1980

Coherency of synchronous generators

James Patrick Hilliard
Iowa State University

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COHERENCY OF SYNCHRONOUS GENERATORS

Iowa State University

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Coherency of synchronous generators

by

James Patrick Hilliard

A Dissertation Submitted to the Graduate Faculty in Partial Fulfillment of the Requirements for the Degree of DOCTOR OF PHILOSOPHY

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Approved:

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LIST OF SYMBOLS AND ABBREVIATIONS

\( A_i, B_i, C_i \) Impedance constants for machine \( i \), used in the calculation of the "M" coefficients

\( a_{ik} \) The negative ratio of the \( ik \) element in the system admittance matrix to the \( ii \) element

\( \text{abc} \) Subscripts, denoting \( abc \)-components

\( \beta \) Angle between the terminal voltage \( V_t \) and the reference axis

\( C_{eq} \) Equivalent coupling constant between generators

\( D \) Generator damping coefficient

\( d \) Subscript, denoting the component of a variable on the \( d \) axis

\( \Delta \) Prefix, denoting incremental change in a variable

\( . \) Dot superscript, representing the time-derivative operator \( \frac{d}{dt} \)

\( \delta \) Angle between the rotor quadrature axis and the synchronous reference

\( E \) Stator equivalent EMF corresponding to the field current

\( EQ \) Subscript, denoting an equivalent variable

\( E_{qf} \) Stator EMF proportional to the field voltage

\( E_{qa} \) Stator EMF referred to the \( q \) axis which, under certain conditions, can be used to represent a synchronous generator as a voltage behind the \( q \) axis reactance

\( E'_{q} \) Stator EMF proportional to the main winding flux linking the stator

\( G_{1ik}, G_{2ik} \) Impedance constants dependent upon \( A_i, B_i, \) and the initial conditions of machines \( i \) and \( k \)

\( G_{3ik}, G_{4ik} \) Impedance constants dependent upon \( A_i, C_i, \) and the initial conditions of machines \( i \) and \( k \)

\( \gamma \) Angular quantity dependent upon system and generator initial conditions
H \quad \text{Inertia constant}

I_a \quad \text{Generator terminal current}

i \quad \text{Subscript, associates variable with generator i}

ii \quad \text{Subscript, denotes variable on the main diagonal of a matrix}

j \quad \text{Subscript, associates variable with generator j}

"K" \quad \text{Coefficients of the "Heffron-Phillips" model}

k \quad \text{Subscript, associates variable with generator k}

K_{A_{ii}} \quad \text{Element of the exciter gain matrix giving the gain of the regulator for generator i}

\lambda \quad \text{Flux linkage}

"M" \quad \text{Coefficients of the extended "Heffron-Phillips" model}

\text{max} \quad \text{Subscript, denoting maximum value of variable}

\text{min} \quad \text{Subscript, denoting minimum value of variable}

n \quad \text{Number of generators in the system}

\omega_i \quad \text{Rotor speed (r/s) of generator i}

\omega_p \quad \text{Base rotor speed (377 r/s)}

P_e \quad \text{Electrical power}

P_L \quad \text{Load power}

P_m \quad \text{Mechanical power}

\phi \quad \text{Angle between the generator terminal voltage and current}

\pi \quad \text{Pi; also used in block diagrams to denote multiplication}

q \quad \text{Subscript, denoting the component of a variable on the q axis}

Q_L \quad \text{Load reactive power}

R_c \quad \text{Real part of the Thevenin equivalent impedance}
\[ s \quad \text{Laplace transform operator} \]
\[ \Sigma \quad \text{Prefix, denotes summation of terms} \]
\[ t \quad \text{Time in seconds} \]
\[ t \quad \text{Subscript, refers to a variable measured at the terminals of a machine} \]
\[ \text{TA}_i \quad \text{Exciter time constant for generator } i \]
\[ \text{THEV} \quad \text{Subscript, denotes a variable obtained through the Thevenin equivalencing process} \]
\[ T_e \quad \text{Electrical torque} \]
\[ T_m \quad \text{Mechanical torque} \]
\[ \tau_E \quad \text{Exciter time constant} \]
\[ \tau_{do} \quad d \text{ axis transient open circuit time constant} \]
\[ \Theta \quad \text{Angle, for elements of the } \dot{Y} \text{ admittance matrix when written in polar form} \]
\[ V_L \quad \text{Load voltage} \]
\[ V_{REF} \quad \text{Reference voltage} \]
\[ V_t \quad \text{Terminal voltage} \]
\[ X_e \quad \text{Reactance portion of the Thevenin equivalent impedance} \]
\[ \text{XFMR} \quad \text{Subscript, abbreviation for transformer} \]
\[ Y_L \quad \text{Load admittance} \]
\[ Y_{ii} \quad \text{Main diagonal term of admittance matrix} \]
\[ Y_{ij} \quad \text{Off diagonal term of admittance matrix} \]
\[ Z_e \quad \text{Thevenin equivalent impedance} \]
\[ 0 \quad \text{Subscript, denotes initial condition} \]
I. INTRODUCTION

A. Background

At the present time the accepted method of transient stability analysis is time solutions of the rotor angles of the synchronous machines. The machines closest to the fault are mathematically modelled in greater detail while those far removed are modelled classically. Sometimes it is possible to combine two or more machines into a single machine without affecting the accuracy of the study. Machines that fall into this category are called "coherent". This term is defined as signifying a group of generating units the rotors of which are moving with approximately the same mode of oscillation. The single equivalent machine that results from combining these coherent generators is defined as a dynamic equivalent. This dynamic equivalent is connected to a bus which is common to the network connecting the terminal buses of all the coherent machines.

The need for accurate stability analysis has required the simulation of synchronous machines by more complex mathematical models, and has prompted the use of increasingly complex computer programs for solution. For a typical power network in North America the cost of running such programs can be quite high, creating an incentive to investigate methods of cutting cost by simplifying the analysis without degrading the results. One of the most effective means is to reduce the size of the network by combining coherent generators located far from the disturbance, into a smaller number of equivalent
generators. Great savings in computer time is realized if the correct dynamic equivalents can be formed quickly and efficiently.

A power system has numerous modes of oscillation which can be broken down into two broad categories. The first category corresponds to the inertial modes, so called because of their high dependence upon the inertia of the rotors. The remaining modes are introduced by the control equipment and are affected mainly by their gains and time constants. Therefore, the different sets of modes are affected primarily by two different sets of parameters. Removal of modes from both groups will result from combining machines. When a group of inertially coherent machines are combined, the inertial modes that are eliminated will have little effect on the accuracy of a transient stability study. However, the extent to which accuracy can be affected by control mode elimination is an area of continuing interest.

B. Objectives

The first objective of this research is to develop a linearized model of a multimachine power system which will provide the information necessary to assess the amount and type of interaction between the machines.

The second objective will be to use the results from this model to separate the inertial modes of oscillation from the exciter modes. Once accomplished, this will allow study of the inertial and exciter effects as independent subsystems. Analysis of the inertial subsystem will then attempt to provide a means of establishing inertial coherency
without resorting to either time studies or eigenvector analysis. The exciter subsystem will be studied for more effective ways of combining the exciters on inertially coherent machines. Modal analysis and reduction will both be used in this regard.

C. Scope

The model just described will be derived in full mathematical detail. The results will then be applied to a four generator test system. Inertial coherency will be determined and tested by modal analysis. A number of cases will be examined for use in developing a systematic method of accurately reducing widely different exciters into an equivalent. These results will also be tested by modal analysis.
II. LITERATURE REVIEW

A. Early Methods

The coherency of synchronous generators has long been of interest in power system transient stability analysis. Prior to 1940, the customary practice involved circuit reduction by means of star-mesh conversions and parallel combinations of elements eliminating all nodes except machine terminals. Then, if some of the machines are in the same plant or if the impedance between them is small and they are remote from the fault location, they were assumed to swing together and combined into a single equivalent generator.

In these early transient stability studies the synchronous generators were all represented classically, i.e., by a constant voltage behind transient reactance. If it was decided to combine two or more they would be bussed together at their terminals with the equivalent machine given the total inertia and a transient reactance found by paralleling transient reactances of the original machines. Each machine was then represented by a second order differential equation. Loads were either treated as synchronous motors or as constant impedances to ground. This approach is still useful today for transient stability studies of small isolated systems.

The advent of the network analyzer answered a great need since power systems had grown and had become interconnected to the point where hand computation was no longer feasible. It was found, however, that equivalencing was still imperative on even the largest analyzer boards.
1. The distribution factors of the J. B. Ward era (1949)

In 1949, J. B. Ward wrote a paper (1) that succinctly summarized the prevailing attitudes toward equivalencing of both passive and active networks. The main thrust of his work was to replace a fixed portion of a network with an equivalent capable of reproducing tie-line flows between the reduced and detailed sections. Recognizing that the reduced section is an exact equivalent only under the operating conditions from which it is derived requires that judgement be used in applying the equivalent when major changes are made in the detailed section.

The equivalents which Ward derived were rigorous applications of Thevenin's and Norton's theorems. However, he notes that some degree of accuracy and rigor can usually be sacrificed in favor of flexibility and simplicity. Many equivalents of that era were developed by cut and try methods at the network analyzer board. The main problem is to determine the proper "distribution factors" by which generation and load from eliminated nodes are spread to the remaining nodes.

Ward recognized that what he had developed was basically a load-flow equivalent and that special circumstances must be considered when developing a stability equivalent. The elimination of nodes is not so straightforward in this case since loads cannot always be treated as constant impedances and machines with radically different inertias cannot be combined if located close to the fault zone.
2. The technique of Brown and Cloues (1955)

In 1955, Brown and Cloues presented a paper (2) which combined the load-flow equivalent with what was then a commonly used stability equivalent. The stability equivalent was obtained by connecting the generators to the same bus with their transient reactances in parallel. A major disadvantage of this equivalent was that it required that a single vector voltage be used for the resultant combined generation. Consequently, this equivalent would not give an accurate load-flow. Essentially, the Brown and Cloues method adjusted the impedances, net loading, and generation within the equivalenced area to produce the proper phase angle relationship at all points where the equivalent network connected to the detailed network. Therefore, their stability equivalent was also an accurate load-flow equivalent.

B. Modern Methods with the Advent of the Digital Computer

Since 1955, the advent of the large digital computer has had a pronounced effect on stability studies and the formation of equivalents. In 1957 Hale and Ward (3) devised a digital computer technique for power system reduction by the elimination of passive nodes. Brown et al. (4) did extensive testing of dynamic equivalents in 1969. The result was a computer program to calculate Ward-type distribution factors for the allocation of generation, inertia, and loads among the retained nodes.

The network analyzer boards, which modelled generators as a
constant voltage behind a reactance, have all given way to the large digital programs which can represent the generator in much more detail and include exciter effects. Whereas the classical model used a second order differential equation to model the generator it became possible, by use of the digital computer, to represent the generator by a much higher order mathematical model. The classical model produces the inertial modes of oscillation of the generators in the system. The frequencies of oscillation are determined by the inertia of the rotors and the network characteristics. By raising the order of generator modelling, new modes are introduced. These new modes superimpose upon the inertial modes and can influence the stability of the machine.

1. **Empirical approach**

Present day practice for the formation of stability equivalents has split into several approaches. The first approach (5-7) is somewhat empirical and, because of its low cost and ease of implementation, is certainly the most popular. This approach involves using high ordered generator models only in the immediate vicinity of the area to be faulted. Generators further from the study area are modelled with fewer equations and, finally, generators far removed are modelled classically. Several cases are then run with faults placed in the study area and the swing curves, i.e., the rotor angle versus time solutions, are calculated by use of the digital computer. These swing curves are then analyzed to find groups of classically
modelled generators which swing together. Once these groups are determined, the individual machines within a group are combined into one equivalent machine by bussing them together at their terminals and by using the sum of their inertias for the equivalent.

The network connecting the equivalent generators to the study area can then be reduced by Kron reduction to minimize the number of nodes outside the study area. Judgement and the use of phase shifting transformers are usually required before the equivalents will produce the same line flows in the study area. Once this has been accomplished, a substantial savings in computer time can be realized. Perhaps the best presentation of this technique is to be found in the 1975 EPRI Report 904 (5) which can be found in condensed form in reference 6.

A further refinement on this approach is treated in reference 7 where the swing curves of the transient are obtained from a simplified, linearized representation of the system. Here, coherency is determined by a clustering algorithm which sorts the swing curves. In this approach the following important assumptions are made:

1. Generator coherency is independent of the size of the disturbance and hence a linearized system model can be used.

2. Generator coherency is independent of model complexity and therefore the classical model is an adequate representation of the generator.

3. By pulsing the mechanical powers of the generators and using the unfaulted network, swing curves can be obtained which reproduce the effects of a fault.

These assumptions are empirical and are based on observation. It is important to note that assumption 2 refers to the determination of the
inertial coherency of machines. It assumes that inertially coherent generators have somewhat similar exciters. Indeed, the authors of reference 7 recommend that inertially coherent machines with radically different exciters not be combined.

The techniques just described represent the most practical and commonly used equivalencing methods available today. They are effective and can be used in conjunction with conventional transient stability programs. Enough computer program software has been developed for these approaches so that the need for engineering judgement on the part of the user has been effectively reduced.

2. Modal elimination

Modal analysis (8-12) has provided another important approach to dynamic equivalencing. This involves determining the fundamental independent frequencies of oscillation present in a system. The relative importance of each of these frequencies to a time solution can be determined by examining their damping and the machines to which they are most closely associated.

An important historical paper (13) was presented by C. A. Desoer in 1960 on mode development in linear time invariant circuits. This was followed in 1966 by the research done by E. J. Davison (14) on reducing the number of simultaneous differential equations required for solution while still retaining the dominant eigenvalues. The first application of this approach to power systems was done by Undrill and Turner (15) and Undrill et al. (16) in 1971. Through modal analysis they developed what are called dynamic electromechanical equivalents.
This approach offered a thorough and effective way of eliminating the least important modes while insuring that the dominant modes remain largely intact in the final solution.

The work on modal analysis has demonstrated the effectiveness of dynamic equivalents. When a system is reduced some information is always lost and modal reduction insures that this information is the least vital to the final time solution.

On the other hand, the dynamic equivalents produced in this fashion often bear little resemblance to any standard model for a synchronous generator. This severely limits their use in the transient stability programs normally available. Another drawback of the modal approach lies in the excessive amount of computer time necessary to compute the eigenvalues and eigenvectors. An attempt to reduce this cost has resulted in extensive statistical programs which analyze swing curves in order to estimate the dominant modes. The work of C. E. Grund (17) and others (18-20) illustrate these attempts.

Research efforts have continued to make the modal dynamic equivalent applicable to transient stability studies. The most successful work to date has been the joint effort (21) of A. J. Germond and R. Podmore. Other prominent researchers who have made contributions to this area are Van Ness et al. (22), Schlueter and Ahn (23), and Schlueter et al. (24). However, much is still to be done in this regard and investigations are continuing.
3. **Singular perturbation techniques**

Developing dynamic equivalents through the use of singular perturbation techniques is receiving attention in the current literature. Excellent introductory texts on the mathematics involved are shown in references 25, 26, and 27. Several papers (28-31) are concerned with methods of detecting and separating out high frequency modes that can be analyzed separately from the rest of the system. This saves much computer time in that numerically stiff systems are avoided and sophisticated integrating routines are not needed. The main difficulty encountered is the need for engineering judgement and "a priori" knowledge of the system. Since a successful application of singular perturbation technique results in a considerable reduction in the order of the system, ongoing efforts are attempting to find a practical means of implementation. A good example of this type of approach is provided by the work of M. A. Pai and Adgaonkar (32).

C. Connection of this Dissertation to Earlier Research

The work reported upon in this dissertation has its foundation in the work of many other researchers. The original Jeffron-Phillips model (33) developed a linearized model of the inertial and field effects of a synchronous generator connected to an infinite bus. Great insight into the interrelationships among the generator variables, and their effect on the excitation system performance, was gained in the remarkable work (34) of F. P. DeMello and C. Concordia.
This model was further developed by C. D. Vournas and R. J. Fleming (35-36) to include multimachine plants where the generators are connected through a reactance to a common bus. They used this model particularly to design a multivariable excitation stabilizer that takes into account the intermachine oscillations. In the work of F. P. deMello and T. F. Laskowski (37) this model was used to examine the effects on loading on generator dynamic stability.

In this dissertation the same linearized model is further expanded for use in a power system containing an arbitrary number of generators, and a general network configuration complete with network resistances. The model thus developed is used to derive special sets of coefficients that determine the various modes of oscillation. Some of these coefficients are primarily responsible for the inertial modes. Others determine the modes introduced by the excitation systems. In addition, the degree of interaction among these modes is determined.

Finally, the results obtained are used to suggest an empirical method of forming dynamic equivalents along conventional lines.
III. NETWORK AND GENERATOR MODELLING

The extent and complexity of the mathematical model used to simulate the power systems dynamic behavior is the first consideration when undertaking stability investigations. The model must be accurate enough to give useful results and yet as simple as possible in order to save engineering and computer time. The boundary between the "useful results" criterion and the "simple model" criterion is not fixed and inevitably involves trade-offs depending upon the specific case. Indeed, the whole concept of coherency is to simplify the mathematical model while still retaining useful results.

A. Network Model

1. Passive bilateral circuits and loads

For the purposes of this research it was decided to model the network as being composed of passive bilateral circuits and loads. This is essential for the use of matrix reduction. This network model is commonly used in stability programs today. Since the loads to be included in the network reduction are far removed from the disturbance, they are adequately represented by constant impedance.

2. Matrix reduction

The elimination of all circuit nodes, except the internal generator nodes, is undertaken in this research. These nodes have zero injection current since the loads are represented by constant impedance. Therefore, they can be eliminated by matrix reduction as
shown below.

The vector of node currents \( \bar{I} \) is related to the vector of node voltages \( \bar{V} \) by

\[
\bar{I} = \bar{Y} \bar{V}
\]  

(3.1)

where \( \bar{Y} \) is the short circuit admittance matrix of the network. If \( n \) is the number of generators in the system, the \( \bar{I} \) matrix can be partitioned as

\[
\bar{I} = \begin{bmatrix}
\bar{I}_n \\
\vdots \\
\bar{0}_r
\end{bmatrix}
\]  

(3.2)

Partitioning the \( \bar{V} \) and \( \bar{Y} \) matrix in the same fashion yields

\[
\begin{bmatrix}
\bar{I}_n \\
\vdots \\
\bar{0}_r
\end{bmatrix} = \begin{bmatrix}
\bar{Y}_{nn} & \cdots & \bar{Y}_{nr} \\
\vdots & \ddots & \vdots \\
\bar{V}_r & \cdots & \bar{Y}_{rr}
\end{bmatrix}
\begin{bmatrix}
\bar{V}_n \\
\vdots \\
\bar{V}_r
\end{bmatrix}
\]  

(3.3)

The subscript \( n \) stands for the number of internal generator nodes while the subscript \( r \) stands for all other nodes. Separating Equation 3.3 along the partition lines and expanding results in

\[
\bar{I}_n = \bar{Y}_{nn} \bar{V}_n + \bar{Y}_{nr} \bar{V}_r
\]

\[
\bar{0}_r = \bar{V}_r \bar{V}_n + \bar{Y}_{rr} \bar{V}_r
\]

(3.4)

Eliminating \( \bar{V}_r \) from Equation 3.4 removes all nodes but the internal generator nodes and leaves
\[ \bar{I}_n = (\bar{Y}_{nn} - \bar{Y}_{nr} \bar{Y}^{-1}_{rr} \bar{Y}_{rn})\bar{V}_n \]  

(3.5)

The new reduced \( \bar{Y} \) matrix is \( (\bar{Y}_{nn} - \bar{Y}_{nr} \bar{Y}^{-1}_{rr} \bar{Y}_{rn}) \) with \( n \times n \) dimensions.

B. One-Axis Generator Model

1. Characteristics

The one-axis model (see Section 4.15 of reference 38) of a synchronous generator is the simplest in which the transient effects are included. The axis in question is the \( d \) axis, obtained from Park's transformation. The field circuit lies on this axis and strongly affects the transient behavior of the generator. In this model, the subtransient (amortisseur) effects are neglected; all transformer-type voltages are set equal to zero; while speed voltages are computed with the simplifying assumption that the angular speed is constant and is equal to the rated speed, i.e., \( \omega = \omega_n \). Figure 3.1 shows a block diagram of the one-axis model.

2. Equations

The following equations form the mathematical basis for the one-axis model. The notation and symbols used here and throughout this dissertation are from reference 38 and Appendix A.

\[ \dot{\lambda}_d = \dot{\lambda}_q = 0 \]

where \( \lambda_d \) and \( \lambda_q \) are the flux linkages along the \( d \) and \( q \) axes, respectively. For a given machine the terminal voltage, \( \bar{V}_t \), and the terminal current, \( \bar{I}_t \), can be defined as
Figure 3.1. Block diagram of the one-axis model
\[
\vec{V}_t = V_q + jV_d \quad \vec{I}_a = I_q + jI_d
\]  
(3.6)

where
\[
V_q = v_c / \sqrt{3} \quad V_d = v_d / \sqrt{3}
\]
\[
I_q = i_q / \sqrt{3} \quad I_d = i_d / \sqrt{3}
\]  
(3.7)

and \(v_d\) and \(v_q\) are the \(d\) and \(q\) axes voltages, while \(i_d\) and \(i_q\) are the \(d\) and \(q\) axes currents. Noting that \(x_d\) and \(x_q\) are the \(d\) and \(q\) axes reactances,

\[
\begin{align*}
V_d &= -x_q I_q \quad V_q = E' + x_d I_d \\
\vec{V}_t &= \vec{E}_{qa} - x_q \vec{I}_a \quad E' = E + (x_d - x_q') I_d \\
E_{qa} &= E_q - (x_q - x_d') I_d \quad T_e = E_{qa} I_q
\end{align*}
\]  
(3.8)

where:

- \(E'\) is the stator per unit EMF proportional to the main winding flux linking the stator
- \(E\) is the stator per unit EMF proportional to the field current
- \(x_d'\) is the \(d\) axis transient reactance
- \(E_{qa}\) is a \(q\) axis per unit voltage which, under steady state conditions with no saturation, can represent a synchronous machine as a voltage behind the \(q\) axis reactance
- \(T_e\) is the per unit electrical torque output of the machine

The differential equations involved are

\[
\begin{align*}
\tau_{do} \dot{E}' &= E_{FD} - E \\
\frac{2H}{\omega_d} \ddot{\delta} + \frac{D}{\omega_d} \dot{\delta} &= T_m - T_e
\end{align*}
\]  
(3.9)  
(3.10)
where:

\( T'_d \) is the d axis transient open circuit time constant
\( E_{FD} \) is the per unit stator EMF proportional to the field voltage
\( H \) is the generator inertia constant in seconds
\( \omega_B \) is the rated synchronous rotor speed in r/s
\( D \) is a damping constant
\( T_m \) is the per unit mechanical torque input of the machine

(Note that time is measured in seconds).

C. Linearized One-Axis Generator Model

1. Description

The linearization is accomplished using first order approximations for the model equations, and consequently is valid only for small perturbations about a quiescent operating point. The resulting equations are linear in the system variables, with coefficients made up of system parameters and quiescent operating conditions (considered as constant).

2. Equations

The following linearized equations consist of constants and incremental variables. However, for convenience, the customary \( \Delta \) prefix to the incremental variables has been omitted. Those constants with a zero subscript stand for the initial value of a variable before linearization. The linearized equations for the one-axis model are
\[ V_d = -x_d I_d \]
\[ V_q = E_q' + x_q' I_d \]
\[ I_a (I_{a0}) = I_d (I_{d0}) + I_q (I_{q0}) \]
\[ V_t (V_{t0}) = V_d (V_{d0}) + V_q (V_{q0}) \]
\[ E'_q = E + (x_d - x_d') I_d \]
\[ E_q a = E_q' - (x_q - x_q') I_d \]
\[ T_e = E_q a (I_{q0}) + I_q (E_{qa0}) \]
\[ V_t = E_q a - x_q' I_a \]
\[ \dot{\delta} = -E + E_{FD} \]
\[ \frac{2H \delta^2}{\omega_B} + \frac{D \delta}{\omega_B} = -T_e \]

### D. Linearized Exciter Model

Exciter modelling can cover a wide range of complexity (38). The purpose of this research was best served with a model consisting of the regulator gain, \( K_A \), and the exciter time constant, \( T_A \). The block diagram for the linearized exciter model is shown in Figure 3.2.

![Block diagram of the linearized exciter model](image-url)
The equation for this model is

\[
(TA)\dot{E}_{FD} = (KA)(V_{REF} - V_t) - E_{FD} \tag{3.12}
\]

where

- \(V_{REF}\) is the reference voltage.

From Equations 3.11 and 3.12 we note that the complete generator-exciter model can be represented by four first order differential equations.
IV. EXTENSION OF THE "HEFFRON-PHILLIPS" MODEL TO A MULTIMACHINE SYSTEM

A. Purpose and Application

In this chapter the Heffron-Phillips model is extended so that it can be applied to a general multimachine system. This will allow analysis and quantification of intermachine relationships and oscillations with a view toward establishing a criterion for coherency.

The Heffron-Phillips model consists of one generator against an infinite bus, with the linearized one-axis model used for the generator (33,34). It provides a highly useful means of analyzing the interaction between the torque angle loop, the exciter, and the stator circuits. In the past, it has been used to advantage to analyze generator response to different exciters and power system stabilizers. Appendix A gives a circuit diagram, and a Fortran program for evaluating the parameters of the Heffron-Phillips model.

In 1978 Vournas and Fleming (35,36) extended the Heffron-Phillips model to multimachine plants. This made possible detailed analysis of intermachine oscillations and the design of a multivariable stabilizer which takes into account the multimachine nature of the plant.

