Sensitivity of the effective viscosity of temperate ice to its water content

Conner Adams

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Sensitivity of the effective viscosity of temperate ice to its water content

by

Conner J.C. Adams

A thesis submitted to the graduate faculty

in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

Major: Geology

Program of Study Committee:
Neal R. Iverson, Major Professor
Jacqueline E. Reber
Peter L. Moore

The student author, whose presentation of the scholarship herein was approved by the program of study committee, is solely responsible for the content of this thesis. The Graduate College will ensure this thesis is globally accessible and will not permit alterations after a degree is conferred.

Iowa State University
Ames, Iowa
2021

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ABSTRACT

Flow speeds of West Antarctic ice streams depend on the shearing resistance of ice in their lateral margins. Ice at depth in these regions is theorized to be at the pressure-melting temperature and thus to contain interstitial meltwater. This water’s influence on the rheology of temperate ice is a major source of uncertainty in efforts to model ice-stream motion. These models rely on scant experimental data indicating effective viscosity decreases by a factor of ~3 as ice water content increases from ~0.01 to 0.8%. Recent modeling incorporating this dependence indicates that the sensitivity of viscosity to water content localizes strain in shear margins, causing strain heating that increases meltwater available for enhancing slip at ice-stream beds.

To expand the database on softening of ice by interstitial water, we conducted a series of shearing experiments on temperate ice under compression with a large viscometer (modified ring-shear device). Ice rings had inner and outer diameters of 0.5 and 0.9 m, respectively, and were approximately ~0.2 m thick. Ice with crystal sizes of 2-4 mm was made by mixing deionized water with sieved snow and maintained at the pressure-melting temperature by a temperature-controlled bath (0.01°C precision). Water content was varied from ~0.2 to 1.7% by imposing various combinations of confining pressure (700-1500 kPa) and shear strain rate (7.1 x 10^{-9} - 1.2 x 10^{-7} s^{-1}). Ice was sheared only until a peak shear stress was attained to avoid the complicating influence of fabric development in tertiary creep. Water content was measured by inducing a freezing front at the ice-ring walls, recording its speed as it moved toward the ice-ring center, and solving the relevant Stefan problem.

Results indicate that effective viscosity is independent of water content above values of ~0.6%. In contrast, at water contents increasing from 0.2 to 0.6%, effective ice viscosity
decreased by a factor of 4.4, about 1.8 times more than in previous experiments conducted to tertiary creep. A stress exponent of $n = 1.1$ was determined for ice water contents above ~0.6% and indicates Newtonian flow, probably resulting from internal melting and refreezing facilitated by wetted grain boundaries. This hypothesis is supported by effective ice viscosities at these high water contents that are the same in secondary and tertiary creep, indicating a deformation mechanism that is independent of ice-fabric development in tertiary creep. The insensitivity of ice effective viscosity, when $n \approx 1$ at low water contents (< 1.7%), is qualitatively consistent with the results of deformation experiments with olivine.

The lack of sensitivity of ice effective viscosity to water content at values greater than ~0.6% suggests that ice-stream models that extrapolate softening data to values greater than 0.6% overestimate the softening effect of water on shear resistance and shear localization at ice-stream margins.
CHAPTER 1. INTRODUCTION

Loss of mass from continental ice sheets is the largest contributor to sea-level rise and source of uncertainty for predicting sea-level rise over the next century. (Rignot et al., 2008; IPCC, 2013, 2019; Feldman and Levermann, 2015; Robel et al., 2019). Rates of mass loss have notably increased in recent decades (IPCC, 2013, 2019) and primarily reflect increased discharges of ice from marine-terminating glaciers into the oceans (Rignot et al., 2011). These high-discharge outlet glaciers, termed ice streams, are the primary drainage conduits of ice from continental ice sheets (Fig. 1.1) (Rignot et al., 2008; Mouginot et al., 2017).

Figure 1.1. Composite mosaic of average flow speeds of the surface of the Antarctic ice sheet between 1992 and 2016 (modified from Mouginot et al., 2017). Some of the fastest moving areas (pink) are floating ice shelves fed by ice streams.
Most of ice lost in Antarctica is discharged primarily through West Antarctic ice streams (Rignot et al., 2008). Ice streams in this region flow at high speeds of 300–2000 m/yr relative to surrounding ice (Figure 1.1) (Cuffey and Paterson, 2010). They are generally ~1 km thick with widths ranging from 20-100 km, and their lengths can extend for 100’s of kilometers inland. They have low surface slopes that drive flow at low gravitational stresses (Cuffey and Paterson, 2010). High flow speeds are primarily facilitated by low basal drag associated with basal slip, usually owing to deformable beds under low effective pressure (Tulaczyk et al., 2001). Consequently, an important component of flow resistance is drag that develops at lateral shear margins, as fast flowing ice moves past more relatively stagnant ice. These zones tend to be a few kilometers in width (Cuffey and Paterson, 2010). Intense shearing localized in these margins has been observed to heat ice at depth (Echelmeyer et al., 1992; Harrison et al., 1998). Heating is sufficient to generate temperate ice, which has been interpreted at depth in shear margins from the results of ground penetrating radar (Clarke et al., 2000). The width of temperate zones is controlled by the degree of strain localization in the margin while the height of the zone is limited by the flow of colder ice into the margin (Haseloff et al. 2019).

In response, numerical models have been developed to assess the development of temperate ice in shear margins and how it may affect flow. Models of shear margins have demonstrated that temperate ice commonly develops at depth (Schoof, 2004; Haseloff et al., 2015, 2018; Perol and Rice, 2015; Minchew et al., 2018). Current ice-stream flow models incorporate temperate ice in shear margin zones (Fig. 1.2). However, the rheology of temperate ice is poorly understood due to scant experimental data on the dependence of the viscosity of ice on its water content (Duval, 1977), and this uncertain dependence is a major source of
Figure 1.2. Conceptual model of the ice stream flow (modified from Haseloff et al., 2019). Temperate ice is caused by strain heating, which is mitigated somewhat by slow flow of cold ice from inter-stream ridges into shear margins.

Uncertainty in the predictions of these models (Haseloff et al., 2019). In only one modeling study (Haseloff et al., 2019) has water content in shear margins been computed and used to calculate ice viscosity. The authors found that both shear resistance and strain localization at ice-stream margins were sensitive to ice water content and the variables that controlled it, most notably strain heating and ice permeability. Water contents of > 5% were predicted in some places within shear margins.

The relevance of ice water content and its effect on ice rheology is not confined to ice-stream shear margins. Understanding the rheology of temperate ice is fundamental for adequately describing the dynamics of temperate and polythermal glaciers (Benjuma et al., 2003). Several field studies have been conducted using ground penetrating radar and seismic wave velocities to estimate water content and its spatial variation in temperate and polythermal
glaciers (Macheret and Glazovsky, 2000; Benjuma et al., 2003; Pettersson et al., 2004; Murray et al., 2007; Endres et al., 2009; Brown et al., 2017). Measurements from these studies generally indicate water contents less than 5%. A few studies have indicated water contents as high as 9% in some temperate glaciers (Macheret and Glazovsky, 2000), but such high values likely reflect water residing in channels, cavities, or crevasses, rather than only intergranular water. In other studies, ice at the beds of glaciers has been directly accessed. On the stoss surface of a roche moutonnée, Carol (1947) noted weak, water-saturated ice that had a different consistency than ice higher above the bed where intergranular water was less conspicuous. In later studies, water

![Figure 1.3](image)

**Figure 1.3.** Bed of Engabreen, a temperate glacier in Norway studied by Cohen (2000). Debris laden ice in contact with the bed had water contents of 2% or greater. Cleaner ice above had water contents of 1% (photograph by Tom Hooyer).
contents up to 2% were measured in basal ice using calorimetric methods (Fig. 1.3) (Lliboutry, 1971; Vallon et al., 1976; Cohen, 2000). Vallon et al. (1976) measured water content across the full thickness of Vallée Blanche Supérieure, a temperate glacier in the French Alps, by analyzing ice cored from the accumulation area (Fig. 1.4). Water content increased with depth as a result of heat generated by ice shear that increased with depth and of downward movement of water. Measured and estimated water contents ranged from near 0% to as high as ~2%, exceeding the range of water contents (0.008-0.8%) measured in the rheology experiments of Duval (1977).

Figure 1.4. Water contents measured at depth from coring samples extracted during drilling at the Vallée Blanche Supérieure (modified from Vallon et al., 1976). Dashed curves are model estimates of water content at depth generated by strain heating.
Experimental results with warm ice

Goldsby and Kohlstedt (2001), who deformed in experiments ultrafine-grained ice at low temperatures, identified four distinct creep mechanisms of polycrystalline ice (Fig. 1.5) that they described with a constitutive equation. These authors predicted diffusional flow at very low stresses and strain rates, grain-boundary sliding (GBS) at intermediate stresses and strain rates, and dislocation creep at high stresses and strain rates. Their constitutive equation does not adequately describe high-temperature creep data; experimental strain rates (Glen, 1955) are three times greater than predicted for temperate ice if dislocation creep is assumed to be the dominant deformation mechanism. Enhancement of strain rates is attributed to increased intergranular meltwater locally at grain boundaries where stresses are concentrated (Goldsby and Kohlstedt, 2001).

Figure 1.5. Strain rate as a function of stress for the four creep mechanisms of polar ice described by Goldsby and Kohlstedt (2001). Values of the stress exponent, n, in the power-law creep rule for each mechanism are indicated by the slopes of the plots (modified from Goldsby and Kohlstedt, 2001).
Prior experimental studies have indicated that the creep rates of ice at or close to its melting temperature are greatly enhanced (Fig. 1.6) (Glen, 1955; Barnes et al., 1971; Colbeck and Evans, 1973; Duval, 1977; Mellor and Testa, 1979; Morgan, 1991; De La Chapelle et al., 1999). Ice samples in these experiments were subjected to uniaxial compression without lateral restraint, except in the experiments of Duval (1977). The viscometer used in his experiments allowed a constant axial torque to be applied to ice samples, in addition to unconfined uniaxial compression. Enhancement of creep in all of these experiments is commonly attributed to increasing amounts of intergranular meltwater as ice approaches its melting temperature. The presence of water at grain boundaries is commonly thought to facilitate recrystallization and sliding.

![Figure 1.6](image)

**Figure 1.6.** High-temperature creep data of Morgan (1991). The sensitivity of strain rate to temperature increases by a factor of ~10 from -0.05°C to -0.01°C. The shear stress in the experiments was 0.1 MPa.
In only the study of Duval (1977) was water content measured during creep experiments. Cores of natural ice containing impurities, collected by Vallon et al. (1976), were sheared to tertiary creep while held at temperatures close to or at the melting temperature. Water content of ice samples varied with temperature. Average grain sizes of ice samples varied between 8-19 mm (Vallon et al., 1976). Ice samples were sheared under a constant axial shear stress of 0.29 MPa (axial torque), which exceeds maximum typical shear stresses in glaciers and ice sheets by at least a factor of 2. (Goldsby and Kohlstedt, 2001). In these experiments conducted at a single, high stress, the strain rate of temperate ice was highly sensitive to its water content (Fig. 1.7). Measured strain rates increased linearly by a factor of ~3 over water contents of 0.008-0.8%.

