Development of a periodic thermal diffusivity method

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DEVELOPMENT OF A PERIODIC THERMAL DIFFUSIVITY METHOD

Iowa State University

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Development of a periodic thermal diffusivity method

by

David Earl Slutz

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DOCTOR OF PHILOSOPHY

Department: Materials Science and Engineering
Major: Ceramic Engineering

Approved:

Signatures have been redacted for privacy

Iowa State University
Ames, Iowa
1985
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NOMENCLATURE

A  Area
A_k  Fourier constants
B_k  Fourier constants
C  Capacitance
C_p  Heat capacity (cal/g°C)
E  Emissivity
e  Percent error on thermal diffusivity
f  Frequency (Hz)
h  Film coefficient
I  Current
K  Thermal conductivity
L  Thickness (cm)
λ  Thickness in wavelengths
N  Number of data points per cycle
n  Number of cycles
p  Perimeter
Q  Heat
Q̇  Heat flux
q  Charge
R  Radius (constant) or resistance
r  Radius (variable)
T  Temperature (°C)
t  Time (s)
$v$

$X$ Distance parallel to heat flow (cm)

$x$ Distance parallel to heat flow in wavelengths

$V$ Voltage

**Greek Symbols**

$\alpha$ Thermal diffusivity (cm$^2$/s)

$\delta$ Phase shift of thermal wave ($2\pi X/\lambda$)

$\Delta$ Error on phase shift

$\varepsilon$ Dielectric constant or error term

$\lambda$ Wavelength (cm)

$\rho$ Density (g/cm$^3$) or electrical resistivity (Vscm$^2$/Ccm)

$\sigma$ Stefan-Boltzman constant

$\mu$ Heat loss

$\omega$ Angular frequency (rad/s)
INTRODUCTION

The trend in modern industry is to increase productivity by increasing the speed of controlled thermal processes. Thermal diffusivity, in turn, controls the speed of thermal processes. This research has to do with improving the accuracy and reducing the drudgery of thermal diffusivity determination procedures.

Thermal diffusivity measurements generally involve time-consuming data acquisition and long, tedious computations. Often the results are inaccurate and imprecise. However, with the advent of low cost microcomputers, experimentalists can be spared the overwhelming task involved in the Fourier analysis of the periodic thermal waves. This research employs the very old and often laborious Fourier method for determining the thermal diffusivity.

Over 100 years ago, Lord Kelvin considered the sun as a 24-hour periodic heat source and used Fourier analysis to determine the thermal diffusivity of the earth. The present research applies the same analysis technique to determine the thermal diffusivity of semi-infinite solid materials. This method has enough precision to warrant its use, and is versatile enough to be applied in situ; however, it is also relatively slow, taking several hours to get a result, and may not be applicable for high conductive materials. The key to success here is the use of a microcomputer which simultaneously controls the apparatus, acquires the data at regular intervals, and completes all the computations.
All the methods now used for measuring thermal conductivity or thermal diffusivity presume a pattern of unidirectional heat flow, but heat is noncontainable and flows in any direction having a significant thermal gradient. Perfect thermal insulation is virtually impossible because the heat losses can occur by conduction, radiation, or both, not only within a material, but between materials as well. Not even a vacuum can prevent radiation losses. To prevent multidirectional heat flow, one must establish a zero temperature gradient in all but the direction of interest and this is attempted in the methods that feature "guarding" arrangements. Other methods seek to account for extraneous heat losses by mathematically adjusting for them. Still others employ geometric designs such as cylinders or spheres to establish a uniform radial heat flow. In summary, all the traditional methods are based on one-dimensional heat flow with some correction for minor flows in the off directions if necessary.

Since all the methods are based on one-dimensional heat flow, it is easier to model thermal transport in a material by using an analogy to electrical transport. Table 1 gives a list of the analogy between thermal and electrical quantities.

From Table 1, it is shown that heat flow is directly analogous to electrical current, while thermal conductivity is inversely analogous to electrical resistivity. Imagine a material composed of a series of resistors each with a resistance \( R = \rho \Delta x / A \) (see Figure 1). Now, impose a voltage at \( x = 0 \) and wait until the current, \( I \), through the material is constant. The voltage at any point, \( x \), is the summation of the
Table 1. Analogous thermal and electrical quantities

<table>
<thead>
<tr>
<th>Thermal</th>
<th>Electrical</th>
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<tr>
<td>Heat, ( Q \text{ (cal)} )</td>
<td>Charge, ( q \text{ (C)} )</td>
</tr>
<tr>
<td>Flux, ( \dot{Q} \text{ (cal/s)} )</td>
<td>Current, ( I \text{ (C/s)} )</td>
</tr>
<tr>
<td>Temperature, ( T \text{ (°C)} )</td>
<td>Voltage, ( V \text{ (V)} )</td>
</tr>
<tr>
<td>Conductivity, ( K \text{ (calcm/cm}^2\text{s°C)} )</td>
<td>Resistivity, ( \rho \text{ (Vscm}^2\text{/Ccm)} )</td>
</tr>
<tr>
<td>Volumetric heat, ( \rho C_p \text{ Capacity (cal/cm}^3\text{°C)} )</td>
<td>Dielectric constant, ( \varepsilon \text{ (Ccm/cm}^2\text{V)} )</td>
</tr>
<tr>
<td>Diffusivity, ( \alpha \text{ (cm}^2\text{/s)} )</td>
<td>Time constant, ( \rho \varepsilon /A \text{ (s/cm}^2\text{)} )</td>
</tr>
</tbody>
</table>

individual voltage drops across each resistor. Now, let the material be continuous by allowing \( \Delta x \to 0 \). Then, the voltage at \( x \) is the integral of \( IR \) from \( L \) to \( x \) or the minus integral from \( x \) to \( L \) as follows:

\[
V = -\int Ip/A \, \Delta x . \tag{1}
\]

Differentiating and rearranging, we get

\[
I = -A/\rho \, \Delta V/\Delta x . \tag{2}
\]

Ohm's law \((V=IR)\) is a simplified specific form of Eq. (2). The principle involved in an ohm meter is to apply a known current through an electrical component, measure the voltage drop, and determine the resistance by the solution of Ohm's law. Now, substituting the thermal
Figure 1. Electrical analog of steady state one-dimensional heat flow

quantities in Eq. (2), we get:

$$\dot{Q} = -KA\Delta T/\Delta X$$  \hspace{1cm} (3)

Equation (3) is Fourier's first law for one-dimensional heat flow under steady state conditions. Thermal conductivity is usually determined from Eq. (3) by establishing a steady state thermal gradient through a material and then measuring the heat flow. This is similar to the principle involved in an ohm meter to measure the resistance, but with a reversed measurement scheme.

From Table 1, heat capacity is seen as the storage of heat in the volume of the material, while the dielectric constant involves the storage of charge. Therefore, the analogy between dielectric constant and heat capacity is the materials' ability to store charge or heat, respectively. If our one-dimensional material of Figure 1 is
Figure 2. Electrical analog of nonsteady state one-dimensional heat flow

able to store charge along x, then, this can be modeled as in Figure 2 where the capacitors have a capacitance $C = \varepsilon \Delta x$. Again, the voltage at any point, $x$, is the summation of the voltage drops across each resistor; however, the current is no longer a constant and it is expressed as follows:

$$i(x) = \sum \varepsilon \frac{\partial V}{\partial t} \Delta x$$  \hspace{1cm} (4)$$

Using Eq. (4) and letting $\Delta x \to 0$, we get

$$V = \int \rho e / A \left[ \int \frac{\partial V}{\partial t} \Delta x \right] \Delta x$$  \hspace{1cm} (5)$$

Differentiating and rearranging Eq. (5), and then differentiating it once again, gives:
\[ \frac{\partial V}{\partial t} = \frac{\partial}{\partial x} \left[ \frac{(\lambda/\rho e)\partial V}{\partial x} \right]. \]  

(6)

Substituting the thermal quantities in Eq. (6) gives:

\[ \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left[ \alpha \frac{\partial T}{\partial x} \right]. \]  

(7)

Equation (7) is the one-dimensional form of Fourier's second law for a nonsteady state, one-dimensional heat flow. The time dependence in Eq. (7) is due to the time dependence of heating or cooling of the differential elements of material and is analogous to the time dependency required to charge or discharge an array of capacitors. Thermal diffusivity, \( \alpha \), is a ratio of thermal conductivity to volumetric heat capacity. Since real materials are a nonseparable continuum of thermal conductivity and volumetric heat capacity, thermal diffusivity is an important quantity for determining heat transport in the materials. One may determine the thermal diffusivity from time versus temperature data alone and may dispense with quantitative determinations of heat flow. This is an obvious advantage, because simple methods for the accurate measurements of time and temperature are readily available, whereas accurate determinations of heat flow are extremely cumbersome and often are inferred indirectly.

From the preceding derivations, it is clear that to measure thermal conductivity we need to know the thermal gradient and heat flow. However, diffusivity measurements require time and temperature data only. In the following sections, methods for measuring thermal
conductivity and properties are reviewed. These methods are divided into steady state and nonsteady state. Steady state methods measure the thermal conductivity by using solutions to Fourier's first law. On the other hand, nonsteady state methods are generally used to determine the thermal diffusivity. The following section briefly reviews some commonly used methods. If the reader wishes to know more details about methodology, I suggest starting with references [1-4]. If the reader wishes to know more details about actual apparatus or techniques for measuring thermal conductivity or thermal diffusivity of materials, then references [5-18] may be helpful. If the reader wishes a more in depth mathematical approach to heat flow problems, then both references [19] and [20] will be helpful. Reference [21] was found to be helpful in understanding the mathematical parts of Reference [19].
LITERATURE SURVEY

Steady State Methods

Measurements of thermal conductivity by steady state methods require solving Eq. (3), which is Fourier's first law of heat flow for one dimension. To facilitate the solution, one may design the apparatus such that a small, steady state thermal gradient ($\partial T/\partial x$) is established across the sample and then, measure the amount of heat flowing through the sample in a given time to infer the heat flow rate ($\dot{Q}$).

This may sound simple, but accurate measurement of heat flow is very difficult and cumbersome. The investigator must make sure that all heat is accounted for. In practice, this requires an experimental design which eliminates all heat flows in directions other than the direction of interest.

Another difficulty encountered in steady state methods is to define exactly the quantity the experimentalist has measured. If great care is taken to establish a one-dimensional heat flow with no heat losses with a very small thermal gradient (1°), then one can infer from this the true thermal conductivity at the temperature involved. However, if the thermal gradient is larger, say 10°, then the inferred conductivity is the average thermal conductivity for this 10° range commonly referred to as the "K-factor". If the "K-factor" is divided by the sample thickness, it is called the thermal conductance of the sample or the "C-factor". Some investigators prefer reporting their results as the thermal resistance or the "R-factor" which is the
inverse of the "C-factor". Other investigators include the heat flow from the sample surfaces to the surrounding atmosphere to measure the heat transmission factor or the "U-factor" of a sample. The above-mentioned quantities can be very useful as long as the conditions of the experiment have been defined and reported.

Steady state methods can be classified into linear heat flow methods and radial heat flow methods based on the type of one-dimensional heat flow imposed on the sample.

Linear methods

Linear heat flow methods involve the solution of Fourier's first law (Eq. 3) of one-dimensional heat flow in cartesian coordinates. The difficulty involved in any linear method is to avoid heat losses in the other directions. This can be accomplished in two ways. The first is to physically guard the sample from heat losses by apparatus design and the second is to design the apparatus such that heat losses can be corrected mathematically.

Laubitz [1] was very concerned with linear heat flow methods and reported that measurements with an error of 10% were common. But to reduce the errors to 1% required a nearly perfect experimental apparatus. He reasoned that this was true since the least possible error due to measurement of temperature, separation distance between thermal measurements, and power input was 1%. He proposed a model of the perfect apparatus which has been reproduced in Figure 3 [1]. It consists of a cylindrical sample (1) heated at one end by a transverse
heater (H1) to establish a longitudinal heat flow. The temperature of the guard (3) is matched to that of the sample (1) by a transverse heater (H3). Transverse heater (H2) shields (H1) and heats the insulation (2) to match that of the sample. The entire apparatus is in a furnace (4) thermally matched to the sample. By thermally matching the furnace, guard, and the insulation to the sample, one has perfectly guarded the sample from any radial heat flow by establishing a zero radial temperature gradient. The following linear methods all
use this principle of guarding the sample, although the specific configurations differ from Laubitz's hypothetical apparatus [1].

Guarded hot plate The basic design has been specified by ASTM C177 [22]. A generalized schematic of the apparatus is reproduced from Pratt [2] in Figure 4. The main heater is guarded and sandwiched by the sample so that practically all the heat produced by the heater passes through the sample. Therefore, heat flow is calculated from the power input to the main heater. The auxiliary heaters, insulation, and heat sinks are for maintaining a constant thermal gradient through the specimen. Difficulties in this method stem from the apparatus design, assembly, and the time involved to achieve one measurement. This method is generally used for materials with a thermal conductance $< 0.6 \text{ watt/cm}^2 \text{ °C}$.

Heat meter The heat meter provides an alternative to the hot plate method. The apparatus design is simpler and requires less time to make a measurement. ASTM C518 [22] is the standardized method, and Figure 5 is a schematic of the apparatus reproduced from Pratt [2]. This method involves a heat flow meter which is a material of known heat capacity, such as copper, to collect the heat that has passed through the specimen. By knowing the specimen dimensions, mass of the heat flow meter, and the temperature of the heat flow meter, one can calculate the heat flow. Therefore, heat flow is calculated from heat exiting from a material as opposed to the hot plate method where heat flow is calculated from heat entering
Figure 4. Guarded hot plate schematic (from Pratt [2])
the sample. Unfortunately, this method is usually limited to soft compressible materials with thermal conductances around 0.12 watt/cm °C.

**Guarded hot box** This method is given in ASTM C236 [22] and is primarily used for construction materials such as walls, roofs, and floors. Both thermal conductance and transmittance can be measured. A schematic is given in Figure 6 from Pratt [2] (originally from ASTM). The heat flow is measured by the amount of heat supplied to the meter box. The heat flow out of the meter box is through the sample to the cold box. All the other sides of the meter box are guarded by the guard box by matching the temperature to that of the meter box.
Figure 6. Guarded hot box schematic (from Pratt [2])
**Calorimeter method**  The calorimeter method for measuring the thermal conductance of a refractory brick has been standardized in ASTM C201 [22]. A schematic picture of the apparatus is shown in Figure 7 from Pratt [2] (originally from ASTM). In this method, heat is supplied to the heating chamber from the silicon carbide heating elements. A silicon carbide slab 1" by 9" by 13 1/2" covers the sample bricks and guards to provide uniform heating. There are three 9" straight sample bricks where one is the specimen and the other two comprise the inner guard. Six soap bricks 9" by 2 1/2" by 2 1/4" surround the test bricks to form the outer guard. Heat flows through all the bricks and the heat is collected and removed by the

![Figure 7. Refractories method schematic (from Pratt [2])](image)
calorimeters. The calorimeters are specially designed wound copper tubing cooled by water. Heat flow is determined by measuring the difference between inlet and outlet water temperatures and knowing the flow rate of water through the calorimeter. Procedures to be used in conjunction with ASTM C201 for particular refractories are as follow [22]: ASTM C182 for insulating bricks, ASTM C202 for fire bricks and ASTM C417 for castable bricks. These are the standard methods for determining the thermal conductance of refractories.

**Radial methods**

Radial methods have an advantage over the linear methods in that the geometric design removes most of our concern of the heat losses. Contrary to linear heat flow, radial heat flow methods usually require isotropic thermal properties. Radial methods utilize Fourier's first law for one-dimensional heat flow in either cylindrical or spherical coordinates.

**Cylindrical methods**

Cylindrical methods utilize the steady state heat flow equation in cylindrical coordinates.

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} = 0 \ .
\]  

(8)

Assume heat flow only in r, then

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) = 0 \ .
\]  

(9)
It follows that

$$T(r) = a \ln(r) + b .$$  \hspace{1cm} (10)

Application of Fourier's first law yields

$$\dot{Q} = -KA \frac{\partial T}{\partial r} ,$$  \hspace{1cm} (11)

where

$$A = 2\pi LR .$$  \hspace{1cm} (12)

Cylindrical methods have a central heat source or sink at $r = 0$. The simplest method is shown in Figure 8, from McElroy and Moore [3]. This is the unguarded cylindrical method from class I of the representative methods. Class II is an improvement over class I in that end guards are used to reduce heat losses from the ends. Class IV is a comparative method which compares the unknown to a known material and it is used primarily for liquid samples. Class V is an example of the methods in which the sample is directly heated with electrical current.

**Spherical or ellipsoidal methods** These methods require a solution to the steady state heat flow equation in spherical coordinates:

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2} = 0 .$$

\hspace{1cm} (13)
Figure 8. Schematic of radial heat flow methods (from McElroy and Moore [3])
Since these methods all employ the use of a central heater, we can then assume that all the heat flow is radial and therefore, Eq. (13) simplifies to:

$$\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial T}{\partial r}) = 0$$  \hspace{1cm} (14)

where

$$T(r) = \frac{a}{r} + b$$ . \hspace{1cm} (15)

Using Eq. (11),

$$A = 4\pi R^2$$ . \hspace{1cm} (16)

The great advantage of these methods is the elimination of the heat losses. However, they do have drawbacks. The sample and apparatus are not easily prepared, heat losses do occur along temperature sensor wires, and it is difficult to achieve a uniform heating source. Figure 8 shows a class III method with an ellipsoidal configuration.

