SUPPLEMENTARY MATERIAL

Phase transitions in Schloegl's second model for autocatalysis on a Bethe lattice

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SM1. Bethe lattice site labeling and neighbor identification

In Sec. 2, we described a scheme for labeling the sites of a Bethe lattice for general z. This scheme was illustrated explicitly in Figure 3 for z = 3. The central site R_0 labeled j = 1 is surrounded by a ring R_1 of 3 sites labeled j = 2, 3, 4; R_1 is surrounded by second ring R_2 with a total of 6 sites labeled j = 5,…,10 such that each site in R_1 is connected to 2 sites in R_2; etc. More generally, the k^{th} ring, R_k, has $3\times2^{k-1}$ sites with labels j satisfying $3\times2^{k-1} - 2 < j \leq 3\times2^k - 2$.

To identify the neighbors of a site labeled j in ring k, it is convenient to define $\delta j = j - (3\times2^{k-1} - 2) = 1, 2, \ldots, 3\times2^{k-1}$. The two neighbors in ring k+1 of this site j are $(3\times2^k - 2) + 2\delta j -1$ and $3\times2^k - 2) + 2\delta j$. The single neighbor of this site in ring k-1 is $(3\times2^{k-2} - 2) + \delta j/2$ if δj is even, and $(3\times2^{k-2} - 2) + (\delta j/ + 1)2$ if δj is odd.

SM2. CONSTANT-p SIMULATIONS OF TIME-EVOLUTION

We present additional results from constant-p simulations for time evolution starting from an initially fully populated state. **Figure S1** shows results for BC_{MF}, and **Figure S2** shows results for BC_{Pc} for a range of *p*.

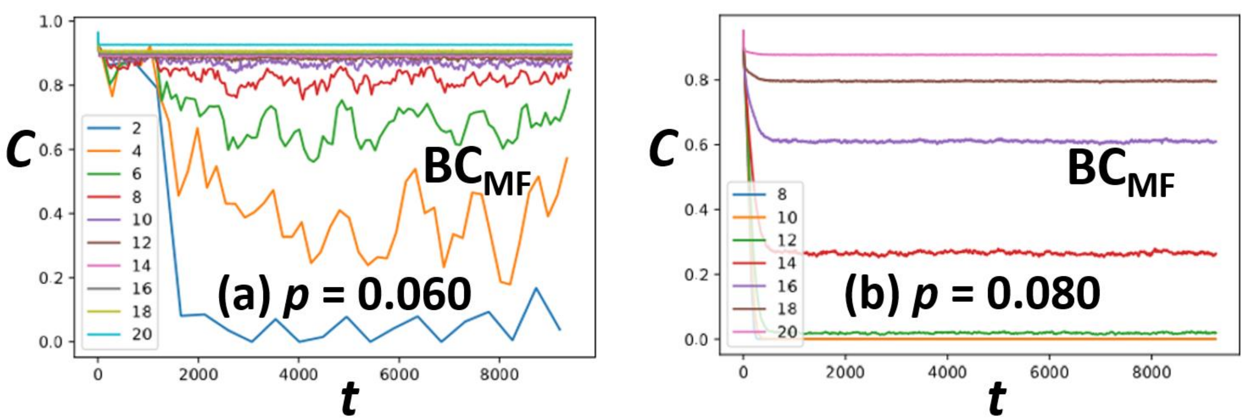

Figure S1. Evolution of concentrations C_k for various rings, k (labelled in the legend) for BC_{MF} boundary conditions for a range of *p* with a populated heterogeneous steady-state.