Duval (1977) explained the correlation by suggesting that interstitial water attenuates strain hardening. During deformation, deviatoric stresses concentrate along grain boundaries (Nye and Mae, 1972) where dislocations pile up, locally depressing the melting temperature.

![Figure 1.7. The tertiary, high-temperature creep data of Duval (1977). The shear stress was 0.29 MPa and temperatures were -0.2 to 0 °C.](image)
Water initially located at grain junctions (Nye and Frank, 1973) will partially freeze, and new meltwater will be generated where stresses are concentrated. This water should both absorb dislocations, reducing dislocation pile-ups and facilitate grain boundary migration. Node and vein networks of water between grains have long been recognized and have been observed in both laboratory grown and natural glacier ice (Fig. 1.8a, b) (Nye and Mae, 1972; Nye and Frank, 1973; Raymond and Harrison, 1975; Nye, 1989; Mader, 1992).

Figure 1.8. (a) Idealized sketch of water vein-node network between ice grains (modified from Nye, 1989, as modified from Smith, 1948). (b) Water veins at three-grain intersections meeting at a junction (water node) in laboratory-grown, temperate ice (photograph from Madder, 1992).

De la Chapelle et al. (1999) conducted a series of experiments on ice with a high water content (~7%) that was not varied to study the value of stress exponent, $n$, in the power-law flow rule of ice (Glen, 1955) over a range of experimental stresses, ~0.1-1.0 MPa. In their experiments synthesized temperate ice was sheared to secondary creep while under a constant uniaxial load. Ice samples had high concentrations of NaCl, so that the temperature of the ice sample could be maintained at -13°C while still containing water. These data were compared to
creep data from experiments with pure ice at -10°C that contained 0% water (Duval and Castelnau, 1995). Strain rates of the watery, saline ice were over one order of magnitude larger than those of the melt-free ice, with the difference being largest at low stresses (< ~0.2 MPa) (Fig. 1.9). The enhancement was thought to be caused by a decrease in the load bearing area of the ice samples due to high melt content, which facilitated basal slip by annihilating dislocations and thus attenuating the internal stress field. Two distinct regimes were identified where creep mechanics differed: a high stress exponent regime (n > 3) associated with dislocation climb or movement along stronger, non-basal glide planes and a low stress exponent regime (n < 2) associated with basal slip.

Figure 1.9. Creep data for saline ice and pure ice at -13°C. The slopes for pure ice are from regression fits (from De La Chapelle et al., 1999).

Experiments with other partially molten aggregates

In the discussion of de la Chapelle et al. (1999), they noted similarity between their results and that of rheology experiments of mantle rock-melt mixtures conducted in the
dislocation creep regime (Fig. 1.10a) (Hirth and Kohlstedt, 1995b; Kohlstedt and Zimmerman, 1996). Strain rates in these experiments increased by more than one order of magnitude over melt contents of 0-10%. The large enhancement of creep rates in these experiments was

**Figure 1.10.** (a) Dislocation creep data for crystalline olivine containing varying amounts of MORB melt (modified from Hirth and Kohlstedt, 1995b). (b) Diffusional creep data for crystalline olivine containing varying amounts of synthetic basalt melt (modified from Hirth and Kohlstedt, 1995a).
attributed to melt wetting grain boundaries and thereby promoting grain boundary sliding and diffusion creep along boundaries (Coble creep). The authors noted the effect was more pronounced in very fine-grained samples (μm scale) where grain boundaries are more effectively wetted by small amounts of melt.

Hirth and Kohlstedt (1995a) completed a second set of experiments in the diffusional creep regime (at lower stresses and strain rates) for mantle rock-melt mixtures (Fig. 1.10b). Of note, the enhancement of the strain rate was much larger for melt contents > 4.5%. This phenomenon, also described by Cooper and Kohlstedt (1986), was attributed to the enhancement of diffusional mechanisms due to high melt content (Fig. 1.11). Diffusional creep mechanisms suggested were grain boundary diffusion, dissociation of ions into melt and precipitation of ions out of melt at grain boundaries, and transport of ions through melt. Above threshold melt contents, wetting of grain boundaries by melt becomes sufficient that transport is no longer confined in melt nodes at grain junctions. The creep rate is thus enhanced because the opening of

![Figure 1.11. The movement of matter conceptualized for diffusional creep mechanisms along grain boundaries and at melt nodes (modified from Cooper and Kohlstedt, 1986).](image-url)
high diffusivity pathways through melt along grain boundaries should consequently shorten the distance matter travels along unwetted grain boundaries (grain boundary diffusion).

Another study (Rosenberg and Handy, 2001) indicated that the dependence of the effective viscosity on the melt fraction of crustal rock-melt mixtures is a non-linear relationship (Fig. 1.12). The sensitivity of the effective viscosity to the melt content was significantly larger for melt fractions below 7% compared to higher melt fractions. This transition from high to low sensitivity correlated with the reduction of new grain boundaries being wetted at higher melt contents. Increasing melt-interconnectivity at grain boundaries was suggested as the driver for the large reduction in effective viscosity until a threshold melt fraction was reached, above which

**Figure 1.12.** Experimental rheological data for partially melted crustal rock. The melt connectivity transition (MCT) marks the transition from a regime of high to low sensitivity of the effective viscosity with respect to the melt fraction (modified from Rosenberg and Handy, 2001; data from van der Molen and Paterson, 1979).
increases in melt content had little effect on effective viscosity due to diminishing increases in grain boundary wetting with increasing melt content.

**Objective**

The goal of the present experiments is to study the dependence of ice rheology on water content in polycrystalline, temperate ice sheared under confined compression. These experiments improve upon Duval’s (1977) dataset through consideration of a wider range of water contents that extends to values significantly larger than those that Duval considered. Water content in ice is varied by imposing different combinations of confining pressure and strain rate. The experiments are rate-controlled, and thus shear stress varies as ice deforms. Except for an initial experiment conducted to tertiary creep, ice is sheared only to peak stresses associated with secondary creep to avoid the complicating influence of ice fabric development in tertiary creep. Ice samples are made with grain sizes comparable to those of many glaciers and to those considered in Duval’s (1977) experiments. Finally, owing to poor understanding of creep mechanisms for temperate ice, particularly at high water contents, no assumption is made about the value of the stress exponent or the prefactor in the power-law flow law of ice (Glen et al. 1955). We instead explore the influence of water content on effective ice viscosity, $\eta = \tau/(2\dot{\varepsilon})$, where $\tau$ is the effective shear stress and $\dot{\varepsilon}$ is the effective strain rate (Duval, 1976; Cuffey and Paterson, 2010). These two parameters are measured in the experiments, so no assumed parameter values are required.

**Hypothesis**

The experimental work on ice and other minerals containing a small melt fraction motivates the guiding hypothesis for this study. Ice is expected to soften with increasing water content over the range studied by Duval (1977) but with effective viscosities that are larger, owing to minimal fabric development in secondary creep. At water contents higher than those
considered by Duval, sensitivity of effective viscosity to water content may change, due to different stress-limiting micro-deformation mechanisms activated by sufficient wetting of grain boundaries.
CHAPTER 2. EXPERIMENTAL METHODS

The experiments were conducted using a ring-shear device designed and used previously to study glacial sliding (Fig. 2.1) (Iverson and Petersen, 2011; Zoet and Iverson, 2016, 2020; Thompson et al., 2020). The device’s ice chamber has been adjusted to study ice rheology in these experiments by installing a toothed platen that grips the ice at the base of the chamber. This modification transforms the chamber into a shear box that deforms ice confined on its sides and held in compression between the rotating upper platen and the stationary lower platen. The upper platen rotates at a controlled speed while a vertical stress, perpendicular to the shearing direction, is applied to the ice ring by a hydraulic ram. The ice chamber resides in a bath

![Figure 2.1. Cross-sectional schematic of ring shear device. Weighted thread (red line) is deflected from an initially vertical position (white dashed line) during shearing. Not pictured are wiring, plumbing, the controller for the hydraulic ram, winch, LVDT, dial gages, upper platen scale, and heating/cooling circulator for the glycol/water mixture.](image)
(cooling tub of Fig. 2.1) with its temperature precisely regulated to hold the ice at the pressure-melting temperature (PMT) without melting the ice too quickly. Temperature, shear stress, normal stress, ice capacitance as a possible proxy for ice water content, ice-ring thickness, and shearing displacement are measured during experiments. Cumulative deformation of ice is measured after experiments are completed from studying the displacement of markers frozen into the ice. In addition, in the final phase of experiments, the water content of the ice is determined by passing a cold front radially through the ice ring and measuring its speed; detailed discussion of that measurement and its analysis is deferred to the next chapter.

**Apparatus**

At the device’s center is an annular chamber of U-shaped cross-section that can accommodate a ring of ice with an outside diameter of 0.9 m diameter, a width of 0.2 m, and a maximum thickness of ~0.21 m (Fig. 2.1). Toothed platens are secured in place at the base and top of the chamber. The lower platen is stationary, while two geared-down electric motors drive the rotation of the upper platen. The platens are made of polyvinyl chloride (PVC) plastic, which has a thermal conductivity about 10-20 times smaller than most rock. This setup inhibits regelation slip and promotes internal deformation of the ice during shearing. The walls of the chamber are smooth except at four locations where wires attached to the data-acquisition system feed into the chamber. Two cameras are affixed outside the walls behind quartz glass windows at 10 cm above the base of the chamber. The chamber has drainage ports at its base and it sides near its base that connect to conduits that extend to atmospheric pressure outside the device.

The chamber is suspended within an approximately 3 m tall loading frame that supports applied forces and torques with minimal flexure. The steel frame consists of two pillars supported at the base by two crossbeams. Two other crossbeams that support the upper platen, motors, and gearing are secured in place during experiments by removable bolts but can be
raised by a motor-driven winch and cables and affixed in place well above the ice chamber during periods between experiments. The ice chamber, cooling tub, and base plate are kept from rotating with the upper platen by the pillars but can move up or down along them. A hydraulic ram sits on the basal crossbeams of the device and applies a constant upward force (controlled to ~2%) on the base plate that supports the ice chamber and tub. Resultant stresses on the ice normal to the plane of shear can be regulated from 250 to ~1400 kPa. The ram extends to accommodate ice volume decreases from melting and drainage.