Nonsteady State Methods

Nonsteady state methods require a complete solution to the heat flow equation including time dependence. Unlike steady state methods, nonsteady state methods generally measure thermal diffusivity, the ratio of thermal conductivity to volumetric heat capacity. Nonsteady state methods are generally concerned with one-dimensional heat flow to simplify the mathematics. Therefore, apparatus design is similar
to steady state methods where the specimens are guarded against heat losses or mathematically accounted for.

Nonsteady state methods can be classified as transient or periodic. Transients are "one shot" methods where the specimen is subjected to a sudden thermal change, while periodic methods are subjected to a repetitive thermal wave.

**Transient methods**

Transient methods start with the specimen at thermal equilibrium and then, one boundary is subjected to a thermal change. Time-temperature measurements are then made at some position along the direction of heat flow. Thermal diffusivity is determined by curve fitting the data to a solution of Eq. (7). Although all transient methods are common in the above characteristics, there are a wide variety of boundary conditions and sample geometries.

Danielson and Sidles [4] characterized boundary conditions and sample geometries of transient methods by assuming heat loss to occur only from radiation. Therefore, the heat balance of a small differential element in a vacuum is as follows:

\[
\dot{Q}_{\text{absorbed}} = \dot{Q}_{\text{conduction}} - \dot{Q}_{\text{radiation}}. \tag{17}
\]

In order to estimate the importance of the heat of radiation, they considered the following ratio:
where \( Q_{\text{r}} \) and \( Q_{\text{a}} \) are the radiating area and volume elements, respectively. Since the curve of temperature vs time is initially concave upwards, the following relationship can be applied:

\[
\int T(t) \, dt < \left( \frac{T_t}{2} \right) t
\]

(19)

where \( T_t \) is the temperature at some time, \( t \). Substituting Eq. (19) into Eq. (18) yields:

\[
R < \left( 2.7 \times 10^{-12} \right) \left( T_0 \right)^3 \frac{E}{\rho C_p} \frac{(p/\rho)(p/A)}{t}.
\]

(20)

This inequality is separated into five components: a constant term, a temperature term, a material properties term, a geometric term, and a time term. The significance of Eq. (20) is that for low \( R \) values, heat losses by radiation are small and heat of conduction can be estimated by measuring heat absorbed. Inspection of Eq. (20) shows that radiation losses can be controlled by temperature \( (T_0)^3 \), geometry \((p/\rho)\), and time \((t)\). Since measurements are made over a wide range of temperatures, experimental setups usually use geometry or time to control radiation heat losses.

Transient methods fall into four main categories depending on the way the heat losses are handled. They are as follows: long rods,
flat plates, cylinders, and flash methods. Although the flash method usually uses flat plates, it differs in that $R$ is minimized by restricting measurements to short times.

**Long rods**  
Long rods inherently have one-dimensional heat flow and exhibit experimental and computational simplicity. The sample design is often desirable from the standpoint of other physical property measurements, such as electrical resistivity [16-17] and Seebeck coefficient. However, at higher temperatures, lateral heat losses become significant.

Heat losses are either suppressed by guards or accounted for in the mathematical solutions used. Errors for long rod methods are reported in the range of 2-5%. Kennedy et al. (1962) (as referenced in Danielson and Sidles [4]) developed a guarded long rod method using a guard of the same material as that of the sample. Figure 9, from Danielson and Sidles [4], shows a schematic of the sample design. Figure 10 [4] is a representation of the time-temperature data obtained by Kennedy et al. (as referenced in Danielson and Sidles [4]). The data from $x=0$ and $x=L$ were used in a finite difference analysis to predict the curve at $x=L/2$. The value of thermal diffusivity which gave the least deviation from the curve was the value reported.

**Flat plates**  
For flat plates, the geometric term is $p/A \ll 1$. Lateral heat losses are small and can be ignored in most cases. However, Eq. (20) is concerned only with lateral heat losses by radiation. In flat plate geometry, heat losses from the front and back faces dominate over the lateral heat losses and they must be
Figure 9. Schematic diagram of a transient guarded long rod method used by Kennedy et al. (1960) (from Danielson and Sidles [4])
accounted for in the boundary conditions as opposed to the additional terms in the heat flow equation. Therefore, flat plate methods, in general, involve a uniform heat source on the front boundary and a heat sink on the back boundary where the boundary conditions are known. Figure 11 is a schematic of Beatty et al.'s (1960) apparatus for measuring the thermal diffusivity (from Danielson and Sidles [4]). Often, the heat sink is used as a heat meter so that thermal conductivity can be measured. Figure 12 is a schematic of Fitch's (1935) apparatus (from Danielson and Sidles [4]) in which thermal
Figure 11. Schematic of Beatty et al. (1950) flat plate method for thermal diffusivity (from Danielson and Sidles [4])

Figure 12. Schematic of Fitch's (1935) flat plate transitory method for measuring thermal conductivity (from Danielson and Sidles [4])
conductivity is measured. Additional flat plate methods can be found by reading references [7] and [8].

**Cylinders**  
Cylinders are relatively free from the problems of heat loss because of the geometry. This is a fair assumption as long as the cylinder is several times longer than its diameter. Configurations range from solid cylinders to hollow tubes and heating uniformly from the outside or from the center. Temperature measurements are usually made at two radii. Errors in cylindrical methods are 2-7%. Some investigators report measurements made up to 1650°C. A great deal of work has been done to adapt cylindrical methods to liquids and low thermal diffusivity materials. A typical cylindrical sample is shown schematically in Figure 13 (from Danielson and Sidles [4]). An example of a cylindrical method is the well-known hot-wire method [5, 6, 10, 15].

**Flash methods**  
Flash methods are independent of the sample geometry. What distinguishes them from other methods is that the time in which the thermal flux is applied is very short compared to the time for the thermal energy to propagate between the temperature sensors. This short time reduces $R$ in Eq. (20) and therefore, radiation heat losses. Reference [12] demonstrates that time is only relatively short compared to the sample properties.

Figure 14, from Danielson and Sidles [4], shows a flash apparatus schematically. A thermal source, either a flash tube or laser, is used to initiate a thermal pulse. Results are obtained by curve fitting to the time-temperature curve similar to Figure 15. Heat
Figure 13. Idealized schematic of transitory cylindrical method (from Danielson and Sidles [4])
Figure 14. Schematic of typical flash method (from Danielson and Sidles [4])
Figure 15. Typical data curve from flash method (from Danielson and Sidles [4])
losses are controlled by reducing the sample thickness, while finite pulse time effects are reduced by a thicker sample. Therefore, there is an optimum sample thickness which minimizes both these effects.

**Periodic methods**

Periodic methods are concerned with establishing a thermal wave inside a material such that the average localized temperature remains constant so the time-dependent temperature can be expressed as:

\[ T(x,t) = A_0 e^{-ax} + A_1 e^{-bx} \cos(\omega t - cx + \epsilon) \]  \hspace{1cm} (21)

This holds true only if the temperature at \( x=0 \) is expressed as a pure sinusoidal wave as follows:

\[ T(x,t) = A_0 + A_1 \cos(\omega t + \epsilon) \]  \hspace{1cm} (22)

Periodic methods are not very popular because pure sinusoidal temperature waves are very difficult to achieve. These difficulties can be overcome and are discussed further in the next section on periodic methods.

**Long rods** Angstrom (1861) (as referenced from Danielson and Sidles [4]) developed the first periodic method to achieve widespread acceptance. His method employed a radiating long, thin rod with a sinusoidal heater at one end. If the temperature difference between the rod and its surroundings is small, the heat losses to the
surroundings can be assumed to vary linearly with respect to the difference in temperature (Newton's law of cooling). Therefore, Eq. (7) can be rewritten to include the heat loss term, where \( \mu \) is a coefficient of surface heat loss:

\[
\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} - \mu T .
\]  

(23)

Using Eq. (21) to solve Eq. (23), we get the following relationships:

\[
a = (\mu/\alpha)^{1/2} \quad \text{(24a)}
\]

\[
b = (1/2\alpha[(\mu^2 + \omega^2)^{1/2} - \mu])^{1/2} \quad \text{(24b)}
\]

\[
c = (1/2\alpha[(\mu^2 + \omega^2)^{1/2} + \mu])^{1/2} \quad \text{(24c)}
\]

\[
v = \omega/c \quad \text{(24d)}
\]

\[
q = e^{b1} \quad \text{(24e)}
\]

Thermal diffusivity can be obtained in several ways. Angstrom (1861) (as referenced from Danielson and Sidles [4]) determined thermal diffusivity by determining the values of \( b \) and \( c \) and using Eqs. (24b) and (24c) to eliminate the heat loss, \( \mu \), as follows:

\[
\alpha = \omega/bc \quad \text{(25)}
\]
King (1915) and Starr (1937) (as referenced in Danielson and Sidles [4]) determined thermal diffusivity by using data from two separate frequencies. King measured the phase relationship utilizing Eqs. (24c) and (24d), while Starr was concerned with amplitude decrements utilizing Eqs. (24b) and (24e).

A schematic of the Angstrom method used by Danielson and Sidles [4] is shown in Figure 16. Abies et al. (1960) (as referenced in Danielson and Sidles [4]) improved on this method by establishing certain limits to reduce problems associated with radiation losses and back face reflections as follows:

\[
\omega r >> \frac{B_o}{E_t^3/C_p^0} \tag{26}
\]

\[
e^{-2bl} \ll 1 \tag{27}
\]

Flat plates

Long rods methods, such as Angstrom's (1861) (as referenced in Danielson and Sidles [4]), generally had frequencies in the range of $10^{-3}$ to $10^{-1}$ Hz. As Abies et al. (1960) (as referenced in Danielson and Sidles [4]) point out, if the frequencies are increased, then shorter samples can be used with reduced heat losses. Flat plate methods utilize this concept by reducing the sample lengths to ~1 cm and increasing the radii to 5-6 cm. In addition, frequencies are increased to $10^{-1}$ to 1000 Hz and are generally applied by radiation or electron beam.
Figure 16. Block diagram of a modified Angstrom method used by Danielson and Sidles [4]
The main difference between long rod methods and flat plate methods is that heat losses for flat plates are accounted for in the boundary condition, as opposed to including additional terms like $\mu T$ in the heat flow, Eq. (23). Cowan (1961) (as referenced in Danielson and Sidles [4]) is one of the few investigators to consider nonsinusoidal temperature distributions on the sample surface.

Semi-infinite solids Most semi-infinite methods employ the same concept as flat plate methods, except that $\mu=0$. This is accomplished by increasing the frequency such that the wave has been damped to at least $e^{-3}$ at the back face. This requires frequencies in the range of 10 to 1000 Hz, depending on the material and its thickness.

The method employed in this research is a semi-infinite solid periodic method. However, it differs from other methods in that frequencies are in the range of $10^{-3}$ to $10^{-2}$ Hz with sample thicknesses the order of 5-6 cm and with a nonsinusoidal temperature distribution imposed on the specimen surface. The next section, Periodic Method, will go into further details on how to handle the above-mentioned conditions to measure thermal diffusivities.
PERIODIC METHOD

This research was primarily concerned with measuring the thermal properties of refractories at high temperatures. Since refractories are generally in the form of bricks, it seems appropriate to investigate the solutions for semi-infinite solids, of which there are three kinds: steady state, transient, and periodic. Periodic methods are statistically superior because accuracy generally increases with the square root of the number of estimates and periodic methods generate one estimate from each thermal cycle, therefore, very high precision is achieved by measurements over many thermal cycles in contrast to steady state or transient methods which obtain one estimate per experimental run. Another difference is that periodic methods yield the thermal diffusivity from which the thermal conductivity may be inferred, whereas steady state and transient methods can yield thermal conductivity directly. However, thermal diffusivity is often the parameter of primary interest where time dependent heat flow problems are of concern.

Any periodic wave propagating into a material can be expressed as a Fourier series. When the periodic wave has achieved a stationary state, i.e., when all transients have ceased to exist, then the temperature inside the material can be expressed as:

\[ T(t, X) = A_0/2 + \sum_{k=1}^{\infty} \left[ A_k \exp(-2\pi X/\lambda_k) \cos(2\pi f k t - 2\pi X/\lambda_k) \right. \]

\[ + B_k \exp(-2\pi X/\lambda_k) \sin(2\pi f k t - 2\pi X/\lambda_k) \]  

(28)
where the wavelength \((\lambda_k)\) depends as follows on the frequency, \(f\), (of the driving function) and \(\alpha\), the thermal diffusivity of the material:

\[
\lambda_k = \left(\frac{4\pi\alpha}{kf}\right)^{1/2}.
\]  

(29)

For simplicity, consider the first sinusoidal component of Eq. (28). Setting \(k=1\) gives us the following equations:

\[
T(X, t) = T_0 \exp(-2\pi X/\lambda)\sin(\omega t - 2\pi X/\lambda)
\]  

(30)

\[
\lambda = \left(\frac{4\pi\alpha}{f}\right)^{1/2}.
\]  

(31)

Figure 17 shows how sharply this "damped thermal wave" attenuates with depth. Indeed, the amplitude at one-half wavelength is only 4% of the original surface wave and a mere 0.2% at a penetration depth of one wavelength. This demonstrates the very strong damping effect materials have on thermal waves. This damping effect is even more severe with the higher frequency components, which means that they get filtered out relative to the fundamental one. The overall effect of this selective filter is clearly demonstrated in Figure 18 which shows what happens to the shape of a square wave driving function as it propagates into a material. Note how quickly the high frequency (corner) components damp away leaving only the lowest frequency one referred to above as the fundamental.
Figure 17. The propagation of a plane, sinusoidal temperature wave into a material. Note the rapid damping. Dotted line is envelope. exp (-2πx)
Figure 18. Decay of square, periodic wave penetrating a material (Carslaw and Jaeger [19])
As a thermal wave propagates into a material, its phase is shifted in time in comparison to the original wave. This phase shift increases with the distance into the material, as can be seen clearly in Figure 19. Also, note the overall amplitude attenuation as well as the rapid rounding of the corners on the triangular wave as it penetrates into the material. Figure 20, which shows how the amplitude attenuates with depth and frequency, was recorded directly from an actual experimental run.

There are two ways to obtain wavelength information from Eq. (30). One can get it from the exponential decay term, which is called the amplitude method, or we can get it from the trigonometric term, whereupon one speaks of the phase method. Once the wavelength is known, the diffusivity can be calculated from Eq. (31). Actually, thermal waves are virtually never a single, well-defined sine wave, but instead are complicated Fourier sums as shown in Eq. (28). This leaves us with an infinite sum of integral wavelengths and frequencies. However, this is not a problem since Fourier analysis can be used to obtain a single wavelength for each frequency component in Eq. (28). Unfortunately, Fourier analysis is an enormous computational task because it requires several hundred computations per thermal cycle, which is one of the reasons why periodic methods have not been very popular. Microcomputers can complete these computational tasks and print out the results of the analysis while the run is in progress. This makes the periodic methods practical and feasible.
Figure 19. Illustration of strong wave damping with depth. Note that depth = 0 is 1/10th scale.
Figure 20. Experimental results for square, periodic power wave. Thermocouples buried in sample at 1/2 inch and 1 1/2 inch
Discrete Fourier analysis [23] is well-suited for computation. All that is necessary is to create two running summations for equally spaced temperature measurements as follows:

\[
\text{sinsum}(X) = \sum_{j=1}^{n} \sum_{i=1}^{N} \frac{2}{nN} T(i,X) \sin(\frac{2\pi ji}{N}) \quad (32)
\]

\[
\text{cossum}(X) = \sum_{j=1}^{n} \sum_{i=1}^{N} \frac{2}{nN} T(i,X) \cos(\frac{2\pi ji}{N}) \quad . \quad (33)
\]

From the summations, the phase shift and amplitude attenuation between two thermocouples at depths \( X_1 \) and \( X_2 \) can be determined. The phase method gives the following solution for the phase shift:

\[
\delta = \arctan\left(\frac{\cos\text{sum}(X_2)}{\sin\text{sum}(X_2)}\right) - \arctan\left(\frac{\cos\text{sum}(X_1)}{\sin\text{sum}(X_1)}\right) \quad (34)
\]

where

\[
\delta = 2\pi(X_2 - X_1)/\lambda \quad .
\]

The amplitude method gives:

\[
\delta = \frac{1}{2} \ln \left(\frac{\cos\text{sum}(X_1)^2 + \sin\text{sum}(X_1)^2}{\cos\text{sum}(X_2)^2 + \sin\text{sum}(X_2)^2}\right) \quad (35)
\]

The thermal diffusivity is then calculated from \( \delta \) as follows:

\[
\alpha = \pi f(X_2 - X_1)^2/\delta^2 \quad . \quad (36)
\]
EXPERIMENTAL ARRANGEMENT

Apparatus

A block diagram of the experimental apparatus is shown in Figure 21. A Commodore 64\(^1\) microcomputer was selected due to its low cost and the user port which is part of a 6526 VIA (versatile interface adapter). The user port provides computer access to the apparatus via the external circuit board shown in Figures 22 and 23. The external I/O (input/output) board allows the computer to send TTL (transistor-transistor logic) trigger pulses to the voltmeters to eliminate timing problems, to send TTL signals to a traic (which, in turn, controls a Kanthal heater to produce periodic heat waves), to control the furnace via reed relays and a West 10 controller, to switch the furnace thermocouples inputs to the voltmeter via a gold contact relay, and finally, to keep the computer program in synchronization with the measurements from the voltmeters.