1. Assumptions

In addition to the assumptions in Chapter III for the one-axis model, the following are added:

1. All generators are represented as variable voltages behind quadrature axis reactances.

2. Salient pole generators ($x_q = x'_q$) are assumed.
3. Saturation effects and armature resistances are not included.

4. The linearization of the equations will reflect the effects of nonlinearities as the actual operating point is changed.

2. **Insight into internal coupling between machines**

   The extended Heffron-Phillips model will have the same type of format as the original. Where the original model defines constants $K_1-K_6$, the extended model will use matrices of constants $K_1-K_9$ and $R_1-R_3$. These matrices will then be combined into the $M_1-M_6$ matrices which will be analogous to the $K_1-K_6$ constants of the original model. The coefficients of the $M$-matrices relate the variables of the $i$th machine to all variables.

B. **System Equivalent from the Terminals of a Single Generator**

The first step in deriving the extended model is to develop the Thevenin equivalent for the system as seen by each generator. Representing each generator by its voltage $\bar{E}_{qa}$ behind the reactance $X_q$, the node currents of an $n$-machine system are given by

\[
\begin{bmatrix}
\bar{I}_1 \\
\vdots \\
\bar{I}_i \\
\vdots \\
\bar{I}_n 
\end{bmatrix} = \begin{bmatrix}
\bar{V}_{11} & \cdots & \bar{V}_{1i} & \cdots & \bar{V}_{1n} \\
\vdots & \ddots & \vdots & \cdots & \vdots \\
\bar{V}_{ii} & \cdots & \bar{V}_{ii} & \cdots & \bar{V}_{in} \\
\vdots & \ddots & \vdots & \cdots & \vdots \\
\bar{V}_{ni} & \cdots & \bar{V}_{ni} & \cdots & \bar{V}_{nn}
\end{bmatrix} \begin{bmatrix}
\bar{E}_{q1} \\
\vdots \\
\bar{E}_{qi} \\
\vdots \\
\bar{E}_{qn}
\end{bmatrix} \quad (4.1)
\]

The Thevenin impedance for machine $i$ seen looking into the system with all other sources shorted to ground, i.e.,
\[ Z_{ei} = R_{ei} + jX_{ei} = (Y_{ii})^{-1} \quad (4.2) \]

is the reciprocal of the driving point admittance. The Thevenin voltage is found by opening the circuit at node \( i \) for generator \( i \) and calculating the resulting voltage with respect to the reference node. Each of the remaining generators contribute to this voltage and the weighting factors can be determined from Equation 4.1, by setting the injection current \( I_i \) equal to zero and solving row \( i \) for \( E_{qai} \).

The Thevenin equivalent voltage thus obtained is given by

\[ E_{THEV(i)} = \sum_{k=1}^{n} \frac{-Y_{ik}}{Y_{ii}} E_{qak} \quad (4.3) \]

These results are schematically illustrated in Figure 4.1. An alternate representation, in which \( x_q \) is removed from \( Z_{ei} \) and displayed separately, is shown in Figure 4.2.

C. Derivation of the "K" and "R" Constants

Using the circuit shown in Figure 4.2 and solving for the \( q \) and \( d \) axis currents for generator \( i \) yields

\[ I_{qi} = A_i(E'_i - \sum_{k=1}^{n} a_{ik}E_{qak} \cos \gamma_{ik}) + B_i(\sum_{k=1}^{n} a_{ik}E_{qak} \sin \gamma_{ik}) \quad (4.4) \]

\[ I_{di} = C_i(\sum_{k=1}^{n} a_{ik}E_{qak} \cos \gamma_{ik} - E'_i) - A_i(\sum_{k=1}^{n} a_{ik}E_{qak} \sin \gamma_{ik}) \quad (4.5) \]
Figure 4.1. The Thevenin equivalent circuit at any generator bus i

Figure 4.2. The Thevenin equivalent circuit at any generator bus i with the reactance \( jx_q \) retained
where

\[ A_i = \frac{R_{ei}}{R_{ei} + X_{ei}(X_{ei} - x_i + x_i') R_{ei} + X_{ei}(X_{ei} - x_i + x_i')} \]

\[ B_i = \frac{-(X_{ei} - x_i + x_i')}{{R_{ei}}^2 + X_{ei}(X_{ei} - x_i + x_i')} \]

\[ C_i = \frac{X_{ei}}{{R_{ei}}^2 + X_{ei}(X_{ei} - x_i + x_i')} \]

\[ a_{ik} = \frac{-Y_{ik}}{Y_{ii}} \]

\[ Y_{ii} = Y_{ii} + a_{ik} \]

\[ Y_{ik} = Y_{ik} + a_{ik} \]

These current equations are then linearized. Note that the incremental \( \Delta \) prefixes to the variables have been omitted for convenience.

\[ I_{qi} = A_i E_i' - \sum_{k=1}^{n} \delta_{ik} a_{ik} E_k q_{k0} G_{1ik} + \sum_{k=1}^{n} E_k a_{ik} G_{2ik} \]

\[ I_{di} = -C_i E_i' + \sum_{k=1}^{n} \delta_{ik} a_{ik} E_k q_{k0} G_{3ik} + \sum_{k=1}^{n} E_k a_{ik} G_{4ik} \]

where

\[ G_{1ik} = A_i \sin \gamma_{ik} + A_i \cos \gamma_{ik} \]

\[ G_{2ik} = \delta_i \sin \gamma_{ik} - A_i \cos \gamma_{ik} \]

\[ G_{3ik} = C_i \sin \gamma_{ik} + A_i \cos \gamma_{ik} \]

\[ G_{4ik} = C_i \cos \gamma_{ik} - A_i \sin \gamma_{ik} \]

The linearized current equations are now substituted into the linearized one-axis model equations from Chapter III. With the current variables eliminated, the results for a multimachine system are placed in matrix form. These matrix equations are shown below in both the general form and in the expanded form for a three machine system.
\[ \dot{\omega} = \omega_B (2H)^{-1} (T_m - T_e) - (2H)^{-1} D \omega \]  
(4.8)

For a three machine system

\[
\begin{bmatrix}
\dot{\omega}_1 \\
\dot{\omega}_2 \\
\dot{\omega}_3 \\
\end{bmatrix} = \begin{bmatrix}
\frac{\omega_B}{2H_1} & 0 & 0 \\
0 & \frac{\omega_B}{2H_2} & 0 \\
0 & 0 & \frac{\omega_B}{2H_3} \\
\end{bmatrix} \begin{bmatrix}
T_{m1} & 0 & 0 \\
0 & T_{m2} & 0 \\
0 & 0 & T_{m3} \\
\end{bmatrix} \begin{bmatrix}
T_{e1} & 0 & 0 \\
0 & T_{e2} & 0 \\
0 & 0 & T_{e3} \\
\end{bmatrix} \begin{bmatrix}
\frac{D_1}{2H_1} & 0 & 0 \\
0 & \frac{D_2}{2H_2} & 0 \\
0 & 0 & \frac{D_3}{2H_3} \\
\end{bmatrix} \begin{bmatrix}
\omega_1 \\
\omega_2 \\
\omega_3 \\
\end{bmatrix}
\]

\[ \dot{\omega}' = \left( \tau_d K_3 \right)^{-1} \left( K_3 E_{FD} - K_3 K_4 \delta + K_3 K_9 E_{qa} - E' \right) \]  
(4.9)

For a three machine system

\[
\begin{bmatrix}
\dot{E}'_{q1} \\
\dot{E}'_{q2} \\
\dot{E}'_{q3} \\
\end{bmatrix} = \begin{bmatrix}
\frac{1}{K_{311} \tau_d \omega_1} & 0 & 0 \\
0 & \frac{1}{K_{322} \tau_d \omega_2} & 0 \\
0 & 0 & \frac{1}{K_{333} \tau_d \omega_3} \\
\end{bmatrix} \begin{bmatrix}
K_{311} & 0 & 0 \\
0 & K_{322} & 0 \\
0 & 0 & K_{333} \\
\end{bmatrix} \begin{bmatrix}
E_{FD1} \\
E_{FD2} \\
E_{FD3} \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
K_{311} & 0 & 0 \\
0 & K_{322} & 0 \\
0 & 0 & K_{333} \\
\end{bmatrix} \begin{bmatrix}
K_{411} & K_{412} & K_{413} \\
K_{421} & K_{422} & K_{423} \\
K_{431} & K_{432} & K_{433} \\
\end{bmatrix} \begin{bmatrix}
\delta_1 \\
\delta_2 \\
\delta_3 \\
\end{bmatrix}
\]
\[
\begin{bmatrix}
K_{311} & 0 & 0 \\
0 & K_{322} & 0 \\
0 & 0 & K_{333}
\end{bmatrix}
+ \begin{bmatrix}
0 & K_{912} & K_{913} \\
K_{921} & 0 & K_{923} \\
K_{931} & K_{932} & 0
\end{bmatrix}
\begin{bmatrix}
\delta_1 \\
\delta_2 \\
\delta_3
\end{bmatrix}
- \begin{bmatrix}
E_{\text{q}a1} \\
E_{\text{q}a2} \\
E_{\text{q}a3}
\end{bmatrix}
= \begin{bmatrix}
\delta_1 \\
\delta_2 \\
\delta_3
\end{bmatrix}
\begin{bmatrix}
K_{211} & 0 & 0 \\
0 & K_{222} & 0 \\
0 & 0 & K_{233}
\end{bmatrix}
\begin{bmatrix}
E_{\text{q}1} \\
E_{\text{q}2} \\
E_{\text{q}3}
\end{bmatrix}
\]

\( T_e = K_1 \delta + K_2 E'_q + K_8 E_{qa} \) \hspace{1cm} (4.10)

For a three machine system

\[
\begin{bmatrix}
T_{e1} \\
T_{e2} \\
T_{e3}
\end{bmatrix}
= \begin{bmatrix}
K_{111} & K_{112} & K_{113} \\
K_{121} & K_{122} & K_{123} \\
K_{31} & K_{32} & K_{33}
\end{bmatrix}
\begin{bmatrix}
\delta_1 \\
\delta_2 \\
\delta_3
\end{bmatrix}
+ \begin{bmatrix}
0 & K_{812} & K_{813} \\
K_{821} & 0 & K_{823} \\
K_\delta & K_\delta & 0
\end{bmatrix}
\begin{bmatrix}
\delta_1 \\
\delta_2 \\
\delta_3
\end{bmatrix}
\begin{bmatrix}
E_{\text{q}a1} \\
E_{\text{q}a2} \\
E_{\text{q}a3}
\end{bmatrix}
\]

\( V_c = K_5 \delta + K_6 E'_q + K_7 E_{qa} \) \hspace{1cm} (4.11)
For a three machine system

\[
\begin{bmatrix}
V_{t1} \\
V_{t2} \\
V_{t3}
\end{bmatrix} =
\begin{bmatrix}
K_{511} & K_{512} & K_{513} \\
K_{521} & K_{522} & K_{523} \\
K_{531} & K_{532} & K_{533}
\end{bmatrix}
\begin{bmatrix}
\delta_1 \\
\delta_2 \\
\delta_3
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 & 0 \\
K_{622} & 0 & 0 \\
0 & K_{633} & 0
\end{bmatrix}
\begin{bmatrix}
E'_{q1} \\
E'_{q2} \\
E'_{q3}
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
0 & K_{712} & K_{713} \\
K_{721} & 0 & K_{723} \\
K_{731} & K_{732} & 0
\end{bmatrix}
\begin{bmatrix}
E_{qa1} \\
E_{qa2} \\
E_{qa3}
\end{bmatrix}
\]

\[
R_1 E_{qa} = R_2 \delta + R_3 E'
\]  \hspace{1cm} (4.12)

For a three machine system

\[
\begin{bmatrix}
1 & R_{112} & R_{113} \\
R_{121} & 1 & R_{123} \\
R_{131} & R_{132} & 1
\end{bmatrix}
\begin{bmatrix}
E_{qa1} \\
E_{qa2} \\
E_{qa3}
\end{bmatrix}
= \begin{bmatrix}
R_{211} & R_{212} & R_{213} \\
R_{221} & R_{222} & R_{223} \\
R_{231} & R_{232} & R_{233}
\end{bmatrix}
\begin{bmatrix}
\delta_1 \\
\delta_2 \\
\delta_3
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
R_{311} & 0 & 0 \\
0 & R_{322} & 0 \\
0 & 0 & R_{333}
\end{bmatrix}
\begin{bmatrix}
E'_{q1} \\
E'_{q2} \\
E'_{q3}
\end{bmatrix}
\]
Therefore, the "K" and "R" constants have the following physical significance: They relate the changes in the electrical quantities of the ith generator to each of the variables of the kth generator, while maintaining all other variables constant. The following definitions provide physical insight into the meaning of the constants.

\[ K_{1ik} = \frac{\Delta T_{ei}}{\Delta \delta_k} \]  

The ith generator synchronizing coefficient due to generator k.

\[ K_{2ii} = \frac{\Delta T_{ei}}{\Delta E'_{qi}} \]  

The change in electrical torque in generator i due to a change in the field flux linkages in generator i.

\[ K_{3ii} = \left[ 1 + C_i (x_{di} - x'_{di}) \right]^{-1} \]  

Impedance factor

\[ K_{4ik} = \frac{-1}{K_{3ii}} \frac{\Delta E'_{qi}}{\Delta \delta_k} \]  

The demagnetizing effect on generator i due to a change in the kth generator angle.

\[ K_{5ik} = \frac{\Delta V_{ti}}{\Delta \delta_k} \]  

The change in terminal voltage of generator i due to a change in rotor angle for generator k.

\[ K_{6ii} = \frac{\Delta V_{ti}}{\Delta E'_{qi}} \]  

The change in terminal voltage of generator i due to a change in the field flux linkages of generator i.

\[ K_{7ik} = \frac{\Delta V_{li}}{\Delta E_{qa}} \]  

The change in terminal voltage of generator i due to a change in \( E_{qa} \) of generator k.

\[ K_{8ik} = \frac{\Delta T_{ei}}{\Delta E_{qa}} \]  

The change in electrical torque for generator i due to a change in \( E_{qa} \) of generator k.

\[ K_{9ik} = \frac{1}{K_{3ii}} \frac{\Delta E'_{qi}}{\Delta E_{qa}} \]  

The change in \( E' \) in generator i due to a change in \( E_{qa} \) in generator k.
\[ R_{ik} = -\frac{\Delta E_{qai}}{\Delta E_{qak}} \]  
\[ R_{ik}^2 = \frac{\Delta E_{qai}}{\Delta \delta_k} \]  
\[ R_{ii}^3 = \frac{\Delta E_{qai}}{\Delta E_{qi}^i} \]

The change in \( E_{qai} \) in generator \( i \) due to a change in \( E_{qak} \) in generator \( k \).

The change in \( E_{qai} \) in generator \( i \) due to a change in \( \delta \) in generator \( k \).

The change in \( E_{qai} \) in generator \( i \) due to a change in the field flux linkages in generator \( i \).

The formulas for the calculation of these constants are as follows:

\[ i \neq k \quad K_{ik} = a_{ik} E_{qak0} [E_{qai0} G_{ik} + I_{qi0} (x_{q1} - x_{d1}) G_{3ik}] \]  \hspace{1cm} (4.13)

\[ K_{ii} = -\sum_{k=1}^{n} K_{ik} \]  \hspace{1cm} (4.14)

\[ i \neq k \quad K_{ik}^2 = 0 \]  \hspace{1cm} (4.15)

\[ K_{ii}^2 = A_i E_{qai0} + [1 + C_i (x_{q1} - x_{d1})] I_{qi0} \]  \hspace{1cm} (4.16)

\[ i \neq k \quad K_{ik}^3 = 0 \]  \hspace{1cm} (4.17)

\[ K_{ii}^3 = [1 + C_i (x_{d1} - x_{d1})]^{-1} \]  \hspace{1cm} (4.18)

\[ i \neq k \quad K_{ik}^4 = a_{ik} E_{qak0} (x_{d1} - x_{q1}^i) G_{3ik} \]  \hspace{1cm} (4.19)

\[ K_{ii}^4 = -\sum_{k=1}^{n} K_{ik}^4 \]  \hspace{1cm} (4.20)
\[ i \neq k \quad K_{5,ik} = \frac{a_{ik}E_{q_{10}G_{ik}}}{v_{q_{10}i0}} \left[ x_{q_{10}i0}G_{1,ik} + x'_{di}V_{q_{10}i0}G_{3,ik} \right] \quad (4.21) \]

\[ K_{5,ii} = - \sum_{k=1}^{n} K_{5,ik} \quad (4.22) \]

\[ i \neq k \quad K_{6,ik} = 0 \quad (4.23) \]

\[ K_{6,ii} = \left( \frac{1}{v_{q_{10}i0}} \right) \left( V_{q_{10}i0} - x_{q_{10}i0}A_{i} - x'_{di}V_{q_{10}i0}C_{1} \right) \quad (4.24) \]

\[ i \neq k \quad K_{7,ik} = \frac{a_{ik}}{v_{q_{10}i0}} \left( x'_{di}V_{q_{10}i0}G_{4,ik} - x_{q_{10}i0}G_{2,ik} \right) \quad (4.25) \]

\[ K_{7,ii} = 0 \quad (4.26) \]

\[ i \neq k \quad K_{8,ik} = a_{ik} \left[ E_{q_{10}G_{2,ik}} - I_{q_{10}}(x_{q_{10}i0} - x_{q_{10}i0}') \right] G_{4,ik} \quad (4.27) \]

\[ K_{8,ii} = 0 \quad (4.28) \]

\[ i \neq k \quad K_{9,ik} = a_{ik}(x_{di} - x'_{di})G_{4,ik} \quad (4.29) \]

\[ K_{9,ii} = 0 \quad (4.30) \]

\[ i \neq k \quad R_{1,ik} = a_{ik}(x_{q_{10}i0} - x_{q_{10}i0}')G_{4,ik} \quad (4.31) \]

\[ R_{1,ii} = 1 \quad (4.32) \]

\[ i \neq k \quad R_{2,ik} = a_{ik}(x_{q_{10}i0} - x_{q_{10}i0}')E_{q_{10}G_{3,ik}} \quad (4.33) \]

\[ R_{2,ii} = - \sum_{k=1}^{n} R_{2,ik} \quad (4.34) \]
i≠k \quad R_{1k}^3 = 0 \quad (4.35)

\quad R_{1i}^3 = 1 + C_i (x_{qi} - x_{di}) \quad (4.36)

D. Derivation of the "M" Coefficients

Using Equation 4.12, the variable \( E_q \) can be eliminated from the matrix Equations 4.9-4.11. This leaves the internal voltage \( E_q^1 \) to represent the EMF of the machines and allows the merging of \( K_7-K_9 \) and \( R_1-R_3 \) with \( K_1-K_6 \) to form the \( M_1-M_6 \) matrices. The system equations thus formed are

\[
\begin{align*}
T_e &= M_1 \delta + M_2 E_q^1 \\
\dot{E}_q^1 &= \frac{T_d}{\tau_{d01}} - \frac{T_d}{\tau_{d01}} M_4 \delta + \frac{T_d}{\tau_{d01}} M_3 E_q^1 \\
V_t &= M_4 \delta + M_4 E_q^1
\end{align*}
\]

where

\[
\begin{align*}
T_d &= \text{Diagonal} \begin{bmatrix} \tau_{d01} & \cdots & \tau_{d0n} \end{bmatrix} \\
M_1 &= K_1 + K_8 R_1^{-1} R_2 \\
M_2 &= K_2 + K_8 R_1^{-1} R_3 \\
M_3 &= -K_3^{-1} + K_9 R_1^{-1} R_3 \\
M_4 &= K_4 - K_9 R_1^{-1} R_2 \\
M_5 &= K_5 + K_7 R_1^{-1} R_2 \\
M_6 &= K_6 + K_7 R_1^{-1} R_3
\end{align*}
\]
Examining Equations 4.37-4.39 we note the similarity between the "M" coefficients developed here and the \( K_1-K_6 \) constants of the original Heffron-Phillips model. The "M" coefficients are therefore in the desired form. Their physical interpretation is given below.

\[
M_{1ik} = \frac{\Delta T_i}{\Delta \delta_k} \quad \text{The synchronizing coefficient of machine } i \text{ due to machine } k \text{ with } E'_{qj}, \ j=1,n \text{ and } \delta_j, \ j=1,n, \ j \neq k \text{ held constant. Note the similarity to the power synchronizing coefficients in Appendix B.}
\]

\[
M_{2ik} = \frac{\Delta T_i}{\Delta E'_{qk}} \quad \text{The change in electrical torque in machine } i \text{ caused by change in the field flux linkage of machine } k.
\]

\[
M_{3ik} = \frac{\Delta E'_{qi}}{\delta_{qi}} \quad \text{The magnetizing effect on the } i\text{th machine due to a change in the } k\text{th machine field flux linkages.}
\]

\[
M_{4ik} = -\tau'_{d0i} \frac{\Delta E'_{qi}}{\Delta \delta_k} \quad \text{The demagnetizing effect on machine } i \text{ due to a change in the angle of the } k\text{th machine.}
\]

\[
M_{5ik} = \frac{\Delta V_i}{\Delta \delta_k} \quad \text{The change in terminal voltage of machine } i \text{ due to a change in the angle of the } k\text{th machine.}
\]

\[
M_{6ik} = \frac{\Delta V_i}{\Delta E'_{qk}} \quad \text{The change in terminal voltage of machine } i \text{ due to a change in the field flux linkages of the } k\text{th machine.}
\]
E. System Block Diagram

When the excitation system is not represented, Equations 4.37, 4.38, and 4.8 describe the entire system in state space form. With \( n \) defined as the number of machines the state vector is defined as

\[
X = [\delta \omega \frac{E' q}{d}]^T \quad (3n \times 1)
\]  
(4.47)

The state equation is defined as

\[
\dot{X} = AX
\]  
(4.48)

where

\[
A = \begin{bmatrix}
0 & I_n & 0 \\
-d_B T_n^{-1} M_1 & -T_n^{-1} D & -d_B T_n^{-1} M_2 \\
-T_d^{-1} M_4 & 0 & T_d^{-1} M_3
\end{bmatrix}
\]  
(4.49)

and

\[
T_n = \text{Diagonal} [2h_1 \ldots 2h_n]
\]  
(4.50)

The block diagram for Equation 4.48 is shown in Figure 4.3.
Figure 4.3. Block diagram of Equation 4.48
V. SEPARATING THE INERTIAL AND EXCITER MODES

A. The "A" Matrix in Terms of the "M" Coefficients

The system modal frequencies are given by the eigenvalues of the "A" matrix. The variables most strongly associated with each modal frequency are determined by examining the eigenvectors. The frequencies associated with \( \delta \) and \( \omega \) are the inertial modes, so called because of their dependence upon the rotor inertias. The exciter modes can be detected by noting the association with the exciter variables.

1. Without exciter

Without exciter, the system "A" matrix is shown in the state equation,

\[
\begin{bmatrix}
\dot{\delta} \\
\dot{\omega} \\
\dot{E}'_q
\end{bmatrix}
= \begin{bmatrix}
0 & \frac{1}{n} & 0 \\
-\frac{\omega_B T_n^{-1} M_1}{n} & -\frac{T_d^{-1} M_1}{n} & -\frac{\omega_B T_n^{-1} M_2}{n} \\
-\frac{T_d^{-1} M_4}{d} & 0 & \frac{T_d^{-1} M_3}{d}
\end{bmatrix}
\begin{bmatrix}
\delta \\
\omega \\
E'_q
\end{bmatrix}
\] (5.1)

The \( E'_q \) equations tend to produce real eigenvalues, while the complex eigenvalues are strongly associated with the \( \delta \) and \( \omega \) variables.
2. With exciter

Adding the exciter model described in Chapter III yields

\[
\begin{bmatrix}
\dot{\delta} \\
\dot{\omega} \\
\dot{E}_d' \\
\dot{E}_{pd}'
\end{bmatrix} =
\begin{bmatrix}
0 & T_n \\
-\omega_B T^{-1}_n M_1 & -T^{-1}_n D \\
-T^{-1}_d M_4 & 0 & T^{-1}_d M_3 & T^{-1}_d \\
-KA T^{-1}_a M_5 & 0 & -KA T^{-1}_a M_6 & -TA^{-1}_a
\end{bmatrix}
\begin{bmatrix}
\delta \\
\omega \\
E_d' \\
E_{pd}'
\end{bmatrix}
\]

(5.2)

with

\[KA = \text{Diagonal } [KA_1 \ldots KA_n]\]  

(5.3)

and

\[TA = \text{Diagonal } [TA_1 \ldots TA_n]\]  

(5.4)

An interesting phenomena now occurs, where the real eigenvalues tend to disappear and are replaced by modal frequencies that are associated with the $E_{pd}$ variables. Analysis of the eigenvectors show that the inertial frequencies are primarily associated with the $\omega$ variables. Since these two groups of modal frequencies are associated with different sets of variables, this suggests that they can be decoupled.

Equation 5.2 has partition lines dividing the matrices into zones of influence that can be attributed to inertial and exciter effects and the coupling between them. The equation can be rewritten as:
The inertial and exciter modes are highly dependent upon matrices \( \mathbf{I} \) and \( \mathbf{IV} \), respectively. The numerical results developed in the next several chapters show that the inertial and exciter modes can indeed be decoupled. This means that the examination of matrices \( \mathbf{I} \) and \( \mathbf{IV} \) individually can produce valuable insight into coherency and the reduction process. We note that matrix \( \mathbf{I} \) is highly dependent upon the M1 coefficients, while matrix \( \mathbf{IV} \) depends upon the M3 and M6 coefficients.

3. Establishing a Reference Generator

The "A" matrix of Equation 5.2 is singular. In this section a technique will be developed for reducing the "A" matrix by establishing a reference generator.