The ring-shear device is housed in a cold room kept at 1 ± 1°C during experiments. To insulate the ice chamber from temperature fluctuations in the cold room and thereby maintain a constant heat flux to the ice ring, the ice chamber is submerged to nearly its full height in an ethylene glycol/water mixture that circulates through the surrounding cooling tub (Fig. 2.1). The chamber sits above the tub’s base on legs to allow the glycol/water mixture to circulate beneath the chamber. The fluid temperature is set to a few hundredths of a degree greater than 0 °C, so that meltwater from conduits draining the bed does not freeze, and is regulated with a precision of 0.01°C by an 18 L heating/cooling circulator (Lauda Proline model RP1840). Fluid is pumped by the circulator into the tub at its base and exits through overflow drainage ports at a height near the top of the ice chamber. The upper and lower PVC platens are 7.6 and 1.9 cm thick, respectively. The aluminum chamber walls are 1.5 cm thick and insulated by a layer of polyethylene closed-cell foam (1 cm thick). This insulation reduces the change in heat flux to the ice-ring boundary for a given change in the temperature of the ethylene glycol/water mixture, thereby allowing more delicate adjustment of ice-ring temperature and melt rates. The temperature regulation system allows prolonged (up to several months) experiments by keeping melting of the ice to a minimum while maintaining it at the PMT.
Within the ice ring, temperature is measured with glass bead thermistors (Measurement Specialties model #55004), and capacitance is measured by two soil moisture sensors (METER 10HS). Thermistors are also embedded at various points flush within the inside surfaces of the chamber walls to measure temperature at the sides of the ice ring (Fig. 2.2a). The temperature of the fluid in the cooling tub is measured by a single high-resolution thermistor (0.0001°C).

Ice thickness is measured by a linear variable differential transformer (LVDT) (RDP Group model LDC1000A) that is affixed magnetically to one of the crossbeams at the base of the device frame. The LVDT records changes in the height of the baseplate, which correspond with changes in the thickness of the ice ring.

A load cell affixed to the top of the hydraulic ram’s piston records the vertical force. A torque sensor (specs) records the torque required to rotate the upper platen and thereby shear the ice. From this torque the shear stress supported by the ice is calculated.

Rotational displacement of the upper platen is measured with a horizontal dial gage (0.01 mm precision) in contact with a vertical gusset on the platen. As a check on the dial gage, the upper platen’s rotation is also read from a scale graduated in millimeters at the outer circumference of the upper platen. The ice chamber rotates a small amount during the early stages of ice shear, owing to the compliance of the chamber, base plate, and loading frame. This rotation is measured with another horizontal dial gage (0.0025 mm precision) in contact with a gusset extending down from the base of the chamber. Ice displacement resulting from ice deformation is recorded by four weighted threads frozen into the ice that are initially vertical but rotate as the ice shears (Fig. 2.3). Ice blocks containing these threads rotated from the vertical are extracted from the ice chamber after experiments and used to measure cumulative shear strain of the ice. Due to slip of temperate ice across the surfaces of the upper and lower platen,
Figure 2.2. (a) Schematic of a horizontal cross-section of an ice ring (top view). Not pictured are leads connecting instruments to the data-acquisition system, wall mounted cameras, beads, plumbing, drainage ports, and vertical thread lines. (b) Schematic of the PVC toothed platens (normal view).
displacement from shear deformation of ice can be less than the platen displacement, so post-experimental measurement of thread rotation is critical.

![Figure 2.3](image_url)

**Figure 2.3.** Photographs of ice samples collected after experiment 11. Threads were deflected from initially vertical positions (dashed lines) due to shearing. Purple arrows indicate orientation and magnitude of ice displacement.

**Experimental procedure**

Each experiment begins by constructing the ice ring within the chamber. The freezer that houses the ring-shear device is set to -7 ± 1°C during construction. Each ring is built incrementally with ice layers ranging from 1.5-2 cm in thickness. To build each layer, chilled,
distilled water is added to the ice chamber and then saturated with sieved snow (2 mm sieve) to promote the growth of randomly oriented crystals as the water freezes. During freezing the cold room is set to -7 °C. Water for two of these experiments was doped with low concentrations of pure sodium chloride ($10^{-6}$ g/g) to experiment with the effect of solutes on controlling water content. At mid height in the ice ring, two horizontal transects of thermistors and two capacitance probes are frozen into place (Fig. 2.2a). The thermistor transects are oriented radially, along the width if the ice ring. Once a ring is built to its full height (approx. 0.17 m), four vertical holes are drilled through the ice ring along its radial centerline (Fig. 2.2b). Weighted threads are suspended in each of the drilled holes so that the weights hanging just above the base of each hole (Fig. 2.1, 2.3). Chilled water is then added to each hole to freeze the threads into position.

A final water layer, seeded with snow, is added to the top of the ice ring immediately before it is brought into contact with the upper platen. A normal stress of 0.25-0.35 MPa is applied with the hydraulic ram while this last water/snow layer freezes in contact with the teeth of the upper platen. Once fully frozen, the cold room is to set to $1 \pm 0.8$°C, and the external circulator is set so that coolant in the cooling tub is just above freezing (+0.015 ± 0.010°C). The ice ring then warms to the PMT for approximately three days. Normal stress on the ice is then increased to the value chosen for shearing. The increased pressure results in a decrease in ice temperature that confirms the ice is at the PMT.

With ice at the PMT and normal stress on the ice set, shear of the ice is initiated by rotating the upper platen. It is set to a speed between 0.1–1.5 m a$^{-1}$ at the centerline of the ring (Fig. 2.2b), depending on the strain rate desired in the experiment. Strain rates are limited by the maximum shearing resistance that the ice chamber can sustain (~0.2 MPa) without causing
unsafe flexure of the loading frame. Slip occurs without resistance between the ice ring and the chamber’s walls due to thin water films that are present. Ice rings are generally sheared until shear stress reaches a peak value and then decreases consistently. Shearing experiments carried out to secondary creep varied considerably in length, owing to variable strain rates, with the shortest experiment lasting just three days and the longest lasting 18 days. A single experiment carried out to tertiary creep lasted 64 days. Once a peak stress is confidently established, the upper platen is then stopped, and a cold-front experiment, as detailed in the next chapter, is immediately conducted to determine water content in the ice. Freezing at the ice chamber walls when a cold front is initiated prohibits measuring water content while the ice is shearing.

After a cold-front experiment is finished, experiments are complete. The bath is set to +3°C to decouple the ice ring from the platens. Once the upper platen has been raised from the upper surface of the ice ring and the ice chamber is accessible, ice is removed from the chamber in one of two ways. After some experiments, the whole ring of ice is lifted out of the chamber with ice screws. In other experiments, specific blocks of ice containing the initially vertical threads are excavated from the chamber. This second approach, although more laborious, allows leads to thermistors in the ice to be left uncut at the interface between the ice ring and the ice chamber walls. Once ice is removed from the chamber, the cold room is set to -7°C again so the ice can be processed. Ice is cut into four blocks that each contain a rotated thread. The threads are then photographed and processed digitally to determine maximum thread rotation among the threads from their starting vertical positions (Fig 2.3).

Strain-rate time series are calculated by taking the derivative of the strain with respect to time; strain is computed from the measured platen rotation with time, assuming that the proportion of platen rotation by ice deformation, as indicated by thread deflection, is constant
with time. As experiments progressed, strain rates generally rise to a maximum stable value as the peak stress is attained, so reported average strain rates are those after peak stresses (Fig. 2.4a, b). Strain rate increases in experiments because as platen displacement proceeds, the compliance of the device decreases and its stiffness increases.

Figure 2.4. (a) Time series of shear stress and (b) strain rate during experiment 11. Vertical dashed lines indicate the averaging window used to calculate reported strain rates following the stress peak.

Grain sizes are assessed by photographing large thin-sections of ice through a cross-polarizing lens, with a universal stage in the freezer, to distinguish grain boundaries that can then be studied digitally. Thin sections are made from ice samples cut in the horizontal plane for unsheared ice (Fig. 2.5a) and in the longitudinal flow plane for sheared ice (Fig. 2.5b). Doing the latter orientation was not possible for unsheared ice because only a horizontal ice surface was assessable to sample while building ice rings. Photographs are analyzed in ImageJ, an open-source image processing program (Schneider et al., 2012). Boundaries of individual grains are delineated in the program to measure the planimetric area of each grain (Fig. 2.6).
The mean planimetric grain area for each thin section is calculated by dividing the sum of planimetric grain areas by the total number of measured grains (Fitzpatrick, 2013). The mean planimetric diameter is then calculated as the diameter of a circle of equivalent area. This diameter is then upscaled by a factor of ~1.5 to estimate the mean volumetric diameter from the two-dimensional image (Durand, 2004). Cut off grains at edges of thin sections are not considered in this analysis, and if possible a minimum of 100 grains are counted in each section so that mean diameters are not underestimated (Durand, 2004).

**Figure 2.5.** Grey boxes denote the orientation of ice thin sections from ice sampled (a) before and (b) after shearing; the shear orientation is indicated by yellow arrows.
Figure 2.6. (a) Close up view of a thin section of ice collected before a shearing experiment. The grid square is 10×10 mm. (b) The same thin section with delineated grain boundaries.
CHAPTER 3. WATER CONTENT ANALYSIS

The calorimetric method of Duval (1976a, 1977) and Cohen (1999) is used to determine the water content of the ice. Cold fronts are initiated at approximately the same time at the inner and outer cylindrical walls of an ice ring. Cold fronts are generated by lowering the temperature of the ethylene glycol/water mixture in the cooling tub that surrounds the ice chamber. This cooling is achieved by re-setting the external heating/cooling circulator to a lower temperature. The insulating PVC platens inhibit heat conduction across the upper and lower surfaces of the ice ring, promoting dominantly lateral heat flow through the ice as the fluid temperature of the glycol/water mixture decreases. The speed at which cold fronts advance through the ice is measured by thermistors embedded in the walls, which track temperature at the inner and outer edges of the ice ring, and by thermistors frozen into ice the along radial transects about

Figure 3.1. Schematic cross section of an ice ring at a thermistor transect (side-view normal to flow) showing cold fronts induced by lowering the temperature of the glycol/water mixture surrounding the ice chamber. Not pictured are wiring, plumbing, and insulation.
mid-height in the ice ring (Fig. 3.1). Cold-front arrival times are selected for each thermistor as the moment when the temperature decreases both abruptly and sustainably (Fig 3.2). The decrease indicates that water in the vicinity of the thermistor has frozen.

The speed of a cold front moving through ice is sensitive to its interstitial water content (equivalent to porosity). Water has a high latent heat of fusion, which requires that significant heat must be withdrawn from a given volume of water before it can freeze. Small differences in water content can thus be detected because cold-front speed decreases measurably with increasing water content. Water contents are determined by fitting numerical solutions of a Stefan (moving boundary) heat conduction model (Asaithambi, 1988; Cohen, 1999) to the arrival times of the cold fronts. Four determinations of water content are made for each experiment, with one calculation for each half-transect of thermistors on either side of the ice-ring centerline.