Figure 21 shows how three voltmeters are used to measure temperature. The first voltmeter measures the furnace temperature at two locations in order to control furnace temperature. The next two voltmeters are used for temperature measurements inside the sample and are triggered from the computer to keep the measurements simultaneous. The voltmeters need to have at least 5 1/2 digit

\(^{1}\text{Commodore Business Machines, Inc., Computer Systems Division, 487 Devon Park Drive, Wayne, PA 19887.}\)
Figure 21. Equipment for development of thermal wave diffusivity apparatus
Figure 22. Block diagram of external I/O board

C1-CAPACITOR (1 MICROFARAD)
I1-SCHMIDT TRIGGER (7414)
I2-SCHMIDT TRIGGER (7414)
P1-24 PIN EDGE CONNECTOR
P2-15 PIN D CONNECTOR
R1-RESISTOR (1 KOHM)
S1-PUSH BUTTON SWITCH (RESET)
S2-DUAL THROW GOLD CONTACT RELAY (SWITCH THERMOCOUPLES)
S3-REED RELAY (FURNACE CONTROL)
S4-REED RELAY (FURNACE CONTROL)
Figure 23. Circuit diagram of external I/O board

DVM- LINES TO DIGITAL VOLTOMETER
FC1 & FC2- LINES TO WEST 10 CONTROLLER TO CONTROL FURNACE
FRE- LINE TO TRIAC TO CONTROL PERIODIC WAVE
TRG- LINE TO VOLTOMETERS TO TRIGGER A READING
TC1 & TC2- THERMOCOUPLE INPUTS TO RELAY (OUTPUT TO DVM)
accuracy with an external trigger feature. A Keithley 192\(^1\) and a Hewlett Packard 3478A\(^2\) DVM (digital voltmeter) were used for our apparatus.

The furnace temperature was measured using two type K thermocouple probes. Temperature measurements inside the sample were made using butt welded 20 mil type K thermocouples. All thermocouples were referenced to the ice point with a K-140-4 ice point box\(^3\).

West 10\(^4\) controllers were used to control the two electric furnace heating elements independent of the computer. The West 10 controllers were used as a safeguard against furnace runaway in case of a computer malfunction.

A Kanthal wound heating element\(^5\) was used to produce the thermal waves on the sample. The power to the heating element was supplied by a constant voltage regulator to assure a constant voltage and to avoid thermal wave drift. An optically isolated, TTL level controlled, zero crossing triac\(^6\) Model 240D 25 was used for on/off control of

\(^1\)Keithley Instruments, Inc., 28775 Aurora Road, Cleveland, OH 97330.
\(^2\)Hewlett Packard, Corvallis Division, 1000 N.E. Circle Blvd., Corvallis, OR 97330.
\(^3\)Kaye Instruments, Inc., 15 De Angelo Dr., Bedford, MA 01730.
\(^5\)C. M., Inc., 103 Dewey St., Bloomfield, NJ 07003.
\(^6\)Opto 22, Huntington Beach, CA 92649.
the heating element.

The video monitor displays the present status of the apparatus and gives instructions for operator use. A floppy disk or audio tape stores the programs and the experimental data. The line printer records the results of experimental runs. The digital counter and oscilloscope were used for design, debugging, and testing of the apparatus. All timing was based on 60 Hz line voltage.

Sample Preparation

Samples used in this experiment were refractory bricks, generally 9" straights. After the bricks were cut in half using a diamond saw, two grooves 20 mils deep and 15 mils wide were sawed into the exposed face. Figure 24 shows a prepared sample schematically. The following procedure was used to determine the thermocouple depth and frequency range for our apparatus.

1) Estimate thermal diffusivity ($\alpha$) of the material to be measured using Table 2.

2) Measure the minimum radius of the heating element ($R$) in centimeters.

3) Measure or estimate the amplitude of the thermal wave produced by the heating element ($A$).

4) Using Figure 25, determine the critical wavelength ($\lambda_c$) of the apparatus (the critical wavelength is the shortest wavelength that the apparatus can produce for a given material based on a maximum frequency of $10^{-2}$ Hz).
Figure 24. Schematic of sample arrangement. Thermocouples are .013 in. butt welded type K
Table 2. Thermophysical properties of various materials [19]

<table>
<thead>
<tr>
<th>Substance</th>
<th>Density (g/cm³)</th>
<th>Specific heat (cal/g-K)</th>
<th>Conductivity (cal/s-cm-K)</th>
<th>Diffusivity (cm²/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver</td>
<td>10.49</td>
<td>0.0556</td>
<td>1.00</td>
<td>1.71</td>
</tr>
<tr>
<td>Gold</td>
<td>19.30</td>
<td>0.0308</td>
<td>0.70</td>
<td>1.18</td>
</tr>
<tr>
<td>Copper</td>
<td>8.94</td>
<td>0.0914</td>
<td>0.93</td>
<td>1.14</td>
</tr>
<tr>
<td>Magnesium</td>
<td>1.74</td>
<td>0.240</td>
<td>0.38</td>
<td>0.91</td>
</tr>
<tr>
<td>Aluminum</td>
<td>2.70</td>
<td>0.206</td>
<td>0.48</td>
<td>0.86</td>
</tr>
<tr>
<td>Zinc</td>
<td>7.14</td>
<td>0.0917</td>
<td>0.27</td>
<td>0.41</td>
</tr>
<tr>
<td>Tin</td>
<td>7.30</td>
<td>0.0534</td>
<td>0.15</td>
<td>0.38</td>
</tr>
<tr>
<td>Brass (70:30)</td>
<td>8.5</td>
<td>0.09</td>
<td>0.25</td>
<td>0.33</td>
</tr>
<tr>
<td>Platinum</td>
<td>21.46</td>
<td>0.0315</td>
<td>0.17</td>
<td>0.25</td>
</tr>
<tr>
<td>Lead</td>
<td>11.34</td>
<td>0.0302</td>
<td>0.084</td>
<td>0.25</td>
</tr>
<tr>
<td>Mild steel (0.1% C)</td>
<td>7.85</td>
<td>0.118</td>
<td>0.11</td>
<td>0.12</td>
</tr>
<tr>
<td>Cast iron</td>
<td>7.4</td>
<td>0.136</td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
<td>Bismuth</td>
<td>9.80</td>
<td>0.0292</td>
<td>0.020</td>
<td>0.070</td>
</tr>
<tr>
<td>Mercury</td>
<td>13.55</td>
<td>0.0335</td>
<td>0.020</td>
<td>0.044</td>
</tr>
</tbody>
</table>

Nonmetals

<table>
<thead>
<tr>
<th>Substance</th>
<th>Density (g/cm³)</th>
<th>Specific heat (cal/g-K)</th>
<th>Conductivity (cal/s-cm-K)</th>
<th>Diffusivity (cm²/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>0.00129</td>
<td>0.240</td>
<td>0.000058</td>
<td>0.187</td>
</tr>
<tr>
<td>Granite</td>
<td>2.6</td>
<td>0.21</td>
<td>0.006</td>
<td>0.011</td>
</tr>
<tr>
<td>Limestone</td>
<td>2.5</td>
<td>0.22</td>
<td>0.004</td>
<td>0.007</td>
</tr>
<tr>
<td>Sandstone</td>
<td>2.3</td>
<td>0.23</td>
<td>0.006</td>
<td>0.011</td>
</tr>
<tr>
<td>Average rock a</td>
<td>---</td>
<td>---</td>
<td>0.0042</td>
<td>0.0118</td>
</tr>
<tr>
<td>Ice</td>
<td>0.92</td>
<td>0.502</td>
<td>0.0053</td>
<td>0.0115</td>
</tr>
<tr>
<td>Glass (crown)</td>
<td>2.4</td>
<td>0.20</td>
<td>0.0028</td>
<td>0.0058</td>
</tr>
<tr>
<td>Concrete (1:2:4)</td>
<td>2.3</td>
<td>0.23</td>
<td>0.0022</td>
<td>0.0042</td>
</tr>
<tr>
<td>Brick (building)</td>
<td>2.6</td>
<td>0.20</td>
<td>0.0020</td>
<td>0.0038</td>
</tr>
<tr>
<td>Snow (fresh)</td>
<td>0.1</td>
<td>0.5</td>
<td>0.00025</td>
<td>0.0050</td>
</tr>
<tr>
<td>Soil (average)</td>
<td>2.5</td>
<td>0.2</td>
<td>0.0023</td>
<td>0.0045</td>
</tr>
<tr>
<td>Soil (sand, dry)</td>
<td>1.65</td>
<td>0.19</td>
<td>0.00063</td>
<td>0.0020</td>
</tr>
<tr>
<td>Soil (sandy, 8% moist)</td>
<td>1.75</td>
<td>0.24</td>
<td>0.0014</td>
<td>0.0033</td>
</tr>
<tr>
<td>Wood (spruce, with grain)</td>
<td>0.41</td>
<td>0.30</td>
<td>0.00055</td>
<td>0.0045</td>
</tr>
<tr>
<td>Wood (spruce, across grain)</td>
<td>0.41</td>
<td>0.30</td>
<td>0.00030</td>
<td>0.0024</td>
</tr>
<tr>
<td>Water</td>
<td>1.0</td>
<td>1.0</td>
<td>0.00144</td>
<td>0.00144</td>
</tr>
<tr>
<td>Ground cork</td>
<td>0.15</td>
<td>0.48</td>
<td>0.0001</td>
<td>0.0014</td>
</tr>
</tbody>
</table>

*Kelvin, cf. § 2.14.*
Figure 25. Figure to assist in determination of critical wavelength
Figure 25 was determined from the following:

\[ \lambda_c = \left( \frac{4\pi \alpha}{f_{\text{max}}} \right)^{1/2} \]  \hspace{1cm} (37)

5) Using Figure 26, determine the maximum depth a thermocouple can be placed using the following procedure:

a) Locate the intersection of \( \lambda_c \) and the solid line corresponding to \( R \). Read the value of depth and call it \( d_R \).

b) Locate the intersection of \( \lambda_c \) and the dashed line corresponding to \( A \). Read the value of depth and call it \( d_A \).

c) The critical depth, \( d_c \), is the smaller value of either \( d_R \) or \( d_A \). Note that Figure 26 is based on the following equations:

\[ \lambda_R = -2\pi \left[ \left( d^2 + R^2 \right)^{1/2} - R \right] / \ln(e) \]  \hspace{1cm} (38)

\[ \lambda_A = 2\pi d / \ln(A) \]  \hspace{1cm} (39)

Equation (38) forms the set of solid lines in Figure 26. Each line represents the maximum wavelength for a given radius \( R \) to produce an error of \( \varepsilon = 0.01 \). Equation (39) forms the set of dashed lines in Figure 26. Each line represents
Geometric Relationship for 1% Edge Effect

Figure 26. Figure to assist in determination of maximum depth and wavelength. 
R = radius of heating element; A = amplitude of surface heat wave (°C)
the minimum wavelength for a given surface amplitude (A) in order to produce a thermal EMF at a depth d of 5 μV.

6) After calculating $d_c$, cut the grooves such that both thermocouples are within $d_c$ of the surface. Practical experience has demonstrated that the first thermocouple should not be placed any closer to the surface than 0.2 cm.

7) Once the thermocouples are in place and the depth of the deepest thermocouple ($d_2$) is known, determine the operating frequency range as follows:

   a) Use Figure 26 to determine the maximum wavelength by finding the intersection of the solid line corresponding to R and $d_2$.

   b) Use the maximum wavelength value obtained in step 7a, the estimate of thermal diffusivity ($\alpha$) from step 1, and Figure 27 to determine the minimum frequency ($f_{\text{min}}$). Therefore, the frequency range will extend from $f_{\text{min}}$ to 0.01 Hz. However, Figure 27 is based on Eq. (37) and the maximum frequency will be less than 0.01 Hz if any of the following conditions are true:

   $d_2 > d_c$,
   actual surface amplitude < A, or
   actual $\alpha$ < estimated $\alpha$. 
Figure 27. Figure to assist in determination of frequency range.
8) Finally, note that when estimating thermal diffusivity ($\alpha$) in step 1 and surface wave amplitude ($A$) in step 3, it is best to underestimate their values.

The foregoing procedure is based on certain assumptions in order to give an estimate of the frequency range which, in practice, often turns out to be larger than the predicted range.
ERROR ANALYSIS

Precision has two connotations: repeatability and reproducibility. Repeatability is the spread of data associated with repeated observations without any system alteration. Reproducibility is the spread of data due to repeated observations with intermediate system alterations. For example, if the repeated measurements of the same specimen are performed with reassembling the specimen between measurements, then the spread in data is a measure of reproducibility. Accuracy, on the other hand, is the deviation of experimental results from the true value.

Precision can be determined experimentally. However, accuracy is not easily obtainable since there are no universal thermal diffusivity standards available. This method has an advantage in that all errors are functions of frequency, as will be demonstrated in this section, with the exception of separation distance. Therefore, if a frequency scan is done on a specimen and there is no dependency on frequency, then all the frequency dependent errors are negligible. Therefore, the accuracy of this method is totally dependent on the accuracy of thermocouple placement.

Timing of Thermal Measurements

A re-examination of Eq. (30) shows that the time dependent part of the equation is in the trigonometric term. This clearly shows that the phase method is sensitive to timing errors, while the amplitude
method is not. Imagine that we have two thermocouples at depths \( X_1 \) and \( X_2 \). Assume the thermocouple at \( X_2 \) for some reason is measured at a time, \( \Delta t \), earlier or later than \( X_1 \). This would give the following relations for the trigonometric terms:

\[
T(X_1, t) = \sin(\omega t - \delta_1) \quad (40)
\]

\[
T(X_2, t) = \sin(\omega t - \delta_2 + \omega \Delta t) . \quad (41)
\]

Solving Eq. (34) and using Eqs. (40) and (41), we will obtain the following for the measured phase shift:

\[
\delta_m = \delta_2 - \delta_1 + \Delta = \delta + \Delta
\]

\[
\Delta = \omega \Delta t . \quad (42)
\]

This gives us a phase shift offset which, in turn, gives an error on the thermal diffusivity as indicated by the following relationship:

\[
\alpha + e\alpha/100 = \pi f(X_2 - X_1)^2/(\delta + \Delta)^2 . \quad (43)
\]

Rearranging the terms in the above equation gives:

\[
e = -100(2\delta + \Delta)\Delta/(\delta + \Delta)^2 . \quad (44)
\]
The results of Eq. (44) are plotted in Figure 28 using dimensionless units. A computer simulation program was used to verify the results obtained in Eq. (44). The computer results verified that the phase method is dependent on timing errors as demonstrated in Figure 29, while the amplitude method is not. In addition, the simulation also demonstrated that both phase and amplitude methods are insensitive to zero sum random timing errors.

The solution to avoid timing errors is to have one voltmeter for each thermocouple and then, trigger the voltmeters simultaneously via a trigger pulse generated from the computer. This was done and timing errors are in the order of ms, which for frequencies in the range of .01-.001 Hz, gives negligible errors.

Thermocouple Calibration

Imagine two thermocouples at depths $X_1$ and $X_2$ in an isothermal material and the ratio of the emf of thermocouple 2 to that of thermocouple 1 is a number $g$ (one if the thermocouples agreed exactly). This gives the following apparent temperatures at the two locations when a material is subjected to a periodic thermal wave:

$$T(X_1, t) = T_0 \exp(-\delta_1)\sin(\omega t - \delta_1)$$  \hspace{1cm} (45)

$$T(X_2, t) = gT_0 \exp(-\delta_2)\sin(\omega t - \delta_2)$$  \hspace{1cm} (46)
Figure 28. Error on thermal diffusivity due to timing error
Simulation Data of 100 ms Timing Error

**Figure 29.** Thermal diffusivity frequency dependency due to timing error
From these equations, it is shown that the phase method is unaffected by this type of error, unlike the amplitude method. The solution to Eq. (35) using Eqs. (45) and (46) gives the following amplitude attenuation:

$$\delta_m = \delta + \ln(g) = \delta + \Delta.$$  \hfill (47)

This gives an error in $\delta$ similar to the timing error and therefore, Eq. (44) holds with $\Delta$ redefined as $\ln(g)$. A plot of the results for dimensionless units is shown in Figure 30. Once again, a computer simulation verified that accuracy for the amplitude method is dependent on thermocouple calibration, as demonstrated in Figure 31, while the phase method is not. Computer simulation also demonstrated that both methods are relatively insensitive to zero sum random errors in the thermal measurements.

The solution to the gain error problem is to calibrate one thermocouple to the other (see Figure 32) so that they have a matching thermal emf characteristic or they are corrected to the same gain. Of course, this is not necessary if we use the phase method alone.

**Frequency Measurement**

From Eq. (36), it is clear that thermal diffusivity is directly proportional to the frequency and therefore, any error in the frequency is directly related to an error in the thermal diffusivity. For example, if the frequency is only known to 1%, then the thermal
Figure 30. Error on thermal diffusivity due to thermocouple EMF gain difference
Simulation Data of 1% Gain Error

Figure 31. Thermal diffusivity frequency dependency due to thermocouple EMF gain error

\[ \alpha = 20.00 \times 10^{-3} \text{ cm/s} \]
Figure 32. Typical thermocouple calibration curve. Note for the above case that 
g=c(2) for thermocouple #2 and g=1 for thermocouple #1
diffusivity, ignoring all other parameters, will be known to 1%.
Measurements of the apparatus's frequency output were made using a
digital counter and an oscilloscope. The results of that
investigation showed that the computer calculated frequency output
agreed with the measured results to within 0.0005%. Therefore, any
errors due to knowing the true frequency can be ignored.

Separation Distance

Let \( \varepsilon \) be the percent error on the diffusivity and \( \Delta \) the percent
error on the separation distance (d) between thermocouple 1 and
thermocouple 2. This gives the following:

\[
e = 100[(1 + \Delta/100)^2 - 1]
\]  \hspace{1cm} (48)

\[
e \approx 2\Delta . \hspace{1cm} (49)
\]

This means that the separation distance must be twice as accurate
as the desired accuracy of the thermal diffusivity. In other words,
for a 1% accuracy on the thermal diffusivity, the separation distance
must be known within 0.5%. This is indeed the limiting factor for
accuracy.