The rotor angle vector for a power system with \( n \) machines is

\[
\begin{bmatrix}
\delta_1 \\
\vdots \\
\delta_p \\
\vdots \\
\delta_n
\end{bmatrix}
\]
with \( i \) varying from 1 to \( n \).

If \( p \) is to be the reference generator the first step is to change the order of terms in the \( \delta_i \) vector. This will form the \( \hat{\delta}_i \) vector as follows:

\[
\hat{\delta}_i = \begin{bmatrix}
\delta_1 \\
\vdots \\
\delta_n \\
\delta_p
\end{bmatrix}
\]

The second step is to premultiply the \( \hat{\delta}_i \) vector by the transformation matrix \( T \):

\[
\delta_{ip} = T \hat{\delta}_i
\]

\[
\begin{bmatrix}
d_1p \\
\vdots \\
d_{n-1}p
\end{bmatrix} = \begin{bmatrix}
I_{n-1} & \vdots & -1 \\
\vdots & \ddots & \vdots \\
0 & \ldots & 1
\end{bmatrix}\begin{bmatrix}
\delta_1 \\
\vdots \\
\delta_n \\
\delta_p
\end{bmatrix}
\]

(5.5)

Where \( I \) is an \( n-1 \) identity matrix and \( T \) has the dimensions \( n-1 \) rows by \( n \) columns. This establishes generator \( p \) as the reference generator in the \( \delta_{ip} \) vector.

To extend this technique to the system the state space equations are first written in this form:
With the selection of generator \( p \) as the reference the equations are now reordered.

\[
\begin{bmatrix}
\dot{\delta}_1 \\
\vdots \\
\dot{\delta}_n \\
\dot{\delta}_p \\
\dot{\omega}_1 \\
\vdots \\
\dot{\omega}_n \\
\dot{\omega}_p
\end{bmatrix} = A
\begin{bmatrix}
\delta_1 \\
\vdots \\
\delta_n \\
\delta_p \\
\omega_1 \\
\vdots \\
\omega_n \\
\omega_p
\end{bmatrix}
\]

(5.6)

The state vector can be placed in the desired form by

\[
\begin{bmatrix}
\delta_{1P} \\
\vdots \\
\delta_{nP} \\
\omega_{1P} \\
\vdots \\
\omega_{nP}
\end{bmatrix} =
\begin{bmatrix}
\begin{array}{c|c}
1 & \vdots \\
\vdots & 0
\end{array}
\end{bmatrix}
\begin{bmatrix}
\delta_1 \\
\vdots \\
\delta_n \\
\delta_p \\
\omega_1 \\
\vdots \\
\omega_n \\
\omega_p
\end{bmatrix}
\]
Now \( \hat{B} \) will be formed by

\[
\hat{B} = \begin{bmatrix}
T & \vdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \vdots & T
\end{bmatrix}
\]

where \( \hat{B} \) has \( n-2 \) rows and \( n \) columns. The columns \( \frac{n}{2} \) and \( n \) must then be deleted to form matrix \( B \) which yields the following desired result.

\[
\begin{bmatrix}
\delta_{ip} \\
\omega_{ip}
\end{bmatrix} = \begin{bmatrix}
\hat{B} \\
\hat{B}
\end{bmatrix}
\]

(5.7)

Here \( B \) is the new "A" matrix with an \( n-2 \) order and dimensions of \( (n-2) \times (n-2) \).

C. Inertial Coherency Determined by the "\( \mu \)" Coefficients

By holding \( E' \) constant, i.e., the incremental variables \( E' \) equal to zero, Equation 4.37 is reduced to

\[
\underline{T_e} = \underline{M1} \underline{\delta}
\]

(5.8)

In its expanded form, for example purposes a three machine system:
The matrix $M_1$ is singular as evidenced by the fact that the sum of any row in $M_1$ is equal to zero. The order of Equation 5.8 can therefore be reduced by a procedure similar to that outlined in the previous section.

By selecting one machine as a reference, the variables are reduced to two angle differences, e.g., $\delta_{21}$ and $\delta_{31}$, or $\delta_{32}$ and $\delta_{12}$, and so on. The electrical torque of any machine can be expressed as a function of these new variables. This suggests the procedure, outlined below, for determining inertial coherency.

Selecting machine 1 as reference we can write

$$\begin{align*}
\begin{bmatrix}
T_{e1} \\
T_{e2} \\
T_{e3}
\end{bmatrix} &= \begin{bmatrix}
M_{11} & M_{12} & M_{13} \\
M_{21} & M_{22} & M_{23} \\
M_{31} & M_{32} & M_{33}
\end{bmatrix} \begin{bmatrix}
\delta_1 \\
\delta_2 \\
\delta_3
\end{bmatrix}
\end{align*}$$

Let us assume that this system is made up of two independent systems: one due to machines 1 and 2 alone, and the second due to machines 1 and 3 alone; with no interaction (or coupling) between the two systems. In that case we can analyze each system independently as a two machine system. The frequency of oscillation for each of the two
systems is the same as that of an equivalent one machine-infinite bus system. For example, we can show that for the system formed by machines 1 and 2, we have

\[
\frac{2H_{eq}}{\omega_B} \delta_{21} + C_{eq}^{22} \delta_{21} = 0
\]

(5.10)

where

\[
H_{eq} = \frac{H_1 H_2}{H_1 + H_2}, \text{ and}
\]

(5.11)

\[
C_{eq}^{22} = M_{L22} - M_{L12}
\]

The frequency of oscillation for this system is given by

\[
\omega_{21} = \left[ \frac{C_{eq}^{22} \omega_B (\frac{H_1 H_2}{H_1 + H_2})}{2} \right]^{1/2}
\]

(5.12)

and similarly for machines 1 and 3

\[
\omega_{31} = \left[ \frac{C_{eq}^{33} \omega_B (\frac{H_1 H_3}{H_1 + H_3})}{2} \right]^{1/2}
\]

where

\[
C_{eq}^{33} = M_{L33} - M_{L13}
\]

We can easily show that the matrix \( C_{eq} \) is given by

\[
C_{eq} = \begin{bmatrix}
C_{eq}^{22} & C_{eq}^{23} \\
C_{eq}^{32} & C_{eq}^{33}
\end{bmatrix}
= \begin{bmatrix}
(M_{L22} - M_{L12}) & (M_{L23} - M_{L13}) \\
(M_{L32} - M_{L12}) & (M_{L33} - M_{L13})
\end{bmatrix}
\]

(5.13)

We note that the diagonal terms of the \( C_{eq} \) matrix are similar to the synchronizing power coefficients. The off-diagonal coefficients
$C_{23}^{eq}$, $C_{32}^{eq}$ are indicative of the cross-coupling between the two modes of oscillation. If their values are small, the two systems 1 and 2, and 1 and 3 approach being two independent systems.

If the angular frequencies of oscillation $\omega_{21}$ and $\omega_{31}$ are nearly equal, then generators 2 and 3 are almost inertially coherent. Thus, selecting a new reference generator and repeating the process of computing and comparing the resulting modes of oscillation, inertial coherency is determined for the various generators. In the next chapter, this procedure will be illustrated for the 4-generator test system.

More information can be extracted from the $C_{eq}$ matrix by observing the off-diagonal terms. These terms give an indication of the strength of cross-coupling between generators other than the reference generator (#2 and #3 in the above example). The two machine system to be considered now is comprised of generators 2 and 3 with a synchronizing torque coefficient of first $C_{23}^{eq}$ and the $C_{32}^{eq}$. The resulting intermachine frequencies are

$$\omega_{23} = \left[ |C_{eq}^{23}(\omega_B \frac{H_2 + H_3}{2}$ \frac{H_2 H_3}{H_2^2 H_3}) | \right]^{1/2} \text{ r/s}$$

and

$$\omega_{32} = \left[ |C_{eq}^{32}(\omega_B \frac{H_2 H_3}{2}$ \frac{H_2 + H_3}{H_2 H_3}) | \right]^{1/2} \text{ r/s}$$

If $\omega_{23} \approx \omega_{32}$ and if these are high frequencies there is strong coupling between generators 2 and 3. The stronger the coupling the more likelihood the two machines will swing together in the event of
a disturbance remote from both machines. Again, this calculation is demonstrated on a numerical example in Chapter VI.

D. Exciter Modes Derived from the Decoupled "A" Matrix

Repeating the $A$ matrix from Equation 5.2 for convenience:

$$A = \begin{bmatrix}
0 & T_n & 0 & 0 \\
-\omega_b & T_n^{-1}M_1 & -T_n^{-1}D & -\omega_bT_n^{-1}M_2 \\
-T_n^{-1}M_4 & 0 & T_n^{-1}M_3 & T_d \\
-KA & TA^{-1}M_5 & 0 & -KA TA^{-1}M_6 & -TA^{-1}
\end{bmatrix}$$  (5.16)

We recall that the upper left hand quadrant represents the inertial part of the system while the lower right hand quadrant represents the exciter. The other two quadrants represent the interaction between the exciter and the inertial effects. The size of the $A$ matrix is $4n \times 4n$ while the order of the system is $4n-2$. The inertial portion is $2n \times 2n$ with an order of $2n-2$ while the exciter portion is $2n \times 2n$ with an order of $2n$.

An eigenvalue and eigenvector analysis of the $A$ matrix will establish $2n-2$ inertial modes and $2n$ exciter modes. To decouple the $A$ matrix successfully, it must be shown that the eigenvalues and eigenvectors of the inertial portion alone correspond to the $2n-2$ inertial modes of the matrix as a whole. Similarly, the eigenvalues and eigenvectors of the exciter portion alone must correspond to the
2n exciter modes of the matrix as a whole.

In Chapter VI an actual system is successfully decoupled using the above criterion, thus considerably reducing the computational cost. Of equal importance in the study of coherency is the fact that this decoupling demonstrates that inertial coherency is, in the general case, independent of the excitors. Consequently, when coherent generators are combined, the machine and exciter parameters should be combined independently.
VI. EXTENDED "HEFFRON-PHILLIPS" MODEL APPLIED TO A FOUR GENERATOR TEST SYSTEM

The extended "Heffron-Phillips" model, developed in the previous chapters, will now be used to examine the modes of oscillation for a four generator test system. This power network is an expanded version of the 9-bus 3-generator test system known in the literature as the WSCC system (see reference 37).

A. System Parameters and Load-Flow Data

The impedance diagram of the test system is shown in Figure 6.1. In this figure, the line and transformer impedances, as well as the line shunt susceptances, are given in per unit to a 100 MVA base. System generator data are given in Table 6.1, where the reactances are given in per unit to a 100 MVA base and the time constants are in seconds.

Load-flow data, for the quiescent operating conditions to be considered in the study, are shown in Figure 6.2. Power flows are in MW and MVAR with positive flow being away from the bus.

The loads are converted to equivalent admittances. From the load bus voltage $V_L$, power $P_L$, and reactive power $Q_L$, the load admittance $Y_L$ is given by

$$Y_L = \frac{P_L}{V_L^2} - j \frac{Q_L}{V_L^2}$$  \hspace{1cm} (6.1)
Figure 6.1. Four generator eleven bus test system impedance diagram; all impedances are in per unit on a 100 MVA base.
Figure 6.2. Load-flow of 11 bus test system; all flows are in MW and MVAR; all tap ratios are fixed at 1.0.
Table 6.1. Generator data

<table>
<thead>
<tr>
<th>Generator</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated MVA</td>
<td>247.5</td>
<td>192.0</td>
<td>128.0</td>
<td>192.0</td>
</tr>
<tr>
<td>H (MW*S)</td>
<td>2364</td>
<td>640</td>
<td>301</td>
<td>640</td>
</tr>
<tr>
<td>KV</td>
<td>16.5</td>
<td>18.0</td>
<td>13.8</td>
<td>18.0</td>
</tr>
<tr>
<td>Power factor</td>
<td>0.72</td>
<td>0.97</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td>Type</td>
<td>Hydro</td>
<td>Steam</td>
<td>Steam</td>
<td>Steam</td>
</tr>
<tr>
<td>Speed (rev/min)</td>
<td>180</td>
<td>3600</td>
<td>3600</td>
<td>3600</td>
</tr>
<tr>
<td>$x'_d$</td>
<td>0.1460</td>
<td>0.8958</td>
<td>1.3125</td>
<td>0.8958</td>
</tr>
<tr>
<td>$x_d$</td>
<td>0.0608</td>
<td>0.1198</td>
<td>0.1813</td>
<td>0.1198</td>
</tr>
<tr>
<td>$x'_q$</td>
<td>0.0969</td>
<td>0.8645</td>
<td>1.2578</td>
<td>0.8645</td>
</tr>
<tr>
<td>$x_q$</td>
<td>0.0969</td>
<td>0.1969</td>
<td>0.25</td>
<td>0.1969</td>
</tr>
<tr>
<td>$x'_L$ (leakage)</td>
<td>0.0336</td>
<td>0.0521</td>
<td>0.0742</td>
<td>0.0521</td>
</tr>
<tr>
<td>$\tau_d0$</td>
<td>8.96</td>
<td>6.00</td>
<td>5.89</td>
<td>6.00</td>
</tr>
<tr>
<td>$\tau_q0$</td>
<td>0</td>
<td>0.535</td>
<td>0.600</td>
<td>0.535</td>
</tr>
</tbody>
</table>

The values for the equivalent shunt admittances for the three loads are:

Load A: $\bar{Y}_{L5} = 1.3340 - j 0.5336$
Load B: $\bar{Y}_{L6} = 0.9333 - j 0.3055$
Load C: $\bar{Y}_{L8} = 0.9764 - j 0.3417$

These values are now combined with the appropriate line shunt.
admittances as shown in Table 6.2. This gives enough information to construct the $Y$ matrix of the system as shown in Table 6.3. A matrix reduction is then performed on the $Y$ matrix and all nodes are eliminated except the internal generator nodes where the generators are modelled as a source with voltage $E_{qa}$ behind the reactance $x_q$. The reactance $x_q$ is included in the $Y$ matrix. The reduced $Y$ matrix is shown in Table 6.4.

B. Application of "Park's Transformation"

To calculate the "$M" coefficients, the $q$ axis for each generator must be located and the corresponding $d$ and $q$ axis quantities calculated. These quantities are displayed in Figure 6.3, where

- $V_{\infty}$ is the infinite bus voltage that a generator "sees" looking into the system (per unit)
- $V_t, I_a$ are the terminal voltage and current (per unit)
- $I_d, I_q$ are the $d$ and $q$ axis generator currents (per unit)
- $\delta$ is the angle between the generator $q$ axis and the infinite bus voltage (degrees)
- $\beta$ is the angle between the generator terminal voltage and the infinite bus voltage (degrees)
- $\phi$ is the angle between generator terminal voltage and terminal current (degrees)

and the subscript $o$ indicates the quiescent operating condition.

A special computer program has been developed to perform these calculations. This program is given in Appendix A, along with sample calculations. The numerical results are shown in Table 6.5.
Table 6.2. System admittance

<table>
<thead>
<tr>
<th>Generators</th>
<th>Bus No.</th>
<th>( R )</th>
<th>( X )</th>
<th>( G )</th>
<th>( B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. 1</td>
<td>1-4</td>
<td>0</td>
<td>0.1545</td>
<td>0</td>
<td>-6.4725</td>
</tr>
<tr>
<td>No. 2</td>
<td>2-7</td>
<td>0</td>
<td>0.9270</td>
<td>0</td>
<td>-1.0787</td>
</tr>
<tr>
<td>No. 3</td>
<td>3-9</td>
<td>0</td>
<td>1.3164</td>
<td>0</td>
<td>-0.7596</td>
</tr>
<tr>
<td>No. 4</td>
<td>11-10</td>
<td>0</td>
<td>0.9245</td>
<td>0</td>
<td>-1.0817</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Transmission lines</th>
<th>Bus No.</th>
<th>( R )</th>
<th>( X )</th>
<th>( G )</th>
<th>( B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-5</td>
<td>0.0100</td>
<td>0.0850</td>
<td>1.3652</td>
<td>-11.6041</td>
<td></td>
</tr>
<tr>
<td>4-6</td>
<td>0.0170</td>
<td>0.0920</td>
<td>1.9422</td>
<td>-10.5107</td>
<td></td>
</tr>
<tr>
<td>5-7</td>
<td>0.0320</td>
<td>0.1610</td>
<td>1.1876</td>
<td>-5.9751</td>
<td></td>
</tr>
<tr>
<td>6-9</td>
<td>0.0390</td>
<td>0.1700</td>
<td>1.2820</td>
<td>-5.5882</td>
<td></td>
</tr>
<tr>
<td>7-8</td>
<td>0.0085</td>
<td>0.0720</td>
<td>1.6171</td>
<td>-13.6980</td>
<td></td>
</tr>
<tr>
<td>8-9</td>
<td>0.0119</td>
<td>0.1008</td>
<td>1.1551</td>
<td>-9.7843</td>
<td></td>
</tr>
<tr>
<td>8-10</td>
<td>0.0119</td>
<td>0.1008</td>
<td>1.1551</td>
<td>-9.7843</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Shunt admittances</th>
<th>Bus No.</th>
<th>( R )</th>
<th>( X )</th>
<th>( G )</th>
<th>( B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load A</td>
<td>5-0</td>
<td>1.3340</td>
<td>-0.4758</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Load B</td>
<td>6-0</td>
<td>0.9333</td>
<td>-0.2432</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Load C</td>
<td>8-0</td>
<td>0.9764</td>
<td>-0.2987</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4-0</td>
<td>0</td>
<td>0.0401</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7-0</td>
<td>0</td>
<td>0.0546</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9-0</td>
<td>0</td>
<td>0.0686</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10-0</td>
<td>0</td>
<td>0.0502</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^a\) The generator reactance \( x_g \) is added to the transformer reactance for each generator.

\(^b\) The appropriate line shunt susceptances are added to each load.
<table>
<thead>
<tr>
<th></th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.08T</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1.07T</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1.07T</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1.07T</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1.07T</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1.07T</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1.07T</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1.07T</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1.07T</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 6.3, X matrix for the 4-machine system.
Figure 6.3. Location of the q axis with reference to the system equivalent Thevenin voltage, $V_\infty$. Positive direction is counterclockwise.
Table 6.4. Reduced $Y$ matrix

<table>
<thead>
<tr>
<th>Node</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0377</td>
<td>0.1733</td>
<td>0.1207</td>
<td>0.1670</td>
</tr>
<tr>
<td></td>
<td>-j2.2162</td>
<td>+j0.4625</td>
<td>+j0.3377</td>
<td>+j0.4189</td>
</tr>
<tr>
<td>2</td>
<td>0.1733</td>
<td>0.0545</td>
<td>0.0326</td>
<td>0.0499</td>
</tr>
<tr>
<td></td>
<td>+j0.4625</td>
<td>-j0.8954</td>
<td>+j0.0843</td>
<td>+j0.1400</td>
</tr>
<tr>
<td>3</td>
<td>0.1207</td>
<td>0.0326</td>
<td>0.0286</td>
<td>0.0350</td>
</tr>
<tr>
<td></td>
<td>+j0.3377</td>
<td>+j0.0843</td>
<td>-j0.6614</td>
<td>+j0.0958</td>
</tr>
<tr>
<td>11</td>
<td>0.1670</td>
<td>0.0499</td>
<td>0.0350</td>
<td>0.0605</td>
</tr>
<tr>
<td></td>
<td>+j0.4189</td>
<td>+j0.1400</td>
<td>+j0.0958</td>
<td>-j0.8603</td>
</tr>
</tbody>
</table>

Table 6.5. Generator initial values

<table>
<thead>
<tr>
<th>Generator</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{a0}$ p.u.</td>
<td>0.8724</td>
<td>0.8510</td>
<td>0.8405</td>
<td>0.8391</td>
</tr>
<tr>
<td>$\delta_{0}$</td>
<td>31.0598°</td>
<td>40.1546°</td>
<td>30.9556°</td>
<td></td>
</tr>
<tr>
<td>$\beta_{0}$</td>
<td>3.1645°</td>
<td>31.0598°</td>
<td>40.1546°</td>
<td>30.9556°</td>
</tr>
<tr>
<td>$V_{d0}$ p.u.</td>
<td>-0.0574</td>
<td>-0.5288</td>
<td>-0.6642</td>
<td>-0.5324</td>
</tr>
<tr>
<td>$V_{q0}$ p.u.</td>
<td>1.0384</td>
<td>0.8780</td>
<td>0.6876</td>
<td>0.6876</td>
</tr>
<tr>
<td>$I_{d0}$ p.u.</td>
<td>-0.6404</td>
<td>-0.5917</td>
<td>-0.6539</td>
<td>-0.5699</td>
</tr>
<tr>
<td>$I_{q0}$ p.u.</td>
<td>0.5923</td>
<td>0.6117</td>
<td>0.5281</td>
<td>0.6158</td>
</tr>
<tr>
<td>$E_{qa0}$ p.u.</td>
<td>1.1005</td>
<td>1.3895</td>
<td>1.6097</td>
<td>1.5803</td>
</tr>
</tbody>
</table>

The next step is to calculate the angle between each of the $q$ axes of the four generators. In Figure 6.4 the $q$ axes and terminal voltage of
two generators are shown. The angle \( \theta_{t10} - \theta_{tk0} \) is the angle between the terminal voltages of the two machines and can be found from the load-flow study. The angle we are now seeking is the angle between the \( q \) axes, \( \delta_{10} - \delta_{k0} \), which will be referred to as \( \delta_{ik0} \). Inspection of Figure 6.4 yields the following equation

\[
\delta_{ik0} = (\delta_{10} - \delta_{10}) + (\delta_{a10} - \delta_{ak0}) - (\delta_{k0} - \delta_{k0})
\]

(6.2)

Substituting numerical values into this equation yields

\[
\delta_{210} = 34.1453°
\]

\[
\delta_{420} = 1.9257°
\]

\[
\delta_{340} = 7.5091°
\]

where the angles between the \( q \) axes are shown in Figure 6.5.

Because of the length and complexity of the data preparation it is important to run the following simple check. Referring now to the Thevenin's equivalent circuit shown in Figure 4.1 the first step is to calculate the Thevenin equivalent voltage each generator "sees" looking into the system. Using Equation 4.3

\[
\bar{E}_{THEV(1)} = -\left[\frac{\bar{V}_{11}^{12}}{\bar{V}_{11}} \bar{E}_{qa20} + \frac{\bar{V}_{11}^{13}}{\bar{V}_{11}} \bar{E}_{qa30} + \frac{\bar{V}_{11}^{14}}{\bar{V}_{11}} \bar{E}_{qa40}\right] = 0.7682 \\text{[}-4.9022°\text{] p.u.}
\]

\[
\bar{E}_{THEV(2)} = -\left[\frac{\bar{V}_{22}^{21}}{\bar{V}_{22}} \bar{E}_{qa10} + \frac{\bar{V}_{22}^{23}}{\bar{V}_{22}} \bar{E}_{qa30} + \frac{\bar{V}_{22}^{24}}{\bar{V}_{22}} \bar{E}_{qa40}\right] = 0.9405 \\text{[}-5.5593°\text{] p.u.}
\]

\[
\bar{E}_{THEV(3)} = -\left[\frac{\bar{V}_{33}^{31}}{\bar{V}_{33}} \bar{E}_{qa10} + \frac{\bar{V}_{33}^{32}}{\bar{V}_{33}} \bar{E}_{qa20} + \frac{\bar{V}_{33}^{34}}{\bar{V}_{33}} \bar{E}_{qa40}\right] = 0.9556 \\text{[}-5.2708°\text{] p.u.}
\]
Figure 6.4. Angular relationship between the q axes

Figure 6.5. Relative q axis positions for the four generators
Once the Thevenin voltages are known, the machine currents can be calculated as follows:

\[
\bar{I}_{a10} = \left( \bar{E}_{qa10} - \bar{E}_{THEV(1)} \right) \bar{Y}_{11} = 0.8724 \quad [-44.1438^\circ] \text{ p.u.}
\]

\[
\bar{I}_{a20} = \left( \bar{E}_{qa20} - \bar{E}_{THEV(2)} \right) \bar{Y}_{22} = 0.8516 \quad [-6.7307^\circ] \text{ p.u.}
\]

\[
\bar{I}_{a30} = \left( \bar{E}_{qa30} - \bar{E}_{THEV(3)} \right) \bar{Y}_{33} = 0.8404 \quad [-4.3679^\circ] \text{ p.u.}
\]

\[
\bar{I}_{a40} = \left( \bar{E}_{qa40} - \bar{E}_{THEV(4)} \right) \bar{Y}_{44} = 0.8392 \quad [-3.6061^\circ] \text{ p.u.}
\]

These currents are then checked against the current values in Table 6.5 and found to be quite accurate.