![Figure 3.2](image-url)

**Figure 3.2.** Temperature records of thermistors, at various distances from the ice-chamber wall, during a cold-front experiment. Cold-front arrival times for each thermistor are indicated by arrows.
Analysis overview

The method used to determine water content follows closely that of Cohen (1999) who modeled a one-dimensional moving phase boundary through temperate ice. The initial condition for the model is the ice at its pressure-melting temperature \((\theta_m)\). A heat sink is created on the inner and outer walls of the ice ring, which creates a freezing phase boundary. The heat sink has a known temperature-time relationship, as measured by thermistors mounted flush with the surfaces of the walls in contact with the ice. The phase boundary moves away from the heat sink and into the ice, thus separating a zone of cold ice near the lateral walls of the ice ring from a zone of temperate ice away from the walls. The following assumptions are made in the analysis:

1. The ice is a homogenous ice-water mixture.
2. Thermal conductivity, specific heat capacity, and density of ice are fixed values.
3. The ice ring is perfectly insulated at its top and bottom surfaces. The problem is, therefore, considered to be one-dimensional, so heat flux is strictly horizontal across the width of the ice ring.
4. The heat sink has a uniform temperature over the area of the ice-ring walls and has perfect thermal contact with the ice (i.e., no drop in temperature across the ice-aluminum contact).

Ice in these experiments is never composed of pure \(H_2O\) due to air bubbles, and in two cases, added NaCl. These impurities are ignored because deviations are mathematically insignificant, such as variations in thermal properties due to trace amounts of NaCl. Water contents are not necessarily spatially uniform, but the analysis applied incrementally across the ice ring’s width can help determine this variability. Heat flux can be approximated as strictly horizontal because, as discussed, the upper and lower PVC platens inhibit heat flow. Cold fronts propagate from both the inner and outer walls of the ice ring at the same time. Cold-front speeds
are analyzed separately over opposing halves of the ice ring’s width to arrive at two water-content determinations for each thermistor transect (Fig. 3.1); cold-front movement on one side of the interior temperate zone is independent of that on the other side because temperate ice cannot serve as a heatsink (i.e., all heat conduction is toward the walls of the ice ring). In the heat sink in our experiments, the aluminum walls of the ice chamber, temperature is not uniform vertically, owing to the overall upward circulation of the ethylene glycol/water mixture in the cooling tub, resulting in vertical temperature differences of a few hundredths of a degree C.

**Mathematical formulation**

The heat equation is used to describe the temperature distribution in the ice with time for a given half-width of the ice ring.

Let,

\[ \theta = \text{temperature} \]
\[ \lambda = \text{geometry constant} \]
\[ \theta_c = \text{heat sink temperature} \]
\[ \rho = \text{density of ice} \]
\[ \theta_m = \text{phase boundary temperature} \]
\[ K = \text{thermal conductivity of ice} \]
\[ t = \text{time} \]
\[ c = \text{specific heat capacity of ice} \]
\[ \zeta = \text{radial position} \]
\[ \alpha = \text{thermal diffusivity} \]
\[ \zeta_c = \text{heat sink radius} \]
\[ L = \text{latent heat of fusion of water} \]
\[ \sigma = \text{moving phase boundary radius} \]
\[ w = \text{water content}. \]

The moving-boundary value or Stefan problem can be described as

\[ \frac{\partial \theta}{\partial t} = \alpha \frac{\partial^2 \theta}{\partial \zeta^2} + \alpha \frac{(\lambda - 1)}{\zeta} \frac{\partial \theta}{\partial \zeta}, \quad \alpha = \frac{K}{\rho c}, \quad (3.1a) \]

with boundary conditions:

\[ \theta = \theta_c(t), \quad \zeta = \zeta_c, \quad (3.1b) \]

at the ice-chamber wall and
at the moving freezing boundary. Equations 3.1b establish the heat sink with temperature, $\theta_c$, (wall thermistor temperature record) that decreases with time, $\tau$, at the ice-wall interface, $\zeta_c$.

Equations 3.1c describe the position, $\sigma$, of the moving freezing boundary, with temperature $\theta_m$, at a specified time step as the function of the latent heat of water, water content, ice density, the change in position of the cold front from its last location, and the thermal conductivity. The initial conditions are

$$\theta(\zeta_c, t) = \theta_c(t), \quad t = 0$$

(3.1d) at the ice-chamber walls and

$$\sigma(t) = \zeta_c, \quad t = 0.$$  

(3.1e) at the moving freezing boundary. Equations 3.1d establish the initial temperature at the ice-chamber wall as the temperature of the heat sink. Equation 3.1e establishes the initial radial position of moving phase boundary at the ice-chamber wall. The geometric constant $\lambda$ has a value of either 1, 2, or 3 for Cartesian, cylindrical, and spherical geometries. The goal is to find $\theta(\zeta, t)$ and $\sigma(t)$, for $\zeta_c \leq \zeta \leq \sigma(t), t > 0$ (Fig. 3.3a).
Figure 3.3. (a) The moving freezing boundary problem without dimensional reduction and coordinate transformation. Circles a and b represent thermistors embedded in the chamber wall and ice, respectively. (b) Temperature records for thermistors a and b showing the passage of the cold front. Black arrows indicate cold-front arrival times.
Dimensional reduction

The problem is simplified by dimensionally reducing parameter values. Temperature, position, and time become

\[
\begin{align*}
u & \equiv \frac{(\theta - \theta_m)}{\theta_o}, \\
x & \equiv \frac{\zeta}{\zeta_o}, \\
t & \equiv \frac{t}{t_o},
\end{align*}
\]

respectively, where

\[
\begin{align*}
\theta_o &= \frac{Lw}{c}, \\
\zeta_o &= \sqrt{\alpha}, \\
t_o &= 1 \text{ second}.
\end{align*}
\]

The heat equation is then written as

\[
\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{(\lambda - 1)}{x} \frac{\partial u}{\partial x'},
\]

with boundary conditions, where \(s\) is the dimensionless moving phase boundary radius,

\[
\begin{align*}
u &= u_c(t), \\x &= x_c, \\
\frac{\partial u}{\partial x} &= \Delta s, \\x &= s(t), \\
u &= 0,
\end{align*}
\]

and initial conditions

\[
\begin{align*}
u(x_c, 0) &= u_c(0), \\
\end{align*}
\]

\[
\begin{align*}
x(t) &= \frac{\sigma(t)}{\zeta_o}.
\end{align*}
\]

The goal is to find \(u(x, t)\) and \(s(t)\), for \(x_c \leq x \leq s(t), t > 0\).
Coordinate transformation

With the variables dimensionally reduced, the moving phase boundary is then eliminated to make the problem tractable. The following transformation was proposed by Crank (1984):

\[ \xi = \frac{x - x_c}{s - x_c}, \quad 0 \leq \xi \leq 1. \tag{3.3} \]

The heat equation is then

\[ \frac{\partial u}{\partial t} = (s - x_c)^{-2} \frac{\partial^2 u}{\partial \xi^2} + \frac{\partial u}{\partial \xi} \left( \frac{(\lambda - 1)}{(s - x_c)(s - x_c) + x_c} + \frac{\xi \Delta s}{s - x_c} \right), \tag{3.4a} \]

with boundary conditions

\[ u = u_c(t), \quad \xi = 0, \tag{3.4b} \]

\[ \frac{\partial u}{\partial \xi} = \Delta s(s - x_c), \quad \xi = 1, \tag{3.4c} \]

and initial conditions

\[ u(x_c, 0) = u_c(0), \tag{3.4d} \]

\[ s(0) = x_c. \tag{3.4e} \]

The problem is then to find \( u(\xi, t) \) and \( s(t) \), for \( 0 \leq \xi \leq 1, t > 0 \).

Finite difference method

Equations 3.4 are solved using a numerical method proposed by Asaithambi (1988) based on the variable time-step method. The time for the phase boundary to move an increment in position is calculated iteratively. Discretization of equations 3.4 yields:
\[ u^{(k)}_{i-1,j+1}(\phi_i^{(k)} - \psi_i^{(k)}) + u^{(k)}_{i,j+1}(1 + 2 \cdot \psi_i^{(k)}) + u^{(k)}_{i+1,j+1}(-\phi_i^{(k)} - \psi_i^{(k)}) = u_{i,j}, \] (3.5a)

where

\[ \psi_i^{(k)} = \frac{\Delta t^{(k)}}{\Delta \xi^2 (s_{j+1} - x_c)^2}, \] (3.5b)

\[ \phi_i^{(k)} = \frac{(\lambda - 1) \Delta t^{(k)}}{2 \Delta \xi (s_{j+1} - x_c)(\xi \star (s_{j+1} - x_c) + x_c)} + \frac{\xi \Delta s}{2 \Delta \xi (s_{j+1} - x_c)}, \] (3.5c)

\[ u_{i,j} = g(t_j + \Delta t^{(k)}), \quad \xi = 1. \] (3.5d)

The discretization includes a central difference approximation for space, and three point forward/backward differences at boundaries. The dimensionless spatial mesh is divided into \( \Delta \xi \) subequal intervals. Index \( i \) denotes each interval. An appropriate value for \( \Delta t^{(k)} \), the timestep, is chosen to initiate the iteration for \( \Delta s \), an increment in the position of the phase boundary. Each increment is denoted by index \( j \). The superscript \( k \) denotes the \( k \)th iteration. Temperature at the phase boundary is set to the pressure-melting temperature by the forcing function \( 3.5d \).

Solving this boundary value problem with a tridiagonal solver produces the temperature distribution across the fixed dimensionless domain. The temperature gradient close to the phase boundary is then used to solve for the next iteration of the timestep using equation \( 3.6 \), which describes the boundary condition at \( \xi = 1 \). The temperature gradient is defined by the temperature at \( \xi = 1 \), \( u^{(k)}_{j+1,n} \), and the two preceding temperatures \( u^{(k)}_{j+1,n-1} \) and \( u^{(k)}_{j+1,n-2} \). The timestep is iterated until a desired convergence (\( \epsilon \)) is achieved.
\[
\Delta t^{(k+1)} = \frac{2(s_{j+1} - x_c) * \Delta s * \Delta \xi}{\sigma_s (3u^{(k)}_{j+1,n} - 4u^{(k)}_{j+1,n-1} + u^{(k)}_{j+1,n-2})} \quad (3.6)
\]

\[
|\Delta t^{(k+1)} - \Delta t^{(k)}| > \varepsilon.
\]

The method is summarized as follows:

1. For a known timestep (\(\Delta t^{(k)}\)) of the next position of the phase boundary (\(s_{j+1}\)), solve equations 3.5 using a tridiagonal solver to obtain the temperature profile (\(u^{(k)}_{i,j+1}\)).