Boundary Conditions for Flat Slab

Since a semi-infinite solid is a definite impossibility, we
assume that a slab of material approximates a semi-infinite solid.
There are now three boundary conditions to be considered for a finite slab; namely, a finite heated region, a finite thickness, and a finite perimeter. Therefore, in order to use the semi-infinite solution, we need to establish some limitations on geometry to eliminate as much error as possible and still be practical.

The diameter of a finite heat source must be large compared to the depth of the two thermocouples and the separation distance between them. This will ensure that the heat flow at the thermocouple locations is still one dimensional. Otherwise, the solution will be a two or three dimensional heat flow problem which is far more complicated. Given a uniform, disk-shaped sinusoidal heated region, the solution of temperature propagation along the axis of symmetry in the material is\(^1\):

\[
T = T_0 \left[ \exp\left(2\pi X/\lambda \right) \sin(\omega t - 2\pi X/\lambda) \right. \\
- \exp\left(-2\pi Z/\lambda \right) \sin(\omega t - 2\pi Z/\lambda) \right]
\]

where \(Z\) is the distance from the edge of the heating element to the thermocouple location and \(X\) is the straight line distance from the center of the heat source to the thermocouple location. Eq. (50) can be interpreted as the wave one would expect for a semi-infinite geometry plus a wave due to the edge of the heated region.

\(^1\)Derivation from Dr. David Martin, Materials Science and Engineering Department, Iowa State University, Ames, IA 50011.
Approximations can be made from Eq. (50) to eliminate the effect of a finite heat source. If the radius of the heated region is one wavelength and the depth of the thermocouple is one-fourth of a wavelength, then the amplitude of the edge wave will be less than 1% of that of the centerline wave.

The model we discussed above assumed a sinusoidal heat wave; however, in reality, thermal waves are rarely sinusoidal. Actual experimental data of the surface wave created by a Kanthal heating element are shown in Figure 33. Clearly, the thermal waves are not sinusoidal, but rather closer to triangular. Therefore, a computer simulation using Eq. (50) and triangular surface waves was used to calculate the data in Figures 34 and 35. Figure 34 shows the error in thermal diffusivity for the phase method versus the radius wavelength ratio, while Figure 35 is a plot of the error for the amplitude method. Both plots clearly demonstrate that the radius of the heat region must be greater than one wavelength. The plots also show that shallower thermocouple depths reduce the error. Eq. (50) was solved in closed form by solving the Fourier integrals for two depths, $X_1$ and $X_2$. Both the amplitude method and the phase method produced an error ($\Delta$) in the actual phase shift ($\delta$) to give an error ($e$) in the thermal diffusivity as follows:

$$e = -100(2\delta + \Delta)/(\delta + \Delta)^2 .$$  \hspace{1cm} (51)
Figure 33. Sample surface temperature with square wave power input to heating plate
GEOMETRIC ERROR RELATIONSHIP
(PHASE METHOD)

\[ \delta = \frac{\gamma X}{R} \]

\( X = \text{DEPTH OF THERMOCOUPLE} \)
\( R = \text{RADIUS OF HEATING ELEMENT} \)

Figure 34. Error on thermal diffusivity (phase method) due to finite disk shaped heating element.
Figure 35. Error on thermal diffusivity (amplitude method) due to finite disk shaped heating element
The error ($\Delta$) of the phase method is:

$$
\Delta = \arctan \left( \frac{\exp(z_2)\sin(z_2)}{1-\exp(z_2)\cos(z_2)} \right) - \arctan \left( \frac{\exp(z_1)\sin(z_1)}{1-\exp(z_1)\cos(z_1)} \right).
$$

(52)

The error ($\Delta$) of the amplitude method is:

$$
\Delta = \frac{1}{2} \ln \left( \frac{1+\exp(2z_1)-2\exp(z_1)\cos(z_1)}{1+\exp(2z_2)-2\exp(z_2)\cos(z_2)} \right)
$$

(53)

where

$$
zi = 2\pi[X_i - (R^2 + X_i^2)^{1/2}] / \lambda \quad (i = 1, 2)
$$

(54)

and

$$
\delta = 2\pi(X_2 - X_1) / \lambda.
$$

(55)

Equations (51), (52), (54), and (55) were used to plot Figures 36 and 37 for the phase method. These plots match the results obtained from the simulation data of Figure 32. Likewise, Eqs. (51), (53), (54), and (55) were used to plot Figures 38 and 39 for the amplitude method. Again, these plots match the results of the simulation data of Figure 35. Equation (50) was used to generate temperature data in a simulated hypothetical experimental run to show the frequency dependency due to a finite heat source. Figure 40 is a plot of the results obtained for a hypothetical sample of thermal diffusivity 0.02 cm$^2$/s and with a heat source radius of 3 cm.
Finite Heat Source Error (phase)

Figure 36. Closed form calculation of thermal diffusivity error due to finite disk-shaped heating element. Above plot is for the phase method with thermocouple #1 at the surface.
Finite Heat Source Error (phase)

\[ x_l/\lambda = 0.1; x_2/\lambda = 1 \]
\[ x_l/\lambda = 0.3; x_2/\lambda = 1 \]
\[ x_l/\lambda = 0.5; x_2/\lambda = 1 \]
\[ x_l/\lambda = 0.7; x_2/\lambda = 1 \]
\[ x_l/\lambda = 0.9; x_2/\lambda = 1 \]

- \( x_1 = \) depth of thermocouple #1
- \( x_2 = \) depth of thermocouple #2
- \( R = \) radius of heat source
- \( \lambda = \) wavelength

Figure 37. Closed form calculation of thermal diffusivity error due to finite disk-shaped heating element. Above plot is for phase method and thermocouple #2 fixed at one wavelength.
Figure 38. Closed form calculation of thermal diffusivity error due to finite disk-shaped heating element. Above plot is for the amplitude method and thermocouple #1 at the surface.
Finite Heat Source Error (amplitude)

Figure 39. Closed form calculation of thermal diffusivity error due to finite disk-shaped heating element. Above plot is for the amplitude method and thermocouple #2 at one wavelength.
Simulation Data for Finite Heat Source \((R=3\text{cm})\)

\[
\begin{align*}
\text{Thermal Diffusivity (cm}^2/\text{s)} \times 10^{-3} \\
\hline
\text{Frequency (mHz)} \\
\end{align*}
\]

- \(\Delta\) - Phase Method
- \(\square\) - Amplitude Method
- \(\alpha = 20.00 \times 10^{-3} \text{ cm}^2/\text{s}\)

Figure 40. Thermal diffusivity frequency dependency due to finite disk-shaped heating element
Obviously, the finite heat source can produce very large errors in thermal diffusivity calculations and that is why the results of this section were used as a part of the procedure for the determination of thermocouple depth.

When a thermal wave encounters a boundary, all or part of the wave is reflected back. The reflected wave then superimposes the original wave with the same frequency as the original. Fourier analysis cannot filter out the reflected wave since it has the same frequency as the original wave. Therefore, one needs to know how thick a material must be in order to ignore the effect of the reflected wave. This can best be analyzed by considering the image technique. This technique assumes that the material is infinite and yet, at some distance L, there is a boundary. At this boundary, there is a reflected (mirror imaged) wave traveling in the opposite direction superimposed on the original wave. The magnitude of this wave will depend on what boundary the original wave crossed. Therefore, one obtains a set of valid solutions superimposed to meet the boundary condition. The four main categories of thermal boundaries are as follows:

1) insulating (no heat flux across boundary),
2) absorbing (boundary at fixed temperature),
3) change of material (composite materials), and
4) film (portion of wave escapes by radiation).

Figures 41 and 42 demonstrate the image technique for an insulating and film boundary, respectively. The solution by the image technique
reflecting boundary

Figure 41. Example of image technique for an insulating boundary. (Courtesy of Dr. David Martin, Materials Science and Engineering Department, Iowa State University, Ames, IA 50011)
Figure 42. Example of image technique for a film boundary. (Courtesy of Dr. David Martin, Materials Science and Engineering Department, Iowa State University, Ames, IA 50011)
for a periodic, one-dimensional thermal wave in a semi-infinite material bounded by X=0 and X=L is:

\[
T = \exp(-2\pi x)[A_0 \sin(\omega t-2\pi x) + B_0 \cos(\omega t-2\pi x)]
\]

\[
+ \exp(-2\pi(21-x))[A_1 \sin(\omega t-2\pi(21-x)) + B_1 \cos(\omega t-2\pi(21-x))]
\]

\[
+ \exp(-2\pi(21+x))[A_2 \sin(\omega t-2\pi(21+x)) + B_2 \cos(\omega t-2\pi(21+x))]
\]

\[
+ \exp(-2\pi(31-x))[A_3 \sin(\omega t-2\pi(31-x)) + B_3 \cos(\omega t-2\pi(31-x))]
\]

\[
+ \text{ etc.} \quad (56)
\]

where \(x\) and \(l\) are in wavelengths. The coefficients related to various boundary types are as follows:

1) insulating: \((A_{n+1} = A_n; B_{n+1} = B_n)\),
2) absorbing (T=0): \((A_{n+1} = -A_n; B_{n+1} = -B_n)\),
3) change material: \((A_{n+1} = K_R A_n; B_{n+1} = K_R B_n)\), and
4) film: \((A_{n+1} = \alpha A_n + \beta B_n; B_{n+1} = \alpha B_n - \beta A_n)\)

where

\[
K_R = (K_1 - K_2)/(K_1 + K_2)
\]

\[
\beta = 2H/(H^2 + 2H + 2)
\]

\(^1\text{Derivation from Dr. David Martin, Materials Science and Engineering Department, Iowa State University, Ames, IA 50011.}\)
\[ \alpha = \frac{(2 - H^2)}{(H^2 + 2H + 2)} \]

\[ H = \frac{2\pi h \lambda}{K} . \]

Examination of Eq. (56) shows that the infinite series decays very rapidly, so only the first few terms are necessary to model a real situation. By choosing a thickness in the order of one wavelength or larger, backside reflections can be ignored. Also, if the backside is in contact with a material of similar thermal conductivity, then reflected waves have very small amplitudes.

The last boundary to be considered is the perimeter. Angstrom (as referenced in Danielson and Sidles [4]) approached this problem from the standpoint of long, thin rods and his solution to the problem is Eqs. (23) and (24a-e). Figure 43 shows the effect of heat losses on the amplitude and phase methods based on Eqs. (24b) and (24c). The heat loss term, \( \mu \), for radiation is:

\[ \mu = 8\sigma ET_0^3(T - T_0)/\rho \rho C_p . \]  

Equation (57)

Angstrom (as referenced in [4]) measured temperatures at the surface of the rods, while this method measures temperatures inside the material on or near the center of symmetry. To explore this situation, a two-dimensional, finite difference dynamic heat flow program\(^1\) was used

---

\(^1\)Program written by Dr. David Martin, Materials Science and Engineering Department, Iowa State University, Ames, IA 50011.
Figure 43. Thermal diffusivity frequency dependency due to lateral heat losses. Plot based on Angstrom method.
to model the experimental setup. Figure 44 shows a typical printout of the temperature profile. From the data obtained, temperature contours were drawn to compare the temperature distributions in finite and infinite samples. Figure 45 is such a contour representing a typical experimental setup. The results obtained from these simulations indicated negligible error if the distance from the thermocouple to the side boundary is at least one wavelength.

Nonstationary State

Previously, we have considered the situation where the specimen is at a stationary state, where the average localized temperature is not varying with time. Now consider a material at thermal equilibrium and at time \( t=0 \), a thermal wave is initiated at the surface of this material. The solution is given by Carslaw and Jaeger [19, p. 64]. Their solution contains a periodic part and a transient term.

How does the transient affect the calculations based on Eqs. (32) and (33)? Assume for the sake of argument that the transient can be expressed as a linear function. Then, we get the following expression:

\[
T(x, t) = T_0 \exp(-2\pi x/\lambda) \sin(\omega t - 2\pi x/\lambda) + bt \quad (58)
\]

Figure 46 represents a temperature profile for \( x=0 \) and \( T_0=1 \). If
Figure 44. Finite difference calculations comparing temperature distribution in a typical insulating fire brick with exposed faces to an infinite slab of the same material. Periodic heating of top surface (wavelength 2 inches). (Courtesy of Dr. David Martin, Materials Science & Engineering Department, Iowa State University, Ames, IA 50011)
Figure 45. Computer generated temperature contours for brick with periodic wave on top and other faces exposed to furnace air. (Courtesy of Dr. David Martin, Materials Science & Engineering Department, Iowa State University, Ames, IA 50011)
Figure 46. Graphic representation of a linear time dependence superimposed on a sinusoidal wave.
Eq. (58) is used in Eq. (32) when $X=0$, then we get $\sin \sum(0)=b/\pi$ instead of zero. This is an error due to the nonstationary state. Similar results are obtained using Eq. (33) for the cossum.

The same finite difference simulation program discussed previously was used to generate time-temperature data to calculate thermal diffusivity. This program starts with the material at thermal equilibrium and then, it imposes a sinusoidal surface heat wave at $t=0$. Figure 47 clearly demonstrates the effect of the transient term for $t < 5000$ sec. The results suggest that the transient term can be ignored after ten thermal cycles.

Figure 48 shows the raw experimental temperature data versus the angular position in the thermal wave for 30 thermal cycles. Clearly, the specimen was not at a stationary state. Figure 49 is another plot of data from Figure 48 at two separate angular positions versus the thermal cycle number. Figure 49 shows the exponential decay of the transient for higher thermal cycles. Both Figures 47 and 49 demonstrate the need for a method to determine when stationary state has been achieved. Both the simulation and experimental data suggest that ten thermal cycles are sufficient. However, a better method is used, and it will be discussed in the next section.

Erroneous Measurement

This method is based on the collection of data for immediate use in Eqs. (32) and (33), without retaining the data. Obviously, it is essential that the measurement be reliable.
Simulation Thermal Diffusivity Data

Figure 47. Computer simulated results of the effect of the transient term on thermal diffusivity calculations. Frequency = 2mHz; separation distance = .5"; element spacing = .05"; thermal diffusivity = 4.8122e
Figure 48. Experimental data showing the nonstationary state
Figure 49. Experimental data showing the decay of the transient term
Figure 50 is a plot of raw temperature data collected for 30 cycles. The plot suggests that the specimen is nearly at a stationary state. On closer examination of angular positions 0.3 rad and 6.2 rad, we see a single point well outside the normal spread in the data. Figure 51 is a plot of the data at 0.3 rad where the erroneous point is clearly seen at cycle number 29. Similarly, the data at 6.2 rad were plotted in Figure 52 showing the erroneous measurement in the third cycle. Figure 53 is a plot of the thermal diffusivity calculated from the data of Figure 50. Clearly, the effect of the erroneous data point shows up in the thermal diffusivity calculations.

To correct for these erroneous temperature measurements, a continuous least squares subroutine was implemented in the computer program. This least squares program does linear fits versus time for each angular position measured. Each data measurement is then compared to its respective population before acceptance. Therefore, erroneous measurements can be rejected. In addition to this, the stationary state can be identified using this method by examining the slopes of each linear fit. From the experimentalist's standpoint, it totally automates the experimental runs.
Figure 50. Experimental data showing erroneous measurements. Frequency = 1.80722892 mHz
Figure 51. Experimental data showing erroneous measurement in the 29th cycle.
Frequency = 1.80722892 mHz
Figure 52. Experimental data showing erroneous measurement in the 3rd cycle. Frequency = 1.80722892 mHz
Figure 53. Thermal diffusivity results showing the effect of erroneous measurements. Frequency = 1.80722892 mHz
RESULTS AND DISCUSSION

In the previous Experimental Arrangement Section, a procedure for sample preparation was discussed. In that procedure, it stated that the maximum frequency was based on a minimum measurable thermal amplitude of 1°C. Also, the procedure defines a minimum frequency due to a finite heat source. These limits were originally determined from the following experimental data.

Table 3 is the experimental data plotted in Figure 54. The term combined refers to the geometric average of the phase and amplitude methods. The amplitude (A2) of the deepest thermocouple is included in Table 3. Note that the amplitude is less than 1°C for frequencies

<table>
<thead>
<tr>
<th>Frequency (mHz)</th>
<th>A2</th>
<th>Thermal Phase</th>
<th>Diffusivity Amplitude</th>
<th>Combined (cm^2/s) x 10^-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>2.66</td>
<td>5.035</td>
<td>5.374</td>
<td>5.202</td>
</tr>
<tr>
<td>5</td>
<td>1.89</td>
<td>4.998</td>
<td>5.465</td>
<td>5.226</td>
</tr>
<tr>
<td>6</td>
<td>1.39</td>
<td>4.933</td>
<td>5.517</td>
<td>5.217</td>
</tr>
<tr>
<td>7</td>
<td>1.06</td>
<td>4.891</td>
<td>5.695</td>
<td>5.276</td>
</tr>
<tr>
<td>8*</td>
<td>0.83</td>
<td>4.821</td>
<td>5.657</td>
<td>5.221</td>
</tr>
<tr>
<td>9*</td>
<td>0.67</td>
<td>4.643</td>
<td>5.753</td>
<td>5.164</td>
</tr>
<tr>
<td>10*</td>
<td>0.55</td>
<td>4.343</td>
<td>5.776</td>
<td>5.003</td>
</tr>
<tr>
<td>Mean (all)</td>
<td>--</td>
<td>4.809</td>
<td>5.605</td>
<td>5.187</td>
</tr>
<tr>
<td>Mean (ex. *)</td>
<td>--</td>
<td>4.964</td>
<td>5.513</td>
<td>5.230</td>
</tr>
<tr>
<td>Std. E. (all)</td>
<td>--</td>
<td>5.05%</td>
<td>2.75%</td>
<td>1.69%</td>
</tr>
<tr>
<td>Std. E. (ex. *)</td>
<td>--</td>
<td>1.30%</td>
<td>2.45%</td>
<td>0.61%</td>
</tr>
</tbody>
</table>
Figure 54. Thermal diffusivity data showing high frequency error due to low thermal amplitude.
greater than 7mHz. At the bottom of all the tables, there are two sets of values for mean and standard error. The first set of values is for all the data, while the second set excludes data with the asterisk. When the data with $A_2 < 1^\circ C$ are excluded in Table 3, the standard error is improved to less than 2% for the phase method. Table 4 and Figure 55 also demonstrate that the standard error reduces to < 2%.