C. Calculation of the "M" Coefficients

Data for the calculation of the K and R matrices are assembled in Table 6.6. The matrices K1-K9 are calculated using Equations 4.13-4.30, with the results shown in Tables 6.7-6.15.
Table 6.6. Data for calculating the "M" coefficients, generator 1 is system reference

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Generator 1</th>
<th>Generator 2</th>
<th>Generator 3</th>
<th>Generator 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_q$</td>
<td>0.0969</td>
<td>0.8645</td>
<td>1.2578</td>
<td>0.8645</td>
</tr>
<tr>
<td>$x_d$</td>
<td>0.1460</td>
<td>0.8958</td>
<td>1.3125</td>
<td>0.8958</td>
</tr>
<tr>
<td>$x_d'$</td>
<td>0.0608</td>
<td>0.1198</td>
<td>0.1813</td>
<td>0.1198</td>
</tr>
<tr>
<td>$V_{t0}$</td>
<td>1.040</td>
<td>1.025</td>
<td>1.030</td>
<td>1.035</td>
</tr>
<tr>
<td>$V_{d0}$</td>
<td>-0.0574</td>
<td>-0.5288</td>
<td>-0.6642</td>
<td>-0.5324</td>
</tr>
<tr>
<td>$V_{q0}$</td>
<td>1.0384</td>
<td>0.8780</td>
<td>0.7872</td>
<td>0.8876</td>
</tr>
<tr>
<td>$E_{qa0}$</td>
<td>1.1005</td>
<td>1.3895</td>
<td>1.6097</td>
<td>1.3803</td>
</tr>
<tr>
<td>$\delta_0$</td>
<td>0.0°</td>
<td>34.1453°</td>
<td>43.5901°</td>
<td>36.0911°</td>
</tr>
<tr>
<td>$I_{q0}$</td>
<td>0.5925</td>
<td>0.6117</td>
<td>0.5281</td>
<td>0.6158</td>
</tr>
</tbody>
</table>

Table 6.7. The K1 matrix

<table>
<thead>
<tr>
<th></th>
<th>1.9408</th>
<th>-0.7228</th>
<th>-0.5648</th>
<th>-0.6532</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.1179</td>
<td>1.7289</td>
<td>-0.2434</td>
<td>-0.3676</td>
<td></td>
</tr>
<tr>
<td>-1.0422</td>
<td>-0.3110</td>
<td>1.6938</td>
<td>-0.3406</td>
<td></td>
</tr>
<tr>
<td>-0.9736</td>
<td>-0.3750</td>
<td>-0.2730</td>
<td>1.6216</td>
<td></td>
</tr>
<tr>
<td>Table 6.8. The K2 matrix</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------------------------</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.8853  0  0  0  0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0  2.0634  0  0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0  0  1.9934  0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0  0  0  1.9462</td>
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</table>

<table>
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<th>Table 6.9. The K3 matrix</th>
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<tbody>
<tr>
<td>0.8297  0  0  0  0</td>
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<tr>
<td>0  0.3241  0  0</td>
</tr>
<tr>
<td>0  0  0.2779  0</td>
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<tr>
<td>0  0  0  0.3499</td>
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<table>
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<tr>
<td>-0.0510  0.0150  0.0217  0.0143</td>
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<tr>
<td>-1.0336  1.2481  -0.0690  -0.1455</td>
</tr>
<tr>
<td>-1.3844  -0.2507  1.8912  -0.2561</td>
</tr>
<tr>
<td>-0.9073  -0.1056  -0.0773  1.1402</td>
</tr>
</tbody>
</table>

<table>
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<th>Table 6.11. The K5 matrix</th>
</tr>
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<tr>
<td>0.0458  -0.0142  -0.0182  -0.0124</td>
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<td>-0.0274  0.1547  -0.0560  -0.0713</td>
</tr>
<tr>
<td>-0.0050  -0.0625  0.1430  -0.0754</td>
</tr>
<tr>
<td>-0.0208  -0.0680  -0.0630  0.1518</td>
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</tbody>
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### Table 6.12. The K6 matrix

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</thead>
<tbody>
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</tr>
<tr>
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<td>0</td>
<td>0</td>
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### Table 6.13. The K7 matrix

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<td>0.0561</td>
</tr>
<tr>
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<td>0.0740</td>
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<tr>
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### Table 6.14. The K8 matrix

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<td>0</td>
<td>-0.1103</td>
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<tr>
<td>0.1692</td>
<td>-0.0930</td>
<td>0</td>
<td>-0.1179</td>
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<tr>
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<td>-0.1135</td>
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### Table 6.15. The K9 matrix

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<td>0</td>
<td>0.2060</td>
</tr>
<tr>
<td>0.6339</td>
<td>0.3057</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.5186</td>
<td>0.2987</td>
<td>0.2149</td>
<td>0</td>
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</table>
The matrices $R_1$-$R_3$ are calculated from Equations 4.31-4.36. The results are shown in Tables 6.16-6.18.

**Table 6.16. The $R_1$ matrix**

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<tr>
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<th>0.0188</th>
<th>0.0129</th>
<th>0.0171</th>
</tr>
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<tbody>
<tr>
<td>0.6373</td>
<td>1</td>
<td>0.1977</td>
<td>0.3165</td>
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<td>0.6032</td>
<td>0.2909</td>
<td>1</td>
<td>0.3378</td>
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<tr>
<td>0.4977</td>
<td>0.2866</td>
<td>0.2062</td>
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</tbody>
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**Table 6.17. The $R_2$ matrix**

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<th>0.0092</th>
<th>0.0061</th>
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<td>-0.0663</td>
<td>-0.1397</td>
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<tr>
<td>-1.3175</td>
<td>-0.2386</td>
<td>1.7997</td>
<td>-0.2437</td>
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<td>-0.8707</td>
<td>-0.1570</td>
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</table>

**Table 6.18. The $R_3$ matrix**

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<td>0</td>
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<tr>
<td>0</td>
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<td>0</td>
<td>2.7830</td>
<td>0</td>
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</tbody>
</table>
The $M$ matrices are formed by using Equations 4.41-4.46. The $M_1$-$M_6$ matrices are given in Tables 6.19-6.24.

**Table 6.19. The $M_1$ matrix**

<p>| | | | |</p>
<table>
<thead>
<tr>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2974</td>
<td>-0.7939</td>
<td>-0.8205</td>
<td>-0.6830</td>
</tr>
<tr>
<td>-0.9339</td>
<td>1.8552</td>
<td>-0.4073</td>
<td>-0.5140</td>
</tr>
<tr>
<td>-0.9218</td>
<td>-0.3919</td>
<td>1.7714</td>
<td>-0.4577</td>
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<tr>
<td>-0.7798</td>
<td>-0.5009</td>
<td>-0.4618</td>
<td>1.7425</td>
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</table>

**Table 6.20. The $M_2$ matrix**

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</thead>
<tbody>
<tr>
<td>2.0926</td>
<td>-0.2997</td>
<td>-0.5201</td>
<td>-0.2056</td>
</tr>
<tr>
<td>0.2294</td>
<td>2.2614</td>
<td>-0.3326</td>
<td>-0.4087</td>
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<tr>
<td>0.2736</td>
<td>-0.2293</td>
<td>2.1131</td>
<td>-0.3053</td>
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<tr>
<td>0.3102</td>
<td>-0.3572</td>
<td>-0.3747</td>
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</tr>
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</table>

**Table 6.21. The $M_3$ matrix**

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</tr>
</thead>
<tbody>
<tr>
<td>-1.2538</td>
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<td>0.0677</td>
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<td>0.5470</td>
<td>-3.5257</td>
<td>0.5701</td>
<td>0.8687</td>
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<td>0.4292</td>
<td>0.7302</td>
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<td>0.8651</td>
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Table 6.22. The $M_4$ matrix

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<td>-0.0090</td>
<td>-0.0029</td>
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<td>-0.6802</td>
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<td>-0.4730</td>
</tr>
<tr>
<td>-1.0184</td>
<td>-0.5238</td>
<td>2.1378</td>
<td>-0.5956</td>
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<td>-0.5100</td>
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<td>-0.4105</td>
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Table 6.23. The $M_5$ matrix

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<th></th>
</tr>
</thead>
<tbody>
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<td>-0.0012</td>
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<tr>
<td>-0.0866</td>
<td>-0.0046</td>
<td>-0.0163</td>
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Table 6.24. The $M_6$ matrix

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<tbody>
<tr>
<td>0.8247</td>
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<td>0.0473</td>
</tr>
<tr>
<td>0.2490</td>
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<tr>
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<td>0.2064</td>
<td>0.1599</td>
<td>0.0894</td>
<td>0.6062</td>
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</table>
D. Calculating the "A" Matrix

Now that the M matrices have been calculated, the $T_d$ and $T_n$ matrices are found using Equations 4.40 and 4.50. Their values are given in Tables 6.25 and 6.26.

Table 6.25. The $T_d$ matrix

<table>
<thead>
<tr>
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</thead>
<tbody>
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<td>0</td>
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<tr>
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<td>0</td>
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<td>0</td>
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</table>

Table 6.26. The $T_n$ matrices

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</thead>
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</table>

The matrices $K_A$ and $T_A$ are then obtained from Equations 5.3–5.4. Tables 6.27 and 6.28 show the exciter gains and time constants, which are contained in the matrices $K_A$ and $T_A$, for the base case. Many other cases have been run and will be discussed in later chapters.
Table 6.27. The $K_A$ matrix, base case

\[
\begin{array}{cccc}
100.0000 & 0 & 0 & 0 \\
0 & 100.0000 & 0 & 0 \\
0 & 0 & 100.0000 & 0 \\
0 & 0 & 0 & 100.0000 \\
\end{array}
\]

Table 6.28. The $T_A$ matrix, base case

\[
\begin{array}{cccc}
1.0000 & 0 & 0 & 0 \\
0 & 1.0000 & 0 & 0 \\
0 & 0 & 1.0000 & 0 \\
0 & 0 & 0 & 1.0000 \\
\end{array}
\]

Note that all time constants are in seconds and that time is measured in seconds throughout this research.

The $A$ matrix can now be calculated using Equation 5.16. The results are given in Table 6.25.
<table>
<thead>
<tr>
<th></th>
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<th>$\delta_2$</th>
<th>$\delta_3$</th>
<th>$\delta_4$</th>
<th>$\omega_1$</th>
<th>$\omega_2$</th>
<th>$\omega_3$</th>
<th>$\omega_4$</th>
</tr>
</thead>
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<tr>
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<td>( E'_{g3} )</td>
<td>( E'_{g4} )</td>
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<td>( E_{FD3} )</td>
<td>( E_{FD4} )</td>
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</tr>
<tr>
<td>-6.7555</td>
<td>-66.6038</td>
<td>9.7955</td>
<td>12.0362</td>
<td>0</td>
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<tr>
<td>-17.1315</td>
<td>14.3563</td>
<td>-132.3310</td>
<td>19.1161</td>
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<td>0</td>
<td>0</td>
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</tr>
<tr>
<td>-9.1367</td>
<td>10.5211</td>
<td>11.0368</td>
<td>-62.9950</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>-0.1399</td>
<td>0.0109</td>
<td>0.0075</td>
<td>0.0076</td>
<td>0.1116</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>0.0912</td>
<td>-0.5876</td>
<td>0.0950</td>
<td>0.1448</td>
<td>0</td>
<td>0.1667</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>0.0729</td>
<td>0.1240</td>
<td>-0.6801</td>
<td>0.1469</td>
<td>0</td>
<td>0</td>
<td>0.1698</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>0.0550</td>
<td>0.1420</td>
<td>0.1085</td>
<td>-0.5498</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.1667</td>
<td></td>
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<tr>
<td>-82.4663</td>
<td>-6.8494</td>
<td>-4.5699</td>
<td>-4.7316</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>-24.9048</td>
<td>-57.6178</td>
<td>-7.6033</td>
<td>-14.7393</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>-29.9065</td>
<td>-15.8908</td>
<td>-43.3797</td>
<td>-17.1278</td>
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<td>0</td>
<td>-1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>-20.6371</td>
<td>-15.9880</td>
<td>-8.9447</td>
<td>-60.6207</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td></td>
</tr>
</tbody>
</table>
E. Establishing Inertial Coherency

Now the techniques for determining inertial coherency that were developed in Chapter V will be applied to this four generator system. In the process, the system will be decoupled and the inertial modes will be separated from the exciter modes.

1. **Eigenvector analysis using the "A" matrix**

   The eigenvalues and eigenvectors for the "A" matrix in Table 6.29 are now found using a special computer library routine. Of the seven pairs of eigenvalues, three pertain to the inertial properties of the system. Table 5.30 shows the eigenvectors associated with these modes. The fact that these are the inertial modes can be seen from the magnitude of the elements of the eigenvectors associated with the angular speeds (ω). The 6.9353 r/s frequency predominates in the rotor speed of generators 2, 3, and 4; the 8.3835 r/s frequency predominates in the rotor speed of generator 2; and the 11.0851 r/s frequency predominates in the rotor speed of generator 3.

2. **Eigenvector analysis using the partitioned "A" matrix**

   The "A" matrix, in Table 6.29 is partitioned into four quadrants. The upper left hand quadrant contains the same terms as shown in Equation 5.16. By separating out this 8x8 matrix it will be possible to determine its eigenvalues and eigenvectors and compare them to their inertial counterparts from the "A" matrix. The upper left hand quadrant of the "A" matrix will henceforth be referred to as the
Table 6.30. Eigenvectors of the inertial modes, full A matrix, base case

<table>
<thead>
<tr>
<th>Variables</th>
<th>(\text{Eigenvalues (r/s)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\delta_1)</td>
<td>(-0.1306 \pm j0.9353) (-0.2180 \pm j8.3835) (-0.2469 \pm j11.0851)</td>
</tr>
<tr>
<td>(\delta_2)</td>
<td>(-0.0004 \pm j0.1134) (-0.0031 \pm j0.1192) (-0.0001 \pm j0.0115)</td>
</tr>
<tr>
<td>(\delta_3)</td>
<td>(-0.0016 \pm j0.0477) (-0.0001 \pm j0.0024) (-0.0020 \pm j0.0902)</td>
</tr>
<tr>
<td>(\delta_4)</td>
<td>(-0.0027 \pm j0.1444) (-0.0022 \pm j0.0959) (-0.0002 \pm j0.0138)</td>
</tr>
<tr>
<td>(\omega_1)</td>
<td>(-0.4306 \pm j0.0128) (-0.0352 \pm j0.0018) (-0.0471 \pm j0.0014)</td>
</tr>
<tr>
<td>(\omega_2)</td>
<td>(0.7863 \pm j0.0118) (1.0000 \pm j0) (-0.1270 \pm j0.014)</td>
</tr>
<tr>
<td>(\omega_3)</td>
<td>(0.3307 \pm j0.0172) (-0.0200 \pm j0.0001) (1.0000 \pm j0)</td>
</tr>
<tr>
<td>(\omega_4)</td>
<td>(1.0000 \pm j0) (-0.0001 \pm j0.0023) (-0.1533 \pm j0.0013)</td>
</tr>
<tr>
<td>(E'_{q1})</td>
<td>(0.0002 \pm j0.0006) (0.0001 \pm j0.0002) (0 \pm j0.0001)</td>
</tr>
<tr>
<td>(E'_{q2})</td>
<td>(0.0056 \pm j0.0067) (0.0056 \pm j0.0037) (-0.0007 \pm j0.0003)</td>
</tr>
<tr>
<td>(E'_{q3})</td>
<td>(0.0021 \pm j0.0044) (-0.0001 \pm j0.0003) (0.0035 \pm j0.0011)</td>
</tr>
<tr>
<td>(E'_{q4})</td>
<td>(0.0069 \pm j0.0087) (-0.0046 \pm j0.0024) (-0.0008 \pm j0.0003)</td>
</tr>
<tr>
<td>(E_{FD1})</td>
<td>(0.0389 \pm j0.0186) (0.0354 \pm j0.0032) (0.0019 \pm j0.0008)</td>
</tr>
<tr>
<td>(E_{FD2})</td>
<td>(0.2859 \pm j0.0958) (0.1964 \pm j0.0441) (-0.0212 \pm j0.0033)</td>
</tr>
<tr>
<td>(E_{FD3})</td>
<td>(0.1784 \pm j0.0651) (0.0124 \pm j0.0017) (0.0793 \pm j0.0152)</td>
</tr>
<tr>
<td>(E_{FD4})</td>
<td>(0.3668 \pm j0.1141) (-0.315 \pm j0.0320) (-0.0238 \pm j0.0038)</td>
</tr>
</tbody>
</table>
inertial matrix and its eigenvalues and eigenvectors for the base case are shown in Table 6.31.

Table 6.31. Eigenvectors of the inertial matrix, base case

<table>
<thead>
<tr>
<th>Variables</th>
<th>Eigenvalues (r/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$0 \pm j6.7757$</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>$0 \pm j0.0666$</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>$0 \pm j0.1130$</td>
</tr>
<tr>
<td>$\delta_3$</td>
<td>$0 \pm j0.0486$</td>
</tr>
<tr>
<td>$\delta_4$</td>
<td>$0 \pm j0.1476$</td>
</tr>
<tr>
<td>$\omega_1$</td>
<td>$-0.4511 \pm j0$</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>$0.7655 \pm j0$</td>
</tr>
<tr>
<td>$\omega_3$</td>
<td>$0.3293 \pm j0$</td>
</tr>
<tr>
<td>$\omega_4$</td>
<td>$1.0000 \pm j0$</td>
</tr>
</tbody>
</table>

3. Comparison of results

Comparing the modes in Table 6.31 with those in Table 6.30 it is apparent that both the frequencies and the mode shapes correspond very closely. This is important because it allows decoupling the inertial modes and permits use of the inertial matrix which has dimensions only half as large as the full "A" matrix.
F. Selection of Inertially Coherent Generators

Using the methods developed in Chapter V the $M_1$ matrix, shown in Table 6.19, has been used to establish inertial coherency. However, from Chapter V it is known that this 4x4 matrix is singular and can be reduced to a 3x3 matrix by selecting one generator as reference. This reduction results in no loss of information and provides the proper formulation for evaluating inertial coherency.

Selecting generator 1 as reference and applying the proper transformation from Chapter V results in the reduced $M_1$ matrix shown in Table 6.32.

Table 6.32. Reduced $M_1$ matrix with generator 1 as reference

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2.6491</td>
<td>0.4132</td>
<td>0.1690</td>
</tr>
<tr>
<td>3</td>
<td>0.4020</td>
<td>2.5919</td>
<td>0.2253</td>
</tr>
<tr>
<td>4</td>
<td>0.2930</td>
<td>0.3587</td>
<td>2.4255</td>
</tr>
</tbody>
</table>

From Equation 5.12 the approximate inertial frequency between machine 2 and 1 is given by

$$\omega_{21} = \sqrt{\left[2.6491 \left(\frac{377}{2}\right) \left(23.64 + 6.40\right)\right] \left(23.64 \cdot 6.40 \right)^{1/2}} = 9.9573 \text{ r/s}$$

In a similar fashion the rest of the direct and cross coupling frequencies can be calculated and are assembled in Table 6.33.
Table 6.33. Approximate inertial frequencies (r/s) with machine 1 as system reference

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>9.9573</td>
<td>6.1682</td>
<td>3.1552</td>
</tr>
<tr>
<td>3</td>
<td>6.0840</td>
<td>13.5272</td>
<td>4.5547</td>
</tr>
<tr>
<td>4</td>
<td>4.1545</td>
<td>5.7470</td>
<td>9.5278</td>
</tr>
</tbody>
</table>

From Table 6.33 it is apparent that machines 2 and 4 have frequencies of oscillation with respect to machine 1 that are nearly the same. Since $\omega_{21} = 9.9573$ r/s and $\omega_{41} = 9.5278$ r/s, we conclude that these two machines are strong candidates for combining into an inertial equivalent. We also note that the cross coupling frequencies between machines 2, 3, and 4 are much less than the frequencies with respect to machine 1 shown on the main diagonal. This fact plus the near symmetry of the matrix indicates that machine 1 is a good choice for system reference. This seems reasonable since machine 1 has much greater inertia than the other machines.

Going through the same calculations using machines 2, 3, and 4 as reference produces Tables 6.34 through 6.39.
Table 6.34. Reduced $M_1$ matrix with generator 2 as system reference

<table>
<thead>
<tr>
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<th>4</th>
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</thead>
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<tr>
<td>1</td>
<td>3.2313</td>
<td>-0.4132</td>
<td>-0.1690</td>
</tr>
<tr>
<td>3</td>
<td>0.0121</td>
<td>2.1787</td>
<td>0.0563</td>
</tr>
<tr>
<td>4</td>
<td>0.1541</td>
<td>-0.0545</td>
<td>2.2565</td>
</tr>
</tbody>
</table>

Table 6.35. Approximate inertial frequencies (r/s) with machine 2 as system reference

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>10.9972</td>
<td>5.4010</td>
<td>2.5150</td>
</tr>
<tr>
<td>3</td>
<td>0.9243</td>
<td>14.1637</td>
<td>2.2768</td>
</tr>
<tr>
<td>4</td>
<td>2.4016</td>
<td>2.2401</td>
<td>11.5292</td>
</tr>
</tbody>
</table>

Table 6.36. Reduced $M_1$ matrix with generator 3 as system reference

<table>
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<th>2</th>
<th>4</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>3.2192</td>
<td>-0.4020</td>
<td>-0.2235</td>
</tr>
<tr>
<td>2</td>
<td>-0.0121</td>
<td>2.2471</td>
<td>-0.0563</td>
</tr>
<tr>
<td>4</td>
<td>0.1420</td>
<td>-0.1090</td>
<td>2.2002</td>
</tr>
</tbody>
</table>
Table 6.37. Approximate inertial frequencies (r/s) with machine 3 as system reference

<table>
<thead>
<tr>
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<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15.0755</td>
<td>3.8789</td>
<td>2.9038</td>
</tr>
<tr>
<td>2</td>
<td>0.6730</td>
<td>14.3843</td>
<td>1.8211</td>
</tr>
<tr>
<td>4</td>
<td>2.3053</td>
<td>2.5339</td>
<td>14.2334</td>
</tr>
</tbody>
</table>

Table 6.38. Reduced $M_1$ matrix with generator 4 as system reference

<table>
<thead>
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<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.0772</td>
<td>-0.2930</td>
<td>-0.3587</td>
</tr>
<tr>
<td>2</td>
<td>-0.1541</td>
<td>2.3561</td>
<td>0.0545</td>
</tr>
<tr>
<td>3</td>
<td>-0.1420</td>
<td>0.1090</td>
<td>2.2332</td>
</tr>
</tbody>
</table>

Table 6.39. Approximate inertial frequencies (r/s) with machine 4 as system reference

<table>
<thead>
<tr>
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<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10.7317</td>
<td>3.3115</td>
<td>5.0323</td>
</tr>
<tr>
<td>2</td>
<td>2.4016</td>
<td>11.7809</td>
<td>2.2401</td>
</tr>
<tr>
<td>3</td>
<td>3.1662</td>
<td>3.1680</td>
<td>14.3397</td>
</tr>
</tbody>
</table>
Comparing Tables 6.33 and 6.37 it is clear that machines 2 and 4 oscillate with nearly identical frequencies against machines 1 and 3. No other combination of machines indicates such likely coherency. From Tables 6.33, 6.35, and 6.39 it is seen that machine 3 oscillates against all other machines with the highest inertial frequency. This is corroborated by examining the eigenvectors shown in Tables 6.30 and 6.31. Analysis of these eigenvectors also indicated coherency of machines 2 and 4. However, use of the Ml coefficients to establish inertial coherency is much more efficient in the amount of computer time needed than is eigenvector analysis. In the next chapter the extent of the coherency of machines 2 and 4 will be tested by combining them into one equivalent machine and examining the new system mode shapes.

G. Finding the Exciter Modes

In Part 2 of Section E the inertial modes were successfully decoupled by using a subset of the "A" matrix. In this section the same procedure is used to decouple the exciter modes.

1. Using the "A" matrix

By using a special computer library routine the eigenvalues and eigenvectors for the "A" matrix shown in Table 6.29 were obtained. Eigenvectors of the modes of oscillation associated with the exciters are shown in Table 6.40. That these are the exciter modes may be verified by observing that the largest elements in the eigenvectors occur
<table>
<thead>
<tr>
<th>Variables</th>
<th>-0.6306 ± j1.9811</th>
<th>-0.6387 ± j2.1938</th>
<th>-0.5814 ± j2.7226</th>
<th>-0.5326 ± j3.6070</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_1$</td>
<td>-0.0091 ± j0.0095</td>
<td>-0.0047 ± j0.0074</td>
<td>0.0033 ± j0.0281</td>
<td>-0.0265 ± j0.0894</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>-0.0116 ± j0.0273</td>
<td>0.0042 ± j0.0841</td>
<td>-0.0001 ± j0.0320</td>
<td>-0.0276 ± j0.0222</td>
</tr>
<tr>
<td>$\delta_3$</td>
<td>0.0065 ± j0.0873</td>
<td>-0.0058 ± j0.0137</td>
<td>-0.0007 ± j0.0160</td>
<td>-0.0240 ± j0.0184</td>
</tr>
<tr>
<td>$\delta_4$</td>
<td>-0.0134 ± j0.0359</td>
<td>-0.0111 ± j0.0768</td>
<td>-0.0030 ± j0.0607</td>
<td>-0.0273 ± j0.0158</td>
</tr>
<tr>
<td>$\omega_1$</td>
<td>0.0246 ± j0.0121</td>
<td>0.0195 ± j0.0055</td>
<td>-0.0783 ± j0.0074</td>
<td>0.3366 ± j0.0480</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>0.0614 ± j0.0057</td>
<td>-0.1873 ± j0.0446</td>
<td>0.0872 ± j0.0183</td>
<td>0.0946 ± j0.0879</td>
</tr>
<tr>
<td>$\omega_3$</td>
<td>-0.1780 ± j0.0426</td>
<td>0.0328 ± j0.0039</td>
<td>0.0441 ± j0.0073</td>
<td>0.0790 ± j0.0769</td>
</tr>
<tr>
<td>$\omega_4$</td>
<td>0.0796 ± j0.0038</td>
<td>0.1757 ± j0.0246</td>
<td>0.1670 ± j0.0272</td>
<td>0.0714 ± j0.0901</td>
</tr>
<tr>
<td>$E'_{q1}$</td>
<td>0.0005 ± j0.0032</td>
<td>0.0064 ± j0.0035</td>
<td>-0.0062 ± j0.0400</td>
<td>-0.0018 ± j0.0183</td>
</tr>
<tr>
<td>$E'_{q2}$</td>
<td>0.0029 ± j0.0158</td>
<td>-0.0166 ± j0.0751</td>
<td>0.0050 ± j0.0356</td>
<td>-0.0043 ± j0.0442</td>
</tr>
<tr>
<td>$E'_{q3}$</td>
<td>-0.0150 ± j0.0834</td>
<td>0.0012 ± j0.0065</td>
<td>0.0043 ± j0.0231</td>
<td>-0.0052 ± j0.0427</td>
</tr>
<tr>
<td>$E'_{q4}$</td>
<td>0.0047 ± j0.0234</td>
<td>0.0081 ± j0.0589</td>
<td>0.0086 ± j0.0576</td>
<td>-0.0050 ± j0.0460</td>
</tr>
<tr>
<td>$E_{FD1}$</td>
<td>-0.0591 ± j0.0040</td>
<td>-0.0715 ± j0.0058</td>
<td>1.0000 ± j0</td>
<td>0.5998 ± j0.0162</td>
</tr>
<tr>
<td>$E_{FD2}$</td>
<td>-0.1921 ± j0.0064</td>
<td>1.0000 ± j0</td>
<td>-0.5889 ± j0.0080</td>
<td>0.9609 ± j0.0078</td>
</tr>
<tr>
<td>$E_{FD3}$</td>
<td>1.0000 ± j0</td>
<td>-0.0870 ± j0.0001</td>
<td>-0.3792 ± j0.0140</td>
<td>0.9164 ± j0.0052</td>
</tr>
<tr>
<td>$E_{FD4}$</td>
<td>-0.2842 ± j0.0161</td>
<td>-0.7840 ± j0.0054</td>
<td>-0.9525 ± j0.0201</td>
<td>1.0000 ± j0</td>
</tr>
</tbody>
</table>
in the $E_{FD}$ rows. Small values in the $\delta$ and $\omega$ rows indicate only minor coupling between the inertial and exciter modes.