2. Compute a refined value of the timestep (\(\Delta t^{(k+1)}\)).

3. Repeat steps 1 and 2 until \(|\Delta t^{(k+1)} - \Delta t^{(k)}| < \varepsilon\).

After desired convergence is achieved, time and position are dimensionally restored by reversing the earlier listed reductions to construct a position-time curve (modeled cold front).

**Fitting model results to experimental data**

Various water contents (0.01\% resolution) are considered in the model to generate a series of modeled cold-front curves for comparison with cold-front arrival times. The sum of squared residuals (RSS) is calculated for each curve in comparison to arrival times to determine which curve has the smallest RSS (best fit). The water content of the best fitting curve is thus selected as representative for the considered half transect. Figure 3.4 displays the results of one such fitting exercise. Residual standard error for the best fit is calculated to assess how well the modeled cold front matches arrival times from the real cold front. Water contents associated with the best fit from each half transect are then averaged to determine a mean total water content for each experiment, and variability is determined by calculating one standard deviation. Spatial uniformity of water content is assessed by fitting model curves to cold-front arrival times from selected thermistors in a transect instead of all thermistors in a transect.
Figure 3.4. Cold-front arrival times best fit with a modeled curve by tuning the ice water content in the model. The cold front initiated at the outer edge of the ice ring (0.45 m, referenced to its center) and propagated inward toward its centerline (0.35). The red dashed arrows indicate tuning directions. A curve with a lower water content would shift to the left and vice versa.
CHAPTER 4. RESULTS

Experiments were conducted by measuring shear resistance at controlled strain rates and various water contents. Ice in an initial experiment was sheared to a strain of ~17% (tertiary creep, Fig. 4.1) and at different rates; water content was measured at the end of that experiment. Ice in subsequent experiments was sheared to only secondary creep (Fig. 4.2a-4.2k), and water contents were measured after peak stresses were attained to minimize effects of recrystallization. Peak stresses in these experiments were attained at strains of 1.7-6.8%, peak stresses varied from 0.059 to 0.20 MPa, and strain rates (during measurement of peak stresses) varied from $1.1 \cdot 10^{-8}$ to $12.0 \cdot 10^{-8}$ s$^{-1}$ (Table 4.1). At the beginning of some experiments, strain rate was set higher to shorten the time to reach the peak stress (Fig. 4.2a-4.2e). Time-averaged strain rates measured from the deflection of initially vertical threads in the ice indicate that slip at the upper and lower platens varied among experiments from 3 to 62% of the total motion of the platens at the centerline (Table 4.1).

![Figure 4.1](image-url)  
**Figure 4.1.** Shear stress plotted against strain for experiment 1. The blue dashed line indicates the peak stress at the initial strain rate. Purple dash-dot lines indicate times when the beginning of steady state creep was inferred. Red dotted lines indicate when strain rate was changed.
Figure 4.2. Shear stress plotted against strain for each peak-stress experiment. Blue dashed lines indicate the timing of the peak stress. Red dotted lines indicate when strain rate was decreased. Black line in experiment 3 indicates a power outage. A strain-rate reduction during experiment 9 was required to avoid stresses in excess of those considered to be safe for the loading frame of the device.
Figure 4.2 continued
Table 4.1. Summary of parameters and measurements during experiments. Strain rates are derived from data collected after peak stresses. Reported errors are ± one standard deviation.

<table>
<thead>
<tr>
<th>Tertiary creep experiment</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp</td>
<td>Confining pressure (MPa)</td>
<td>Shear duration (days)</td>
<td>Total strain (%)</td>
<td>Total motion from slip (%)</td>
<td>Strain rate ($10^{-8}$ s$^{-1}$)</td>
<td>Steady-state shear stress (MPa)</td>
</tr>
<tr>
<td>1</td>
<td>0.70 ± &lt;0.01</td>
<td>63.8</td>
<td>17.0</td>
<td>57</td>
<td>1.3 ± 0.6</td>
<td>0.10 ± &lt;0.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2.4 ± &lt;0.1</td>
<td>0.14 ± &lt;0.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3.5 ± 0.1</td>
<td>0.14 ± &lt;0.01</td>
</tr>
<tr>
<td>Peak stress experiments</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Peak shear stress (MPa)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.70 ± &lt;0.01</td>
<td>63.8</td>
<td>17.0</td>
<td>57</td>
<td>1.1 ± 0.5</td>
<td>0.13</td>
</tr>
<tr>
<td>2</td>
<td>0.71 ± &lt;0.01</td>
<td>11.7</td>
<td>3.7</td>
<td>7</td>
<td>2.9 ± 0.7</td>
<td>0.094</td>
</tr>
<tr>
<td>3</td>
<td>0.69 ± &lt;0.01</td>
<td>13.0</td>
<td>3.0</td>
<td>34</td>
<td>2.0 ± 0.1</td>
<td>0.11</td>
</tr>
<tr>
<td>4</td>
<td>1.30 ± &lt;0.01</td>
<td>7.0</td>
<td>3.3</td>
<td>3</td>
<td>3.3 ± 0.4</td>
<td>0.070</td>
</tr>
<tr>
<td>5</td>
<td>0.70 ± &lt;0.01</td>
<td>10.0</td>
<td>1.9</td>
<td>54</td>
<td>1.5 ± &lt;0.1</td>
<td>0.10</td>
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<tr>
<td>6</td>
<td>0.91 ± &lt;0.01</td>
<td>18.0</td>
<td>4.4</td>
<td>2</td>
<td>1.8 ± 0.1</td>
<td>0.085</td>
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<tr>
<td>7</td>
<td>1.19 ± &lt;0.01</td>
<td>16.0</td>
<td>6.9</td>
<td>24</td>
<td>2.9 ± 0.3</td>
<td>0.059</td>
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<tr>
<td>8</td>
<td>1.20 ± &lt;0.01</td>
<td>11.0</td>
<td>8.4</td>
<td>13</td>
<td>6.0 ± 0.3</td>
<td>0.13</td>
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<tr>
<td>9</td>
<td>1.40 ± &lt;0.01</td>
<td>3.3</td>
<td>5.4</td>
<td>21</td>
<td>12.0 ± 0.6</td>
<td>0.20</td>
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<tr>
<td>10</td>
<td>1.40 ± &lt;0.01</td>
<td>5.0</td>
<td>5.5</td>
<td>46</td>
<td>8.6 ± 0.4</td>
<td>0.20</td>
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<tr>
<td>11</td>
<td>1.40 ± &lt;0.01</td>
<td>8.1</td>
<td>6.4</td>
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<td>5.8 ± 0.2</td>
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<tr>
<td>12</td>
<td>0.33 ± &lt;0.01</td>
<td>19.0</td>
<td>2.9</td>
<td>62</td>
<td>1.1 ± &lt;0.1</td>
<td>0.10 ± &lt;0.01</td>
</tr>
</tbody>
</table>

Notes: Exp 2 & 3 had initial salinities of $1 \cdot 10^{-6}$ & $2 \cdot 10^{-6}$ g/g respectively

Water content

Different combinations of confining pressure and strain rate were applied to attempt to vary the water content among experiments through as wide a range as possible. Resultant mean water contents varied from 0.21% to 1.70% (Table 4.2), which significantly exceeded the upper limit of water content measured by Duval (1977) (0.008-0.80%). Efforts in experiments 2 and 3 (Table 4.1) to vary water content by making ice with different salt concentrations were not successful. Salt concentrations in the ice changed with time owing to preferential release of salt early in experiments (Fig. 4.3).
Table 4.2. Water content determined from each experiment by fitting the cold-front model to cold-front arrival times along thermistor transects spanning 9 cm. IW and OW denote whether the inner or outer wall was the origin of the cold front. Reported errors are ± one standard deviation.

<table>
<thead>
<tr>
<th>Exp</th>
<th>Transect ID</th>
<th>Residual standard error of fit (hour)</th>
<th>Cold front arrival time at 9 cm (hour)</th>
<th>Water content (%)</th>
<th>Mean water content (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1-OIW</td>
<td>&lt;0.1</td>
<td>1.6</td>
<td>0.25</td>
<td>0.31 ± 0.08</td>
</tr>
<tr>
<td></td>
<td>2-OIW</td>
<td>0.2</td>
<td>1.9</td>
<td>0.37</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2-IW</td>
<td>1.7</td>
<td>6.1</td>
<td>0.46</td>
<td>0.45 ± 0.01</td>
</tr>
<tr>
<td></td>
<td>2-OIW</td>
<td>0.8</td>
<td>4.9</td>
<td>0.44</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2-IW</td>
<td>0.9</td>
<td>6.3</td>
<td>0.41</td>
<td>0.44 ± 0.04</td>
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<tr>
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<td>1.4</td>
<td>5.0</td>
<td>0.46</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1-IW</td>
<td>3.8</td>
<td>10.2</td>
<td>1.58</td>
<td>1.70 ± 0.23</td>
</tr>
<tr>
<td></td>
<td>1-OIW</td>
<td>3.2</td>
<td>11.1</td>
<td>2.00</td>
<td></td>
</tr>
<tr>
<td></td>
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<td>3.1</td>
<td>10.6</td>
<td>1.47</td>
<td></td>
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<tr>
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<td>9.8</td>
<td>1.74</td>
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<td>0.38</td>
<td>0.34 ± 0.03</td>
</tr>
<tr>
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<td>3.9</td>
<td>0.31</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2-IW</td>
<td>0.6</td>
<td>4.9</td>
<td>0.32</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2-OIW</td>
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<td>4.4</td>
<td>0.34</td>
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<td>6</td>
<td>1-IW</td>
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<td>5.0</td>
<td>0.45</td>
<td>0.55 ± 0.09</td>
</tr>
<tr>
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<td>5.6</td>
<td>0.58</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2-OIW</td>
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<td>5.4</td>
<td>0.61</td>
<td></td>
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<tr>
<td>7</td>
<td>1-IW</td>
<td>1.7</td>
<td>6.1</td>
<td>0.80</td>
<td>0.75 ± 0.09</td>
</tr>
<tr>
<td></td>
<td>2-IW</td>
<td>1.5</td>
<td>6.1</td>
<td>0.80</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2-OIW</td>
<td>0.6</td>
<td>4.9</td>
<td>0.64</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>1-IW</td>
<td>1.4</td>
<td>5.8</td>
<td>0.71</td>
<td>0.64 ± 0.05</td>
</tr>
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<td>1-OIW</td>
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<td>4.7</td>
<td>0.63</td>
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</tr>
<tr>
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<td>2-IW</td>
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<td>6.0</td>
<td>0.60</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2-OIW</td>
<td>1.4</td>
<td>4.9</td>
<td>0.62</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>1-IW</td>
<td>2.6</td>
<td>9.2</td>
<td>1.30</td>
<td>1.22 ± 0.07</td>
</tr>
<tr>
<td></td>
<td>1-OIW</td>
<td>1.6</td>
<td>8.0</td>
<td>1.22</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2-IW</td>
<td>1.5</td>
<td>9.8</td>
<td>1.20</td>
<td></td>
</tr>
<tr>
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<td>2-OIW</td>
<td>2.9</td>
<td>7.8</td>
<td>1.14</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1-IW</td>
<td>1.3</td>
<td>9.2</td>
<td>1.23</td>
<td>1.33 ± 0.07</td>
</tr>
<tr>
<td></td>
<td>1-OIW</td>
<td>0.2</td>
<td>9.5</td>
<td>1.38</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2-IW</td>
<td>1.5</td>
<td>10.0</td>
<td>1.35</td>
<td></td>
</tr>
<tr>
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<td>2-OIW</td>
<td>&lt;0.1</td>
<td>9.4</td>
<td>1.35</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>1-IW</td>
<td>3.1</td>
<td>9.2</td>
<td>1.39</td>
<td>1.65 ± 0.26</td>
</tr>
<tr>
<td></td>
<td>1-OIW</td>
<td>2.1</td>
<td>10.6</td>
<td>1.90</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2-IW</td>
<td>2.1</td>
<td>11.0</td>
<td>1.46</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2-OIW</td>
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<td>10.7</td>
<td>1.85</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>1-IW</td>
<td>0.4</td>
<td>3.2</td>
<td>0.22</td>
<td>0.21 ± 0.02</td>
</tr>
<tr>
<td></td>
<td>1-OIW</td>
<td>0.2</td>
<td>3.0</td>
<td>0.21</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2-IW</td>
<td>0.4</td>
<td>3.3</td>
<td>0.22</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2-OIW</td>
<td>0.5</td>
<td>2.7</td>
<td>0.18</td>
<td></td>
</tr>
</tbody>
</table>