Table 5 is the experimental data of Figure 56. When the data with $A_2 < 1^\circ C$ are excluded, the standard error improves, but is still larger than 2%. Closer examination of the data in Table 5 shows a low frequency dependency on the thermal diffusivity values. This suggests that the finite heat source is affecting the results. Using the

<table>
<thead>
<tr>
<th>Table 4. Experimental data (shown in Figure 55) illustrating high frequency error due to low thermal amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>(S4-KH-503-JUN)</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>8*</td>
</tr>
<tr>
<td>9*</td>
</tr>
<tr>
<td>10*</td>
</tr>
<tr>
<td>Mean (all)</td>
</tr>
<tr>
<td>Mean (ex. *)</td>
</tr>
<tr>
<td>Std. E. (all)</td>
</tr>
<tr>
<td>Std. E. (ex. *)</td>
</tr>
</tbody>
</table>
Figure 55. Thermal diffusivity data showing high frequency error due to low thermal amplitude.
Table 5. Experimental data (shown in Figure 56) illustrating the combined effects of low thermal amplitude at high frequency and boundary effects at low frequencies

<table>
<thead>
<tr>
<th>(S3-KH-555-MAY) frequency (mHz)</th>
<th>A2</th>
<th>Thermal phase</th>
<th>Diffusivity amplitude (cm²/s) x 10⁻³ combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25.49</td>
<td>5.015</td>
<td>5.043</td>
</tr>
<tr>
<td>2</td>
<td>8.01</td>
<td>4.933</td>
<td>4.960</td>
</tr>
<tr>
<td>3</td>
<td>3.52</td>
<td>4.732</td>
<td>4.646</td>
</tr>
<tr>
<td>4</td>
<td>1.80</td>
<td>4.514</td>
<td>4.549</td>
</tr>
<tr>
<td>5</td>
<td>1.00</td>
<td>4.515</td>
<td>4.550</td>
</tr>
<tr>
<td>6*</td>
<td>0.60</td>
<td>3.807</td>
<td>4.290</td>
</tr>
<tr>
<td>7*</td>
<td>0.36</td>
<td>3.296</td>
<td>4.123</td>
</tr>
<tr>
<td>8*</td>
<td>0.24</td>
<td>2.640</td>
<td>4.404</td>
</tr>
<tr>
<td>9*</td>
<td>0.15</td>
<td>2.088</td>
<td>4.721</td>
</tr>
<tr>
<td>10*</td>
<td>0.11</td>
<td>1.966</td>
<td>5.897</td>
</tr>
</tbody>
</table>

Mean (all) -- 3.751 4.718 4.139
Mean (ex. *) -- 4.742 4.750 4.745
Std. E. (all) -- 31.5% 10.6% 16.8%
Std. E. (ex. *) -- 4.89% 4.95% 4.89%

procedure for determining the minimum frequency on the sample in Table 5, one gets $f_{\text{min}} = 3.2$ mHz. Therefore, only two measurements were in the correct frequency range, 4mHz and 5mHz. Figure 57 is a plot of the data in Table 6 which strongly demonstrates the low frequency dependency. The calculated minimum frequency for this sample is 4.4 mHz.

Table 7 and Figure 58 demonstrate the low frequency finite heat source problem and the high frequency low amplitude problem.

The data in Table 8 are plotted in Figure 59. Uncalibrated thermocouples were used and the calculated frequency range is 2-10 mHz.
Figure 56. Thermal diffusivity data showing combined effects of low thermal amplitude at high frequencies and boundary effects at low frequencies.
Table 6. Experimental data illustrating boundary effect at low frequencies (shown in Figure 57)

<table>
<thead>
<tr>
<th>(S1-KH-434-MAY) frequency (mHz)</th>
<th>A2</th>
<th>Thermal phase</th>
<th>Diffusivity amplitude</th>
<th>(cm²/s) x 10⁻³ combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>1*</td>
<td>25.31</td>
<td>6.318</td>
<td>2.655</td>
<td>4.096</td>
</tr>
<tr>
<td>2*</td>
<td>12.25</td>
<td>6.259</td>
<td>3.631</td>
<td>4.767</td>
</tr>
<tr>
<td>3*</td>
<td>7.55</td>
<td>5.672</td>
<td>4.374</td>
<td>4.981</td>
</tr>
<tr>
<td>4*</td>
<td>5.27</td>
<td>5.100</td>
<td>4.631</td>
<td>4.859</td>
</tr>
<tr>
<td>5</td>
<td>3.95</td>
<td>4.765</td>
<td>4.704</td>
<td>4.734</td>
</tr>
<tr>
<td>6</td>
<td>3.11</td>
<td>4.620</td>
<td>4.728</td>
<td>4.674</td>
</tr>
<tr>
<td>7</td>
<td>2.53</td>
<td>4.495</td>
<td>3.726</td>
<td>4.609</td>
</tr>
<tr>
<td>8</td>
<td>2.07</td>
<td>4.438</td>
<td>4.691</td>
<td>4.563</td>
</tr>
<tr>
<td>9</td>
<td>1.76</td>
<td>4.437</td>
<td>4.768</td>
<td>4.600</td>
</tr>
<tr>
<td>10</td>
<td>1.49</td>
<td>4.339</td>
<td>4.683</td>
<td>4.507</td>
</tr>
<tr>
<td>Mean (all)</td>
<td>--</td>
<td>5.044</td>
<td>4.359</td>
<td>4.639</td>
</tr>
<tr>
<td>Mean (ex. *)</td>
<td>--</td>
<td>4.516</td>
<td>4.717</td>
<td>4.615</td>
</tr>
<tr>
<td>Std. E. (all)</td>
<td>--</td>
<td>15.2%</td>
<td>15.8%</td>
<td>5.14%</td>
</tr>
<tr>
<td>Std. E. (ex. *)</td>
<td>--</td>
<td>3.39%</td>
<td>0.66%</td>
<td>1.74%</td>
</tr>
</tbody>
</table>
Figure 57. Thermal diffusivity data showing low frequency error due to boundary effects
Table 7. Experimental data illustrating errors due to a finite heat source at low frequencies and low thermal amplitude at high frequencies (shown in Figure 58)

<table>
<thead>
<tr>
<th>S4-KH-504-JUN)</th>
<th>A2</th>
<th>Thermal Phase</th>
<th>Diffusivity Amplitude</th>
<th>(cm²/s) x 10⁻³ combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>frequency (mHz)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3*</td>
<td>4.83</td>
<td>9.109</td>
<td>8.549</td>
<td>8.825</td>
</tr>
<tr>
<td>4</td>
<td>2.94</td>
<td>8.917</td>
<td>8.700</td>
<td>8.792</td>
</tr>
<tr>
<td>5</td>
<td>1.95</td>
<td>8.824</td>
<td>8.679</td>
<td>8.751</td>
</tr>
<tr>
<td>6</td>
<td>1.38</td>
<td>8.862</td>
<td>8.736</td>
<td>8.797</td>
</tr>
<tr>
<td>7</td>
<td>1.00</td>
<td>8.821</td>
<td>8.728</td>
<td>8.773</td>
</tr>
<tr>
<td>8*</td>
<td>0.75</td>
<td>8.748</td>
<td>8.887</td>
<td>8.816</td>
</tr>
<tr>
<td>9*</td>
<td>0.58</td>
<td>8.669</td>
<td>8.801</td>
<td>8.732</td>
</tr>
<tr>
<td>Mean (all)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean (ex. *)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std. E. (all)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std. E. (ex. *)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 58. Thermal diffusivity data showing the in range and out of range data
Table 8. Experimental data illustrating amplitude method errors due to uncalibrated thermocouples (shown in Figure 59)

<table>
<thead>
<tr>
<th>(S2A-KH-415-MAY) frequency (mHz)</th>
<th>A2</th>
<th>Thermal phase</th>
<th>Diffusivity amplitude</th>
<th>(cm²/s) x 10⁻³ combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>1*</td>
<td>21.58</td>
<td>20.62</td>
<td>12.24</td>
<td>15.89</td>
</tr>
<tr>
<td>2</td>
<td>10.48</td>
<td>21.02</td>
<td>14.02</td>
<td>17.16</td>
</tr>
<tr>
<td>3</td>
<td>6.42</td>
<td>21.00</td>
<td>15.04</td>
<td>17.78</td>
</tr>
<tr>
<td>4</td>
<td>4.46</td>
<td>21.00</td>
<td>15.65</td>
<td>18.13</td>
</tr>
<tr>
<td>5</td>
<td>3.34</td>
<td>21.05</td>
<td>16.06</td>
<td>18.39</td>
</tr>
<tr>
<td>6</td>
<td>2.59</td>
<td>21.06</td>
<td>16.32</td>
<td>18.54</td>
</tr>
<tr>
<td>7</td>
<td>2.11</td>
<td>20.95</td>
<td>16.73</td>
<td>18.72</td>
</tr>
<tr>
<td>8</td>
<td>1.73</td>
<td>21.33</td>
<td>17.05</td>
<td>19.07</td>
</tr>
<tr>
<td>9</td>
<td>1.46</td>
<td>20.54</td>
<td>16.88</td>
<td>18.62</td>
</tr>
<tr>
<td>10</td>
<td>1.23</td>
<td>20.87</td>
<td>17.09</td>
<td>18.88</td>
</tr>
<tr>
<td>Mean (all)</td>
<td>--</td>
<td>20.94</td>
<td>15.71</td>
<td>18.12</td>
</tr>
<tr>
<td>Mean (ex. *)</td>
<td>--</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Std. E. (all)</td>
<td>--</td>
<td>1.08%</td>
<td>9.94%</td>
<td>5.32%</td>
</tr>
<tr>
<td>Std. E. (ex. *)</td>
<td>--</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>

Figure 59 demonstrates strong frequency dependency for the amplitude method and no such dependency for the phase method. To prove that the dependency is due to uncalibrated thermocouples, the thermocouples were then calibrated and the run repeated.

Table 9 and Figure 60 are the results after thermocouple calibration. Figure 60 does not show any frequency dependency in the phase or amplitude methods and Table 9 shows standard errors less than 1%.

In order to determine repeatability and reproducibility of the apparatus, the following test was conducted. The same sample used
Figure 59. Thermal diffusivity data showing no frequency dependency for the phase method. Frequency dependency for the amplitude method is due to EMF gain error.
Table 9. Experimental data showing no frequency dependence (shown in Figure 60)

<table>
<thead>
<tr>
<th>(S2-KH-415-MAY) frequency (mHz)</th>
<th>A2</th>
<th>Thermal phase</th>
<th>Diffusivity amplitude</th>
<th>(cm²/s) x 10⁻³ combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>1*</td>
<td>22.72</td>
<td>20.65</td>
<td>21.43</td>
<td>21.04</td>
</tr>
<tr>
<td>2</td>
<td>11.14</td>
<td>21.01</td>
<td>21.06</td>
<td>21.03</td>
</tr>
<tr>
<td>3</td>
<td>6.84</td>
<td>21.05</td>
<td>21.10</td>
<td>21.08</td>
</tr>
<tr>
<td>4</td>
<td>4.73</td>
<td>21.04</td>
<td>21.03</td>
<td>21.04</td>
</tr>
<tr>
<td>5</td>
<td>3.54</td>
<td>21.01</td>
<td>20.82</td>
<td>20.92</td>
</tr>
<tr>
<td>6</td>
<td>2.76</td>
<td>20.99</td>
<td>20.88</td>
<td>20.94</td>
</tr>
<tr>
<td>7</td>
<td>2.24</td>
<td>21.04</td>
<td>20.94</td>
<td>20.99</td>
</tr>
<tr>
<td>8</td>
<td>1.83</td>
<td>21.18</td>
<td>20.86</td>
<td>21.02</td>
</tr>
<tr>
<td>9</td>
<td>1.53</td>
<td>20.58</td>
<td>20.83</td>
<td>20.70</td>
</tr>
<tr>
<td>10</td>
<td>1.30</td>
<td>20.96</td>
<td>20.67</td>
<td>20.81</td>
</tr>
<tr>
<td>Mean (all)</td>
<td>--</td>
<td>20.95</td>
<td>20.96</td>
<td>20.96</td>
</tr>
<tr>
<td>Mean (ex. *)</td>
<td>--</td>
<td>20.98</td>
<td>20.91</td>
<td>20.95</td>
</tr>
<tr>
<td>Std. E. (all)</td>
<td>--</td>
<td>0.89%</td>
<td>1.00%</td>
<td>0.57%</td>
</tr>
<tr>
<td>Std. E. (ex. *)</td>
<td>--</td>
<td>0.78%</td>
<td>0.65%</td>
<td>0.59%</td>
</tr>
</tbody>
</table>

in Table 9 was used at a temperature of 516°C and a frequency of 5 mHz. Five measurements of thermal diffusivity were made without changing the apparatus to measure repeatability. After the fifth measurement, the sample was allowed to cool to room temperature and then, was reheated to 516°C. Another five measurements were then done. This procedure was repeated so that four sets of data were obtained and appear in Table 10. The repeatability was less than 1% for all runs except one. The mean values in Table 10 appear in Table 11 to determine the reproducibility. Both methods showed errors less than 1%. Both Tables 10 and 11 demonstrate that the precision of the apparatus is at least with 2% and is usually better than 1%.
Figure 60. Thermal diffusivity data showing no frequency dependency. (Thermocouples were calibrated following the data of Figure 59)
Table 10. Experimental data illustrating repeatability (sample cooled to room temperature between tests)

<table>
<thead>
<tr>
<th>Run Number</th>
<th>Phase</th>
<th>Amplitude 1</th>
<th>Amplitude 2</th>
<th>Combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>18.84</td>
<td>18.81</td>
<td>18.83</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>18.66</td>
<td>18.50</td>
<td>18.58</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>18.88%</td>
<td>18.61%</td>
<td>18.75%</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>18.51%</td>
<td>18.72%</td>
<td>18.61%</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>18.62%</td>
<td>18.24%</td>
<td>18.43%</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td>18.70%</td>
<td>18.58%</td>
<td>18.64%</td>
</tr>
<tr>
<td>Std. E.</td>
<td></td>
<td>0.83%</td>
<td>1.19%</td>
<td>0.83%</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>18.57%</td>
<td>18.53%</td>
<td>18.55%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>18.74%</td>
<td>18.75%</td>
<td>18.74%</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>18.80%</td>
<td>18.94%</td>
<td>18.87%</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>18.79%</td>
<td>18.72%</td>
<td>18.76%</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>18.58%</td>
<td>18.53%</td>
<td>18.55%</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td>18.70%</td>
<td>18.69%</td>
<td>18.69%</td>
</tr>
<tr>
<td>Std. E.</td>
<td></td>
<td>0.60%</td>
<td>0.92%</td>
<td>0.75%</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>18.75%</td>
<td>18.54%</td>
<td>18.65%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>18.79%</td>
<td>18.69%</td>
<td>18.74%</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>18.80%</td>
<td>18.60%</td>
<td>18.70%</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>18.90%</td>
<td>18.71%</td>
<td>18.80%</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>18.47%</td>
<td>18.42%</td>
<td>18.43%</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td>18.74%</td>
<td>18.59%</td>
<td>18.66%</td>
</tr>
<tr>
<td>Std. E.</td>
<td></td>
<td>0.86%</td>
<td>0.68%</td>
<td>0.76%</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>18.51%</td>
<td>18.76%</td>
<td>18.63%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>18.98%</td>
<td>18.74%</td>
<td>18.86%</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>18.85%</td>
<td>18.60%</td>
<td>18.72%</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>19.17%</td>
<td>18.64%</td>
<td>18.90%</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>18.40%</td>
<td>18.48%</td>
<td>18.44%</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td>18.78%</td>
<td>18.64%</td>
<td>18.71%</td>
</tr>
<tr>
<td>Std. E.</td>
<td></td>
<td>1.71%</td>
<td>0.61%</td>
<td>0.99%</td>
</tr>
</tbody>
</table>
Table 11. Date of Table 10 summarized to show reproducibility

<table>
<thead>
<tr>
<th>Run</th>
<th>Thermal phase</th>
<th>Diffusivity amplitude</th>
<th>Combination $(cm^2/s) \times 10^{-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>18.70</td>
<td>18.58</td>
<td>18.64</td>
</tr>
<tr>
<td>2</td>
<td>18.70</td>
<td>18.69</td>
<td>18.69</td>
</tr>
<tr>
<td>3</td>
<td>18.74</td>
<td>18.59</td>
<td>18.66</td>
</tr>
<tr>
<td>4</td>
<td>18.78</td>
<td>18.64</td>
<td>18.71</td>
</tr>
<tr>
<td>Mean</td>
<td>18.73</td>
<td>18.64</td>
<td>18.68</td>
</tr>
<tr>
<td>Std. E.</td>
<td>0.20%</td>
<td>0.27%</td>
<td>0.17%</td>
</tr>
</tbody>
</table>

The same sample was then raised to a temperature of 878°C to determine if the flat frequency curves could be reproduced at high temperatures. Figure 61 is a plot of the data obtained in Table 12. Again, the amplitude has a frequency dependency. After the run, the sample was removed for inspection. The thermocouples showed signs of corrosion. From these data, we can conclude that the thermocouple calibration changes over extended periods of time at high temperatures. Therefore, the phase method is superior to the amplitude method in that thermocouple calibration does not affect the thermal diffusivity measurement.