2. **Decoupling the "A" matrix**

In Equation 5.2 the "A" matrix is partitioned to allow decoupling of the inertial and exciter modes. The inertial matrix has already been tested and yielded quite a close approximation to the inertial modes. The lower right quadrant of the "A" matrix, shown in Table 6.29, was then examined to see if it could provide close approximations of the exciter modes shown in Table 6.40. This quadrant of the "A" matrix will henceforth be called the exciter matrix and its eigenvalues and eigenvectors are shown in Table 6.41.

3. **Analysis and comparison of results**

Comparing the results of Table 6.41 with Table 6.40 it is easy to identify corresponding modes. Though the fit is not as precise as for the inertial case, it retains sufficient accuracy for use in exciter modal analysis. We note that each machine has a separate identifiable exciter mode associated with it. Unlike the inertial case, there is no evident coherency among the exciters. Consequently, combining the exciters for the inertially coherent machines 2 and 4 must mean the elimination of one of these independent exciter modes. The means of accomplishing this is explored in Chapter VIII.
<table>
<thead>
<tr>
<th>Variables</th>
<th>Mode 1</th>
<th>Eigenvalues (r/s)</th>
<th>Mode 2</th>
<th>Eigenvalues (r/s)</th>
<th>Mode 3</th>
<th>Eigenvalues (r/s)</th>
<th>Mode 4</th>
<th>Eigenvalues (r/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E'_{q1}$</td>
<td>-0.0033 + j0.0052</td>
<td>-0.0054 + j0.0060</td>
<td>-0.0065 + j0.0085</td>
<td>-0.0028 + j0.0123</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E'_{q2}$</td>
<td>0.0051 + j0.0075</td>
<td>-0.0030 + j0.0062</td>
<td>0.0176 + j0.0183</td>
<td>-0.0036 + j0.0392</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E'_{q3}$</td>
<td>-0.0048 + j0.0093</td>
<td>-0.0045 + j0.0057</td>
<td>0.0168 + j0.0028</td>
<td>-0.0026 + j0.0391</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E'_{q4}$</td>
<td>0.0028 + j0.0163</td>
<td>0.0126 + j0.0479</td>
<td>-0.0063 + j0.0456</td>
<td>-0.0037 + j0.0431</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E'_{FD1}$</td>
<td>-0.0940 + j0.1043</td>
<td>-0.1107 + j0.1653</td>
<td>1.0000 + j0</td>
<td>0.4369 + j0.0337</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E'_{FD2}$</td>
<td>-0.1176 + j0.0854</td>
<td>1.0000 + j0</td>
<td>-0.3145 + j0.2727</td>
<td>0.9114 + j0.0058</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E'_{FD3}$</td>
<td>1.0000 + j0</td>
<td>-0.0924 + j0.0757</td>
<td>-0.0470 + j0.2541</td>
<td>0.8920 + j0.0011</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E'_{FD4}$</td>
<td>-0.2456 + j0.0466</td>
<td>-0.7855 + j0.1657</td>
<td>-0.7924 + j0.1338</td>
<td>1.0000 + j0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
VII. ELEVEN BUS TEST SYSTEM WITH A CONVENTIONAL GENERATOR EQUIVALENT

In Chapter VI it was established that machines 2 and 4 are inertially coherent. In most transient stability studies this is sufficient grounds to combine the two machines into an equivalent generator. However, this will mean the elimination of one of the four pairs of independent exciter modes calculated in Chapter VI. In this chapter a conventional generator equivalent is developed. The inertial and exciter modes to be eliminated, and their effect on the remaining modes, are determined.

A. Constructing a New Admittance Matrix

To maintain the steady state load-flow precisely as in the original system, it is necessary to replace the transformers for machines 2 and 4 (of the test system) with the approximating phase shifting pi equivalent shown schematically in Figure 7.1. The change in the system circuit diagram is shown in Figure 7.2. The phase shift is needed because buses 2 and 11 of the original system are connected together to form bus "A" which carries the average of the voltages and angles of buses 2 and 11. Equations 7.1-7.3 give the parameters of these phase shifting transformer equivalents. In Figure 7.1, note that node P represents the terminal of the equivalent generator.
Figure 7.1. Phase shifting transformer equivalent.
5. Figure 7.2. Phase shifting transformers: a) original system, b) phase shifting transformer equivalents with the equivalent generator
\[
\bar{Y}_{Y1} = \left[ \left( \frac{\bar{V}}{\bar{V}_p} \right) \left[ \frac{1}{a} \begin{pmatrix} 0 \ - \ \frac{1}{a} \ \\
1 \ - \ 0 \end{pmatrix} \right] + \left( \frac{1-a}{a^2} \right) \bar{Y}_{pq} \right] \tag{7.1}
\]

\[
\bar{Y}_{Z1} = \left[ \left( \frac{\bar{V}}{\bar{V}_q} \right) \left[ \frac{1}{a} \begin{pmatrix} 0 \ - \ \frac{1}{a} \ \\
1 \ - \ 0 \end{pmatrix} \right] + \left( \frac{1-a}{a^2} \right) \bar{Y}_{pq} \right] \tag{7.2}
\]

\[
\bar{Y}_X = \left( \frac{1}{a} \right) \bar{Y}_{pq} \tag{7.3}
\]

where \( a = \frac{\bar{V}_p}{\bar{V}_t} \) and \( \bar{Y}_{pq} \) is the original transformer admittance.

Applying these equations to the transformers of machines 2 and 4, the data in Table 7.1 are obtained.

**Table 7.1. Parameters of transformer equivalents for machines 2 and 4**

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{Y}_{Y1} )</td>
<td>-0.2772 + j0.0939</td>
<td>0.2953 - j0.0929</td>
</tr>
<tr>
<td>( \bar{Y}_{Z1} )</td>
<td>0.2300 - j0.0605</td>
<td>-0.2975 + j0.0693</td>
</tr>
<tr>
<td>( \bar{Y}_X )</td>
<td>0 - j15.9223</td>
<td>0 - j16.7476</td>
</tr>
</tbody>
</table>

It is now possible to form the admittance matrix for the resulting three machine system. This modified admittance matrix is shown in Table 7.2.
Table 7.2. Y matrix for the equivalent 3-machine system

<table>
<thead>
<tr>
<th>Node</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(1)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(j6.4725)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>'2'</td>
<td>0</td>
<td>(-j2.3135)</td>
<td>0</td>
<td>(j2.3135)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(-j10.7596)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(j0.7596)</td>
</tr>
<tr>
<td>4</td>
<td>(j6.4725)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(3.3074)</td>
<td>(-1.3652)</td>
<td>(-1.9422)</td>
<td>(-j28.5477)</td>
<td>(+j11.6041)</td>
<td>(j10.5107)</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(+j11.6041)</td>
<td>(-j18.0511)</td>
<td>0</td>
<td>(-1.1876)</td>
<td>0</td>
<td>(+j5.9751)</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(+j10.5107)</td>
<td>0</td>
<td>(-j16.3457)</td>
<td>0</td>
<td>(-1.2820)</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(j15.9223)</td>
<td>0</td>
<td>(-1.1876)</td>
<td>(+j5.9751)</td>
<td>0</td>
<td>(3.0907)</td>
<td>(-1.6171)</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(-1.6171)</td>
<td>0</td>
<td>(6.0588)</td>
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<tr>
<td>9</td>
<td>0</td>
<td>0</td>
<td>(j0.7596)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(-1.2820)</td>
<td>(+j5.5882)</td>
<td>0</td>
<td>(-1.1551)</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(j16.7476)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(-2.3102)</td>
<td>0</td>
<td>(2.0126)</td>
</tr>
</tbody>
</table>
The Y matrix can now be reduced by eliminating all but the internal generator nodes. The result is shown in Table 7.3.

Table 7.3. The reduced admittance matrix for the equivalent 3-machine system

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>A</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.2116 - j2.0591</td>
<td>0.2386 + j0.5282</td>
<td>0.1550 + j0.3738</td>
</tr>
<tr>
<td>A</td>
<td>0.2386 + j0.5282</td>
<td>0.0786 - j0.9060</td>
<td>0.0466 + j0.1068</td>
</tr>
<tr>
<td>3</td>
<td>0.1550 + j0.3738</td>
<td>0.0466 + j0.1068</td>
<td>0.0349 - j0.6547</td>
</tr>
</tbody>
</table>

B. Forming a Conventional Generator Equivalent

In this section, machines 2 and 4 will be combined into one conventional generator equivalent. The procedure to be applied is that used in reference 5. The machine will be examined in two parts; first the generator proper and then the exciter.

1. Combining the Generators

The equivalent for machines 2 and 4 will have an inertia constant H equal to the sum of the inertia constants of the two machines. The other new parameters for the equivalent will be found by paralleling $x'_q$, $x'_d$, and $x'_d$ from machines 2 and 4. The direct axis open circuit transient time constant $\tau'_{do}$ for the equivalent is found by taking the log average as shown in Equation 7.4

$$\text{Equivalent } \tau'_{do} = \text{Inv } \log \left[ \frac{1}{2} (\log \tau'_{do2} + \log \tau'_{do4}) \right] \quad (7.4)$$
Since machines 2 and 4 are identical, the equivalent will have parameters half as large as the original machines except for $H$ which will be doubled.

2. Combining the exciters

The accepted method today for combining exciters is the log average method. With the first order exciter models used here, the reasoning for this combining technique is demonstrated in Figure 7.3 and 7.4. In Figure 7.3 it can be seen that the assumption is made that the two exciters are operating in parallel and the intention is to replace them in such a way as to duplicate the response as closely as possible. The Bode plots in Figure 7.4 show that using the log average of the gains and time constants yields a frequency response that lies equidistant from both the original exciters. This tends to minimize the error in the frequency response and is the basis for the log average method. The equations for finding the log averages of the exciter gains and time constants are now shown for machines 2 and 4.

\[
K_{A_{EQ}} = \text{Inv} \log \left[ \frac{1}{2} \left( \log K_{A_2} + \log K_{A_4} \right) \right] \quad (7.5)
\]

\[
T_{A_{EQ}} = \text{Inv} \log \left[ \frac{1}{2} \left( \log T_{A_2} + \log T_{A_4} \right) \right] \quad (7.6)
\]

The case presently under consideration is the normal case in which machines 2 and 4 both have the same exciter. For this case the equivalent exciter is simply equal to the original exciters.
Figure 7.3. Exciter block diagram: a) original exciters for machines 2 and 4, b) equivalent exciter

Figure 7.4. Bode plots of separate exciters and their equivalent
However in Chapter VIII, cases with widely different exciters will be investigated.

C. Application of "Park's Transformation"

The Fortran program, shown in Appendix A, is now used to transform the machine steady state voltage and current to the corresponding d and q axis quantities. The pertinent results are tabulated in Table 7.4.

Table 7.4. Parameters of the equivalent 3-machine system, generator 1 is system reference

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Generator 1</th>
<th>Generator 2</th>
<th>Generator 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_q$</td>
<td>0.0969</td>
<td>0.4323</td>
<td>1.2578</td>
</tr>
<tr>
<td>$x_d$</td>
<td>0.1460</td>
<td>0.4479</td>
<td>1.3125</td>
</tr>
<tr>
<td>$x'_d$</td>
<td>0.0608</td>
<td>0.0599</td>
<td>0.1313</td>
</tr>
<tr>
<td>$V_{r1}$</td>
<td>1.0400</td>
<td>1.0300</td>
<td>1.0300</td>
</tr>
<tr>
<td>$V_{d0}$</td>
<td>-0.0574</td>
<td>-0.5306</td>
<td>-0.6642</td>
</tr>
<tr>
<td>$V_{q0}$</td>
<td>1.0384</td>
<td>0.8828</td>
<td>0.7872</td>
</tr>
<tr>
<td>$E_{eq0}$</td>
<td>1.1005</td>
<td>1.3849</td>
<td>1.6097</td>
</tr>
<tr>
<td>$\delta_0$</td>
<td>0.0°</td>
<td>35.1086°</td>
<td>43.5801°</td>
</tr>
<tr>
<td>$I_{q0}$</td>
<td>0.5925</td>
<td>1.2276</td>
<td>0.5281</td>
</tr>
<tr>
<td>$I_{a0}$</td>
<td>0.8724</td>
<td>1.6900</td>
<td>0.8405</td>
</tr>
</tbody>
</table>

*aThe equivalent machine is labelled as machine 2 in this table.
Before proceeding with the calculations leading to the "M" coefficients, it is important to perform the check used in Chapter VI. This gives confidence in the numerous calculations made up to this point. Using the procedure of Chapter VI the following quantities are calculated. It is to be noted that in subsequent calculations the equivalent machine is defined as machine 2.

\[
\begin{align*}
E_{\text{THEV}(1)} &= - \left[ \frac{12}{Y_{11}} E_{qa2} + \frac{13}{Y_{11}} E_{qa3} \right] = 0.7697 \angle 4.8872^\circ \text{ p.u.} \\
E_{\text{THEV}(2)} &= - \left[ \frac{21}{Y_{22}} E_{qa1} + \frac{23}{Y_{22}} E_{qa3} \right] = 0.8610 \angle -16.4603^\circ \text{ p.u.} \\
E_{\text{THEV}(3)} &= - \left[ \frac{31}{Y_{33}} E_{qa1} + \frac{32}{Y_{33}} E_{qa2} \right] = 0.9565 \angle -5.2615^\circ \text{ p.u.} \\
I_{a10} &= (E_{qa1} - E_{\text{THEV}(1)}) Y_{11} = 0.8730 \angle -44.0870^\circ \text{ p.u.} \\
I_{a20} &= (E_{qa2} - E_{\text{THEV}(2)}) Y_{22} = 1.6901 \angle -5.2007^\circ \text{ p.u.} \\
I_{a30} &= (E_{qa3} - E_{\text{THEV}(3)}) Y_{33} = 0.8404 \angle -4.3730^\circ \text{ p.u.}
\end{align*}
\]

These currents compare quite closely with those found in Table 7.4. With this satisfactory check, work can now proceed toward finding the "M" coefficients.
D. Calculation of the "M" Coefficients

Data for calculating the $K_1$ through $K_9$ matrices and the $R_1$ through $R_9$ matrices are given in Table 7.4. Applying the equations and the detailed procedure already employed in Chapter VI yields these matrices as shown in Table 7.5.

Using the matrices in Table 7.5 and the Equations 4.41 through 4.46 the "M" coefficients can be calculated. These are displayed in matrix form in Table 7.6.

E. Finding the "A" Matrix

Using the matrices shown in Tables 7.5 and 7.6 and Equation 5.16 the "A" matrix for the normal case of the reduced 3-machine system can be calculated. The result is shown in Table 7.7. Note that the equivalent machine is shown as machine 2.

F. Eigenvector Analysis of the Resulting Modes

Now that the "A" matrix has been calculated, two of its subsets, the inertial matrix and the exciter matrix can be examined. The eigenvalues and eigenvectors of these two matrices will now be determined and compared to their counterparts from the full four machine system. Only the normal case will be considered in this chapter but a variety of different exciters will be reviewed in Chapter VIII.
<table>
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<tr>
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<th>TA</th>
<th></th>
<th>KA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.8900</td>
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<td>0</td>
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<td>6.0000</td>
<td>0</td>
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<td>9.6000</td>
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<td>0</td>
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<td>1.0804</td>
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<td>0</td>
</tr>
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<td>0.3161</td>
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<td>0.4111</td>
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<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td>0.6110</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 7.5. Matrices R1-K2, R1-K3, TA, and KA.
Table 7.6. The M1-M6 matrices for the reduced 3-machine system

\[
\begin{array}{ccccccc}
M1 & M2 & M3 \\
2.3130 & -1.4872 & -0.8258 & 2.9993 & -0.5082 & -0.5233 & -1.2548 & 0.1665 & 0.0677 \\
-1.7198 & 2.5873 & -0.8675 & 0.5498 & 3.6248 & -0.7114 & 0.4393 & -2.5516 & 0.6104 \\
-0.9249 & -0.8476 & 1.7725 & 0.2773 & -0.5376 & 2.1120 & 0.4297 & 1.5949 & -4.0050 \\

M4 & M5 & M6 \\
0.0275 & -0.0185 & -0.0090 & -0.0085 & 0.0060 & 0.0025 & 0.8240 & 0.1164 & 0.0458 \\
-0.5987 & 0.9804 & -0.3817 & -0.0839 & 0.1007 & -0.0168 & 0.2298 & 0.7434 & 0.0818 \\
-1.0223 & -1.1170 & 2.1393 & -0.0819 & -0.0074 & 0.0893 & 0.3012 & 0.3285 & 0.4332 \\
\end{array}
\]
Table 7.7. The partitioned "A" matrix for the base case of the equivalent 3-machine system.  
The equivalent machine is shown as machine 2

<table>
<thead>
<tr>
<th>$\delta_1$</th>
<th>$\delta_2$</th>
<th>$\delta_3$</th>
<th>$\omega_1$</th>
<th>$\omega_2$</th>
<th>$\omega_3$</th>
<th>$E'_{q1}$</th>
<th>$E'_{q2}$</th>
<th>$E'_{q3}$</th>
<th>$E_{FD1}$</th>
<th>$E_{FD2}$</th>
<th>$E_{FD3}$</th>
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<td>0</td>
<td>0</td>
</tr>
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<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
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</tr>
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<td>0.0021</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>-0.1400</td>
<td>0.0186</td>
<td>0.0076</td>
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<td>0</td>
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<td>0.0998</td>
<td>-0.1634</td>
<td>0.0636</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.0732</td>
<td>-0.4253</td>
<td>0.1017</td>
<td>0</td>
<td>0.1667</td>
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</tr>
<tr>
<td>0.1736</td>
<td>0.1896</td>
<td>-0.3632</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.0730</td>
<td>0.2708</td>
<td>-0.6800</td>
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<td>0.1698</td>
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<td>0.8421</td>
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<td>0</td>
<td>0</td>
<td>-82.3968</td>
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<td>-32.8471</td>
<td>-43.3224</td>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
</tbody>
</table>
1. Inertial modes from the decoupled "A" matrix

Decoupling the "A" matrix along the partition lines shown in Table 7.7 provides both the inertial and exciter matrices as previously defined. Selecting a library computer routine and finding the eigenvalues and eigenvectors of the inertial matrix produces the results shown in Table 7.8. The equivalent is represented by machine 2.

Table 7.8. Eigenvectors of the inertial matrix for the equivalent 3-machine system

<table>
<thead>
<tr>
<th>Variables</th>
<th>Eigenvalues (r/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 ± j0.0749</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0 ± j0.1468</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>0 ± j0.0535</td>
</tr>
<tr>
<td>$\omega_1$</td>
<td>-0.3103 ± j0</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>1.0000 ± j0</td>
</tr>
<tr>
<td>$\omega_3$</td>
<td>0.3640 ± j0</td>
</tr>
</tbody>
</table>

When the values for the reduced case in Table 7.8 are compared to the original values in Table 6.31 it is apparent that machines 2 and 4 are indeed very coherent. The mode shapes match almost exactly and it is clear that very little information concerning the inertial response has been lost by combining the two machines. This provides corroboration and validity to the use of the Ml coefficients in establishing inertial coherency.
2. **Exciter modes from the decoupled "A" matrix**

The eigenvalues and eigenvectors of the exciter matrix, shown in the lower right quadrant of Table 7.7, are shown in Table 7.9. The equivalent is represented as machine 2.

Comparing the values in Table 7.9 to the values for the full 4-machine system shown in Table 6.41 reveal that the exciter modes have been altered considerably. An analysis of this case and other cases covering a wide range of exciters is contained in the next chapter.
Table 7.9. Eigenvectors of the exciter matrix for the equivalent 3-machine system, base case

<table>
<thead>
<tr>
<th>Variables</th>
<th>Eigenvalues (r/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.8675 + j2.4540</td>
</tr>
<tr>
<td>$E'_q1$</td>
<td>-0.0031 + j0.0050</td>
</tr>
<tr>
<td>$E'_q2$</td>
<td>0.0038 + j0.0120</td>
</tr>
<tr>
<td>$E'_q3$</td>
<td>-0.0038 + j0.0692</td>
</tr>
<tr>
<td>$E_{FD1}$</td>
<td>-0.0896 + j0.0982</td>
</tr>
<tr>
<td>$E_{FD2}$</td>
<td>-0.1836 + j0.0648</td>
</tr>
<tr>
<td>$E_{FD3}$</td>
<td>1.0000 + j0</td>
</tr>
</tbody>
</table>
VIII. REDUCTION AND MODAL ANALYSIS
OF THE EXCITER MATRIX

In Chapter VII it was seen that combining the two inertially coherent machines resulted in the elimination of one of two very similar modes. It had little effect on the remaining inertial modes. However, for the exciter modes, the reduction had a much more drastic outcome. None of the original exciter modes were similar and, after the reduction, the remaining mode shapes were considerably altered.

In this chapter a technique to determine the proper exciter mode to be eliminated is developed. This modal reduction process will then be examined for useful insights that can provide improvements in the reduction technique that is in common use today. Finally, the results will be tested in a number of different cases employing exciters with very different parameters.