Notes: 1-OIW and 2-OIW thermistor lines of experiment 1 were 8.4 and 8.1 cm long, respectively.
**Figure 4.3.** Record of salt draining from the ice ring of Experiment 3 with time. The ring initially contained 1.5 grams of salt with a total concentration of $2 \times 10^{-6}$ g/g. The dashed line marks the beginning of the shearing experiment.

Although water content was not tightly correlated to any single control variable, ice pressure had the clearest influence on the average water content of the ice ring (Table 4.2). Positive correlation between water content and confining pressure is demonstrated by multiple groups of experiments with similar strain rates (Fig. 4.4a). Water content also correlates positively to the durations of experiments with the same ice pressure (Fig. 4.4b) but only strongly for short experiments with high water contents. Water content was inversely related to strain rate in sets of experiments with the same ice pressure (Fig. 4.4c), but again only strongly for short experiments (high strain rates) with high water contents.

Water content was not uniform in ice rings; rather, it generally increased towards the edges of the ring (Fig. 4.5). Fits of the cold-front model to selected thermistors in a transect, rather than to all the thermistors of a transect, allow water content to be estimated as function of distance along the transect. Experiments 10 and 11 were exceptions, wherein water content was highest midway between the centerline and edges (Fig. 4.5). Although water contents have
Figure 4.4. Mean water content as a function of (a) confining pressure, (b) shear duration, and (c) strain rate. Error bars are ± one standard deviation. Experiments in part a are grouped by strain rate, and experiments in parts b and c are grouped by confining pressure.
considerable variability, there is a clear trend toward higher water content near the ice-ring edges.

Capacitance probes in the ice, aimed at providing a continuous proxy measurement of water content, provided data poorly correlated to water content determined using the calorimetric (cold-front) method. One capacitance probe yielded a direct correlation but with two major outliers (Fig. 4.6a). The other probe yielded no significant correlation (Fig. 4.6b). As a result, data from the probes cannot be used to determine water contents or their changes with time.
Figure 4.6. Summary of time-averaged output from the two capacitance probes (a and b) after peak stress compared to mean water content measured using the calorimetric (cold-front) method. Error bars indicate ± one standard deviation.

Grains

Mean grain diameters during the experiments conducted to secondary creep increased by a factor of 2.7-4.2. Initial mean grain diameters averaged 2.8-3.9 mm for experiments in which ice was sampled prior to shearing and thin-sectioned, and standard deviations of diameters were 1.0-1.6 mm (Table 4.3, Fig. 4.7a, b). Significant differences for initial mean grain sizes of unstudied ice samples are not expected because the variation of grain sizes among studied samples is small (Table 4.3, Fig 4.8a), and ice-ring construction techniques did not vary among experiments. Mean grain diameters measured at the ends of experiments carried out to secondary creep averaged 8.3-12.5 mm, with standard deviations of 4-8 mm (Table 4.3, Fig. 4.7c, d). The distribution of planimetric grain areas before and after shearing highlights how grain size and its variability increased during deformation (Fig. 4.8a-d).

Grain sizes at the end of experiments correlated inversely to water content and strain rate, while correlating positively to shearing duration (Fig. 4.9a-c). All correlations, however, were
Table 4.3. Summarized grain data from thin sections of ice samples collected before and after shearing. Reported errors are ± one standard deviation.

<table>
<thead>
<tr>
<th>Exp</th>
<th>Mean volumetric grain diameter (mm)</th>
<th>Number of grains sampled (count)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Un-sheared</td>
<td>Sheared</td>
</tr>
<tr>
<td>1</td>
<td>3.9 ± 1.4</td>
<td>12.6 ± 7.4</td>
</tr>
<tr>
<td>2</td>
<td>3.9 ± 1.6</td>
<td>11.0 ± 6.3</td>
</tr>
<tr>
<td>3</td>
<td>12.5 ± 8.0</td>
<td>10.3 ± 6.4</td>
</tr>
<tr>
<td>4</td>
<td>-</td>
<td>10.4 ± 5.7</td>
</tr>
<tr>
<td>5</td>
<td>3.9 ± 1.5</td>
<td>10.4 ± 5.5</td>
</tr>
<tr>
<td>6</td>
<td>3.1 ± 1.3</td>
<td>10.5 ± 5.8</td>
</tr>
<tr>
<td>7</td>
<td>3.6 ± 1.4</td>
<td>10.1 ± 5.3</td>
</tr>
<tr>
<td>8</td>
<td>-</td>
<td>8.3 ± 4.2</td>
</tr>
<tr>
<td>9</td>
<td>2.8 ± 1.1</td>
<td>10.9 ± 6.1</td>
</tr>
<tr>
<td>10</td>
<td>2.9 ± 1.2</td>
<td>10.2 ± 5.1</td>
</tr>
<tr>
<td>11</td>
<td>2.9 ± 1.0</td>
<td>11.9 ± 6.9</td>
</tr>
</tbody>
</table>

The character of grain boundaries differed before and after shearing to secondary creep. Un-sheared ice grains were relatively equant with straight, smooth grain boundaries (Fig 4.8a, b). Grain boundaries of large crystals (~ >10 mm diameter) after shearing were highly irregular and boundaries of large grains in some cases intruded into adjacent grains. The shapes of smaller sheared crystals were more like that of the unsheared grains, but boundaries were more irregular (Fig 4.8c, d).
Figure 4.7. Photographs of un-sheared ice grains (a and b, top view) and sheared ice grains (c and d, side view of longitudinal flow plane) from experiments 6 (a and c) and 12 (b and d) under polarized light. Grid squares are 10 × 10 mm. Arrows indicate the orientation of shear.
Figure 4.8. Planimetric grain area distributions from ice thin sections of samples collected before (a) and after shearing (b). The bottom and top of the boxes mark the 25th and 75th percentiles, respectively (50% of grains fall within). The horizontal black bars within the boxes indicate the median grain area. Whiskers indicate the 10th and 90th percentiles (80% of grains fall within). Black dots indicate grain areas below and above the whisker percentiles (remaining 20% of grains).
Figure 4.9. Mean volumetric grain diameter (sheared ice from peak-stress experiments) as a function of (a) water content, (b) strain rate, and (c) shear duration. Error bars are ± one standard deviation.
Effective viscosity

Effective ice viscosity for each experiment was computed to characterize the effect of water content on ice rheology. Glen’s power-law flow rule for the case of isothermal ice reduces to

\[ \dot{\varepsilon} = A\tau^n, \]

(4.1)

where \( \dot{\varepsilon} \) is the effective strain rate, \( A \) is a prefactor that scales inversely with ice viscosity, \( \tau \) is effective shear stress, and \( n \) is the stress exponent (Glen, 1955). Although the value of \( n \) is usually taken to be 3.0, herein no assumption is made about its value and hence the prefactor \( A \) in the flow law cannot be calculated from measured strain rates and stresses. Alternatively, the flow rule can be written as

\[ \tau = 2\eta\dot{\varepsilon}, \]

(4.2)

where the effective ice viscosity, \( \eta \), is given by

\[ \eta = \frac{1}{2} [A\tau^{n-1}]^{-1}. \]

(4.3)

Writing the flow law this way combines the two unknown variables \( A \) and \( n \) together into a single variable, \( \eta \), that can be calculated from the measured strain rate and peak stress using Equation 4.2 (Cuffey and Paterson, 2010).

Plotting effective viscosity as a function of the mean water content of the ice ring indicates two regimes of sensitivity to water content (Fig. 4.10). At water contents below \(~0.6\%\), effective viscosity decreases markedly with increasing water content. At water contents above this value, effective viscosity is essentially independent of water content. Effective viscosities computed from Duval’s data (Duval, 1977), collected in shear experiments with solute-bearing glacier ice and conducted to tertiary creep, are a factor of 1.2 to 3.6 lower than effective viscosities measured in peak-stress experiments over the equivalent water-content range (Fig. 4.10).
Regressing both data sets with exponential relations over the range of water content explored by Duval (up to 0.8%) indicates lower sensitivity to water content in Duval’s experiments over that range by a factor of 1.8. The one experiment from this study conducted to tertiary creep plots closest to the trend of Duval’s data but is not considered in either regression.

Figure 4.10. Effective viscosity as a function of water content. Error bars are ± one standard deviation. The Duval (1977) data reflect stresses and strain rates in tertiary creep during shear of glacier ice with the grain sizes of predominately 4-10 mm. The one tertiary-creep data point from the experiments described herein (red) is not considered in either regression.
CHAPTER 5. DISCUSSION

Controls on water content

Measured water content reflects the balance between internal meltwater production and flow of water out of the ice ring. Meltwater production within the ring is driven by at least two sources: strain heating and the thermodynamic requirement that the pressure-melting temperature decreases with increasing pressure. Water flow toward the edges of the ring is driven by the normal stress on the ice squeezing out water toward water films at the perimeter of the ice ring. These water films, which divide the ice ring from the walls of the ice chamber, connect to drainage ports at the ice-ring’s base where water can escape to atmospheric pressure.