The test temperature reported for all the data is the time averaged temperature for both thermocouples over the entire run. Each thermal diffusivity measurement was the accumulation of data over at least nine thermal cycles, or between 1800 and 8000 data points.
Table 12. Experimental data taken at 878°C

<table>
<thead>
<tr>
<th>(S2B-KH-878-JUN) frequency (mHz)</th>
<th>A2</th>
<th>Thermal phase</th>
<th>Diffusivity amplitude</th>
<th>(cm²/s) x 10⁻³ combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>2*</td>
<td>12.56</td>
<td>12.56</td>
<td>8.147</td>
<td>10.11</td>
</tr>
<tr>
<td>3</td>
<td>5.75</td>
<td>12.48</td>
<td>8.809</td>
<td>10.49</td>
</tr>
<tr>
<td>4</td>
<td>3.86</td>
<td>12.41</td>
<td>9.338</td>
<td>10.77</td>
</tr>
<tr>
<td>5</td>
<td>2.75</td>
<td>12.31</td>
<td>9.709</td>
<td>10.93</td>
</tr>
<tr>
<td>6</td>
<td>2.07</td>
<td>12.34</td>
<td>9.917</td>
<td>11.06</td>
</tr>
<tr>
<td>7</td>
<td>1.62</td>
<td>12.21</td>
<td>10.260</td>
<td>11.19</td>
</tr>
<tr>
<td>8</td>
<td>1.31</td>
<td>12.31</td>
<td>10.460</td>
<td>11.35</td>
</tr>
<tr>
<td>9</td>
<td>1.08</td>
<td>12.27</td>
<td>10.580</td>
<td>11.39</td>
</tr>
<tr>
<td>10*</td>
<td>0.86</td>
<td>12.04</td>
<td>10.850</td>
<td>11.42</td>
</tr>
<tr>
<td>Mean (all)</td>
<td>--</td>
<td>12.33</td>
<td>9.766</td>
<td>10.97</td>
</tr>
<tr>
<td>Mean (ex. *)</td>
<td>--</td>
<td>12.33</td>
<td>9.868</td>
<td>11.03</td>
</tr>
<tr>
<td>Std. E. (all)</td>
<td>--</td>
<td>1.23%</td>
<td>9.07%</td>
<td>4.07%</td>
</tr>
<tr>
<td>Std. E. (ex. *)</td>
<td>--</td>
<td>0.72%</td>
<td>6.46%</td>
<td>2.93%</td>
</tr>
</tbody>
</table>
Figure 61. Thermal diffusivity data showing no frequency dependency for the phase method. Amplitude method demonstrates a thermocouple EMF gain error. This data demonstrates that thermocouple EMF gains can change with time at elevated temperatures.
CONCLUSION

This method has proven to have very high precision, < 2%, when care is taken to avoid errors. The precision of the phase method proved to be superior in comparison to the amplitude method. This is due to the fact that time can be measured and controlled to a higher precision than can the calibration of the thermocouples.

Our apparatus has successfully measured a range of thermal diffusivities from 0.002 to 0.021 cm$^2$/s. Although these measurements were all made at temperatures in the range of 300-1000°C, it should not be difficult to extend the temperature range much higher. Unfortunately, our apparatus was not capable of achieving temperatures greater than 1000°C.

Type K thermocouples were used primarily due to availability. Other thermocouples can be substituted for particular applications. However, one should select thermocouples with the largest thermal emf possible for the temperature range of interest since the amplitude of the thermal waves is usually only a few degrees Celsius.

The accuracy of this apparatus proved to be better than 2% and usually less than 1% as demonstrated from the flat frequency dependency curves. Therefore, if we assume that the accuracy of the separation distance between thermal measurements can be measured to 1%, then the accuracy of the thermal diffusivity measurement will be 3-4%. Certainly, the accuracy of this method is comparable to the accuracy of either the commonly used hot-wire method or the flash
method, yet sample design and preparation are much simpler. In addition, the thermal diffusivity is obtained during the experimental run as opposed to completing the experimental run and then estimating the thermal diffusivity.

In conclusion, this method is a useful alternative to other methods and in many respects, much simpler to implement.
BIBLIOGRAPHY


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The author also thanks both Dr. Wilder and Dr. Martin for giving him a second chance at graduate studies by readmittance in March 1981.

The author extends his appreciation to Barb Dubberke for her excellent job in typing and editing, and knowledge of the Thesis Office rules and regulations.

Finally, the author extends his most dearest appreciation to his family, namely, Diane, Stephanie, Shawn, Wade, Agnas, Bert, and Nora, for their patience, love, understanding and encouragement through the rough years as a graduate student.
APPENDIX A: THERMAL DIFFUSIVITY CONTROL PROGRAM
100 REM PERIODIC THERMAL DIFFUSIVITY CONTROL PROGRAM
101 REM DEVELOPED & WRITTEN BY
102 REM DAVID SLUTZ
103 REM IOWA STATE UNIVERSITY
104 REM AMES, IA 50010
105 :----------------------------------------------------------------------------------:
106 REM TABLE OF VARIABLE NAMES
107 REM A1-AMPLITUDE FOR TC1 (°C)
108 REM A2-AMPLITUDE FOR TC2 (°C)
109 REM AA-DIFFUSIVITY (CM²/S) BY AMPLITUDE
110 REM AB-FLAG TO ABORT RUN
111 REM AC-DIFFUSIVITY (CM²/S) COMBINED METHOD
112 REM AD-DELTA PHASE SHIFT AMPLITUDE METHOD
113 REM AN-ANGLE OF PRIMARY PERIODIC WAVE (FOURIER FILTER)
114 REM AP-DIFFUSIVITY (CM²/S) BY PHASE SHIFT
115 REM B1-BANDWIDTH LOWER FURNACE
116 REM B2-BANDWIDTH UPPER FURNACE
117 REM C1-SETPOINT LOWER FURNACE
118 REM C2-SETPOINT UPPER FURNACE
119 REM C3-MINIMUM SET POINT
120 REM C4-MAXIMUM SET POINT
121 REM C5-TEMPERATURE STEP SIZE
122 REM D-SAMPLE THICKNESS (CM)
123 REM DE-DEVIATION FOR FURNACE CONTROL
124 REM DA-DAY
125 REM DR-PRINTER DEVICE NUMBER
126 REM D$-DATE
127 REM DT(I)-ARRAY OF DAYS IN THE MONTH
128 REM F-FREQUENCY (HZ)
129 REM F1-FLAG FOR CHANGING FREQUENCY
130 REM F2-MAXIMUM FREQUENCY ALLOWED
131 REM F3-MINIMUM FREQUENCY ALLOWED
132 REM F4-FREQUENCY STEP SIZE
133 REM F5-NUMBER OF RUNS @ SINGLE F
134 REM F6-AVERAGE PHASE DIFFUSIVITY
135 REM F7-AVERAGE AMPLITUDE DIFFUSIVITY
136 REM F8-AVERAGE COMBINED DIFFUSIVITY
137 REM FF-FLAG TO END ITERATION
138 REM FR-OLD FREQUENCY
139 REM FT-TIME FACTOR (# SECONDS PER READING)
140 REM G1-GAIN OF THERMOCOUPLE #1
141 REM G2-GAIN OF THERMOCOUPLE #2
142 REM IC(1)-TC1 INTEGRAL VALUE * COS(FILTER)
143 REM IC(2)-TC2 INTEGRAL VALUE * COS(FILTER)
144 REM IS(1)-TC1 INTEGRAL VALUE * SIN(FILTER)
145 REM IS(2)-TC2 INTEGRAL VALUE * SIN(FILTER)
1460 REM JJ-PAPER ADVANCE COUNTER
1470 REM MO-MONTH
1480 REM 01-OFFSET OF THERMOCOUPLE #1
1490 REM 02-OFFSET OF THERMOCOUPLE #2
1500 REM P1-PHASE SHIFT TC1 (RAD)
1510 REM P2-PHASE SHIFT TC2 (RAD)
1520 REM P1%-PROPORTIONAL POWER LOWER FURNACE
1530 REM P2%-PROPORTIONAL POWER UPPER FURNACE
1540 REM PD-DELTA PHASE SHIFT BETWEEN TC1 & TC2
1550 REM R1-RESET LOWER FURNACE
1560 REM R2-RESET UPPER FURNACE
1570 REM S1*-STANDARD ERROR TC#1
1580 REM S2*-STANDARD ERROR TC#2
1590 REM SE( )-ARRAY USED FOR STANDARD ERRORS
1600 REM SS( )-ARRAY USED FOR LEAST SQUARES
1610 REM SF-FREQUENCY ON CNT PIN
1620 REM TB7-TIMERB VALUE (TRIGGER OUTPUT)
1630 REM TBX-TIMERB VALUE (IE #TRIGGERS/PERIOD) (PERIODIC OUTPUT)
1640 REM TR*-TRANSFER STRING VARIABLE (UTILITY)
1650 REM TT$-SUM OF ALL TEMPERATURE MEASUREMENTS
1660 REM V1-DEVICE # FOR VOLTOMETER OF TCI
1670 REM V2-DEVICE # FOR VOLTOMETER OF TC2
1680 REM V3-DEVICE # FOR VOLTOMETER OF FURNACE
1690 REM XX( )-ARRAY USED FOR LEAST SQUARES
1700 REM XX-VARIABLE FLAG FOR LEAST SQUARES.
1710 REM YR-YEAR
1720 REM ZC-PURE COS FILTER VALUE
1730 REM ZS-PURE SIN FILTER VALUE
1740 REM 6526 REGISTERS
1750 SYSSC728:CLR:REM ENABLE IEEE
1760 PB=56577:REM PORT B
1770 DB=56579:REM DATA DIRECTION REGISTER
1780 TA=56580:REM TIMER A LOW BYTE
1790 TB=56582:REM TIMER B LOW BYTE
1800 IR=56589:REM ICR REGISTER
1810 CA=56590:REM CONTROL A
1820 CB=56591:REM CONTROL B
1830 M1=829:REM MEMORY FOR LOWER FURNACE
1840 M2=829:REM MEMORY FOR UPPER FURNACE
1850 CD=898:REM FURNACE CONTROL ENABLED
1860 CI=898:REM FURNACE CONTROL ENABLED
1870 CD=898:REM FURNACE CONTROL DISABLED
1880 F:"OFF":HS:"OFF"
1890 REM MESSAGE STRINGS
1900 T$="TIMERS OFF"
1910 P7$="PB7 SET HIGH"
1920 P6$="PB6 SET HIGH"
1930 M$="TIMERA=0"
1940 FC$="OFF":HS$="OFF"
1950 SYSSC728:CLR:REM ENABLE IEEE
1960 PB=56577:REM PORT B
1970 DB=56579:REM DATA DIRECTION REGISTER
1980 TA=56580:REM TIMER A LOW BYTE
1990 TB=56582:REM TIMER B LOW BYTE
2000 IR=56589:REM ICR REGISTER
2010 CA=56590:REM CONTROL A
2020 CB=56591:REM CONTROL B
2030 M1=829:REM MEMORY FOR LOWER FURNACE
2040 M2=829:REM MEMORY FOR UPPER FURNACE
2050 CD=898:REM FURNACE CONTROL ENABLED
2060 CI=898:REM FURNACE CONTROL ENABLED
2070 CD=898:REM FURNACE CONTROL DISABLED
2080 F:"OFF":HS:"OFF"
1970 REM PARAMETERS FOR THERMOCOUPLE EQUATION V=MICROVOLTS; T='C
1980 A(0)=.226584602; A(1)=2415.2; A(2)=67233.4248
1990 A(3)=210340.682; A(4)=860963714.9; A(5)=4.835E10
2000 A(6)=1.18452E12; A(7)=1.3E13; A(8)=-6.33708E13
2010 REM SET GAIN & OFFSET OF THERMOCOUPLES 1&2 TO UPPER FURNACE TC
2020 61=1:01=0:REM TC#1
2030 G2=1:02=0:REM TC#2
2040 63=1:03=0:REM FURNACE TC
2050 :::::::::::::::::::::::::::::::::::::::::::
2060 REM DEFINITIONS
2070 DEF FNT(V)=A(0)+A(1)*V+A(2)*V^2+A(3)*V^3+A(4)*V^4+FNTT(V)
2080 DEF FNTT(V)=A(5)*V^5+A(6)*V^6+A(7)*V^7+A(8)*V^8
2090 DEF FNT1(F)=SF/TB%/F/2-1:REM TIMER A VALUE
2100 DEF FNF(T)=SF/TB%/F(T+1)/2:REM FREQUENCY OUTPUT ON PB7
2110 :::::::::::::::::::::::::::::::::::::::::::
2120 REM DIMENSION ARRAYS
2130 DIM DT(12),SS(100,3,2),XX(2,3),MF(100)
2140 DT ( 1 ) =31 : DT (2) =28: DT (3) =31 : DT (4) =30: DT (5) =31 : DT (6) =30: DT (7) =31
2150 DT(8)=31:DT(9)=30:DT(10)=31:DT(11)=30:DT(12)=31
2160 :::::::::::::::::::::::::::::::::::::::::::
2170 REM MESSAGE ROUTINE
2180 POKE53280,6:POKE33280,6:PRINTCHR*(144)CHR*(14)CHR*(8)
2190 REM SET SCREEN COLOR
2200 PRINT"*THIS PROGRAM WILL PUT A SQUARE WAVE OUT ON PB6 AND PB7."
2210 PRINT"THE FREQUENCY OF PB6 IS A MULTIPLE OF PB7. THE PURPOSE"
2220 PRINT"IS TO HAVE PB6 TRIGGER A VOLTMETER. WHILE PB7 CONTROLS"
2230 PRINT"THE HEAT SOURCE. IN ADDITION, PB6 WILL DRIVE THE FLAG"
2240 PRINT"BIT IN THE ICR REGISTER FOR USE OF THE WAIT COMMAND."
2250 PRINT"THIS WILL ALLOW THE BASIC PROGRAM TO STAY IN SYNC.";
2260 :::::::::::::::::::::::::::::::::::::::::::
2270 REM MAIN SELECTION ROUTINE
2280 :::::PRINT"#MAIN SELECTION MENU#"
2290 :::::PRINT 1) FURNACE CONTROL SELECTION"
2300 :::::PRINT 2) HEATING ELEMENT SELECTION"
2310 :::::PRINT 3) EXAMINE THERMOCOUPLES"
2320 :::::PRINT 4) LOOK AT TIMER VALUES"
2330 :::::PRINT 5) PERIODIC THERMAL DIFFUSIVITY"
2340 :::::PRINT 6) CALIBRATE THERMOCOUPLES"
2350 :::::PRINT 7) EXIT PROGRAM"
2360 :::::PRINT SELECT ONE"
2470 ::GOSUB 5420 :REM PRESENT STATUS
2480 GET E$:PRINTE$;
2490 IF VAL(E$)=0 OR VAL(E$)>7 THEN GOSUB7660:GOSUB3300
PRINT"***";GOSUB5450:GOTO2480
2500 ::ON VAL(E$) GOSUB 2860,2530,4940,5590,3720,7920,6160
2510 TO=TI;GOTO2370:REM NEXT INPUT
2520 :::::::::::::::::::::::::::::::
2530 ::::PRINT"*HEATING ELEMENT SELECTION#"
2540 :::::PRINT" L) HEAT SOURCE IS "HS$
2550 :::::PRINT" 2) SELECT FREQUENCY F=";F"Hz"
2560 :::::PRINT" 3) SELECT TRIGGER RATE FT=";FT"S/READING"
2570 :::::PRINT" 4) RETURN MAIN MENU"
2580 GET A$:N=VAL(A$):IF N=0 THEN GOSUB3300:GOSUB7660:GOTO2580
2590 PRINTN:IF N>4 THEN RETURN
2600 ON N GOSUB 2650,3130,2630
2610 GOTO2530
2620 :::::::::::::::::::::::::::::::
2630 INPUT"TRIGGER RATE (# SECONDS PER READING)";FT:GOTO3160
2640 :::::::::::::::::::::::::::::::
2650 IF HS$="OFF"THEN 2700
2660 IF HS$="ON" THEN 2820
2670 IF HS$="PERIODIC" THEN 2760
2680 RETURN
2690 :::::::::::::::::::::::::::::::
2700 REM PB7 LOW
2710 POKECB,(PEEK(CB)AND253):REM PB7 DISABLED
2720 POKEPB,(PEEK(PB)AND127):REM SET PB 7 LOW
2730 P7$="PB7 SET LOW":HS$="ON"
2740 RETURN
2750 :::::::::::::::::::::::::::::::
2760 REM PB7 HIGH
2770 POKECB,(PEEK(CB)AND253):REM PB7 DISABLED
2780 POKEPB,(PEEK(PB)OR128):REM SET PB7 HIGH
2790 P7$="PB7 SET HIGH":HS$="OFF"
2800 RETURN
2810 :::::::::::::::::::::::::::::::
2820 REM PB7 SQUARE WAVE
2830 POKECB,(PEEK(CB)OR2):REM ENABLE PB7
2840 P7$="PB7 SQUARE WAVE":HS$="PERIODIC":RETURN
2850 :::::::::::::::::::::::::::::::
2860 REM FURNACE CONTROL SELECTION ROUTINE
2870 ::::PRINT"*FURNACE CONTROL SELECTION#"
2880 ::::PRINT" 1) COMPUTER CONTROL IS "FC$
2890 ::::PRINT" 2) UPPER FURNACE SET POINT=";C2
2900 ::::PRINT" 3) UPPER FURNACE BANDWIDTH";B2
2910 ::::PRINT" 4) UPPER FURNACE RESET";R2
2920 ::::PRINT" 5) LOWER FURNACE SET POINT=";C1
2930 ::::PRINT" 6) LOWER FURNACE BANDWIDTH";B1
2940 ::::PRINT" 7) LOWER FURNACE RESET";R1
2950 ::::PRINT" 8) RETURN MAIN MENU"
2960 ::::PRINT"SELECT ONE";
GET A$=VAL(A$): IF N=0 THEN GOSUB3300: GOSUB7660: GOTO2970
PRINT: IF N<>0 THEN RETURN
ON N GOSUB 3920, 3960, 3970, 3980, 3990, 3100, 3110
GOTO 2660