A. Modes Present Before Reduction

A listing of the different cases to be studied is shown in Table 8.1. Note that the exciter gains and time constants for the coherent machines 2 and 4 are the only parameters that are varied in this table. A tabulation of the eigenvalues for each case, derived from the exciter matrix of the full four machine system, is shown in Table 8.2. As a sample, the eigenvectors for case 10 are shown in Appendix C.
Table 8.1. Exenter values to be examined

<table>
<thead>
<tr>
<th>Case</th>
<th>$K_{A_1}$</th>
<th>$T_{A_1}$</th>
<th>$K_{A_2}$</th>
<th>$T_{A_2}$</th>
<th>$K_{A_3}$</th>
<th>$T_{A_3}$</th>
<th>$K_{A_4}$</th>
<th>$T_{A_4}$</th>
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</thead>
<tbody>
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<td>100</td>
<td>1.0</td>
<td>100</td>
<td>1.0</td>
<td>100</td>
<td>1.0</td>
</tr>
<tr>
<td>1</td>
<td>100</td>
<td>1.0</td>
<td>200</td>
<td>1.0</td>
<td>100</td>
<td>1.0</td>
<td>100</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
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<td>50</td>
<td>1.0</td>
<td>100</td>
<td>1.0</td>
<td>100</td>
<td>1.0</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
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<td>100</td>
<td>1.2</td>
<td>100</td>
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<td>100</td>
<td>1.0</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
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<td>100</td>
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<td>100</td>
<td>1.0</td>
<td>100</td>
<td>1.0</td>
</tr>
<tr>
<td>5</td>
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<td>1.0</td>
<td>100</td>
<td>1.0</td>
<td>100</td>
<td>1.0</td>
<td>200</td>
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<td>1.0</td>
<td>100</td>
<td>1.0</td>
<td>50</td>
<td>1.0</td>
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<td>100</td>
<td>1.0</td>
<td>100</td>
<td>1.0</td>
<td>100</td>
<td>1.2</td>
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<td>1.0</td>
<td>100</td>
<td>1.0</td>
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<td>0.5</td>
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<td>9</td>
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<td>0.15</td>
<td>100</td>
<td>1.0</td>
<td>100</td>
<td>2.15</td>
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<tr>
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<td>100</td>
<td>2.15</td>
<td>100</td>
<td>1.0</td>
<td>100</td>
<td>0.15</td>
</tr>
<tr>
<td>11</td>
<td>100</td>
<td>1.0</td>
<td>20</td>
<td>1.0</td>
<td>100</td>
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<td>200</td>
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</tr>
<tr>
<td>12</td>
<td>100</td>
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<td>1.0</td>
<td>100</td>
<td>1.0</td>
<td>20</td>
<td>1.0</td>
</tr>
<tr>
<td>13</td>
<td>100</td>
<td>1.0</td>
<td>20</td>
<td>2.15</td>
<td>100</td>
<td>1.0</td>
<td>200</td>
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<td>14</td>
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<td>0.15</td>
<td>100</td>
<td>1.0</td>
<td>20</td>
<td>2.15</td>
</tr>
<tr>
<td>Case</td>
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<td>Machine 2</td>
<td>Machine 1</td>
<td>Machine 4</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Base</td>
<td>-0.8664 + j2.4512</td>
<td>-0.8454 + j2.6833</td>
<td>-0.6311 + j2.8221</td>
<td>-0.6357 + j3.8413</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>1</td>
<td>-0.8597 + j2.4555</td>
<td>-0.7860 + j3.0850</td>
<td>-0.6374 + j2.8560</td>
<td>-0.6959 + j4.6698</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-0.8521 + j2.4597</td>
<td>-0.8423 + j2.0005</td>
<td>-0.6488 + j2.8336</td>
<td>-0.6353 + j3.6111</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-0.8623 + j2.4344</td>
<td>-0.7756 + j2.5313</td>
<td>-0.6444 + j2.8224</td>
<td>-0.6131 + j3.7483</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-0.8601 + j2.4566</td>
<td>-0.8967 + j3.1374</td>
<td>-0.6369 + j2.8481</td>
<td>-1.0850 + j4.5716</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>-0.8597 + j2.4775</td>
<td>-0.7096 + j2.7391</td>
<td>-0.7216 + j3.1425</td>
<td>-0.6878 + j4.7417</td>
<td></td>
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</tr>
<tr>
<td>6</td>
<td>-0.8527 + j2.5109</td>
<td>-0.8448 + j2.0382</td>
<td>-0.6448 + j2.7330</td>
<td>-0.6364 + j3.5975</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>-0.8486 + j2.4153</td>
<td>-0.8296 + j2.5960</td>
<td>-0.6051 + j2.7781</td>
<td>-0.6121 + j3.7400</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>-0.8637 + j2.4794</td>
<td>-0.8189 + j3.1671</td>
<td>-0.7010 + j2.7501</td>
<td>-1.0952 + j4.6508</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>-0.5963 + j2.0244</td>
<td>-0.7430 + j2.5485</td>
<td>-0.6653 + j3.0747</td>
<td>-3.5400 + j7.4283</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>-0.5870 + j1.9682</td>
<td>-0.7893 + j2.5194</td>
<td>-0.6416 + j3.1132</td>
<td>-3.5267 + j7.6387</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>-0.8277 + j1.2872</td>
<td>-0.8144 + j2.5028</td>
<td>-0.6253 + j3.0173</td>
<td>-0.7114 + j4.6154</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>-0.8153 + j1.3209</td>
<td>-0.7977 + j2.5339</td>
<td>-0.6408 + j2.9579</td>
<td>-0.7249 + j4.5281</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>-0.5587 + j0.8872</td>
<td>-0.8030 + j2.5169</td>
<td>-0.6232 + j3.0697</td>
<td>-3.5596 + j11.2116</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>-0.5437 + j0.9166</td>
<td>-0.7875 + j2.5345</td>
<td>-0.6355 + j3.0400</td>
<td>-3.5779 + j10.9189</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
It is interesting to note that, even with the wide changes in the exciter gains and time constants, the eigenvalues are still quite comparable and easily identifiable. As shown in Table 8.2, each of the eigenvalues is predominantly associated with one machine and that association continues over the wide range of cases. It is interesting to note that wide differences in the time constants $T_{A_2}$ and $T_{A_4}$ tends to raise the highest exciter frequency.

B. Modal Reduction of the 4-Machine Exciter Matrix

The exciter matrix is placed in equation form as shown in Equation 8.1. For modal reduction purposes the exciter matrix is reduced from an 8x8 to a 6x6 matrix by combining the elements pertaining to machines 2 and 4. This procedure begins with Equation 8.2 where two columns in the exciter matrix have been eliminated. This was done by defining

$$
\begin{align*}
\hat{P}_{q24} & \triangleq \hat{P}_{q2} - \hat{P}_{q4}, & \hat{P}_{q26} & \triangleq \hat{P}_{q2} + \hat{P}_{q4}, \\
\hat{P}_{FD24} & \triangleq \hat{P}_{FD2} - \hat{P}_{FD4}, & \hat{P}_{FD26} & \triangleq \hat{P}_{FD2} + \hat{P}_{FD4}.
\end{align*}
$$

This constrains the field flux linkages of the $d$ axes of the coherent machines to be equal. With this assumption it is possible to combine like terms within the matrix and eliminate two columns.

The next step is to add the two rows containing $\hat{P}_{q24}$ and combine like terms. The two rows containing $\hat{P}_{FD24}$ are added in the same manner. The desired 6x6 matrix, shown in Equation 8.3, is thus obtained. We note that this procedure, as evidenced by the $1/2$
<table>
<thead>
<tr>
<th>$E_{q1}$</th>
<th>$E_{q2}$</th>
<th>$E_{q3}$</th>
<th>$E_{q4}$</th>
<th>$E_{FD1}$</th>
<th>$E_{FD2}$</th>
<th>$E_{FD3}$</th>
<th>$E_{FD4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T^{-1}M_{11}$</td>
<td>$T^{-1}M_{12}$</td>
<td>$T^{-1}M_{13}$</td>
<td>$T^{-1}M_{14}$</td>
<td>$T^{-1}M_{15}$</td>
<td>$T^{-1}M_{16}$</td>
<td>$T^{-1}M_{17}$</td>
<td>$T^{-1}M_{18}$</td>
</tr>
<tr>
<td>$T^{-1}M_{21}$</td>
<td>$T^{-1}M_{22}$</td>
<td>$T^{-1}M_{23}$</td>
<td>$T^{-1}M_{24}$</td>
<td>$T^{-1}M_{25}$</td>
<td>$T^{-1}M_{26}$</td>
<td>$T^{-1}M_{27}$</td>
<td>$T^{-1}M_{28}$</td>
</tr>
<tr>
<td>$T^{-1}M_{31}$</td>
<td>$T^{-1}M_{32}$</td>
<td>$T^{-1}M_{33}$</td>
<td>$T^{-1}M_{34}$</td>
<td>$T^{-1}M_{35}$</td>
<td>$T^{-1}M_{36}$</td>
<td>$T^{-1}M_{37}$</td>
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</tr>
<tr>
<td>$T^{-1}M_{41}$</td>
<td>$T^{-1}M_{42}$</td>
<td>$T^{-1}M_{43}$</td>
<td>$T^{-1}M_{44}$</td>
<td>$T^{-1}M_{45}$</td>
<td>$T^{-1}M_{46}$</td>
<td>$T^{-1}M_{47}$</td>
<td>$T^{-1}M_{48}$</td>
</tr>
</tbody>
</table>

(8.1)
\[
\begin{align*}
\dot{E}_{q1}^i & = T_{d1}^{-1} M_{31}^{-1} + T_{d1}^{-1} (M_{2,12}^{-1} + M_{3,14}^{-1}) + T_{d1}^{-1} M_{313} + T_{d1}^{-1} M_{313} + T_{d1}^{-1} 0 + 0 & E_{q1}^i \\
\dot{E}_{q24}^i & = T_{d2}^{-1} M_{321}^{-1} + T_{d2}^{-1} (M_{2,22}^{-1} + M_{3,24}^{-1}) + T_{d2}^{-1} M_{323} + 0 + T_{d2}^{-1} 0 & E_{q24}^i \\
\dot{E}_{q3}^i & = T_{d3}^{-1} M_{331}^{-1} + T_{d3}^{-1} (M_{2,32}^{-1} + M_{3,34}^{-1}) + T_{d3}^{-1} M_{333} + 0 + 0 + T_{d3}^{-1} 0 & E_{q3}^i \\
\dot{E}_{q24}^i & = T_{d4}^{-1} M_{341}^{-1} + T_{d4}^{-1} (M_{2,42}^{-1} + M_{3,44}^{-1}) + T_{d4}^{-1} M_{343} + 0 + T_{d4}^{-1} 0 & E_{FD1}^i \\
\dot{E}_{FD1}^i & = -K_{A_1} T_{A_1}^{-1} M_{611}^{-1} - K_{A_1} T_{A_2}^{-1} (M_{6,12}^{-1} + M_{6,14}^{-1}) - K_{A_1} T_{A_1}^{-1} M_{613}^{-1} T_{A_1}^{-1} 0 + 0 & E_{FD1}^i \\
\dot{E}_{FD24}^i & = -K_{A_2} T_{A_2}^{-1} M_{621}^{-1} - K_{A_2} T_{A_2}^{-1} (M_{6,22}^{-1} + M_{6,24}^{-1}) - K_{A_2} T_{A_2}^{-1} M_{623}^{-1} 0 + T_{A_2}^{-1} 0 & E_{FD24}^i \\
\dot{E}_{FD3}^i & = -K_{A_3} T_{A_3}^{-1} M_{631}^{-1} - K_{A_3} T_{A_3}^{-1} (M_{6,32}^{-1} + M_{6,34}^{-1}) - K_{A_3} T_{A_3}^{-1} M_{633}^{-1} 0 + 0 + T_{A_3}^{-1} 0 & E_{FD3}^i \\
\dot{E}_{FD24}^i & = -K_{A_4} T_{A_4}^{-1} M_{641}^{-1} - K_{A_4} T_{A_4}^{-1} (M_{6,42}^{-1} + M_{6,44}^{-1}) - K_{A_4} T_{A_4}^{-1} M_{643}^{-1} 0 + T_{A_4}^{-1} 0 & E_{FD24}^i
\end{align*}
\]

(8.2)
\[
\begin{align*}
\dot{E}^{\prime}_{q_1} &= \begin{bmatrix}
T_{d_1}^{-1}M_{3,11} & T_{c_1}^{-1}(M_{3,12} + M_{3,14}) & T_{d_3}^{-1}M_{3,31} & T_{d_1}^{-1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
T_{d_2}^{-1}M_{3,21} + T_{d_4}^{-1}M_{3,41} & (\frac{1}{2})T_{c_2}^{-1}(M_{3,22} + M_{3,24}) & (\frac{1}{2})T_{c_4}^{-1}(M_{3,22} + M_{3,24}) & 0 & (\frac{1}{2})T_{d_2}^{-1} & 0 \\
T_{d_3}^{-1}M_{3,31} & T_{c_3}^{-1}(M_{3,32} + M_{3,34}) & T_{d_3}^{-1}M_{3,33} & 0 & 0 & 0 \\
-\Lambda_1^{-1}T_{1}^{-1}M_{6,11} & -\Lambda_1^{-1}(M_{6,12} + M_{6,14}) & -\Lambda_1^{-1}T_{1}^{-1}M_{6,13} & -\Lambda_1^{-1} & 0 & 0 \\
(-\frac{1}{2})K_{d_2}T_{d_2}^{-1}M_{6,21} & (-\frac{1}{2})K_{c_2}T_{c_2}^{-1}(M_{6,22} + M_{6,24}) & (-\frac{1}{2})K_{c_4}T_{c_4}^{-1}(M_{6,22} + M_{6,24}) & 0 & (-\frac{1}{2})T_{a_2}^{-1} & 0 \\
+\Lambda_4^{-1}M_{6,41} & +\Lambda_4^{-1}M_{6,44} & +\Lambda_4^{-1}M_{6,44} & 0 & 0 & 0 \\
-\Lambda_3^{-1}T_{d_3}^{-1}M_{6,31} & -\Lambda_3^{-1}T_{d_3}^{-1}(M_{6,32} + M_{6,34}) & -\Lambda_3^{-1}T_{d_3}^{-1}M_{6,33} & 0 & 0 & -\Lambda_3^{-1} \\
\end{bmatrix}
\end{align*}
\]

(8.3)
multipliers shown in Equation 8.3, has essentially performed an arithmetic average of the matrix terms associated with the coherent machines 2 and 4.

1. Arithmetic averaging of the exciter matrix terms

Using the equation for the exciter matrix shown in Equation 8.3, the eigenvalues and eigenvectors for each case have been calculated. The eigenvalues are shown in Table 8.3 and, after examination of the eigenvectors, have been identified with the corresponding mode from the unreduced system.

Upon comparing this table with Table 8.2 it is apparent that the exciter mode associated with machine 2 has been eliminated. Examination of the reduced eigenvectors (see sample in Appendix C) shows very little change in the retained mode shapes. The mode shape for the eliminated mode, mode 2, shown in Table 6.41, indicates that this mode is predominant only in the coherent machines 2 and 4. However, the other mode that is considered as a possible candidate for elimination, mode 4, is also strongly associated with machines 1 and 3. Consequently, the proper mode was eliminated and the reduced model should provide a close approximation to the original.

This procedure, therefore, has accomplished the desired result. Based on the constraints of equal d axis flux linkage and equal field voltages, a particular mode has been eliminated and the remaining modes have not been significantly altered. The precise nature of this mode elimination is the reason for terming this procedure a "modal
Table 8.3. Eigenvalues (r/s) from the modally reduced exciter matrix with arithmetic averaging of the matrix terms

<table>
<thead>
<tr>
<th>Case</th>
<th>Machine 3</th>
<th>Machine 1</th>
<th>Machine equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>-0.8680 + j2.4536</td>
<td>-0.6187 + j2.8084</td>
<td>-0.6360 + j3.8416</td>
</tr>
<tr>
<td>1</td>
<td>-0.8457 + j2.4918</td>
<td>-0.6178 + j2.8919</td>
<td>-0.6592 + j4.4849</td>
</tr>
<tr>
<td>2</td>
<td>-0.8807 + j2.3859</td>
<td>-0.6232 + j2.7186</td>
<td>-0.6188 + j3.5457</td>
</tr>
<tr>
<td>3</td>
<td>-0.8667 + j2.4367</td>
<td>-0.6092 + j2.7846</td>
<td>-0.6051 + j3.7411</td>
</tr>
<tr>
<td>4</td>
<td>-0.8545 + j2.5017</td>
<td>-0.6659 + j2.8909</td>
<td>-0.8722 + j4.4523</td>
</tr>
<tr>
<td>5</td>
<td>-0.8453 + j2.4843</td>
<td>-0.6168 + j2.9062</td>
<td>-0.6606 + j4.5192</td>
</tr>
<tr>
<td>6</td>
<td>-0.8831 + j2.3894</td>
<td>-0.6528 + j2.6963</td>
<td>-0.6168 + j3.5356</td>
</tr>
<tr>
<td>7</td>
<td>-0.8674 + j2.4384</td>
<td>-0.6090 + j2.7789</td>
<td>-0.6046 + j3.7365</td>
</tr>
<tr>
<td>8</td>
<td>-0.6545 + j2.4933</td>
<td>-0.6726 + j2.9056</td>
<td>-0.8755 + j4.4873</td>
</tr>
<tr>
<td>9</td>
<td>-0.8289 + j2.5281</td>
<td>-0.6542 + j2.9625</td>
<td>-1.9225 + j6.4553</td>
</tr>
<tr>
<td>10</td>
<td>-0.8349 + j2.5103</td>
<td>-0.6637 + j2.9842</td>
<td>-1.9271 + j6.6214</td>
</tr>
<tr>
<td>11</td>
<td>-0.8600 + j2.4572</td>
<td>-0.6178 + j2.8572</td>
<td>-0.6449 + j4.0026</td>
</tr>
<tr>
<td>12</td>
<td>-0.8666 + j2.4754</td>
<td>-0.6183 + j2.8114</td>
<td>-0.6397 + j3.9439</td>
</tr>
<tr>
<td>13</td>
<td>-0.8262 + j2.5057</td>
<td>-0.6752 + j2.9981</td>
<td>-1.9542 + j9.1747</td>
</tr>
<tr>
<td>14</td>
<td>-0.8210 + j2.5236</td>
<td>-0.6320 + j2.9762</td>
<td>-1.9526 + j8.9172</td>
</tr>
</tbody>
</table>
reduction".

The modal reduction procedure will now be used to develop insights that might provide a way to improve the existing conventional technique of combining exciters. Equation 8.3 indicates that arithmetic averaging of the exciter gains and of the reciprocals of the exciter time constants may yield a better result than the logarithmic averaging currently used in the conventional reduction. That indication will be tested throughout the rest of this chapter.

2. Logarithmic averaging used in the modal reduction

In this approach the exciter gains and time constants for machines 2 and 4 are replaced with their logarithmic averages. The same modal reduction, as shown in Equation 8.3, is then performed. The eigenvalues of the resulting reduced exciter matrix are shown in Table 8.4.

3. Comparison

The original modal frequencies shown in Table 8.2 will now be compared to Table 8.3 for the normal modal reduction and to Table 8.4 for the modal reduction using logarithmic averages. Careful scrutiny of these eigenvalues and their associated eigenvectors show little difference in the first eight cases. In these cases only one parameter is varied, and there is little difference between the arithmetic and logarithmic averages.

However, in the last six cases, the exciter gains and time constants
<table>
<thead>
<tr>
<th>Case</th>
<th>Machine 3</th>
<th>Eigenvalues</th>
<th>Machine equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>-0.8680 + j2.4536</td>
<td>-0.6187 + j2.8084</td>
<td>-0.6360 + j3.8416</td>
</tr>
<tr>
<td>1</td>
<td>-0.8481 + j2.4851</td>
<td>-0.6176 + j2.8896</td>
<td>-0.6570 + j4.3903</td>
</tr>
<tr>
<td>2</td>
<td>-0.8816 + j2.3667</td>
<td>-0.6292 + j2.6853</td>
<td>-0.6149 + j3.4962</td>
</tr>
<tr>
<td>3</td>
<td>-0.8669 + j2.4367</td>
<td>-0.6086 + j2.7804</td>
<td>-0.6035 + j3.7342</td>
</tr>
<tr>
<td>4</td>
<td>-0.8569 + j2.4935</td>
<td>-0.6424 + j2.8885</td>
<td>-0.8305 + j4.3638</td>
</tr>
<tr>
<td>5</td>
<td>-0.8481 + j2.4851</td>
<td>-0.6176 + j2.8896</td>
<td>-0.6570 + j4.3903</td>
</tr>
<tr>
<td>6</td>
<td>-0.8816 + j2.3667</td>
<td>-0.6292 + j2.6853</td>
<td>-0.6149 + j3.4962</td>
</tr>
<tr>
<td>7</td>
<td>-0.8669 + j2.4367</td>
<td>-0.6086 + j2.7804</td>
<td>-0.6035 + j3.7342</td>
</tr>
<tr>
<td>8</td>
<td>-0.8569 + j2.4935</td>
<td>-0.6424 + j2.8885</td>
<td>-0.8305 + j4.3638</td>
</tr>
<tr>
<td>9</td>
<td>-0.8483 + j2.5052</td>
<td>-0.6475 + j2.9212</td>
<td>-1.0073 + j4.7818</td>
</tr>
<tr>
<td>10</td>
<td>-0.8483 + j2.5052</td>
<td>-0.6475 + j2.9212</td>
<td>-1.0073 + j4.7818</td>
</tr>
<tr>
<td>11</td>
<td>-0.8761 + j2.3186</td>
<td>-0.6361 + j2.6446</td>
<td>-0.6105 + j3.4258</td>
</tr>
<tr>
<td>12</td>
<td>-0.8761 + j2.3186</td>
<td>-0.6361 + j2.6446</td>
<td>-0.6105 + j3.4258</td>
</tr>
<tr>
<td>13</td>
<td>-0.8935 + j2.4969</td>
<td>-0.6904 + j2.8364</td>
<td>-0.9192 + j3.9149</td>
</tr>
<tr>
<td>14</td>
<td>-0.8935 + j2.4969</td>
<td>-0.6904 + j2.8364</td>
<td>-0.9192 + j3.9149</td>
</tr>
</tbody>
</table>
for the coherent machines are varied over a wider range. This produces a substantial difference between the results obtained with the arithmetic and logarithmic averaging of the exciter constants. The results clearly favor the original matrix reduction involving the arithmetic averaging of the matrix terms. This is especially evident in the cases involving a wide range of exciter time constants. In that instance a heavily damped mode appears at the upper end of the exciter frequency range. The constraints of the modal reduction indicates that this mode should be retained in the reduced model. It is clear therefore, that the arithmetic averaging method approximates both the damping and the frequency of this mode to a much greater degree than the logarithmic averaging method. While the retention of a heavily damped higher frequency mode would seem to have little effect on a time solution of the system, it is important not to replace that mode with a lighter damped lower frequency mode that was not present in the solution of the original system.

C. Conventional Reduction

Although the modal reduction technique is useful for gaining insight into the effects of reduction, it is difficult to apply directly to transient stability studies. The most common type of reduction performed is the conventional reduction demonstrated in Chapter VII. In this section the results of the modal reduction are used to provide insights which may be useful in performing the conventional reduction. These results have demonstrated the superiority of
arithmetic averaging to logarithmic averaging in approximating the
dynamic performance of the original system. This is especially true
in cases involving a wide difference in exciter time constants. This
idea will be tested on the reduced conventional model developed in
Chapter VII.

1. Arithmetic averaging

The $M_3$ and $M_6$ matrices are obtained from the conventional reduction
performed in Chapter VII. Then, using the arithmetic averages for the
exciter constants of machine 2 and machine 4, the exciter matrix is
formed according to Equation 5.16. The eigenvalues for the cases under
consideration are shown in Table 8.5. A sample of the mode shapes is
contained in Appendix C.

As noted in Chapter VII, the conventional reduction has a much more
drastic effect of the modal frequencies and mode shapes than the modal
reduction. The order of magnitude of the frequencies associated with
machines 1 and 2 has been reversed as shown by examining the mode
shapes in Tables 6.41 and 7.9. Further distortion has occurred since
all of the modes of the conventionally reduced model are predominantly
associated with machine 1, while in the original system only two of
the modes are strongly connected with machine 1. On the other hand,
while the modes from the original system have been distorted by this
reduction, it is still apparent that the same mode has been eliminated
as in the modal reduction.

It is also observed in Table 8.5 that, for the cases involving
Table 8.5. Eigenvalues (r/s) from the conventionally reduced exciter matrix with arithmetic averaging

<table>
<thead>
<tr>
<th>Case</th>
<th>Machine 1</th>
<th>Eigenvalues</th>
<th>Machine equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>-0.6196 + j2.8061</td>
<td>-0.8675 + j2.4540</td>
<td>-0.6356 + j3.8387</td>
</tr>
<tr>
<td>1</td>
<td>-0.6184 + j2.8965</td>
<td>-0.8445 + j2.4874</td>
<td>-0.6597 + j4.4977</td>
</tr>
<tr>
<td>2</td>
<td>-0.6235 + j2.7019</td>
<td>-0.8818 + j2.3884</td>
<td>-0.6173 + j3.5391</td>
</tr>
<tr>
<td>3</td>
<td>-0.6097 + j2.7771</td>
<td>-0.8668 + j2.4380</td>
<td>-0.6045 + j3.7363</td>
</tr>
<tr>
<td>4</td>
<td>-0.6459 + j2.8957</td>
<td>-0.8532 + j2.4969</td>
<td>-0.8735 + j4.4651</td>
</tr>
<tr>
<td>5</td>
<td>-0.6184 + j2.8965</td>
<td>-0.8445 + j2.4874</td>
<td>-0.6597 + j4.4977</td>
</tr>
<tr>
<td>6</td>
<td>-0.6235 + j2.7019</td>
<td>-0.8818 + j2.3884</td>
<td>-0.6173 + j3.5391</td>
</tr>
<tr>
<td>7</td>
<td>-0.5097 + j2.7771</td>
<td>-0.8668 + j2.4380</td>
<td>-0.6045 + j3.7363</td>
</tr>
<tr>
<td>8</td>
<td>-0.5459 + j2.8957</td>
<td>-0.8532 + j2.4969</td>
<td>-0.8735 + j4.4651</td>
</tr>
<tr>
<td>9</td>
<td>-0.5500 + j2.9725</td>
<td>-0.8307 + j2.5172</td>
<td>-1.9251 + j6.5317</td>
</tr>
<tr>
<td>10</td>
<td>-0.5500 + j2.9725</td>
<td>-0.8307 + j2.5172</td>
<td>-1.9251 + j6.5317</td>
</tr>
<tr>
<td>11</td>
<td>-0.6195 + j2.8315</td>
<td>-0.8612 + j2.4659</td>
<td>-0.6420 + j3.9697</td>
</tr>
<tr>
<td>12</td>
<td>-0.6195 + j2.8315</td>
<td>-0.8612 + j2.4559</td>
<td>-0.6420 + j3.9697</td>
</tr>
<tr>
<td>13</td>
<td>-0.6455 + j2.9751</td>
<td>-0.8290 + j2.5161</td>
<td>-1.9312 + j6.8620</td>
</tr>
<tr>
<td>14</td>
<td>-0.6455 + j2.9751</td>
<td>-0.8290 + j2.5161</td>
<td>-1.9312 + j6.8620</td>
</tr>
</tbody>
</table>
widely different exciter time constants, the same heavily damped higher frequency mode appears. Therefore, the conventional reduction has retained the proper mode although with much more distortion in all mode shapes than occurred in the modal reduction.