Steady-state water content was likely not achieved as experiments to peak stresses progressed. Meltwater retention increased with experimental duration for experiments with equivalent confining pressures (Fig. 4.4b). Water content was measured shortly after stress peaked in each experiment, so the measured dependence of ice viscosity on water content is, nevertheless, valid (Fig. 4.10).

External heat that flows to the ice ring can melt only its edges, so the only obvious source of heat within the ice during experiments is from strain heating; however, a positive dependence of water content on strain heating was not observed (Fig. 4.4c). For simple shear, the rate of strain heating (\( \dot{\varepsilon} = \text{work/time} \)) can be described as

\[
\dot{\varepsilon} = \tau \cdot dx \cdot dy \cdot \dot{\varepsilon} \cdot dz,
\]

where \( \tau \cdot dx \cdot dy \) is the exerted force equal to the shear stress, \( \tau \), applied over an area, \( dx \cdot dy \), and \( \dot{\varepsilon} \cdot dz \) is the average shearing speed, \( u \), with the strain rate, \( \dot{\varepsilon} = \frac{1}{2} du/dz \) (Hooke, 2020).

If water flow out of the ice ring is neglected, Equation 5.1 allows the maximum water accumulation in the ice to be calculated for each experiment. The change in ice water content, \( w \)
(= ice porosity) with time, \( \partial w / \partial t \), is calculated by dividing Equation 5.1 by the ice density, \( \rho \) (916 kg m\(^{-3}\)), latent heat of fusion, \( L \) \((3.34 \times 10^5 \text{ J kg}^{-1}\)) (Hooke, 2020), and ice volume, \( dx \cdot dy \cdot dz \) to obtain

\[
\frac{\partial w}{\partial t} = \frac{\tau \cdot \dot{e}}{\rho \cdot L}.
\] (5.2)

The maximum water content is then the product of \( \partial w / \partial t \) and the duration of shearing. Water content generated by strain heating using peak stress and strain rates from each experiment is negligibly small, < 0.01\%, for all experiments (Table 5.1), and much smaller than measured values (Table 4.2). Therefore, experimental results do not show a positive dependence on strain heating (Fig. 4.4c) because meltwater produced by this mechanism is less than the resolution of the measurement technique and apparently masked by the dependence of water content on other parameters.

**Table 5.1.** Cumulative interstitial meltwater produced by strain heating alone assuming no flow of water out of the ring.

<table>
<thead>
<tr>
<th>Exp</th>
<th>Water content (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0091</td>
</tr>
<tr>
<td>2</td>
<td>0.0011</td>
</tr>
<tr>
<td>3</td>
<td>0.0009</td>
</tr>
<tr>
<td>4</td>
<td>0.0005</td>
</tr>
<tr>
<td>5</td>
<td>0.0005</td>
</tr>
<tr>
<td>6</td>
<td>0.0008</td>
</tr>
<tr>
<td>7</td>
<td>0.0009</td>
</tr>
<tr>
<td>8</td>
<td>0.0024</td>
</tr>
<tr>
<td>9</td>
<td>0.0023</td>
</tr>
<tr>
<td>10</td>
<td>0.0025</td>
</tr>
<tr>
<td>11</td>
<td>0.0016</td>
</tr>
<tr>
<td>12</td>
<td>0.0006</td>
</tr>
</tbody>
</table>

Water content increased with confining pressure (Fig. 4.4a). This is expected, if again flow of water out of the ice ring is neglected, because the temperature of melting ice decreases with increasing pressure; thus, ice must melt to remove the sensible heat from the ice and store it as latent heat in meltwater. The change in water content per unit change in pressure is described as

\[
\frac{dw}{d\sigma_n} = \frac{P_T c}{L},
\] (5.3)

where \( P_T \) is the depression of the pressure-melting temperature with pressure, \( c \) is the specific heat capacity of ice at 0°C (2096 J kg\(^{-1}\) K\(^{-1}\)) (Hooke, 2020), and \( L \) is the latent heat of fusion for
water. Harrison (1972) found that $P_T = 0.098 \text{ K MPa}^{-1}$ for pure ice containing air bubbles (Harrison, 1972; Hooke, 2020). Thus, if no water is expelled from ice, water content is expected to linearly increase with increasing pressure. However, this effect can account for only a tenth of measured water-content increase with pressure at lower pressures and does not account for the apparent nonlinear increase of water content at higher confining pressures (Fig. 4.4a).

Ultimately a good theoretical estimate of water content and its dependence on experimental duration, strain rate, pressure, or other factors is not possible because data necessary to estimate the discharge of water into or out of the ice are not available. Most notably, ice permeability and its dependence on water content would need to be known. Despite these uncertainties measured water contents were significantly higher in the experiments than can be justified from ice melting due to either strain heating or changes in pressure on the ice. A possible but untestable explanation is that meltwater, generated at the upper surface of the ice ring and heated slightly above the pressure-melting temperature, may have flowed downward into the ice and out towards the walls; this flow could have caused significant melting by both advection of heat and viscous dissipation.

**Distribution of water content in the ice ring**

There was a general tendency for water content to increase toward the lateral walls of the ice ring (Fig. 4.5). Shear stress measured by the torque sensor during experiments reflects stress integrated over the full plan-view area of the ice ring, so the non-uniform distribution of water content did not affect the measurement of stress, as long as variations were symmetric about the centerline of the ice ring. The increase in water content toward the inner and outer walls of the ice chamber likely is a result of a component of water flow toward the walls through the ice. Water discharge through veins at three-grain intersections (Mader, 1992) would increase toward the walls because the spatial integral of upstream meltwater production would increase toward
the walls. Assuming a uniform potential gradient driving water flow, this increasing discharge would require veins with larger cross-sectional areas near the walls and hence higher water contents.

**Crystal growth**

Empirically determined crystal growth rates at the PMT in these experiments were orders of magnitude higher than have been measured in cold ice, presumably due to melt films at grain boundaries aiding the diffusion of matter during recrystallization (Fig. 5.1) (Jacka and Li, 1994; Cuffey and Patterson, 2010). Planimetric growth rate \( k_g \), in experiments was determined from

\[
D^2 = D_0^2 + k_g \Delta t,
\]

where the difference between mean diameters of grains before \( D_0 \) and after \( D \) shearing experiments (Table 4.3) depended on the duration of shearing, \( \Delta t \) (Cuffey and Patterson, 2010). Growth rates varied from ~2000-8000 mm\(^2\) yr\(^{-1}\), orders of magnitude larger than rates measured at slightly colder temperatures but consistent with rough extrapolations of those rates to the pressure-melting temperature (Fig. 5.1). Measured rates from our experiments include crystal growth during short periods when ice was not shearing: before shearing as ice warmed to the pressure-melting temperature and also after shearing before ice was extracted from the ice chamber. Even after subtracting growth during these periods, growth rates would still have been of order \( 10^3 \) mm\(^2\) yr\(^{-1}\).

Under deviatoric stress and associated strain, crystal growth is a function of temperature and cumulative deformation (Jacka and Li, 1994). The development of large grains with interlocking, irregular boundaries (Fig. 4.7c, d) suggests that grain-boundary migration recrystallization, a process favored at high temperatures, occurred during these experiments (Duval and Castelnau, 1995; Cuffey and Patterson, 2010). Grain boundaries migrate due to
Figure 5.1. Crystal growth rates in polycrystalline ice from this study and others at various temperatures up to ~0°C (1000/T = 3.66 K⁻¹). Note that crystal growth rates reported by Jacka and Li (1994) and Cuffey and Patterson (2010) reflect growth in non-deforming synthesized ice and polar firn, respectively. Growth rates at the pressure-melting temperature from the present study vary considerably because they are driven by dynamic recrystallization and thus are not directly comparable to static growth. Nevertheless, they do suggest a substantial increase in activation energy, probably of the order 10⁵ kJ mol⁻¹ at 0°C.

contrasts in lattice strain energy that develop among grains as ice deforms. During this process, larger less deformed crystals consume smaller more deformed ones. Consequently, strain rate, which controls the rate of boundary migration, and crystal growth are positively correlated, as observed in these experiments (Fig. 5.2). Jacka and Li (1994) explored the relationship between the creep rate of ice and normal crystal growth. They noted that the two processes share similar temperature dependencies, expressed as activation energy, up to -0.1°C. They presented no crystal growth data above -0.1°C but did conduct creep experiments up to -0.01°C. They
observed that the activation energy for creep increased greatly in that range, possibly up to 
~33,000 kJ mol$. Activation energy, $Q$, for both differences of ice creep, $\dot{\varepsilon}$, and crystal growth, $\dot{\varepsilon}_g$, can be estimated as temperature, $\theta$, changes using Poirier’s (1985) formula:

$$Q = -R \frac{\ln \frac{\dot{\varepsilon}_2}{\dot{\varepsilon}_1}}{\frac{1}{\theta_2} - \frac{1}{\theta_1}},$$  \hspace{1cm} (5.5)$$

where $R$ is the gas constant. By applying the maximum observed creep activation energy from -0.1 to -0.01 conservatively to normal crystal growth from -0.1 to 0°C yields a growth rate of 
~8000 mm$^2$ yr$^{-1}$ at 0°C. Thus, the high rates of grain growth in the present study are expected.
**Influence of water content on effective viscosity**

Sensitivity of effective ice viscosity to water content is different over different ranges of water content (Fig. 4.10). Below ~0.6% water content, effective viscosity decreases with increasing water content. Above this threshold water content, effective viscosity becomes insensitive to water content. Effective viscosities of Duval’s tertiary-creep experiments are less sensitive over the equivalent water content range by a factor of 1.8 when comparing the exponential fits to the two data sets.

The simplest explanation for the larger effective ice viscosities measured in this study, relative to those measured by Duval, are that his data were based on shear of natural ice specimens to tertiary creep. Recrystallization during tertiary creep results in alignment of basal glide planes (development of ice fabric) that reduces shearing resistance and hence effective ice viscosity by as much as a factor of 10 (Hudleston, 2015). In the one experiment of this study in which ice was sheared to tertiary creep, the effective viscosity indeed more closely aligns with Duval’s data (Fig. 4.10).

The effect of ice fabric on softening of ice decreases with increasing water content (Fig. 4.10). Softening from fabric development in tertiary creep can be described with an enhancement factor (Cuffey and Patterson, 2010; Hudleston, 2015). Enhancement, $E$, has a value equal to or greater than 1.0 and can be defined as

$$E = \frac{\eta_s}{\eta_t}$$

(5.5)

to describe the decrease in effective ice viscosity in tertiary creep, $\eta_t$, relative to secondary creep, $\eta_s$. Comparing the new experimental data with the tertiary-creep data of Duval allows $E$ to be calculated. The enhancement factor approaches 1.0 as water content increases toward ~0.8%
(Fig. 5.3). This suggests that ice softening from fabric development diminishes to almost zero above a threshold water content of ~0.6-0.8%.