IF FC$="ON" THEN FC$="OFF": SYS(CD): POKEPB, (PEEK(PB) OR 3): RETURN
IF FC$="OFF" AND B1>0 AND B2>0 AND F>0 THEN FC$="ON"
SYS(CI): RETURN

PRINTN: IF N>-8 THEN RETURN

INPUT"SETPOINT UPPER FURNACE"%C2: RETURN
INPUT"BANDWIDTH UPPER FURNACE"%B2: RETURN
INPUT"RESET UPPER FURNACE"%R2: RETURN
INPUT"SETPOINT LOWER FURNACE"%C1: RETURN
INPUT"BANDWIDTH LOWER FURNACE"%B1: RETURN
INPUT"RESET LOWER FURNACE"%R1: RETURN

REM ROUTINE TO SET PERIODIC HEAT WAVE
INPUT"DESIRED FREQUENCY"%F
IF FT<-0 THEN 2630
IF F<-0 THEN 3140
TB% INT(1/F/FT): IF TB%>100 THEN TB%-100
GOSUB6110: REM LOAD TIMER B FOR TRIGGER
H-FNT1(F): IF H<0 OR H>32767 THEN PRINT: INPUT AGAIN": GOTO3140
GOSUB3250: POKETA, L%: POKETA+1, H%: TA%»256*H%+L%
F-FFN(TA%): GOSUB6040: REM ENABLE PB7 PB6 & START TIMERS
M$="TIMER A-"+STR$(TAX)
RETURN

REM SUBROUTINE TO CALCULATE BYTE VALUES
HX-H/256
L%-H-256*H%
RETURN

REM UPDATE CALENDAR
IF TF<TF THEN TF=TF: RETURN
TF=TF: MOD=VAL(D$): YR=VAL(RIGHT$(D$,2))
FOR KK=1 TO LEN(D$)
IF MID$(D$, KK, 1)="/" THEN DA=VAL(MID$(D$, KK+1)): KK=LEN(D$)
NEXT KK
DA=DA+1
IF DA>DT(MO) AND MO<2 THEN DA=1: MO=MO+1: GOTO3380
IF DA>DT(MO) AND MO>2 THEN GOSUB3420
IF MO=13 THEN MO=1: YR=YR+1
DS=MID$(STR$(MO), 2)+"/"+MID$(STR$(DA), 2)+"/"+MID$(STR$(YR), 2)
RETURN

REM CHECK FOR LEAP YEAR
IF DA=30 THEN DA=1: MO=MO+1: RETURN
LP=YEAR/4: FOR I=0 TO 24
LP=LP+1: NEXT I
RETURN

DA=1: MO=MO+1: RETURN
REM HEADER OUTPUT
PRINT#DR,"FREQUENCY="F'HZ',;I
PRINT#DR,"PERIOD=1/F'S"
PRINT#DR,"# OF POINTS=TB%;I
PRINT#DR,"TIME=TI$"
PRINT#DR,"#CYCLES TEMP PHASE1 PHASE2 DELTA DIFFUSIVITY"
PRINT#DR,"TIME A TIME C AMPL.1 AMPL.2 DELTA DIFFUSIVITY"
PRINT#DR,"STD E1 STD E2 COMBINED DIFFUSIVITY"
JJ=5:RETURN

REM ADVANCE PAPER TO NEXT PAGE
IF JJ=-66 THEN JJ=JJ-66:GOTO3600
IF JJ=0 THEN3630
FOR I=1 TO 66-JJ:PRINT#DR:NEXTI
RETURN

REM UPDATE FREQUENCY
IF Fl=0 THEN RETURN
FR=FR+F1*F4:F-FR
GOSUB3160:REM RESET PERIOD
GOSUB4940:REM WAIT
RETURN

REM THERMAL CONDUCTIVITY CALCULATION ROUTINE
REM BY DISCRETE FOURIER DATA ANALYSIS
PRINT"PRESENT FREQUENCY=",F'Z"
PRINT$:PRINTP$:PRINTP$:PRINTM$:PRINT"NUMBER DATA POINTS=",TB%
INPUT"DO YOU WISH TO MAKE A CHANGE (Y/N)";A$
ZPOKEPB,(PEEK(PB)OR4):REM RESET THERMOCOUPLE RELAY
IF A$=<"N- OR F-0 THEN RETURN
INPUT-OUTPUT DEVICE # (CRT-3);DR:OPENDR,DR
INPUT"THICKNESS SAMPLE (CM)";D
PRINT"DO YOU WANT TEMPERATURE TO VARY (Y/N) ?";INPUTA$
IF A$<"N" THEN C6=0:GOTO3860
IF A$<"Y" THEN 3810
INPUT"TEMPERATURE (MAX,MIN) ";C4,C3
INPUT"TEMPERATURE STEP SIZE ";C5:C6=1
PRINT"DO YOU WANT FREQUENCY VARY (Y/N) ?";INPUTA$
IF A$="N" THEN F1=0:F2=0:GOTO 3910
IF A$<"Y" THEN 3860
INPUT"FREQUENCY (MAX,MIN) Hz";F2,F3:F1=-1
INPUT"FREQUENCY STEP SIZE (Hz)";F4:FR=F2
INPUT"HOW MANY RUNS @ SAME F";F5:IF F5<1 THEN 3910
Ab=0:REM RESET ABORT FLAG
GOSUB3490:REM OUTPUT HEADER
GOSUB 4070:REM CALCULATE THERMAL DIFFUSIVITY
GOSUB 3300:REM UPDATE CALANDER
GOSUB 3590:REM ADVANCE PAPER
IF Ab=1 THEN CLOSEDR:RETURN:REM ABORT PROGRAM
3990 IF F1=0 AND THEN 4010:REM END SINGLE RUN
4000 IF FR>F3 THEN GOSUB3650:GOTO3930:REM UPDATE FREQUENCY
4010 IF C6=0 THEN CLEODR:RETURN:REM END FREQUENCY SCAN
4020 IF C2>-C4 THEN C2=C3:C1=C3:CLOSEDR:RETURN:REM END TEMPERATURE SCAN
4030 C2=C2+C5:C1=C1+C5:FR=F2:GOSUB3160:REM RESET FREQUENCY AND TEMPERATURE
4040 IF W<(Cl-.5) OR W>(Cl+.5) THEN GOSUB3690:GOTO4040:REM STEADY STATE
4050 GOTO3930:REM CONTINUE
4060 :::::::::::::::::::::::::::::::::
4070 REM ITERATION SUBROUTINE
4080 F6=0:F7=0:F8=0
4090 FOR WW=1TOF5
4100 PRINT"WAIT!!":GOSUB7540:REM INITIALIZE ARRAY
4110 PRINT"M TEMP MAX MIN AVERAGE"
4120 PRINT"###############################"
4130 T1=PEEK<IR):WAIT IR,16:T0=T1:REM WAIT FOR TRIGGER
4140 J=1
4150 FF=0:XT=0:YT=0:UT=0:T2=T1:WT=0
4160 XM(0)=0:YM(0)=0:UM(0)=0:XN(1)=1E10:YN(1)=1E10
4170 UM(1)=1E10:WN(0)=0:WM(1)=1E9
4170 FOR I=1TO4
4180 IF I=I THEN GOSUB 3300
4190 NEXTI
4200 ::FOR I=1TOTB%
4210 ::GOSUB6480:REM INPUT TEMP & ANGLE & NEW SUM
4220 ::::IF XX OR SS(I,2,2)<7 THEN FF=FF+1
4230 ::::IS(I)<IS(I)+X*ZS:IC(I)=IC(I)+X*ZC:REM TC1 DECREASE DATA
4240 ::::IS(I)«IS(I)+Y*ZS:IC(I)«IC(I)+Y*ZC:REM TC2 DECREASE DATA
4250 ::::REM CORRECTION INTEGRALS FOR SLOPE
4260 ::::IS(3)<IS(3)+XX(I,0)*XX(0,1)*ZS
4270 ::::IC(3)<IC(3)+XX(I,0)*XX(0,1)*ZC
4280 ::::IS(4)<IS(4)+XX(I,1)*XX(0,1)*ZS
4290 ::::IC(4)<IC(4)+XX(I,1)*XX(0,1)*ZC
4300 ::::GOSUB 5140:REM SCREEN OUTPUT TEMPERATURES & STATUS
4310 ::::GET A$:IF A$="S" THEN AB=1:RETURN
4320 ::::IF FF THEN J=J+1:GOSUB4150
4330 ::::REM CORRECT FOR SLOPE
4350 ::::FORI=1TO2
4360 ::::IS(I)=2*(IS(I)-IS(I+2))/TB%
4370 ::::IC(I)«IC(I)-2*(IC(I)-IC(I+2))/TB%
4380 ::::NEXTI
4390 ::::GOSUB4470:REM OUTPUT CORRECTION
4400 ::::NEXTWW
4410 PRINT#DR,"AVERAGES"
4420 PRINT#DR,"PHASE","AMPLITUDE","COMBINED"
4430 PRINT#DR,F6/F5,F7/F5,F8/F5
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4440 JJ=JJ+3
4450 :GOSUB 7540 :REM CLEAR LS ARRAYS
4460 RETURN
4470 REM **** TIME & TEMPERATURE ****
4480 ::IF T1<T0 THEN T1=T1+24*60+3:REM ADJUST FOR JIFFY ROLL OVER
4490 ::TR$=STR$(INT(5*(T1-T0)/3+.5)/100):GOSUB4830:TS$=TR$ :REM REAL TIME
4500 ::TR$=STR$(J/F):GOSUB4830:T4$=TR$:REM CALCULATE TIME
4510 ::TR$=STR$(INT((XT+YT)/100)/100):GOSUB4830:TT$=TR$:REM TEMPERATURE
4520 ::TR$=STR$(J):GOSUB4830:J$=TR$:REM CYCLE NUMBER
4530 REM **** PHASE ANGLE METHOD ****
4540 ::REM CALCULATE PHASE ANGLES & CORRECT FOR DISCONTINUITIES
4550 ::PI=ATN(IC(1)/IS(1)):P2=ATN(IC(2)/IS(2))
4560 :::::IF PI<0 THEN PI=PI+rr
4570 :::::IF P2<PI THEN P2=PI
4580 :::::PI»rr
4590 :::::P2»rr
4600 :::::AP»rr
4610 :::::P1=rr
4620 :::::P2=rr
4630 REM **** AMPLITUDE METHOD ****
4640 :::::A1=SQRT(IC(1)^2+IS(1)^2)
4650 :::::A2=SQRT(IC(2)^2+IS(2)^2)
4660 :::::A1=rr
4670 :::::A2=rr
4680 :::::AP=rr
4690 :::::P1=rr
4700 :::::P2=rr
4710 REM *** COMBINED METHOD ****
4720 ::::AC=rr
4730 ::::SE=rr
4740 ::::S1=rr
4750 ::::S2=rr
4760 ::::AC=rr
4770 ::::SE=rr
4780 ::::S1=rr
4790 ::::S2=rr
4800 ::::JT=JJ+3
4810 RETURN
4820 :FOR II=1 TO LEN(TR$)
4830 ::::PS=MID$(TR$,II,1):IF PS="" THEN TR$=3+II-LEN(TR$)
4840 :NEXTII
4850 ::IF TR$=0 THEN TR$=TR$+" ":GOTO4910
4860 ::FORII=1 TO TR$:TR$=TR$+" ":NEXTII
4870 ::IF LEN(TR$)=10 THEN RETURN
4920 FOR II=LEN(TR$) TO 9:TR$="+TR$:NEXTII:RETURN
4930 :::::::::::::::::::::::::::::::::::::::::::
4940 REM SUBROUTINE TO LOOK AT THERMOCOUPLES
4950 IF F=0 THEN 3130
4960 POKEPB,(PEEK(PB)OR4):REM RESET THERMOCOUPLE RELAY
4970 PRINT"###TEMP MAX MIN AVERAGE###"
4980 J=1:60SUB 7540:REM NEW SUM
4990 XM(0)=0:YM(0)=0:UM(0)=0;XM(1)=1E10;YM(1)=1E10
   UM(1)=1E10;WM(0)=0;WM(1)=1E9
5000 XT=0;YT=0;UT=0;WT=0
5010 FOR I=1TOTBY.
5020 ::::GOSUB6470:REM INPUT TEMP & ANGLE
5040 :::GOSUB5140:REM SCREEN OUTPUT
5050 ::::GET A$:IF A$="S" THEN RETURN
5060 ::SS(I,3,0)=0
5070 :NEXTI
5080 :GOSUB3300:REM UPDATE CALANDER
5090 ::IF ABS((XT+YT)/2/TB%)-C2)<20 THEN 5100
5100 ::IF E$="3" THEN J=J+1:GOTO5000
5110 ::IF E$="5" AND J<10 THEN J=J+1:GOTO5000
5120 RETURN
5130 :::::::::::::::::::::::::::::::::::
5140 REM SCREEN OUTPUT
5150 PRINT"
5160 PRINT"
5170 PRINT"
5180 PRINT":PRINT"####"
5190 PRINTINT(100*W)/100,WM(0),WM(1),INT(100*WT/I)/100
5200 PRINTINT(100*Y)/100,YM(0),YM(1),INT(100*YT/I)/100
5210 PRINTINT(100*X)/100,XM(0),XM(1),INT(100*XT/I)/100
5220 PRINTINT(100*U)/100,UM(0),UM(1),INT(100*UT/I)/100
5230 PRINT"
5240 PRINT"J=J"I="I
5250 60SUB 5420:REM REST OF OUTPUT
5260 PRINT"####"
5270 PRINT"ANGLE="INT(1800*AN/+.5)/100
5280 PRINT"SLOPE TC#1="XX(1,0)
5290 PRINT"INTER TC#1="INT(100*XX(1,1))/100,"STD.E=";
5300 IF SS(I,3,0)<3 THEN PRINT"???";GOT05320
5310 PRINTSE(3)
5320 PRINT"SLOPE TC#2="XX(1,2)
5330 PRINT"INTER TC#2="INT(100*XX(1,3))/100,"STD.E=";
5340 IF SS(I,3,0)<3 THEN PRINT"???";GOT05380
5350 PRINTSE(4)
5360 PRINT"AVG. STD. ERROR TC#1="SE(1)/I
5370 PRINT"AVG. STD. ERROR TC#2="SE(2)/I
5380 PRINT"SP#2=",C2"DEG SP#1=";INT(100*C1)/100"DEG"
5390 PRINT"####"
5400 RETURN
5410 :::::::::::::::::::::::::::::::::::
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5420 REM PRINT PRESENT STATUS  
5430 PRINT "PRESENT FREQUENCY=";F"Hz";PRINT " DATA POINTS/CYCLE=";TB%  
5440 PRINT T$;PRINT P7$; " P6$";PRINT T$  
5450 PRINT T$  
5460 PRINT "TIME=";TI$" RUN TIME="XX(0,1)  
5470 RETURN  
5480 :::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::

5490 REM START TIMERS  
5500 T$="TIMERS ON"  
5510 POKECA,(PEEK(CA)OR17):POKECB,(PEEK(CB)OR17):REM START TIMERS  
5520 RETURN  
5530 :::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::

5540 REM STOP TIMERS  
5550 POKECA,(PEEK(CA)AND254):REM TURN OFF TIMER A  
5560 POKECB,(PEEK(CB)AND254):REM TURN OFF TIMER B  
5570 T$="TIMERS OFF":RETURN  
5580 :::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::