2. Logarithmic averaging

The exciter matrix of the conventional reduction is now formed with the logarithmic average of the exciter gains and time constants. The eigenvalues for all cases are shown in Table 8.6 and a sample of the eigenvectors are shown in Appendix C. As with the arithmetic averaging, the frequencies are shifted and the mode shapes altered. The proper mode is retained but with much less damping and a much lower frequency than occurred in the original system. This mode also shows much lower damping and frequency than occurred in the reduction using arithmetic averaging.
<table>
<thead>
<tr>
<th>Case</th>
<th>Eigenvalues Machine 1</th>
<th>Eigenvalues Machine 3</th>
<th>Machine equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>-0.6196 + j2.8041</td>
<td>-0.8675 + j2.4540</td>
<td>-0.6356 + j3.8387</td>
</tr>
<tr>
<td>1</td>
<td>-0.6187 + j2.8867</td>
<td>-0.8472 + j2.4847</td>
<td>-0.6568 + j4.3860</td>
</tr>
<tr>
<td>2</td>
<td>-0.6265 + j2.6794</td>
<td>-0.8817 + j2.3677</td>
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</tr>
<tr>
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<tr>
<td>4</td>
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<td>-0.8557 + j2.4932</td>
<td>-0.8301 + j4.3595</td>
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<tr>
<td>5</td>
<td>-0.6187 + j2.8867</td>
<td>-0.8472 + j2.4847</td>
<td>-0.6568 + j4.3860</td>
</tr>
<tr>
<td>6</td>
<td>-0.6265 + j2.6794</td>
<td>-0.8817 + j2.3677</td>
<td>-0.6144 + j3.4948</td>
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<tr>
<td>7</td>
<td>-0.6092 + j2.7757</td>
<td>-0.8667 + j2.4372</td>
<td>-0.6032 + j3.7318</td>
</tr>
<tr>
<td>8</td>
<td>-0.6440 + j2.8856</td>
<td>-0.8557 + j2.4932</td>
<td>-0.8301 + j4.3595</td>
</tr>
<tr>
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<td>-1.0070 + j4.7767</td>
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<tr>
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<td>-0.8762 + j2.3196</td>
<td>-0.6101 + j3.4248</td>
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<tr>
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<td>-0.8762 + j2.3196</td>
<td>-0.6101 + j3.4248</td>
</tr>
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<td>-0.9182 + j3.9112</td>
</tr>
<tr>
<td>14</td>
<td>-0.6931 + j2.8323</td>
<td>-0.8918 + j2.4975</td>
<td>-0.9182 + j3.9112</td>
</tr>
</tbody>
</table>
IX. SUMMARY AND CONCLUSIONS

A. Main Contributions

1. Extension of the "Heffron-Phillips" model

As shown in Chapter IV, this research has developed an extended "Heffron-Phillips" model which is applicable to a general power system with an arbitrary number of generators, and which takes into account network resistances. This is a linearized model in which the generators are represented by the one-axis model and the network and loads are represented by constant impedances. A group of constants, called the "M" coefficients are derived which relate the torque, $E'_q$, and terminal voltage of each machine to the rotor angles and $E''_q$'s of the various machines. These coefficients provide much insight into the intermachine interactions, and are potentially useful in many applications. In this investigation they have been used as a tool for determining and analyzing coherency.

2. Decoupling of the inertial and exciter modes

In Chapter V certain operations on the system "A" matrix are explored. Modal analysis of the matrix reveals that some of the modal frequencies are strongly associated with the generators' $\omega$ variables, while other modal frequencies are associated with the exciters' $E_{PB}$ variables. The former are the system's inertial modes of oscillation, and the latter are the exciter modes.

The strength of the coupling between the inertial and exciter modal
frequencies was investigated. By partitioning the "A" matrix, a subset was designated the "inertial matrix" which accounts for the inertial effects, and another subset was designated the "exciter matrix" which accounts for the exciter effects. It was found that the modal frequencies obtained from the partitioned matrices compare quite favorably with those obtained from the original, and complete, "A" matrix. This suggests that the system's inertial and exciter frequencies can be decoupled into independent groups.

3. Establishing inertial coherency

The "Ml" coefficients, a subset of the "M" coefficients which are analogous to the familiar synchronizing power coefficients, are contained in the inertial matrix. A technique using the "Ml" coefficients to establish inertial coherency is developed in Chapter V. This technique does not require time solutions or eigenvectors but involves analyzing the coupling and resulting frequencies of oscillation between machines. In so doing, the tendency of coherent machines to "swing together" can be determined with a minimum amount of computation. The analysis requires establishing different reference generators. In the 4-machine test system each generator was alternately used as reference. However, in a large system, it should suffice to perform these calculations employing only a few large generators, remote from the fault location, as system reference.
4. **Exciter reduction**

The modes and mode shapes of the exciter matrix in a given power system may all be distinct and show no evidence of coherency. When some of the inertial modes are coherent, and modal reduction is to be used to combine the inertially coherent generators, a similar procedure is needed to combine their exciters, even though they are not coherent. In Chapter VIII a modal reduction procedure was developed that resulted in the removal of the mode of oscillation occurring almost entirely between the two exciters to be combined. The remaining modes and mode shapes were hardly affected. This procedure was developed by using the mathematical constraints of coherency to reduce the exciter matrix. It resulted in taking the arithmetic average of terms in the rows and columns pertaining to the inertially coherent machines. Use of this technique provides information on what the ideal reduction would accomplish, and can be used as a criterion against which a conventional reduction can be judged.

Development of the modal reduction procedure provided insights which were successfully incorporated into the conventional reduction method. Conventional exciter reductions normally use the logarithmic average of the exciter gains and time constants as shown in Chapter VII and reference 5. However, the modal reduction indicated that arithmetic averaging of the exciter gains and of the reciprocals of the exciter time constants was appropriate. Changing the averaging technique was easily adapted within the overall framework of the conventional exciter reduction.
B. Numerical Results

The extended "Heffron-Phillips" model was applied to a 4-machine test system. The "M" coefficients and the resulting "A" matrix were calculated. The "A" matrix was partitioned yielding the inertial and exciter matrices which were then examined by modal analysis. When this analysis was compared to the modal analysis of the "A" matrix as a whole it became clear that the system could be decoupled into independent inertial and exciter subsets.

The test for inertial coherency was applied to this system and showed that two of the four generators are coherent. This coherency was then corroborated by examining the eigenvectors of the inertial matrix. The two machines exhibited inertial coherency by the similarity of their modal frequencies and respective mode shapes. While combining these two machines into an equivalent machine would eliminate one of these modes, the resultant time solution of a disturbance remote from the equivalent would show very little error.

The two inertially coherent machines were then combined into a conventional equivalent machine, reducing the test system to a 3-generator system in Chapter VII. The "M" coefficients and "A" matrix were formed for the resulting 3-machine system, and modal analysis was performed on the inertial and exciter matrices to determine which modes were lost by the system reduction, and to what extent the mode shapes were altered. The results showed that the inertial subsystems combined quite nicely, with one of the two similar modes eliminated and the remaining modes and mode shapes left intact. To the exciter
matrix one mode was eliminated and the remaining modes and mode shapes were considerably altered.

In Chapter VIII the modal reduction was tested on the exciter matrix of the 4-machine system. The result was the elimination of the inter-exciter mode of the two inertially coherent machines with very little effect on the remaining exciter modes and mode shapes. The arithmetic averaging technique employed in the modal reduction was adapted for the conventional exciter reduction method. A number of cases using different combinations of exciter gains and time constants were performed in which both the logarithmic and arithmetic averaging techniques were used. The results of the exciter reductions favored arithmetic averaging, especially in cases of widely different time constants.

C. Discussion

1. Theory versus test system results

The extended "Heffron-Phillips" model performed quite well on the test system. It produced the "M" coefficients which provided substantial insight into the intermachine relationships and coupling of the four generators.

The conventional generator reduction technique performed precisely on the inertial portion but not so well for the exciters. The modal reduction method was developed to provide a criterion by which exciter reduction could be judged. This method did provide a clear idea of what constitutes a desirable exciter equivalent. It also indicated an
improvement which was tried on the conventional reduction method. Many cases employing the conventional method were examined using different combinations of exciter gains and time constants. They showed that both the modal frequencies and mode shapes were better approximated using arithmetic instead of logarithmic averaging. This validated the prediction inferred by the modal reduction technique.

2. Suggestions for further study

Further study based on this research is indicated in several areas. One area of interest is the need to describe the effect of fault location on generator coherency. If this effect could be quantified as a function of the \"M\" coefficients it would make possible the combining of generators close to the fault a routine matter.

Another area worth exploring is the sensitivity of the \"M\" coefficients to varying generator operating points and network conditions. If two machines are coherent it would be useful to know the extent of variation possible in the generation levels and network configuration before loss of coherency. This would eliminate the need for computing new generator equivalents for every system change.

The use of more complicated exciter models could also provide an extension of this work. More effective methods of combining advanced exciter models may possibly be developed by using the modal analysis technique of Chapter VIII.
X. REFERENCES


XI. ACKNOWLEDGMENTS

The author would like to express his thanks and appreciation to the members of his committee, Dr. A. A. Fouad, Dr. A. A. Mahmoud, Dr. K. C. Kruempel, Dr. R. G. Brown, and Dr. G. W. Smith. A special thanks is extended to his major professor, Dr. A. A. Fouad, for his patience and guidance. Also much appreciated is the advice received from Dr. K. C. Kruempel and Dr. M. A. Pai.

The Power Affiliate Committee also deserves thanks for financial help and encouragement.

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In Chapter III, Equations 3.6-3.8 define the internal variables of a synchronous machine. These variables are generated from the stator quantities by use of a modified Park's transformation. This transformation $P$ is orthogonal, power invariant, and features the $d$ axis leading the $q$ axis by $90^\circ$ as shown in Figure 12.1. Chapter 4 in reference 38 discusses this technique in detail.

1. The "Heffron-Phillips" model

The Heffron-Phillips model results from describing a synchronous generator with the linearized one-axis model connected to an infinite bus (Figure 12.2). Excellent development of this subject is given in references 33, 34 and 38.

2. Fortran program

The following Fortran program applies Park's transformation to the stator quantities and calculates the Heffron-Phillips constants. The conventions and equations are the same as used in Chapters 4 and 6 of reference 38. The program, based upon one machine against an infinite bus, also calculates the voltage behind transient reactance, the power synchronizing coefficient, the regulated and unregulated synchronous torque, various frequencies and damping ratios defined in the documentation, flux linkages, coefficients of the characteristic polynomials for the regulated and unregulated cases, Routh's criterion
Figure 12.1. Pictorial diagram of a synchronous generator

Figure 12.2. One machine against an infinite bus
analysis, the maximum and minimum regulator gain permissable for stability, and the eigenvalues for the regulated and unregulated cases.

**DOCUMENTATION**

The input quantities are numbered and defined below:

1. \( P_g \), power supplied by the generator in per unit
2. \( \Phi \), power factor angle at machine terminals in degrees. A negative sign infers leading power factor
3. \( x_d \)
4. \( x'_d \)
5. \( x_q \)
6. \( r \), armature resistance
7. \( x_L \), leakage reactance
8. \( R_e \)
9. \( X_e \)
10. \( \Omega \)
11. \( D \), damping constant
12. \( K_E \), exciter gain
13. \( \tau_E \), exciter time constant in seconds
14. \( \tau'_{d0} \), d-axis transient open circuit time constant in seconds
15. \( V_t \), when known, the per unit terminal voltage will be punched in the 15th position on the input cards. When unknown, 0. will be punched in the 15th position on the input cards. If unknown, \( V_t \) will be calculated and printed out as number 22 of the output.
16. \( V_\infty \), when known, the per unit infinite bus voltage will be punched in the 16th position on the input cards. The printout would then show \( V_\infty \) as number 15 of the input. When unknown, 0. will be punched in the 16th position on the input cards. If unknown, \( V_\infty \) will be calculated and printed out as number 22 of the output.

The computer output lists the results in the following order.

1. \( I_a \)
2. \( I_r \)
3. \( I_x \)
4. PF, power factor
5. \( \delta-\beta \), in degrees
6. \( V_d \)
7. \( \sqrt{3} V_d = v_d \)
8. \( V_q \)
9. \( \sqrt{3} V_q = v_q \)
10. \( I_d \)
11. \( \sqrt{3} I_d = i_d \)
12. \( I_q \)
13. \( \sqrt{3} I_d = i_d \)
14. \( E \), per unit stator equivalent EMF corresponding to \( i_F \)
15. \( \sqrt{3} I_F = i_F \)
16. \( \lambda_d \)
17. \( \lambda_{AD} \)
18. \( \lambda_q \)
19. \( \lambda_{AO} \)
20. \( \lambda_F \)
21. $P_{\text{CHK}}$, power check: must equal $P_G$
22. $V_t$ or $V_\infty$, prints out $V_t$ if $V_\infty$ is given in the input or prints out $V_\infty$ if $V_t$ is given in the input.
23. $\alpha$-
24. $\delta$-
25. $E_{qa}$
26. $K_1$
27. $K_l$
28. $K_2$
29. $K_3$
30. $K_4$
31. $K_5$
32. $K_6$
33. $E$, per unit constant voltage behind transient reactance, classical model with $V_\infty = V_\infty|_0^0$
34. $\delta$, angle in degrees for number 33
35. $P_s$, power synchronizing coefficient
36. $T_{su}$, unregulated synchronous torque
37. $T_{SR}$, regulated synchronous torque
38. $\omega_{n1}$, $\omega_{n1} = (\omega_B K_1/2H)^{1/2}$
39. $\omega_{n2}$, $\omega_{n2} = [(\omega_B/2H)(K_1-K_2K_5/K_6)]^{1/2}$
40. $\omega_{n3}$, $\omega_{n3} = [(\omega_B/2H)(K_1-K_2K_3K_4)]^{1/2}$
41. $\xi_{n1}$, $\xi_{n1} = (D/2)/(2HK_1\omega_B)^{1/2}$
42. $\xi_{n2}$, $\xi_{n2} = (D/2)/[2H(K_1-K_2K_5/K_6)\omega_B]^{1/2}$
43. $\xi_{n3}$, $\xi_{n3} = (D/2)/[2H(K_1-K_2K_3K_4)\omega_B]^{1/2}$
44. \( \omega_{x1}, \omega_{x2} = \left( \frac{K_6 K_e}{\tau d_0 \tau e} \right)^{1/2} \)

45. \( \omega_{x1}, \omega_{x2} = \left[ \frac{(1+K_3 K_6 K_e)}{(K_3 \tau d_0 \tau e)} \right]^{1/2} \)

46. \( \xi_{x1}, \xi_{x1} = \left( \frac{\tau e + K_3 \tau d_0}{2 \omega_{x1}} \right) \)

47. \( \xi_{x2}, \xi_{x2} = \left( \frac{\tau e + K_3 \tau d_0}{2 \omega_{x1}} \right) \)

48. \( K_{\text{max}}, \) maximum value for \( K_e \) or system goes unstable

49. \( K_{\text{min}}, \) minimum value for \( K_e \) or system goes unstable

50. \( A_2 \) These are the coefficients for the unregulated characteristic polynomial

51. \( A_1 \)

52. \( A_0 \)

53. \( S^3 \) This is the first column of the Routh criterion array for the unregulated case

54. \( S^2 \)

55. \( S^1 \)

56. \( S^0 \)

57. \( A_3 \) These are the coefficients for the regulated characteristic polynomial

58. \( A_2 \)

59. \( A_1 \)

60. \( A_0 \)

61. \( S^4 \) This is the first column of the Routh criterion array for the regulated case

62. \( S^3 \)

63. \( S^2 \)

64. \( S^1 \)

65. \( S^0 \)

Finally, the eigenvalues for the unregulated and regulated cases are shown.
In order to run this program the following data cards must be prepared:

1. The first data card contains only one integer number. This number is the number of cases to be run and should contain no decimal point. It can appear anywhere on the card.

2. The second card contains the heading for the 1st case. Everything punched on this card from Col. 2 through Col. 71 will be shown on the printout as the heading for the first case.

3. The third data card (or cards) contains the 16 input real numbers as described earlier. You can start anywhere on the card but each number must contain a decimal point. The numbers must be in the proper order and be separated by commas. You can use as many cards as you need to show all 16 numbers.

4. For the second case repeat steps 2 and 3 and do the same for each succeeding case.

A listing of the program follows:
FUNCTION C46(I,X,F,CT.S.A4,A1,A2,A3,A0)

CALL H26B(N,1,1,FCT,S,L6,4,A4,A3,A2,A1,A0)
IF(L.0..1) RETURN
DO 1 I = 1,N
1 PRINT*,S(I)
7 FORMAT(*'0,2F17.7')
GO TO 9
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254 \[ Y_2 = f(1, x, FC, 5, f_A, f_A, f_A, f_A, f_A, f_A) \]
255 \[ F_A = f(1, x, FC, 5, f_A, f_A, f_A, f_A, f_A, f_A) \]
256 \[ G_A = \{1, x, FC, 5, f_A, f_A, f_A, f_A, f_A, f_A\}(Y_2-Y_1) - \{(f_A)^2\}(Y_1-Y_0) \]
257 \[ A_A = -2, bY_2, f_A, f_A, f_A, f_A, f_A \]
258 \[ H_A = \{(f_A)^2\}(Y_2-Y_1) - \{(f_A)^2\}(Y_1-Y_0) \]
259 \[ C_A = f_A \]
260 \[ C_A = f_A \]
261 \[ f(f_A) = \{(f_A)^2\}(Y_2-Y_1) - \{(f_A)^2\}(Y_1-Y_0) \]
262 \[ C_A = f_A \]
263 \[ C_A = f_A \]
264 \[ f(f_A) = \{(f_A)^2\}(Y_2-Y_1) - \{(f_A)^2\}(Y_1-Y_0) \]
265 \[ C_A = f_A \]
266 \[ C_A = f_A \]
267 \[ f(f_A) = \{(f_A)^2\}(Y_2-Y_1) - \{(f_A)^2\}(Y_1-Y_0) \]
268 \[ C_A = f_A \]
269 \[ C_A = f_A \]
270 \[ C_A = f_A \]
271 \[ f(f_A) = \{(f_A)^2\}(Y_2-Y_1) - \{(f_A)^2\}(Y_1-Y_0) \]
272 \[ C_A = f_A \]
273 \[ C_A = f_A \]
274 \[ f(f_A) = \{(f_A)^2\}(Y_2-Y_1) - \{(f_A)^2\}(Y_1-Y_0) \]
275 \[ C_A = f_A \]
276 \[ C_A = f_A \]
277 \[ f(f_A) = \{(f_A)^2\}(Y_2-Y_1) - \{(f_A)^2\}(Y_1-Y_0) \]
278 \[ C_A = f_A \]
279 \[ C_A = f_A \]
280 \[ f(f_A) = \{(f_A)^2\}(Y_2-Y_1) - \{(f_A)^2\}(Y_1-Y_0) \]
281 \[ C_A = f_A \]
282 \[ C_A = f_A \]
283 \[ f(f_A) = \{(f_A)^2\}(Y_2-Y_1) - \{(f_A)^2\}(Y_1-Y_0) \]
284 \[ C_A = f_A \]
285 \[ C_A = f_A \]
286 \[ f(f_A) = \{(f_A)^2\}(Y_2-Y_1) - \{(f_A)^2\}(Y_1-Y_0) \]
287 \[ C_A = f_A \]
288 \[ C_A = f_A \]
289 \[ f(f_A) = \{(f_A)^2\}(Y_2-Y_1) - \{(f_A)^2\}(Y_1-Y_0) \]
290 \[ C_A = f_A \]
291 \[ C_A = f_A \]
292 \[ f(f_A) = \{(f_A)^2\}(Y_2-Y_1) - \{(f_A)^2\}(Y_1-Y_0) \]
293 \[ C_A = f_A \]
294 \[ C_A = f_A \]
295 \[ f(f_A) = \{(f_A)^2\}(Y_2-Y_1) - \{(f_A)^2\}(Y_1-Y_0) \]
296 \[ C_A = f_A \]
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298 \[ f(f_A) = \{(f_A)^2\}(Y_2-Y_1) - \{(f_A)^2\}(Y_1-Y_0) \]
299 \[ C_A = f_A \]
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301 \[ f(f_A) = \{(f_A)^2\}(Y_2-Y_1) - \{(f_A)^2\}(Y_1-Y_0) \]
302 \[ C_A = f_A \]
303 \[ C_A = f_A \]
304 \[ f(f_A) = \{(f_A)^2\}(Y_2-Y_1) - \{(f_A)^2\}(Y_1-Y_0) \]
305 \[ C_A = f_A \]
306 \[ C_A = f_A \]
307 \[ f(f_A) = \{(f_A)^2\}(Y_2-Y_1) - \{(f_A)^2\}(Y_1-Y_0) \]
308 \[ C_A = f_A \]
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310 \[ f(f_A) = \{(f_A)^2\}(Y_2-Y_1) - \{(f_A)^2\}(Y_1-Y_0) \]
311 \[ C_A = f_A \]
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313 \[ f(f_A) = \{(f_A)^2\}(Y_2-Y_1) - \{(f_A)^2\}(Y_1-Y_0) \]
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316 \[ f(f_A) = \{(f_A)^2\}(Y_2-Y_1) - \{(f_A)^2\}(Y_1-Y_0) \]
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319 \[ f(f_A) = \{(f_A)^2\}(Y_2-Y_1) - \{(f_A)^2\}(Y_1-Y_0) \]
320 \[ C_A = f_A \]
321 \[ C_A = f_A \]
322 \[ f(f_A) = \{(f_A)^2\}(Y_2-Y_1) - \{(f_A)^2\}(Y_1-Y_0) \]
323 \[ C_A = f_A \]
324 \[ C_A = f_A \]
325 \[ f(f_A) = \{(f_A)^2\}(Y_2-Y_1) - \{(f_A)^2\}(Y_1-Y_0) \]
326 \[ C_A = f_A \]
327 \[ C_A = f_A \]
328 \[ f(f_A) = \{(f_A)^2\}(Y_2-Y_1) - \{(f_A)^2\}(Y_1-Y_0) \]
329 \[ C_A = f_A \]
330 \[ C_A = f_A \]
3. 4-Machine system

The computer output for each of the four synchronous generators of the modified 11 bus WSCC system follows:
<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>0.50</td>
<td>1.40</td>
<td>0.40</td>
<td>0.00</td>
<td>0.00</td>
<td>1.40</td>
<td>0.40</td>
<td>1.90</td>
</tr>
<tr>
<td>2.</td>
<td>0.40</td>
<td>1.40</td>
<td>0.40</td>
<td>0.00</td>
<td>0.00</td>
<td>1.40</td>
<td>0.40</td>
<td>1.90</td>
</tr>
<tr>
<td>3.</td>
<td>0.30</td>
<td>1.40</td>
<td>0.40</td>
<td>0.00</td>
<td>0.00</td>
<td>1.40</td>
<td>0.40</td>
<td>1.90</td>
</tr>
<tr>
<td>4.</td>
<td>0.20</td>
<td>1.40</td>
<td>0.40</td>
<td>0.00</td>
<td>0.00</td>
<td>1.40</td>
<td>0.40</td>
<td>1.90</td>
</tr>
<tr>
<td>5.</td>
<td>0.10</td>
<td>1.40</td>
<td>0.40</td>
<td>0.00</td>
<td>0.00</td>
<td>1.40</td>
<td>0.40</td>
<td>1.90</td>
</tr>
</tbody>
</table>

**Dynamic Stability Analysis: One Machine Against an Infinite Bus**

**Case:** Single Machine

**Data:**
- **O.C.:** Output Characteristics
- **E.I.:** Excitation Inductance
- **G.H.:** Generator Characteristics
- **F.H.:** Armature Characteristics
- **K.H.:** Transformer Characteristics
- **A.H.:** System Characteristics

**Evaluation Notes:**
- The eigenvalues for the unregulated case are:
- The eigenvalues for the regulated case are:
<table>
<thead>
<tr>
<th>INPUT</th>
<th>OUTPUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( P ) = 0.450000</td>
<td>26. ( K ) = 1.747955</td>
</tr>
<tr>
<td>2. ( M ) = 12.691750</td>
<td>27. ( K ) = 1.075673</td>
</tr>
<tr>
<td>3. ( X ) = 0.899000</td>
<td>28. ( K ) = 1.441030</td>
</tr>
<tr>
<td>4. ( X ) = 0.112000</td>
<td>29. ( K ) = 0.538044</td>
</tr>
<tr>
<td>5. ( X ) = 0.746100</td>
<td>30. ( K ) = 1.396290</td>
</tr>
<tr>
<td>6. ( X ) = 0.203000</td>
<td>31. ( K ) = 0.123463</td>
</tr>
<tr>
<td>7. ( X ) = 0.134000</td>
<td>32. ( K ) = 0.569110</td>
</tr>
<tr>
<td>8. ( X ) = 0.747100</td>
<td>33. ( K ) = 1.355256</td>
</tr>
<tr>
<td>9. ( X ) = 0.347100</td>
<td>34. ( K ) = 0.723140</td>
</tr>
<tr>
<td>10. ( X ) = 0.533000</td>
<td>35. ( K ) = 0.314651</td>
</tr>
<tr>
<td>11. ( X ) = 0.332000</td>
<td>36. ( K ) = 0.751432</td>
</tr>
<tr>
<td>12. ( X ) = 0.223000</td>
<td>37. ( K ) = 1.159217</td>
</tr>
<tr>
<td>13. ( X ) = 1.000000</td>
<td>38. ( K ) = 4.809712</td>
</tr>
<tr>
<td>14. ( X ) = 0.000000</td>
<td>39. ( K ) = 0.869311</td>
</tr>
<tr>
<td>15. ( X ) = 1.025000</td>
<td>40. ( K ) = 0.011922</td>
</tr>
<tr>
<td>16. ( X ) = 0.520000</td>
<td>41. ( K ) = 0.009000</td>
</tr>
<tr>
<td>17. ( X ) = 1.974295</td>
<td>42. ( K ) = 0.007100</td>
</tr>
<tr>
<td>18. ( X ) = 0.915950</td>
<td>43. ( K ) = 0.006000</td>
</tr>
<tr>
<td>19. ( X ) = 0.403760</td>
<td>44. ( K ) = 3.950304</td>
</tr>
<tr>
<td>20. ( X ) = 1.786971</td>
<td>45. ( K ) = 3.443171</td>
</tr>
<tr>
<td>21. ( X ) = 0.992949</td>
<td>46. ( K ) = 0.247143</td>
</tr>
<tr>
<td>22. ( X ) = 0.915432</td>
<td>47. ( K ) = 0.024850</td>
</tr>
<tr>
<td>23. ( X ) = 1.094740</td>
<td>48. ( K ) = 0.450300</td>
</tr>
<tr>
<td>24. ( X ) = 0.895100</td>
<td>49. ( K ) = -2.166760</td>
</tr>
<tr>
<td>25. ( X ) = 1.015250</td>
<td>50. ( K ) = 0.341515</td>
</tr>
</tbody>
</table>

