0&1 \\ 0&1&0&1&0&1&0 \\ 0&0&1&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&0 \\ 0&1&1&1&0&0&0 \\ 0&0&0&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&0 \\ 0&1&0&1&0&0&0 \\ 0&0&1&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&1 \\ 0&1&0&1&0&1&1 \\ 0&0&1&1&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&1 \\ 0&1&0&1&0&0&0 \\ 0&0&1&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&1 \\ 0&1&0&1&0&1&0 \\ 0&0&1&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&1 \\ 0&1&0&0&1&1&1 \\ 0&0&0&1&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&1&1 \\ 0&1&0&0&1&1&0 \\ 0&0&0&1&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&1 \\ 0&1&0&0&0&1&0 \\ 0&0&0&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&1 \\ 0&1&0&0&0&0&1 \\ 0&0&0&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&1&0 \\ 0&1&0&0&0&0&1 \\ 0&0&1&1&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&1&1 \\ 0&1&0&1&1&1&0 \\ 0&0&1&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&0 \\ 0&1&0&0&1&1&0 \\ 0&0&1&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&0 \\ 0&1&1&0&1&0&0 \\ 0&0&0&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&0&1 \\ 0&1&0&0&1&0&0 \\ 0&0&1&1&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&1&0 \\ 0&1&0&0&0&0&0 \\ 0&0&0&0&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&1 \\ 0&1&0&0&0&0&1 \\ 0&0&0&0&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&0 \\ 0&1&0&1&1&0&0 \\ 0&0&1&0&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&1 \\ 0&1&0&0&0&1&1 \\ 0&0&1&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&0 \\ 0&0&1&0&1&0&0 \\ 0&0&0&1&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&1&0 \\ 0&1&0&1&1&1&1 \\ 0&0&1&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&0 \\ 0&1&0&1&1&0&1 \\ 0&0&1&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&0 \\ 0&1&0&1&0&1&1 \\ 0&0&1&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&1 \\ 0&1&0&1&0&1&1 \\ 0&0&1&1&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&0&1 \\ 0&1&0&1&1&1&0 \\ 0&0&1&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&0&1 \\ 0&1&0&0&0&1&0 \\ 0&0&1&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&1 \\ 0&1&0&0&1&1&1 \\ 0&0&1&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&1&1&0&0&0&1 \\ 0&0&0&1&0&1&0 \\ 0&0&0&0&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&0 \\ 0&1&1&0&0&1&0 \\ 0&0&0&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&1 \\ 0&1&0&1&0&0&1 \\ 0&0&1&1&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&1 \\ 0&0&0&1&0&0&0 \\ 0&0&0&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&1&1&1 \\ 0&1&1&0&0&1&1 \\ 0&0&0&1&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&0&0 \\ 0&0&1&1&0&0&0 \\ 0&0&0&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&0 \\ 0&0&1&0&0&1&0 \\ 0&0&0&1&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&0 \\ 0&0&1&1&0&1&1 \\ 0&0&0&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&1 \\ 0&1&1&0&0&1&1 \\ 0&0&0&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&1 \\ 0&1&1&0&0&1&0 \\ 0&0&0&1&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&1&1&0&1&1 \\ 0&1&1&0&0&1&0 \\ 0&0&0&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&0 \\ 0&1&0&1&0&1&1 \\ 0&0&1&1&0&0&0 \end{pmatrix} \begin{pmatrix} 1&1&0&0&1&1&0 \\ 0&0&1&0&1&0&1 \\ 0&0&0&1&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&1&1 \\ 0&0&1&0&1&1&0 \\ 0&0&0&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&0 \\ 0&1&0&0&0&0&0 \\ 0&0&1&1&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&0 \\ 0&1&0&0&1&0&0 \\ 0&0&1&0&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&1&1 \\ 0&1&0&1&0&1&1 \\ 0&0&1&1&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&1 \\ 0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&0&0 \\ 0&1&1&0&1&0&0 \\ 0&0&0&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&1&0&0&0&0&0 \\ 0&0&1&0&1&0&0 \\ 0&0&0&1&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&1 \\ 0&1&1&0&0&1&1 \\ 0&0&0&0&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&1&1&1&0\\ 0&0&1&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&1&0&1\\ 0&1&0&0&0&1&0\\ 0&0&0&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&0&1&1&0\\ 0&0&1&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&0&0&0&1\\ 0&0&1&1&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&1&0&1\\ 0&1&0&0&1&1&0\\ 0&0&0&1&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&1&1&0\\ 0&1&0&0&0&0&1\\ 0&0&0&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&1&0&1&0&1\\ 0&1&0&0&1&1&1\\ 0&0&0&1&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&0&0&0&1\\ 0&0&1&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&1&0&0&1&1\\ 0&0&0&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&1\\ 0&0&0&1&0&1&1\\ 0&0&0&0&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&1\\ 0&1&0&0&1&1&0\\ 0&0&1&1&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&1\\ 0&1&0&0&1&1&0\\ 0&0&0&1&0&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&0&1&0\\ 0&1&0&0&0&0&1\\ 0&0&0&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&1\\ 0&0&1&0&0&1&1\\ 0&0&0&1&0&1&1 \end{pmatrix} \begin{pmatrix} 0&1&0&0&1&0&0\\ 0&0&1&0&1&1&0\\ 0&0&0&1&0&0&0 \end{pmatrix} \begin{pmatrix} 0&0&1&0&0&0&0\\ 0&0&0&0&1&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&0&1&0&1\\ 0&0&1&0&1&0&1\\ 0&0&0&1&1&1&0 \end{pmatrix} \begin{pmatrix} 0&1&0&0&0&1&1\\ 0&0&1&0&1&0&0\\ 0&0&0&1&0&1&0 \end{pmatrix} \begin{pmatrix} 0&1&1&0&0&1&1\\ 0&0&0&1&0&1&1\\ 0&0&0&0&1&0&0 \end{pmatrix} \begin{pmatrix} 0&1&0&0&1&1&0\\ 0&0&1&0&0&1&0\\ 0&0&0&1&1&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&0&0&0&0\\ 0&0&1&1&0&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&0&0&1\\ 0&0&1&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&1&0&0&0\\ 0&0&0&0&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&1&0&0\\ 0&0&0&1&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&1&1&0\\ 0&0&1&0&0&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&0&0&0\\ 0&0&1&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&0&1&0&0&1&1\\ 0&0&0&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&1&0&1&0&0\\ 0&0&0&1&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&1&1&0\\ 0&1&0&0&1&1&0\\ 0&0&0&1&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&0\\ 0&1&0&0&1&1&0\\ 0&0&1&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&0&1&1&1&0\\ 0&0&1&0&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&0&0\\ 0&1&0&0&0&0&1\\ 0&0&0&1&1&1&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&1&0&0\\ 0&1&0&0&0&1&0\\ 0&0&0&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&1\\ 0&1&0&0&0&0&1\\ 0&0&1&1&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&1&1\\ 0&1&0&0&1&1&1\\ 0&0&0&1&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&1\\ 0&1&0&0&0&0&1\\ 0&0&0&0&0&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&1&0&1&1&1\\ 0&1&0&0&1&1&1\\ 0&0&0&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&1&0&0\\ 0&1&1&0&1&0&1\\ 0&0&0&1&0&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&1&1&0&0\\ 0&0&0&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&0&1&0\\ 0&1&0&0&1&1&0\\ 0&0&0&1&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&1&1&0\\ 0&1&0&1&1&1&1\\ 0&0&1&0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&1\\ 0&0&1&0&0&1&1\\ 0&0&0&1&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&1&0&1&0&0\\ 0&1&0&0&1&0&1\\ 0&0&0&1&1&1&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&0&1\\ 0&0&1&0&0&0&1\\ 0&0&0&1&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&0&0&0\\ 0&1&0&1&0&0&0\\ 0&0&1&0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0&0&1&0\\ 0&0&0&0&1&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&0&1&0\\ 0&1&0&0&0&1&1\\ 0&0&1&1&1&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&1&1&0&1\\ 0&1&0&1&0&0&1\\ 0&0&1&1&1&1&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&1&1&0&0\\ 0&1&1&1&1&0&0\\ 0&0&0&0&0&1&1 \end{pmatrix}$$