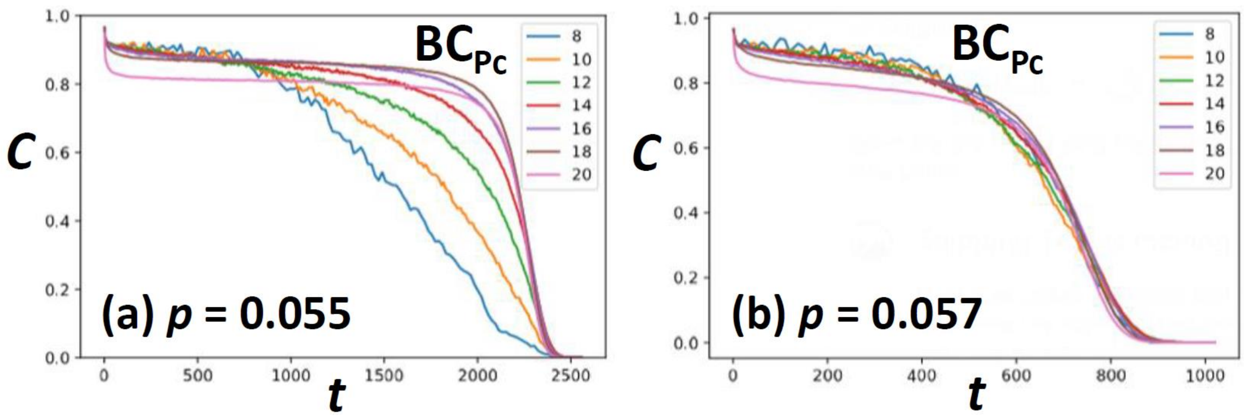

Figure S2. Evolution of concentrations C_k for various rings, k (labelled in the legend) with BC_{Pc} for a range of p above the discontinuous transition showing evolution to the vacuum state.

SM3. DROPLET EVOLUTION IN ANALYTIC TRUNCATION APPROXIMATIONS

For completeness, in **Figure S3**, we shown the results from a MF analysis for propagation of a vacuum droplet for z = 4 and z = 6. In these cases, the droplet shrinks for all $p \leq 0.25$, and the shrinkage velocity, V, is even finite at $p = 0.25$, i.e., there is no equistability point as for z = 3. The same is true for analysis in the pair approximation.

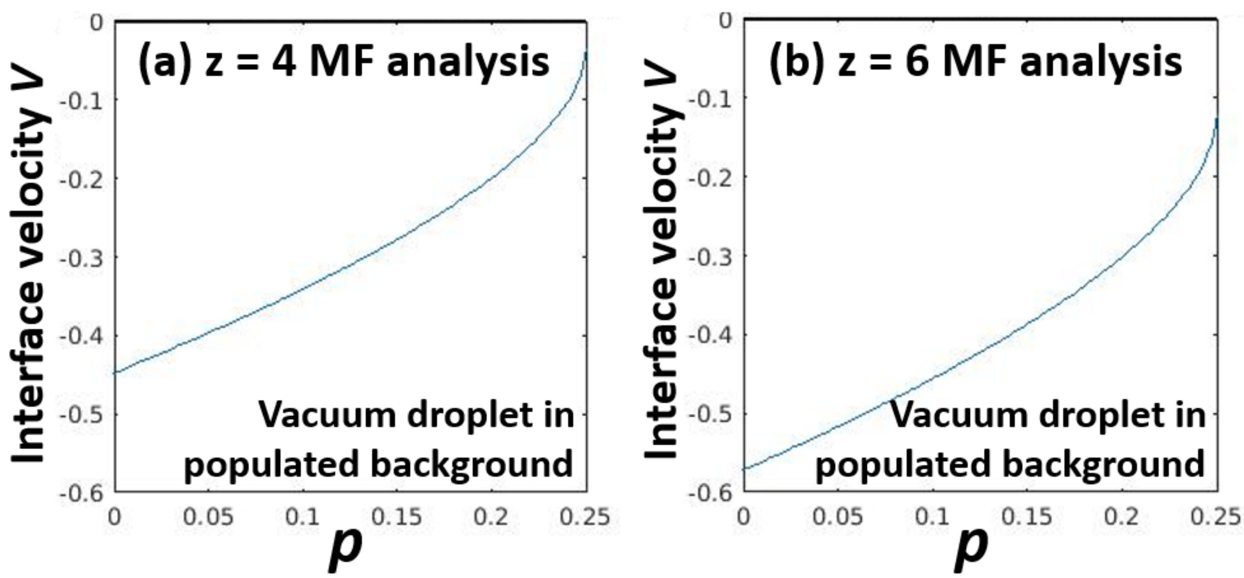

Figure S3. MF estimate of the propagation velocity, V, versus p of a large vacuum droplet embedded in the populated steady state for: (a) z = 4; (b) z = 6.