5590 REM ROUTINE TO LOOK AT TIMERS  
5600 IF F=0 THEN RETURN  
5610 ::PRINT "THE VALUES ON THE LEFT ARE TIMER A WHILE THOSE ON THE":  
5620 ::PRINT "RIGHT ARE TIMER B. TO START PRESS G KEY. TO STOP":  
5630 ::PRINT "PRESS G KEY. TA TB B7 ANGLE PERIOD":  
5640 ::GET A$:IF A$<"G" THEN GOSUB 33300:GOTO 5640  
5650 ::T0-TI:AA=0:WAIT IR,16  
5660 FOR I=1 TO 20  
5670 WAIT IR,16:REM WAIT FOR TRIGGER  
5680 A<256*:PEEK(TA+1)+PEEK(TA)  
5690 B<256*:PEEK(TB+1)+PEEK(TB):C=(PEEK(PB)AND128)/128  
5700 ::::IF C=1 AND AA=1 THEN AA=0:T0-TI  
5710 :::::T1=1/F-(TI-T0)/60  
5720 ::::IF C=0 AND AA=0 THEN AA=1  
5730 ::::AN=(C1-B/TB)*2BN=TB*/AN*/2  
5740 ::::PRINT A; B; C; INT(100*AN)/100; INT(100*T1)/100  
5750 GET A$:IF A$="S" THEN RETURN  
5760 GOSUB 3300:NEXTI  
5770 ::PRINT "THE VALUES ON THE LEFT ARE TIMER A WHILE THOSE ON THE":  
5780 ::PRINT "RIGHT ARE TIMER B. TO START PRESS G KEY. TO STOP":  
5790 ::PRINT "PRESS G KEY. TA TB B7 ANGLE PERIOD":  
5800 60T05660  
5810  
5820 REM PB6 LOW  
5830 POKECA,(PEEK(CA)AND253):REM PB6 DISABLED  
5840 POKEPB,(PEEK(PB)AND191):REM SET PB 6 LOW  
5850 P6$="PB6 SET LOW"  
5860 RETURN  
5870  
5880 REM PB6 HIGH  
5890 POKECA,(PEEK(CA)AND253):REM PB6 DISABLED  
5900 POKEPB,(PEEK(PB)OR64):REM SET PB6 HIGH  
5910 P6$="PB6 SET HIGH"  
5920 RETURN
5930 :::::::::::::::::::::::::::::::::::::
5940 REM PB6 SQUARE WAVE
5950 POKECA, (PEEK(CA) OR2):REM ENABLE PB6
5960 P6$="PB6 SQUARE WAVE":RETURN
5970 :::::::::::::::::::::::::::::::::::::
5980 REM TURN ALL OFF
5990 GOSUB 5540:REM TIMERS OFF
6000 GOSUB 2760:REM PB7 HIGH
6010 GOSUB 5880:REM PB6 HIGH
6020 RETURN
6030 :::::::::::::::::::::::::::::::::::::
6040 REM ALL ON
6050 GOSUB 5980:REM IDLE STATE
6060 GOSUB 5490:REM TIMERS ON
6070 GOSUB 2820:REM PB7 ON
6080 GOSUB 5940:REM PB6 ON
6090 RETURN
6100 :::::::::::::::::::::::::::::::::::::
6110 REM SET THE TRIGGER RATE
6120 H-TB%-1:GOSUB 3250:REM CALCULATE BYTE VALUE
6130 POCKETB, L%: POCKETB+1, M%: REM LOAD TIMERB
6140 POKECB, (PEEK(CB) OR 16):REM FORCE LOAD TIMERB
6150 RETURN
6160 :::::::::::::::::::::::::::::::::::::
6170 REM ROUTINE TO EXIT PROGRAM
6180 GOSUB 5980:REM TURN ALL OFF
6190 POKECC, (PEEK(CC) AND 127):REM TURN OFF ALARM
6200 POKECA, 0: POKECB, 0:REM CLEAR CONTROL REGISTER
6210 SYS(CD): POKEPB, 255:END
6220 :::::::::::::::::::::::::::::::::::::
6230 REM INITIALIZE VOLTMETERS
6240 V1=24:0PENV1,V1:REM OPEN CHANNEL TO KEITHLEY FOR TC1
6250 V2=15:0PENV2,V2:REM OPEN CHANNEL TO KEITHLEY FOR TC2
6260 V3=23:0PENV3,V3:REM OPEN CHANNEL TO KEITHLEY FOR FURNACE
6270 :::::::::::::::::::::::::::::::::::::
6280 VM=V2
6290 BS="F0R1Z0T5S1W1Q0M0K0X"
6300 PRINT#VM, BS IF VM=V1 THEN PRINT#VM, "T1X"
6310 PRINT#VM, "YUX"
6320 INPUT#VM, BS IF ST=2 THEN 6320
6330 PRINT"KEITHLEY#" VM"STATUS="; BS
6340 PRINT#VM, "Y:UX"
6350 PRINT"KEITHLEY#" VM"STATUS=000000"
6360 PRINT"KEITHLEY#" VM"STATUS=1B"
6370 INPUT#VM, BS IF ST=2 THEN 6370
6380 PRINT"KEITHLEY#" VM"STATUS="; BS
6390 :::::::::::::::::::::::::::::::::::::
6400 PRINT#V3, "F1RANST2Z1KM00"
6410 PRINT#V1, "$0R0Z0T4K1G1X"
6420 PRINT"DE"
6430 PRINT#V1, "$0X"


6440 INPUT#V1,B$: IF ST=2 THEN 6440
6450 PRINTB$
6460 RETURN
6470 :::::::::::::::::::::::::::::::::::
6480 REM CALCULATE FILTER VALUE & INPUT TEMPERATURES
6490 WAIT IR,16:T1=TI: REM WAIT FOR TRIGGER PULSE
6500 B=1+256*PEEK(TB+1)+PEEK(TB):C=(PEEK(PB)AND128)/128
6510 AN=#(*C+1-B/128)
6520 Z=SIN(AN-π/2):IC=COB(AN-π/2)
6530 :::::::::::::::::::::::::::::::::::
6540 INPUT#V3,V$: IF ST<>0 THEN 6540
6550 V=VAL(V$):X=81*FNT(V)+01:XT=XT+X
6560 REM PRINT#V3, "D3"INT(100*X)/100"DEB"
6570 IF X=XH(0) THEN XM(0)=INT(100*X+.5)/100
6580 IF X=XH(1) THEN XM(1)=INT(100*X+.5)/100
6590 :::::::::::::::::::::::::::::::::::
6600 REM ROUTINE TO INPUT TC2
6610 INPUT#V2,V$: IF ST=2 THEN 6610
6620 IF LEFT$(V$,4)<"NDCV" THEN 6610
6630 V=VAL(MID$(V$,5,16)):Y=Q2*FNT(V)+02:YT=YT+Y
6640 IF Y=YM(0) THEN YM(0)=INT(100*Y+.5)/100
6650 IF Y=YM(1) THEN YM(1)=INT(100*Y+.5)/100
6660 :::::::::::::::::::::::::::::::::::
6670 REM ROUTINE TO INPUT FURNACE TEMP 1 (LOWER)
6680 INPUT#V1,V$: IF ST=2 THEN 6680
6690 V=VAL(V$):X=G1*FNT(V)+01:XT=XT+X
6700 IF X>XM(0) THEN XM(0)=INT(100*X+.5)/100
6710 IF X<XM(1) THEN XM(1)=INT(100*X+.5)/100
6720 POKEPB,(PEEK(PB)AND251): IF FC$="OPF" THEN 6790
6730 DE=C1-U: IF ABS(DE)>B1 THEN R1=-B1: IF DE<0 THEN P1%=255: 6770
6740 IF ABS(DE)<B1 THEN P1%=P1%-DE/20
6750 IF (DE+R1)<0 THEN P1%=0: 6770
6760 IF P1%=(R1+DE)/B1<255 THEN P1%=255
6770 POKEM1,P1%
6780 :::::::::::::::::::::::::::::::::::
6790 REM NEW SUMS
6800 XX(0,0)=X: XX(0,1)=(I/TB%+J-1)/F:60:XX(0,2)=1:XX(0,3)=Y:XX=0:SS=0
6810 MF=MF(1)
6820 B=0: 6910
6830 DOSUB7400: REM GET SLOPE, INTERCEPT AND PREDICTION
6840 IF SS(I,J)>3 OR XX=3 THEN 6910
6850 IF (XX(0,0)-X)>0.3 THEN XX(0,0)=X+.3:MF=.65:XX=2
6860 IF (XX(0,0)-X)<-.3 THEN XX(0,0)=X-.3:MF=.65:XX=2
6870 IF (XX(0,3)-Y)>0.3 THEN XX(0,3)=Y+.3:MF=.65:XX=2
6880 IF (XX<2 AND MF<.95 THEN MF=MF+.1
6890 MF(1)=MF
6900 :::::::::::::::::::::::::::::::::::
6910 REM SUMMATIONS FOR TIME AND TC#1
6920 :IF XX=3 THEN XX=0
6930 :FOR KK=0 TO 2
6940 :FOR II=KK TO 2
6950 :: SS(I,KK,II) = XX(0,KK) * XX(0,II) + MF * SS(I,KK,II)
6960 :: NEXTII
6970 :: NEXTKK
6980 :: SS(I,3,0) = XX(0,3) * XX(0,KK) + MF * SS(I,KK,0)
7000 :: FOR KK = 1 TO 2
7010 :: SS(I,KK,0) = XX(0,3) * XX(0,KK) + MF * SS(I,KK,0)
7030 :: SS(I,2,1) = XX(0,3) * XX(0,3) + MF * SS(I,2,1)
7040 :: GOSUB 7400: REM SLOPE
7050 :: IF XX < 3 THEN 7150
7060 :: SS(I,3,1) = (XX(0,3) - X)^2 + MF * SS(I,3,1): REM STANDARD ERROR TC#1
7070 :: SS(I,3,2) = (XX(0,3) - Y)^2 + MF * SS(I,3,2): REM STANDARD ERROR TC#2
7080 :: IF SS(I,3,0) < 3 THEN 7150
7100 :: IF SE(KK+2) > 0.1 THEN XX = 1
7120 :: NEXT KK
7130 :: NEXTTC
7140 :: REM ROUTINE TO INPUT FURNACE TEMP 2 (UPPER)
7160 :: INPUT#V1, V$: IF ST = 2 THEN 7160
7170 :: V = VAL(V$): W = FNT(V): W = W + W + W
7180 :: IF W = W THEN WH(0) = INT(10^0 * W + 0.5) / 100
7190 :: IF KK = W THEN WH(1) = INT(10^0 * W + 0.5) / 100
7200 :: POKE PB, (PEEK(PB) OR 4): IF FC$ = "OFF" THEN RETURN
7210 :: DE = C2 - W: IF ABS(DE) > 2 THEN R2 = 0: IF DE > 0 THEN P2% = 255: GOTO 7250
7220 :: IF DE < 0 THEN R2 = R2 + DE / 20
7230 :: IF DE + R2 < 0 THEN P2% = 0: GOTO 7250
7240 :: P2% = (R2 + DE) / 255: IF P2% > 255 THEN P2% = 255
7250 :: POKE M2, P2%
7260 :: RETURN
7270 :: REM MEANS, XBAR, YBAR
7290 :: IF SS(1,2,2) < 0 THEN XX(2,0) = SS(1,0,2) / SS(1,2,2): REM TC#1
7300 :: IF SS(1,2,2) < 0 THEN XX(2,1) = SS(1,2,0) / SS(1,2,2): REM TC#2
7310 :: RETURN
7320 :: REM CORRELATION COEFFICIENT SQR, RSQ
7340 :: SS = (SS(I,1,1) * SS(I,2,2) - SS(I,1,2)^2) * (SS(I,0,0) * SS(I,2,2) - SS(I,0,2)^2)
7350 :: IF SS < 0 THEN XX(2,2) = (SS(I,0,1) * SS(I,2,2) - SS(I,1,2)^2) / SS
7360 :: SS = (SS(I,1,1) * SS(I,2,2) - SS(I,1,2)^2) * (SS(I,1,2) * SS(I,2,2) - SS(I,1,2)^2)
7370 :: IF SS < 0 THEN XX(2,3) = (SS(I,1,0) * SS(I,1,2) - SS(I,1,2)^2) / SS
7380 :: RETURN
134

7420 IF SS=0 THEN 7490
7430 REM SLOPE INT OF TC#1
7440 XX(1,0)=(SS(I,0,1)-SS(I,0,2)*SS(I,1,2)/SS(I,2,2))/SS
7450 XX(1,1)=(SS(I,0,2)-XX(1,0)*SS(I,1,2)/SS(I,2,2))
7460 REM SLOPE INT OF TC#2
7470 XX(1,2)=(SS(I,1,0)-SS(I,1,2)*SS(I,0,2)/SS(I,2,2))/SS
7480 XX(1,3)=(SS(I,1,2)-XX(1,0)*SS(I,0,2))/SS(I,2,2)
7490 REM PREDICTION OF X & Y
7500 IF SS(I,3)<2 THEN XX=3:RETURN
7510 X=XX(1,0)*XX(0,1)+XX(1,1):Y=XX(1,2)*XX(0,1)+XX(1,3)
7520 RETURN
7530 :
7540 REM INITIALIZE ARRAYS
7550 MF=.95:REM MEMORY FACTOR
7560 FOR II=1 TO TB
7570 ;FOR KK=0 TO 3
7580 ::FOR LL=0 TO 2
7590 ::::SS(II,KK,LL)=0:XX(LL,KK)=0
7600 ::NEXT LL
7610 ::NEXT KK
7620 :MF<II>=MF
7630 NEXT II
7640 RETURN
7650 :
7660 REM ROUTINE TO INPUT FURNACE TEMPERATURES
7670 IF FC$="OFF" OR FT<»0 THEN RETURN
7680 IF TI<T0+FT*30 THEN RETURN
7690 T0=TI:IF T0+FT<518400 THEN T0=T0-518400
7700 IF (PEEK(PB)AND4)»0 THEN 7820
7710 :::::REM ROUTINE TO INPUT FURNACE TEMP 1 (LOWER)
7720 :::::INPUT#V1,V$:IF ST=2 THEN 7720
7730 V=VAL(V$):U=3*FNT(V)+03
7740 POKEPB,(PEEK(PB)AND251)
7750 DE=C1-U:IF ABS(DE)>B1 THEN R1=0:IF DE>0 THEN P1%=255:GOTO7800
7760 IF ABS(DE)<B1 THEN R1=R1+DE/20
7770 IF (DE+R1)<0 THEN P1%=0:GOTO7800
7780 P1%=(R1+DE)/B1*255:IF P1%>255 THEN P1%«255
7790 POKEM1,P1%:RETURN
7800 POKEPB,(PEEK(PB)AND251)
7810 :::::REM ROUTINE TO INPUT FURNACE TEMP 2 (UPPER)
7820 :::::INPUT#V1,V$:IF ST=2 THEN 7820
7830 V=VAL(V$):W=FNT(V)
7840 POKEPB,(PEEK(PB)OR4)
7850 DE=C2-W:IF ABS(DE)>B2 THEN R2=0:IF DE>0 THEN P2%=255:GOTO7900
7860 IF ABS(DE)<B2 THEN R2=R2+DE/20
7870 IF (DE+R2)<0 THEN P2%=0:GOTO7900
7880 P2%=(R2+DE)/B2*255:IF P2%>255 THEN P2%«255
7890 POKEPB,P2%:RETURN
7900 :::::REM THERMOCOUPLE CALIBRATION ROUTINE
7910 :::::...
7920 :::::REM THERMOCOUPLE CALIBRATION ROUTINE
7930. IF FC$="OFF" OR FT<=0 THEN RETURN 7940 PRINT"**"
7950 INPUT"TEMPERATURE SCAN (MIN/MAX)";C3,C4
7960 INPUT"TEMPERATURE STEP SIZE";C5
7970 INPUT"OUTPUT DEVICE ";DR
7980 OPENDR,DR:PRINT#DR,"FURNACE TC1 TC2":C2=C3
7990 GOSUB4940:REM LOOK AT TC'S
8000 J=J+1:GOSUB5000:IF A$="S" THEN 8000:REM CONTINUE TO LOOK A TC'C
8010 IF ABS(C2-WT/TB%)>1 THEN 8020
8020 GOSUB4940:REM LOOK AT TC'S
8030 IF A$="S" THEN 8000
8040 PRINT#DR,INT(100*WT/TB%+.5)/100,INT(100*XT/TB%+.5)/100,
8050 PRINT#DR,INT(100*YT/TB%+.5)/100
8060 C2=C2+C5:C1=C1+C5:IF C2=C4 THEN 7990
8070 IF C1=20:1=C1=20
8080 CLOSEDR:RETURN

*—clear home
#—home
•—reverse field on
•—reverse field off
1—cursor down
^—cursor up
APPENDIX B: FURNACE CONTROL ASSEMBLY PROGRAM
FURNACE CONTROL ROUTINE FOR C-64
USING THE INTERNAL JIFFY CLOCK
PRECISION OF 0-255 OF 4SEC
WRITTEN AND DEVELOPED BY
DAVID SLUTZ
ISU
JANUARY 9, 1985

M1=828 ;FURNACE#1 CONTROL FACTOR LOCATION
M2=829 ;FURNACE#2 CONTROL FACTOR LOCATION
CLOCK=$FFEA ;UPDATE JIFFY CLOCK ROUTINE
PORTB=$DD01 ;USER PORTB ADDRESS
LOWCK=$A2 ;LOW BYTE OF JIFFY CLOCK
IRQ=$0314 ;POINTER TO IRQ ROUTINE
REST=$EA34 ;IRQ ROUTINE AFTER UPDATE OF JIFFY CLOCK
**$033E ;START OF NEW IRQ ROUTINE

START JSR CLOCK ;UPDATE JIFFY CLOCK
LDA LOWCK ;FETCH LOW BYTE JIFFY CLOCK
BNE TEST1 ;IF EQUAL TURN ON FURNACES 1 & 2
LDA PORTB
AND #$FC ;TURN BOTH ON
STA PORTB
LDA LOWCK ;RESTORE A WITH LOW BYTE OF JIFFY CLOCK
TEST1 CMP M1 ;TEST JIFFY FOR TURNING FURNACE#1 OFF
BNE TEST2 ;LEAVE FURNACE ALONE
CMP #$FF ;TEST IF FURNACE STAYS ON FULL
BEQ TEST2 ;LEAVE FURNACE #1 ON
LDA PORTB
ORA #$01 ;TURN FURNACE#1 OFF
STA PORTB
LDA LOWCK ;RESTORE LOW BYTE OF JIFFY CLOCK INN A
TEST2 CMP M2 ;TEST FOR TURNING FURNACE#2 OFF
BNE DONE ;LEAVE FURNACE ALONE
CMP #$FF ;TEST FOR FURNACE FULL ON
BEQ DONE ;LEAVE FURNACE#2 ON
LDA PORTB
ORA #$02 ;TURN FURNACE#2 OFF
STA PORTB
DONE JMP REST ;DO REST OF IRQ ROUTINE
BRK
ENABLE LDA #$3E ;LOW BYTE START SYS(887)
STA IRQ
LDA #$03 ;HIGH BYTE START
STA IRQ+1 ;RESET IRQ VECTOR
RTS
DISABLE LDA #$31 ;LOW BYTE IRQ SYS(898)
STA IRQ
LDA #$EA
STA IRQ+1 ;NORMAL IRQ ROUTINE
RTS