**Notes:**
- The eigenvalues for the unregulated case are:
  - \( \lambda = 0.002000 \)
- The eigenvalues for the regulated case are:
  - \( \lambda = 0.003000 \)

**Further Details:**
- The system is analyzed for stability against an infinite bus case.
### Dynamic Stability Analysis: One Machine Against an Infinite Bus

**Input:**

1. $P_g = 0.050000$
2. $Q_f = 10.920520$
3. $X_0 = 1.012500$
4. $P_f = 0.400000$
5. $Q_0 = 1.257000$
6. $R = 0.000000$
7. $X_L = 0.000000$
8. $E = 0.000000$
9. $X_E = 0.281270$
10. $N$ = \[3 \times 3\]
11. $U = 0.000000$
12. $K_I = 100.000000$
13. $T_E = 1.000000$
14. $R = 5.000000$
15. $V_{A_x} = 1.000000$
16. $A_0 = 1.105310$
17. $K_A = 0.000000$
18. $K = 1.150214$
19. $K = 1.032559$
20. $K_2 = 1.177132$
21. $PHCA = 0.131309$
22. $V = 0.440000$
23. $A = -11.01499$
24. $Q = 52.317010$
25. $E = 1.059458$

**Output:**

1. $I_A = 0.440463$
2. $I_B = 0.257243$
3. $E = -0.159223$
4. $P_f = 0.901091$
5. $D = 0.154630$
6. $V = -0.641990$
7. $S_{BW} = -1.150424$
8. $E = 0.107236$
9. $S_{RD} = 1.901392$
10. $E = -0.653056$
11. $S_{IM} = -1.032510$
12. $E = 0.570063$
13. $S_{TM} = 0.014612$
14. $E = 1.045424$
15. $S_{TH} = 2.101503$
16. $E = 1.165510$
17. $E = 1.037503$
18. $E = 1.150874$
19. $E = 1.032559$
20. $E = 1.177132$
21. $E = 0.131309$
22. $E = 0.440000$
23. $E = -11.01499$
24. $E = 52.317010$
25. $E = 1.059458$

---

**The Eigenvalues for the Unregulated Case are:**

-0.2331472 0.0000000
-0.1006701 -1.290043
-0.1006701 -1.290043
-0.2175354
-0.2175354

**The Eigenvalues for the Regulated Case are:**

-0.5070060 0.2409349
-0.5070060 0.2409349
-0.1098664 -1.2938749
-0.1098664 -1.2938749

---

**Generator #3: 4-Machine System, Base Case**
# Dynamic Stability Analysis: One Machine Against an Infinite Bus

**Generation #4: 4-Machine System, Base Case**

<table>
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<tr>
<th>Input</th>
<th>Output</th>
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<td>1. PG= 0.850000</td>
<td>1. IA= 0.839070</td>
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<td>2. MH= 11.027490</td>
<td>2. IR= 0.821256</td>
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<td>3. XD= 0.895000</td>
<td>3. IX= -0.117491</td>
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<td>4. X01= 0.119490</td>
<td>4. PF= 0.979769</td>
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<td>5. XQ= 0.064590</td>
<td>5. Q-B= 0.955590</td>
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<td>6. N= 0.000000</td>
<td>6. VD= -0.532376</td>
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<tr>
<td>7. KL= 0.051900</td>
<td>7. SH3VD= -0.629102</td>
</tr>
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<td>8. RE= 0.001271</td>
<td>8. VQ= 0.087501</td>
</tr>
<tr>
<td>9. KE= 0.292159</td>
<td>9. SH3VD= 1.537384</td>
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<td>10. H= 6.400000</td>
<td>10. ID= -0.569917</td>
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<td>11. D= 0.000000</td>
<td>11. SH3ID= -0.997124</td>
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<td>12. KE= 100.000000</td>
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<td>13. TE= 1.000000</td>
<td>13. SH3ID= 1.088629</td>
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<td>14. TO= 6.000000</td>
<td>14. E= 1.380112</td>
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<td>15. VA= 1.035000</td>
<td>15. SH3IF= 2.870214</td>
</tr>
<tr>
<td>16. 9= 1.537332</td>
<td>16. ZN= 0.000000</td>
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The Eigenvalues for the Unregulated Case are:

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<td></td>
</tr>
<tr>
<td>0.1146890</td>
<td>-6.9106004</td>
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</table>

The Eigenvalues for the Regulated Case are:

<table>
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<tr>
<th>Eigenvalue</th>
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<tr>
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<tr>
<td>-0.5523014</td>
<td>-2.7050514</td>
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</table>

*Note: The table continues with additional values.*
4. **Equivalent 3-machine system**

After the system is reduced to three machines the Fortran program is applied as follows:
## Dynamic Stability Analysis: One Machine Against An Infinite Bus
### Jan-Aug 78

**Generator #1: 3-Machine System, Base Case**

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<td>1. TI = 0.072444</td>
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<td>3. KO = 0.014600</td>
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<tr>
<td>4. KO = 0.000000</td>
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<tr>
<td>5. KO = 0.094690</td>
<td>29. K3 = 0.899000</td>
</tr>
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<td>6. KO = 0.000000</td>
<td>30. K4 = -0.051333</td>
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<tr>
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</table>

The Eigenvalues for the Unregulated Case are:

-0.140165

0.000000

The Eigenvalues for the Regulated Case are:

-0.140165

0.000000
### Dynamic Stability Analysis: One Machine Against an Infinite Bus

#### Table: Generator Equivalent One-Machine System, Base Case

<table>
<thead>
<tr>
<th>Index</th>
<th>P(MW)</th>
<th>Q(MVAR)</th>
<th>G(MVA)</th>
<th>B(MVAR)</th>
<th>K1</th>
<th>A1</th>
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</tbody>
</table>

#### Table: Eigenvalues for the Unregulated Case

<table>
<thead>
<tr>
<th>Index</th>
<th>( \lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.1901497</td>
</tr>
<tr>
<td>2</td>
<td>-0.0944544</td>
</tr>
<tr>
<td>3</td>
<td>-0.0944544</td>
</tr>
</tbody>
</table>

#### Table: Eigenvalues for the Regulated Case

<table>
<thead>
<tr>
<th>Index</th>
<th>( \lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.4093015</td>
</tr>
<tr>
<td>2</td>
<td>-0.4093015</td>
</tr>
<tr>
<td>3</td>
<td>-0.2002794</td>
</tr>
<tr>
<td>4</td>
<td>-0.2002794</td>
</tr>
</tbody>
</table>
### Dynamic Staility Analysis: One Machine Against an Infinite Bus

**Generator #1: 3 Machine System, Base Case**

<table>
<thead>
<tr>
<th>INPUT</th>
<th>OUTPUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( D_0 = 0.493000 )</td>
<td>1. ( L_A = 0.849463 )</td>
</tr>
<tr>
<td>2. ( D_{10} = 1.020000 )</td>
<td>2. ( K_1 = 1.524428 )</td>
</tr>
<tr>
<td>3. ( X_D = 1.310000 )</td>
<td>3. ( K_2 = 1.099319 )</td>
</tr>
<tr>
<td>4. ( C_T = 0.101000 )</td>
<td>4. ( K_3 = 0.277665 )</td>
</tr>
<tr>
<td>5. ( X_O = 1.297000 )</td>
<td>5. ( K_4 = 1.093169 )</td>
</tr>
<tr>
<td>6. ( D_{1} = 0.006000 )</td>
<td>6. ( K_5 = 0.142310 )</td>
</tr>
<tr>
<td>7. ( X_L = 0.274200 )</td>
<td>7. ( K_6 = 0.525161 )</td>
</tr>
<tr>
<td>8. ( X_F = 0.364310 )</td>
<td>8. ( K_7 = 1.069394 )</td>
</tr>
<tr>
<td>9. ( X_E = 0.086790 )</td>
<td>9. ( K_8 = 1.335311 )</td>
</tr>
</tbody>
</table>

#### Input:
- \( D_0 = 0.493000 \)
- \( D_{10} = 1.020000 \)
- \( X_D = 1.310000 \)
- \( C_T = 0.101000 \)
- \( X_O = 1.297000 \)
- \( D_{1} = 0.006000 \)
- \( X_L = 0.274200 \)
- \( X_F = 0.364310 \)
- \( X_E = 0.086790 \)

#### Output:
- \( L_A = 0.849463 \)
- \( K_1 = 1.524428 \)
- \( K_2 = 1.099319 \)
- \( K_3 = 0.277665 \)
- \( K_4 = 1.093169 \)
- \( K_5 = 0.142310 \)
- \( K_6 = 0.525161 \)
- \( K_7 = 1.069394 \)
- \( K_8 = 1.335311 \)

#### Eigenvalues for the Unregulated Case:
- \(-0.2333481 \quad 0.000000 \)
- \(-0.1032579 \quad 2.274194 \)
- \(-0.000086 \quad 6.296920 \)
- \(-0.000086 \quad 13.416920 \)
- \(-0.000086 \quad 42.512372 \)
- \(-0.000086 \quad 63.566566 \)

#### Eigenvalues for the Regulated Case:
- \(-0.1032579 \quad 2.274194 \)
- \(-0.000086 \quad 6.296920 \)
- \(-0.000086 \quad 13.416920 \)
- \(-0.000086 \quad 42.512372 \)
- \(-0.000086 \quad 63.566566 \)
XIII. APPENDIX B

A. Power Synchronizing Coefficients

The power synchronizing coefficient, $P_{sij}$, of synchronous generator $i$ relative to $j$ is defined as the change in power output of $i$ resulting from an incremental change in the relative rotor angle. This involves holding all other variables constant while allowing small changes in the relative rotor angle $\delta_{ij}$ about a quiescent operating point $\delta_{ij0}$, where $\delta_{ij}$ equals $\delta_i - \delta_j$. The physical meaning is given in mathematical terms as

$$P_{sij} = \frac{\delta P_{ij}}{\delta \delta_{ij}}|_{\delta_{ij0}}$$

The calculation of $P_{sij}$ is given by

$$P_{sij} = E_i E_j (B_{ij} \cos \delta_{ij0} - G_{ij} \sin \delta_{ij0}) \quad (13.1)$$

where,

$$P_{sij} : \text{per unit power/radian}$$
$$E_i : \text{per unit voltage behind transient reactance of generator} i$$
$$E_j : \text{per unit voltage behind transient reactance of generator} j$$
$$G_{ij} + jB_{ij} : \text{an off diagonal term of the network short circuit admittance matrix } Y, \text{ in per unit}$$

Equation 13.1 is derived in Chapter 3 of reference 38.
1. 4-Machine test system

The data for solving Equation 13.1 for the 4-generator case are shown in Table 13.1. The results of the calculation, \( P_{si,j} \), are also shown in the table.

<table>
<thead>
<tr>
<th>( i,j )</th>
<th>( V_i )</th>
<th>( V_j )</th>
<th>( G_{i,j} )</th>
<th>( B_{i,j} )</th>
<th>( \delta_{i,j} )</th>
<th>( P_{si,j} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>1.0775</td>
<td>1.0526</td>
<td>0.1637</td>
<td>1.1208</td>
<td>-9.6394°</td>
<td>1.2843</td>
</tr>
<tr>
<td>21</td>
<td>1.0526</td>
<td>1.0775</td>
<td>0.1637</td>
<td>1.1208</td>
<td>9.6394°</td>
<td>1.2221</td>
</tr>
<tr>
<td>13</td>
<td>1.0775</td>
<td>1.0694</td>
<td>0.1153</td>
<td>0.9269</td>
<td>-12.6056°</td>
<td>1.0714</td>
</tr>
<tr>
<td>31</td>
<td>1.0694</td>
<td>1.0775</td>
<td>0.1153</td>
<td>0.9269</td>
<td>12.6056°</td>
<td>1.0134</td>
</tr>
<tr>
<td>14</td>
<td>1.0775</td>
<td>1.0602</td>
<td>0.1002</td>
<td>0.7705</td>
<td>-11.5781</td>
<td>0.8853</td>
</tr>
<tr>
<td>41</td>
<td>1.0602</td>
<td>1.0775</td>
<td>0.1002</td>
<td>0.7705</td>
<td>11.5781</td>
<td>0.8393</td>
</tr>
<tr>
<td>23</td>
<td>1.0526</td>
<td>1.0694</td>
<td>0.0945</td>
<td>0.7162</td>
<td>-2.9662</td>
<td>0.8106</td>
</tr>
<tr>
<td>32</td>
<td>1.0694</td>
<td>1.0526</td>
<td>0.0945</td>
<td>0.7162</td>
<td>2.9662</td>
<td>0.7996</td>
</tr>
<tr>
<td>24</td>
<td>1.0526</td>
<td>1.0602</td>
<td>0.1280</td>
<td>1.1311</td>
<td>-1.9387</td>
<td>1.2664</td>
</tr>
<tr>
<td>42</td>
<td>1.0602</td>
<td>1.0526</td>
<td>0.1280</td>
<td>1.1311</td>
<td>1.9387</td>
<td>1.2567</td>
</tr>
<tr>
<td>34</td>
<td>1.0694</td>
<td>1.0602</td>
<td>0.0982</td>
<td>0.8258</td>
<td>1.0275</td>
<td>0.9341</td>
</tr>
<tr>
<td>43</td>
<td>1.0602</td>
<td>1.0694</td>
<td>0.0982</td>
<td>0.8258</td>
<td>-1.0275</td>
<td>0.9381</td>
</tr>
</tbody>
</table>

2. 3-Machine reduced test system

The 3-generator reduced WSCC system is solved for \( P_{si,j} \). The results and data are shown in Table 13.2.
Table 13.2. Power synchronizing coefficients for the reduced 3-machine WSCC system

<table>
<thead>
<tr>
<th>(ij)</th>
<th>(V_i)</th>
<th>(V_j)</th>
<th>(G_{ij})</th>
<th>(B_{ij})</th>
<th>(\delta_{ij0})</th>
<th>(P_{sij})</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>1.0776</td>
<td>1.0564</td>
<td>0.2715</td>
<td>1.9005</td>
<td>-10.6078</td>
<td>2.1834</td>
</tr>
<tr>
<td>21</td>
<td>1.0564</td>
<td>1.0776</td>
<td>0.2715</td>
<td>1.9005</td>
<td>10.6078</td>
<td>2.0696</td>
</tr>
<tr>
<td>13</td>
<td>1.0776</td>
<td>1.0694</td>
<td>0.1173</td>
<td>0.9311</td>
<td>-12.6054</td>
<td>1.0766</td>
</tr>
<tr>
<td>31</td>
<td>1.0694</td>
<td>1.0776</td>
<td>0.1173</td>
<td>0.9311</td>
<td>12.6054</td>
<td>1.0176</td>
</tr>
<tr>
<td>23</td>
<td>1.0564</td>
<td>1.0694</td>
<td>0.1891</td>
<td>1.5396</td>
<td>-1.9976</td>
<td>1.7457</td>
</tr>
<tr>
<td>32</td>
<td>1.0694</td>
<td>1.0564</td>
<td>0.1891</td>
<td>1.5396</td>
<td>1.9976</td>
<td>1.7308</td>
</tr>
</tbody>
</table>

B. Inertial Modal Frequencies with Classical Modelling

With the generators modelled classically (38) the state space formulation for a linearized 4-generator system with no driving function is \( \dot{X} = AX \) or

\[
\begin{bmatrix}
\ddot{\delta}_{14} \\
\ddot{\delta}_{24} \\
\ddot{\delta}_{34} \\
\ddot{\omega}_{14} \\
\ddot{\omega}_{24} \\
\ddot{\omega}_{34}
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
-\alpha_{11} & -\alpha_{12} & -\alpha_{13} & 0 & 0 & 0 \\
-\alpha_{21} & -\alpha_{22} & -\alpha_{23} & 0 & 0 & 0 \\
-\alpha_{31} & -\alpha_{32} & -\alpha_{33} & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\delta_{14} \\
\delta_{24} \\
\delta_{34} \\
\omega_{14} \\
\omega_{24} \\
\omega_{34}
\end{bmatrix}
\tag{13.2}
\]

where machine 4 is used as reference.

The equations for the \( \alpha \) constants shown in Equation 13.2 are derived using the method shown in Chapter 3 of reference 38. These
equations are shown below:

\[
\begin{align*}
\alpha_{11} &= \frac{\omega_B}{2H_1} P_{s14} + \frac{\omega_B}{2H_4} P_{s41} + \frac{\omega_B}{2H_1} P_{s12} + \frac{\omega_B}{2H_1} P_{s13} \\
\alpha_{12} &= \frac{\omega_B}{2H_4} P_{s42} - \frac{\omega_B}{2H_1} P_{s12} \\
\alpha_{13} &= \frac{\omega_B}{2H_4} P_{s43} - \frac{\omega_B}{2H_1} P_{s13} \\
\alpha_{21} &= \frac{\omega_B}{2H_4} P_{s41} - \frac{\omega_B}{2H_2} P_{s21} \\
\alpha_{22} &= \frac{\omega_B}{2H_4} P_{s24} + \frac{\omega_B}{2H_4} P_{s42} + \frac{\omega_B}{2H_2} P_{s21} + \frac{\omega_B}{2H_2} P_{s23} \\
\alpha_{23} &= \frac{\omega_B}{2H_4} P_{s43} - \frac{\omega_B}{2H_2} P_{s23} \\
\alpha_{31} &= \frac{\omega_B}{2H_4} P_{s41} - \frac{\omega_B}{2H_3} P_{s31} \\
\alpha_{32} &= \frac{\omega_B}{2H_4} P_{s42} - \frac{\omega_B}{2H_3} P_{s32} \\
\alpha_{33} &= \frac{\omega_B}{2H_3} P_{s34} + \frac{\omega_B}{2H_4} P_{s43} + \frac{\omega_B}{2H_3} P_{s31} + \frac{\omega_B}{2H_3} P_{s32}
\end{align*}
\]

(13.3)

For a 3-generator system the equations for finding the \( \alpha \) constants are given in Chapter 3 of reference 38.

1. 4-Machine test system

From Equations 13.2 and 13.3 the "A" matrix for the 4-generator case is:
The inertial modal frequencies are given by the eigenvalues of this "A" matrix and are shown below. The units are r/s.

\[ 0 + j13.9196 \]
\[ 0 + j11.4408 \]
\[ 0 + j7.7302 \]

2. **3-Machine reduced test system**

For the 3-generator reduced system the "A" matrix is

\[
\begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
-89.7213 & -90.9807 & 0 & 0 \\
-33.2487 & -164.5769 & 0 & 0 \\
\end{bmatrix}
\]

and the eigenvalues in r/s are

\[ 0 + j13.9168 \]
\[ 0 + j7.7860 \]
XIV. APPENDIX C

A. Eigenvector Samples

In case 10 the exciters for machines 2 and 4 have the following parameters:

\[ K_A^2 = 100 \quad T_A^2 = 2.15 \quad K_A^4 = 100 \quad T_A^2 = 0.15 \]

Exciters 2 and 4 are combined into an equivalent exciter which is designated as exciter 2 in the reduced system.

Taking the arithmetic average of the exciter gains and of the reciprocals of the exciter time constants yields

\[ K_A^2 = 100 \quad T_A^2 = 0.2802 \]

The logarithmic average of the exciter gains and time constants yields

\[ K_A^2 = 100 \quad T_A^2 = 0.5679 \]

Tables 14.7 through 14.4 show the eigenvectors for case 10.
### Table 14.1. Eigenvectors of the exciter matrix, 4-machine system, case 10

<table>
<thead>
<tr>
<th>Variable</th>
<th>Eigenvector (r/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-3.5267 ± j7.6387</td>
</tr>
<tr>
<td>$E'_{q1}$</td>
<td>-0.0002 ± j0.0001</td>
</tr>
<tr>
<td>$E'_{q2}$</td>
<td>-0.0006 ± j0.0002</td>
</tr>
<tr>
<td>$E'_{q3}$</td>
<td>-0.0012 ± j0.0001</td>
</tr>
<tr>
<td>$E'_{q4}$</td>
<td>-0.0074 ± j0.0189</td>
</tr>
<tr>
<td>$E_{FD1}$</td>
<td>0.0085 ± j0.0106</td>
</tr>
<tr>
<td>$E_{FD2}$</td>
<td>0.0109 ± j0.0140</td>
</tr>
<tr>
<td>$E_{FD3}$</td>
<td>0.0310 ± j0.0355</td>
</tr>
<tr>
<td>$E_{FD4}$</td>
<td>1.0000 ± j0.0</td>
</tr>
</tbody>
</table>
Table 14.2. Eigenvectors (mode 1) of the exciter matrix, reduced system, case 10

<table>
<thead>
<tr>
<th>Variable</th>
<th>Modal Reduction</th>
<th>Conventional reduction</th>
<th>Conventional reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Arithmetic eigenvalue (r/s)</td>
<td>Logarithmic</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-1.9271 + j0.6674</td>
<td>-1.9251 + j0.6532</td>
<td>-1.0070 + j4.7767</td>
</tr>
<tr>
<td>$E'_{q1}$</td>
<td>-0.0007 + j0.0006</td>
<td>-0.0007 + j0.0006</td>
<td>-0.0019 + j0.0031</td>
</tr>
<tr>
<td>$E'_{q2}$</td>
<td>-0.0055 + j0.0239</td>
<td>-0.0056 + j0.0242</td>
<td>-0.0045 + j0.0342</td>
</tr>
<tr>
<td>$E'_{q3}$</td>
<td>-0.0032 + j0.0024</td>
<td>-0.0034 + j0.0025</td>
<td>-0.0064 + j0.0123</td>
</tr>
<tr>
<td>$E_{FD1}$</td>
<td>0.0470 + j0.0268</td>
<td>0.0485 + j0.0283</td>
<td>0.1485 + j0.0496</td>
</tr>
<tr>
<td>$E_{FD2}$</td>
<td>1.0000 + j0.0</td>
<td>1.0000 + j0</td>
<td>1.0000 + j0</td>
</tr>
<tr>
<td>$E_{FD3}$</td>
<td>0.1281 + j0.0693</td>
<td>0.1308 + j0.0724</td>
<td>0.3658 + j0.1014</td>
</tr>
<tr>
<td>Variable</td>
<td>Modal Reduction</td>
<td>Conventional Reduction</td>
<td></td>
</tr>
<tr>
<td>----------</td>
<td>-----------------</td>
<td>------------------------</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Eigenvalue (r/s)</td>
<td>Arithmetic</td>
<td>Logarithmic</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E'_{q_1}$</td>
<td>-0.6437 $\pm$ 12.9842</td>
<td>-0.6500 $\pm$ 12.9723</td>
<td>-0.6491 $\pm$ 12.9189</td>
</tr>
<tr>
<td>$E'_{q_2}$</td>
<td>-0.0061 $\pm$ j0.0364</td>
<td>-0.0063 $\pm$ j0.0366</td>
<td>-0.0064 $\pm$ j0.0372</td>
</tr>
<tr>
<td>$E'_{q_3}$</td>
<td>0.0031 $\pm$ j0.0174</td>
<td>0.0038 $\pm$ j0.0185</td>
<td>0.0043 $\pm$ j0.0235</td>
</tr>
<tr>
<td>$E_{PD1}$</td>
<td>0.0251 $\pm$ j0.0594</td>
<td>0.0265 $\pm$ j0.0382</td>
<td>0.0278 $\pm$ j0.0274</td>
</tr>
<tr>
<td>$E_{PD2}$</td>
<td>1.0000 $\pm$ j0.0</td>
<td>1.0000 $\pm$ j0</td>
<td>1.0000 $\pm$ j0</td>
</tr>
<tr>
<td>$E_{PD3}$</td>
<td>-0.3276 $\pm$ j0.0726</td>
<td>-0.3489 $\pm$ j0.0816</td>
<td>-0.3613 $\pm$ j0.0767</td>
</tr>
<tr>
<td></td>
<td>0.6958 $\pm$ j0.4301</td>
<td>0.6709 $\pm$ j0.4438</td>
<td>0.4725 $\pm$ j0.4518</td>
</tr>
</tbody>
</table>
Table 14.4  Eigenvectors (mode 3) of the exciter matrix, reduced system, case 10

<table>
<thead>
<tr>
<th>Variable</th>
<th>Modal Reduction</th>
<th>Conventional reduction</th>
<th>Arithmetic Eigenvalue (r/s)</th>
<th>Logarithmic</th>
</tr>
</thead>
<tbody>
<tr>
<td>F'_q1</td>
<td>-0.0041 + j0.0077</td>
<td>-0.0044 + j0.0082</td>
<td>-0.0044 + j0.0071</td>
<td></td>
</tr>
<tr>
<td>F'_q2</td>
<td>0.0031 + j0.0059</td>
<td>0.0032 + j0.0049</td>
<td>0.0041 + j0.0068</td>
<td></td>
</tr>
<tr>
<td>F'_q3</td>
<td>-0.0033 + j0.0677</td>
<td>-0.0033 + j0.0675</td>
<td>-0.0036 + j0.0679</td>
<td></td>
</tr>
<tr>
<td>F'_FD1</td>
<td>-0.1473 + j0.1372</td>
<td>-0.1569 + j0.1469</td>
<td>-0.1316 + j0.1394</td>
<td></td>
</tr>
<tr>
<td>F'_FD2</td>
<td>-0.0926 + j0.0700</td>
<td>-0.0775 + j0.0747</td>
<td>-0.1080 + j0.0824</td>
<td></td>
</tr>
<tr>
<td>F'_FD3</td>
<td>1.0000 + j0.0</td>
<td>1.0000 + j0</td>
<td>1.0000 + j0</td>
<td></td>
</tr>
</tbody>
</table>