![Figure 5.3](image_url)

**Figure 5.3.** Enhancement factor, E (black line), as a function of water content, calculated as the ratio of the effective-viscosity regressions for data collected during secondary creep (this study) and tertiary creep (Duval, 1977). Both of the data sets are also shown.

At higher water contents between 0.6-1.7%, sensitivity of ice viscosity to water content is near zero (Fig. 4.10). Effective viscosity is essentially uniform over this range. Plotting strain rate as a function of stress for our experiments for the higher water content range indicates nearly Newtonian behavior with a stress exponent of $n = 1.1$ (Fig. 5.4). At lower water contents, the high sensitivity of effective viscosity to water content (Figs. 4.10, 5.3) does not allow determination of the value of $n$; water content would have needed to have been constant in experiments at different strain rates to determine $n$. 
Figure 5.4. Strain rate as a function of stress for experiments at water contents of 0.6-1.7%, compared with data from uniaxial compression tests on saline ice (2 mm grain size) with a water content of 7% and at a temperature of -13°C (de la Chapelle et al., 1999).

The stress exponent near 1.0 at high water contents implies a micro-deformation mechanism in which peak stress is not limited by either dislocation creep (n = 4) or grain boundary sliding (n = 1.8) (Goldsby and Kohlstedt, 2001). Barnes et al. (1972) noted similarly large grains and rapid grain growth rates during high-temperature deformation experiments and noted that migration recrystallization occurred. They suggested that internal melting and recrystallization, with meltwater forming at grain boundaries where stress is high and diffusing to zones of low stress where there is refreezing, aids migration recrystallization in deforming temperate ice. Such a process, analogous to pressure solution in rocks, would yield a stress exponent of n = 1 if it rate-limits deformation (Hirth and Kohlstedt, 1995a; Kohlstedt and Zimmerman, 1996) or limits stress in rate-controlled experiments like those of the present study.
Duval (1977) in his study of ice softening by interstitial water did not suggest that such a process rate-limits the deformation of temperate ice because he found the stress exponent in the flow law to be 3 in another series of experiments (Duval, 1976b). In these earlier experiments (Duval, 1976b), stress was varied from 0.05 to 0.37 MPa to measure the resultant strain rate of ice held at -0.05°C. Water content was not measured. However, it was presumably steady among experiments because the stress exponent could be determined with high certainty and was likely in the lower half of the measured range of Duval’s later study (1977), likely 0.1-0.4% as indicated by similar experimental conditions. Duval (1977) agreed that during deformation, meltwater would be produced in areas of high stress that locally depress the melting temperature (i.e., at grain boundaries where dislocations pile up). He thought this water could facilitate grain boundary migration and soften ice by absorbing dislocation tangles but not rate-limit deformation through a grain-boundary process.

However, experiments with cold, saline ice at a much higher water content (7%), conducted later by Duval’s group (de la Chapelle et al., 1999), yielded the same stress exponent as the present study, n = 1.1. In their original interpretation (Fig. 1.9), they suggested that at stresses below ~0.3 MPa, the stress exponent is less than 2. At higher stresses, they suggested that n = 3. Neither of these suggestions, however, was based on regression of their data. Replotting and regressing their data yields stress exponents of n = 1.1 at stresses lower than 0.3 MPa (Fig. 5.4), as in the present study, and n = 4.2 at higher stresses. This latter value of n is consistent with dislocation creep (Goldsby and Kohlstedt, 2001). The stress-exponent transition from low-stress and high-stress behavior is higher for ice containing significant water than for ice with little water (Duval and Castelnau, 1995).
At lower stresses that are more representative of glaciers and like those of the present study, sufficiently high interstitial meltwater concentrations may aid internal melting and recrystallization sufficiently so that it becomes the rate- or stress-limiting micro-deformation mechanism with \( n \approx 1 \) (Fig. 5.4). Basal slip accommodated by grain boundary sliding, \( n = 1.8 \), and dislocation movement along easy basal glide planes that is optimized by development of ice fabric, \( n= 4 \), likely both occur but do not apparently limit peak stresses in temperate ice containing meltwater concentrations greater than \( \approx 0.6\% \) (Fig. 4.10, 5.3).

The value of \( n \approx 1 \) at water contents greater than 0.6% implies diffusional creep, such as that studied for crystalline olivine with varying percentages of basaltic melt (Hirth and Kohlstedt, 1995a; Kohlstedt and Zimmerman, 1996). In these studies across a range of melt contents from \(~1\%\) to \(2\%\), effective viscosity is insensitive to melt content (see Fig. 1.10b), just as in the present study. Between melt contents of 2 and 4.5% in these studies, effective viscosity becomes highly sensitive to melt, such that at a melt content of about 7%, olivine effective viscosity is reduced by about an order of magnitude relative to values at 1% melt content. Interestingly, a comparably large decrease in effective ice viscosity occurs for ice with a water content of 7%, as indicated by the data of de la Chapelle et al (1999) (Fig. 1.9). An explanation for this behavior is that as the concentration of interstitial melt increases above some threshold between 2 to 4.5%, melt films thicken sufficiently to extend along grain boundaries, thereby providing high-diffusivity pathways (Hirth and Kohlstedt, 1995a). Thus existing, albeit limited, effective-viscosity data for ice at high water contents (\(> 0.6\%)\) are consistent with effective-viscosity data for olivine with basaltic melts in which diffusional creep dominates behavior.

Interstitial meltwater aiding internal melting and recrystallization may also extend the range of stress and strain rate over which the micro-deformation mechanism with a low stress
exponent applies (Fig. 1.9) (de la Chapelle, 1999), with the degree of the shift dependent on the meltwater concentration. Meltwater localized at zones of high stress is thought to absorb dislocation pileups (Duval, 1977) that contribute to the high stress exponent of $n > 3$. Melt films at grain boundaries also attract moving dislocations due to low lattice strain energy at the wetted boundaries (Cuffey and Patterson, 2010). Both of these effects combined at sufficient meltwater concentrations may nullify dislocation creep as the rate-limiting process at intermediate stresses, as hinted by our results (Fig. 4.10), and thus extend the zone in which a low stress exponent is operative to higher stresses (Fig. 1.9). At still higher stresses, dislocation creep, enhanced by interstitial meltwater, is likely the rate-limiting creep mechanism for the range of experimentally studied water contents in temperate ice (de la Chapelle et al., 1999).

Creep rate would be sensitive to the grain size if internal melting and recrystallization of grain boundaries is rate-limiting as suggested by the present study. As in rock partial melts (Kohlstedt and Zimmerman, 1996), smaller steady-state grains should yield larger deformation rates—a hypothesis that could be tested with experiments conducted to tertiary creep in which grain size is varied.
CHAPTER 6. CONCLUSIONS AND IMPLICATIONS

By exploring a higher range of water content than previously studied in experiments with pure ice, we have identified two distinct creep regimes above and below a threshold interstitial meltwater content of ~0.6% (Fig. 4.10). From water contents of 0.2% to 0.6%, ice effective viscosity decreases by a factor of 4.4. This sensitivity is 1.8 times greater than for the case of ice deformed to tertiary creep (Duval, 1977). In this regime, the enhancement factor of creep at peak stress relative to tertiary creep (Duval, 1977) converges to ~1, and thus the development of ice crystal fabric during tertiary creep has a negligible effect on softening ice at water contents above ~0.6% (Fig. 5.3). Above ~0.6%, ice effective viscosity becomes essentially constant and does not depend on water content. As a result, at these higher water contents, the stress exponent in Glen’s flow law (1955) could be determined, with $n = 1.1$ (Fig. 5.4).

This finding that $n \approx 1$ indicates Newtonian flow, in contrast to previous data collected at mostly colder temperatures indicating $n = 1.8-4.0$, depending upon the magnitude of the stress. The micro-deformation mechanism responsible for $n = 1$ may be internal melting and refreezing at grain boundaries, with melting occurring in areas of high stress and refreezing in low stress areas, as originally suggested by Barnes et al. (1971). Thin-section analysis of ice from these experiments supports this hypothesis because grain boundary migration and recrystallization were extensive in deformed ice samples, as evidenced by high grain growth rates and irregular, infringing grain boundaries. At higher stresses, other studies indicate that dislocation creep, characterized by $n > 3$, rate-limits deformation for ice (Duval, 1977; de la Chapelle et al., 1999).

The results of this study will be important to consider in future efforts to model ice-stream shear margins. Efforts to estimate water contents of shear margins indicate values commonly in excess of 0.6% and as high as 8% (Haseloff et al., 2019). Results of the present
study indicate that at water contents of 0.6-1.7% there is little or no sensitivity of effective ice viscosity to water content. This result implies that ice-stream models that rely only on the Duval (1977) experimental data can overestimate the softening effect of water in ice and thereby overestimate a principal feedback that has been invoked to localize strain in shear margins and reduce shearing resistance there (Haseloff et al., 2019). On the other hand, if at still higher water contents of 2.0-4.5% effective-viscosity sensitivity to melt content is high, as in olivine with basaltic melt (Hirth and Kohlstedt, 1995a), then effects of ice water content in ice stream models may be reasonable or even underestimated. This uncertainty highlights the need for measuring ice effective viscosity at water contents higher than those that could be explored in this study.

Nevertheless, at water contents less than 0.6%, values that seem to typify much glacier ice (Vallon et al., 1976), sensitivity of effective viscosity to water content is substantial and consistent with the sensitivity measured by Duval (1977) (Fig. 4.10) if fabric development in his study is taken into consideration; this softening effect has relevance beyond ice-stream shear margins. Water produced preferentially near the beds of glaciers by strain heating and movement of water toward the bed in glaciers should reduce viscous drag exerted by bedrock obstacles on sliding ice, thereby tending to increase glacier sliding speeds. On the other hand, softer ice at glacier beds will reduce the extent of ice-bed separation (i.e., sizes of subglacial cavities) and act to increase drag and reduce sliding speeds (Zoet and Iverson, 2016). So, it is unclear if the softening effect of water in ice should necessarily enhance sliding velocities of glaciers on hard (i.e., bedrock) beds. The effect of ice softening by water on ice motion due to internal ice deformation (e.g., Cuffey and Paterson, 2010) is much clearer than for sliding. For a given glacier thickness and surface slope, ice softened by water will be less viscous and deform more rapidly than ice not subject to this effect. Owing to water content that increases with depth in
glaciers (e.g., Vallon et al., 1976), the softening effect of water will result in longitudinal
deformation profiles where shear strain near glacier beds is more focused than if this effect were
not operative.

The sensitivity of ice viscosity to water content below 0.6%, the absence of this
sensitivity up to at least 1.7%, and possible sensitivity at higher water contents point to the
importance of better estimating water content in glaciers. Although strain heating and associated
water production are readily calculated, estimates of water content will be highly uncertain until
flow rates of interstitial water through ice can be estimated. Such estimations will require direct
measurements of the permeability of glacier ice (Schoof and Hewitt, 2016).
REFERENCES