APPENDIX B. ISOTROPIC POINTS FROM EXAMPLE 15

Symplectic $1\frac{1}{2}$-design points for $v = 3, m = 2, q = 2$

Vertex number	Vertex Basis Matrix for v = 3,q=2,m=2	Vertex number	Vertex Basis Matrix for v = 3,q=2,m=2
0	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$	1	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$
2	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$	3	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$
4	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}$	5	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$
6	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$	7	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$
8	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$	9	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$
10	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}$	11	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$
12	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}$	13	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix}$
14	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$	15	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$
16	$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$	17	$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$
18	$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}$	19	$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$
20	$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$	21	$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix}$
22	$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix}$	23	$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$
24	$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$	25	$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$
26	$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}$	27	$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$
28	$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$	29	$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}$
30	$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix}$	31	$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$
32	$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}$	33	$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$
34	$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}$	35	$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix}$

Table B.1 Points for example 15

36	$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$	37	$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix}$
38	$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$	39	$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$
40	$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$	41	$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$
42	$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$	43	$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$
44	$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}$	45	$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$
46	$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$	47	$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix}$
48	$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$	49	$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$
50	$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$	51	$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$
52	$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}$	53	$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}$
54	$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$	55	$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$
56	$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$	57	$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$
58	$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$	59	$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$
60	$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}$	61	$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix}$
62	$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$	63	$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$
64	$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$	65	$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$
66	$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$	67	$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$
68	$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}$	69	$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$
70	$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$	71	$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$
72	$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$	73	$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$
74	$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}$	75	$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$
76	$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}$	77	$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix}$
78	$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$	79	$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$
80	$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$	81	$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$
82	$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}$	83	$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$
84	$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$	85	$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix}$

86	$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix}$	87	$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$
88	$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$	89	$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$
90	$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}$	91	$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$
92	$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$	93	$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}$
94	$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix}$	95	$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$
96	$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}$	97	$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$
98	$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}$	99	$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix}$
100	$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$	101	$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix}$
102	$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$	103	$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$
104	$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$	105	$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$
106	$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$	107	$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$
108	$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}$	109	$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$
110	$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$	111	$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix}$
112	$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$	113	$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$
114	$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$	115	$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$
116	$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}$	117	$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}$
118	$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$	119	$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$
120	$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$	121	$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$
122	$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$	123	$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$
124	$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}$	125	$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix}$
126	$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$	127	$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$
128	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$	129	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$
130	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$	131	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$
132	$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$	133	$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$
134	$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$	135	$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$

136	$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$	137	$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$
138	$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$	139	$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$
140	$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$	141	$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$
142	$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$	143	$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$
144	$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$	145	$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$
146	$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$	147	$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$
148	$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$	149	$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$
150	$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$	151	$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$
152	$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$	153	$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$
154	$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$	155	$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$
156	$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$	157	$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$
158	$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$	159	$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$
160	$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$	161	$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$
162	$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$	163	$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$
164	$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$	165	$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$
166	$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$	167	$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$
168	$\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$	169	$\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$
170	$\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$	171	$\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$
172	$\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$	173	$\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$
174	$\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$	175	$\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$
176	$\begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$	177	$\begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$
178	$\begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$	179	$\begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$
180	$\begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$	181	$\begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$
182	$\begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$	183	$\begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$
184	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$	185	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$

186	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$	187	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$
188	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$	189	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$
190	$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$	191	$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$
192	$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$	193	$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$
194	$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$	195	$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$
196	$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$	197	$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$
198	$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$	199	$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$
200	$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$	201	$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$
202	$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$	203	$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$
204	$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$	205	$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$
206	$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$	207	$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$
208	$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$	209	$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$
210	$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$	211	$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$
212	$\begin{pmatrix} 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$	213	$\begin{pmatrix} 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$
214	$\begin{pmatrix} 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$	215	$\begin{pmatrix} 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$
216	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$	217	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$
218	$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$	219	$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$
220	$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$	221	$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$
222	$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$	223	$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$
224	$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$	225	$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$
226	$\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$	227	$\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$
228	$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$	229	$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$
230	$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$	231	$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$
232	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	233	$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$
234	$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	235	$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

236	$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	237	$\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$
238	$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	239	$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$
240	$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$	241	$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$
242	$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$	243	$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$
244	$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$	245	$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$
246	$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$	247	$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$
248	$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$	249	$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$
250	$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$	251	$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$
252	$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$	253	$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$
254	$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$	255	$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$
256	$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$	257	$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$
258	$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$	259	$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$
260	$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$	261	$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$
262	$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$	263	$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$
264	$\begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$	265	$\begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$
266	$\begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$	267	$\begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$
268	$\begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$	269	$\begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$
270	$\begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$	271	$\begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$
272	$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$	273	$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$
274	$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$	275	$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$
276	$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$	277	$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$
278	$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$	279	$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$
280	$\begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$	281	$\begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$
282	$\begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$	283	$\begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$
284	$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$	285	$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$

286	$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$	287	$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$
288	$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$	289	$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$
290	$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$	291	$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$
292	$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	293	$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$
294	$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	295	$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$
296	$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$	297	$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$
298	$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$	299	$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$
300	$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$	301	$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$
302	$\begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$	303	$\begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$
304	$\begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$	305	$\begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$
306	$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$	307	$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$
308	$\begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$	309	$\begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$
310	$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	311	$\begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$
312	$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$	313	$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$
314	$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$		